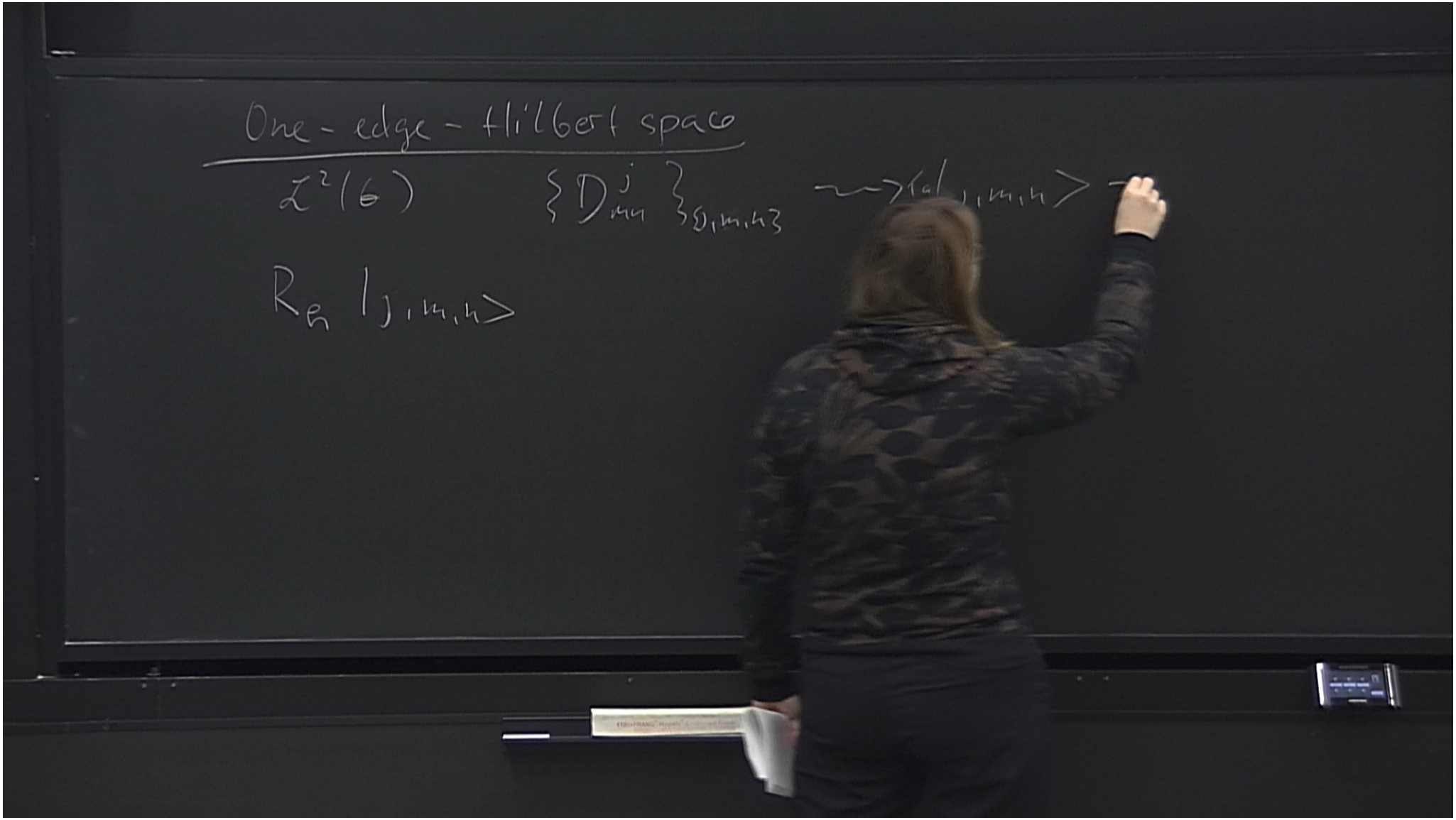
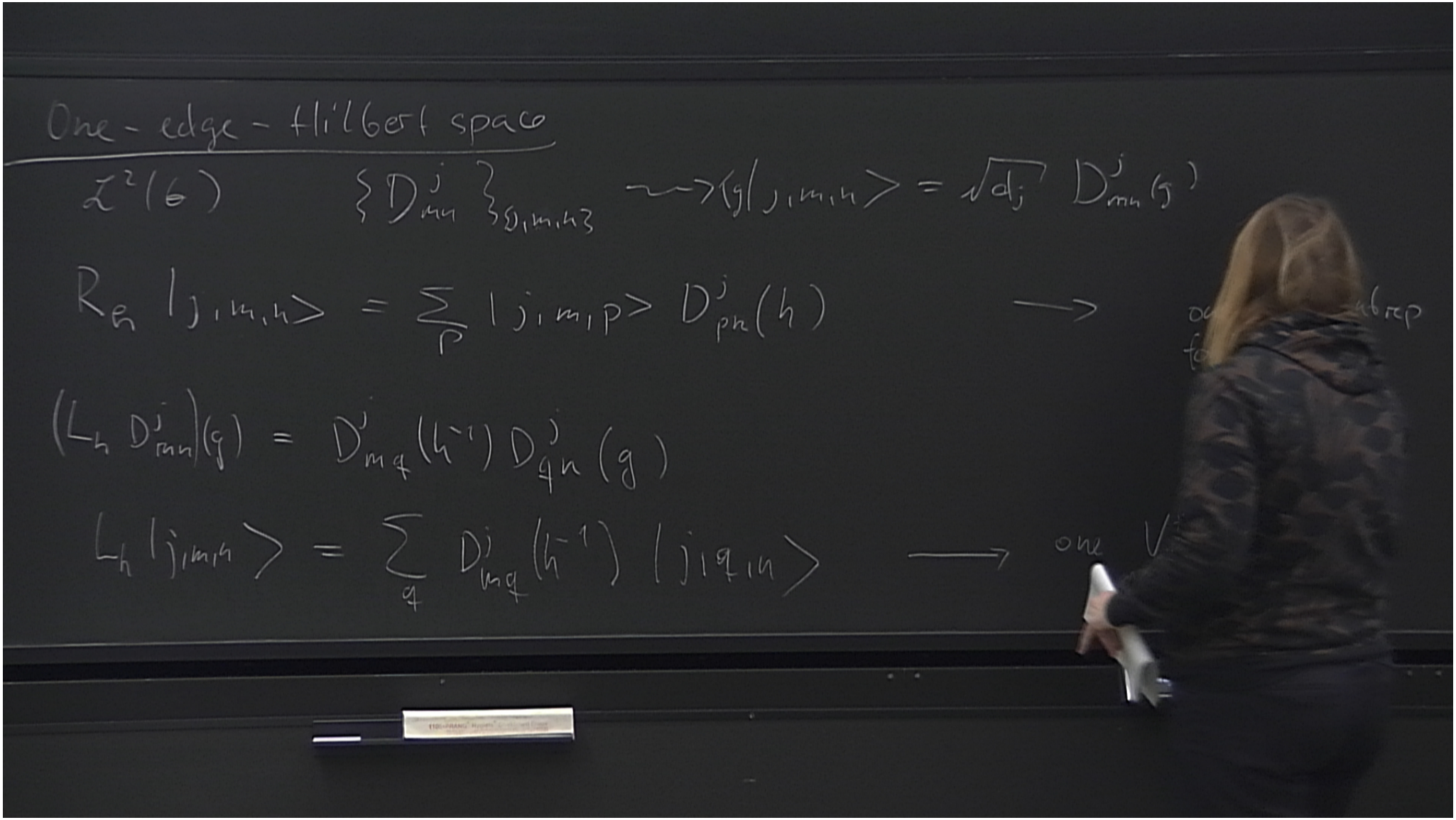
Title: 13/14 PSI - Quantum Gravity Review - Lecture 11

