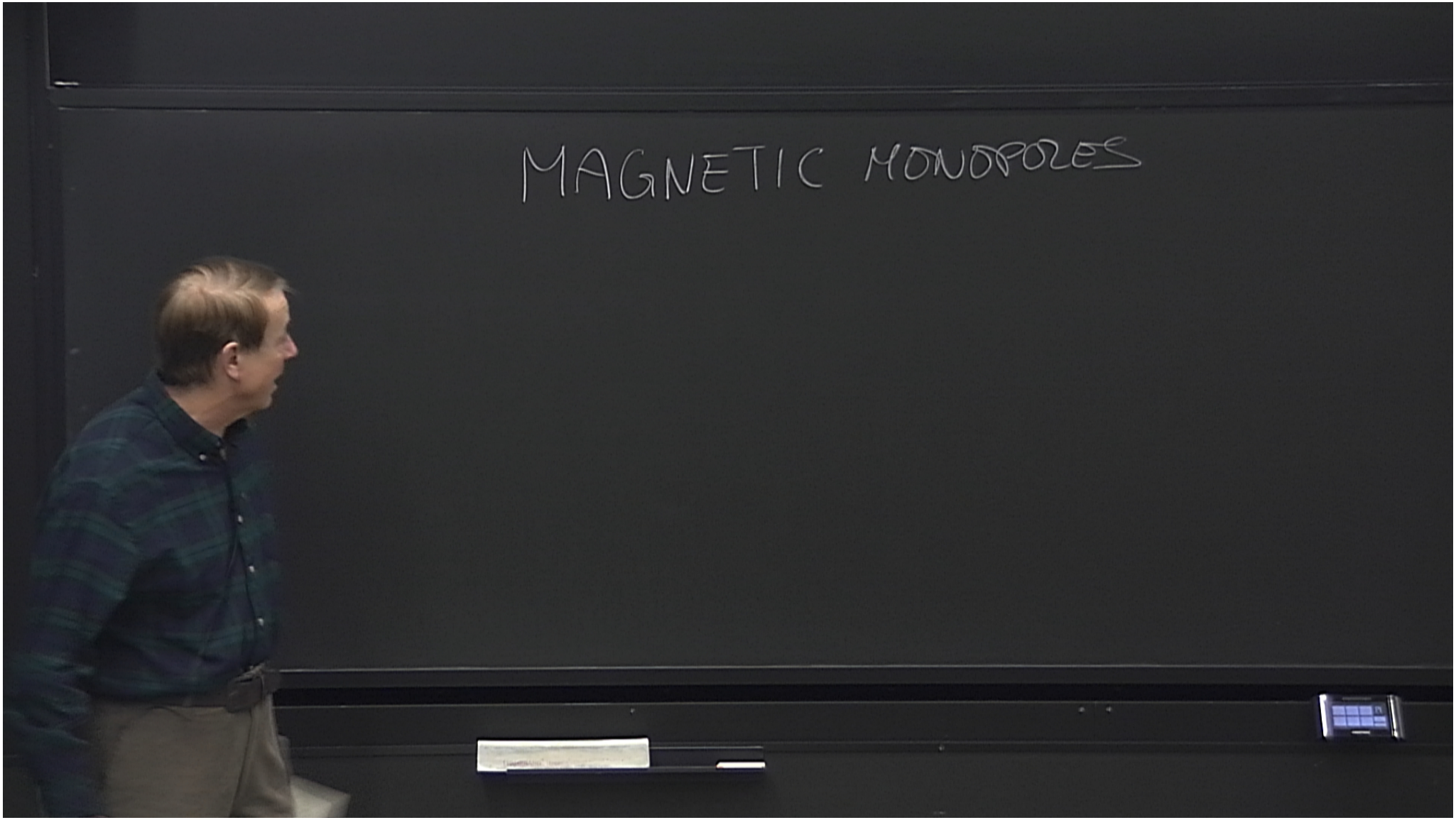
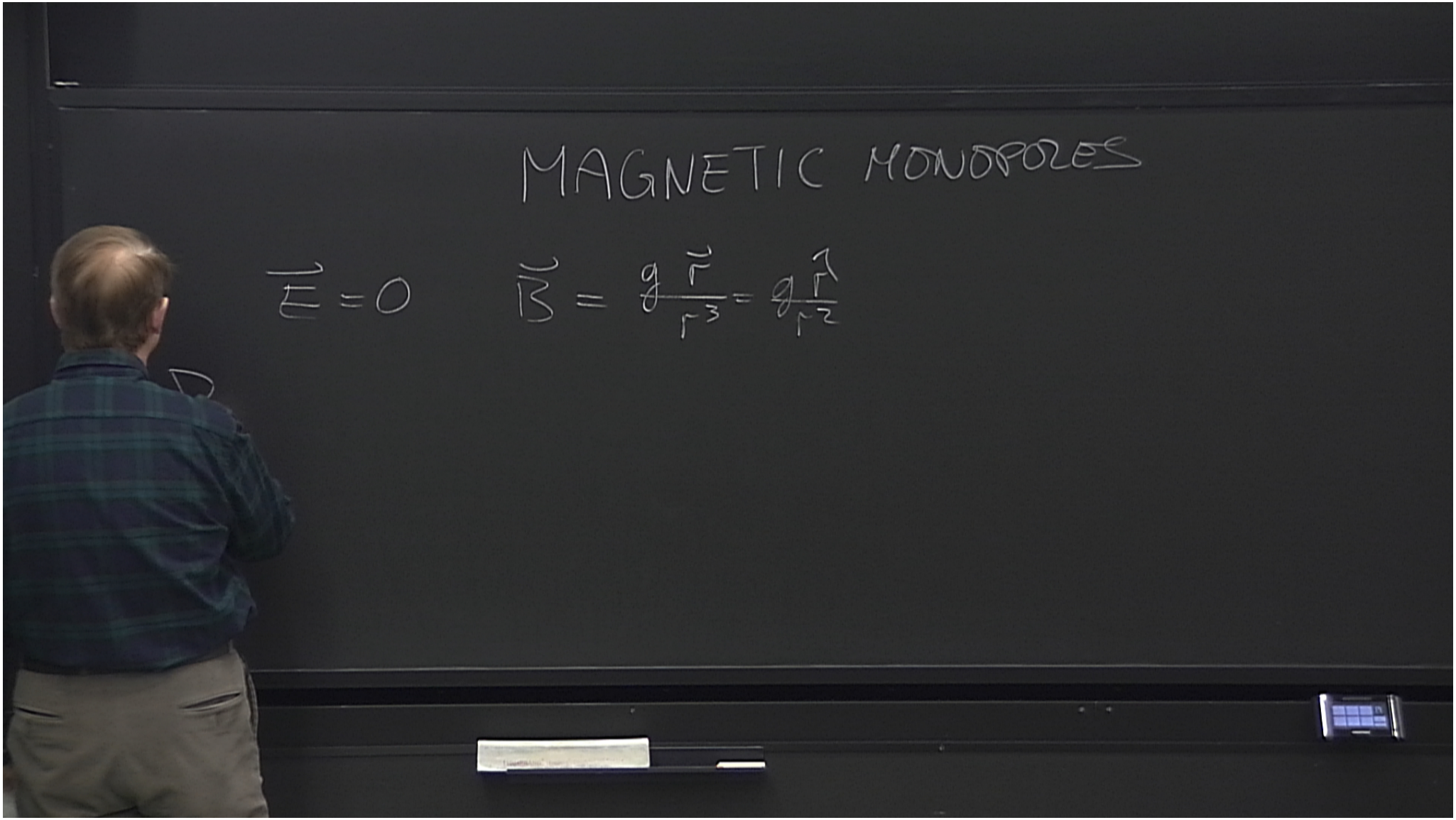
Title: 13/14 PSI - Beyond the Standard Model - Lecture 13

