

Title: 13/14 PSI - Quantum Gravity Review - Lecture 14

Date: Mar 07, 2014 10:15 AM

URL: <http://pirsa.org/14030021>

Abstract:

Solving flatness (Hamiltonian) constraints

$$\langle \gamma_{s,j} | \gamma_{s,j} \rangle = \prod_i \sqrt{d_i d_{j,i}} \{6_{j,i}\} \{6_j\}$$



Solving flatness (Hamiltonian) constraints

$$\langle \psi_{S,j} | \psi_{S,j} \rangle = \prod_i \sqrt{d_i d_{j_i}} \{6_{j_i}\} \{6_j\}$$



Solving flatness (Hamiltonian) constraints

$$\langle \psi_{Sj} | \psi_{Sj} \rangle_{\text{phys}} = \prod_i \sqrt{d_i d_{ji}} \{6_j\} \{6_j\} \\ = A_{Sj} A_{Sj}$$



One physical state $\psi(j) = \prod_i \sqrt{d_i} \{6_j\}$

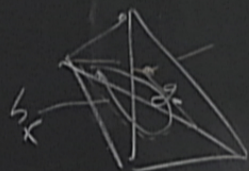
Path - integ

$$\int e^{i \int B F} dB \\ \int e^{i(P \cdot \dot{q})} dp \sim$$

Path - integral for 3D.

$$\int e^{i \int B F} dB dA = \int \prod_{\text{faces}} \delta(h_{\text{face}}) dg_e$$

$$\int e^{i P \cdot f(g)} dP \sim \delta(f(g))$$



Path - integral for 3D.

$$\int e^{i \int B F} dB dA = \int \prod_{\text{faces}} \delta(h_{\text{face}}) dg_e = \sum_k \prod_{\text{faces}} \delta(h_{\text{face}})$$

$\uparrow$
 group
 former tifo

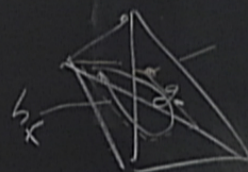
$\{6_j\}$



Path - integral for 3D.

$$\int e^{i \int B F} dB dA = \int \prod_{\text{faces}} (\dots) dg_e$$

$$\int e^{i P \cdot f(g)} dp \sim \delta(f(g))$$



$$= \sum_k \prod_{\text{faces}} \prod_{e \in \text{face}} \sqrt{d_j e}$$

↑
group
faces into

$\{b_j\}$

$$\{b_j\} \sim e^{i S_{\text{Regge}}}$$

Path - integral for 3D.

$$\int e^{i \int B F} dB dA = \int$$

$$\int e^{i P \cdot L(q)} dp \sim \delta(\dots)$$



$$\prod_{\text{faces}} \delta(h_{\text{face}}) dg_e$$

$$\sum_j d_j \chi_j(h_{\text{face}})$$

$$= \sum_j d_j \sum D_{n_1 n_1}^j(g_1) \dots D_{n_4 n_4}^j(g_4)$$

group
further info

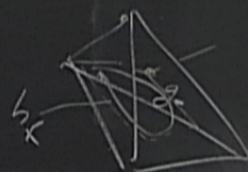
$$\sum_k \prod_{\text{faces}} \delta(h_{\text{face}}) dg_e \{b_j\}$$

$$\{b_j\} \sim e^{i S_{\text{Regge}}}$$

Path - integral for 3D.

$$\int e^{i \int B F} dB dA =$$

$$\int e^{i P \cdot f(g)} dP \sim \delta(f(g))$$



$$\prod_{\text{faces}} \delta(h_{\text{face}}) dg_e$$

$$\sum_j d_j \chi_j(h_{\text{face}})$$

$$= \sum_j d_j \sum D_{h_{j1}}^j(g_1) \dots D_{h_{jn}}^j(g_n)$$

group
factors

$$\sum_k \prod_{\text{faces}} \delta(h_{\text{face}}) \{b_j\}$$

$$\{b_j\} \sim e^{i S_{\text{Regge}}}$$

4 D gravity

Ashtekar variables

q_{ab} , K_{ab}

4 D gravity

Ashthekar variables

^{3D}
 q_{ab} , K_{ab} extrinsic curvature

$$q_{ab} = e_a^i e_b^j \leftarrow \text{SU}(2)$$

$$K_a^i = K_{ab} e^{bi}$$

$$E_j^a = \sqrt{\det(q)} e_j^a$$

$$A_a^j = \omega_a^j + \gamma K_a^j$$

spin-connection
Barbero-
Immirzi-
parameter

$$A_a^j = \underbrace{\omega_a^j}_{\text{spin-connection}} + \gamma \underbrace{K_a^j}_{\text{Barbero-Immirzi-parameter}}$$