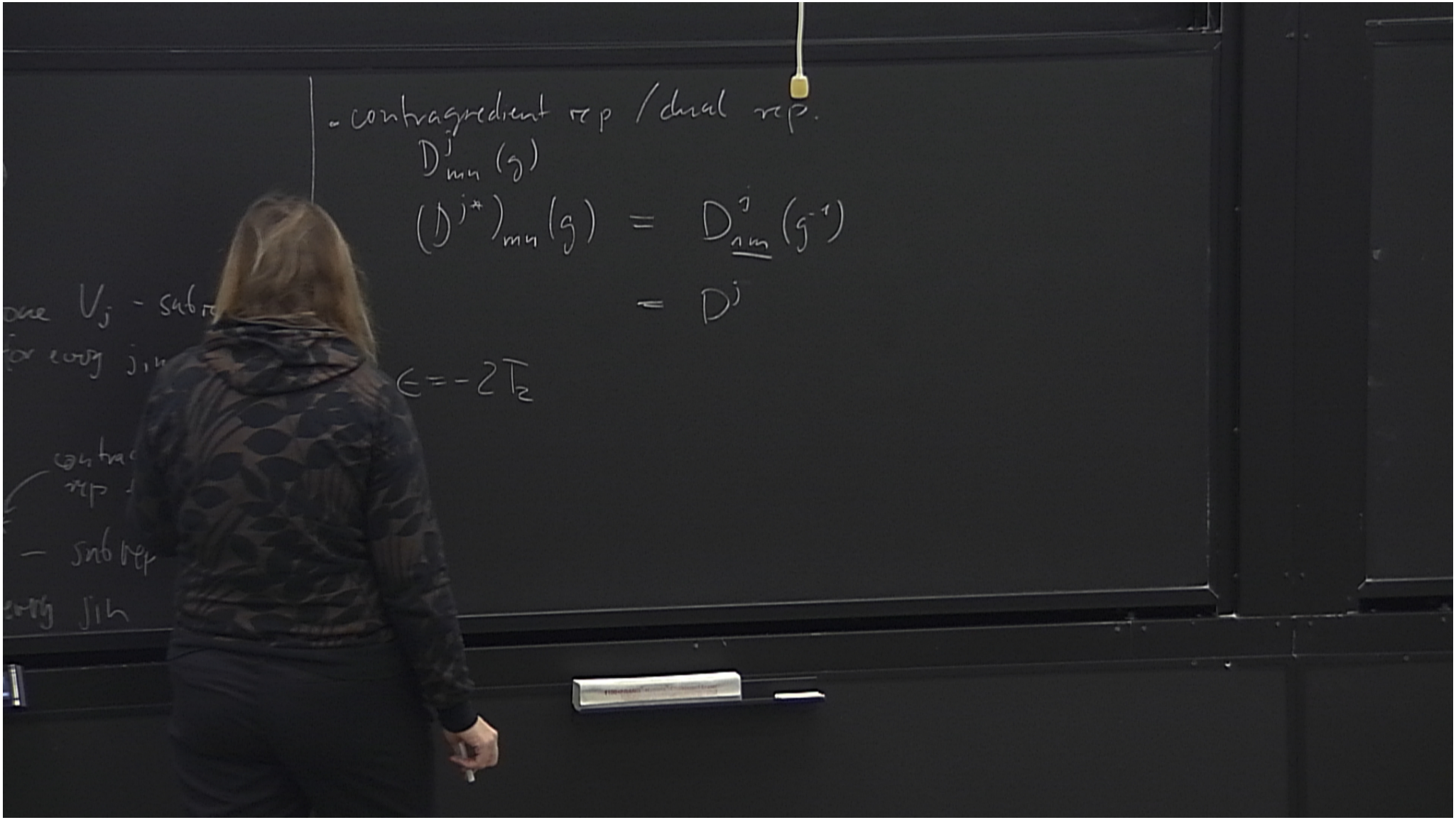
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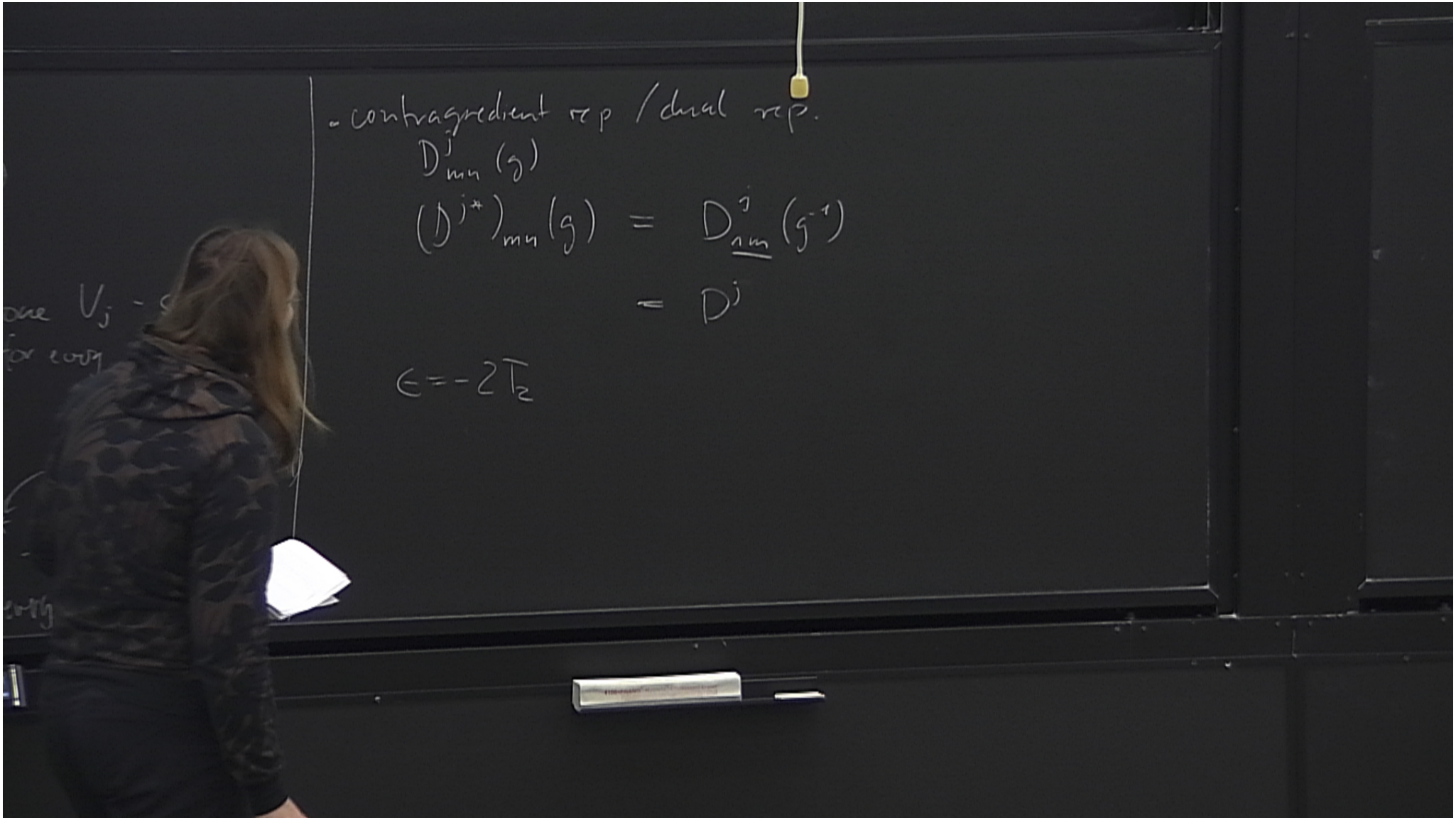
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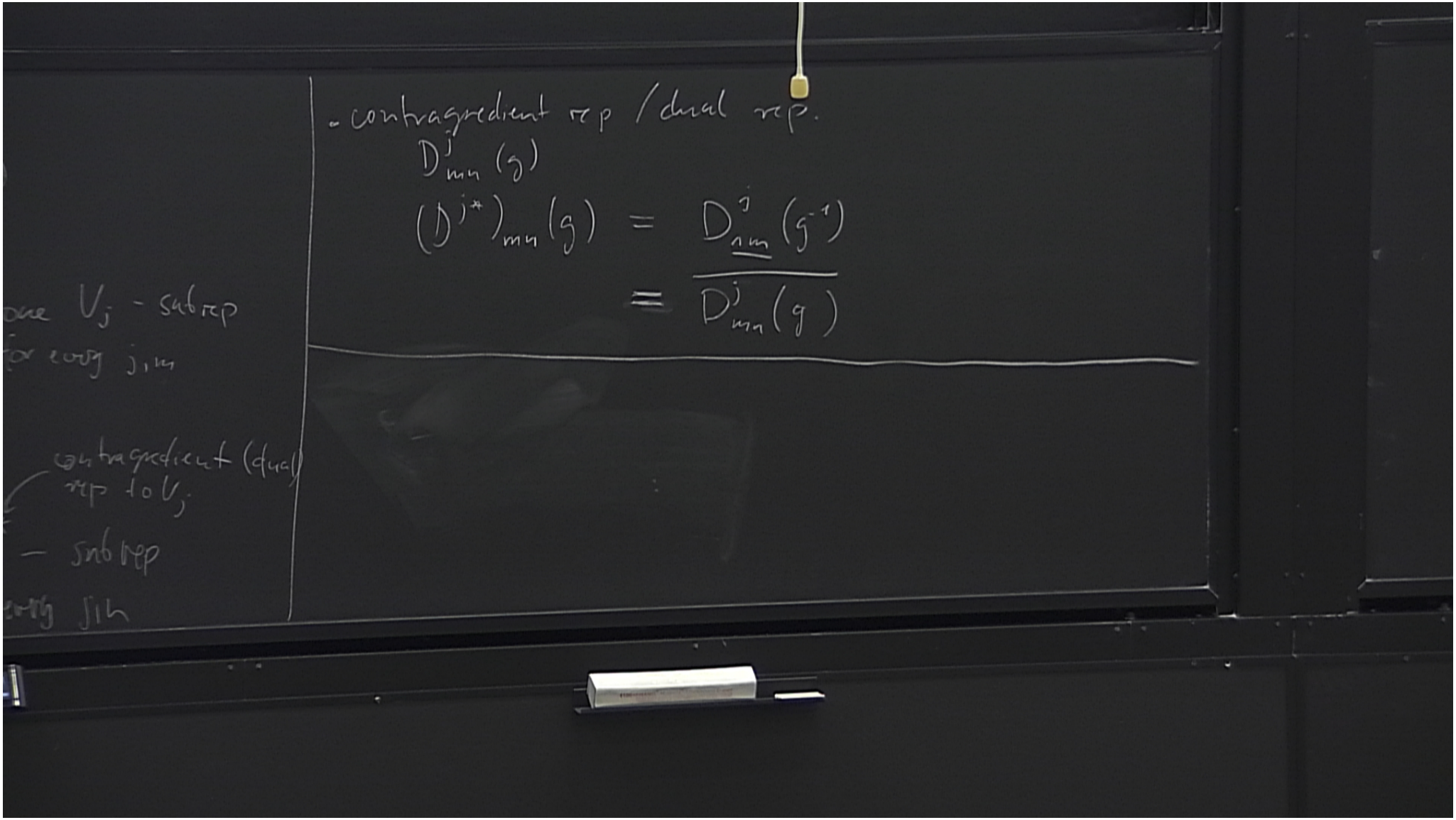
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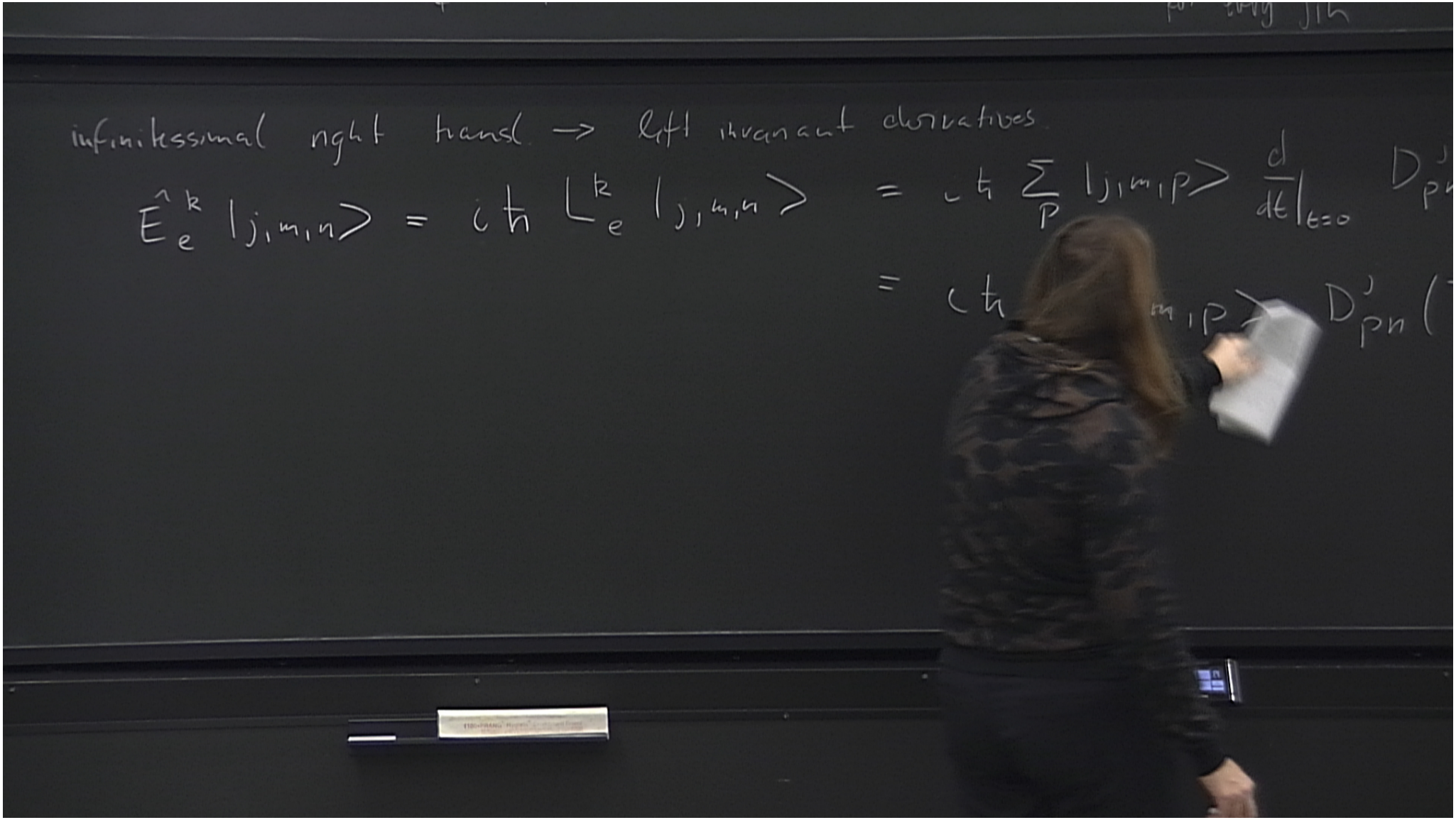

