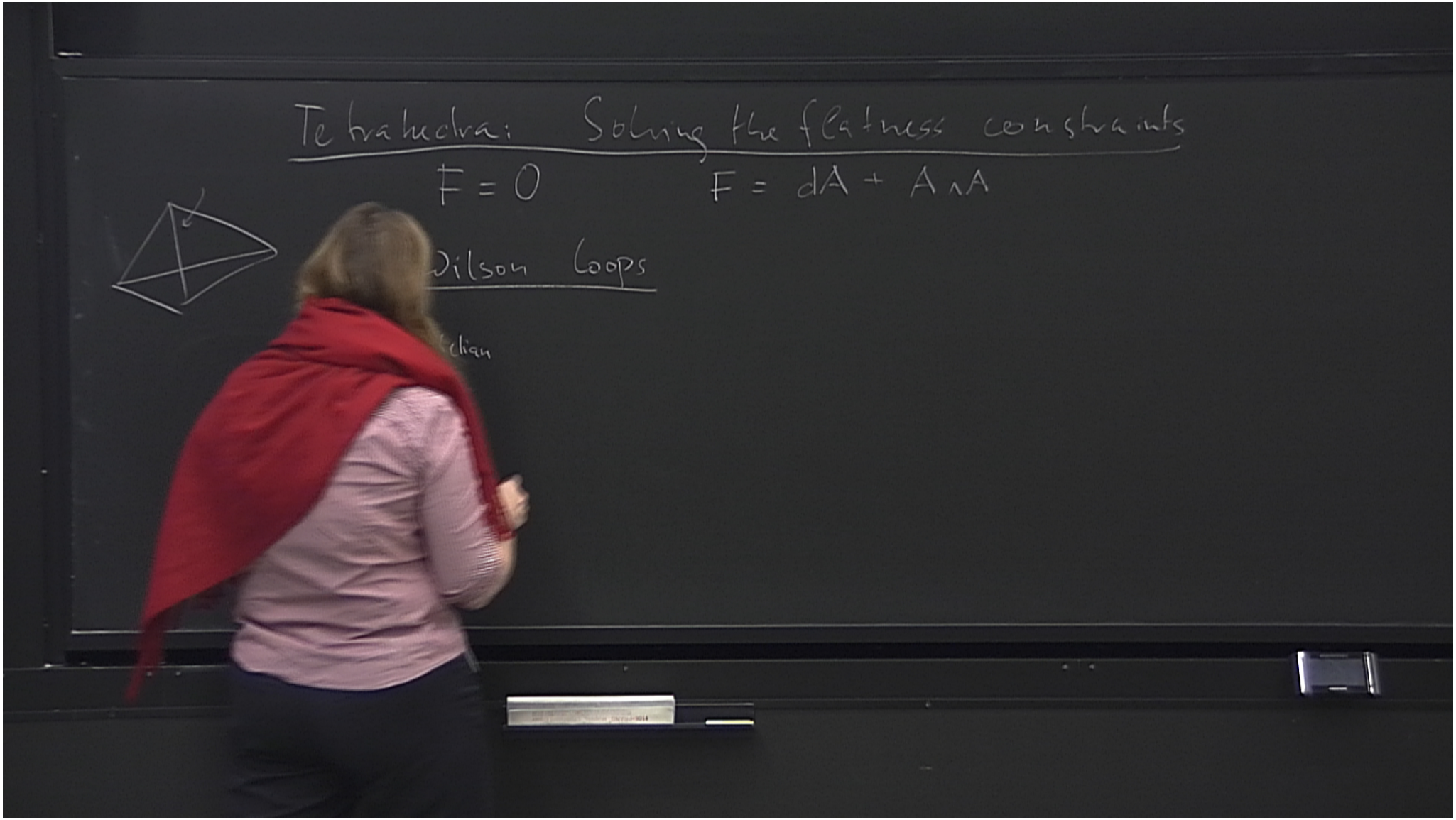
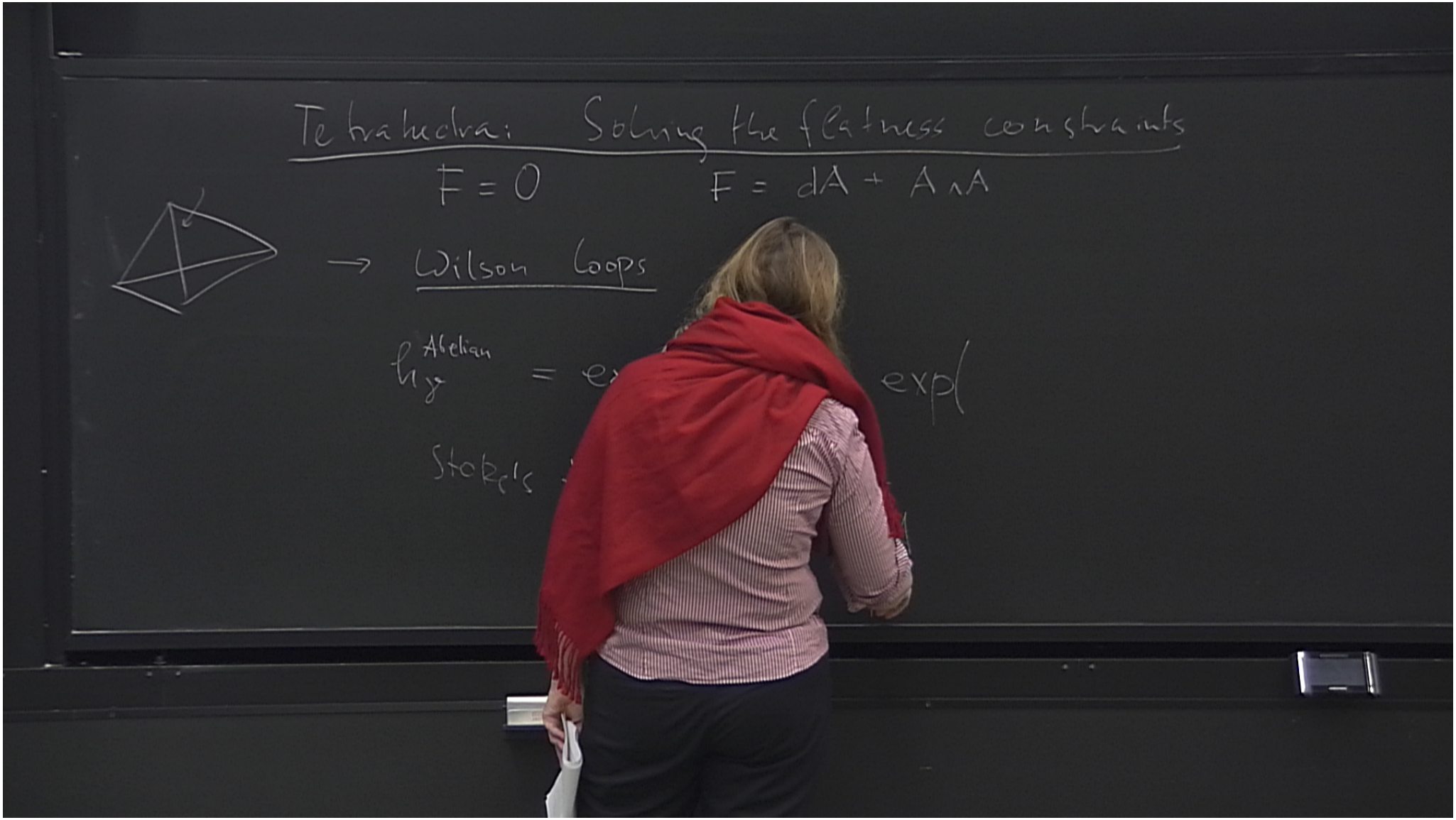
Title: 13/14 PSI - Quantum Gravity Review - Lecture 13

Date: Mar 06, 2014 10:15 AM

URL: http://pirsa.org/14030020

Abstract:


Tetrahedra: Solving the flatness constraints
F = 0
F = dA + A∧A
Wilson Loops


Tetrahedra: Solving the flatness constraints
F = 0
F = dA + A∧A
→ Wilson loops
Abelian
exp(
Stokes

Tetrahedra: Solving the flatness constraints

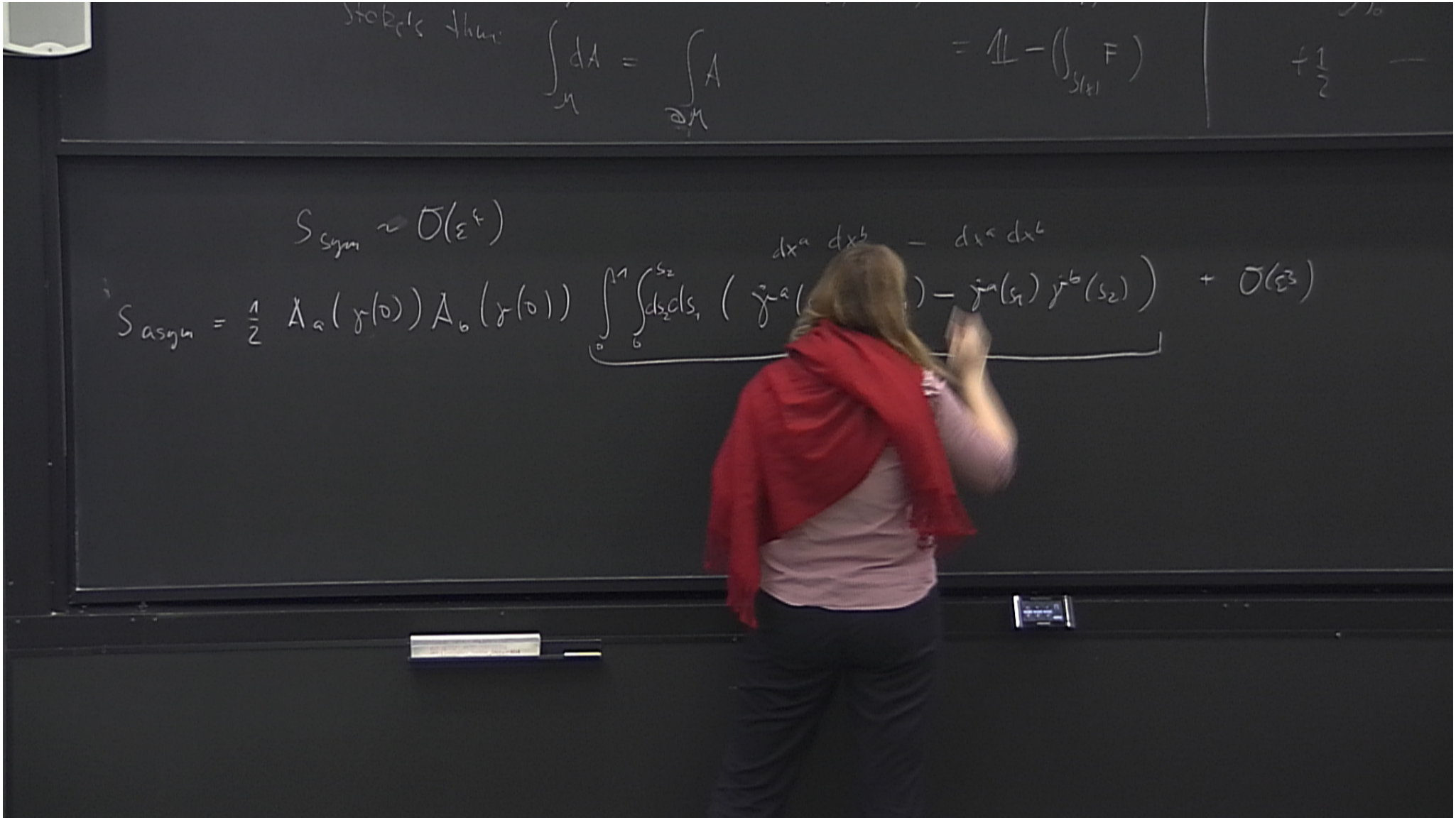
$F = 0 \qquad F = dA + A \wedge A$

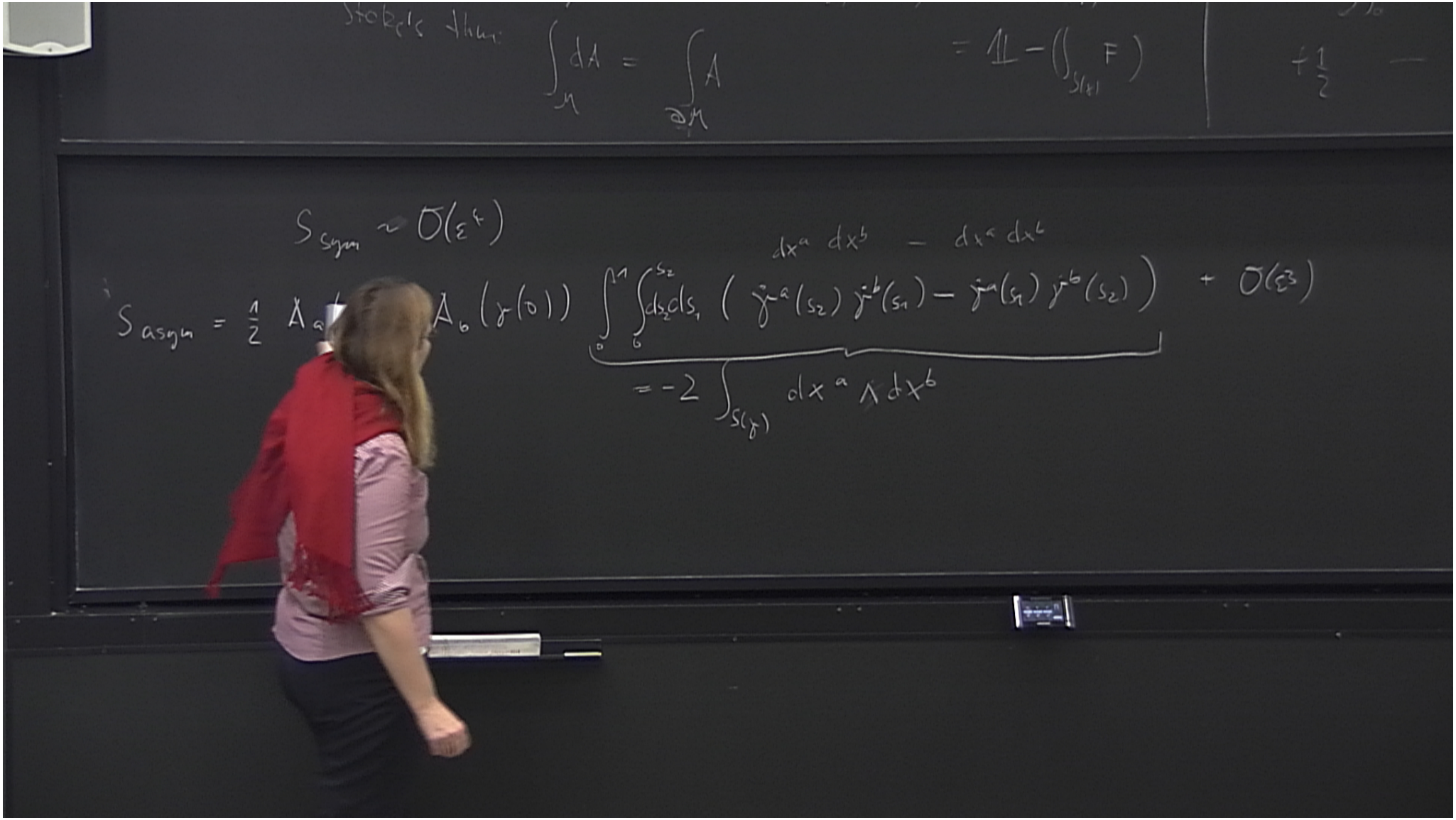


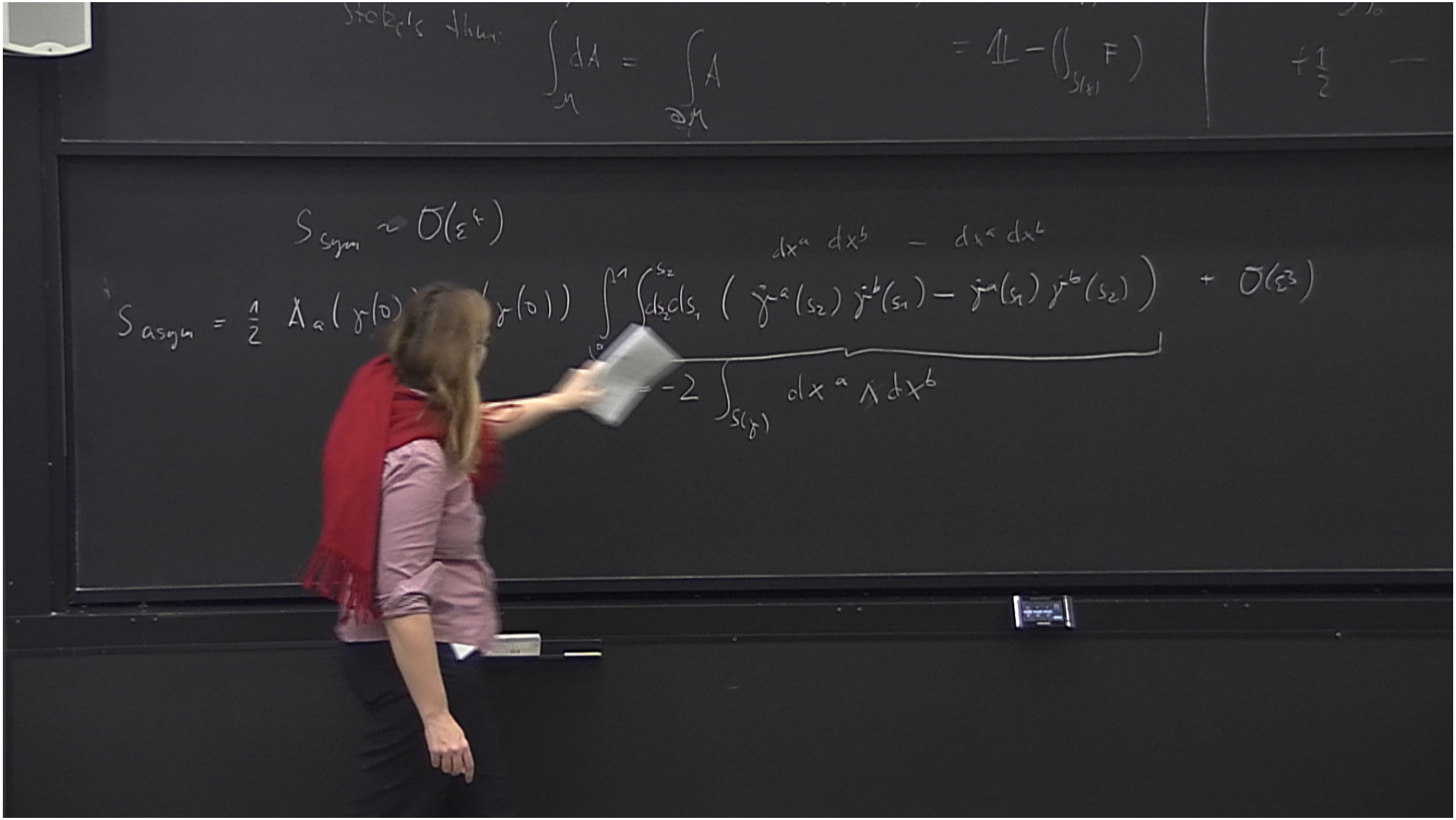
→ Wilson Loops

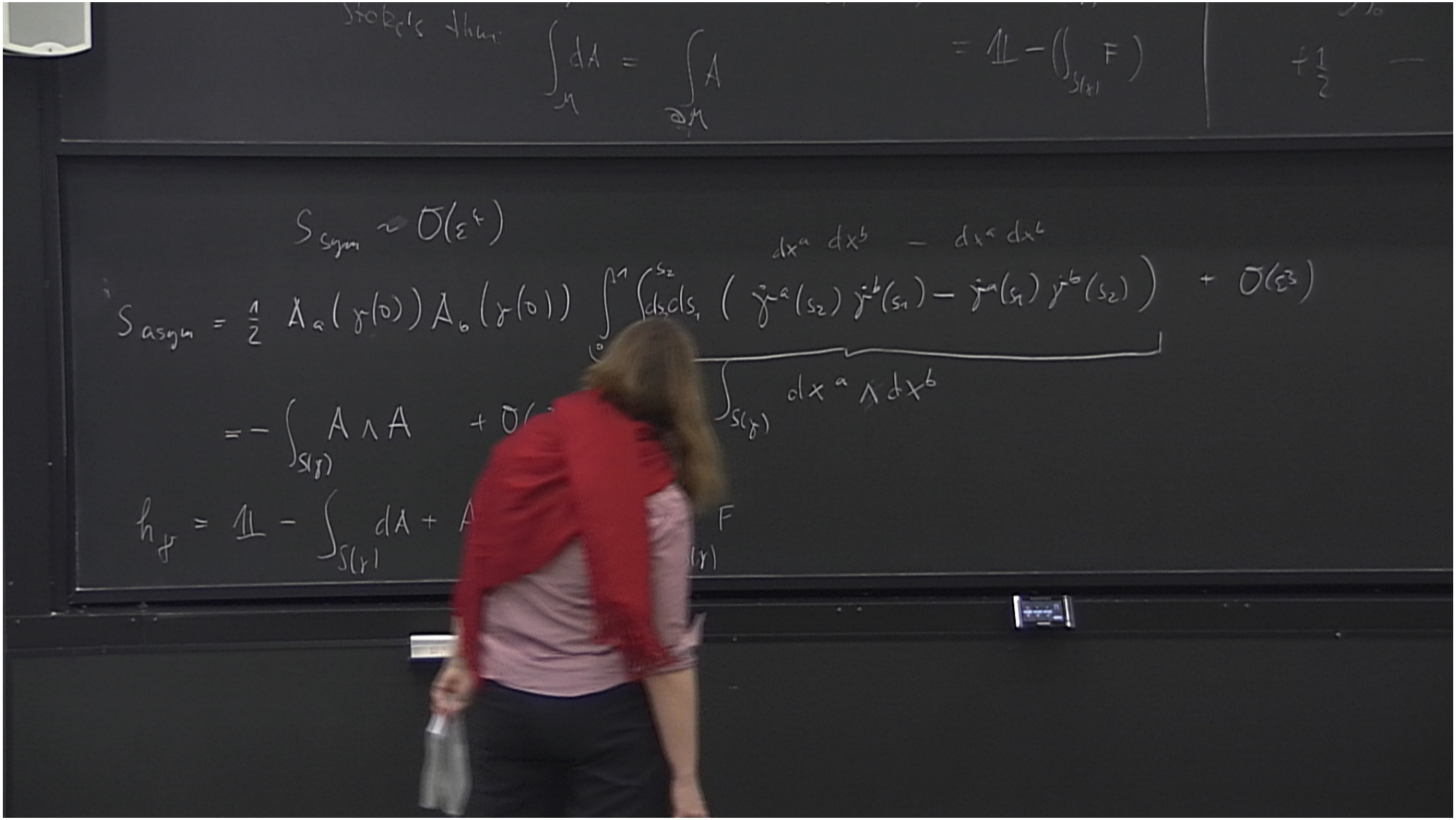
$$\text{Abelian } \text{hol}_\gamma = \exp\left(-\int_\gamma A\right) = \exp\left(-\int_{S(\gamma)} dA\right) = \exp\left(-\int_{S(\gamma)} F\right)$$

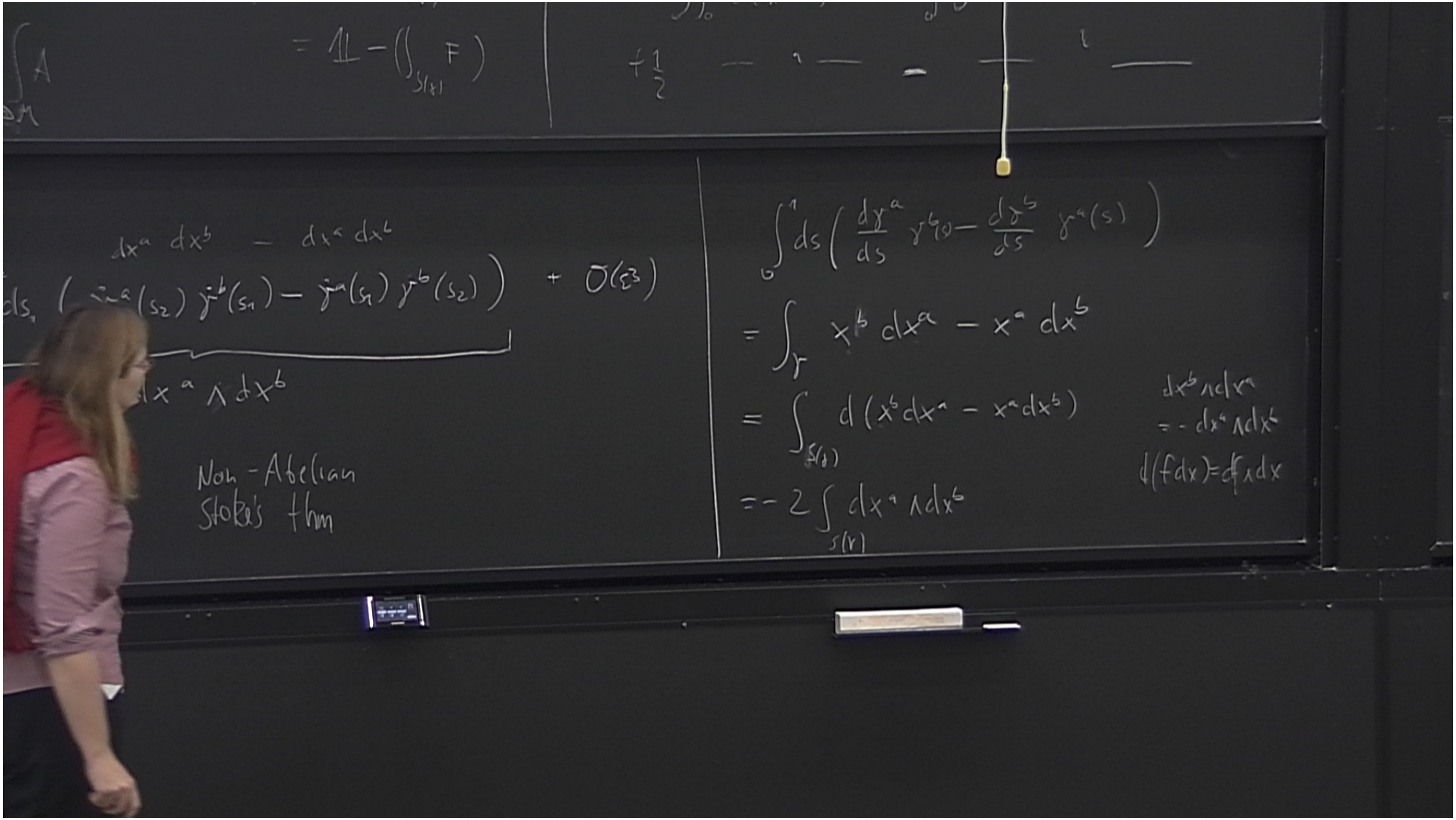
Stokes thm: $\int_M dA = \int_{\partial M} A \qquad = \mathbb{1} - \left(\int_{S(\gamma)} F\right)$