$$\{A_a^j(x),$$

$$A_a^j = \underbrace{\omega_a^j}_{\text{spin-connection}} + \gamma \underbrace{K_a^j}_{\text{Barbero-Immirzi-parameter}}$$

$$\{A_a^j(x), A_b^k(y)\} = 0$$

$$\{E_a^i(x), E_b^j(y)\} = 0$$

$$\{A_a^j(x), E_b^k(y)\} = \gamma \delta_k^j \delta_a^b \delta(x, y)$$

$$A_a^j = \underbrace{\omega_a^j}_{\text{spin-connection}} + \gamma \underbrace{K_a^j}_{\text{Barbero-Imzi-Muho}}$$

$$\{A_a^j(x), A_b^j(x)\}$$

$$\{E_a^j(x), E_b^j(x)\}$$

$$\{A_a^j(x), E_b^j(x)\}$$

$$\delta_a^j \delta_b^k \delta(x,y)$$

Quantize

$A \rightarrow \text{holonomies } h_p$

$$E(S)^j = \int_S (h_p(*E) h_p^{-1})^j$$

$$*E = \epsilon_{a_1 a_2 a_3} E^{a_1} E^{a_2} E^{a_3}$$

$$A_a^j = \omega_a^j + \gamma K_a^j$$

spin-connection

Barbero-
Immirzi-
parameter

$$\{A_a^j(x), A_b^j(x)\}$$

$$\{E_a^j(x), E_b^k(x)\}$$

$$\{A_a^j(x), E_b^k(x)\}$$

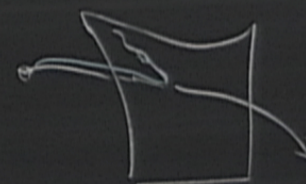
$$\int d^3x \delta_a^b \delta(x,y)$$

Quantize

$A \rightarrow$ holonomies h_p

$$E(S)^j = \int_S (h_p(*E) h_p^{-1})^j$$

$$*E = \epsilon_{a_1 a_2 a_3} E^a$$



$$A_a^j = \underbrace{\omega_a^j}_{\text{spin-connection}} + \gamma \underbrace{K_a^j}_{\text{Barbero-Immirzi-parameter}}$$

$$\{A_a^j(x), A_b^k(y)\} = 0$$

$$\{E_a^i(x), E_b^j(y)\} = 0$$

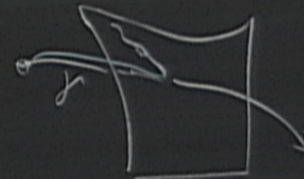
$$\{A_a^j(x), E_b^i(y)\} = \delta_k^j \delta_a^b \delta(x,y)$$

Quantize

$A \rightarrow$ holonomies h_p

$$E(S)^j = \int_S (h_p(*E) h_p^{-1})^j$$

$$*E = \epsilon_{a_1 a_2 a_3} E^a$$



$$A_a^j = \underbrace{\omega_a^j}_{\text{spin-connection}} + \gamma \underbrace{K_a^j}_{\substack{\text{Barbero-} \\ \text{Immirzi-} \\ \text{parameter}}}$$

$$\{A_a^j(x), \dots\} = 0$$

$$\{E^a_j(x), \dots\} = 0$$

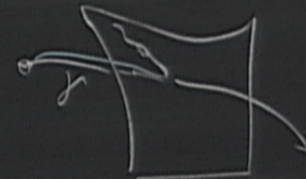
$$\{A_a^j(x), \dots\} = \gamma \delta_k^j \delta_a^b \delta(x, y)$$