One-edge-Hilbert space
$L^2(G)$
$\{D^j_{mn}\}_{j,m,n}$
$R_n |j,m,n\rangle$


One-edge-Hilbert space
$\mathcal{L}^2(G)$ $\{D^j_{mn}\}_{j,m,n}$ $\rightsquigarrow |g|j,m,n\rangle = \sqrt{d_j}\, D^j_{mn}(g)$
$R_n |j,m,n\rangle = \sum_p |j,m,p\rangle D^j_{pn}(h)$
$(L_h D^j_{mn})(g) = D^j_{mq}(h^{-1}) D^j_{qn}(g)$
$L_h |j,m,n\rangle = \sum_q D^j_{mq}(h^{-1}) |j,q,n\rangle$


- contragredient rep / dual rep.
$D^j_{mn}(g)$
$(D^{j*})_{mn}(g) = D^j_{nm}(g^{-1})$
$= D^j$
$\epsilon = -2T_z$


- contragredient rep / dual rep.
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$\epsilon = -2\bar{T}_2$


- contragredient rep / dual rep.
$D^j_{mn}(g)$
$(D^{j*})_{mn}(g) = D^j_{nm}(g^{-1})$
$= \overline{D^j_{mn}(g)}$
one V_j - subrep
for every j, m
contragredient (dual) rep to V_j
- subrep
every j, m


infinitessimal right transl → left invariant derivatives
$\hat{E}_e^k |j,m,n\rangle = i\hbar L_e^k |j,m,n\rangle = -i\hbar \sum_p |j,m,p\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}$
$= i\hbar$ $m,p\rangle$ D^j_{pn}

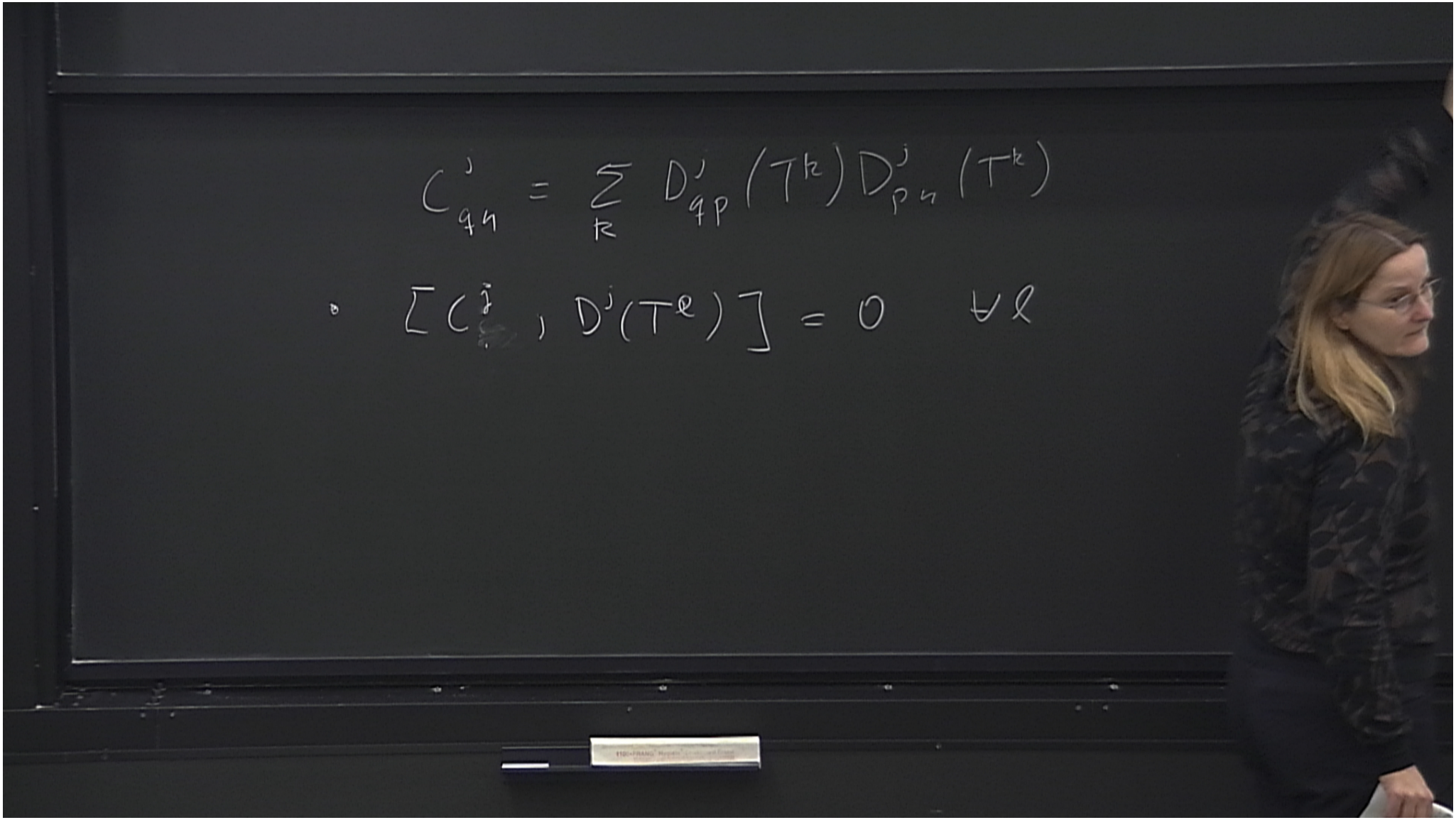
infinitessimal right transl → left invariant derivatives

$$\hat{E}^k_e |j,m,n\rangle = i\hbar L^k_e |j,m,n\rangle = i\hbar \sum_p |j,m,p\rangle$$

$$= i\hbar \sum_p |j,m,p\rangle$$

The length operator

$$\hat{\ell}^2_{ex} |j,m,n\rangle := \sum_k \hat{E}^k_e \hat{E}^k_e |j,m,n\rangle$$


$C^j_{qn} = \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$[C^j, D^j(T^\ell)] = 0 \quad \forall \ell$

$$C^j_{q\,q'} = \sum_k D^j_{q\,p}(T^k)\, D^j_{p\,q'}(T^k)$$

- $[C^j, D^j(T^\ell)] = 0 \quad \forall \ell$

"intertwiner" of j-representation.

→ Schur's lemma: $C^j = \begin{cases} 0 \\ \lambda \mathbb{1} \end{cases}$

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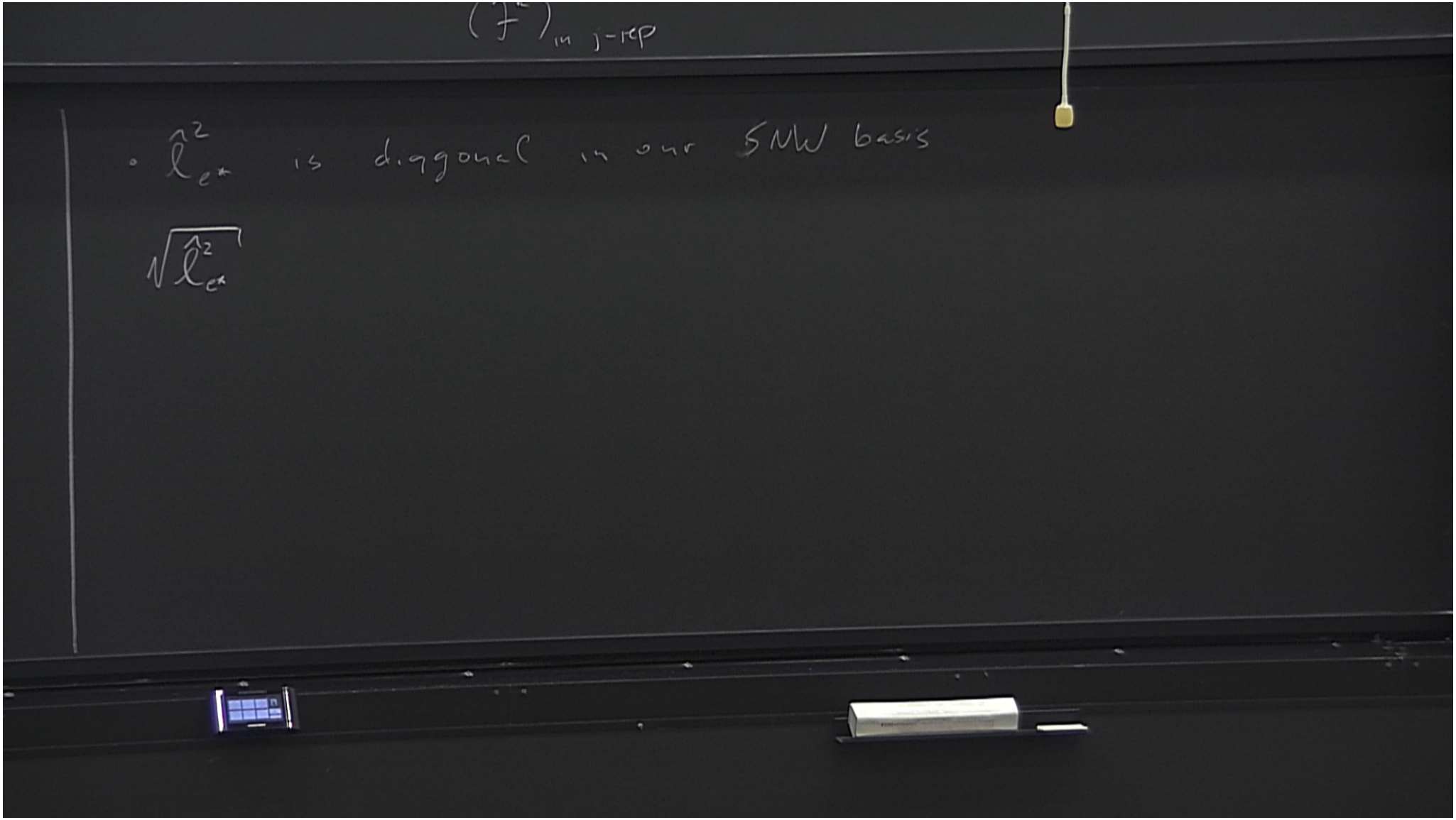
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- $\hat{\ell}^2_{cm}$ is diagon