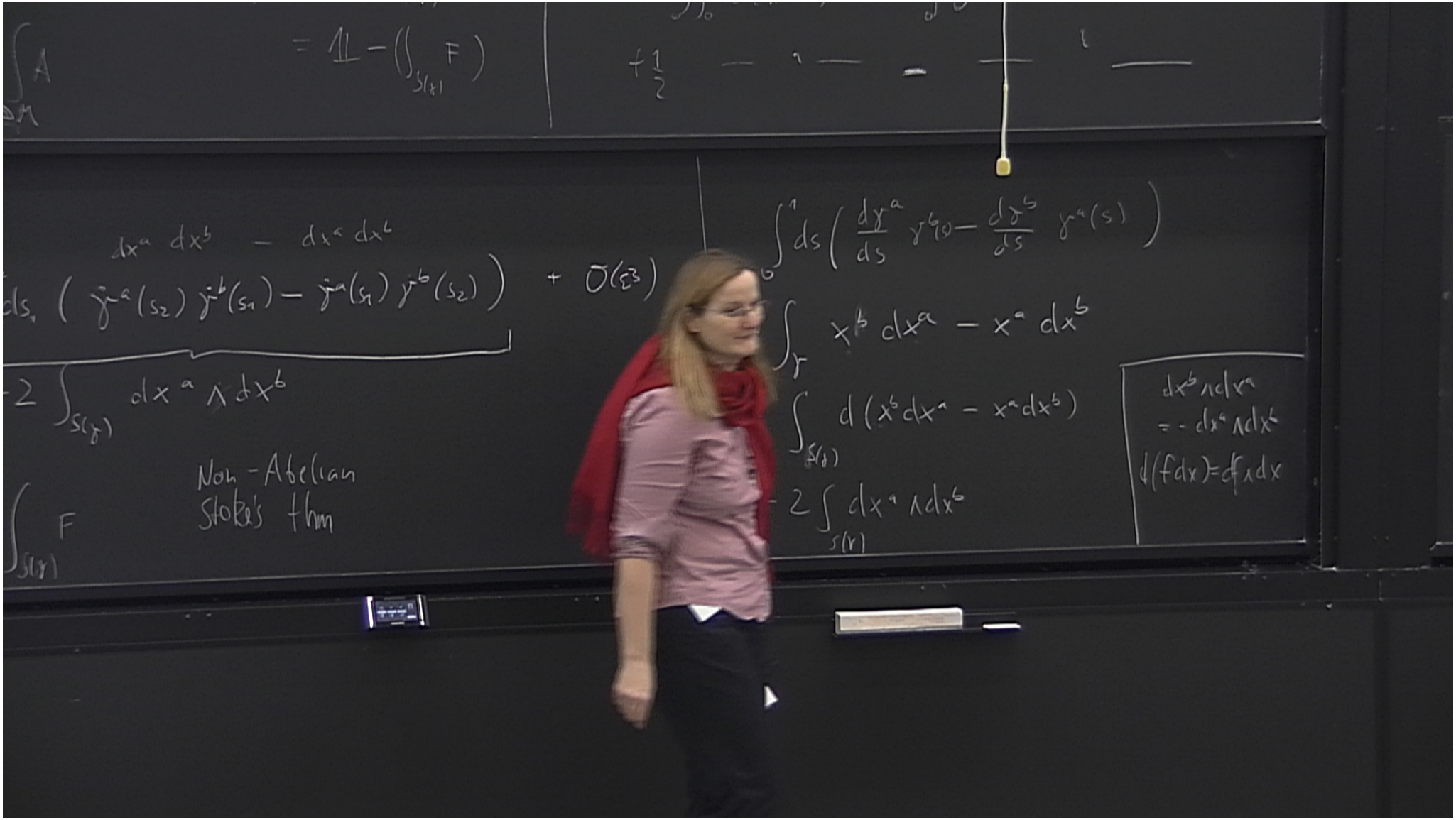

Stokes' thm
$\int_M dA = \int_{\partial M} A$
$= 1\!\!1 - \left(\int_{S(t)} F\right)$
$+\frac{1}{2}$
$S_{sym} \sim O(\varepsilon^4)$
$S_{asym} = \frac{1}{2} A_a(\gamma(0)) A_b(\gamma(0)) \int_0^1 \int_0^{s_2} ds_2 ds_1 \left(\dot{\gamma}^a(\right.$
$dx^a\, dx^b - dx^a\, dx^b$
$\left.) - \dot{\gamma}^a(s_1)\, \dot{\gamma}^b(s_2) \right) + O(\varepsilon^3)$

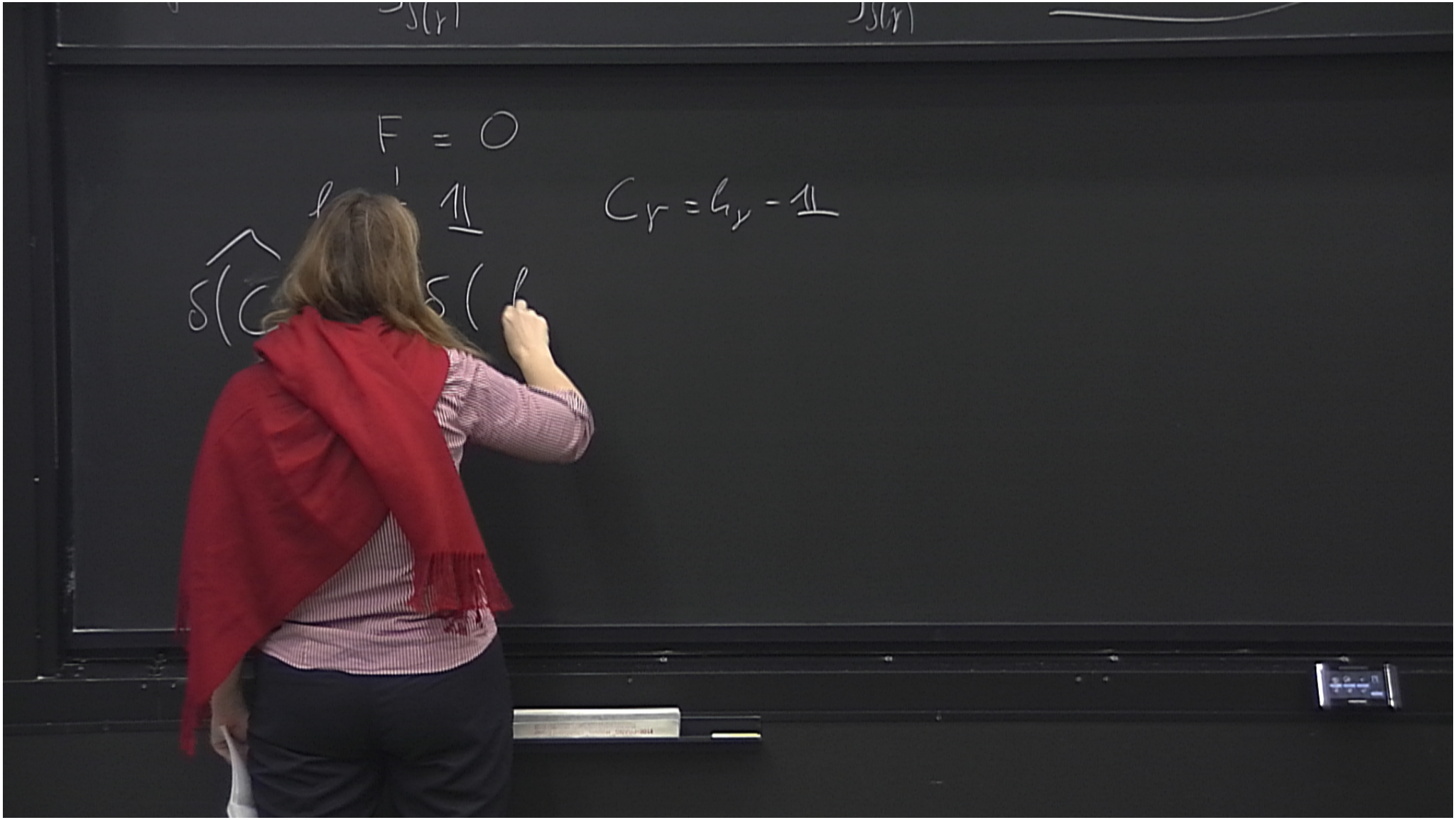

Stokes' thm
$\int_M dA = \int_{\partial M} A$
$= 1\!\!1 - \left(\int_{S(\gamma)} F\right)$
$+\frac{1}{2}$ —
$S_{sym} \sim O(\varepsilon^4)$
$dx^a\, dx^b - dx^a\, dx^b$
$S_{asym} = \frac{1}{2} A_a A_b(\gamma(0)) \int_0^1 \int_0^{s_2} ds_2 ds_1 \left(\dot\gamma^a(s_2)\dot\gamma^b(s_1) - \dot\gamma^a(s_1)\dot\gamma^b(s_2) \right) + O(\varepsilon^3)$
$= -2 \int_{S(\gamma)} dx^a \wedge dx^b$

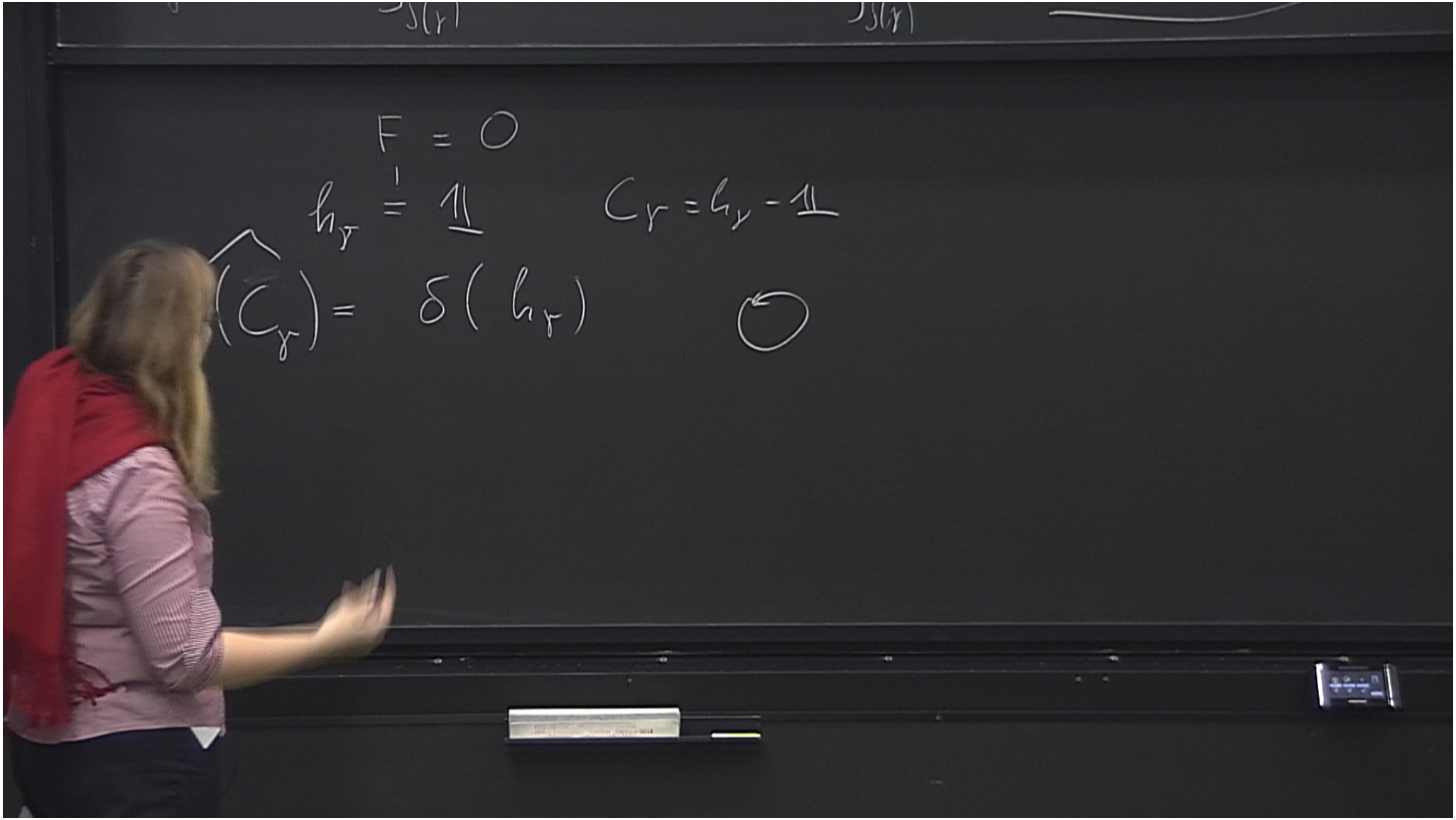

Stokes' thm.
$\int_M dA = \int_{\partial M} A$
$= 1\!\!1 - \left(\int_{S(\gamma)} F\right)$
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$= -2 \int_{S(\gamma)} dx^a \wedge dx^b$

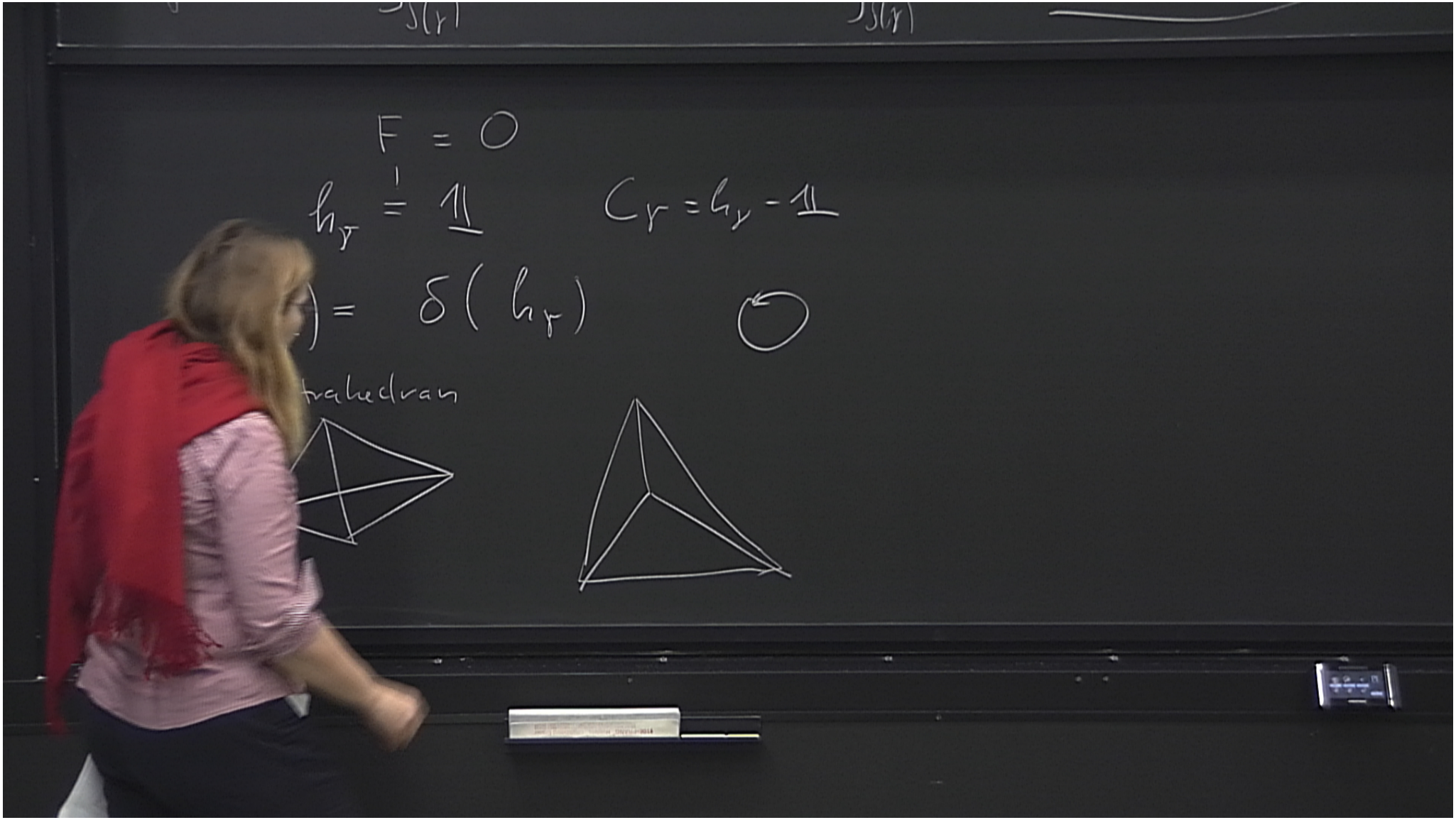

Stokes' thm.
$\int_M dA = \int_{\partial M} A$
$= \mathbb{1} - \left(\int_{S(\gamma)} F\right)$
$+\frac{1}{2}$
$S_{sym} \sim O(\varepsilon^4)$
$S_{asym} = \frac{1}{2} A_a(\gamma(0)) A_b(\gamma(0)) \int_0^T \int_{s_1}^{s_2} ds_2 ds_1 \left(\dot{\gamma}^a(s_2)\dot{\gamma}^b(s_1) - \dot{\gamma}^a(s_1)\dot{\gamma}^b(s_2) \right) dx^a dx^b - dx^a dx^b + O(\varepsilon^3)$
$= -\int_{S(\gamma)} A \wedge A + O($
$\int_{S(\gamma)} dx^a \wedge dx^b$
$h_\gamma = \mathbb{1} - \int_{S(\gamma)} dA + A$
F
(γ)

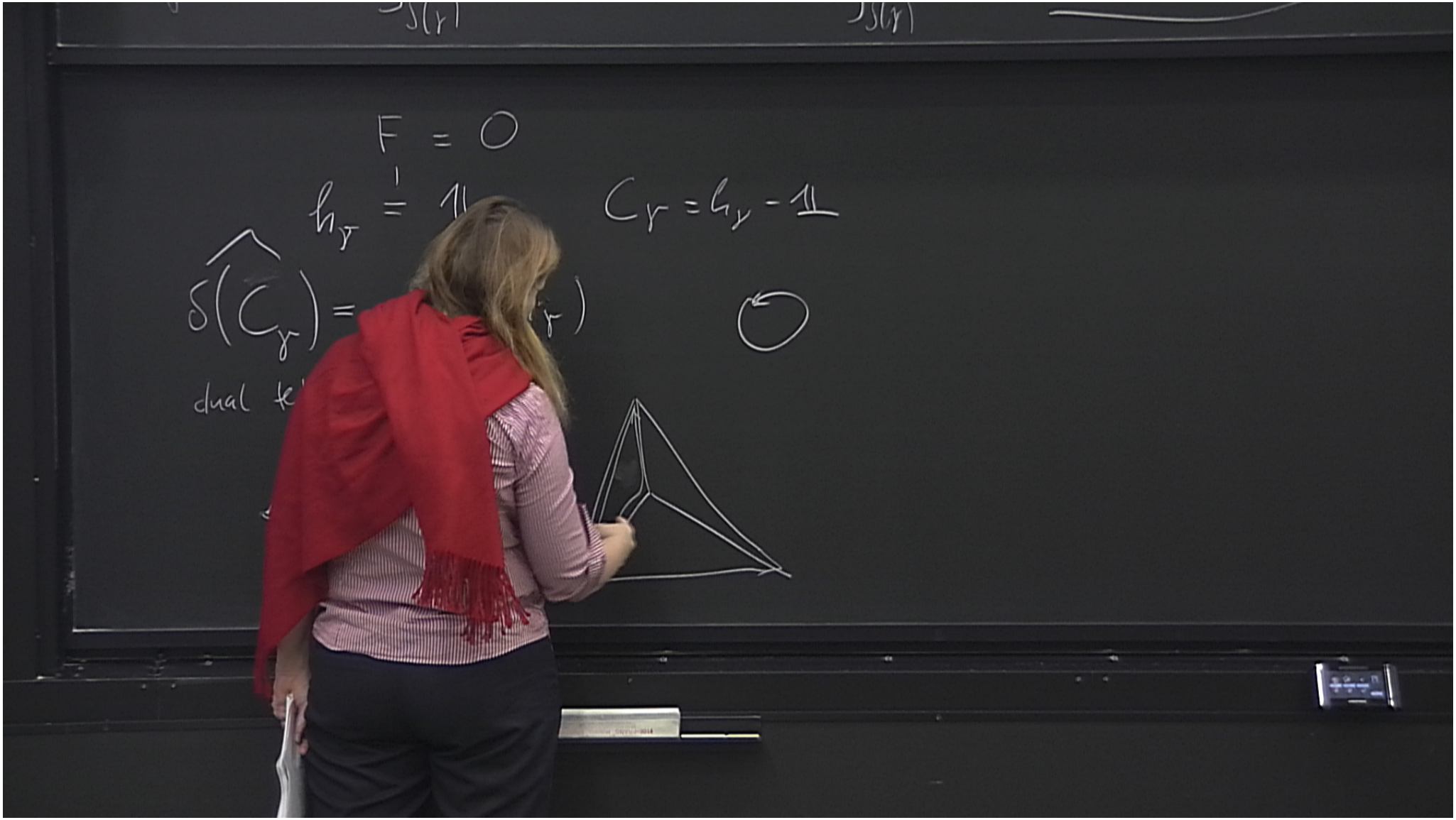

Non-Abelian
Stoke's thm


Non-Abelian
Stoke's thm
dx^a ∧ dx^b = - dx^a ∧ dx^b
d(f dx) = df ∧ dx


F = 0
$C_\gamma = G_\gamma - \mathbb{1}$


F = 0
$h_\gamma \stackrel{!}{=} \mathbb{1}$
$C_\gamma = h_\gamma - \mathbb{1}$
$\widehat{(C_\gamma)} = \delta(h_\gamma)$


F = 0
$h_\gamma \stackrel{!}{=} \mathbb{1}$
$C_\gamma = h_\gamma - \mathbb{1}$
$\delta(h_\gamma)$
tetrahedron

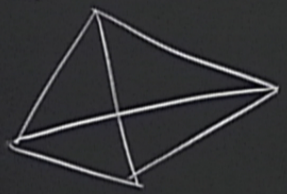

F = 0
$h_\gamma \stackrel{!}{=} 1$
$C_\gamma = G_\gamma - 1$
$\delta(C_\gamma) =$
dual

$F = 0$

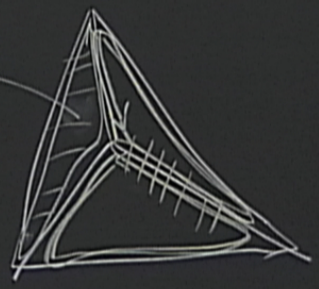
$h_\gamma \overset{!}{=} \mathbb{1}$ $\quad C_\gamma = h_\gamma - \mathbb{1}$

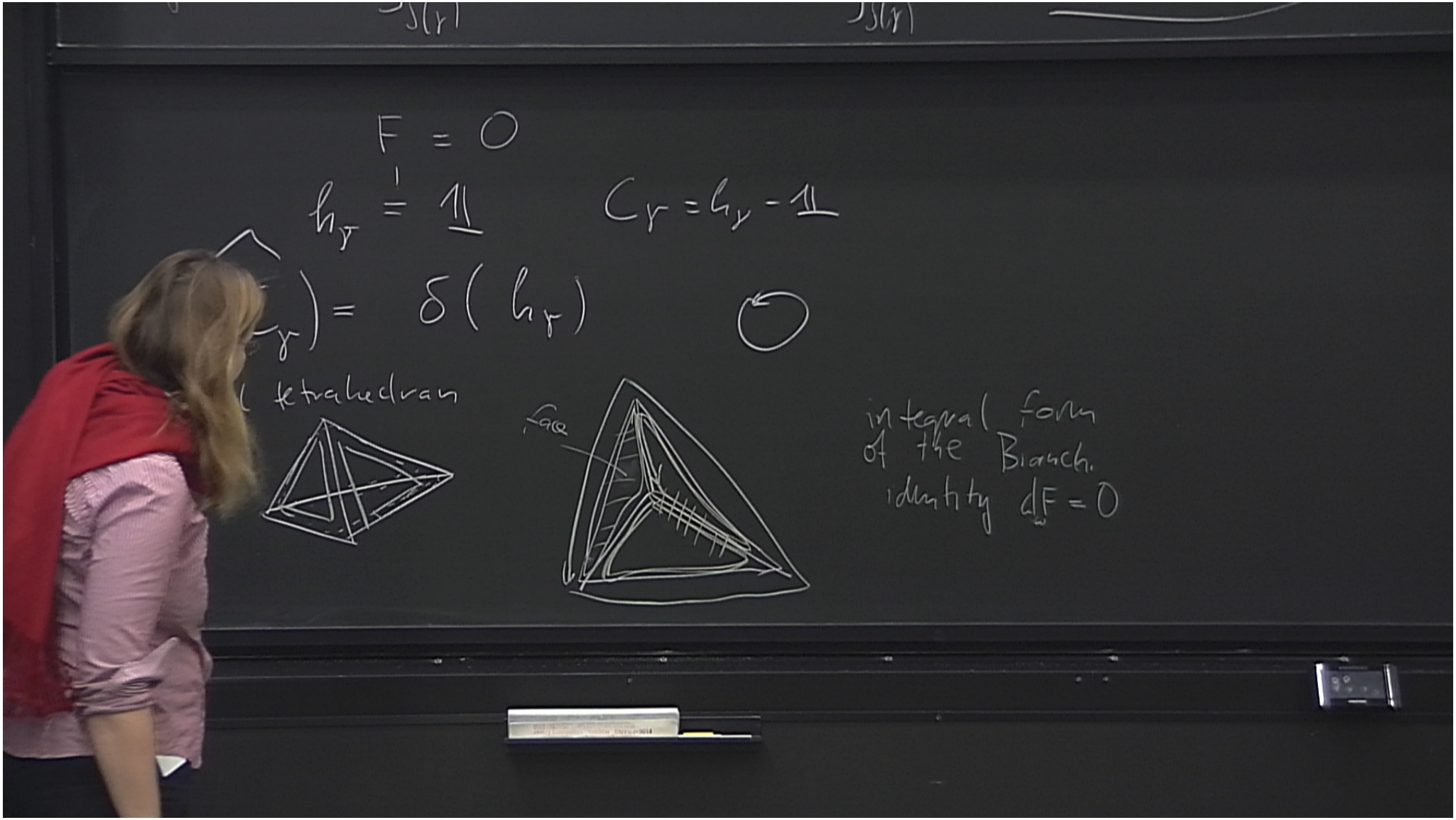
$\delta(C_\gamma) = \delta(h_\gamma)$ $\quad \bigcirc$