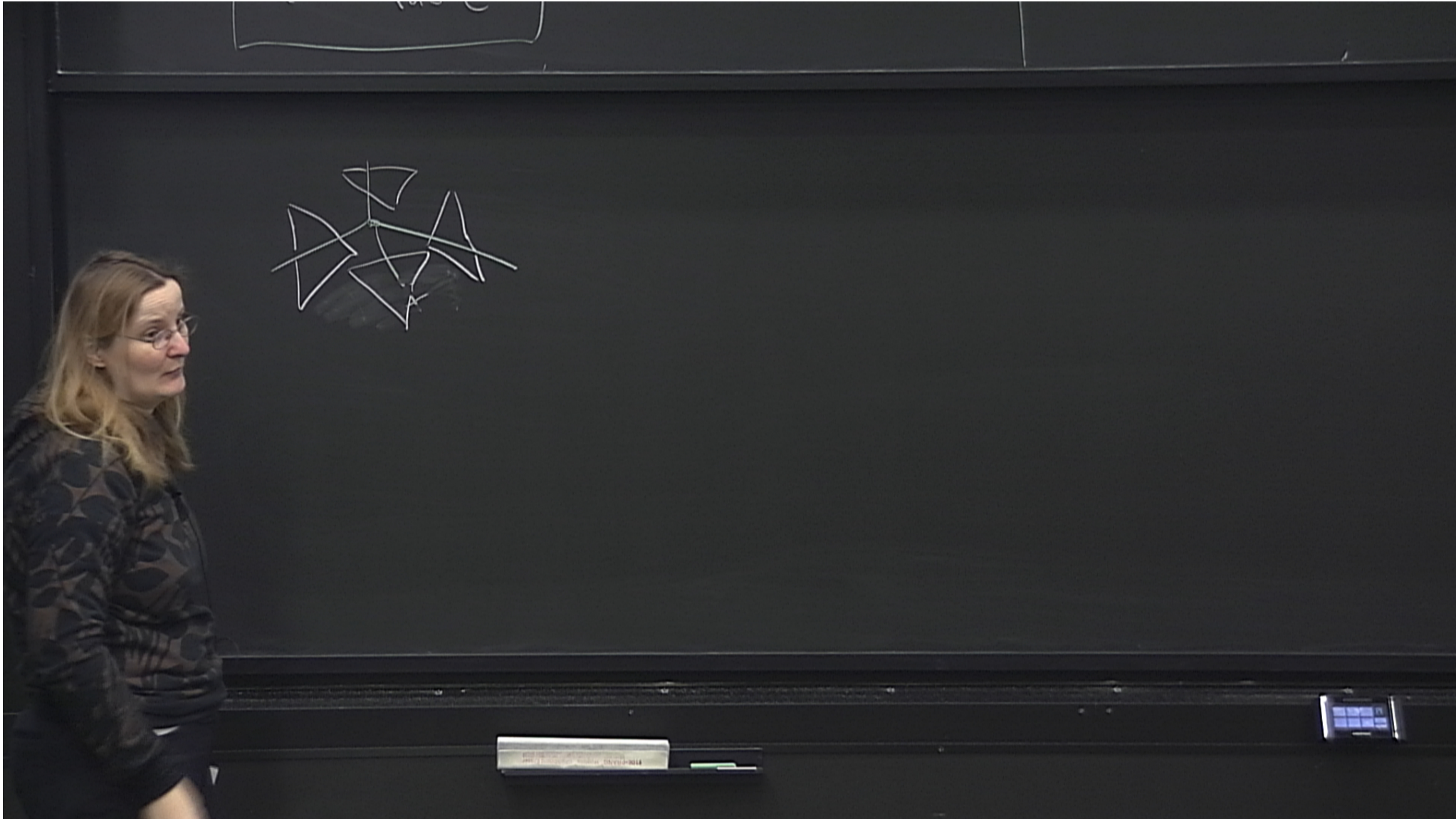
Quantize

$A \rightarrow$ holonomies h_p

$$\underbrace{E(S)^j}_{\text{fluxes}} = \int_S (h_p(*E) h_p^{-1})^j$$

$$*E = \epsilon_{a_1 a_2} E^a_j$$







Gauss - constraints:
→ closed tetrahedra

$\bar{t}(s) =$ normals
to the surface
(triangle)

Con

Constraints

$$g_j = \partial_a E_j^a + \epsilon_{jka} \underbrace{(\omega_a^k + \gamma K_a^k)}_{A_a^k} E_l^a$$

Constraints

$$g_j = \partial_a E_j^a + \epsilon_{jka} \underbrace{(\omega_a^k + \gamma K_a^k)}_{A_a^k} E_k^a$$

Diffeom. constraint:

$$H_a = -\underset{\substack{\uparrow \\ \text{signature} \\ \text{of spacetime}}}{s} F_{ab}^j E_j^b$$