$(\hat{f}^2)_{\text{in j-rep}}$
$\hat{l}^2_{ext}$ is diagonal in our SNW basis
$\sqrt{\hat{l}^2_{ext}}$

$(\hat{J}^2)_{\text{in } j\text{-rep}}$

- $\hat{\ell}^2_{ex}$ is diagonal in our SNW basis

$$\sqrt{\hat{\ell}^2_{ex}} = \underbrace{\hbar \kappa_{3D}}_{\ell^{3D}_{Planck}} \sqrt{j(j+1)}$$

$$S = \frac{1}{\kappa_{3D}} \int R \, d^3x$$

$(\hat{J}^2)_{\text{in } j\text{-rep}}$

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- (spatial)

$(\hat{J}^2)_{\text{in } j\text{-rep}}$

- $\hat{\ell}^2_{ea}$ is diagonal in our SNW basis

$$\sqrt{\hat{\ell}^2_{ea}} = \underbrace{\hbar \kappa_{3D}}_{\ell^{3D}_{Planck}} \sqrt{j(j+1)}$$

$$S = \frac{1}{\kappa_{3D}} \int R \, d^3x$$

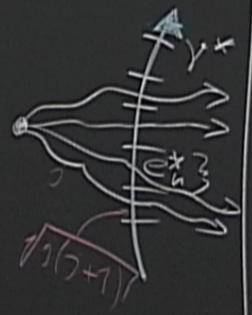
- (spatial) geom. operators have discrete spectrum.

$(\hat{f}^2)_{\text{in } j\text{-rep}}$

- $\hat{\ell}^2_{e^*}$ is diagonal in our SNW basis

$$\sqrt{\hat{\ell}^2_{e^*}} = \underbrace{\hbar \kappa_{3D}}_{\ell^{3D}_{\text{Planck}}} \sqrt{j(j+1)}$$

$$S = \frac{1}{\kappa_{3D}} \int R \, d^3x$$



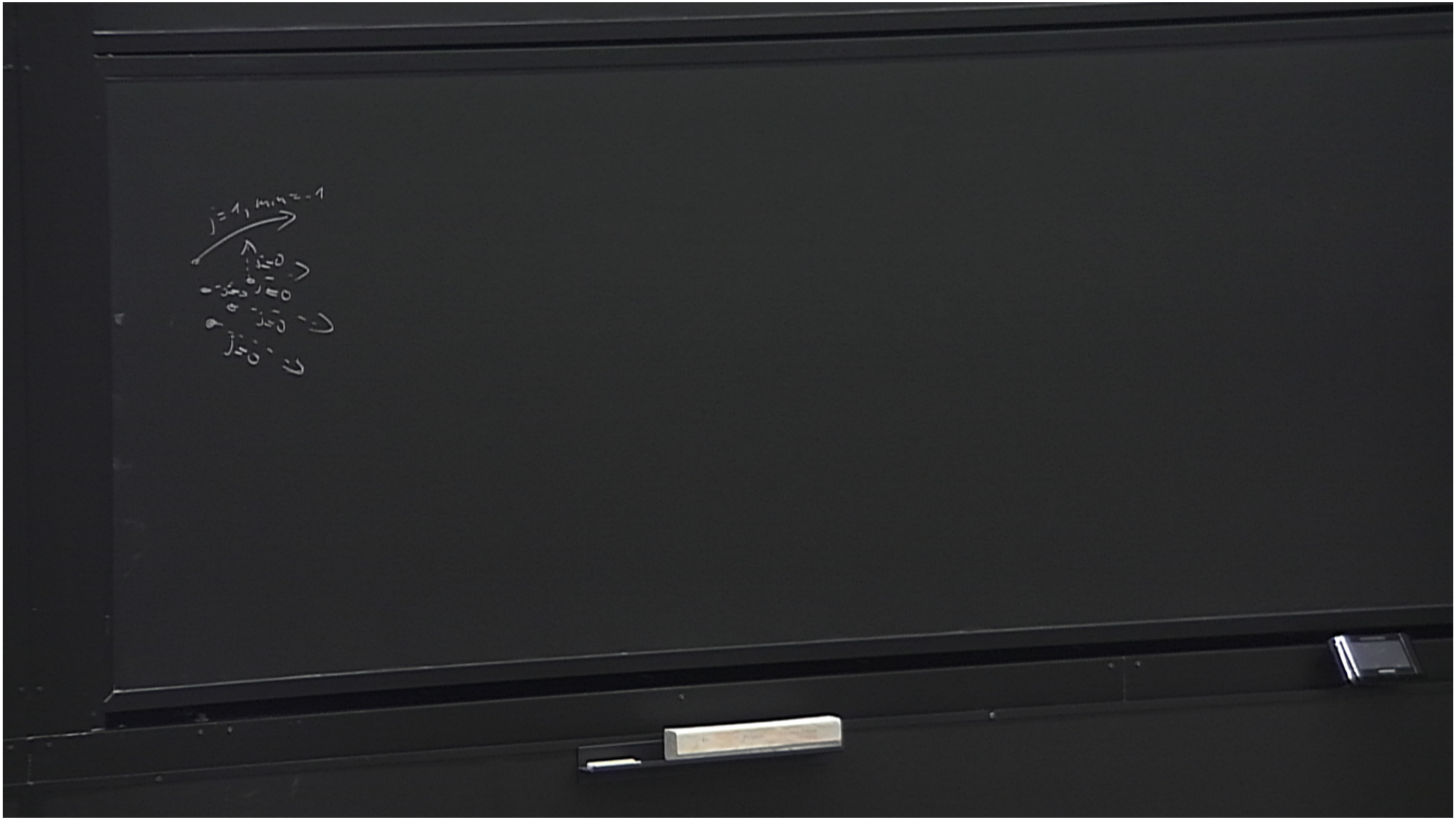
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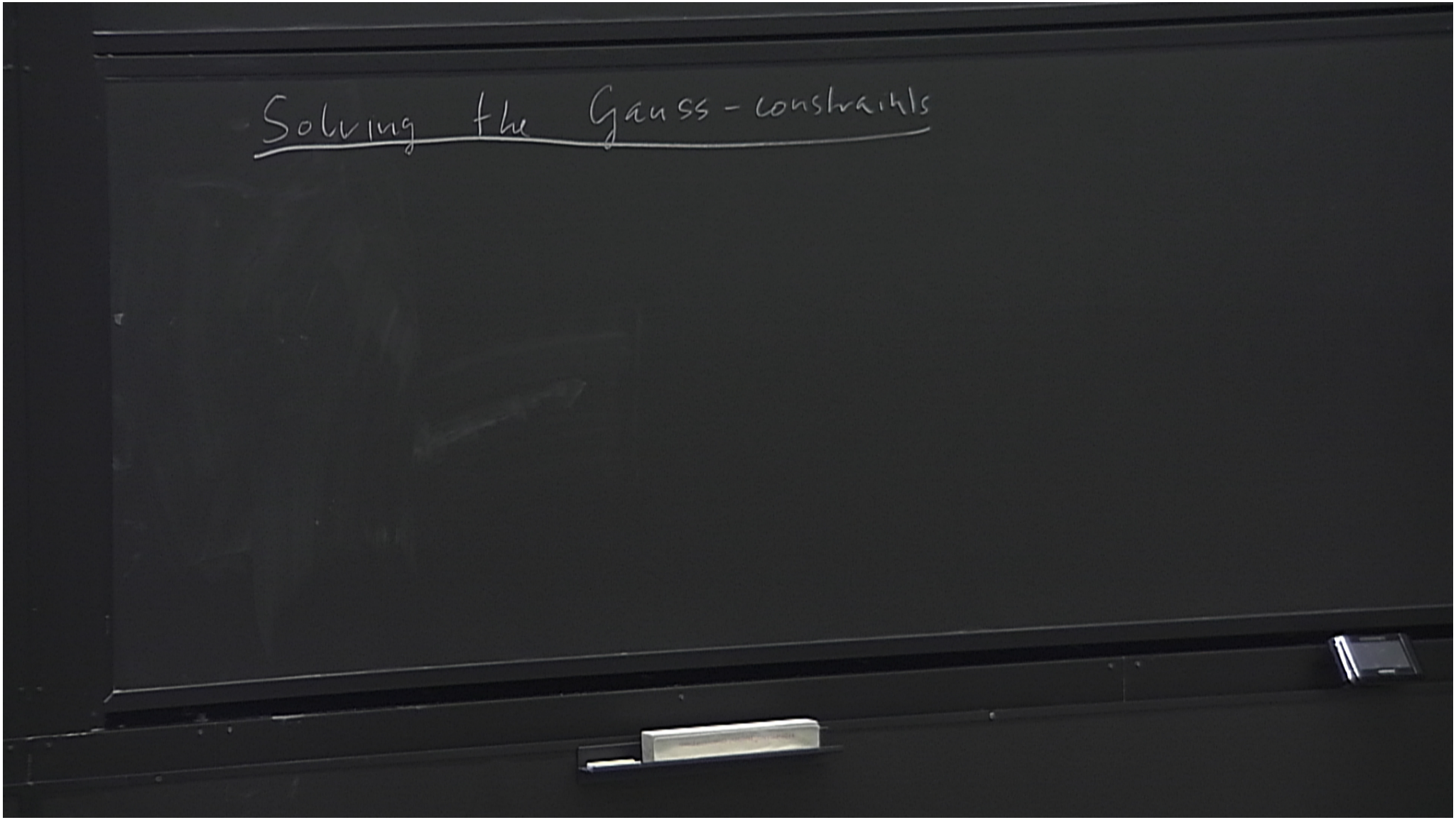
$$\ell_{j^*} = \int_{j^*} \sqrt{\dot{j}^{*a} \, \dot{j}^{*b} \, q_{ab}} \, ds$$