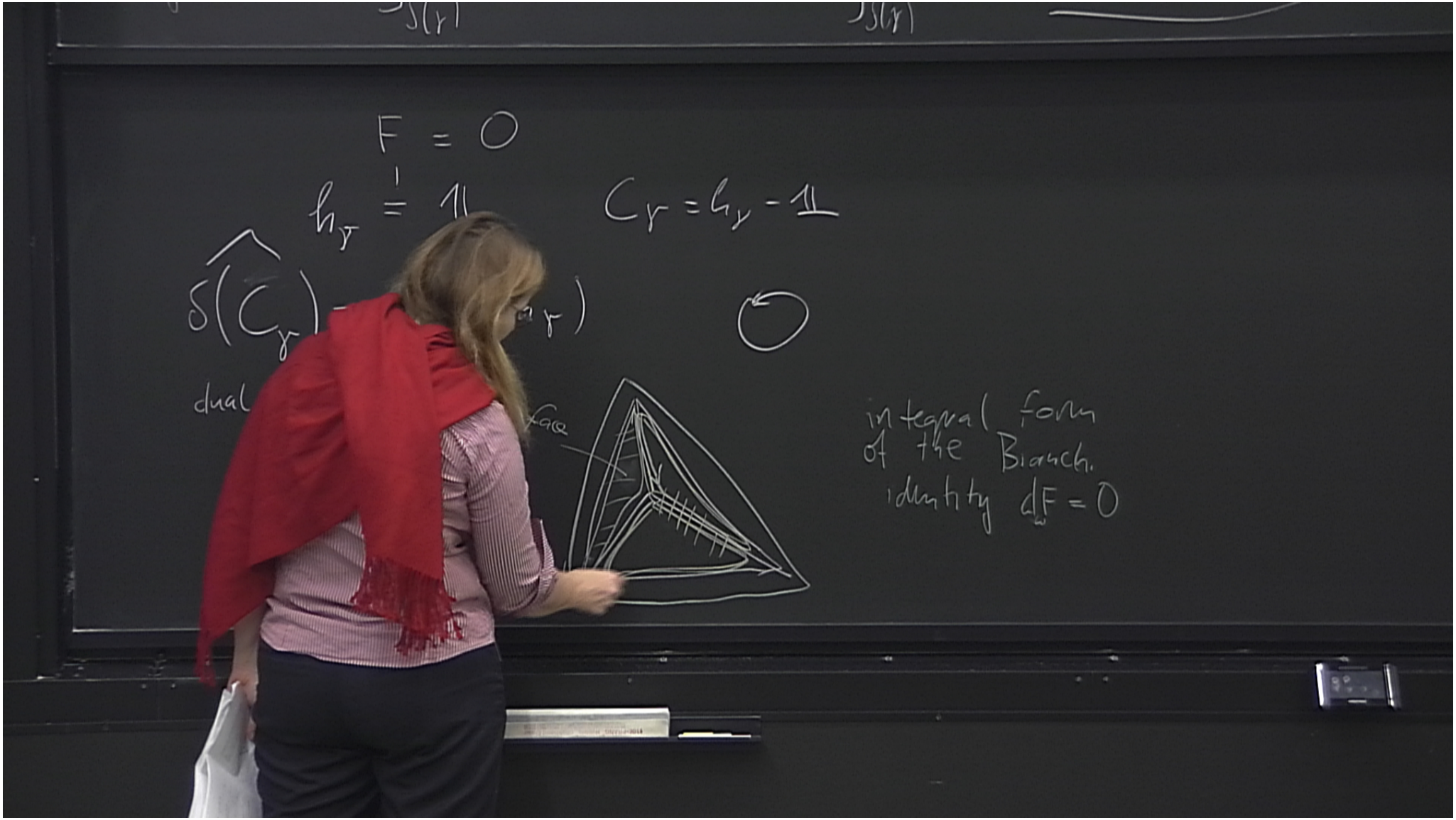
dual tetrahedron

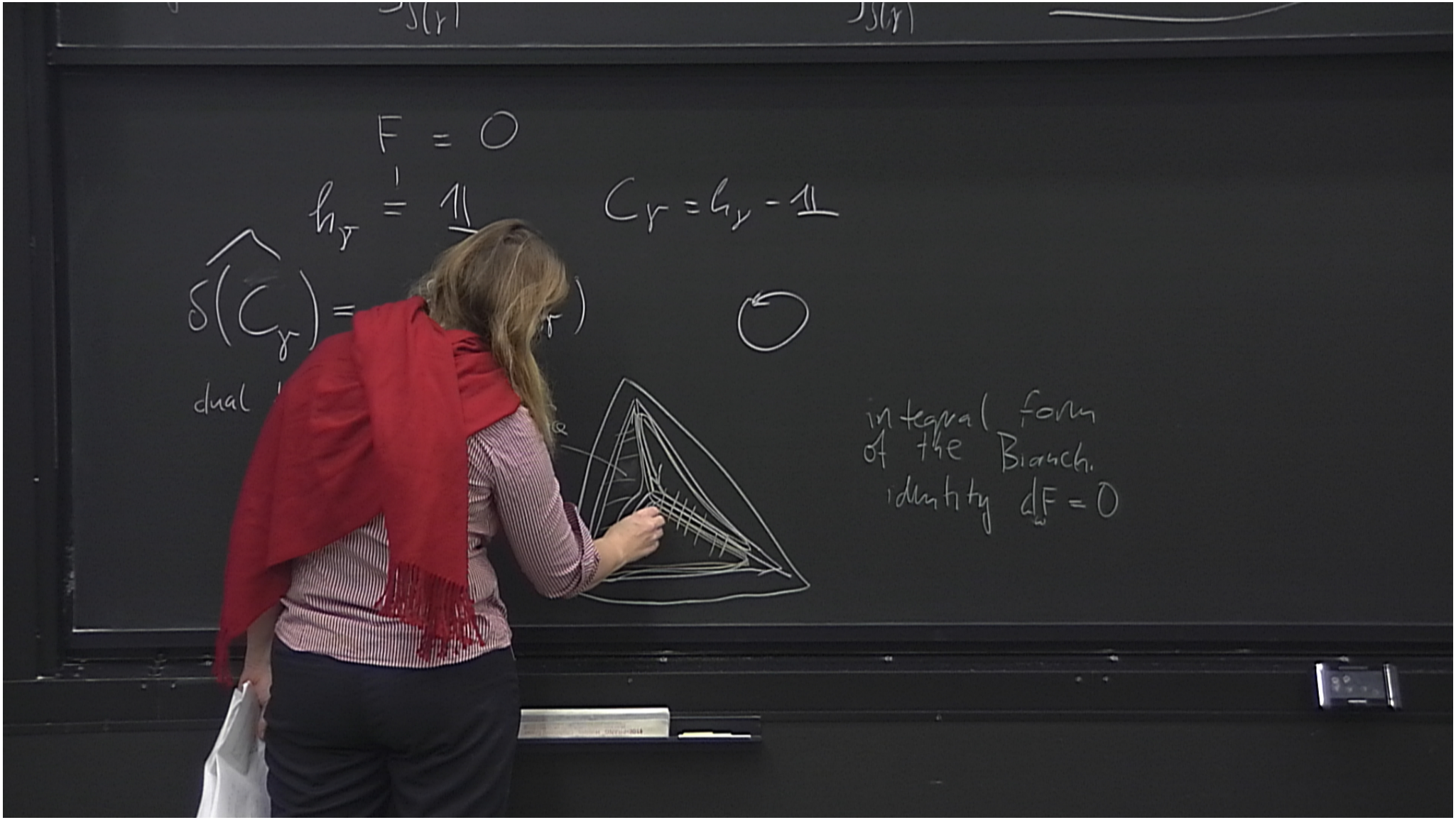


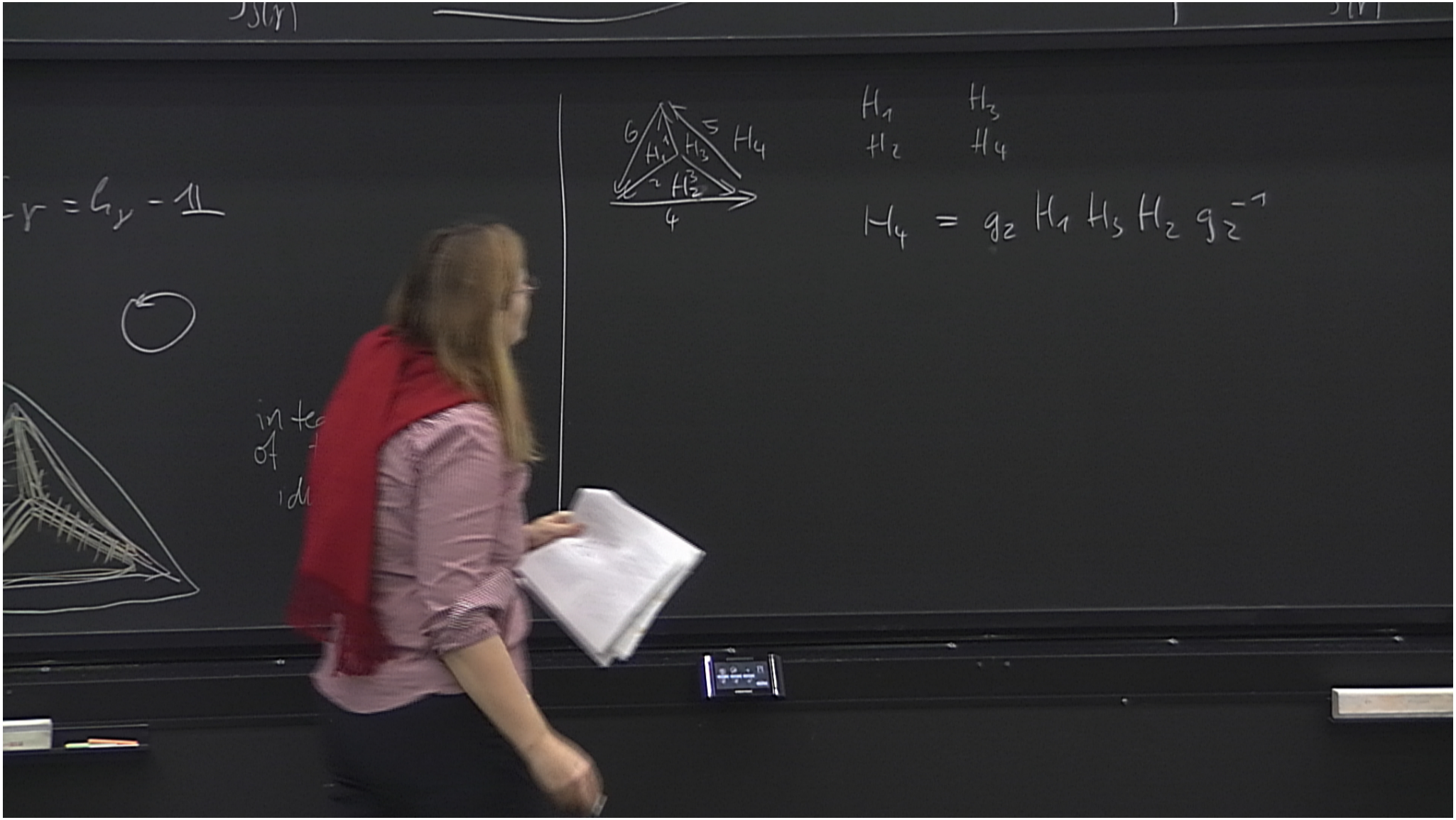
face

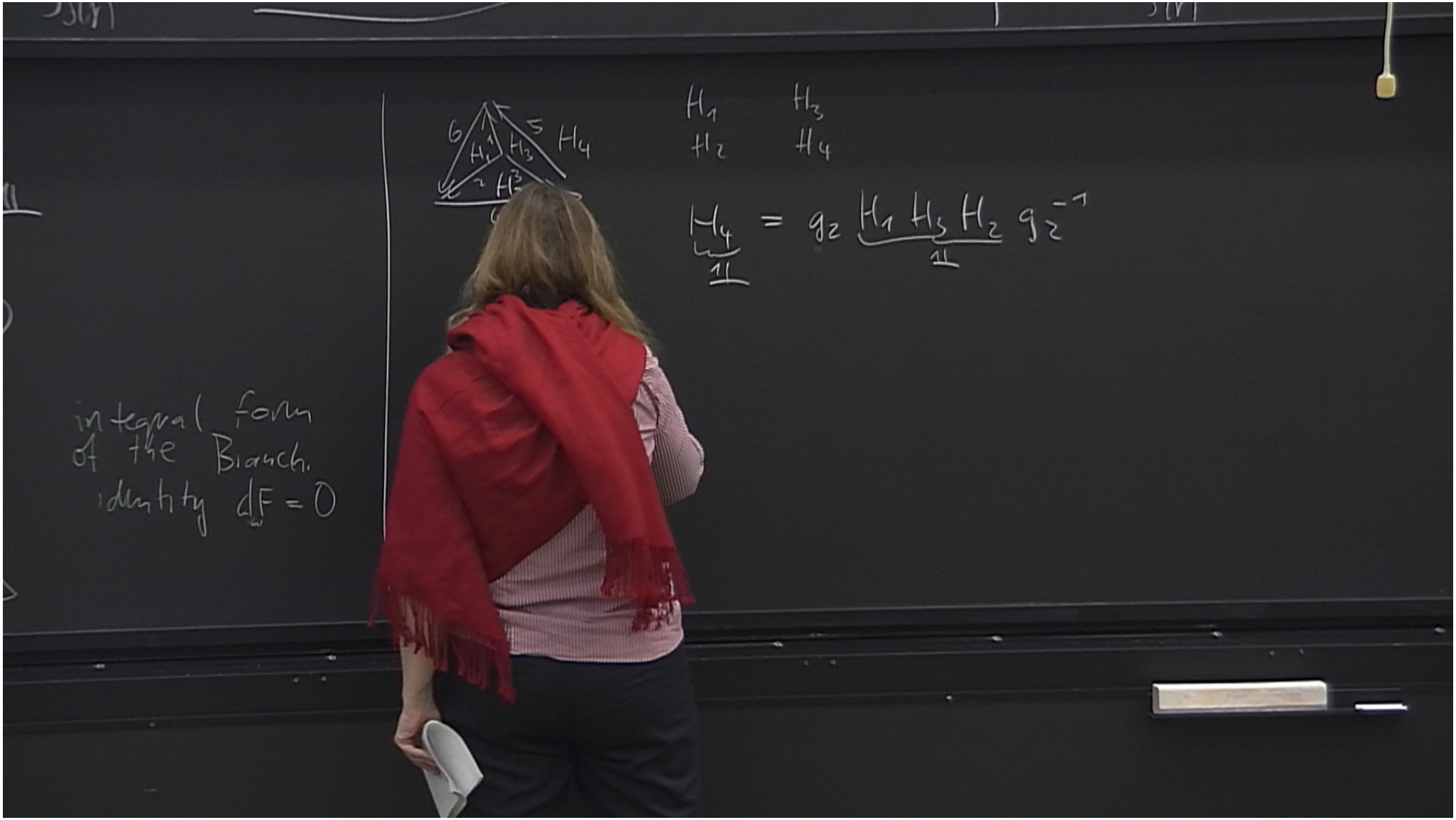


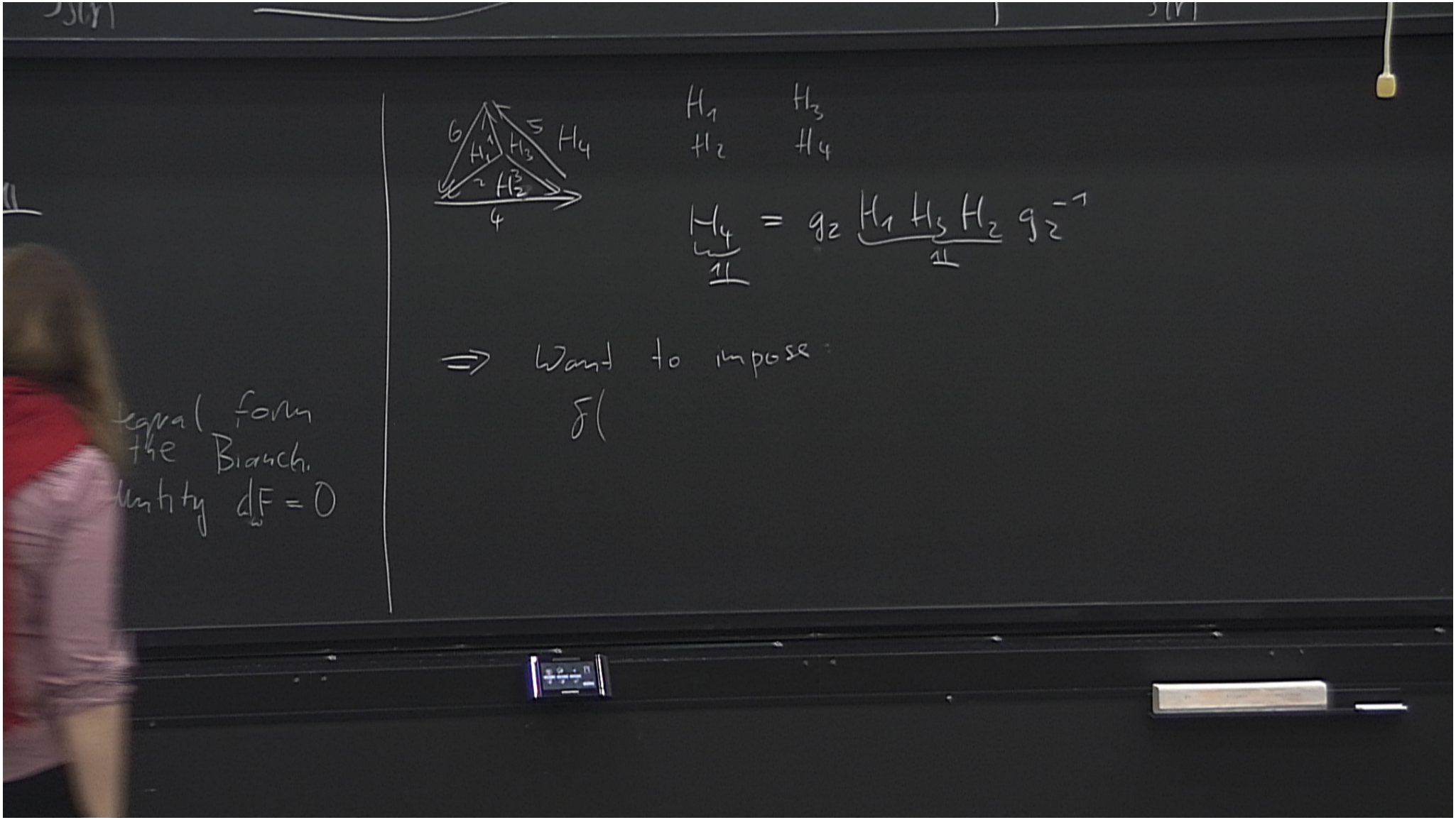

F = 0
$h_\gamma = \mathbb{1}$
$C_\gamma = h_\gamma - \mathbb{1}$
$\delta(h_\gamma)$
tetrahedron
face
integral form of the Bianchi identity $d_\omega F = 0$

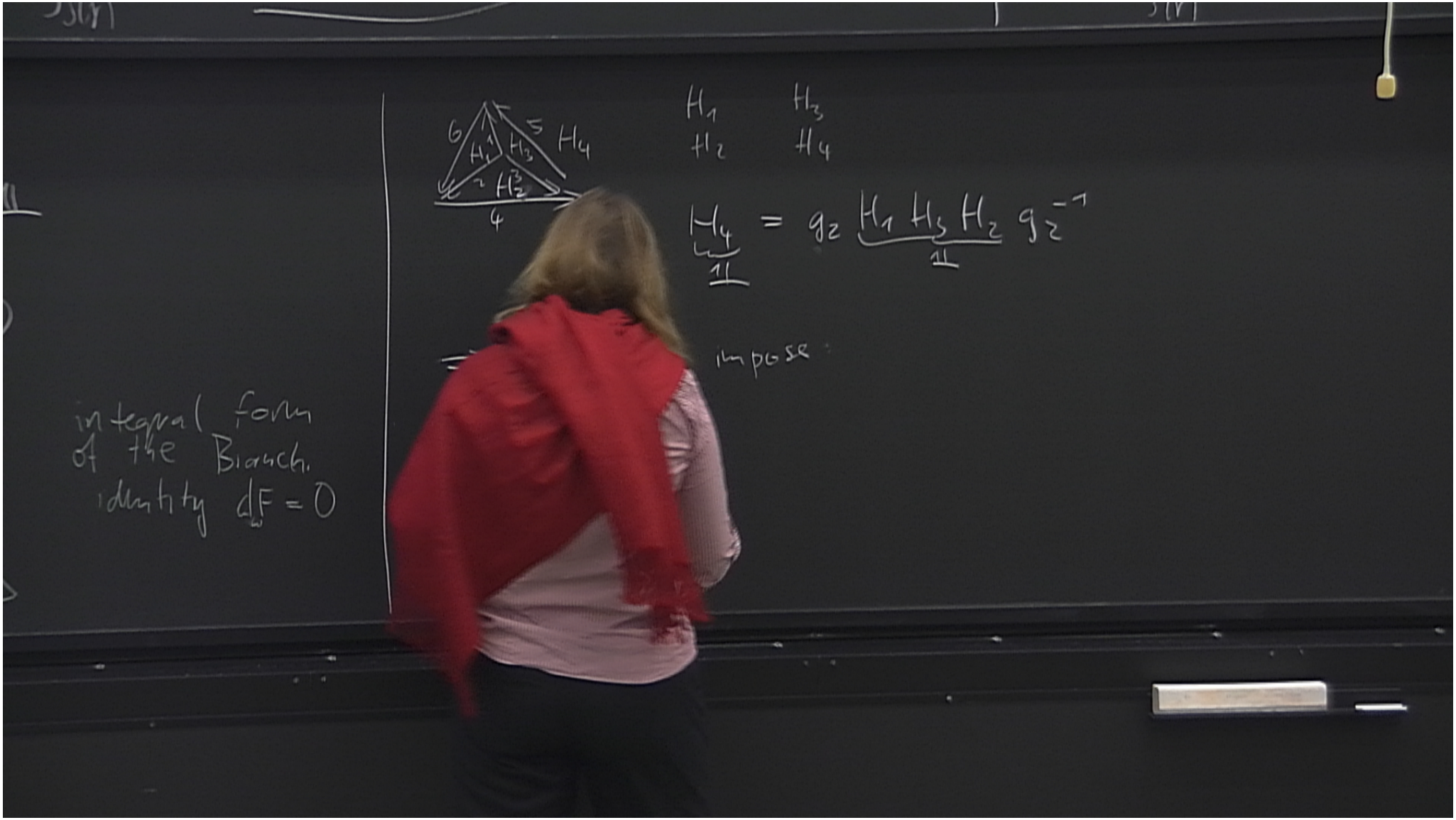

F = 0
$h_\gamma \stackrel{!}{=} 1$
$C_\gamma = h_\gamma - 1$
integral form of the Bianchi identity $d_\omega F = 0$

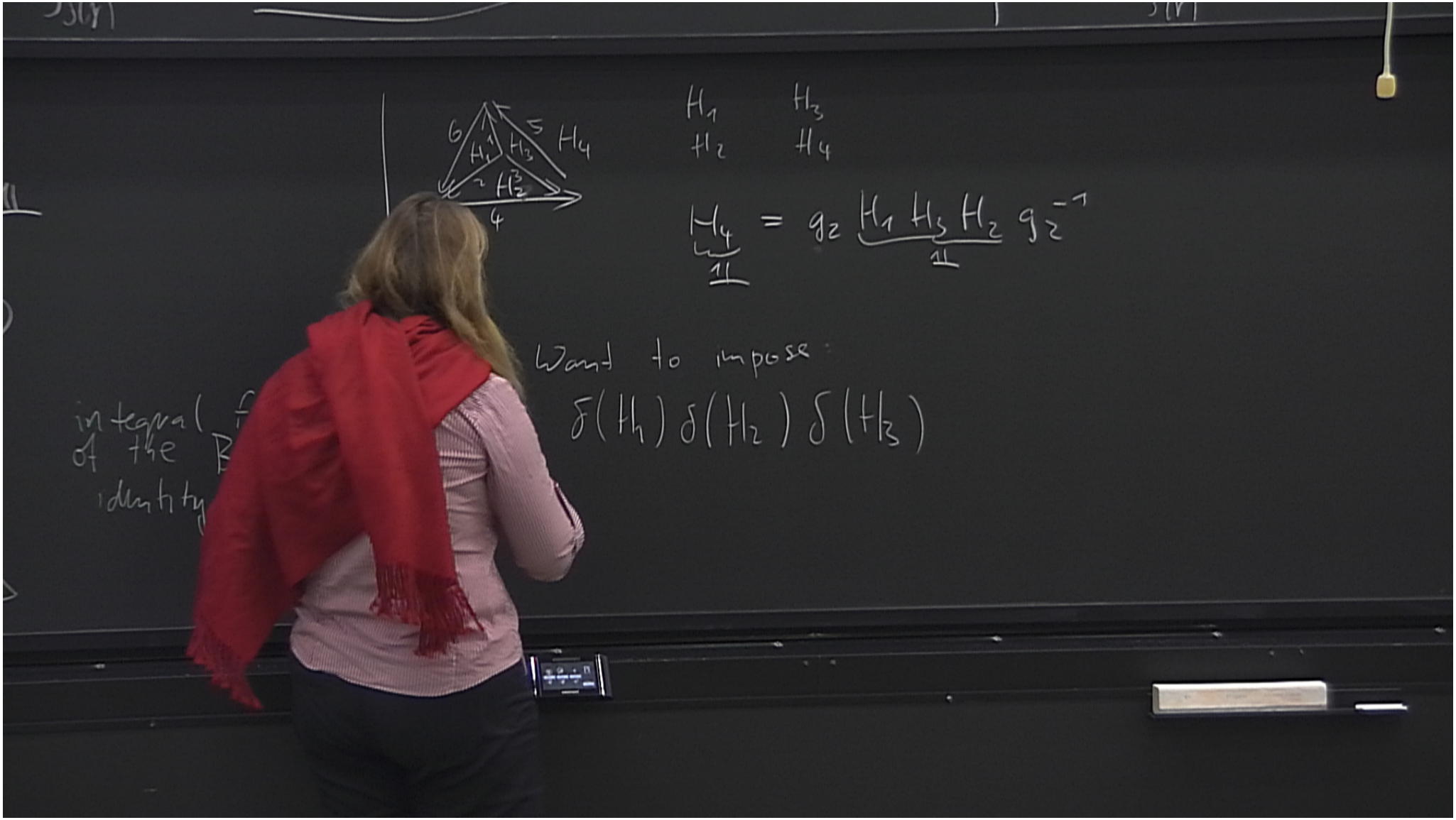

F = 0
$h_\gamma = \mathbb{1}$
$C_\gamma = h_\gamma - \mathbb{1}$
$\delta(C_\gamma)$
integral form of the Bianchi identity $d_\omega F = 0$