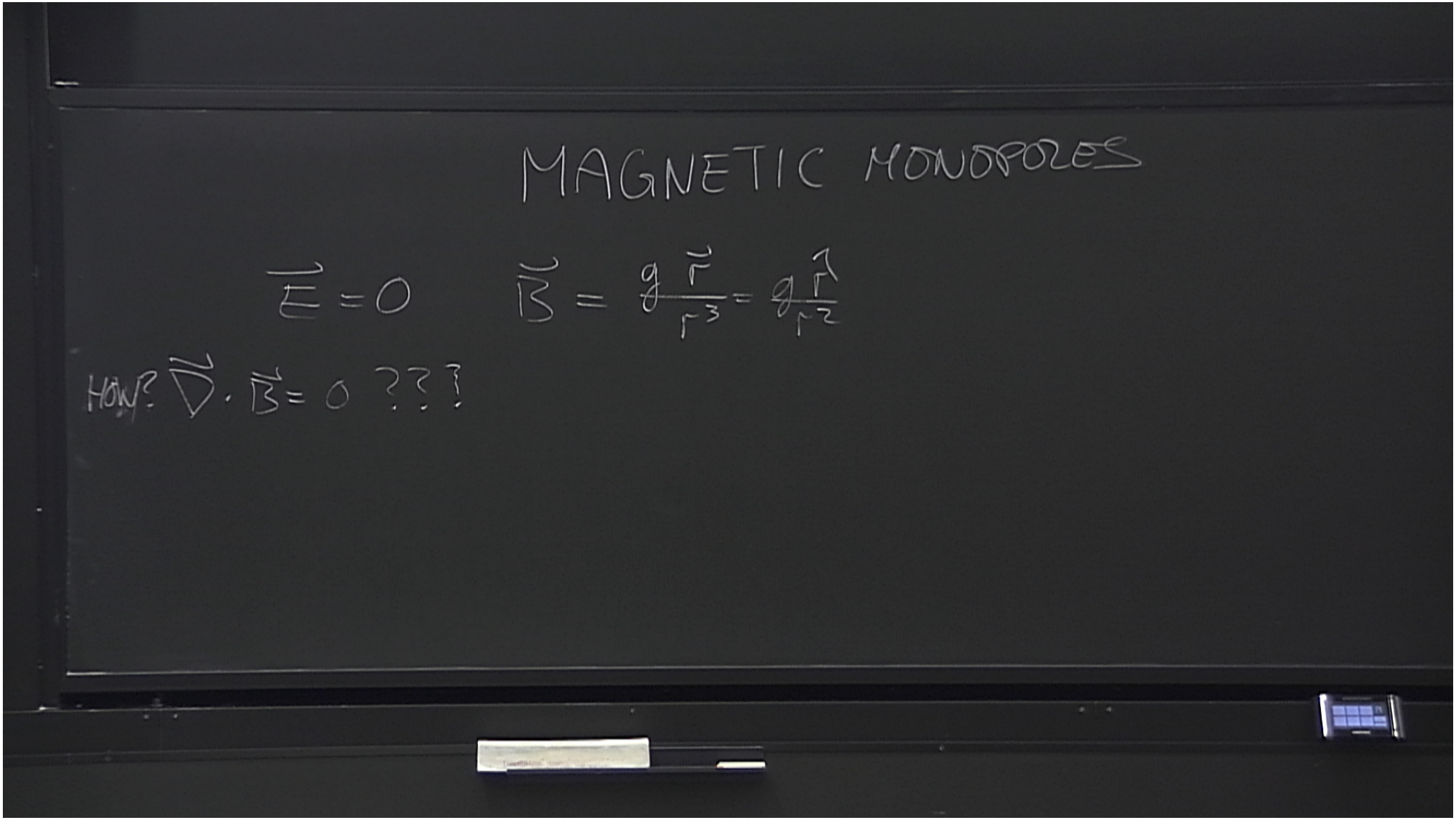
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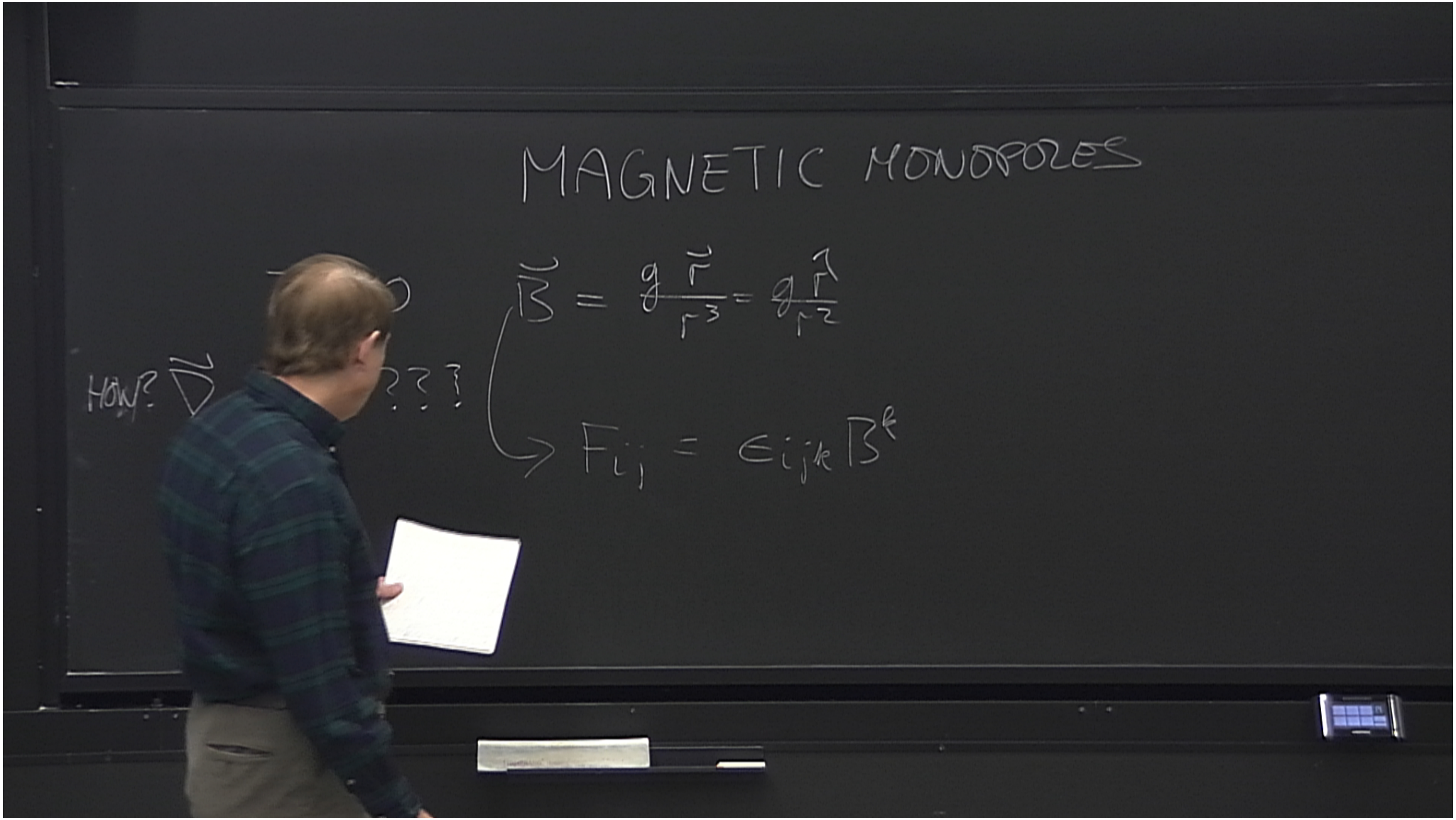
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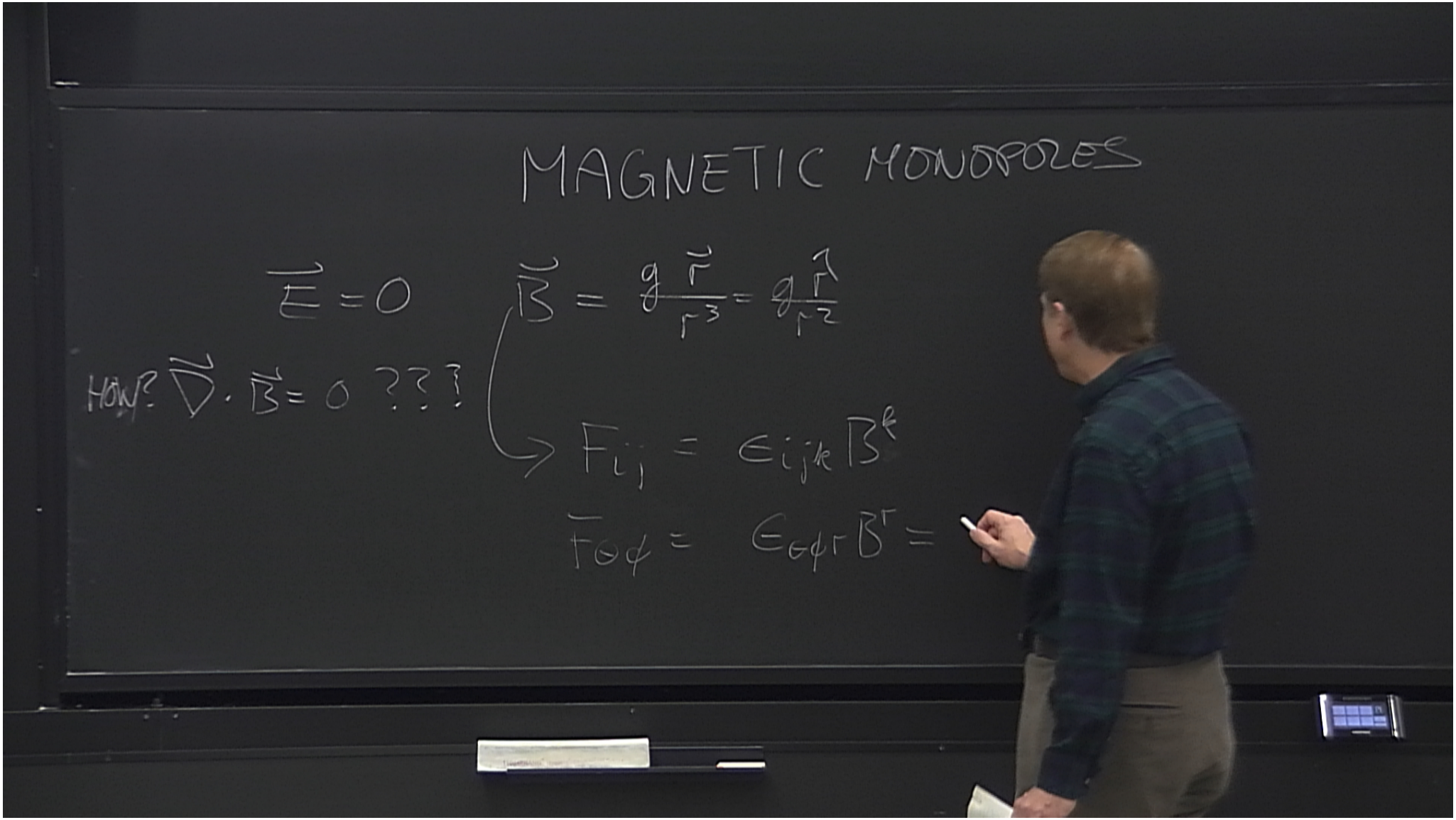
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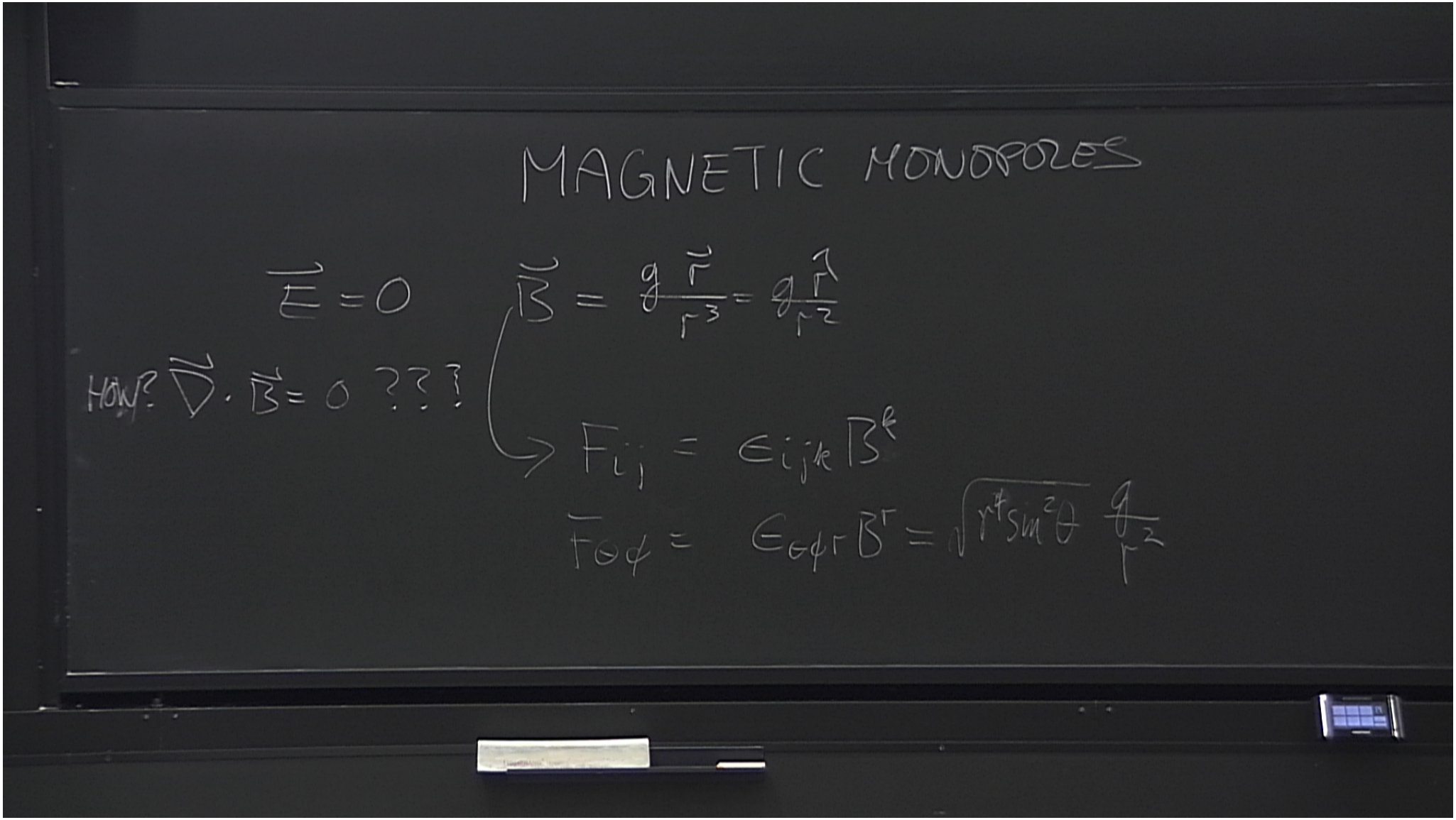

MAGNETIC MONOPOLES

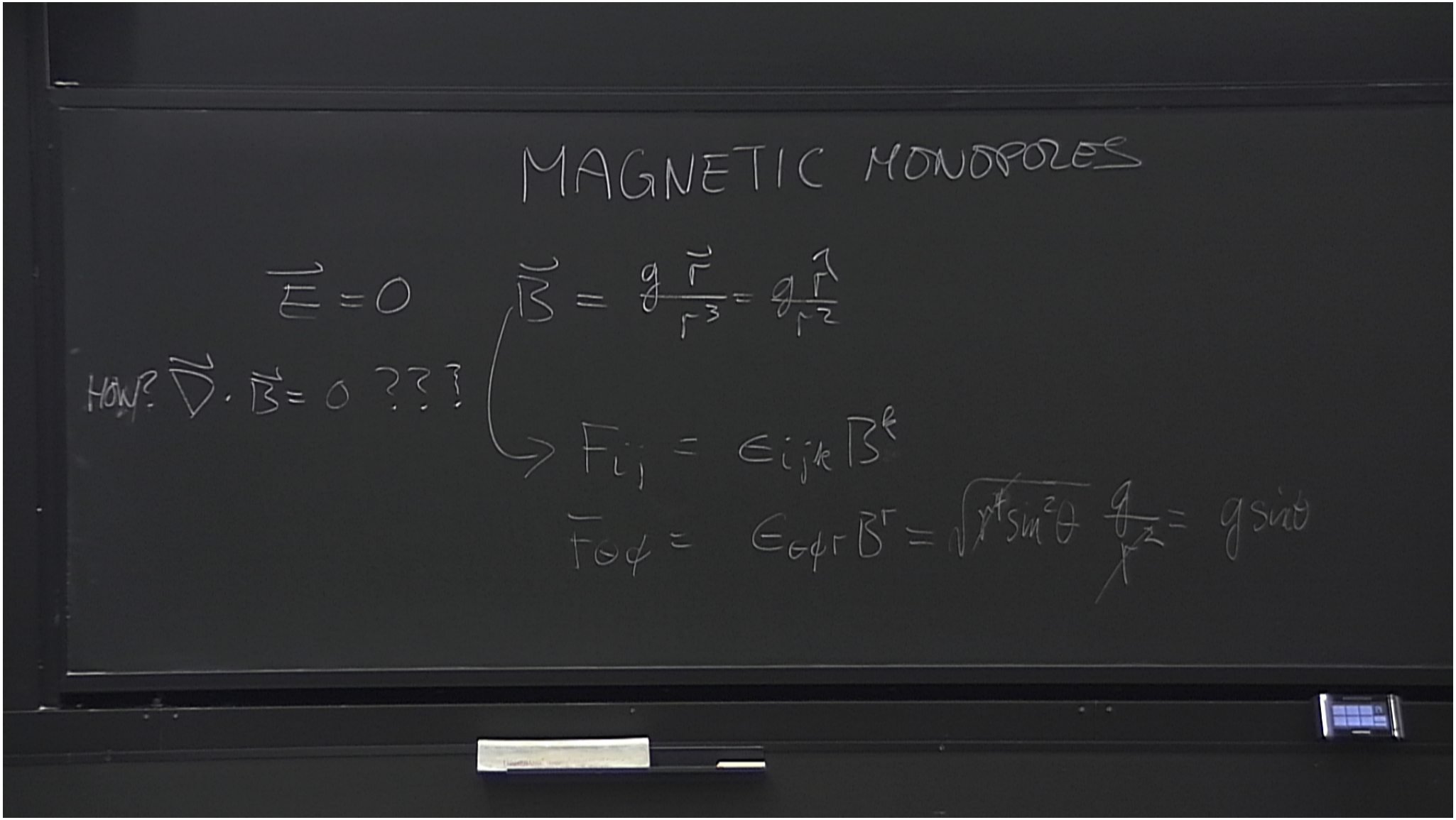

MAGNETIC MONOPOLES
$\vec{E} = 0$
$\vec{B} = g\frac{\vec{r}}{r^3} = g\frac{\hat{r}}{r^2}$