Constraints

$$G_j = \partial_a E_j^a + \epsilon_{jka} \underbrace{(\omega_a^k + \gamma K_a^k)}_{A_a^k} E_l^a$$

from constraint.

$$H_a = -s \overset{j}{F}_{ab} E_j^b$$

signature of spacetime

$$H = F_{ab}^j \epsilon_{jkl} \frac{E_k^a E_l^b}{\sqrt{\det g}}$$

Constraints

$$G_j = \partial_a E_j^a + \epsilon_{jka} \underbrace{(\omega_a^k + \gamma K_a^k)}_{A_a^k} E_l^a$$

Diffeom.

$$H_a = -s F_{ab}^j E_j^b$$

signature
of spacetime

$$\epsilon_{jkl} \frac{E_k^a E_l^b}{\sqrt{\det q}} - (\gamma^2 - s)$$

Constraints

$$g_j = \partial_a E_j^a + \epsilon_{jkl} \underbrace{(\omega_a^k + \gamma K_a^k)}_{A_a^k} E_l^a$$

Diffeom. constraint:

$$H_a = -s \overset{\substack{\uparrow \\ \text{signature} \\ \text{of spacetime}}}{F_{ab}^j} E_j^b$$

$$H = F_{ab}^j \epsilon_{jkl} \frac{E_k^a E_l^b}{\sqrt{\det q}} - (\gamma^2 - s) K_a^m K_b^n \epsilon_{jkl} \frac{E_k^a E_l^b}{\sqrt{\det q}}$$

Constraints

$$g_j = \partial_a E_j^a + \epsilon_{jka} \underbrace{(\omega_a^k + \gamma K_a^k)}_{A_a^k} E_l^a$$

Diffeom. constraint.

$$H_a = -s F_{ab}^j E_j^b$$

signature
of spacetime

Hamiltonian constraint

$$H = F_{ab}^j \epsilon_{jkl} \frac{E_k^a E_l^b}{\sqrt{\det q}} - (\gamma^2 - s) K_a^m K_b^n \epsilon_{jkl} \frac{E_k^a E_l^b}{\sqrt{\det q}}$$

Constraints

$$g_j = \partial_a E_j^a + \epsilon_{jka} \underbrace{(\omega_a^k + \gamma K_a^k)}_{A_a^k} E_l^a$$

Diffeom. constraint:

$$H_a = -s F_{ab}^j E_j^b$$

signature
of spacetime

Hamiltonian constraint

$$H = F_{ab}^j \epsilon_{jkl} \frac{E_k^a E_l^b}{\sqrt{\det q}} - (\gamma^2 - s) K_a^m K_b^n \epsilon_{jkl} \frac{E_k^a E_l^b}{\sqrt{\det q}}$$

$$\epsilon_{jke} \left(\underbrace{\omega_a^k + \gamma K_a^k}_{A_a^k} \right) E_l^a$$

$$H_a = -s F_{ab}^j E_j^b$$

signature
of spacetime

$$\frac{E_k^a E_l^b}{\sqrt{\det g}} - (\gamma^2 - s) K_a^m K_b^n E_{mn}$$

$$C = \int \frac{N}{\sqrt{\det g}} F_{ab}^j \epsilon_{jkl} E_k^a E_l^b d^3x$$

$$= \int \frac{N}{\sqrt{\det g}} \text{tr} (F_{ab} [E^a, E^b])$$

$$\epsilon_{jke} \left(\underbrace{\omega_a^k + \gamma K_a^k}_{A_a^k} \right) E_l^a$$

$$H_a = -s F_{ab}^j E_j^b$$

signature
of spacetime

$$\frac{E_k^a E_l^b}{\sqrt{\det g}}$$

$$- (\gamma^2 - 1)$$

$$\frac{E_k^a E_l^b}{\sqrt{\det g}}$$

$$C = \int \frac{N}{\sqrt{\det g}} F_{ab}^j \epsilon_{jkl} E_k^a E_l^b d^3x$$

$$= \int \frac{N}{\sqrt{\det g}} \text{tr} (F_{ab} [E^a, E^b])$$

Thiemann trick

$$\frac{[E^a, E^b]}{\sqrt{\det g}} = 2 \epsilon^{abc} \{V(R), A_a^c\}$$

$$\epsilon_{jke} \left(\underbrace{\omega_a^k + \gamma K_a^k}_{A_a^k} \right) E_l^a$$

$$H_a = -s F_{ab}^j E_j^b$$

signature
of spacetime

$$\frac{E_k^a E_l^b}{\sqrt{\det g}} - (\gamma^2 - s) K_a^m K_b^n \epsilon_{jkl} \frac{E_k^a E_l^b}{\sqrt{\det g}}$$