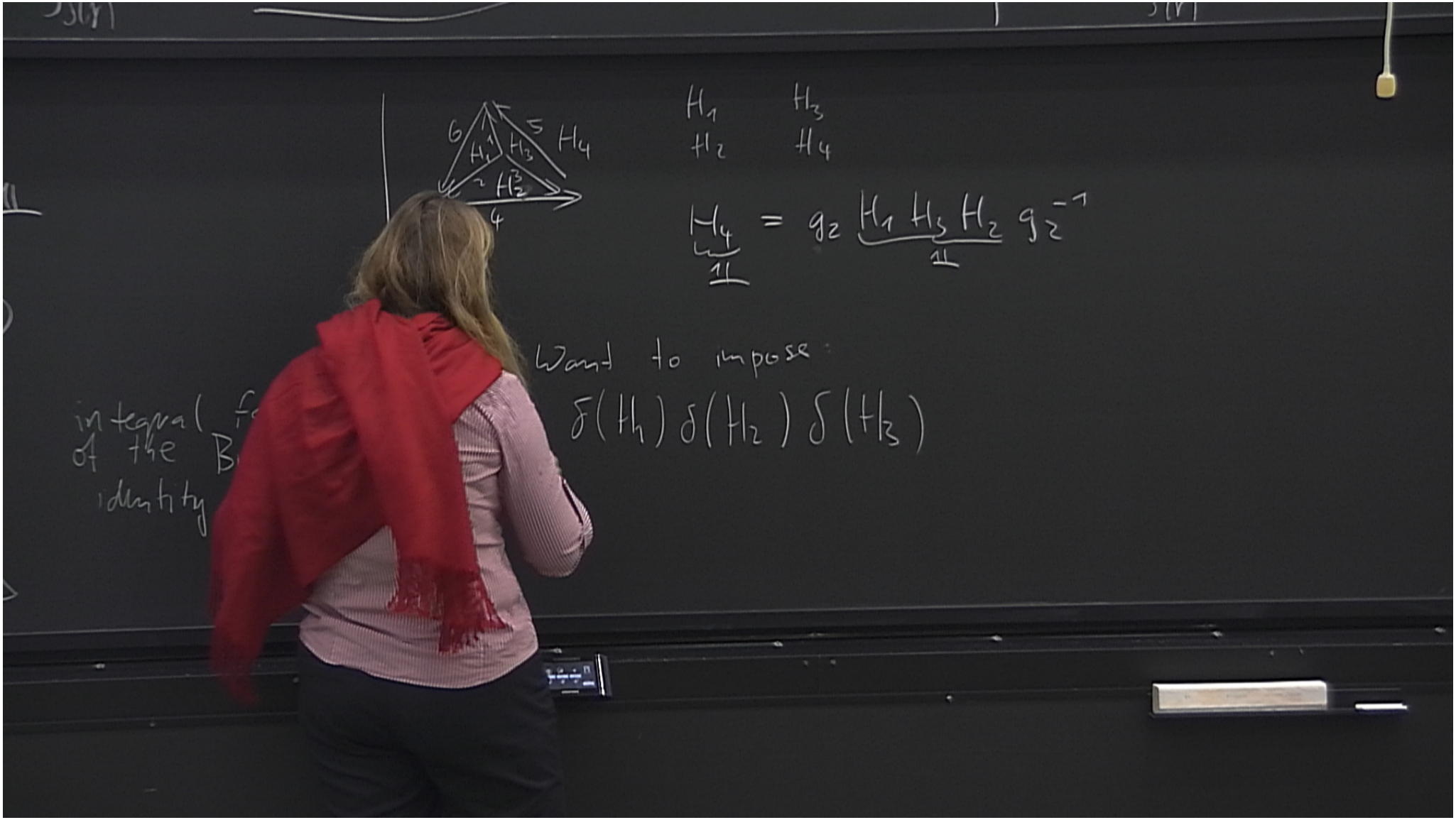

H_1 H_3
H_2 H_4
$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$

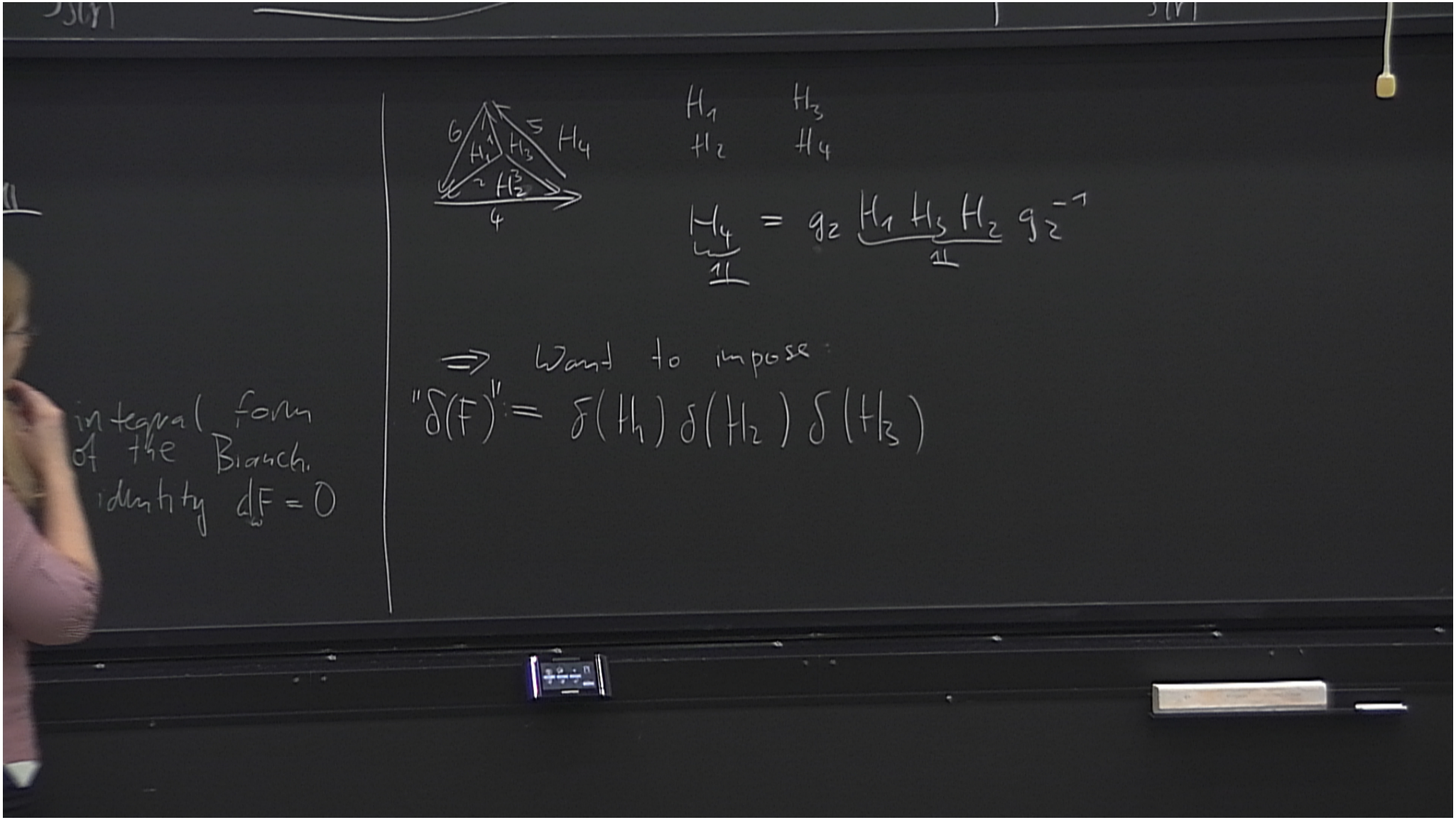

integral form of the Bianchi identity $d_\omega F = 0$
H_1 H_3
H_2 H_4
$\underbrace{H_4}_{\parallel} = g_2 \underbrace{H_1 H_3 H_2}_{\parallel} g_2^{-1}$


H_1 H_3
H_2 H_4
$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$
$\Rightarrow$ Want to impose
$\delta($
tegral form
the Bianchi
entity $dF = 0$


integral form of the Bianchi identity $d_\omega F = 0$
H_1 H_3
H_2 H_4
$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$
impose

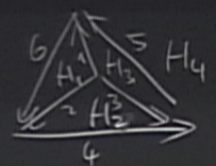

H_1 H_3
H_2 H_4
$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$
Want to impose:
$\delta(H_1)\,\delta(H_2)\,\delta(H_3)$


H_1 H_3
H_2 H_4
$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$
Want to impose:
$\delta(H_1)\,\delta(H_2)\,\delta(H_3)$


integral form of the Bianchi identity $d_\omega F = 0$
H_1 H_3
H_2 H_4
$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$
$\Rightarrow$ Want to impose:
"$\delta(F)$" $= \delta(H_1)\,\delta(H_2)\,\delta(H_3)$

Stokes thm

$= -2 \int_{S(r)} dx^4 \wedge dx^5$



$H_1 \quad H_3$
$H_2 \quad H_4$

$H_4 = g_2 \, H_1 H_3 H_2 \, g_2^{-1}$
$\quad \in \mathbb{1} \qquad \in \mathbb{1}$

$C = P \quad \mathbb{R}$
$\hookrightarrow \delta(p)\, 4 \rightarrow$ not

$\Rightarrow$ Want to impose

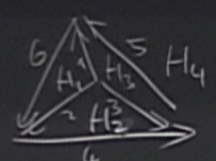
$"\delta(F)" = \delta(H_1)\,\delta(H_2)\,\delta(H_3)$

Problem: Fluxes generates non-compact orbits

$F = 0$

Stokes thm

$= -2 \int_{S(r)} dx^a \wedge dx^5$



$H_1 \quad H_3$
$H_2 \quad H_4$

$\underbrace{H_4}_{\mathbb{1}} = g_2 \underbrace{H_1 H_3 H_2}_{\mathbb{1}} g_2^{-1}$

$\Rightarrow$ Want to impose

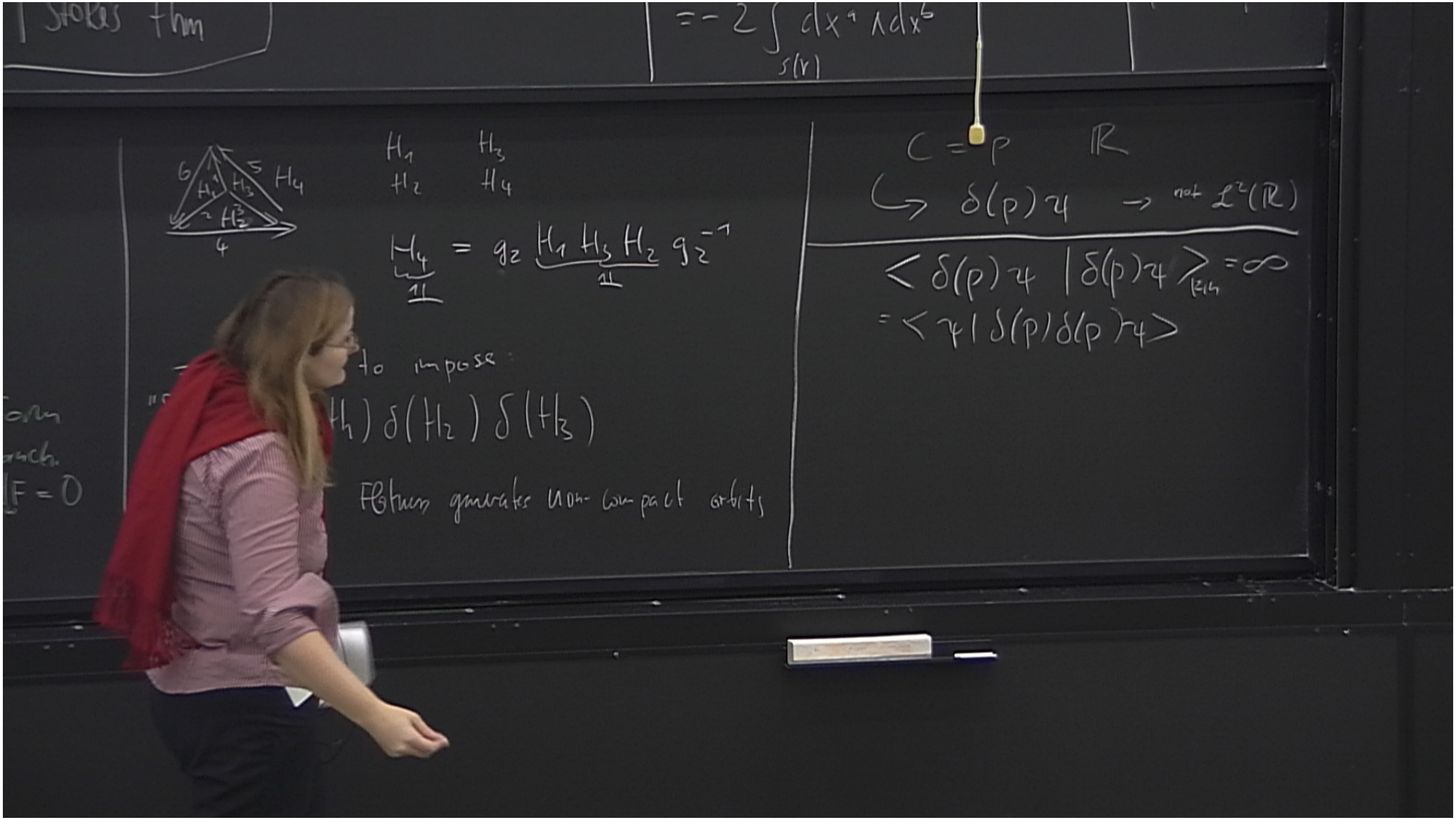
$"\delta(F)" = \delta(H_1)\,\delta(H_2)\,\delta(H_3)$

Problem: Fluctuations generates non-compact orbits

$C = P \quad \mathbb{R}$

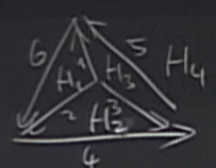
$\hookrightarrow \delta(p) \to$ not $\mathcal{L}^2(\mathbb{R})$

$\langle \delta(p) \ldots \delta(p) \rangle = \infty$


Stokes thm
$= -2 \int_{S(r)} dx^a \wedge dx^b$
H_1 H_3
H_2 H_4
$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$
$H_4 = \mathbb{1}$
$H_1 H_3 H_2 = \mathbb{1}$
to impose
$\delta(H_4)\,\delta(H_2)\,\delta(H_3)$
Fluxes generates non-compact orbits
$C = p \quad \mathbb{R}$
$\hookrightarrow \delta(p)\,\eta \rightarrow$ not $L^2(\mathbb{R})$
$\langle \delta(p)\,\eta \,|\, \delta(p)\,\eta \rangle_{\mathbb{R}/\mathbb{Z}} = \infty$
$= \langle \eta \,|\, \delta(p)\,\delta(p)\,\eta \rangle$

Stokes thm

$= -2 \int_{S(\gamma)} dx^a \wedge dx^b$



$H_1 \quad H_3$
$H_2 \quad H_4$

$\underbrace{H_4}_{\mathbb{1}} = g_2 \underbrace{H_1 H_3 H_2}_{\mathbb{1}} g_2^{-1}$

$C = P \quad \mathbb{R}$

$\hookrightarrow \delta(p)\gamma \rightarrow$ not $L^2(\mathbb{R})$

$\delta(p)\gamma \,|\, \delta(p)\gamma\rangle_{kin} = \infty$

$\gamma | \delta(p)\delta(p)\gamma\rangle$

$|\, \delta(p)\gamma\rangle_{kin}$

$\Rightarrow$ Want to impose:

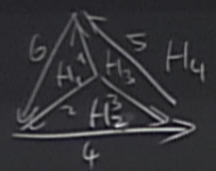
$"\delta(F)" = \delta(H_1)\,\delta(H_2)\,\delta(H_3)$

Problem: Fluxes generates non-compact

orm
ach.
$F = 0$

Stokes thm

$= -2 \int_{S(r)} dx^a \wedge dx^b$



$H_1 \quad H_3$
$H_2 \quad H_4$

$\underbrace{H_4}_{\mathbb{1}} = g_2 \underbrace{H_1 H_3 H_2}_{\mathbb{1}} g_2^{-1}$

$\Rightarrow$ Want to impose

$"\delta(F)" = \delta(H_1)\,\delta(H_2)\,\delta(H_3)$

Problem: Flows generates non-compact orbits

$C = P \quad \mathbb{R}$

$\hookrightarrow \delta(p)\,\mathcal{U} \rightarrow$ not $L^2(\mathbb{R})$

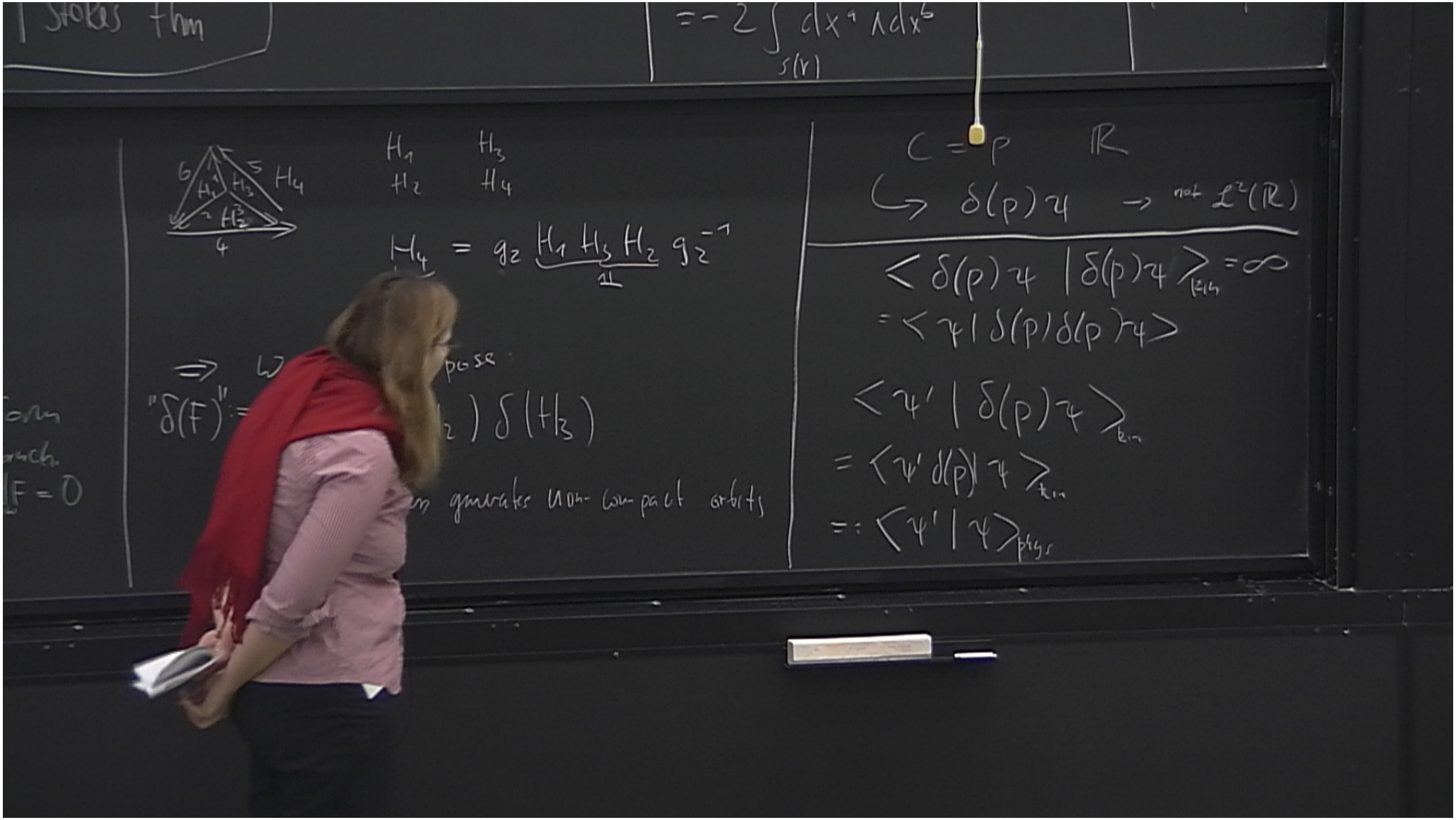
$\langle \delta \mid \delta(p)\,\mathcal{U} \rangle_{kin} = \infty$

$= \langle \quad \delta(p)\,\mathcal{U} \rangle$

$\rangle_{kin}$

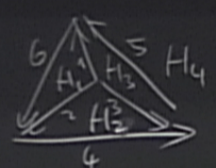
$=$

$=$


Stokes thm
$= -2 \int_{S(V)} dx^a \wedge dx^b$
H_1 H_3
H_2 H_4
$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}$
$C = \mathbb{R}$ $\mathbb{R}$
$\hookrightarrow \delta(p)\psi \rightarrow$ not $L^2(\mathbb{R})$
$\langle \delta(p)\psi \mid \delta(p)\psi \rangle_{kin} = \infty$
$= \langle \psi \mid \delta(p)\delta(p)\psi \rangle$
$\langle \psi' \mid \delta(p)\psi \rangle_{kin}$
$= \langle \psi' \delta(p) \mid \psi \rangle_{kin}$
$=: \langle \psi' \mid \psi \rangle_{phys}$
$F = 0$

Stokes thm

$= -2 \int_{S(r)} dx^a \wedge dx^b$



$H_1 \quad H_3$
$H_2 \quad H_4$

$H_4 = g_2 \underbrace{H_1 H_3 H_2}_{\mathbb{1}} g_2^{-1}$

$\underbrace{H_4}_{\mathbb{1}}$

$\Rightarrow$ Want to impose:

$\delta(F)'' := \delta(H_1)\,\delta(H_2)\,\delta(H_3)$

Problem: Fluxes generates non-compact orbits

$C = p \quad \mathbb{R}$

$\hookrightarrow \delta(p)\,\psi \rightarrow$ not $L^2(\mathbb{R})$

$\langle \delta(p)\psi \,|\, \delta(p)\psi \rangle_{kin} = \infty$

$= \langle \psi \,|\, \delta(p)\delta(p)\,\psi \rangle$

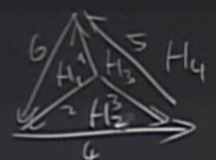
$\langle \psi' \,|\, \delta(p)\,\psi \rangle_{kin}$

$= \langle \psi' \delta(p) \,|\, \psi \rangle_{kin}$

$=: \langle \psi' \,|\, \psi \rangle_{phys}$

Stokes thm

$= -2 \int_{S(r)} dx^a \wedge dx^5$



$H_1 \quad H_3$
$H_2 \quad H_4$

$H_4 = g_2 \underbrace{H_1 H_3 H_2}_{\mathbb{1}} g_2^{-1}$

$\Rightarrow$ Want to impose:

$\delta(F) := \delta(H_1)\,\delta(H_2)\,\delta(H_3)$

Problem: FG-trans generates non-compact orbits

$G = \mathbb{R}$

$\hookrightarrow \delta(p)\,\psi \rightarrow$ not $\in L^2(\mathbb{R})$

$\langle \delta(p)\psi \mid \delta(p)\psi \rangle_{kin} = \infty$

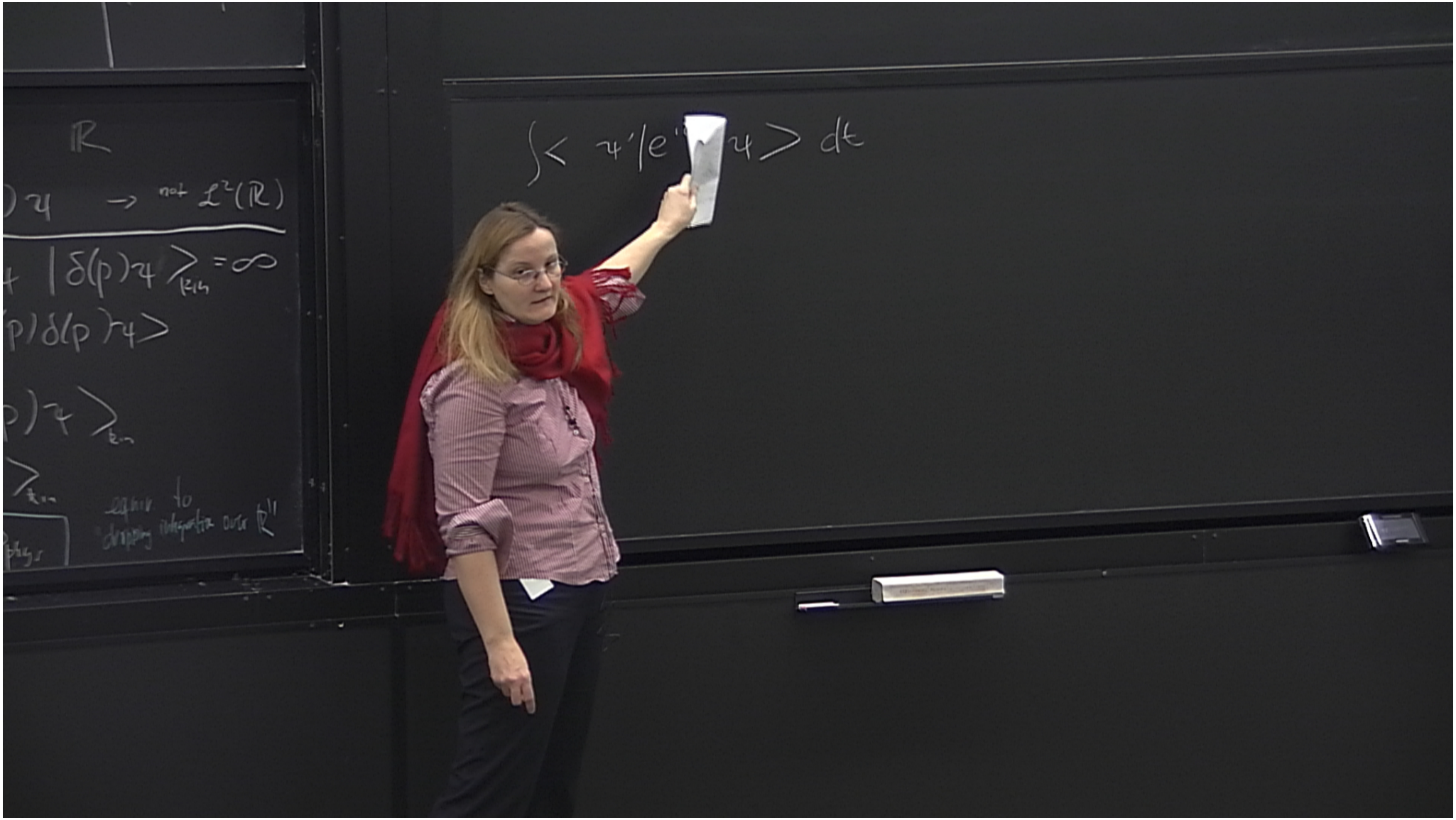
$= \langle \psi \mid \delta(p)\delta(p)\psi \rangle$

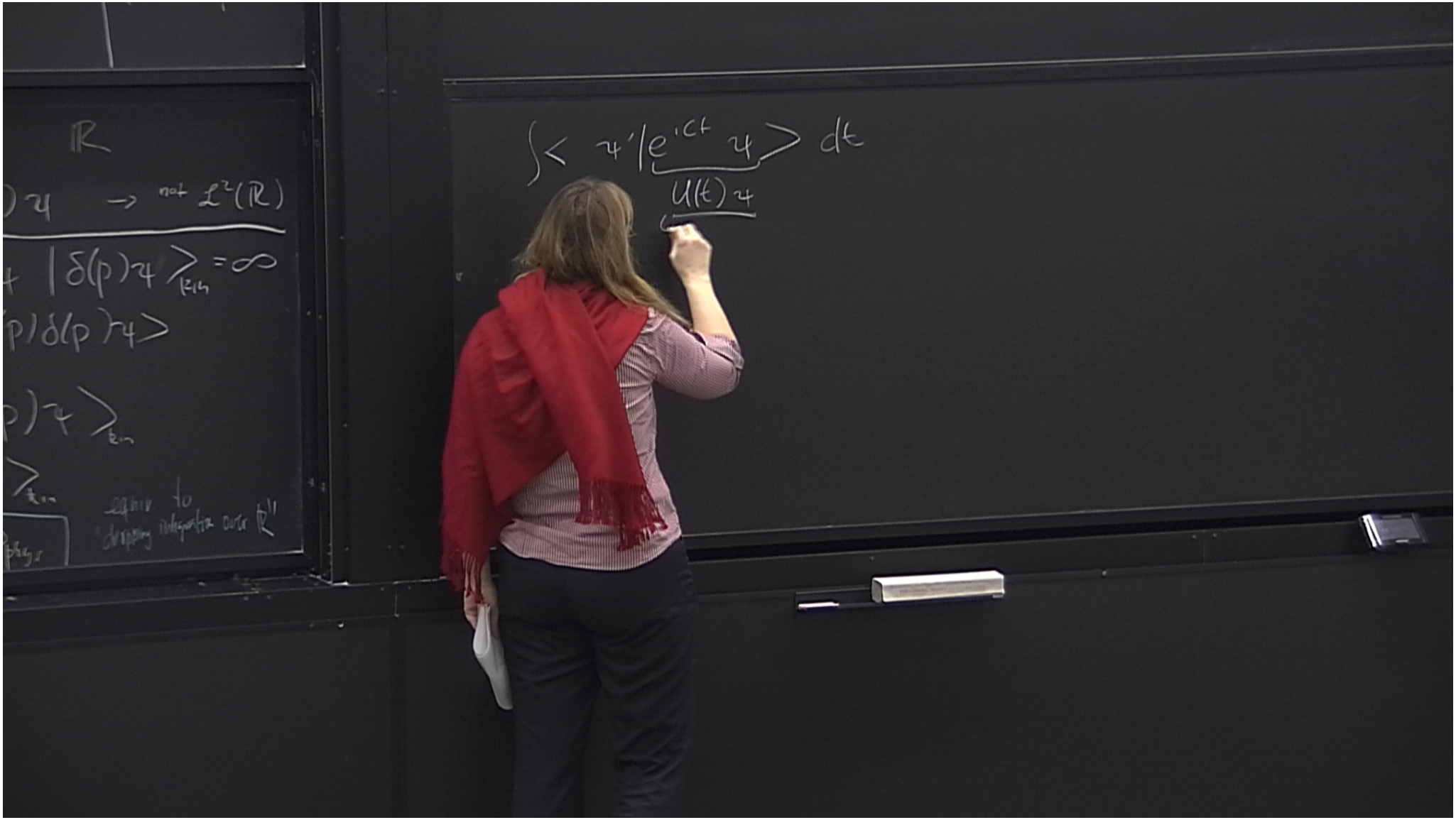
$\langle \psi' \mid \delta(p)\psi \rangle_{kin}$