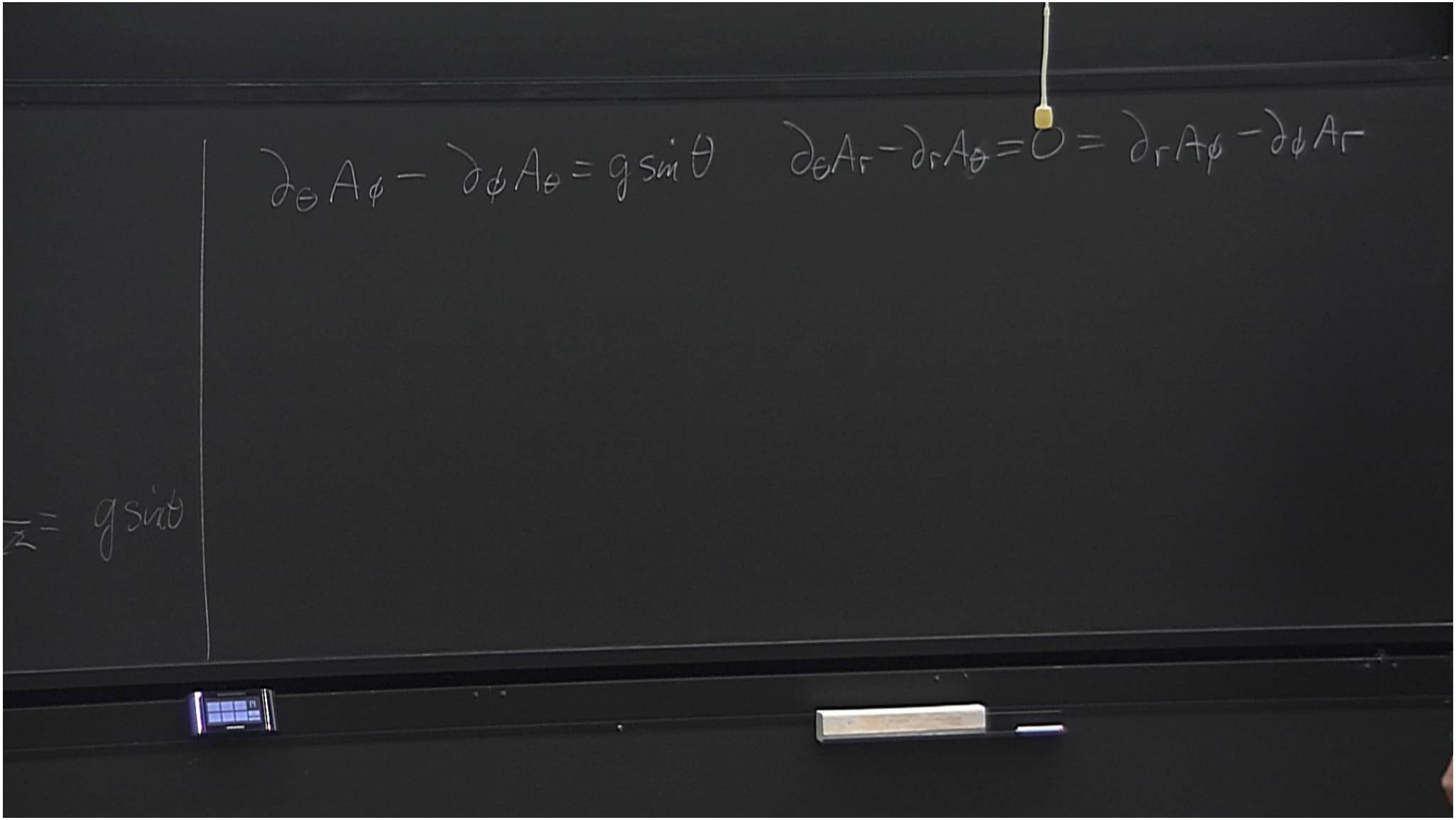

MAGNETIC MONOPOLES
$\vec{E} = 0$
$\vec{B} = g\frac{\vec{r}}{r^3} = g\frac{\hat{r}}{r^2}$
HOW? $\vec{\nabla} \cdot \vec{B} = 0$???

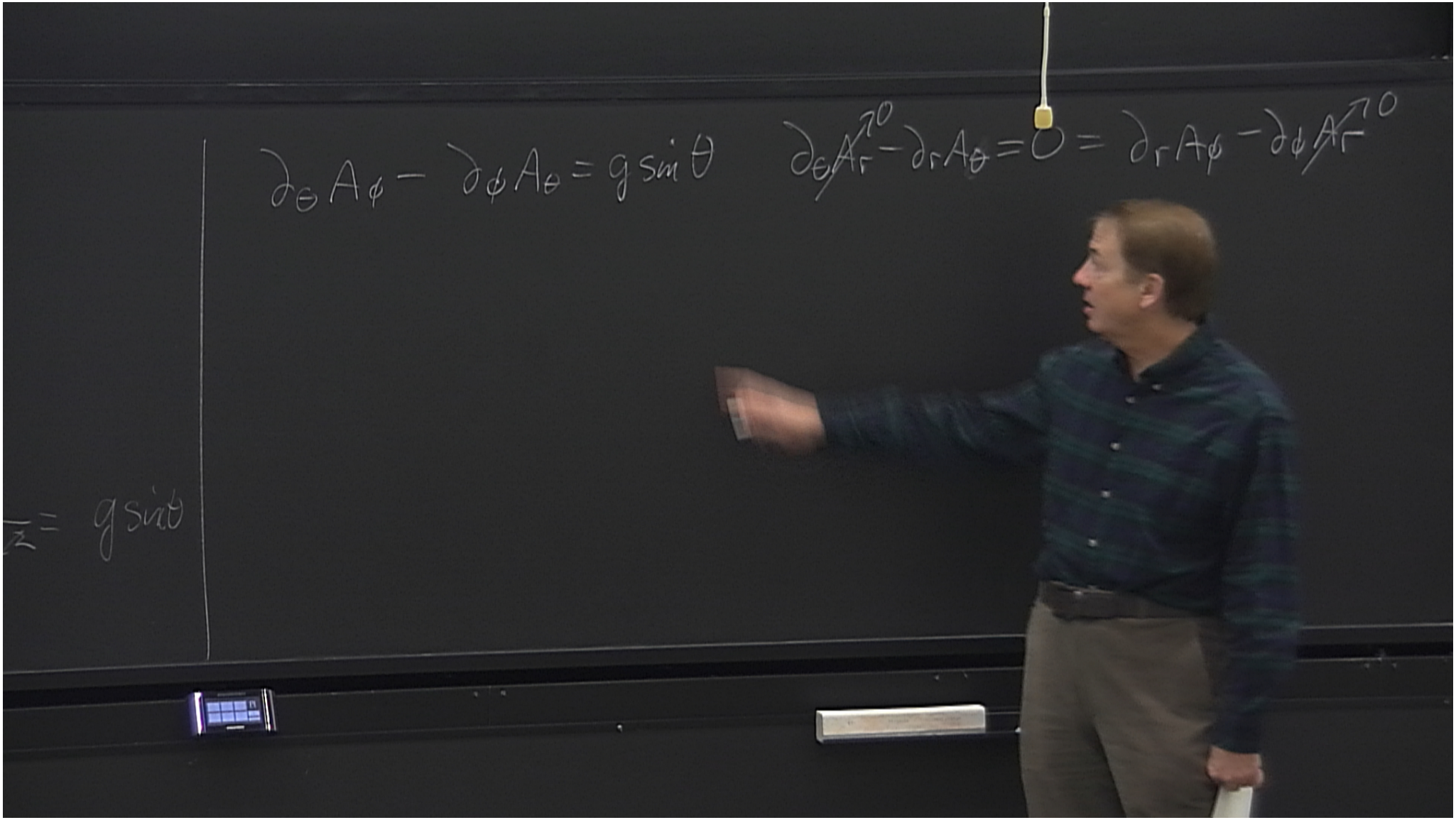

MAGNETIC MONOPOLES
HOW?
???
$\vec{B} = g\frac{\vec{r}}{r^3} = g\frac{\hat{r}}{r^2}$
$F_{ij} = \epsilon_{ijk} B^k$

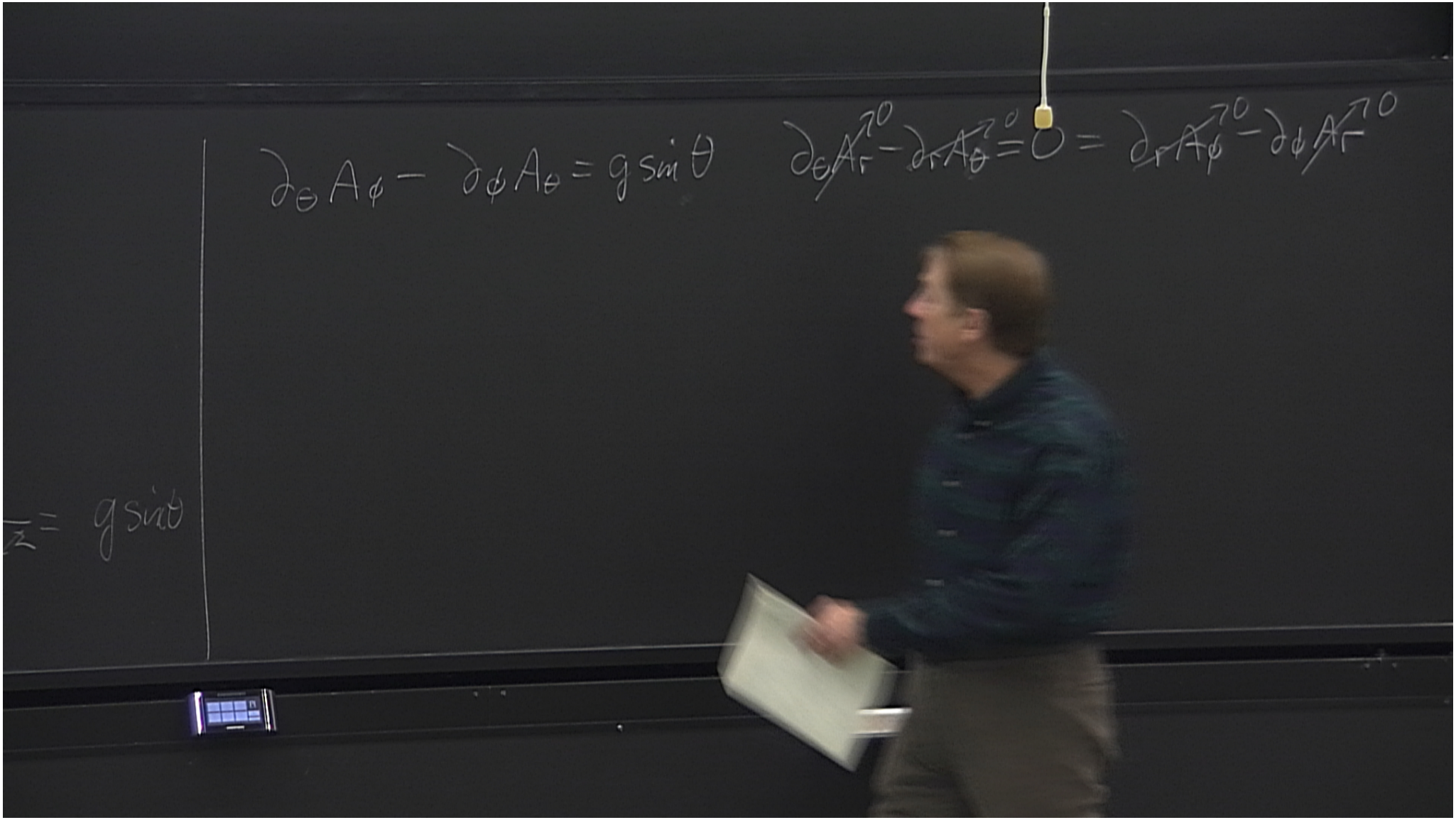

MAGNETIC MONOPOLES
$\vec{E} = 0$
$\vec{B} = g\frac{\vec{r}}{r^3} = g\frac{\hat{r}}{r^2}$
HOW? $\vec{\nabla} \cdot \vec{B} = 0$???
$F_{ij} = \epsilon_{ijk} B^k$
$F_{\theta\phi} = \epsilon_{\theta\phi r} B^r =$