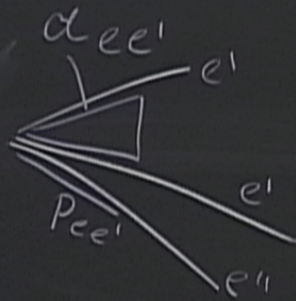
$$C = \int \frac{N}{\sqrt{\det g}} F_{ab}^j \epsilon_{jkl} E_k^a E_l^b d^3x$$

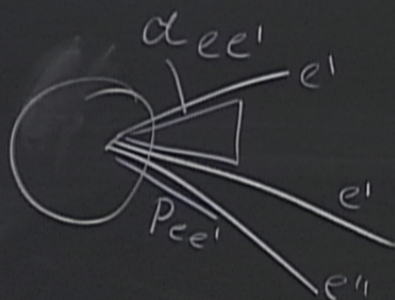
$$= \int \frac{N}{\sqrt{\det g}} \text{tr} (F_{ab} [E^a, E^b])$$

holonomy

Thiemann trick

$$\frac{[E^a, E^b]}{\sqrt{\det g}} = 2 \epsilon^{abc} \{V(R), A_a^c\}$$





$$\{V(R), A'_a(x)\}$$

$$= h_{p_{ee'}} [h_{p_{ee'}}^{-1}, V(R)]$$

$$\hat{C}[N] = \sum_{\text{vertices}} N(v) S_v \quad \swarrow \text{Symmetry factor}$$

$$\sum_{\text{edge triples at } v} \{ \text{Tr} [(h_{a_{ee'}} - h_{ee'}^{-1})] \}$$

$$\epsilon_{jke} \underbrace{(\omega_a^k + \gamma K_a^k)}_{A_a^k} E_e^a$$

$$H_a = -s F_{ab}^j E_j^b$$

signature
of spacetime

$$\frac{E_k^a E_l^b}{\sqrt{\det g}} - (\gamma^2 - s) K_a^m K_b^n \epsilon_{jkl} \frac{E_k^a E_l^b}{\sqrt{\det g}}$$

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$$= \int \frac{N}{\sqrt{\det g}} \text{tr} (F_{ab} [E^a, E^b])$$

holonomy

Threumann trick

$$\frac{[E^a, E^b]_c}{\sqrt{\det g}} = 2 \epsilon^{abc} \{ \dots \}$$

$$\epsilon_{jke} \underbrace{(\omega_a^k + \gamma K_a^k)}_{A_a^k} E_e^a$$

$$H_a = -s F_{ab}^j E_j^b$$

signature
of spacetime

$$\frac{E_k^a E_l^b}{\sqrt{\det g}} - (\gamma^2 - s) K_a^m K_b^n \epsilon_{jkl} \frac{E_b^a E_l^b}{\sqrt{\det g}}$$