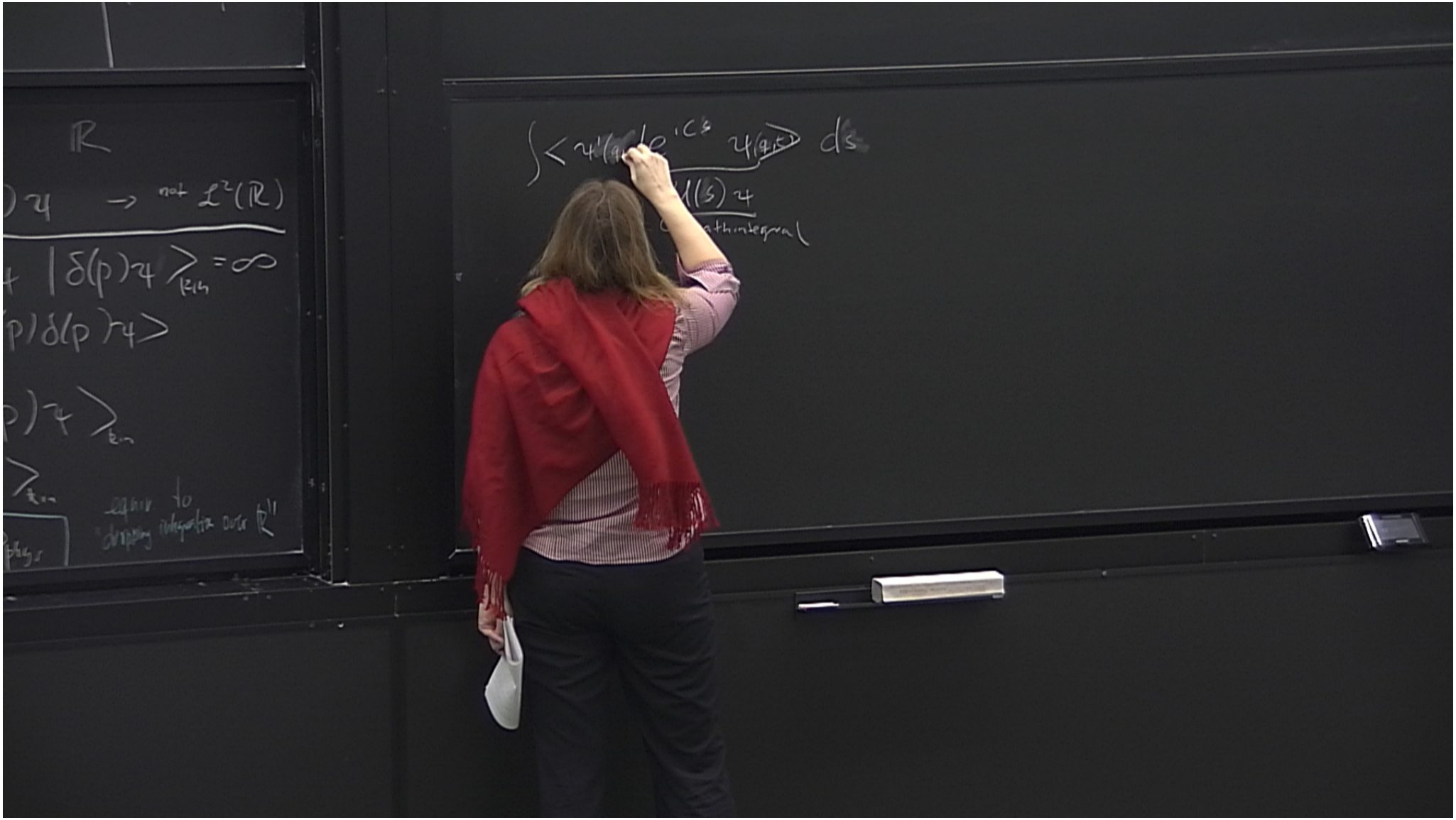
$= \langle \psi' \delta(p) \mid \psi \rangle_{kin}$

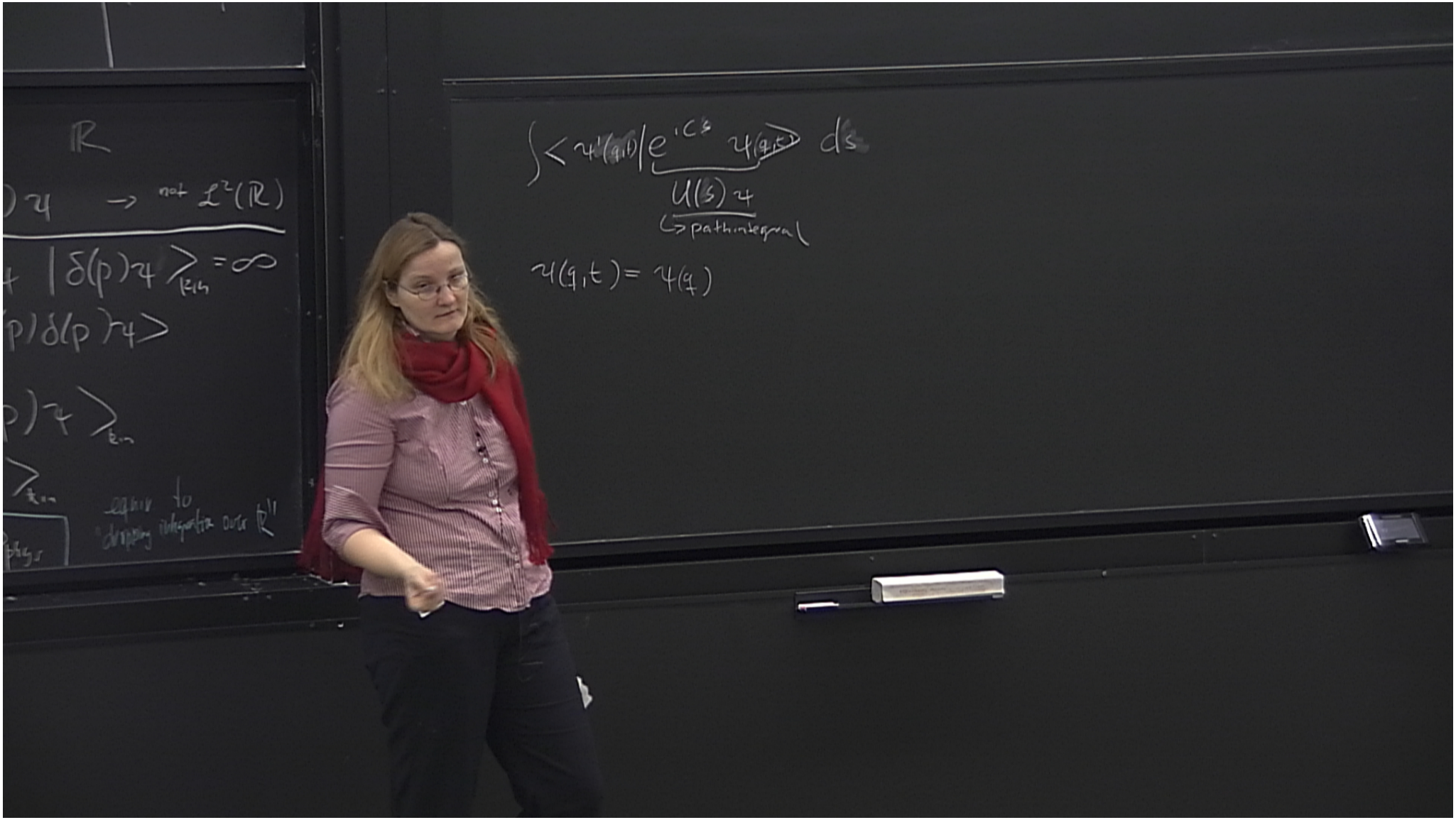
$=: \langle \psi' \mid \psi \rangle_{phys}$

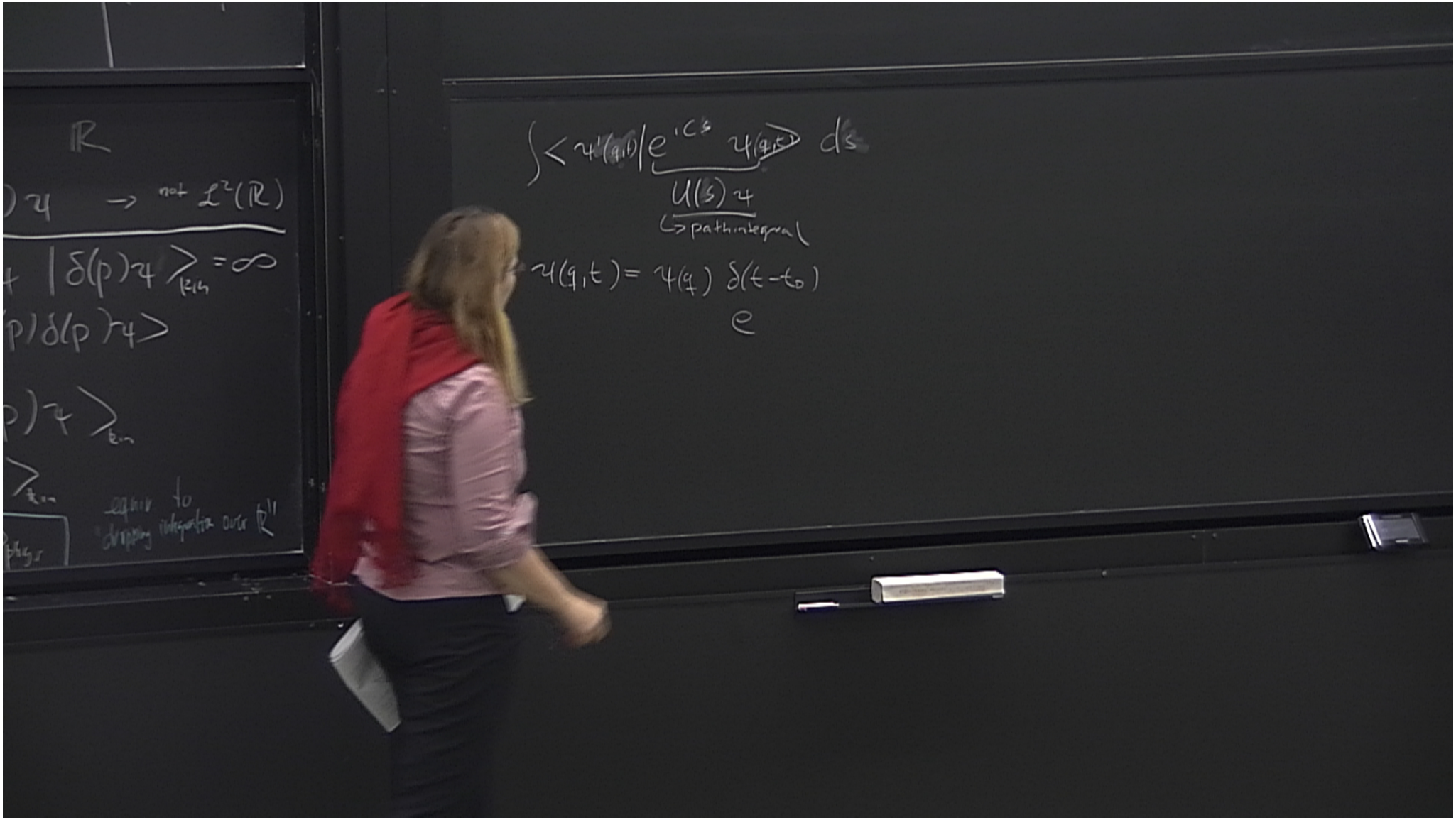
equiv to "dropping integration over $\mathbb{R}$"

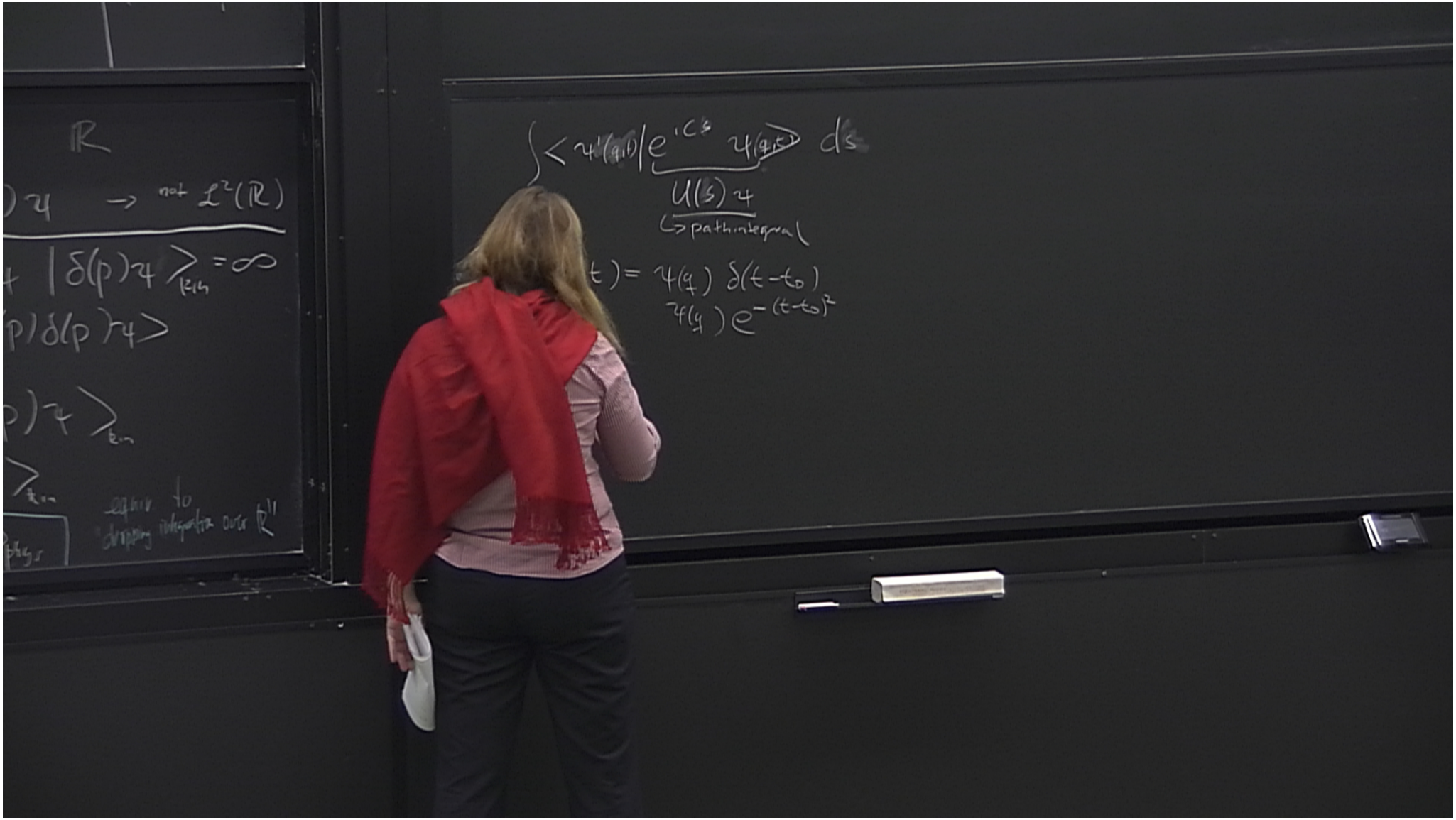

IR
not $\mathcal{L}^2(\mathbb{R})$
$|\delta(p)\psi\rangle = \infty$
$\int \langle \psi' | e^{} | \psi \rangle \, dt$

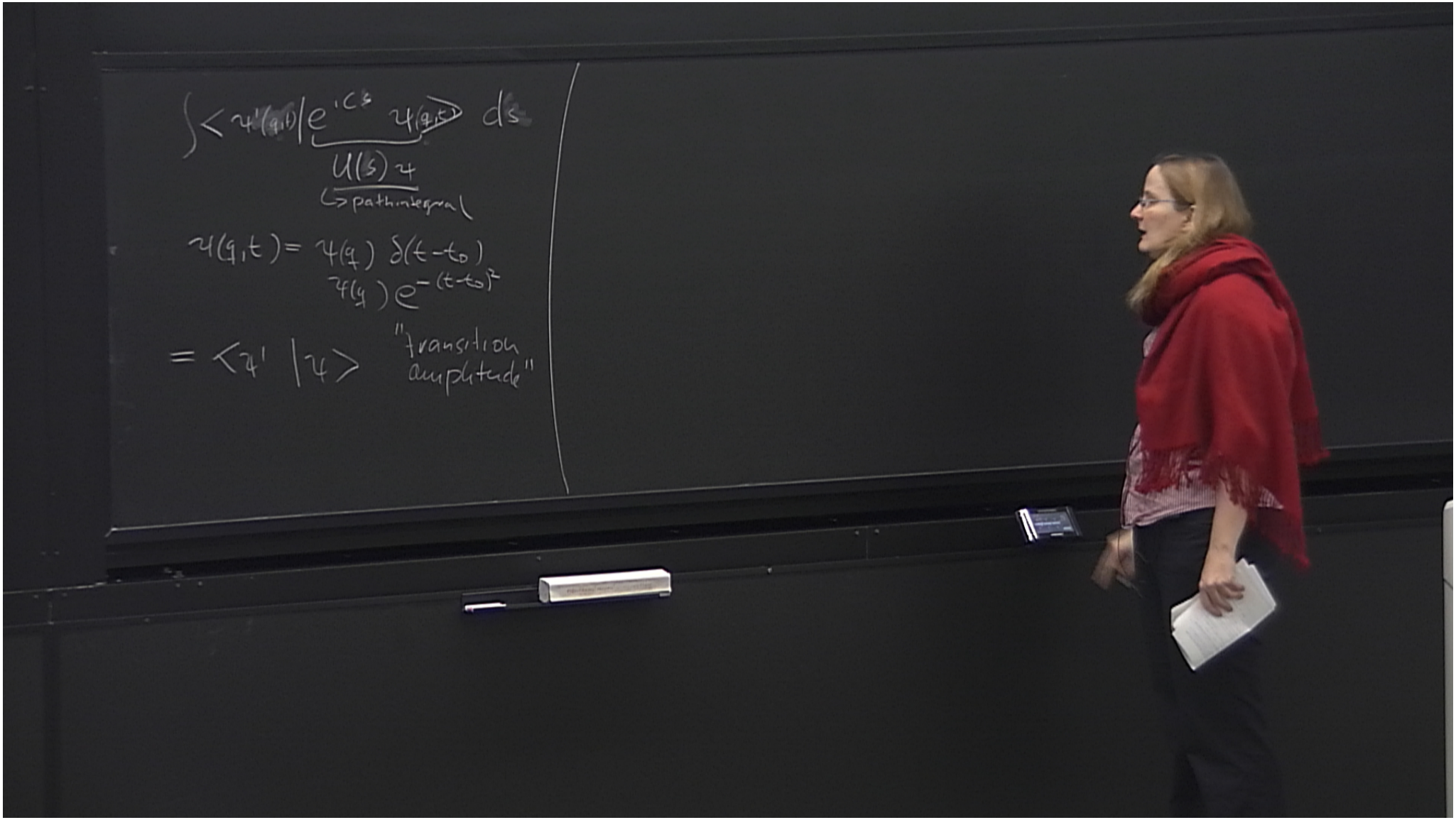

$\int \langle \psi' | e^{iHt} \psi \rangle dt$
$U(t)\psi$

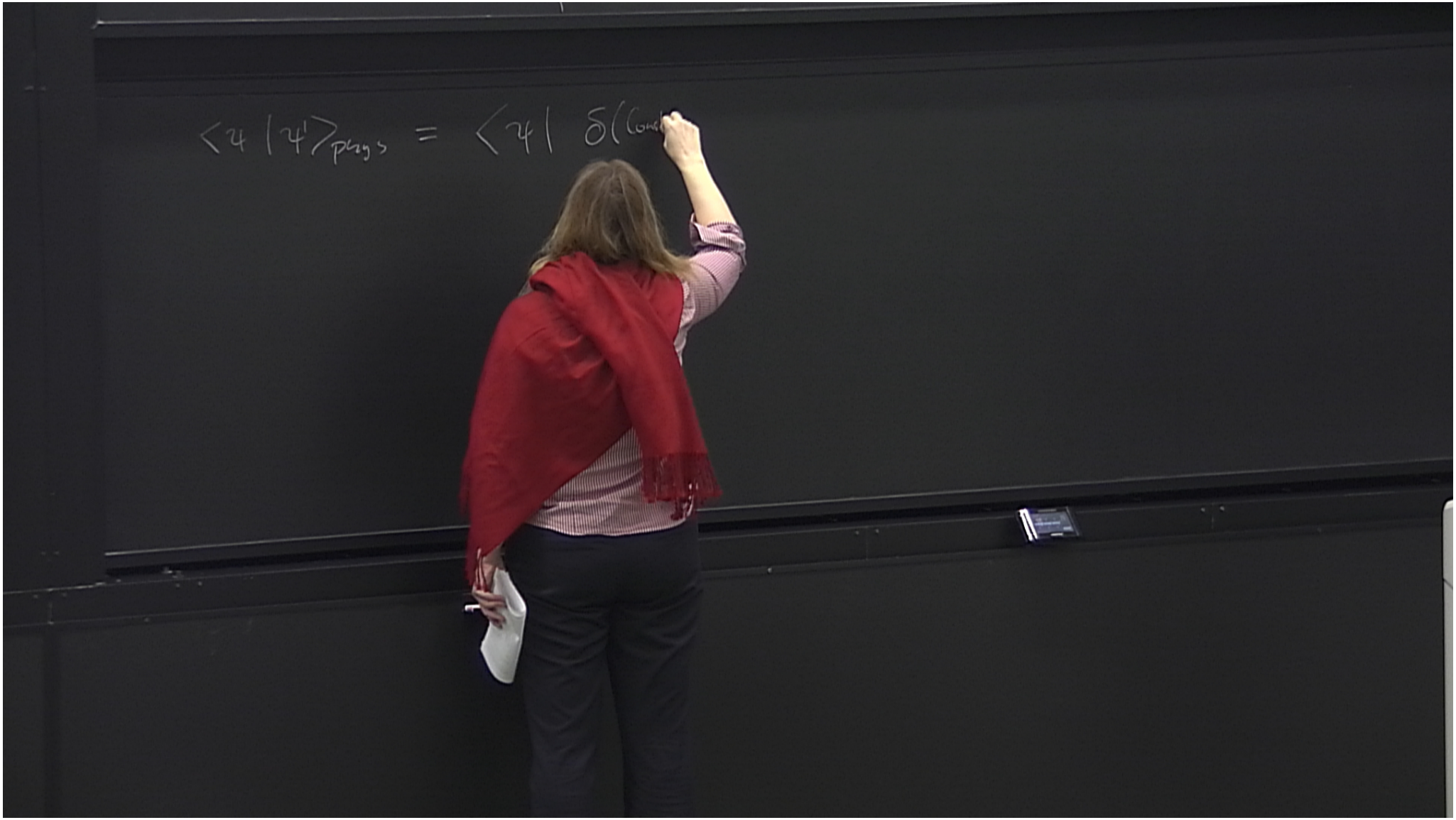

IR
not $\mathcal{L}^2(\mathbb{R})$
$|\delta(p)\psi\rangle_{kin} = \infty$
$\int ds$

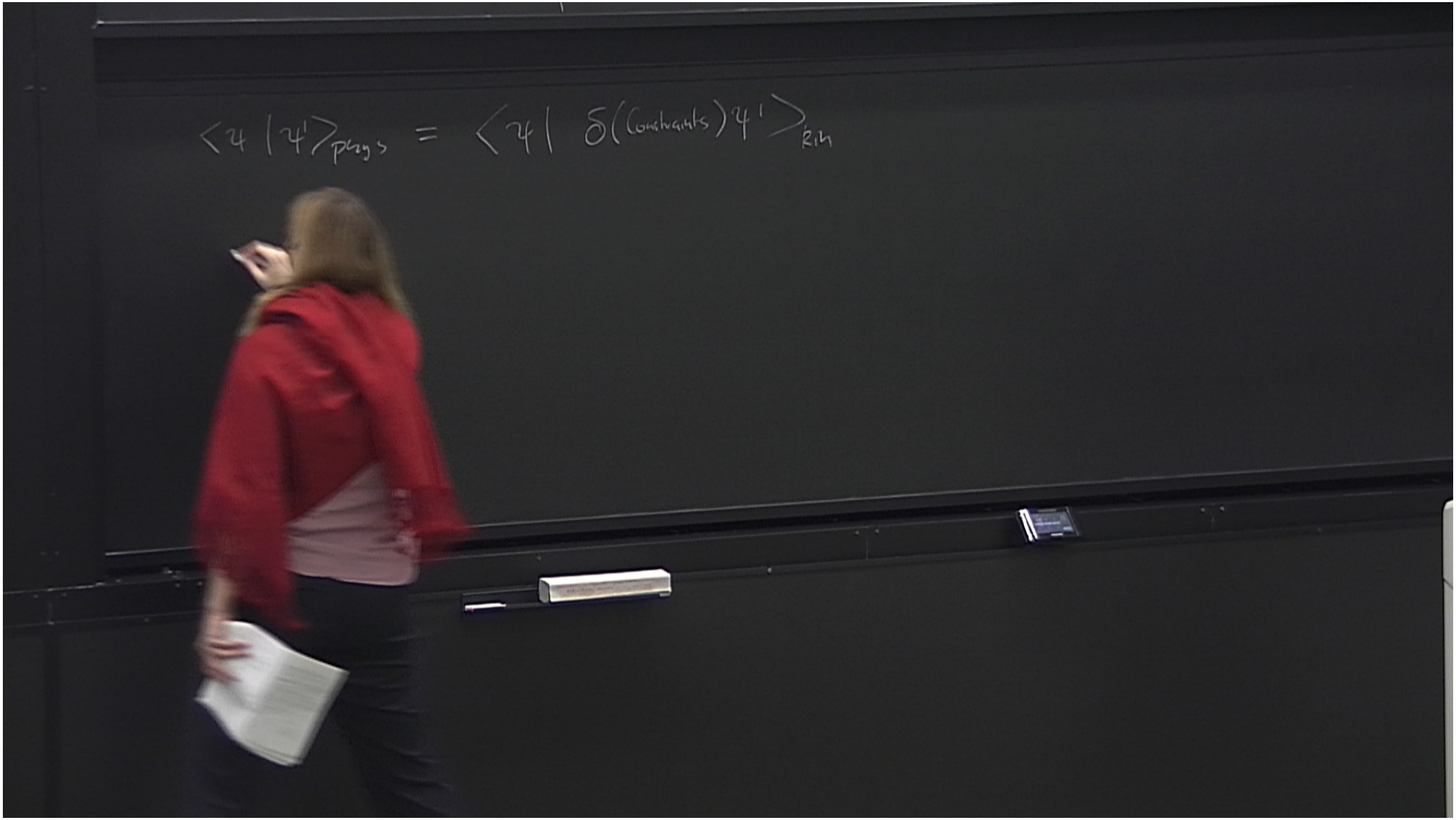

IR
not $\mathcal{L}^2(\mathbb{R})$
$|\delta(p)\psi\rangle_{kin} = \infty$
$\int \langle \psi(q,0) | e^{iCs} \psi(q,s) \rangle ds$
$U(s)\psi$
↳ pathintegral
$\psi(q,t) = \psi(q)$