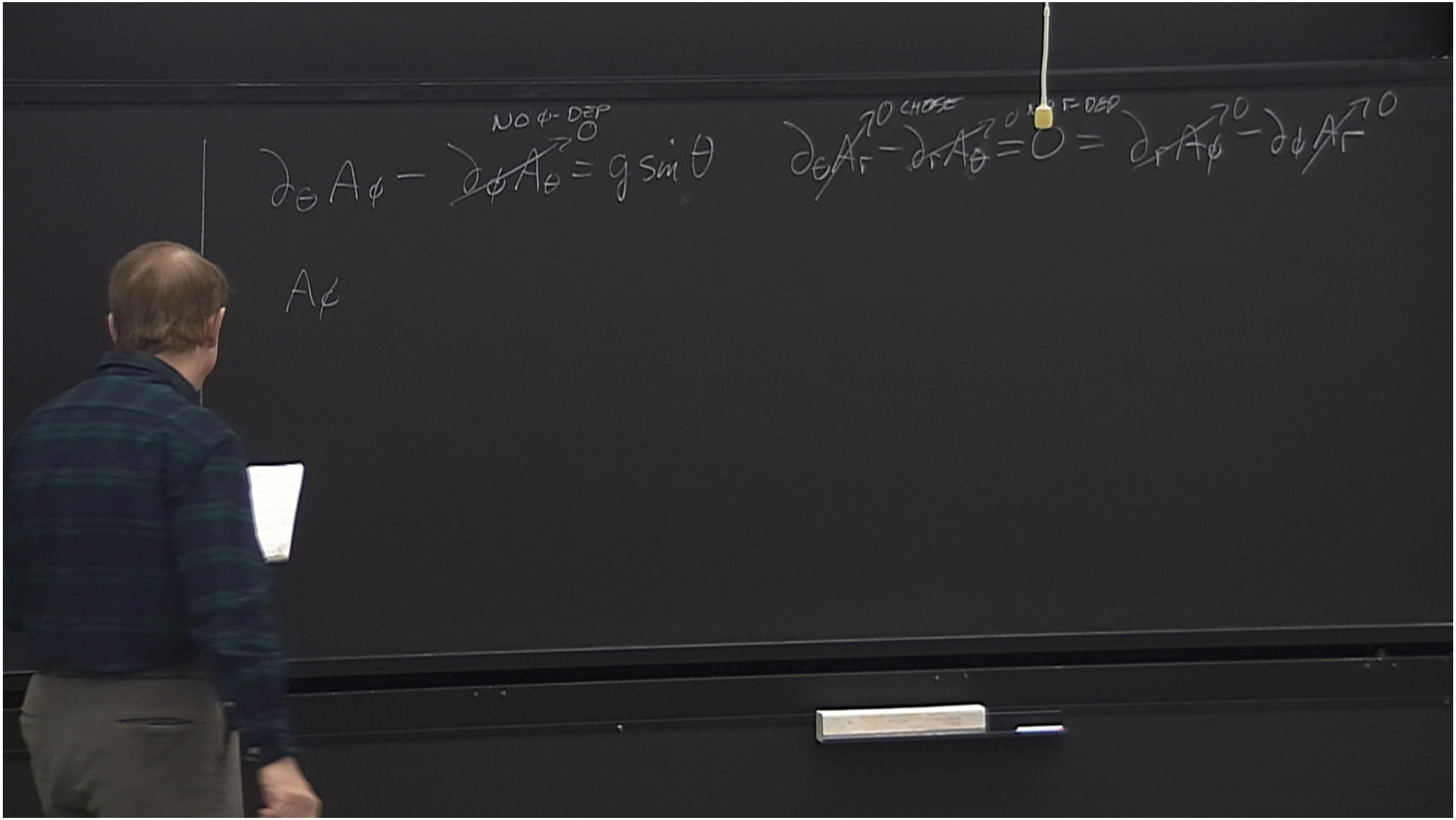

MAGNETIC MONOPOLES
$\vec{E} = 0$
$\vec{B} = g\frac{\vec{r}}{r^3} = g\frac{\hat{r}}{r^2}$
HOW? $\vec{\nabla}\cdot\vec{B} = 0$???
$F_{ij} = \epsilon_{ijk}B^k$
$F_{\theta\phi} = \epsilon_{\theta\phi r}B^r = \sqrt{r^4\sin^2\theta}\,\frac{g}{r^2}$

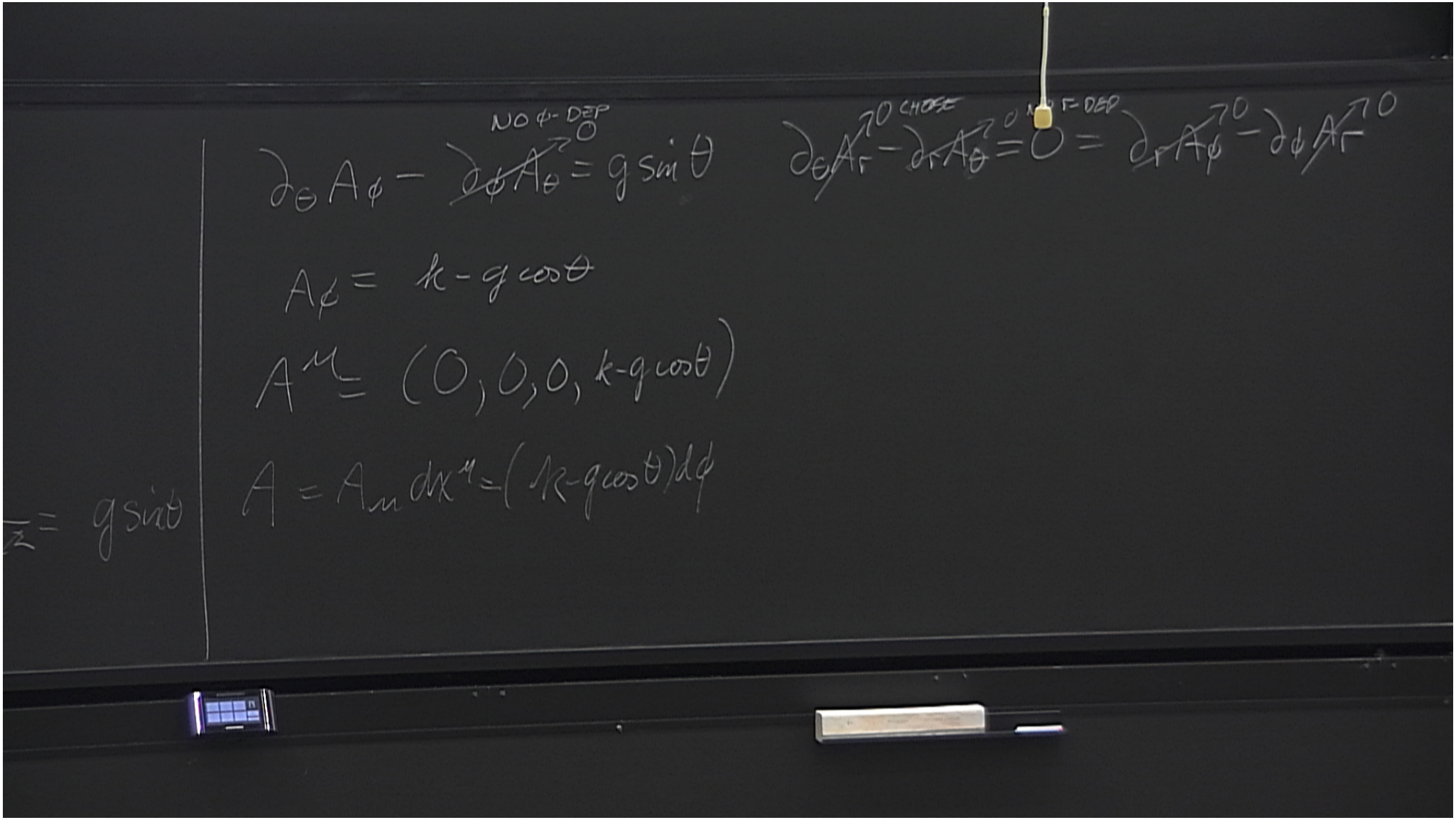

MAGNETIC MONOPOLES
$\vec{E} = 0$
$\vec{B} = g\frac{\vec{r}}{r^3} = g\frac{\hat{r}}{r^2}$
HOW? $\vec{\nabla}\cdot\vec{B} = 0$???
$F_{ij} = \epsilon_{ijk}B^k$
$F_{\theta\phi} = \epsilon_{\theta\phi r}B^r = \sqrt{r^4\sin^2\theta}\,\frac{g}{r^2} = g\sin\theta$


$\partial_\theta A_\phi - \partial_\phi A_\theta = g \sin\theta$
$\partial_\theta A_r - \partial_r A_\theta = 0 = \partial_r A_\phi - \partial_\phi A_r$
$= g \sin\theta$


$\partial_\theta A_\phi - \partial_\phi A_\theta = g \sin\theta$
$\partial_\theta A_r^{\nearrow 0} - \partial_r A_\theta = 0 = \partial_r A_\phi - \partial_\phi A_r^{\nearrow 0}$
$= g \sin\theta$


$\partial_\theta A_\phi - \partial_\phi A_\theta = g \sin\theta$
$\partial_\theta \cancel{A_r^{\nearrow 0}} - \partial_r A_\theta^{\nearrow 0} = 0 = \partial_r A_\phi^{\nearrow 0} - \partial_\phi \cancel{A_r^{\nearrow 0}}$
$= g \sin\theta$


NO φ- DEP
$\partial_\theta A_\phi - \partial_\phi A_\theta = g \sin\theta$
A_ϕ
CHOOSE
$\partial_\theta A_r - \partial_r A_\theta = 0 = \partial_r A_\phi - \partial_\phi A_r$
r-DEP


NO φ- DEP
$\partial_\theta A_\phi - \partial_\phi A_\theta = g \sin\theta$
$\partial_\theta A_r - \partial_r A_\theta = 0 = \partial_r A_\phi - \partial_\phi A_r$
CHOOSE
$A_\phi = k - g\cos\theta$
$A^\mu = (0, 0, 0, k - g\cos\theta)$
$A = A_\mu dx^\mu = (k - g\cos\theta) d\phi$
$= g \sin\theta$

NO ϕ-DEP

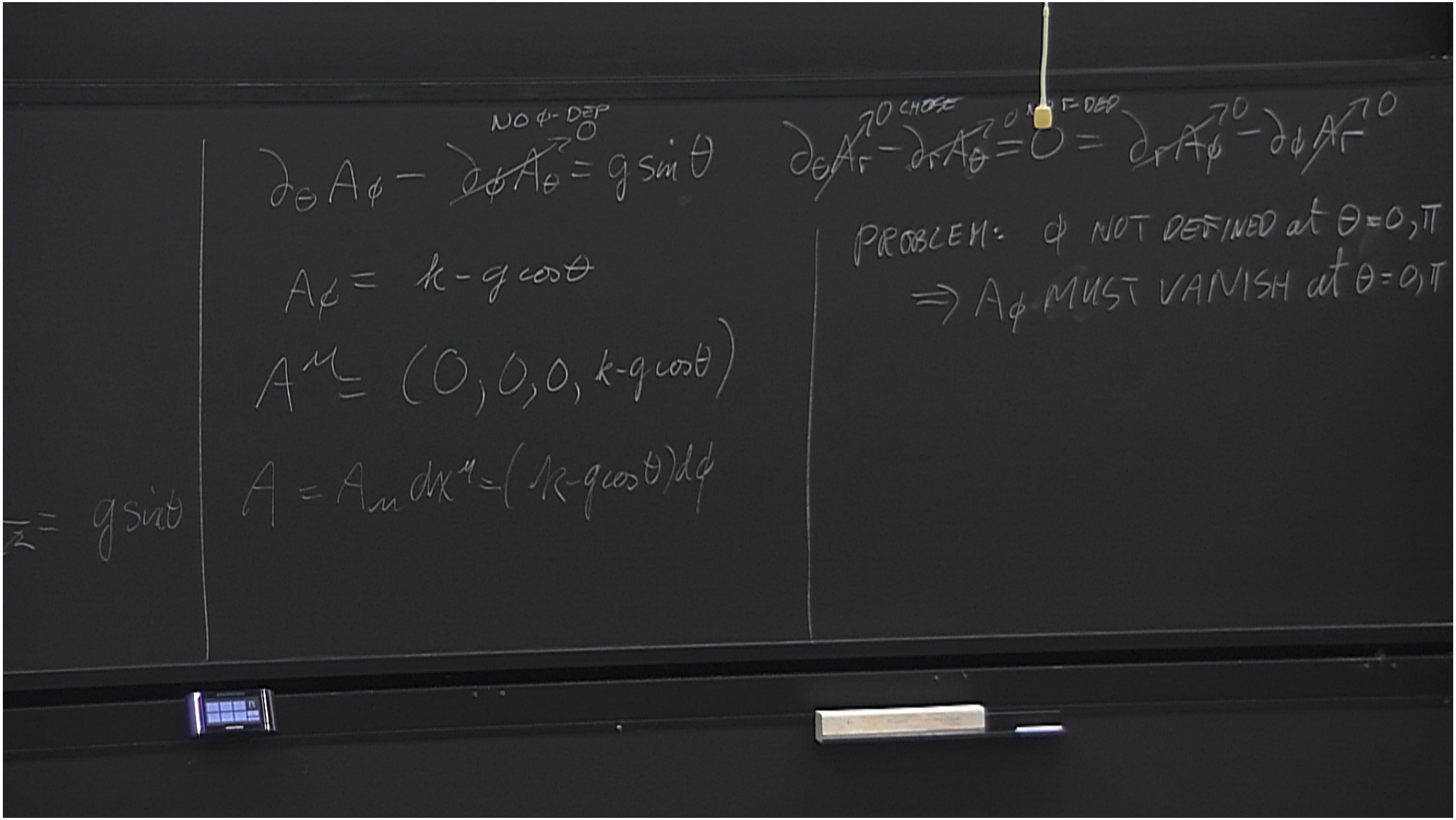
$\partial_\theta A_\phi - \cancel{\partial_\phi A_\theta}^{\to 0} = g \sin\theta$