$$C = \int \frac{N}{\sqrt{\det g}} F_{ab}^j \epsilon_{jkl} E_k^a E_l^b d^3x$$

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holonomy

Threumann trick

$$\frac{[E^a, E^b]_c}{\sqrt{\det g}} = 2 \epsilon^{abc} \{V(R), A_a^c\}$$

$$\hat{C}[N] = \sum_{\text{vertices}} N(v) S_v$$

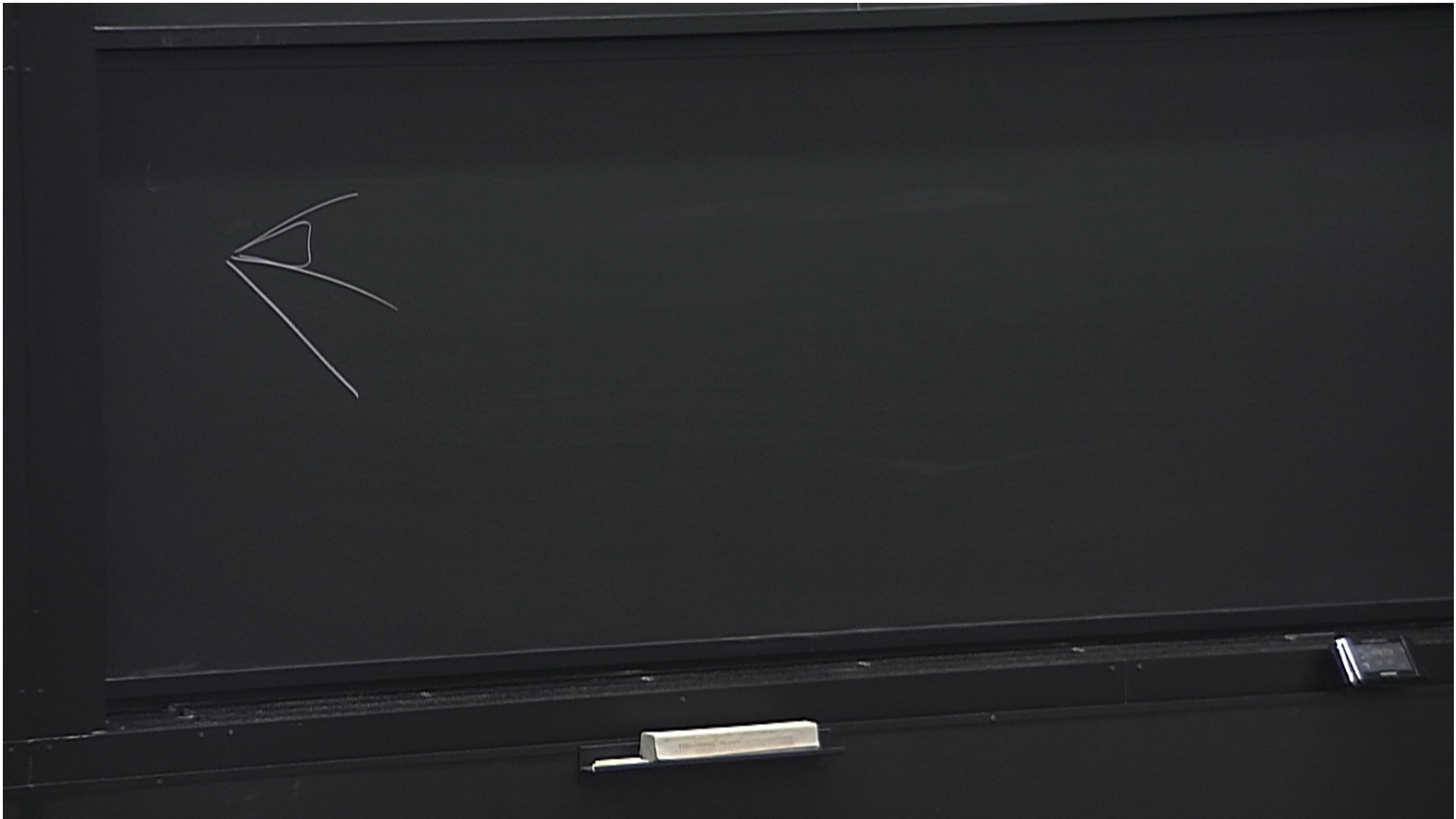
Symmetry factors

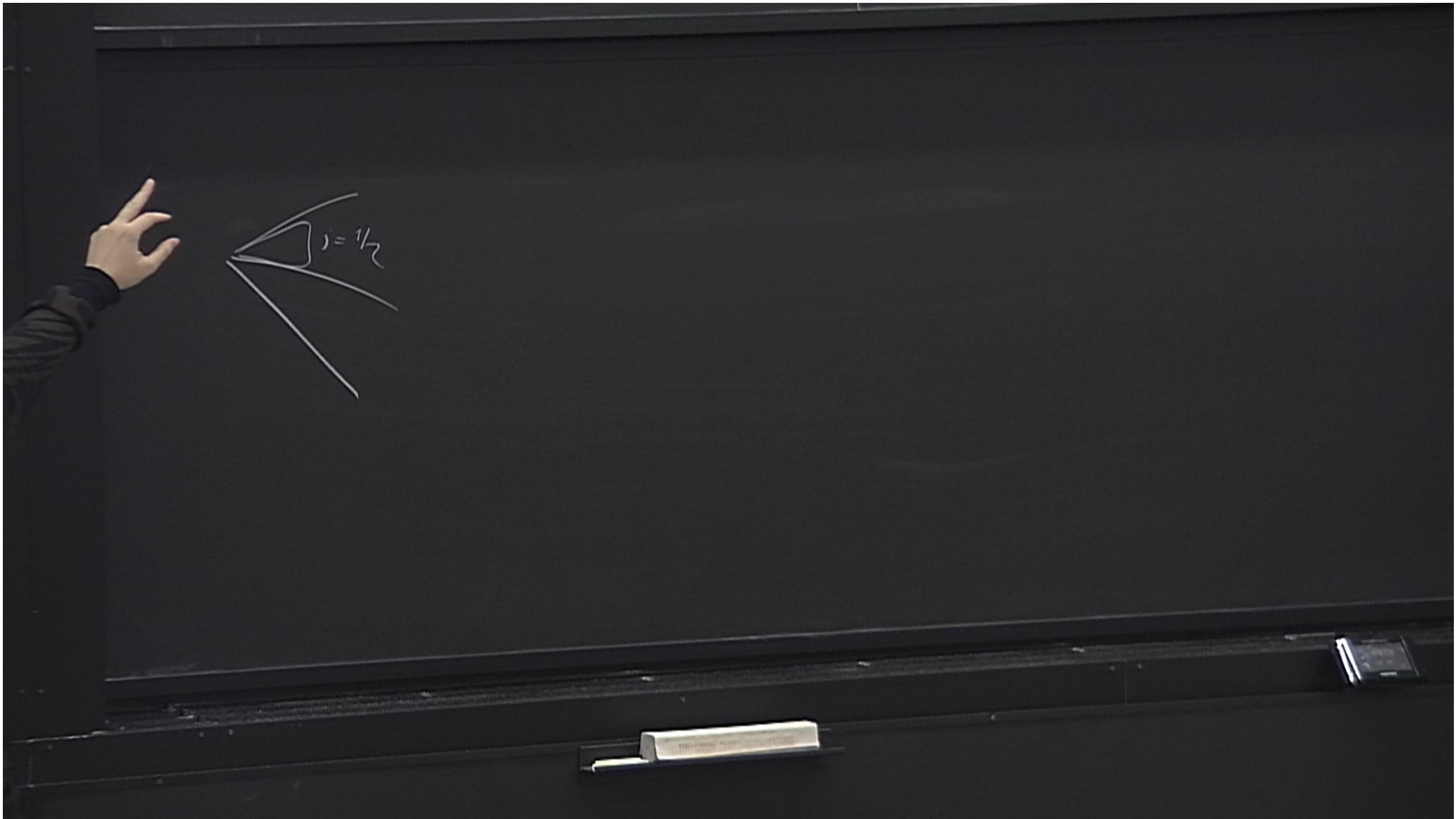
$$\sum_{\text{edge triples at } v} \left\{ \text{Tr} \left[(h_{a_{ee'}} - h_{ee'}^{-1}) \right. \right. \\ \left. \left. h_{p_{ee'}} [h_{p_{ee'}}^{-1}, \hat{U}_v] \right] \right\}$$

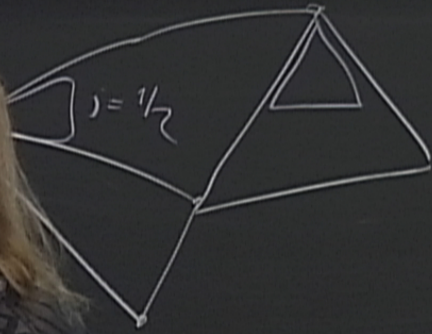
$$\hat{C}[N] = \sum_{\text{vertices}} N(v) S_v \quad \leftarrow \text{Symmetry factor}$$

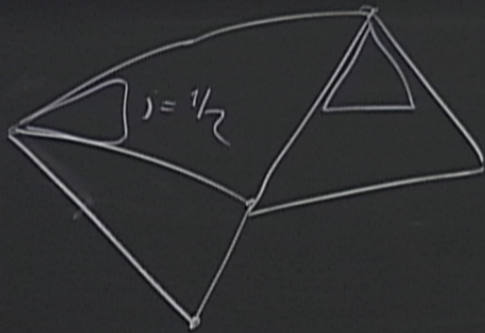
$$\sum_{\text{edge triples at } v} \left\{ \text{Tr} \left[(h_{a_{ee'}} - h_{ee'}^{-1}) \right. \right. \\ \left. \left. h_{p_{ee'}} [h_{p_{ee'}}^{-1}, \hat{U}_v] \right] \right\}$$