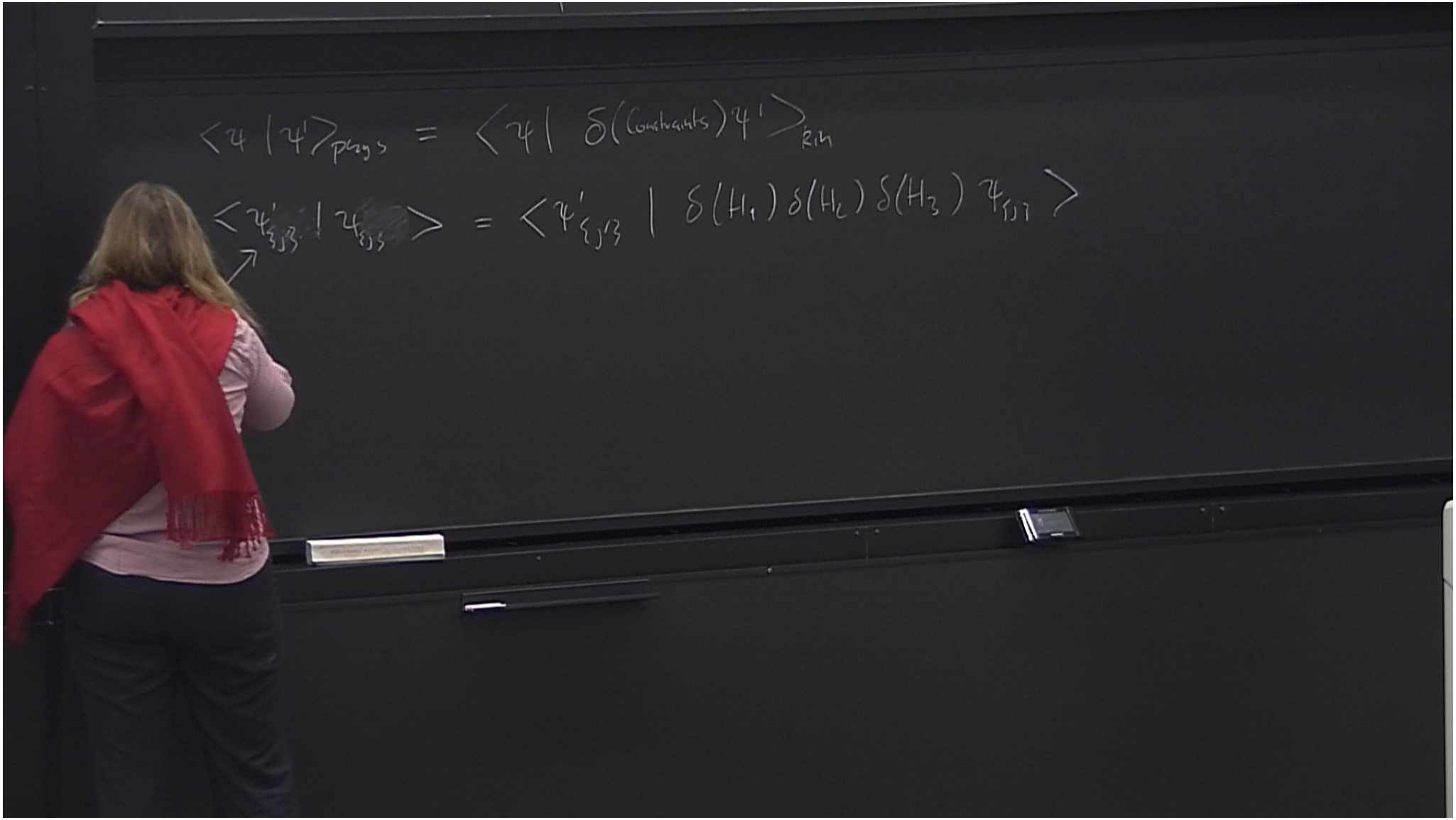

IR
$\mathcal{L}^2(\mathbb{R})$
$\int \langle \psi(q,t) | e^{iCs} \psi(q,s) \rangle ds$
$U(s)\psi$
↳ pathintegral
$\psi(q,t) = \psi(q)\ \delta(t-t_0)$
e

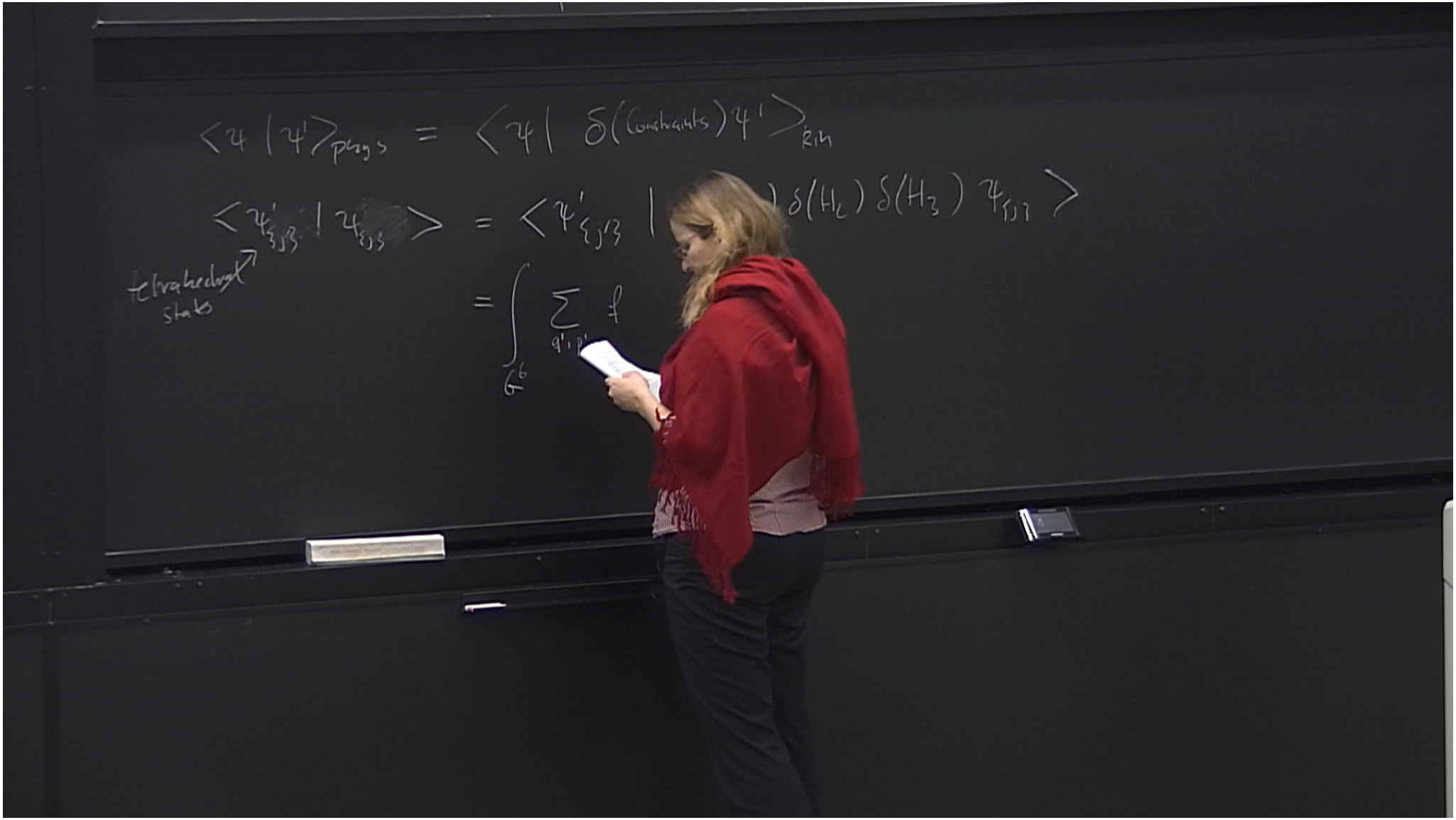

IR
not $\mathcal{L}^2(\mathbb{R})$
$\int \langle \psi(q,0) | e^{iCs} \psi(q,s) \rangle ds$
$U(s)\psi$
↳ pathintegral
$\psi(q) \delta(t-t_0)$
$\psi(q) e^{-(t-t_0)^2}$

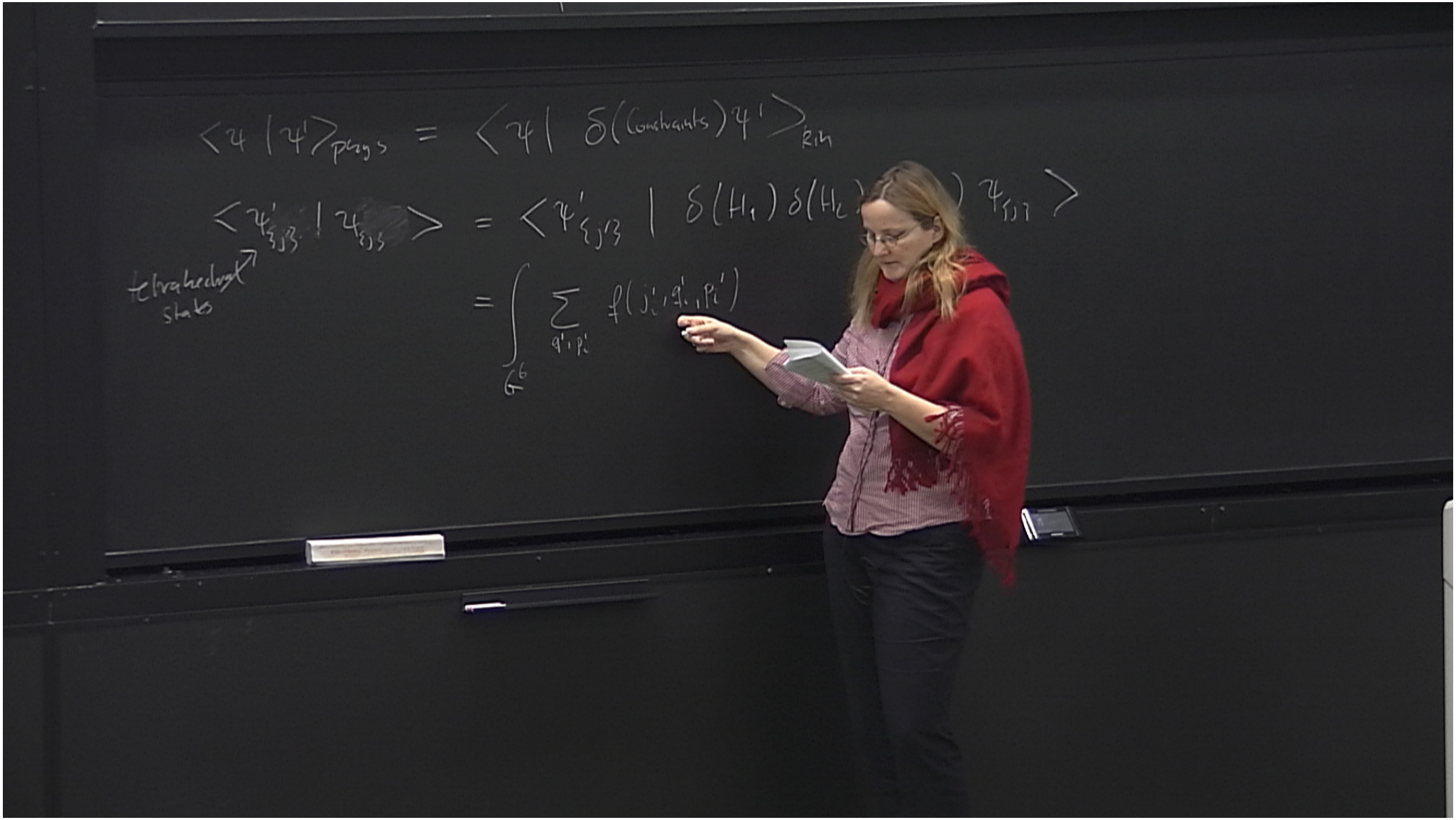

$\int \langle \psi'(q,t) | e^{iCs} \psi(q,t) \rangle ds$
$U(s)\psi$
↳ path integral
$\psi(q,t) = \psi(q)\,\delta(t-t_0)$
$\psi(q)\,e^{-(t-t_0)^2}$
$= \langle \psi' | \psi \rangle$ "transition amplitude"

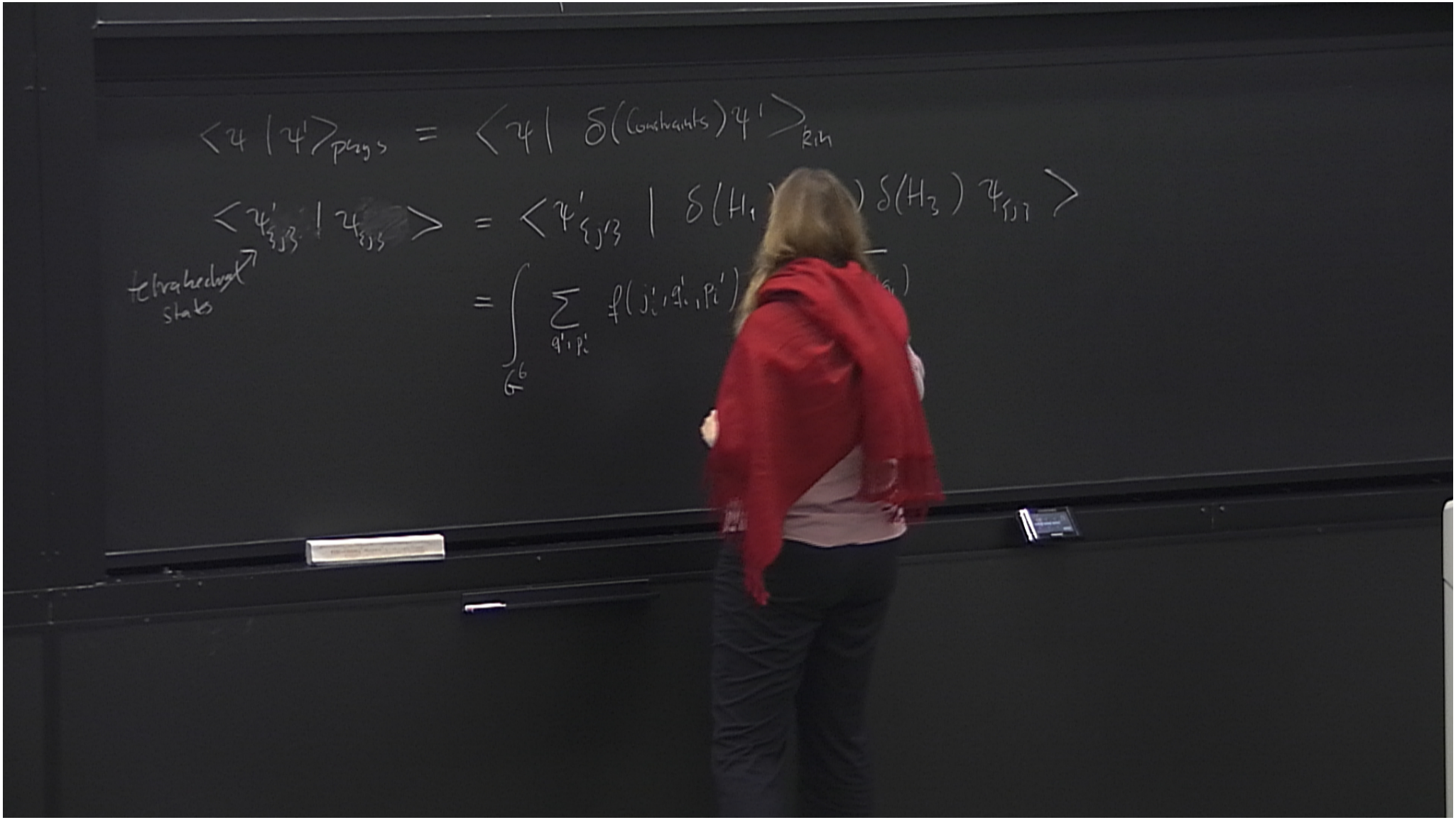

$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta($

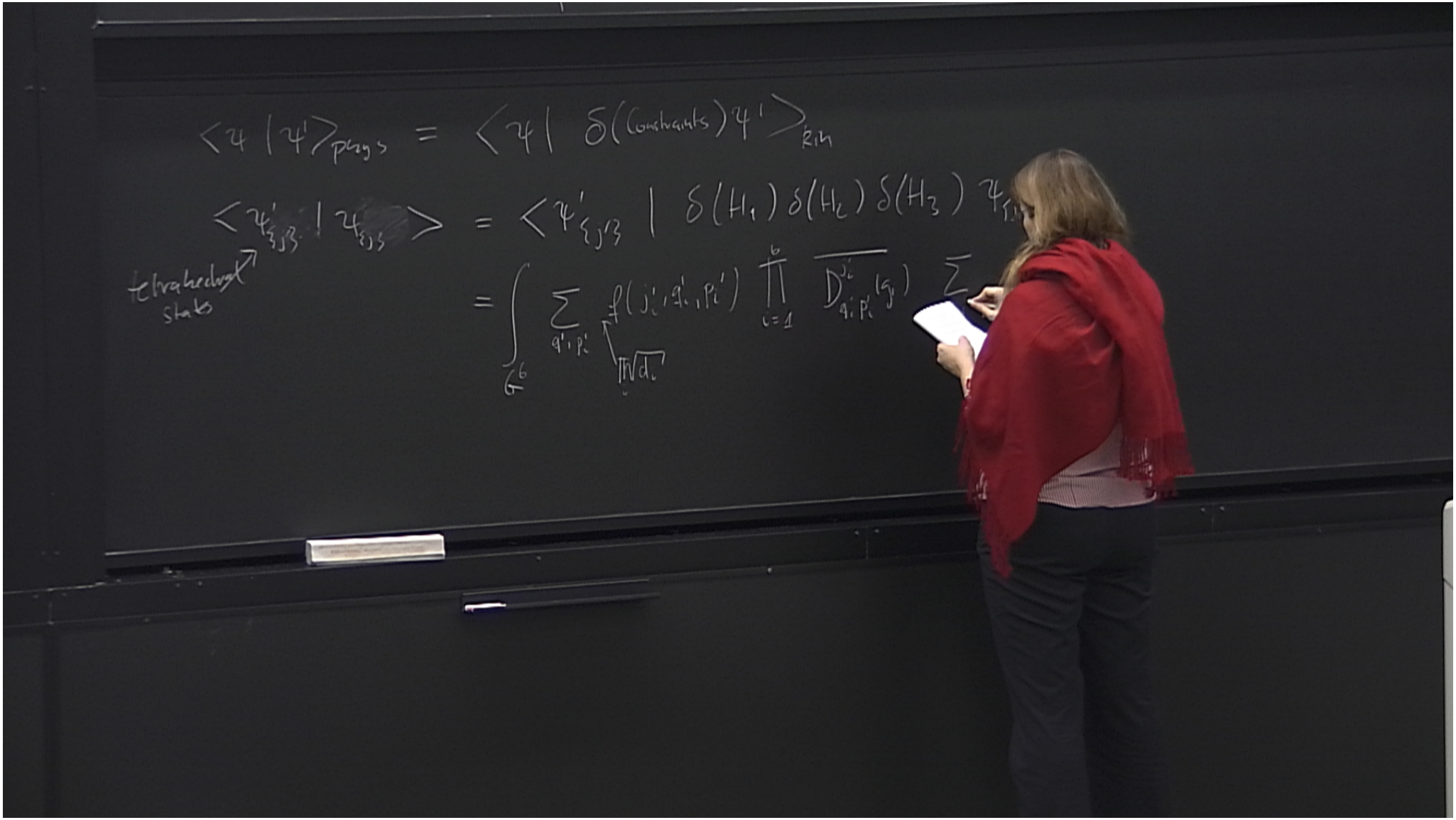

$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{constraints}) \psi' \rangle_{Kin}$

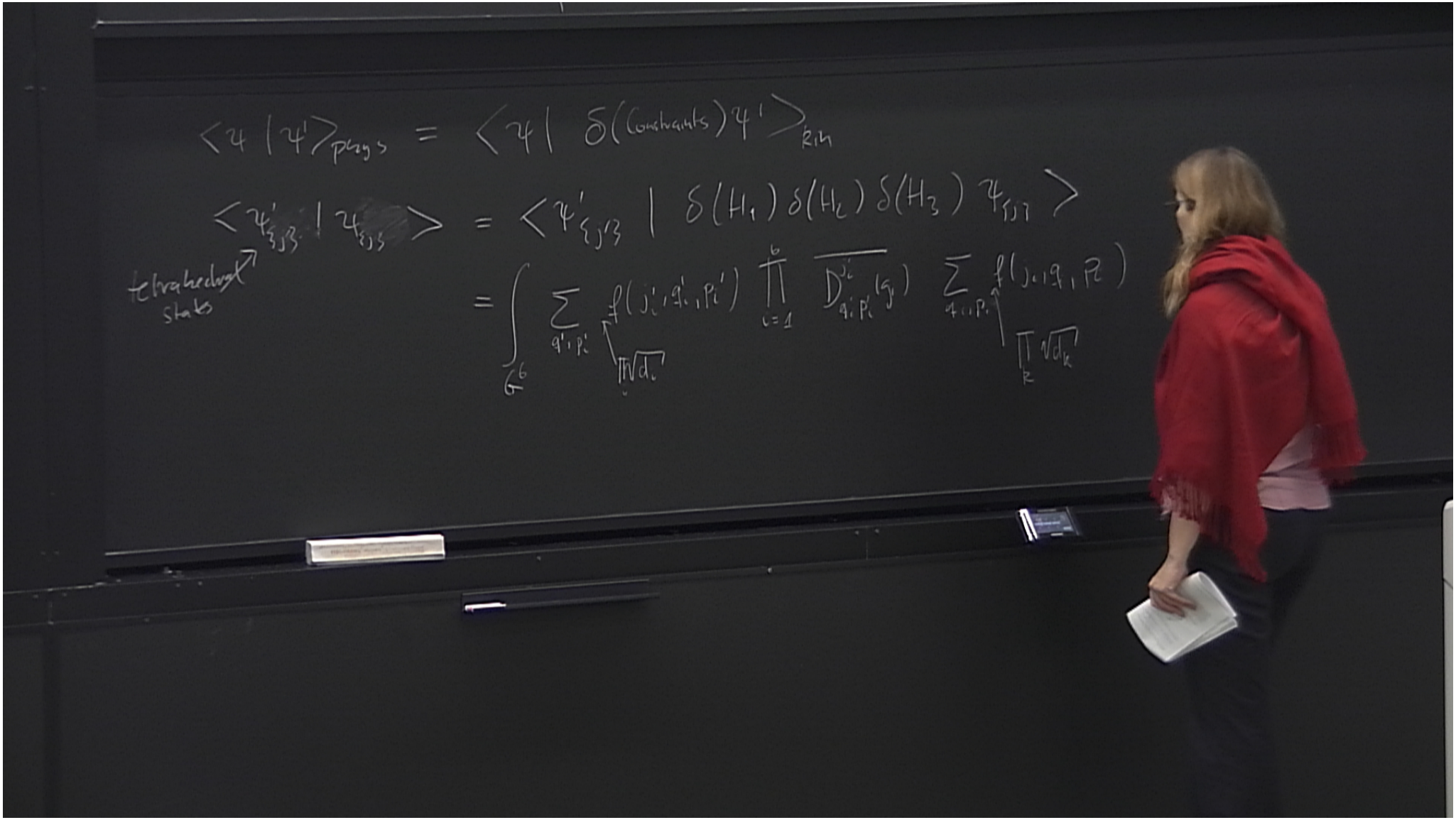

$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{Constraints}) \psi' \rangle_{Kin}$
$\langle \psi'_{\{j\}_3} | \psi_{\{j\}_3} \rangle = \langle \psi'_{\{j\}_3} | \delta(H_1)\delta(H_2)\delta(H_3) \psi_{\{j\}_7} \rangle$

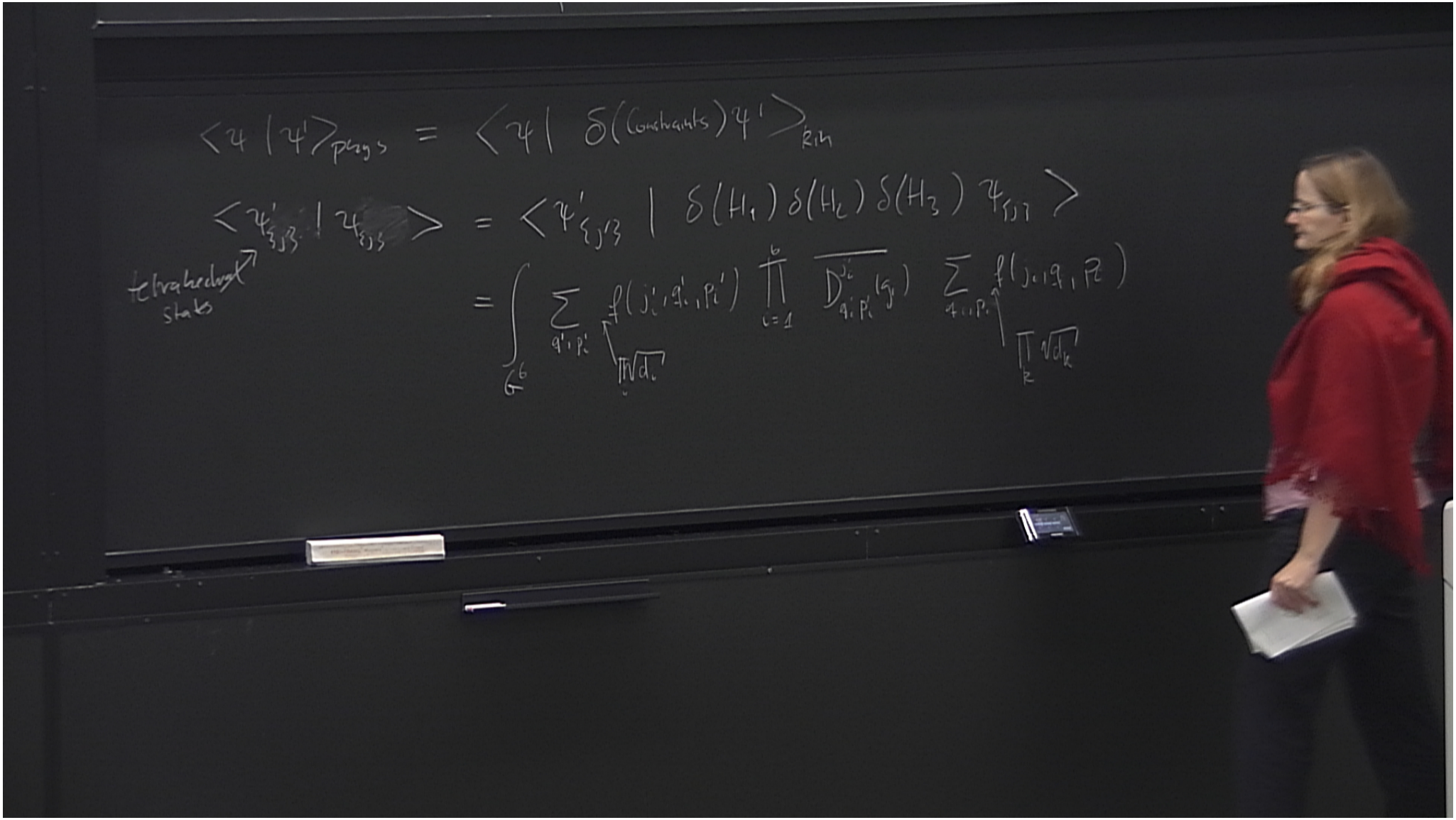

$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{constraints}) \psi' \rangle_{Kin}$
$\langle \psi'_{\{j\}_3} | \psi_{\{j\}_3} \rangle = \langle \psi'_{\{j\}_3} | \delta(H_2)\,\delta(H_3)\,\psi_{\{j\}_7} \rangle$
tetrahedron stabs
$= \int_G \sum_{g', h'} f$