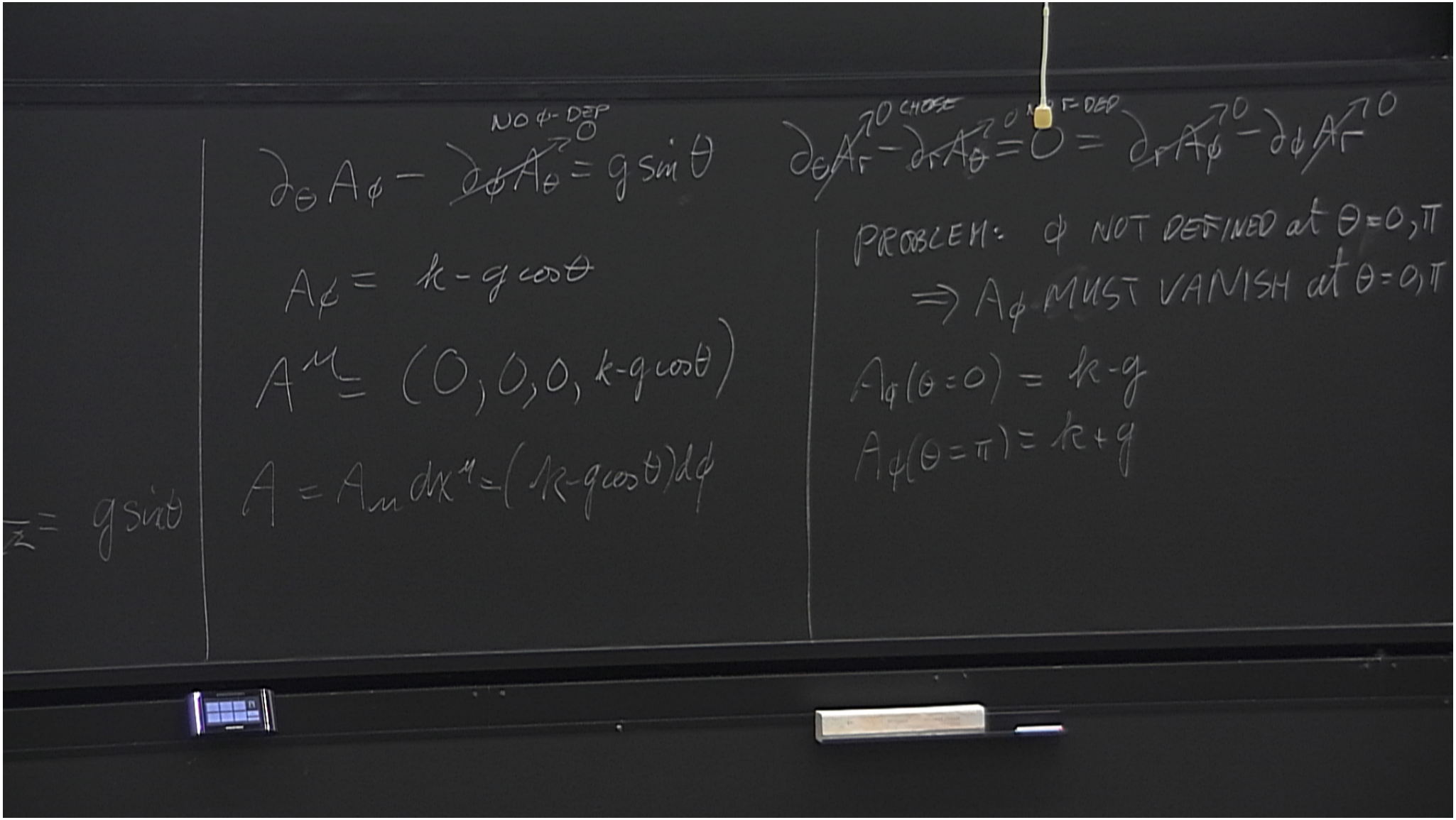
$\partial_\theta \cancel{A_r}^{\to 0\ \text{CHOSE}} - \partial_r \cancel{A_\theta}^{\to 0\ \text{NO r-DEP}} = 0 = \partial_r \cancel{A_\phi}^{\to 0} - \partial_\phi \cancel{A_r}^{\to 0}$

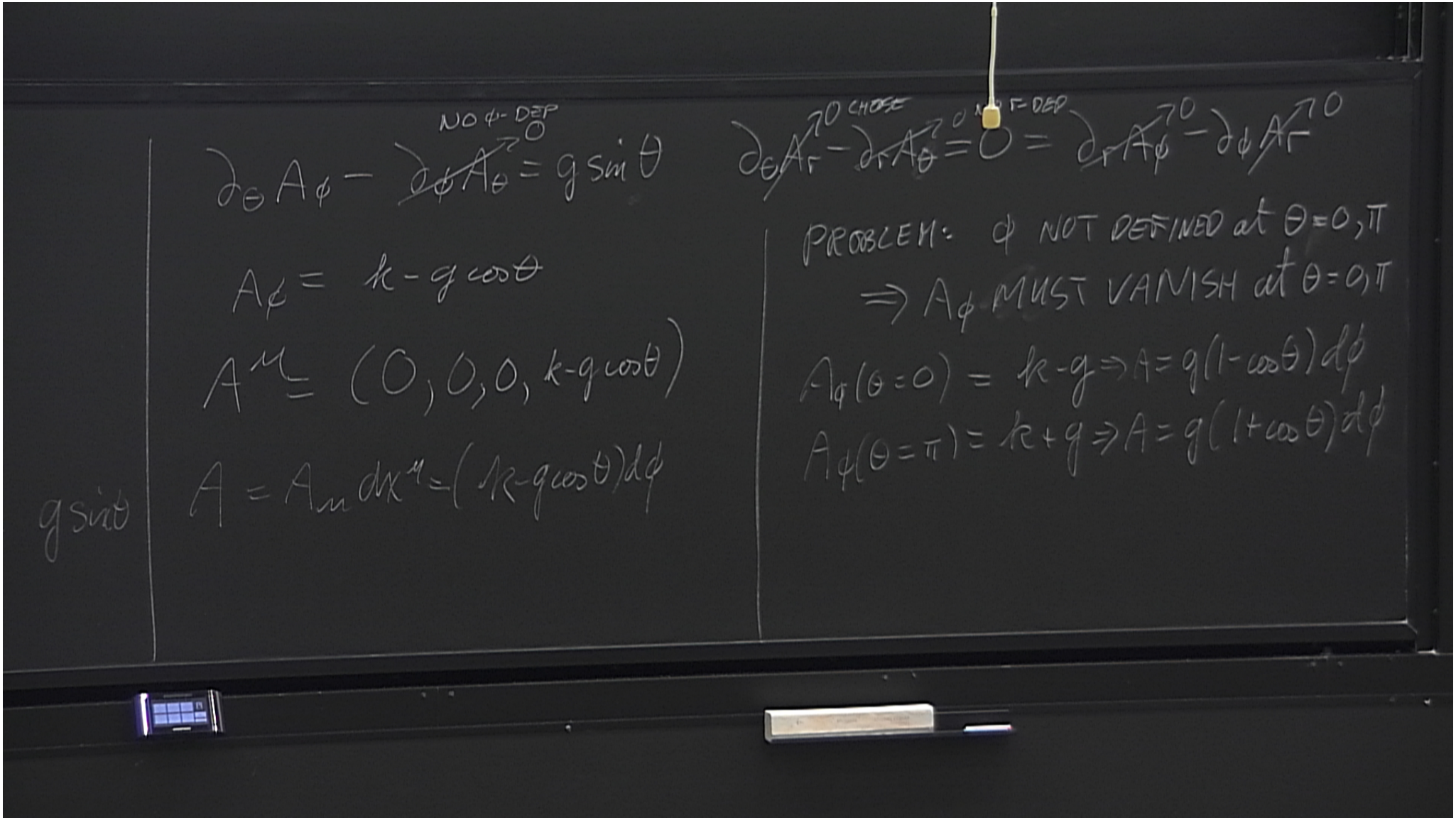
$A_\phi = k - g\cos\theta$

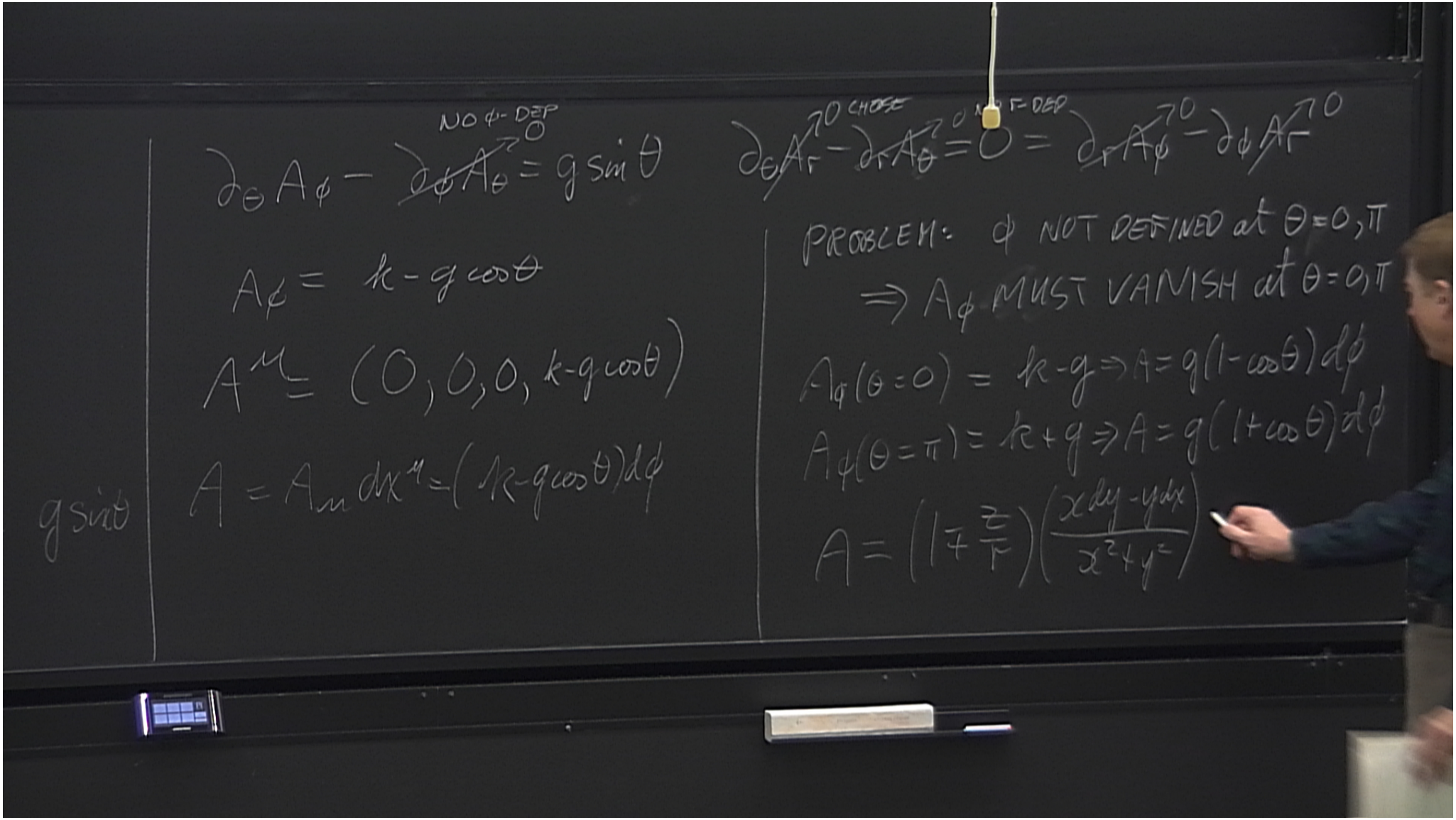
$A^\mu = (0, 0, 0, k - g\cos\theta)$

$A = A_\mu dx^\mu = (k - g\cos\theta)\, d\phi$


NO φ-DEP
$\partial_\theta A_\phi - \partial_\phi A_\theta = g \sin\theta$
$A_\phi = k - g\cos\theta$
$A^\mu = (0, 0, 0, k - g\cos\theta)$
$A = A_\mu dx^\mu = (k - g\cos\theta) d\phi$
$= g\sin\theta$
$\partial_\theta A_r - \partial_r A_\theta = 0 = \partial_r A_\phi - \partial_\phi A_r$
PROBLEM: ϕ NOT DEFINED at $\theta = 0, \pi$
$\Rightarrow A_\phi$ MUST VANISH at $\theta = 0, \pi$