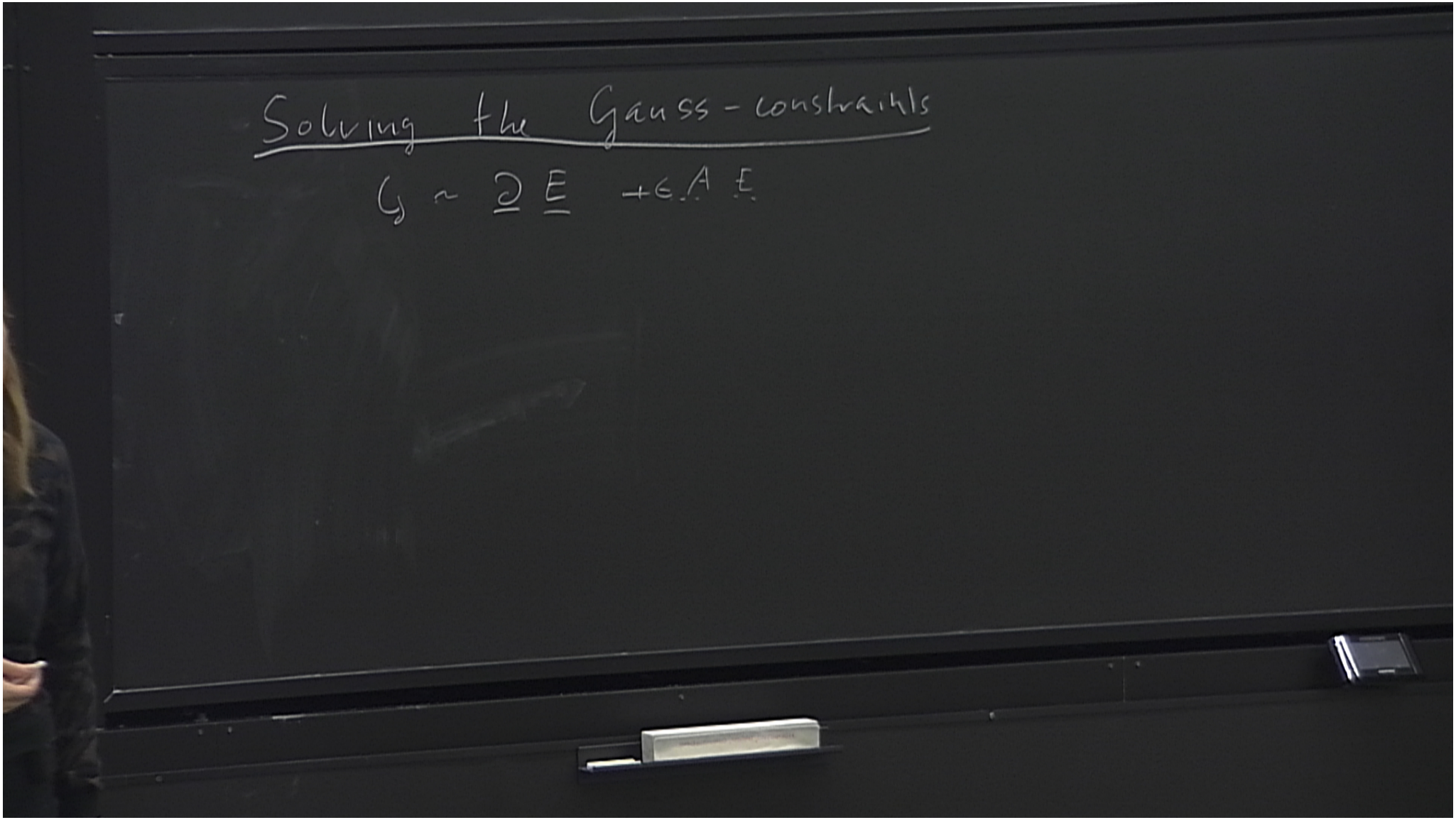
$$= \int \sqrt{\dot{j}^{*a} \, \dot{j}^{*b} \, e^k_a \, e_{bk}}$$

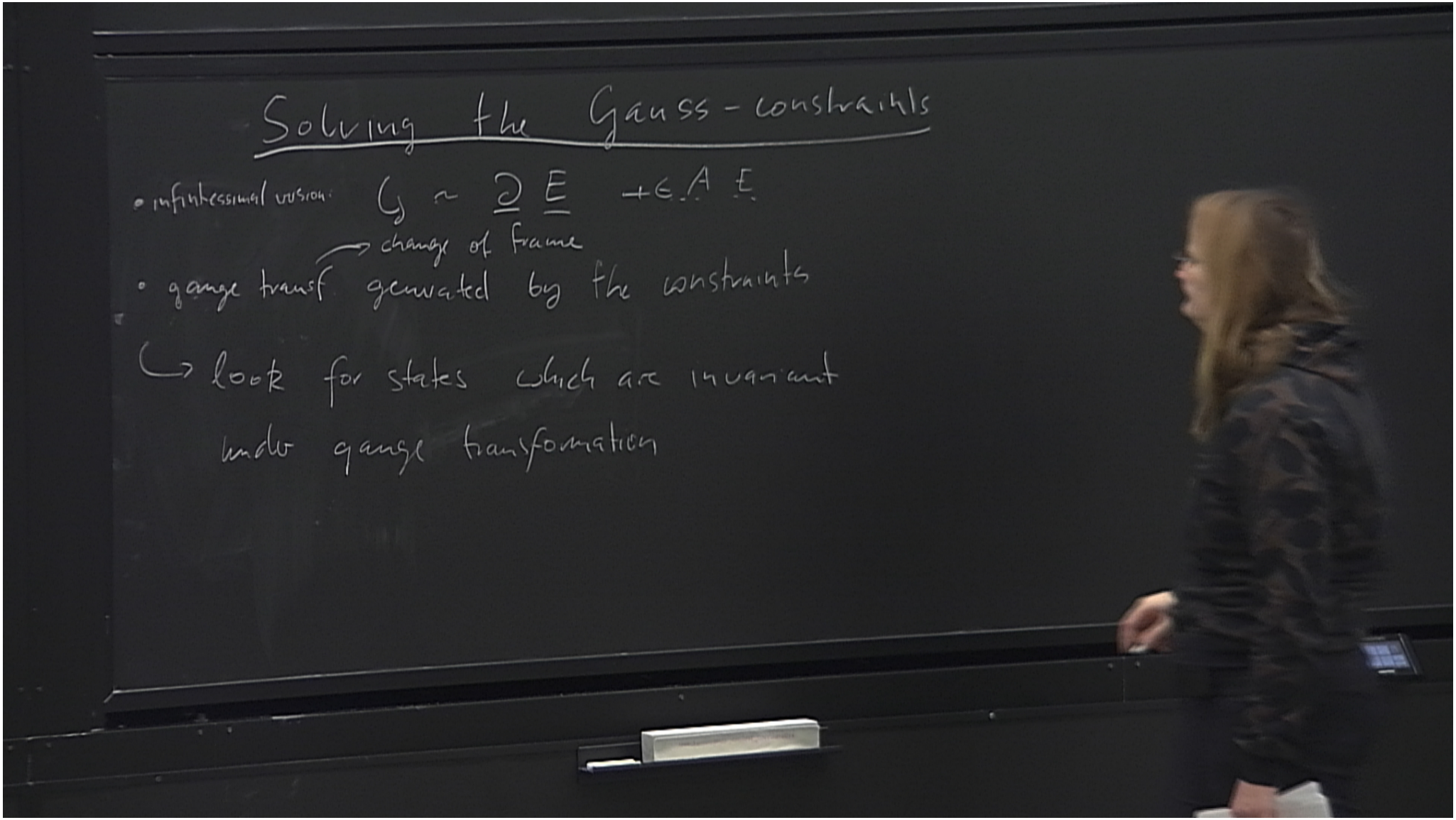
$$\ell_{j^*} \sim \sum_{n=0}^{N} \sqrt{\sum_k \left(\int_{e^*_n} \dot{e}^{*a} \, \dot{e}^k_a(s) \, ds \right)^2}$$