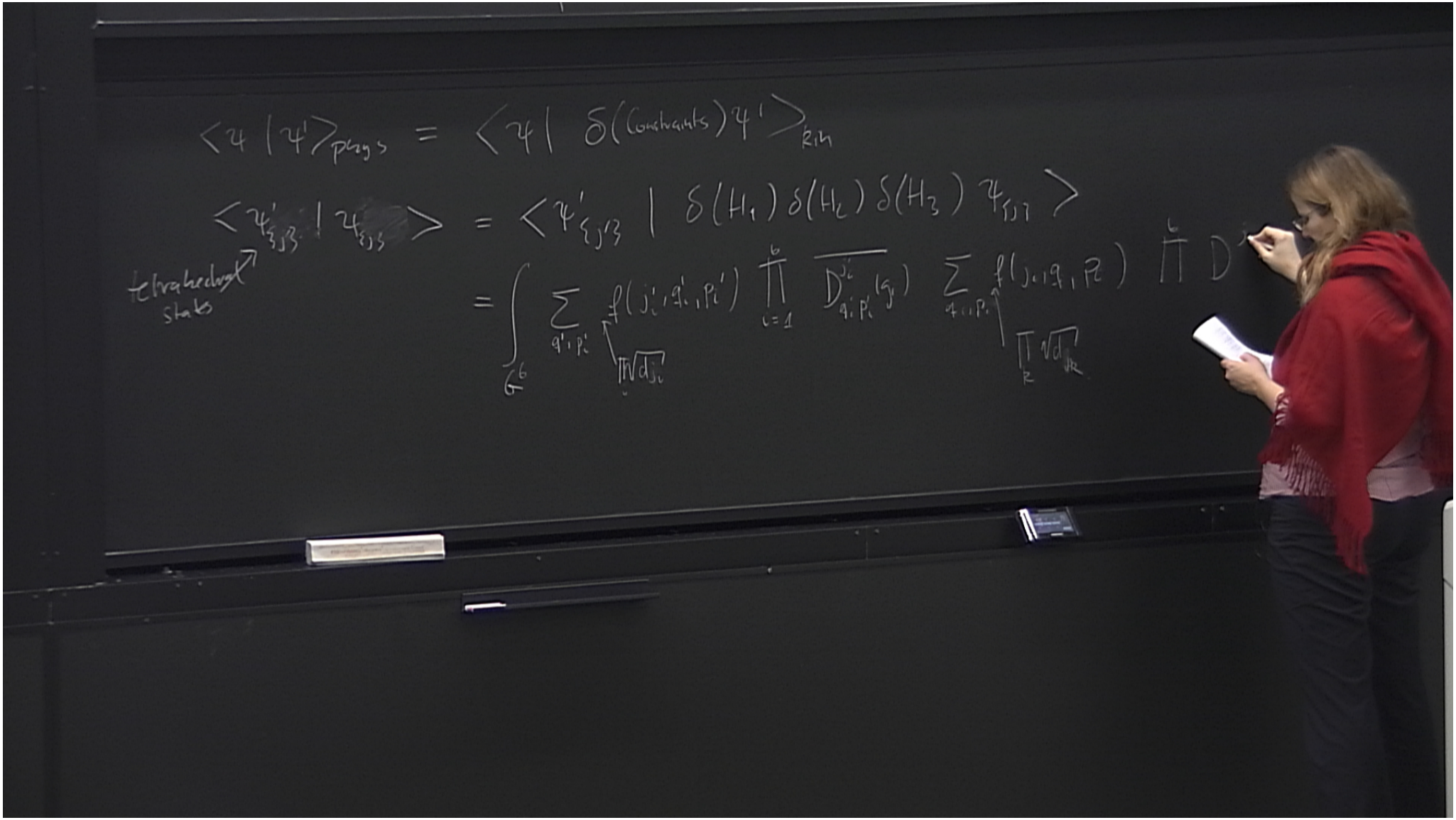

$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{constraints}) \psi' \rangle_{Kin}$
$\langle \psi'_{\{j\}} | \psi_{\{j\}} \rangle = \langle \psi'_{\{j\}} | \delta(H_1) \delta(H_2$
$\psi_{\{j\}} \rangle$
$= \int_G \sum_{g', p_i} f(j_i, g'_i, p'_i)$
tetrahedron
states

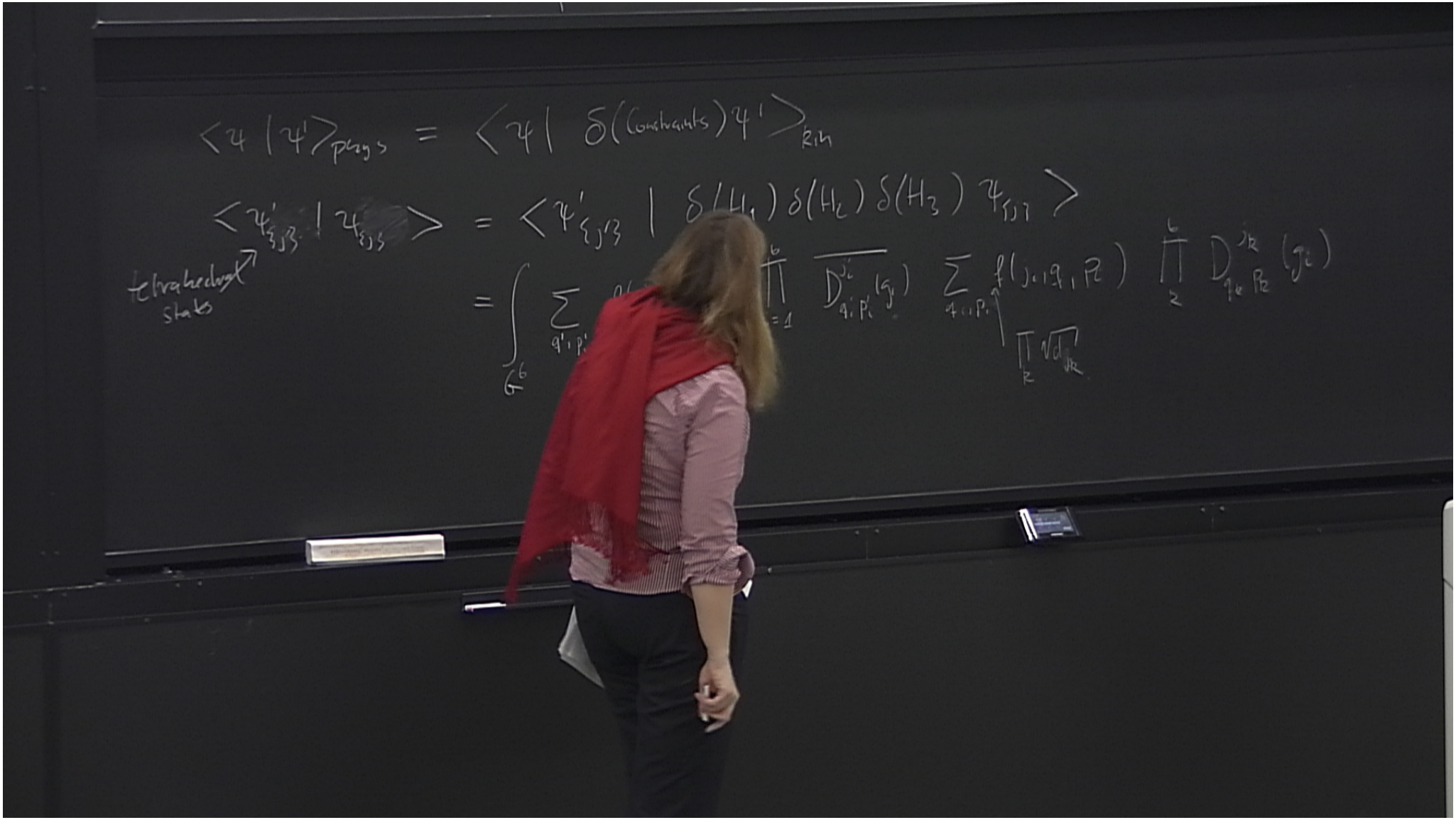

$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{constraints}) \psi' \rangle_{Kin}$
$\langle \psi'_{\{j\}_3} | \psi_{\{j\}_3} \rangle = \langle \psi'_{\{j\}_3} | \delta(H_1$
$\delta(H_3)\, \psi_{\{j\}_7} \rangle$
$= \int_{G^6} \sum_{q_i', p_i'} f(j_i', q_i', p_i')$
tetrahedron
states

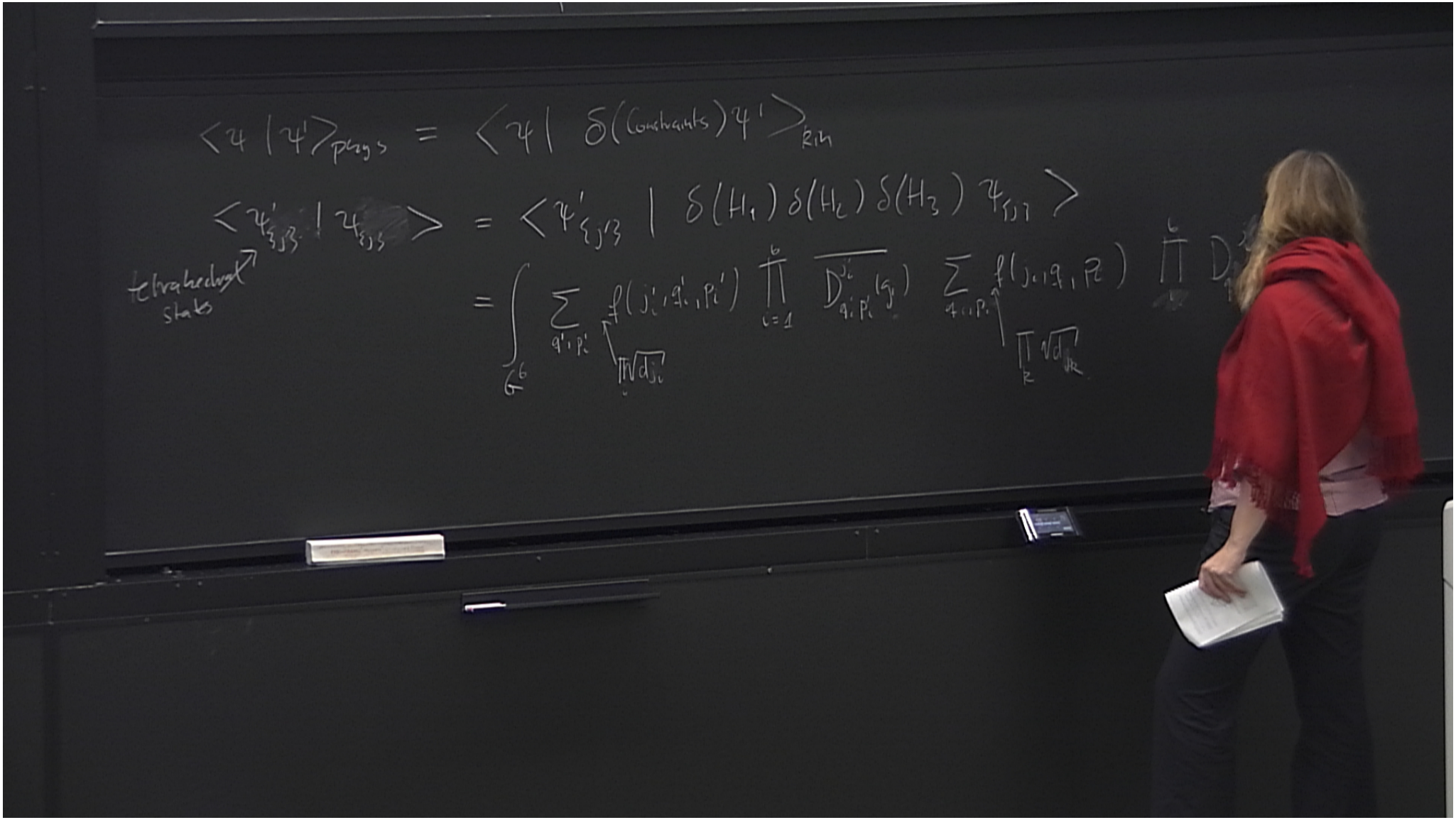

$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{constraints}) \psi' \rangle_{Kin}$
$\langle \psi'_{\{j\}\beta} | \psi_{\{j\}\beta} \rangle = \langle \psi'_{\{j\}\beta} | \delta(H_1)\delta(H_2)\delta(H_3)\psi_{\{j\}\beta} \rangle$
$= \int_G \sum_{q_i', p_i'} f(j_i', q_i', p_i') \prod_{i=1}^{b} \overline{D^{j_i}_{q_i' p_i'}(g)} \sum$
tetrahedron
states

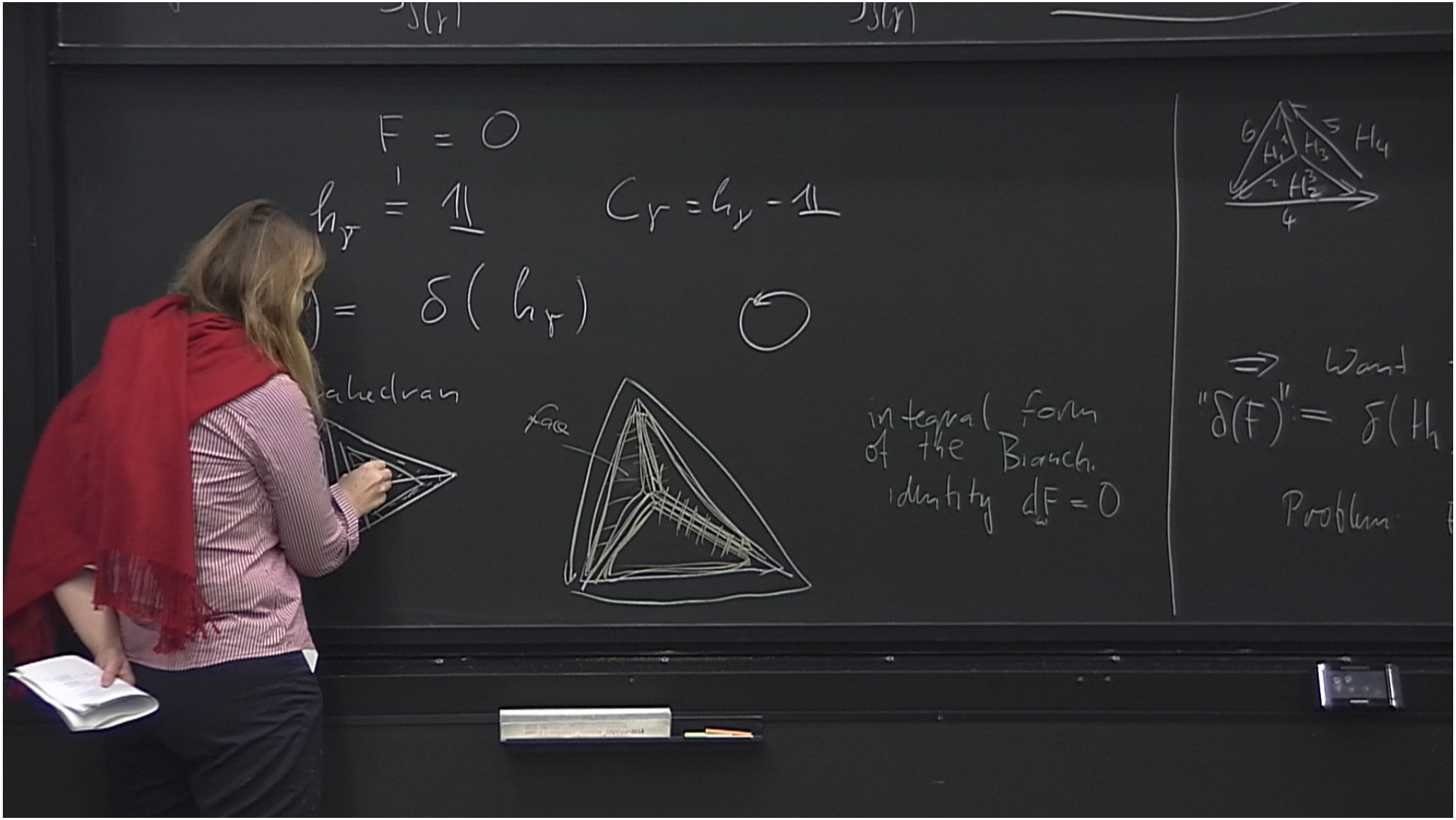

$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{constraints}) \psi' \rangle_{Kin}$
$\langle \psi'_{\{j\}\{i\}} | \psi_{\{j\}\{i\}} \rangle = \langle \psi'_{\{j\}\{i\}} | \delta(H_1)\delta(H_2)\delta(H_3) \psi_{\{j\}\{i\}} \rangle$
tetrahedron stabs
$= \int_{G^6} \sum_{q'_i, p'_i} \frac{f(j'_i, q'_i, p'_i)}{\prod_i \sqrt{d_{j'_i}}} \prod_{i=1}^{6} \overline{D^{j'_i}_{q'_i p'_i}(g_i)} \sum_{q_i, p_i} \frac{f(j_i, q_i, p_i)}{\prod_k \sqrt{d_{j_k}}}$


$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{constraints}) \psi' \rangle_{Kin}$
$\langle \psi'_{\{j\}\beta} | \psi_{\{j\}\alpha} \rangle = \langle \psi'_{\{j\}\beta} | \delta(H_1)\delta(H_2)\delta(H_3) \psi_{\{j\}\gamma} \rangle$
$= \int_G \sum_{q_i' p_i'} \frac{f(j_i', q_i', p_i')}{\prod_i \sqrt{d_{j_i}}} \prod_{i=1}^{b} \overline{D^{j_i}_{q_i' p_i'}(g)} \sum_{q_i p_i} \frac{f(j_i, q_i, p_i)}{\prod_k \sqrt{d_{j_k}}}$
tetrahedron
state


$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{constraints}) \psi' \rangle_{Kin}$
$\langle \psi'_{\{j\}\{\beta\}} | \psi_{\{j\}\{\alpha\}} \rangle = \langle \psi'_{\{j\}\{\beta\}} | \delta(H_1)\delta(H_2)\delta(H_3) \psi_{\{j\}\{\alpha\}} \rangle$
tetrahedron
Stabs
$= \int_{G^6} \sum_{q_i, p_i} f(j'_i, q'_i, p'_i) \prod_{i=1}^{6} \overline{D^{j_i}_{q'_i p'_i}(g_i)} \sum_{q_i, p_i} f(j_i, q_i, p_i) \prod_{i=1}^{6} D$
$\prod_i \sqrt{d_{j_i}}$
$\prod_i \sqrt{d_{j_i}}$


$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{constraints}) \psi' \rangle_{Kin}$
$\langle \psi'_{\{j\}_3} | \psi_{\{j\}_3} \rangle = \langle \psi'_{\{j\}_3} | \delta(H_1)\delta(H_2)\delta(H_3) \psi_{\{j\}_3} \rangle$
tetrahedron
states
$= \int_{G^6} \sum_{q',P'} \prod_{e=1}^{6} \overline{D^{j_e}_{q'_e P'_e}(g_e)} \sum_{q_e,P_e} f(j_e, q_e, P_e) \prod_{e} D^{j_e}_{q_e P_e}(g_e)$
$\prod_e \sqrt{d_{j_e}}$

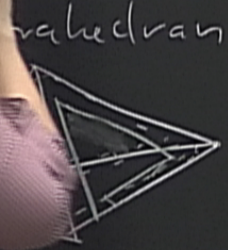

$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{constraints}) \psi' \rangle_{Kin}$
$\langle \psi'_{\{j\}\beta} | \psi_{\{j\}\alpha} \rangle = \langle \psi'_{\{j\}\beta} | \delta(H_1)\delta(H_2)\delta(H_3) \psi_{\{j\}\gamma} \rangle$
$= \int_G \sum_{q_i', p_i'} f(j_i', q_i', p_i') \prod_{i=1}^{b} \overline{D^{j_i}_{q_i' p_i'}(g)} \sum_{q_i, p_i} f(j_i, q_i, p_i) \prod_{i=1}^{b} D$
$\prod_i \sqrt{d_{j_i}}$
$\prod_k \sqrt{d_{j_k}}$
kinematical states

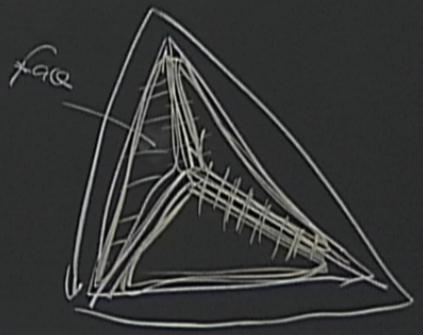

F = 0
$h_\gamma = \mathbb{1}$
$C_\gamma = h_\gamma - \mathbb{1}$
$= \delta(h_\gamma)$
integral form of the Bianchi identity $d_\omega F = 0$
$\Rightarrow$ Want
"$\delta(F)$" $= \delta(H_4$
Problem

$F = 0$

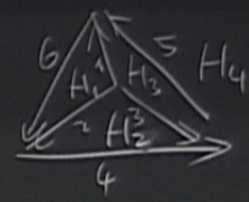
$h_\gamma = \mathbb{1}$ $\quad C_\gamma = h_\gamma - \mathbb{1}$

$\delta(h_\gamma)$





integral form of the Bianchi identity $dF = 0$



$\Rightarrow$ Want

"$\delta(F)$" $= \delta(H_$

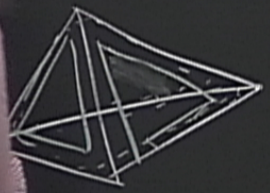
Problem

$F = 0$

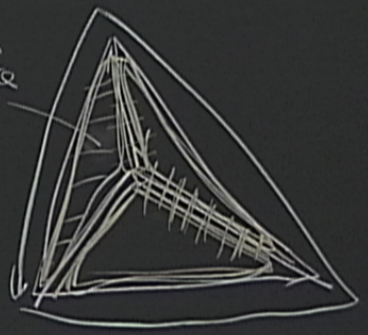
$h_\gamma = 1\!\!1$ $\quad C_\gamma = h_\gamma - 1\!\!1$

$\chi(C_\gamma) = \delta(h_\gamma)$

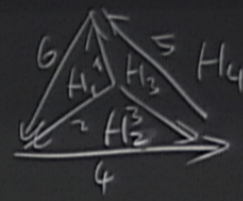
tetrahedron



face



integral form of the Bianchi identity $dF = 0$



$\Rightarrow$ Want

"$\delta(F)$" $= \delta(H_4$

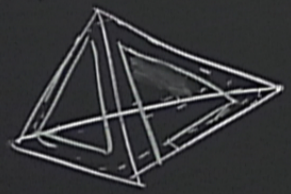
Problem

$F = 0$

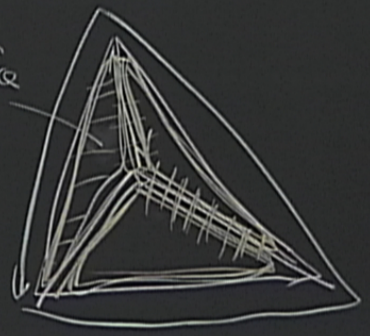
$h_\gamma \overset{!}{=} 1\!\!1 \qquad C_\gamma = h_\gamma - 1\!\!1$

$\widehat{\delta}(C_\gamma) = \delta(h_\gamma)$

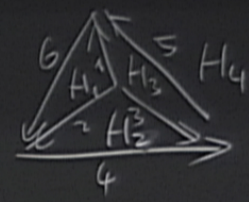
al tetrahedran



face



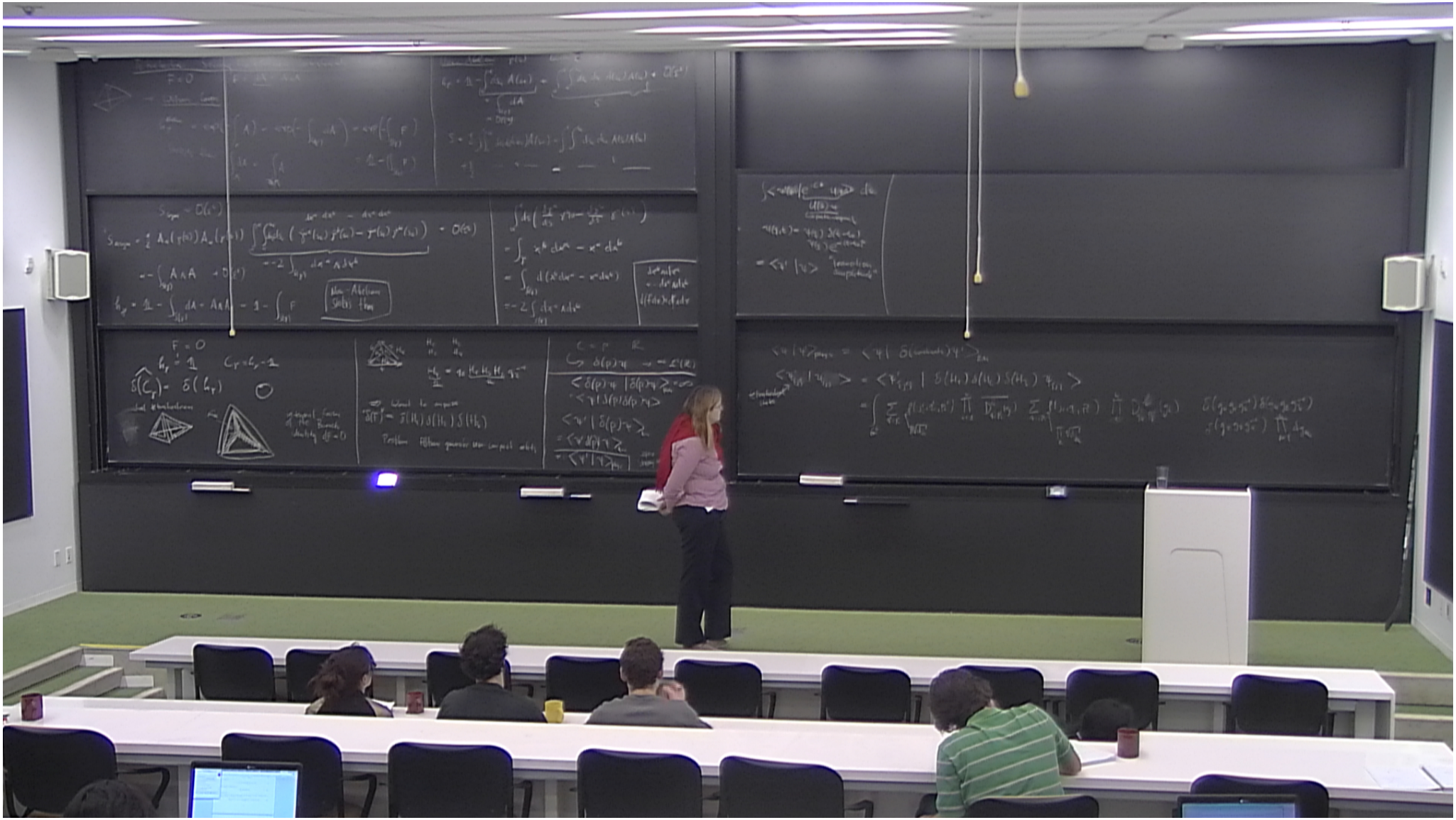
integral form of the Bianchi identity $d_\omega F = 0$