NO φ-DEP
$\partial_\theta A_\phi - \partial_\phi A_\theta = g \sin\theta$
$A_\phi = k - g\cos\theta$
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$\partial_\theta A_r - \partial_r A_\theta = 0 = \partial_r A_\phi - \partial_\phi A_r$
PROBLEM: ϕ NOT DEFINED at $\theta = 0, \pi$
$\Rightarrow A_\phi$ MUST VANISH at $\theta = 0, \pi$
$A_\phi(\theta = 0) = k - g$
$A_\phi(\theta = \pi) = k + g$


NO φ-DEP
$\partial_\theta A_\phi - \partial_\phi A_\theta = g\sin\theta$
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$A^\mu = (0, 0, 0, k - g\cos\theta)$
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$g\sin\theta$
$\partial_\theta A_r - \partial_r A_\theta = 0 = \partial_r A_\phi - \partial_\phi A_r$
PROBLEM: ϕ NOT DEFINED at $\theta = 0, \pi$
$\Rightarrow A_\phi$ MUST VANISH at $\theta = 0, \pi$
$A_\phi(\theta = 0) = k - g \Rightarrow A = g(1 - \cos\theta)d\phi$
$A_\phi(\theta = \pi) = k + g \Rightarrow A = g(1 + \cos\theta)d\phi$


NO φ-DEP
$\partial_\theta A_\phi - \partial_\phi A_\theta = g\sin\theta$
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$A = \left(1 \mp \frac{z}{r}\right)\left(\frac{x\,dy - y\,dx}{x^2 + y^2}\right)$

NO ϕ-DEP

$\partial_\theta A_\phi - \partial_\phi A_\theta = g \sin\theta$

$A_\phi = k - g\cos\theta$

$A^\mu = (0, 0, 0, k - g\cos\theta)$

$A = A_\mu dx^\mu = (k - g\cos\theta)\, d\phi$

$g \sin\theta$

CHOOSE

$\partial_\theta A_r - \partial_r A_\theta = 0 = \partial_r A_\phi - \partial_\phi A_r$

PROBLEM: ϕ NOT DEFINED at $\theta = 0, \pi$

$\Rightarrow A_\phi$ MUST VANISH at $\theta = 0, \pi$

$A_\phi(\theta = 0) = k - g \Rightarrow A = g(1 - \cos\theta)\, d\phi$

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$A = \left(1 \mp \frac{z}{r}\right)\left(\frac{x\,dy - y\,dx}{x^2 + y^2}\right) = \begin{cases} 0 & z = +r \\ \infty & z = -r \end{cases}$

$g \sin\theta$

NO ϕ-DEP

$\partial_\theta A_\phi - \partial_\phi A_\theta = g \sin\theta$

$A_\phi = k - g\cos\theta$

$A^\mu = (0, 0, 0, k - g\cos\theta)$

$A = A_\mu dx^\mu = (k - g\cos\theta) d\phi$

$\partial_\theta A_r - \partial_r A_\theta = 0 = \partial_r A_\phi - \partial_\phi A_r$

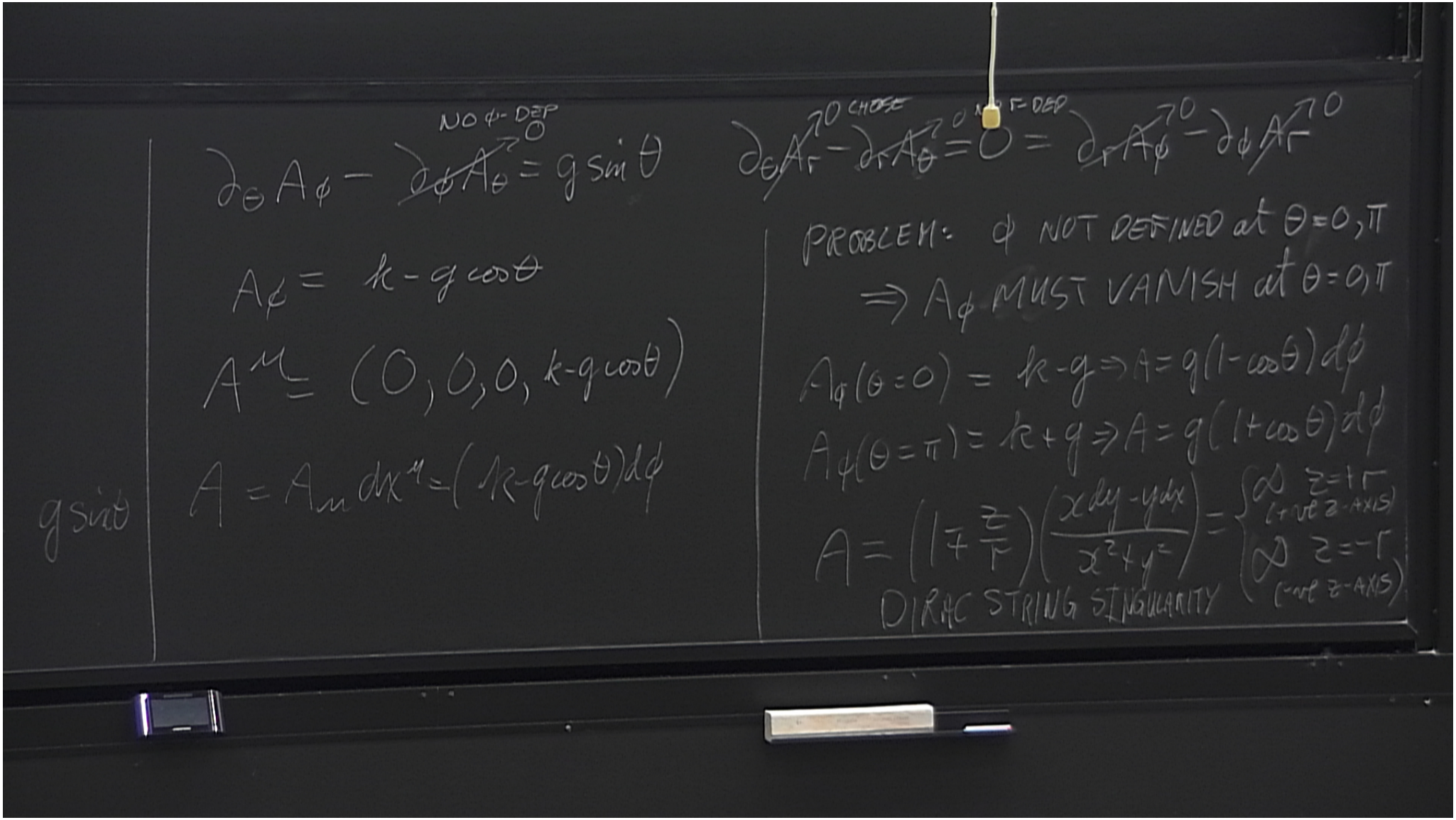
PROBLEM: ϕ NOT DEFINED at $\theta = 0, \pi$

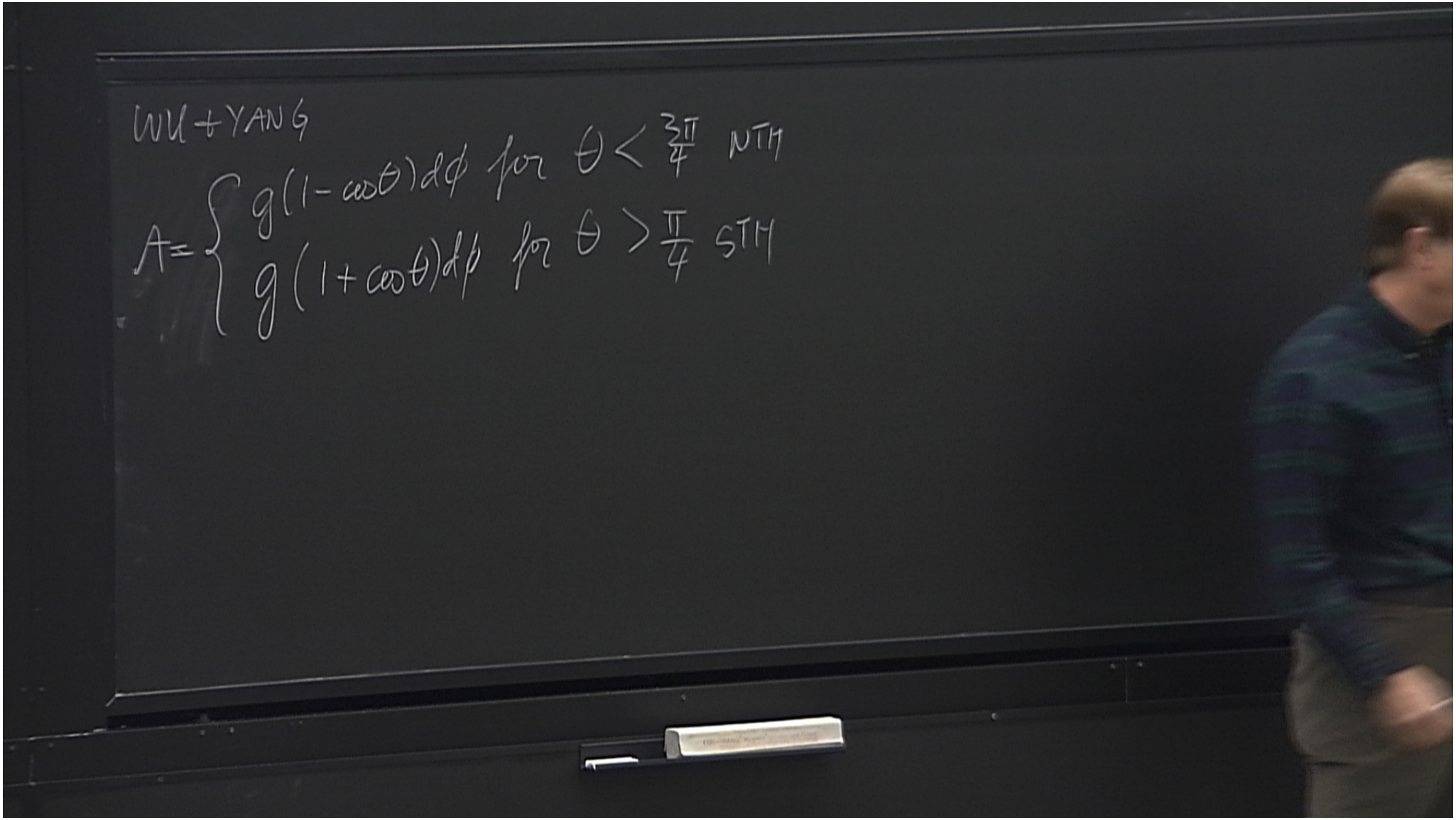
$\Rightarrow A_\phi$ MUST VANISH at $\theta = 0, \pi$