$$\sim \sum_{n=0}^{N} \sqrt{\sum_k E^k_{e_n} \, \bar{E}^k_{e_n}}$$


j=1, m=-1
j=0
j=0
j=0
j=0


Solving the Gauss-constraints


Solving the Gauss-constraints
G ~ ∂ E + ε A E

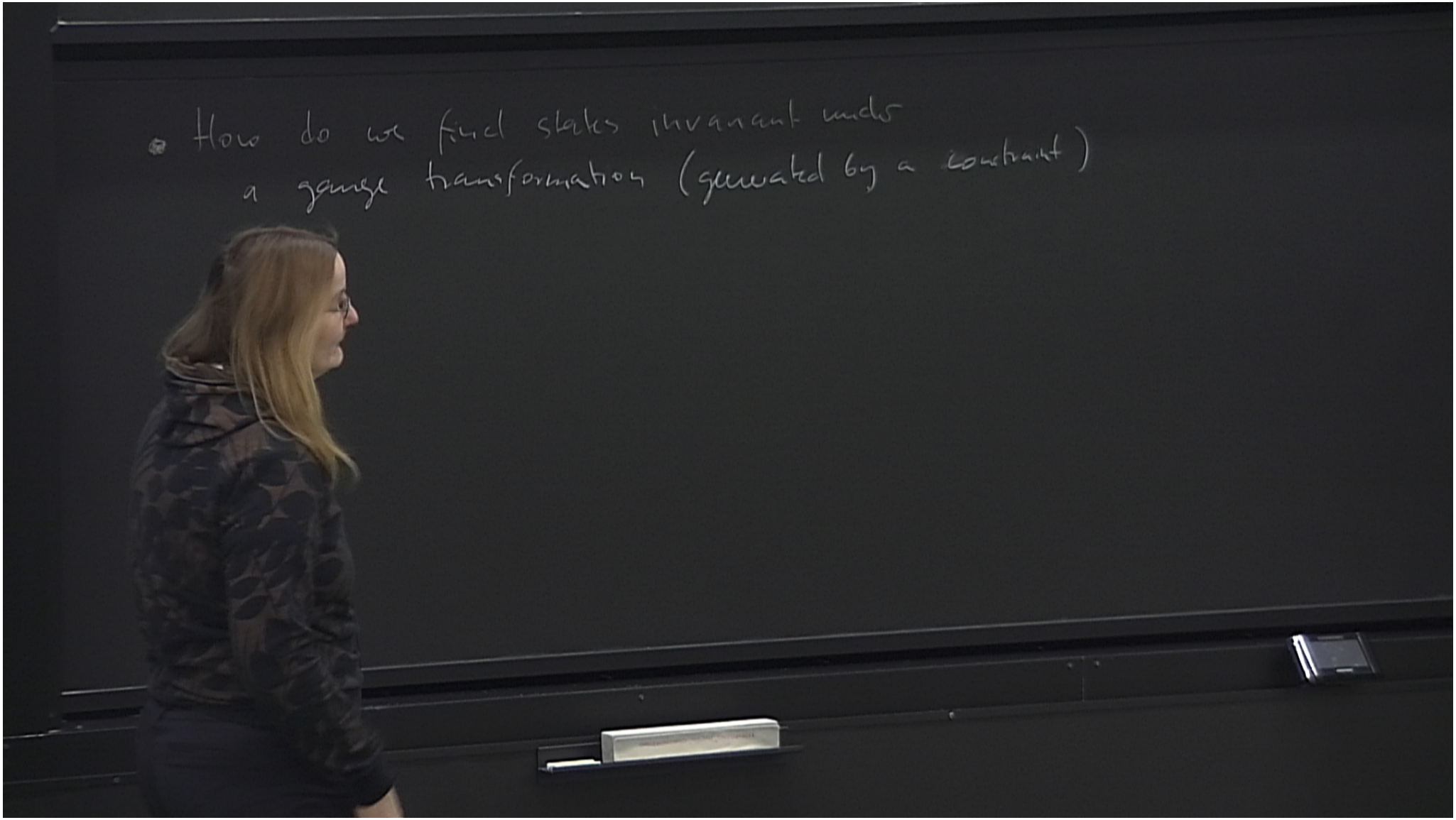

Solving the Gauss-constraints
• infinitessimal version: $G \sim \partial \underline{E} + \epsilon A \underline{E}$
→ change of frame
• gauge transf generated by the constraints
↳ look for states which are invariant
under gauge transformation

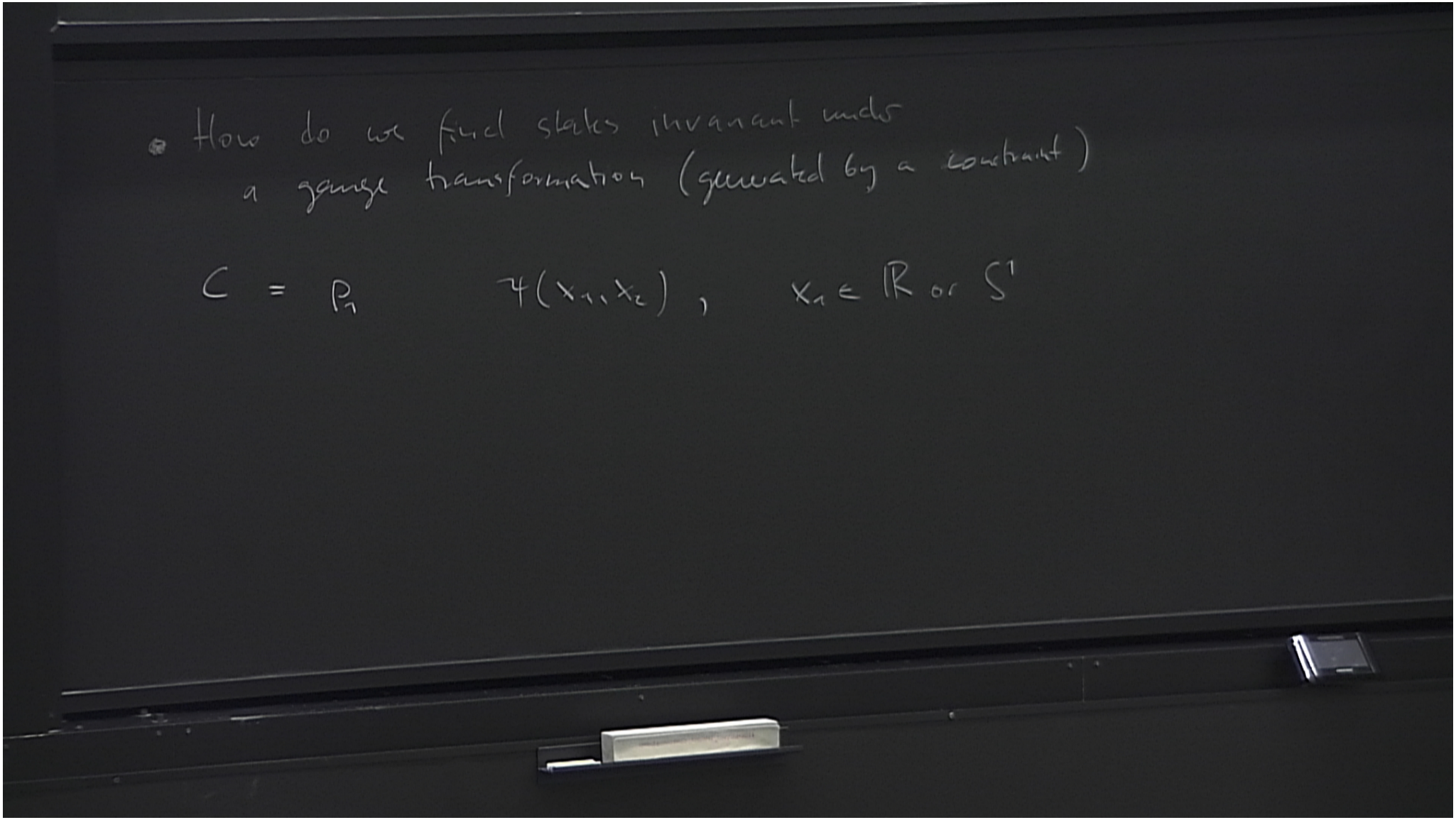
Solving the Gauss-constraints

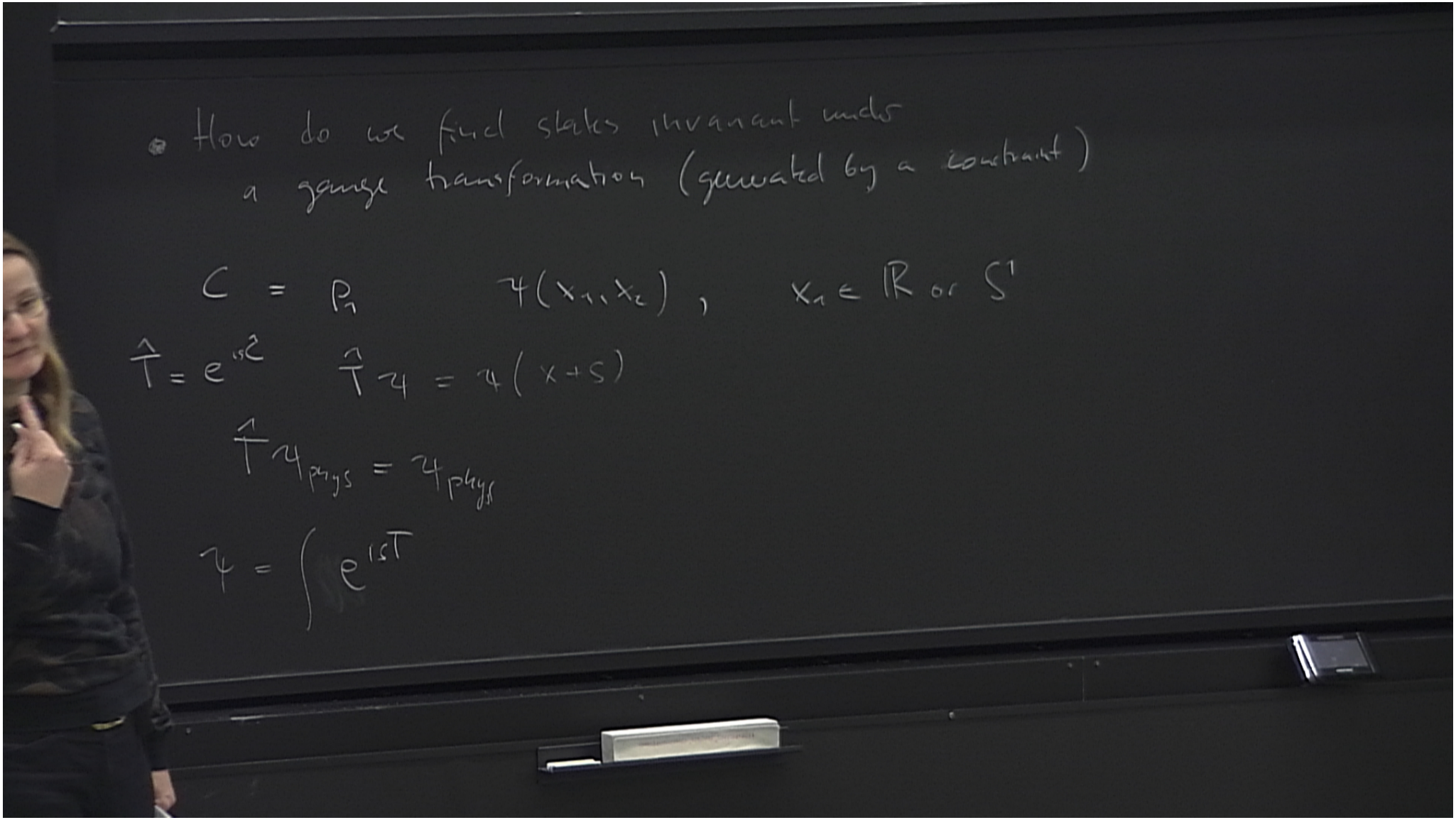
- infinitessimal version: $G_j \sim \partial \underline{E} + \epsilon A \underline{E}$
  → change of frame
- gauge transf generated by the constraints: $h_e \to g(v_t)\, h_e\, g(v_s)^{-1}$