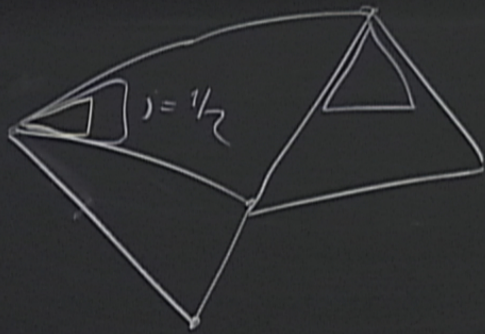
+ cyclic permutation in e, e', e''

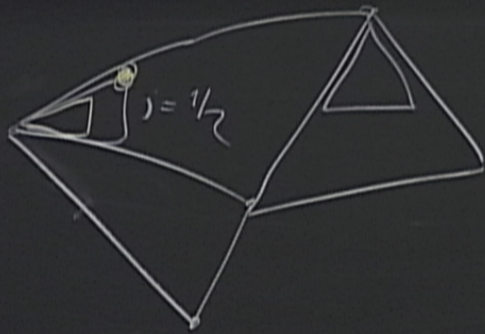


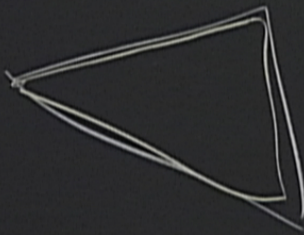
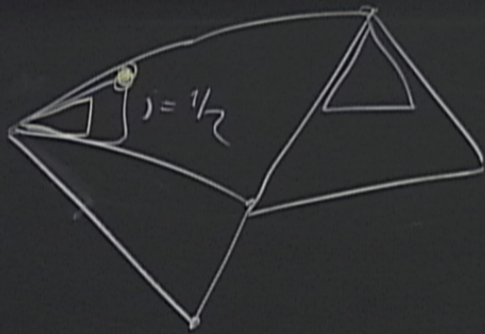


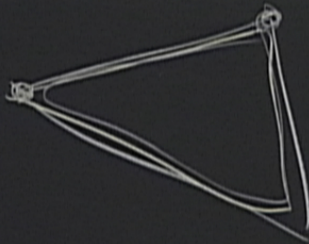
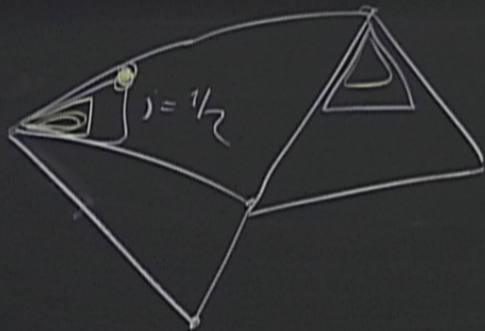


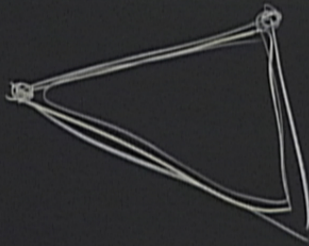
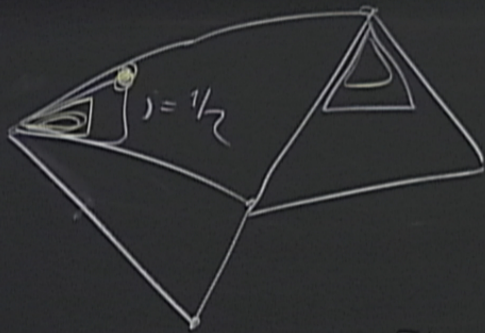




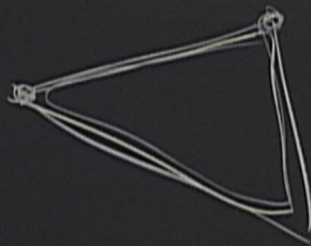
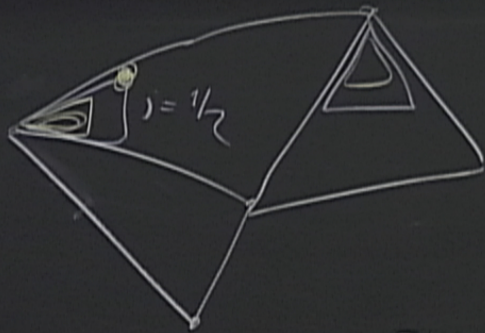








$$[H_0, H_1] \neq 0$$



$[H_1, H_2] \neq 0$
anomalous algebra