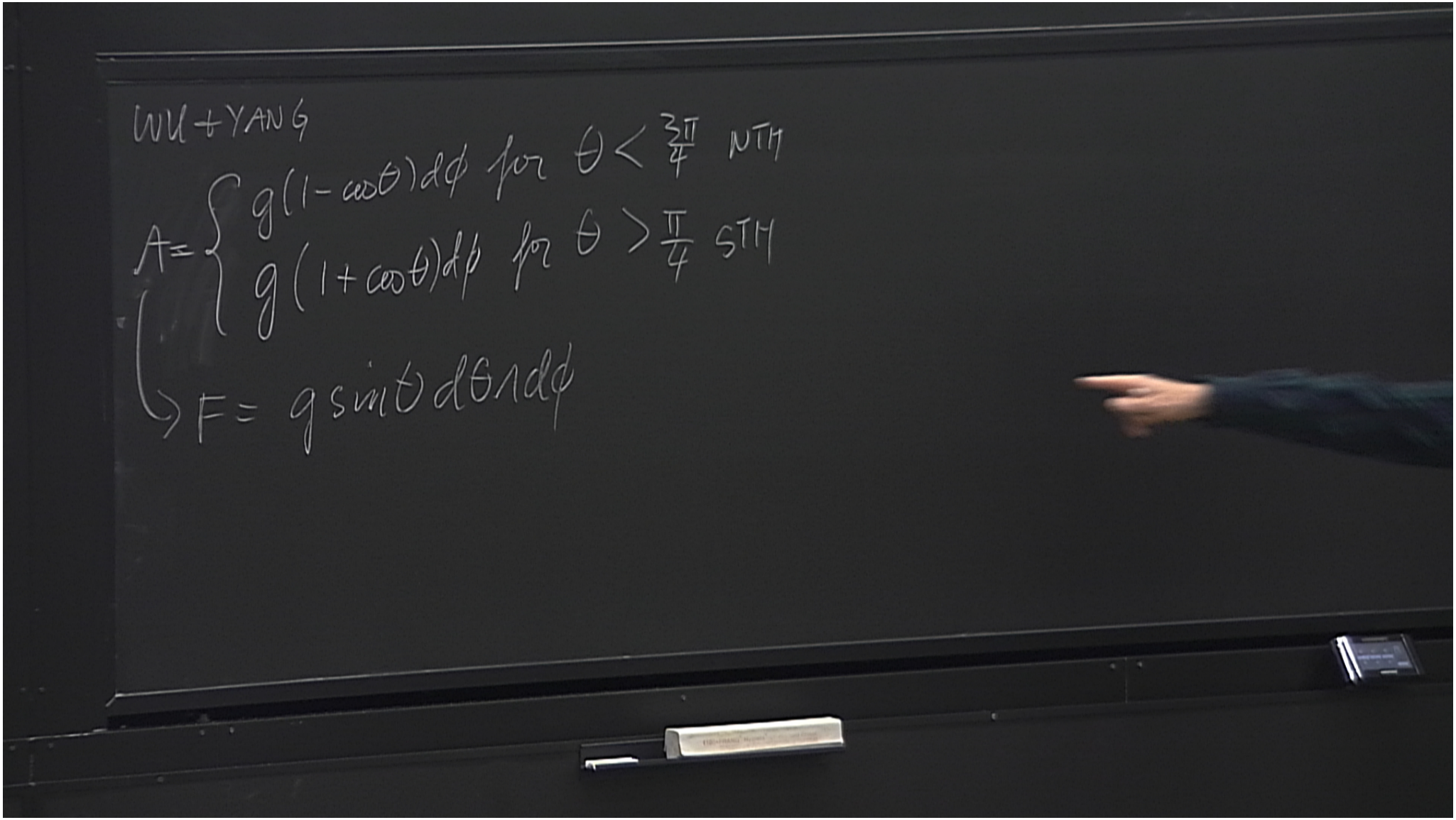
$A_\phi(\theta = 0) = k - g \Rightarrow A = g(1 - \cos\theta) d\phi$

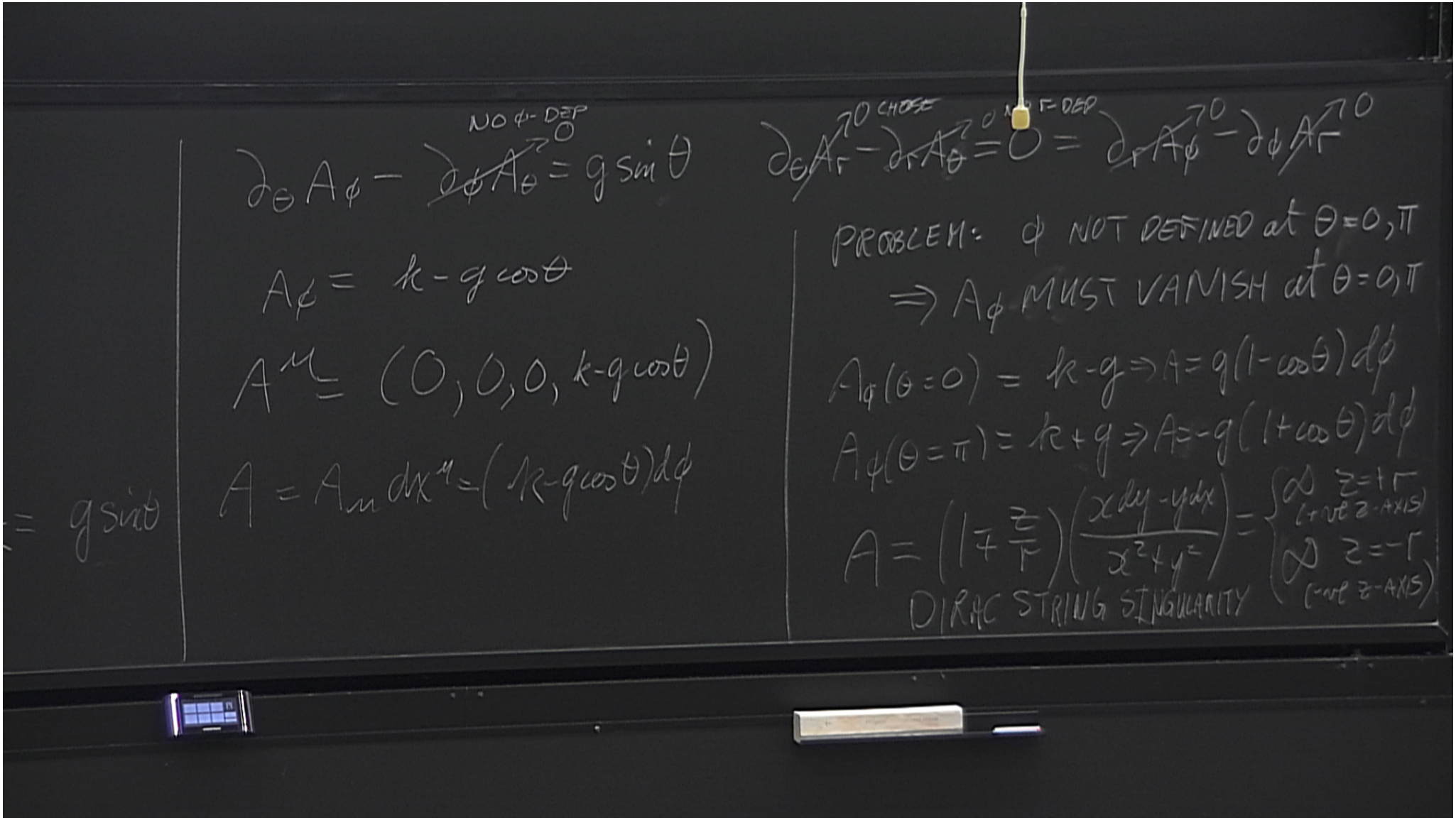
$A_\phi(\theta = \pi) = k + g \Rightarrow A = g(1 + \cos\theta) d\phi$

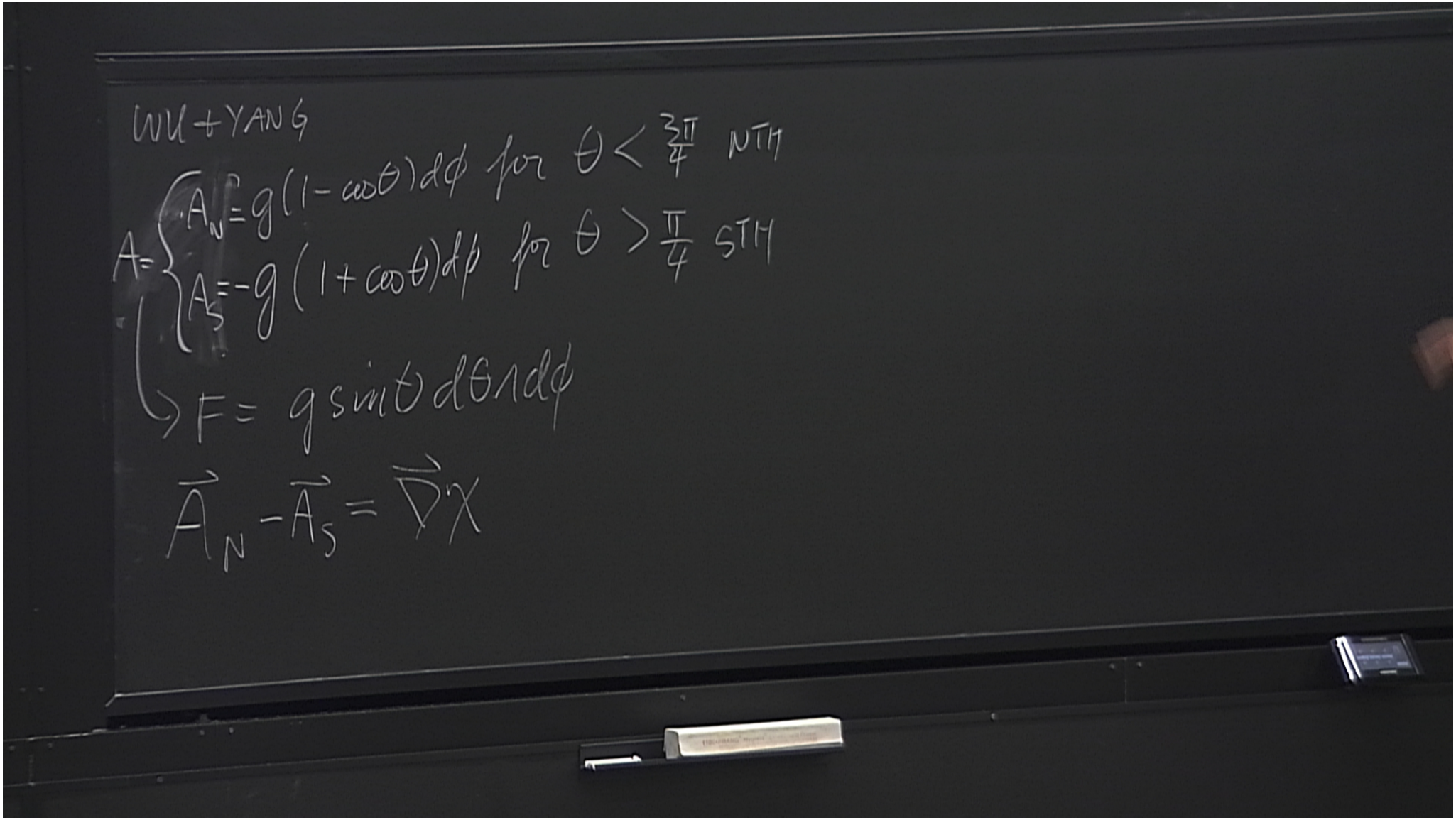
$A = \left(1 \mp \frac{z}{r}\right)\left(\frac{x\,dy - y\,dx}{x^2 + y^2}\right) = \begin{cases} \infty & z = +r \\ & (+ve\ z\text{-AXIS}) \\ \infty & z = -r \\ & (-ve\ z\text{-AXIS}) \end{cases}$

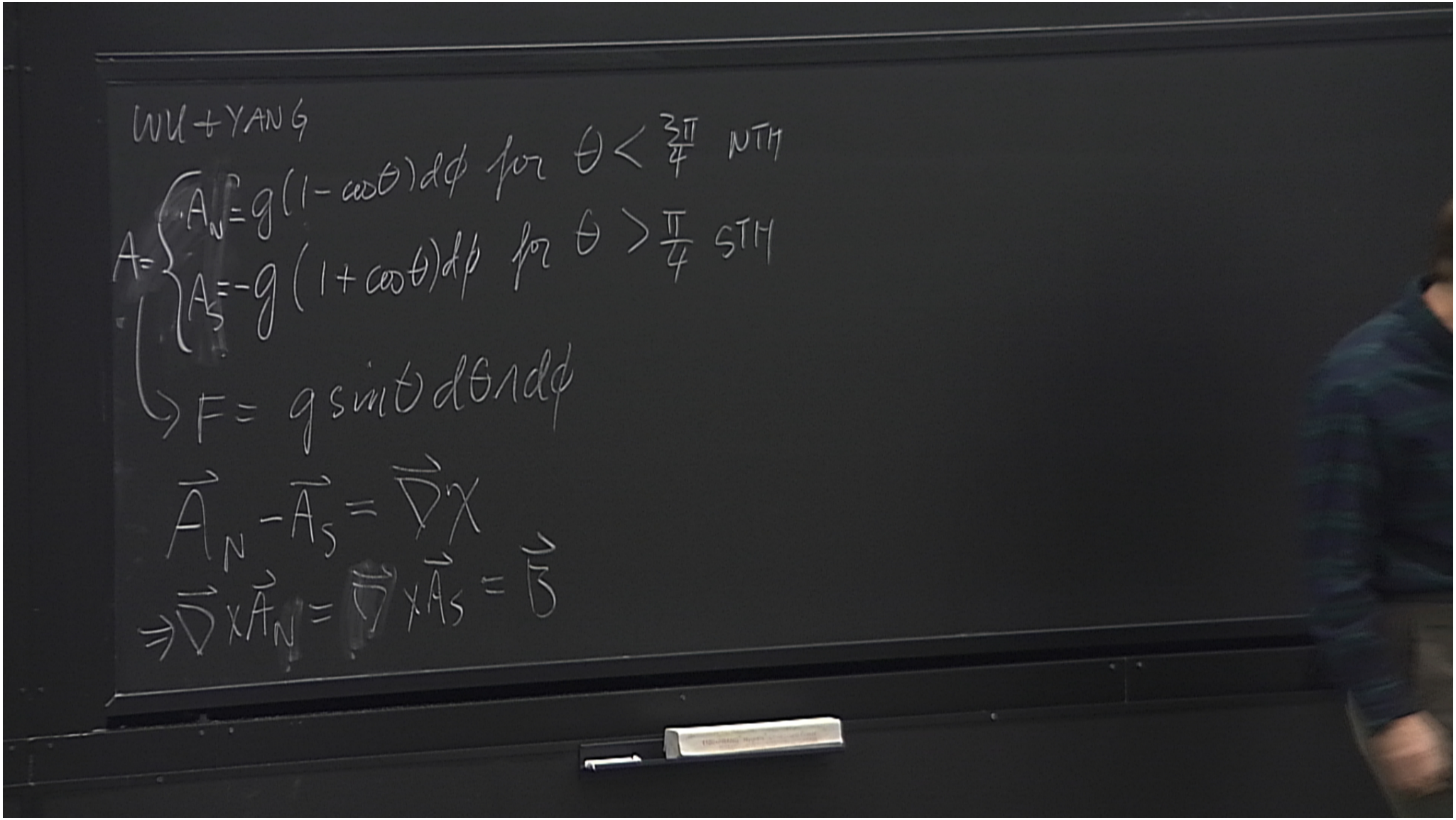

NO φ-DEP
$\partial_\theta A_\phi - \partial_\phi A_\theta = g \sin\theta$
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DIRAC STRING SINGULARITY