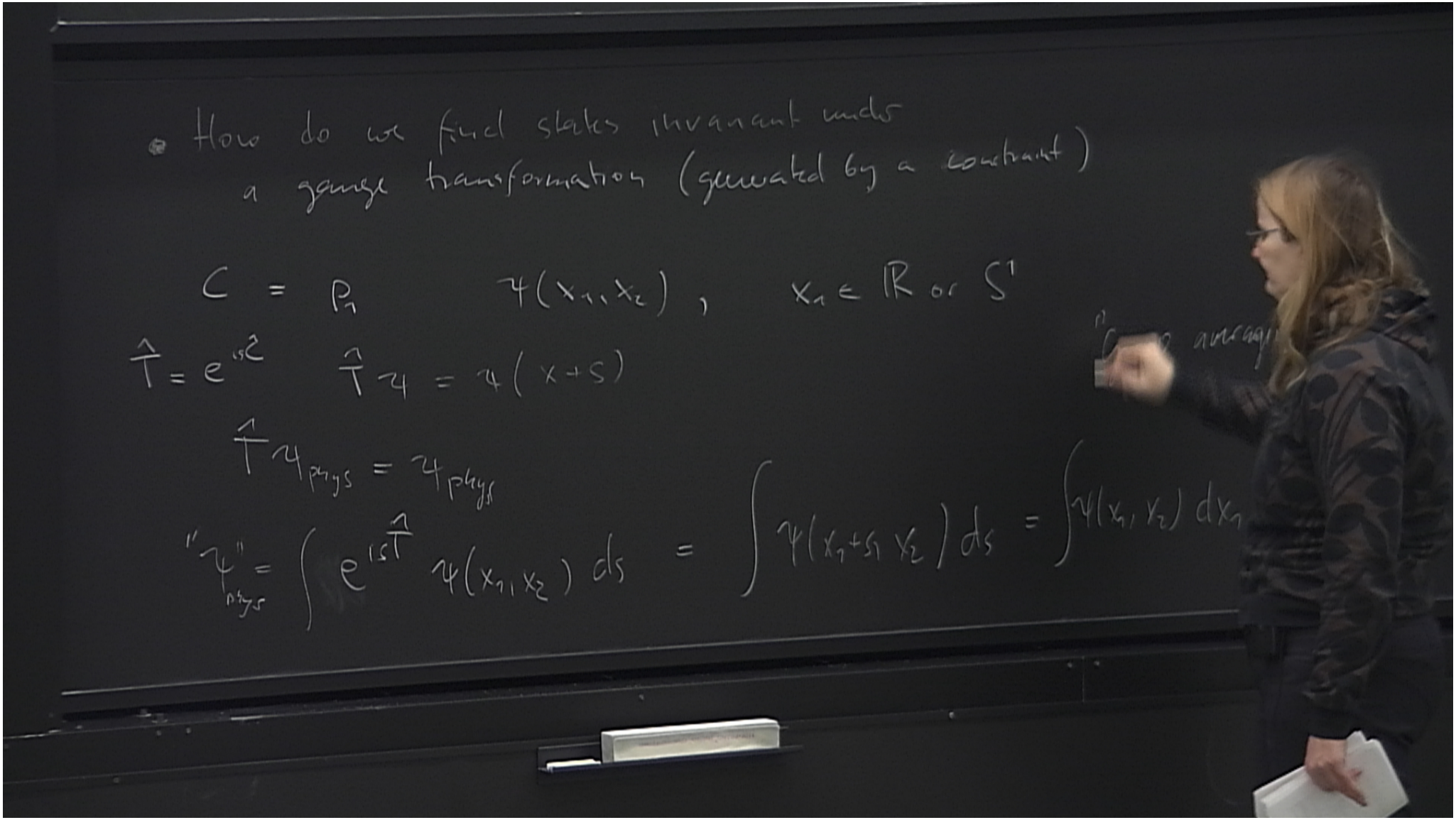


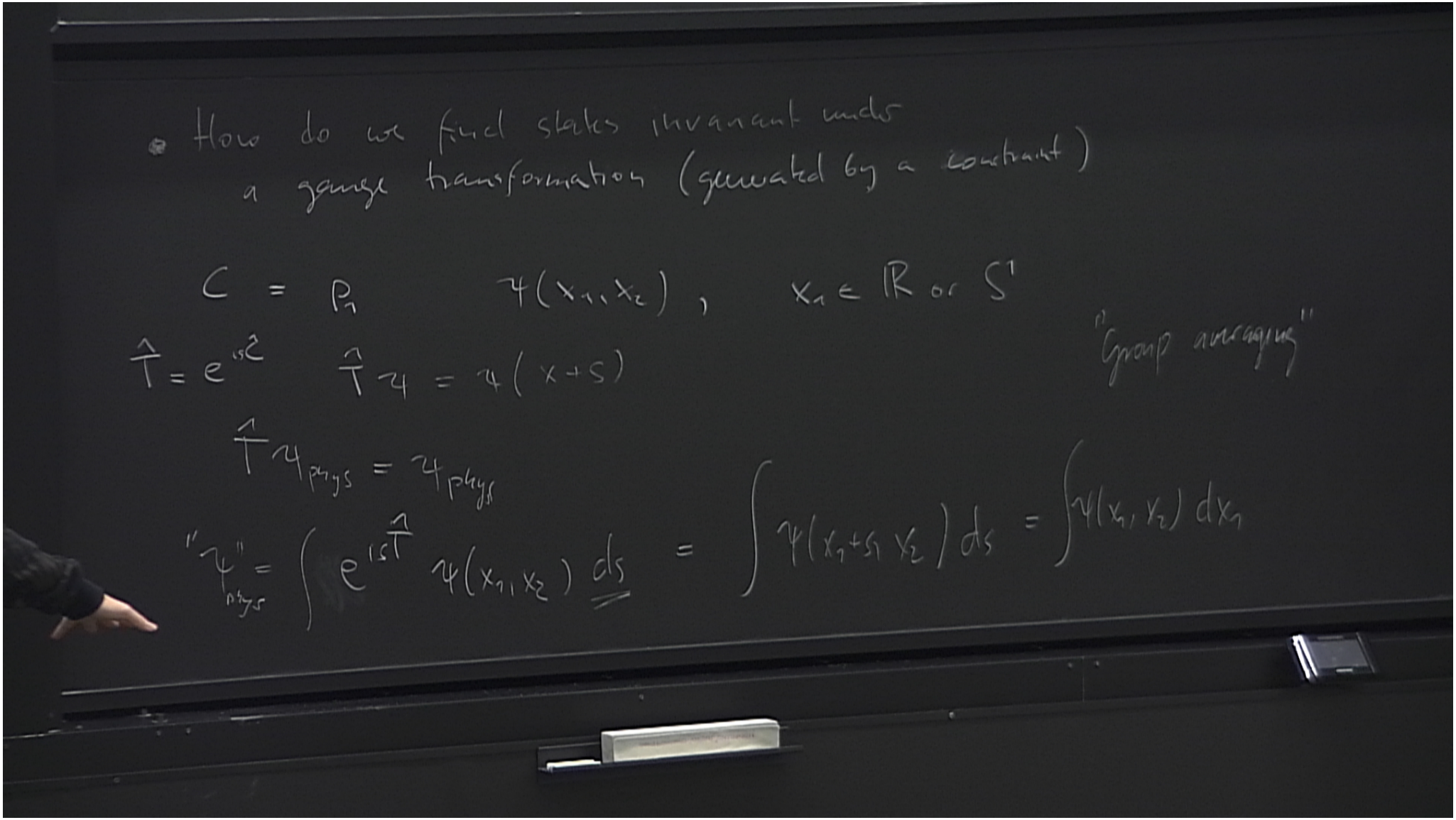
↳ look for states which are invariant under gauge transformation


• How do we find states invariant under
a gauge transformation (generated by a constraint)


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$C = P_1 \quad \psi(x_1, x_2), \quad x_1 \in \mathbb{R}$ or S^1


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$C = P_1 \quad \psi(x_1, x_2), \quad x_1 \in \mathbb{R}$ or S^1
$\hat{T} = e^{is\hat{C}} \quad \hat{T}\psi = \psi(x+s)$
$\hat{T}\psi_{phys} = \psi_{phys}$
$\psi = \int e^{isT}$


• How do we find states invariant under a gauge transformation (generated by a constraint)
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"ψ" $= \int e^{is\hat{T}} \psi(x_1, x_2)\, ds = \int \psi(x_1+s, x_2)\, ds = \int \psi(x_1, x_2)\, dx_1$


• How do we find states invariant under a gauge transformation (generated by a constraint)
C = p_1
ψ(x_1, x_2), x_1 ∈ ℝ or S^1
T̂ = e^{isĈ}
T̂ ψ = ψ(x+s)
"Group averaging"
T̂ ψ_phys = ψ_phys
"ψ"_phys = ∫ e^{isT̂} ψ(x_1, x_2) ds = ∫ ψ(x_1+s, x_2) ds = ∫ ψ(x_1, x_2) dx_1