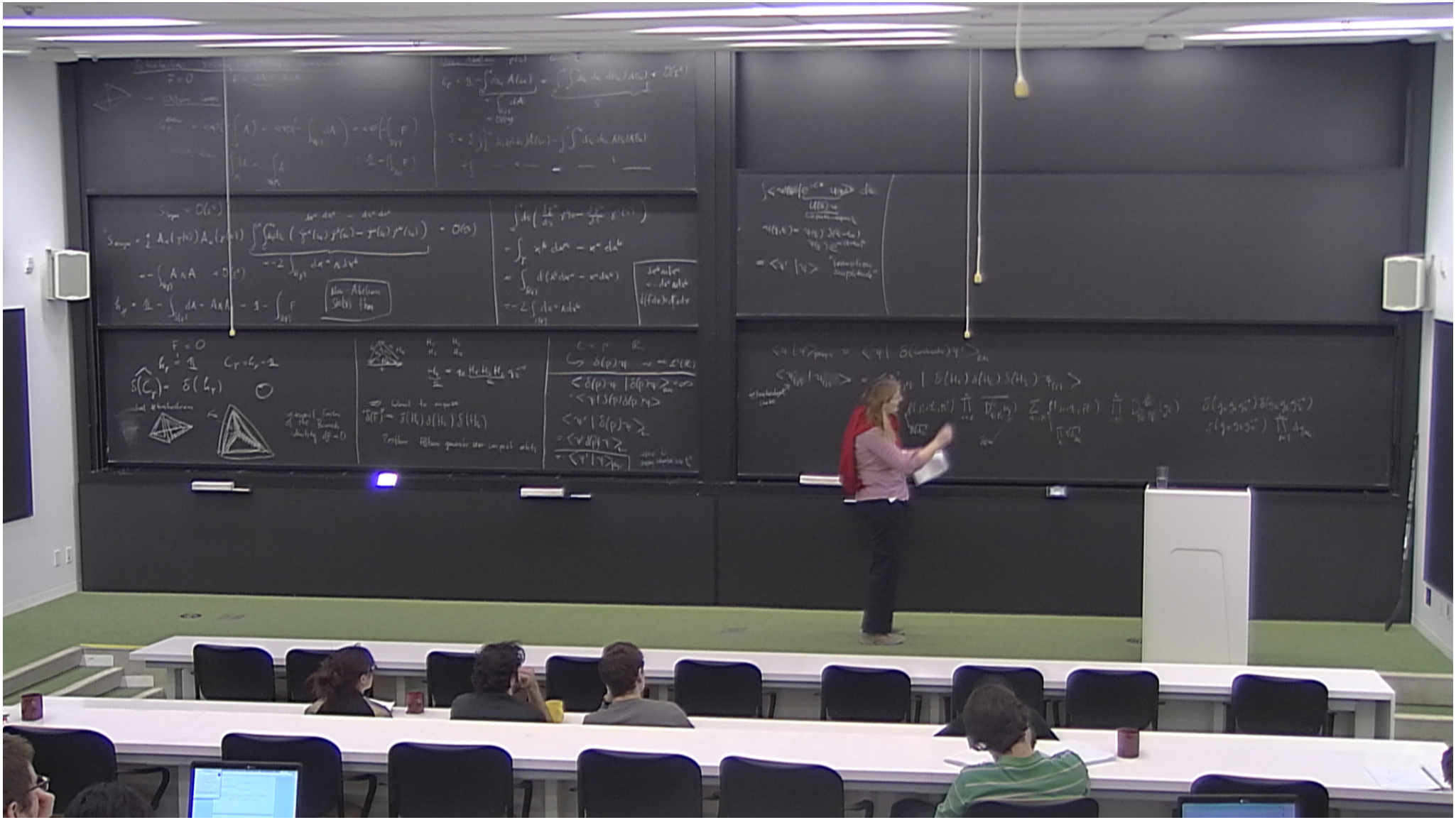


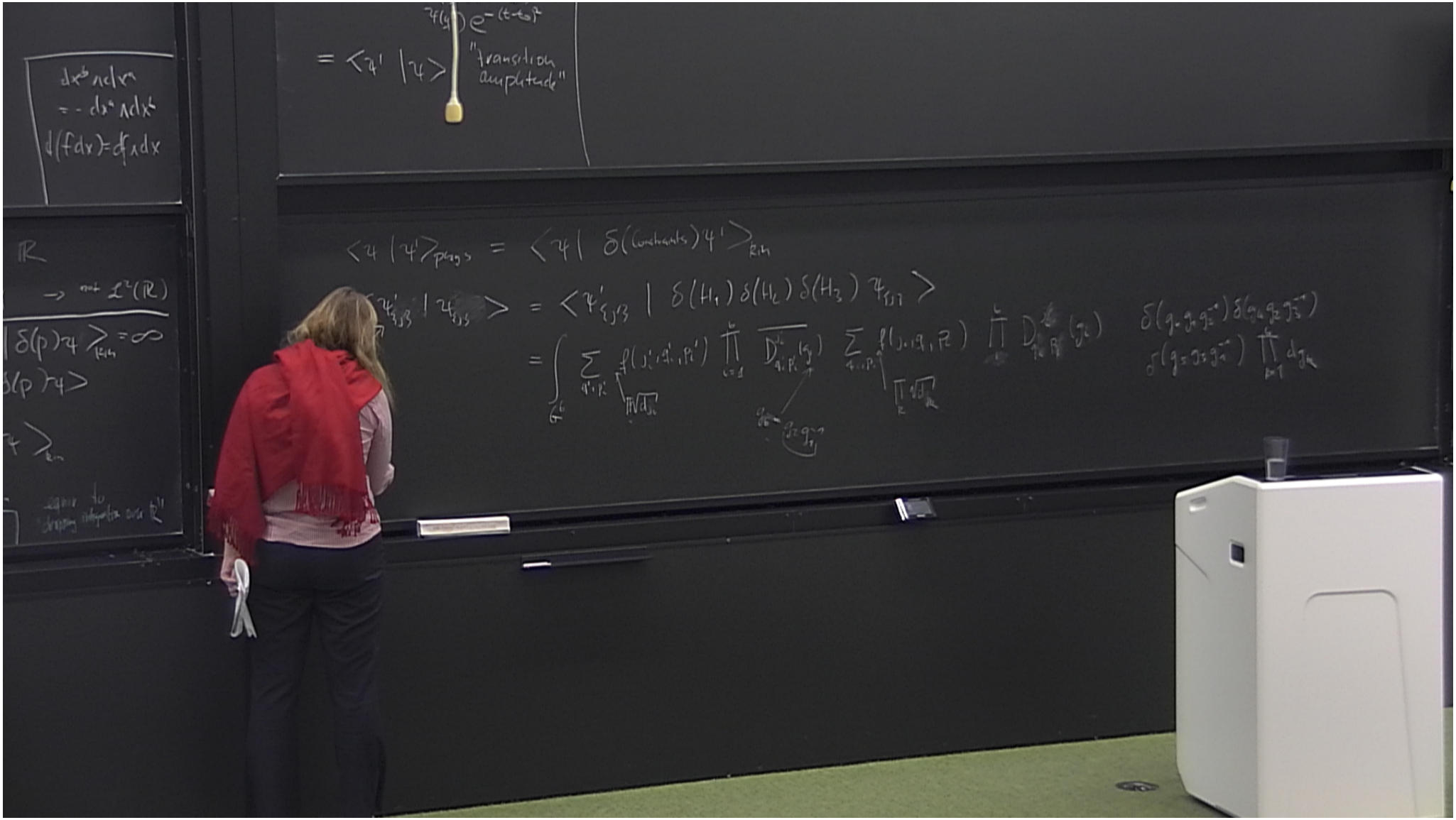
$\Rightarrow$ Want

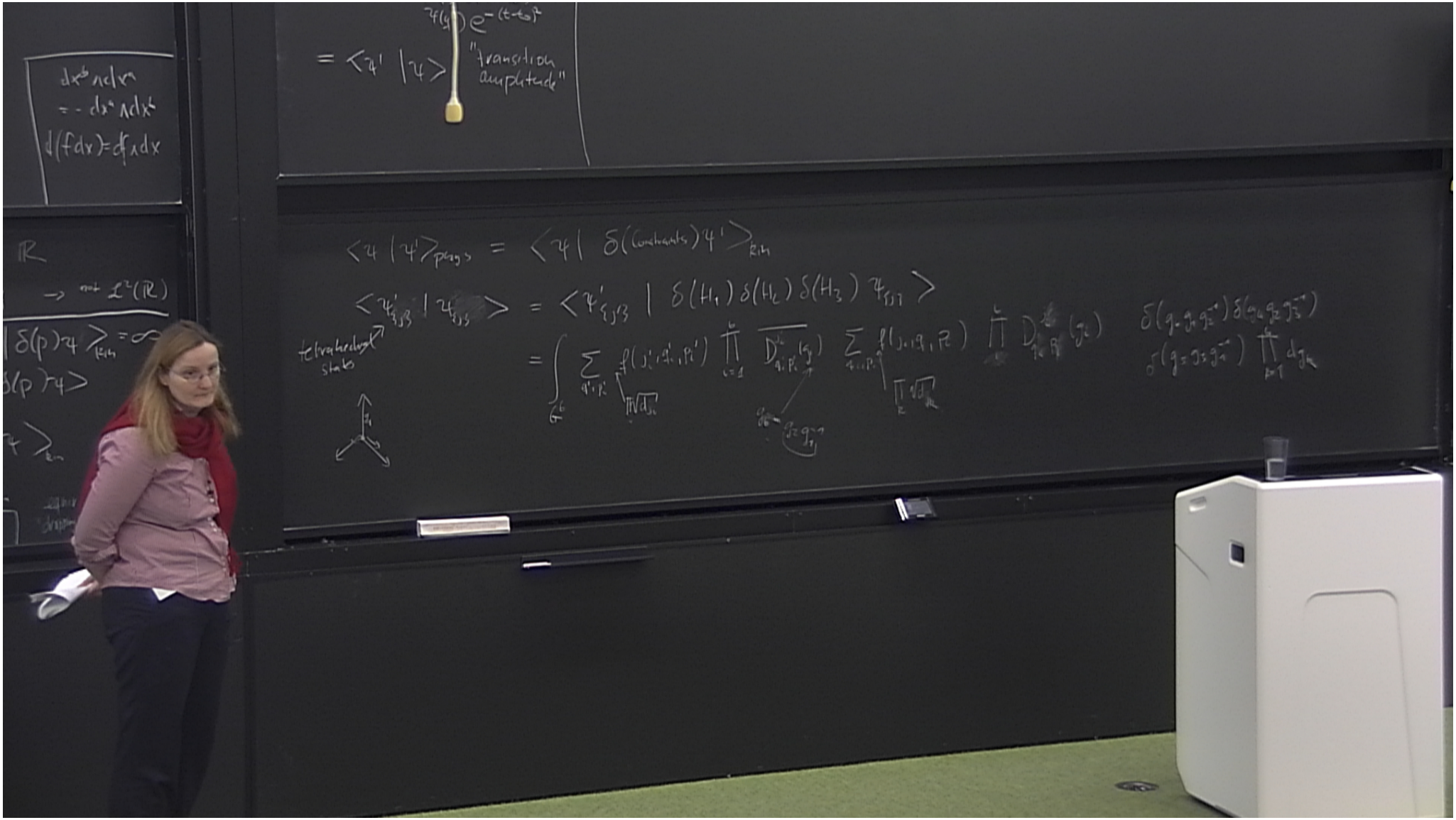
"$\delta(F)$" $= \delta(H_4$

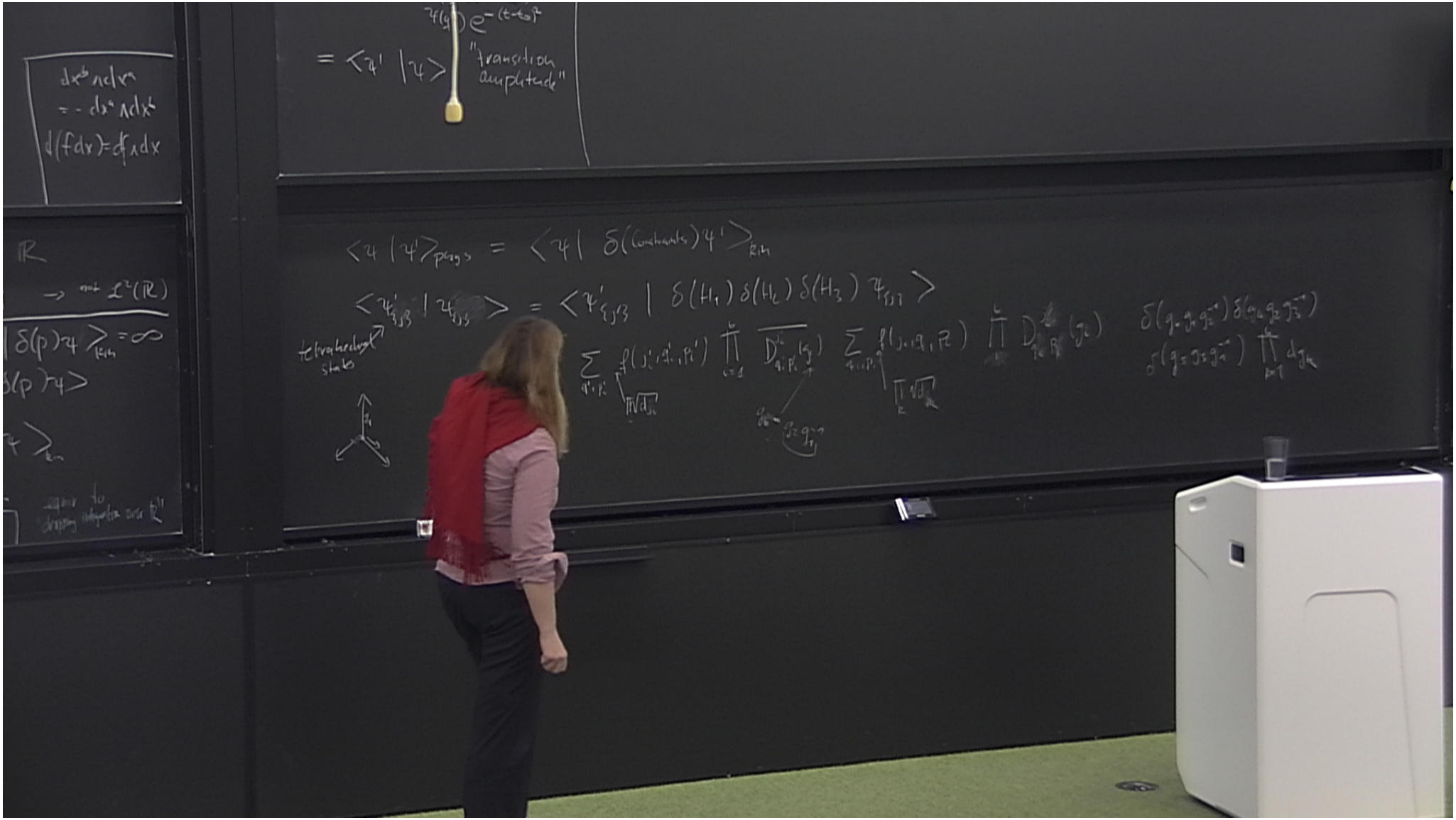
Problem

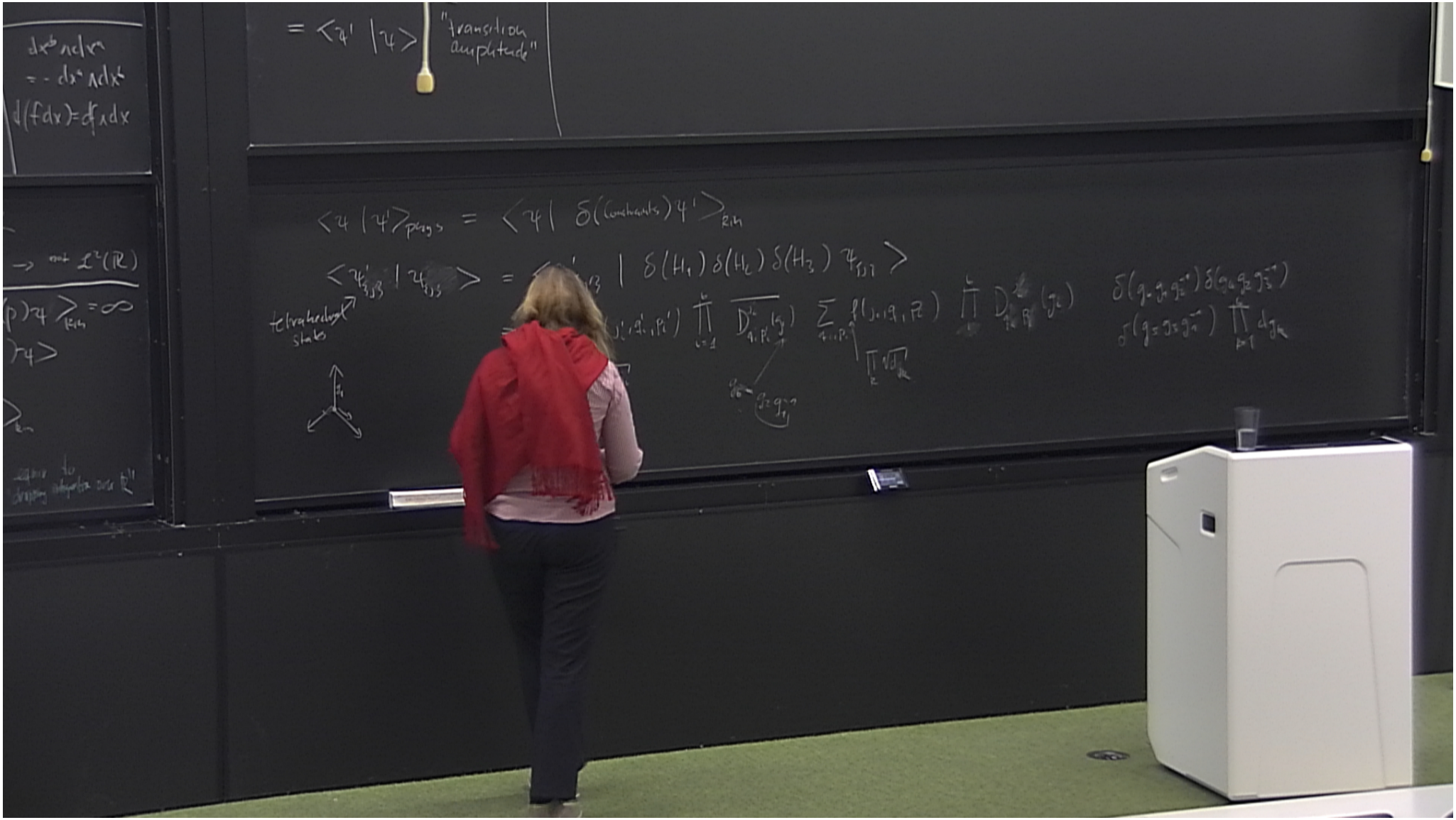


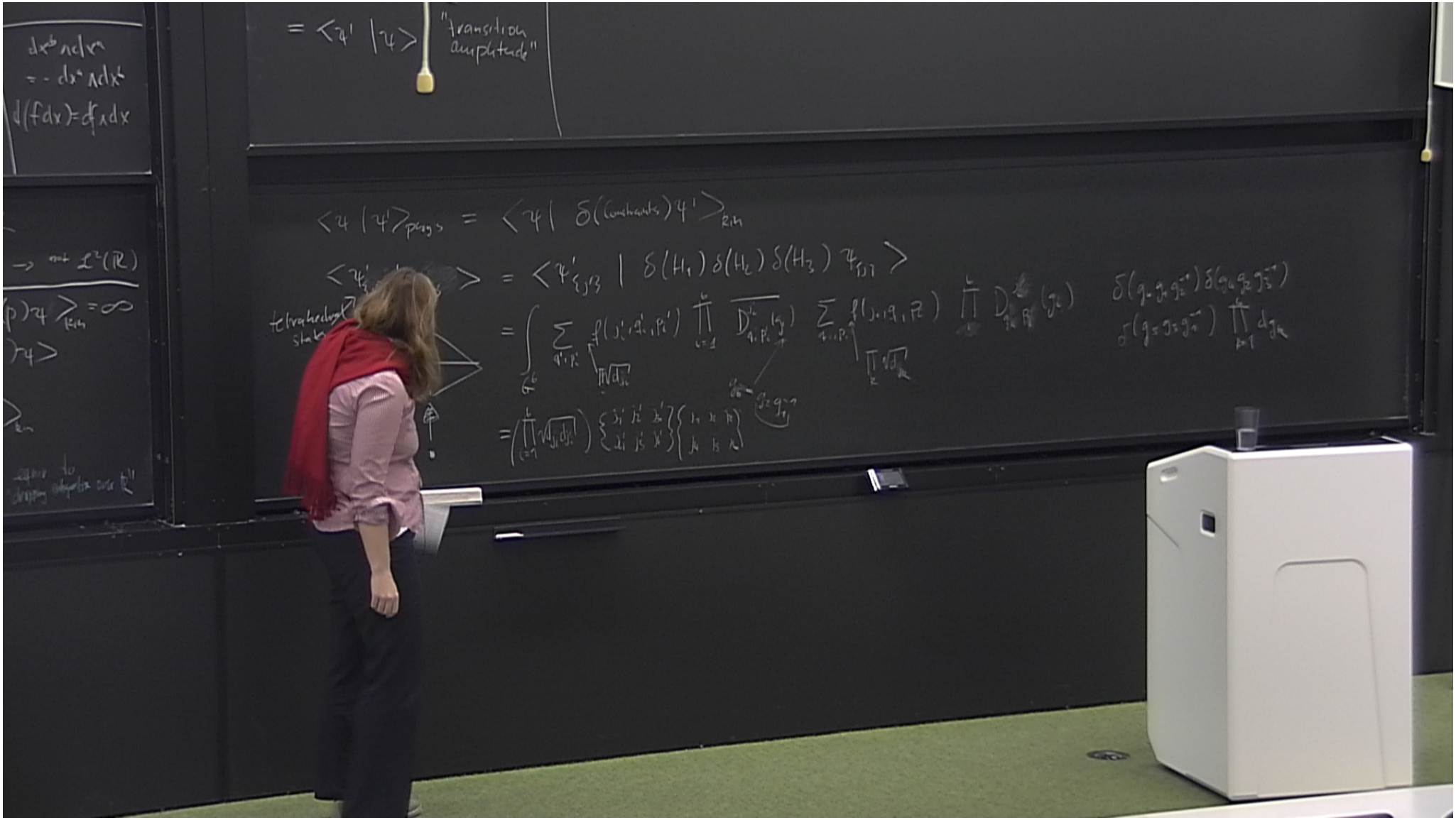


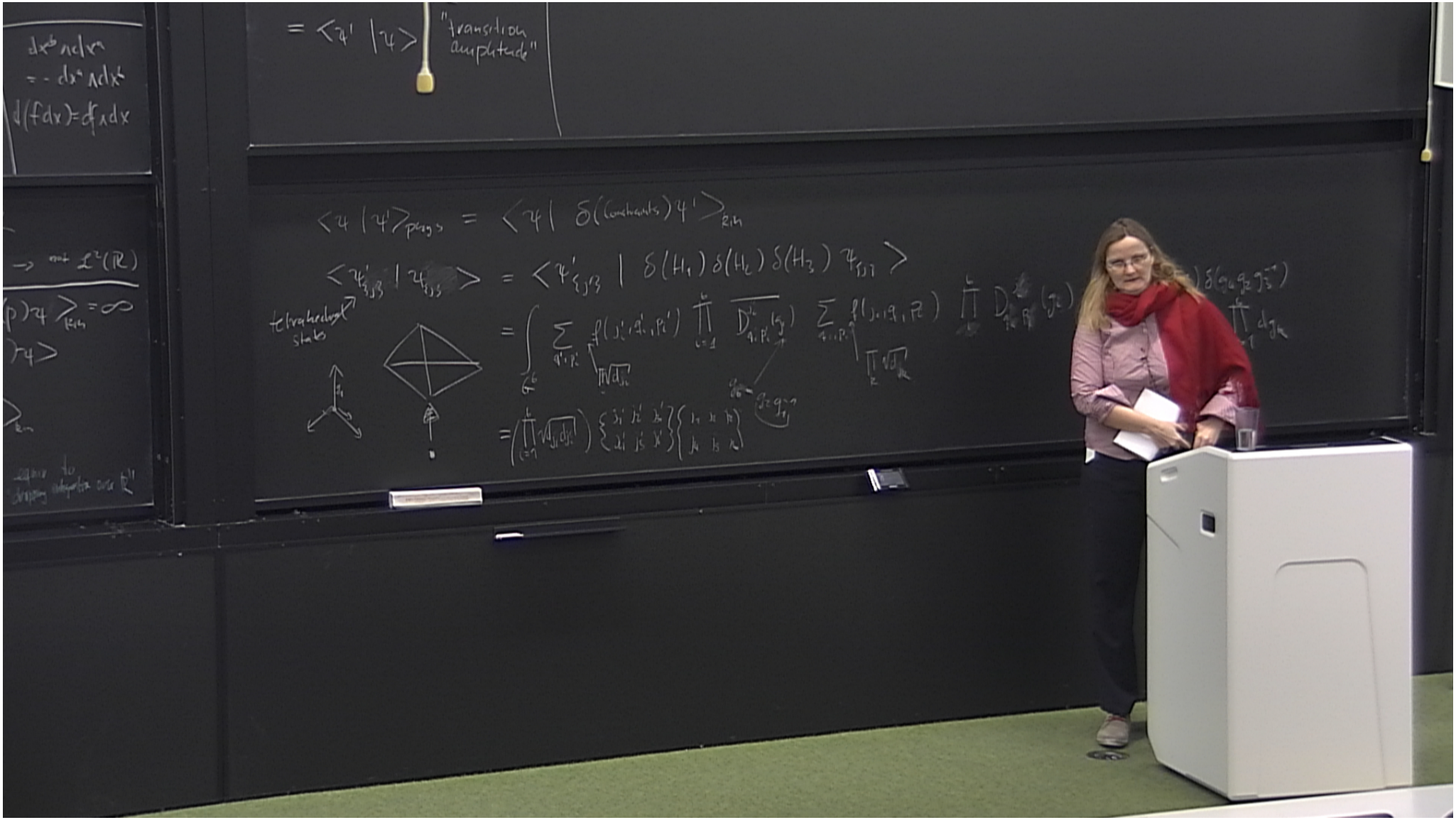

"transition amplitude"
$\langle \psi' | \psi \rangle$
$\langle \psi | \psi' \rangle_{phys} = \langle \psi | \delta(\text{constraints}) \psi' \rangle_{Kin}$
$\langle \psi'_{s_3 s_3} | \psi_{s_3 s_3} \rangle = \langle \psi'_{s_3 s_3} | \delta(H_1)\delta(H_2)\delta(H_3)\psi_{s_3 s_3} \rangle$
$\mathbb{R}$
$\to$ not $\mathcal{L}^2(\mathbb{R})$
$\delta(p)\psi\rangle_{Kin} = \infty$
$\delta(p)\psi\rangle$
$\psi\rangle_{Kin}$
$dx^a \wedge dx^b = -dx^b \wedge dx^a$
$d(fdx) = df \wedge dx$


"transition amplitude"
tetrahedron states


"transition amplitude"
tetrahedron shape


"transition amplitude"
tetrahedral shape


"transition amplitude"
tetrahedron
state


"transition amplitude"
tetrahedral state