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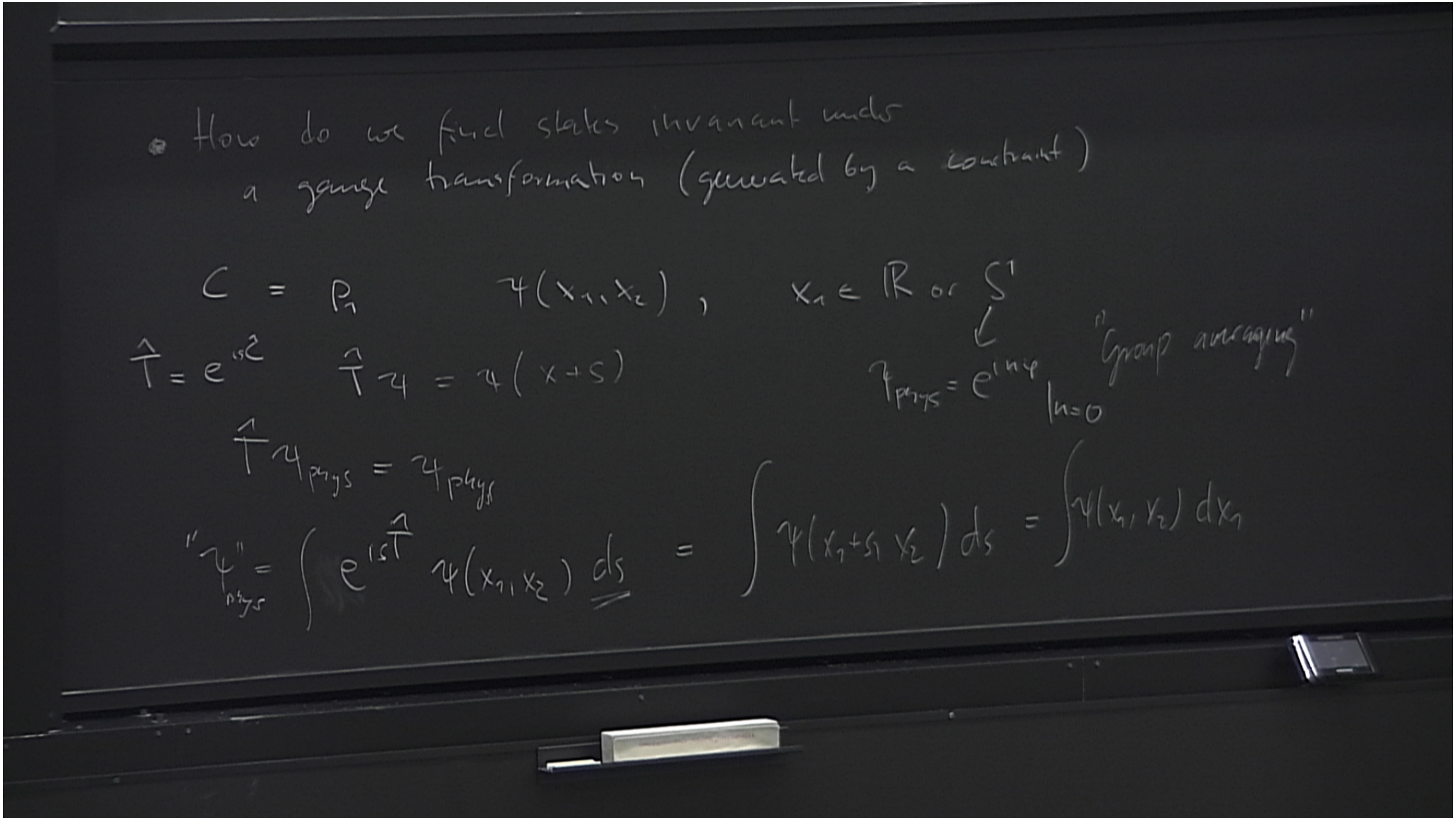
$C = P_1$ $\quad \psi(x_1, x_2)$, $\quad x_1 \in \mathbb{R}$ or S^1

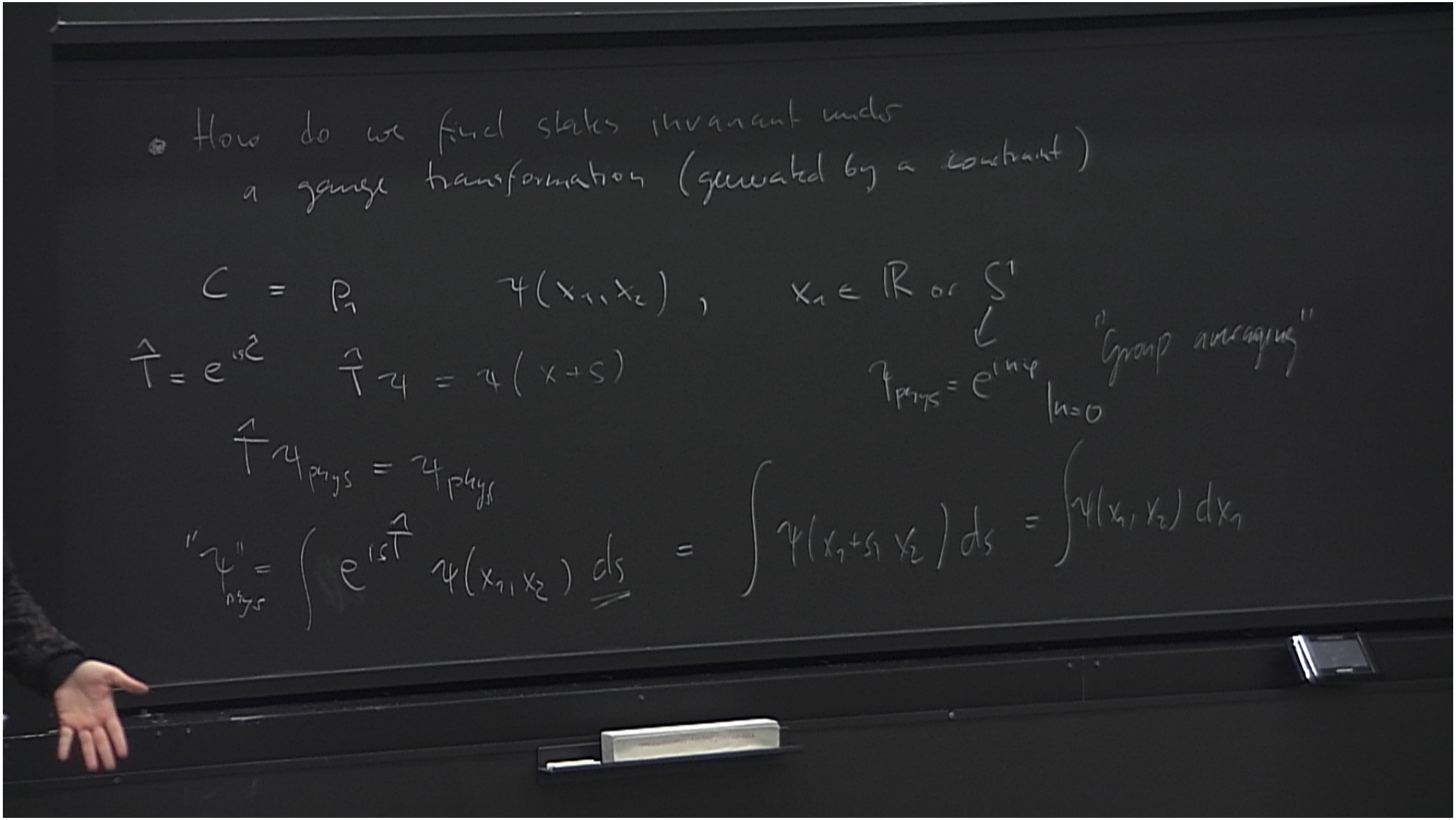
$\hat{T} = e^{is\hat{C}}$ $\quad \hat{T}\psi = \psi(x+s)$

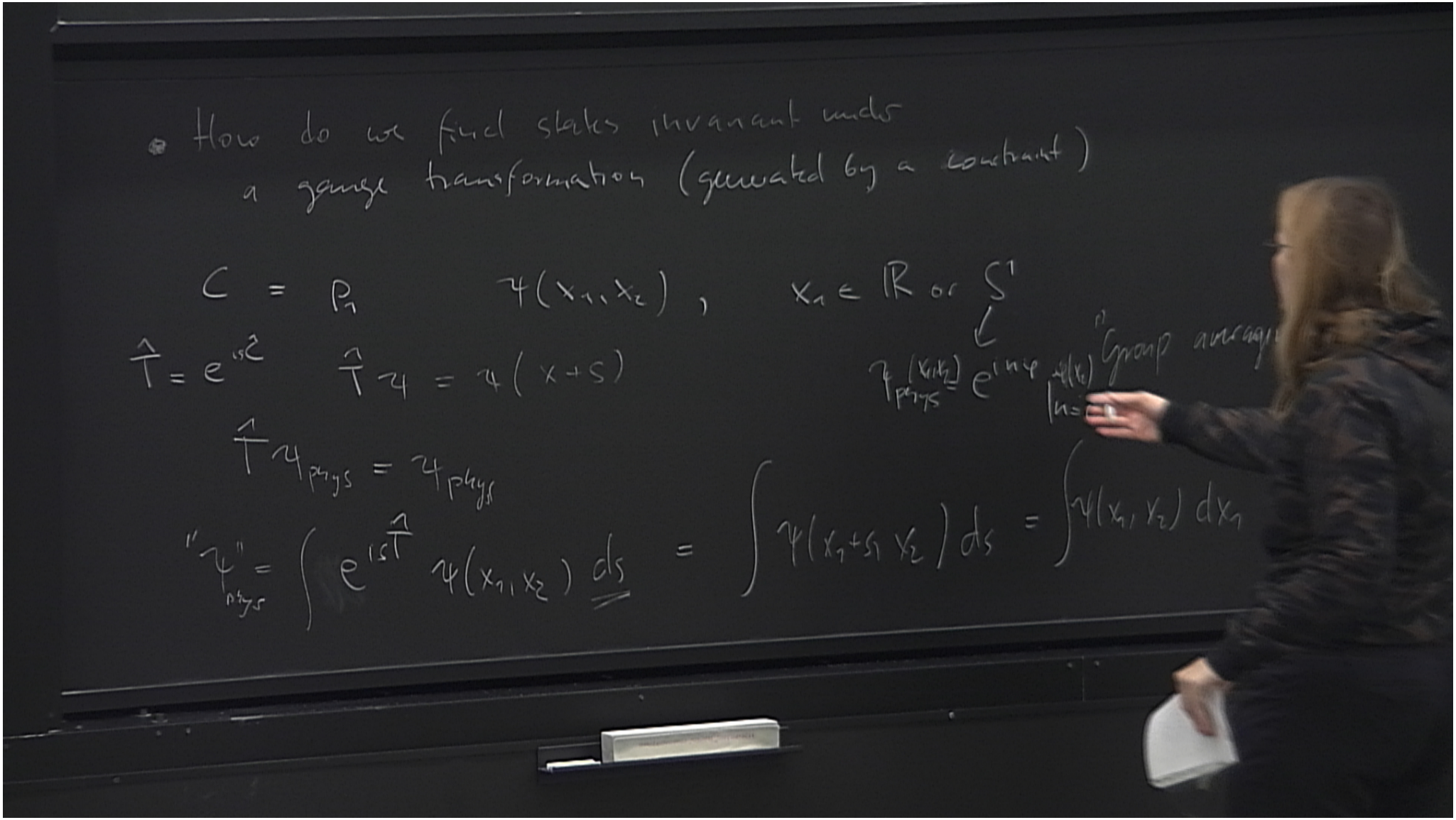
"Group averaging"

$\hat{T}\psi_{phys} = \psi_{phys}$

$$\text{"}\psi\text{"}_{phys} = \int e^{is\hat{T}}\psi(x_1, x_2)\,ds = \int \psi(x_1+s, x_2)\,ds = \int \psi(x_1, x_2)\,dx_1$$


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$\hat{T} = e^{is\hat{C}}$ $\hat{T}\psi = \psi(x+s)$
$\psi_{phys} = e^{in\varphi}|_{n=0}$ "Group averaging"
$\hat{T}\psi_{phys} = \psi_{phys}$
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