WU + YANG
$A = \begin{cases} g(1-\cos\theta)\,d\phi & \text{for } \theta < \frac{3\pi}{4} \text{ NTH} \\ g(1+\cos\theta)\,d\phi & \text{for } \theta > \frac{\pi}{4} \text{ STH} \end{cases}$


WU + YANG
$A = \begin{cases} g(1-\cos\theta)\,d\phi & \text{for } \theta < \frac{3\pi}{4} \text{ NTH} \\ g(1+\cos\theta)\,d\phi & \text{for } \theta > \frac{\pi}{4} \text{ STH} \end{cases}$
$\hookrightarrow F = g \sin\theta\, d\theta \wedge d\phi$


NO φ-DEP
$\partial_\theta A_\phi - \partial_\phi A_\theta = g\sin\theta$
$A_\phi = k - g\cos\theta$
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$A = \left(1 \mp \frac{z}{r}\right)\left(\frac{x\,dy - y\,dx}{x^2 + y^2}\right) = \begin{cases} \infty & z = +r \\ & (+ve\ z\text{-AXIS}) \\ \infty & z = -r \\ & (-ve\ z\text{-AXIS}) \end{cases}$
DIRAC STRING SINGULARITY


WU + YANG
$A = \begin{cases} A_N = g(1-\cos\theta)\,d\phi \text{ for } \theta < \frac{3\pi}{4} \text{ NTH} \\ A_S = -g(1+\cos\theta)\,d\phi \text{ for } \theta > \frac{\pi}{4} \text{ STH} \end{cases}$
$F = g \sin\theta\, d\theta \wedge d\phi$
$\vec{A}_N - \vec{A}_S = \vec{\nabla}\chi$


WU + YANG
$A = \begin{cases} A_N = g(1-\cos\theta)\,d\phi \text{ for } \theta < \frac{3\pi}{4} \text{ NTH} \\ A_S = -g(1+\cos\theta)\,d\phi \text{ for } \theta > \frac{\pi}{4} \text{ STH} \end{cases}$
$\hookrightarrow F = g\sin\theta\, d\theta \wedge d\phi$
$\vec{A}_N - \vec{A}_S = \vec{\nabla}\chi$
$\Rightarrow \vec{\nabla} \times \vec{A}_N = \vec{\nabla} \times \vec{A}_S = \vec{B}$

WU + YANG

$$A = \begin{cases} A_N = g(1-\cos\theta)\,d\phi \text{ for } \theta < \frac{3\pi}{4} \text{ NTH} \\ A_S = -g(1+\cos\theta)\,d\phi \text{ for } \theta > \frac{\pi}{4} \text{ STH} \end{cases}$$

$$\hookrightarrow F = g\sin\theta\, d\theta \wedge d\phi$$

$$\vec{A}_N - \vec{A}_S = \vec{\nabla}\chi \Rightarrow 2g\,d\phi = \vec{\nabla}\chi \cdot d\vec{z}$$

$$\Rightarrow \vec{\nabla} \times \vec{A}_N = \vec{\nabla} \times \vec{A}_S = \vec{B} \quad \Rightarrow \chi = 2g\phi$$

NOT SINGLE VALUED

WU + YANG

$$A = \begin{cases} A_N = g(1-\cos\theta)\,d\phi \text{ for } \theta < \frac{3\pi}{4} \text{ NTH} \\ A_S = -g(1+\cos\theta)\,d\phi \text{ for } \theta > \frac{\pi}{4} \text{ STH} \end{cases}$$

$$\hookrightarrow F = g\sin\theta\, d\theta \wedge d\phi$$

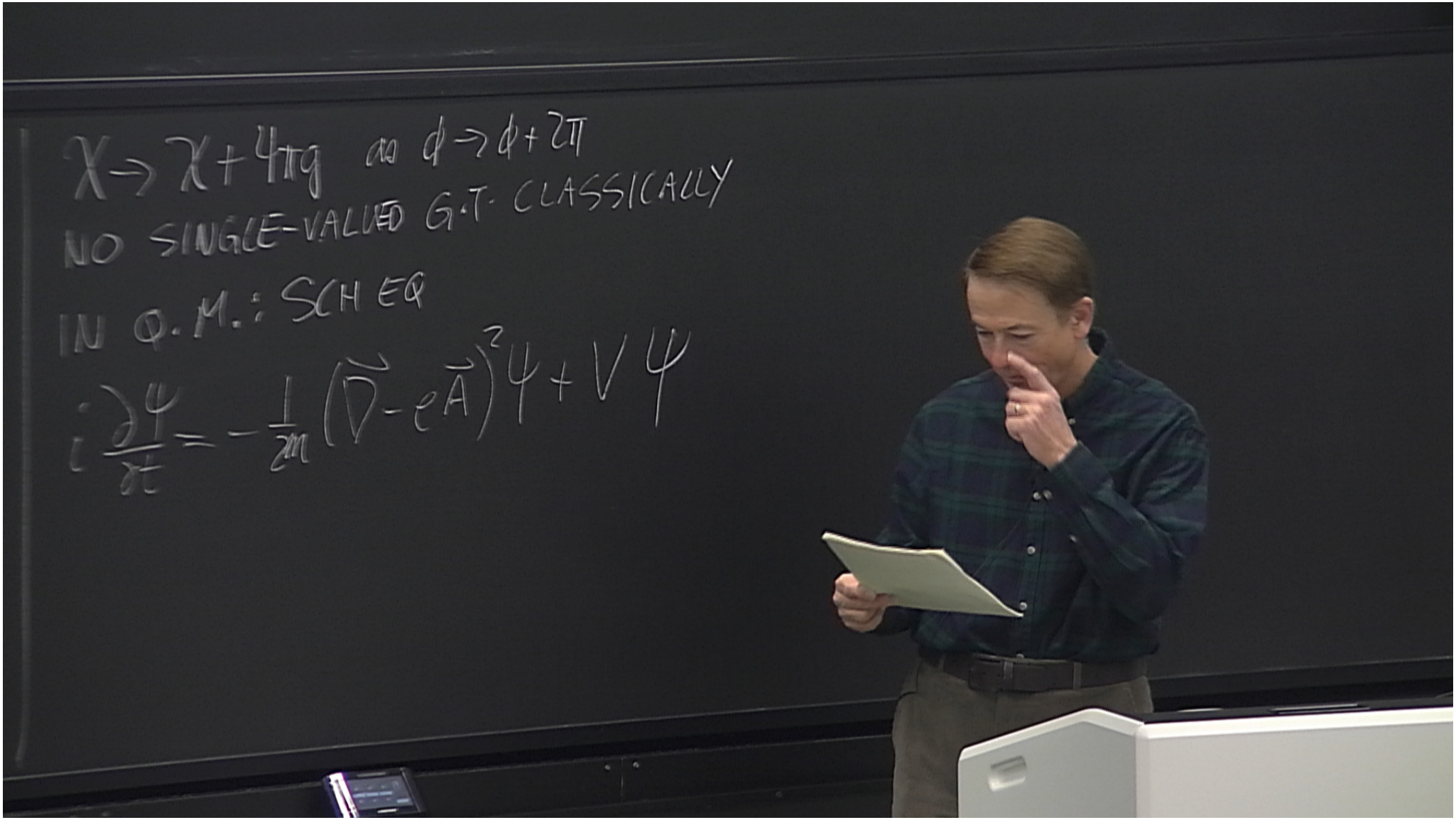
$$\vec{A}_N - \vec{A}_S = \vec{\nabla}\chi \Rightarrow 2g\,d\phi = \vec{\nabla}\chi \cdot d\vec{x}$$

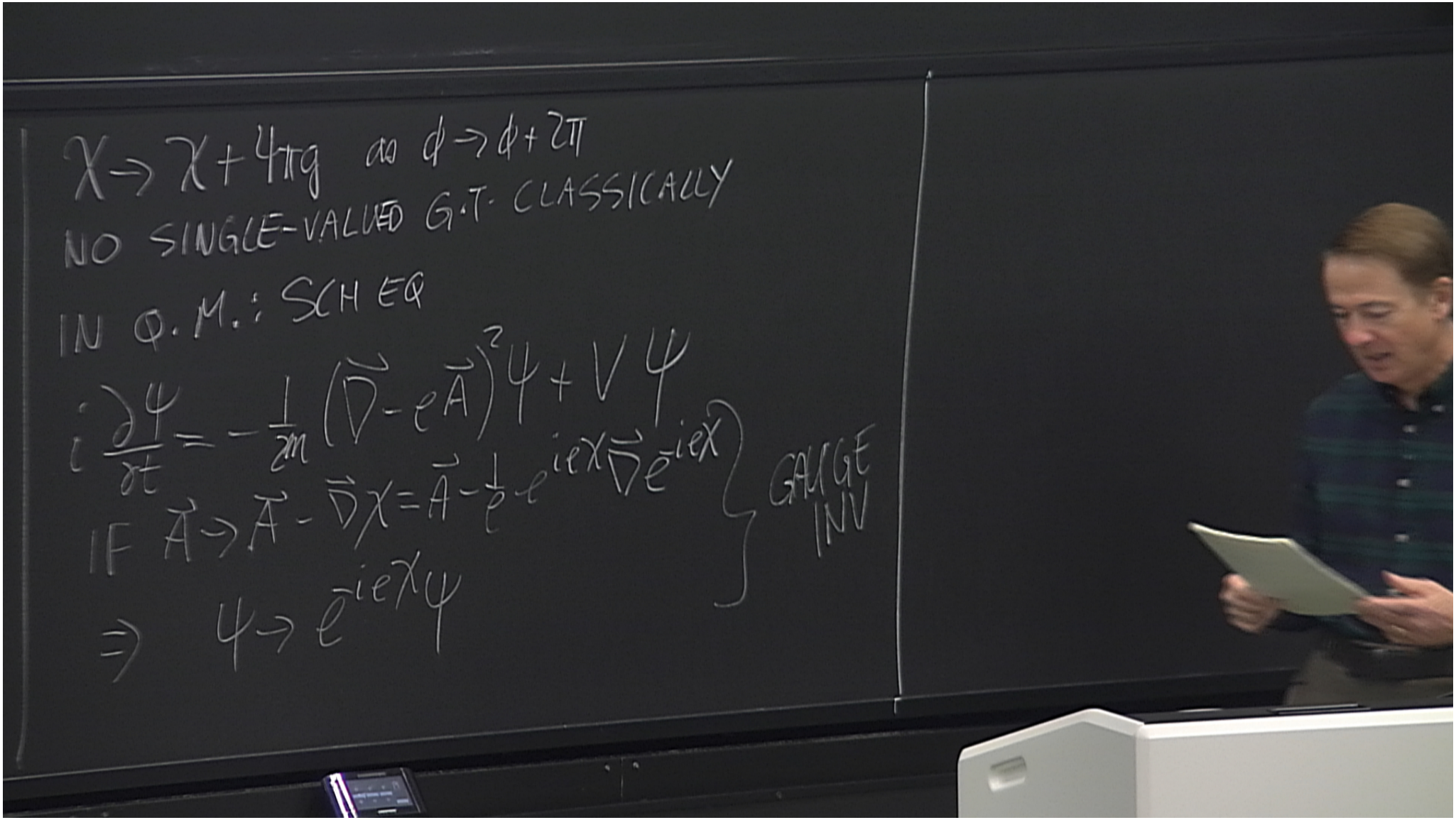
$$\Rightarrow \vec{\nabla} \times \vec{A}_N = \vec{\nabla} \times \vec{A}_S = \vec{B} \quad | \quad \Rightarrow \chi = 2g\phi$$

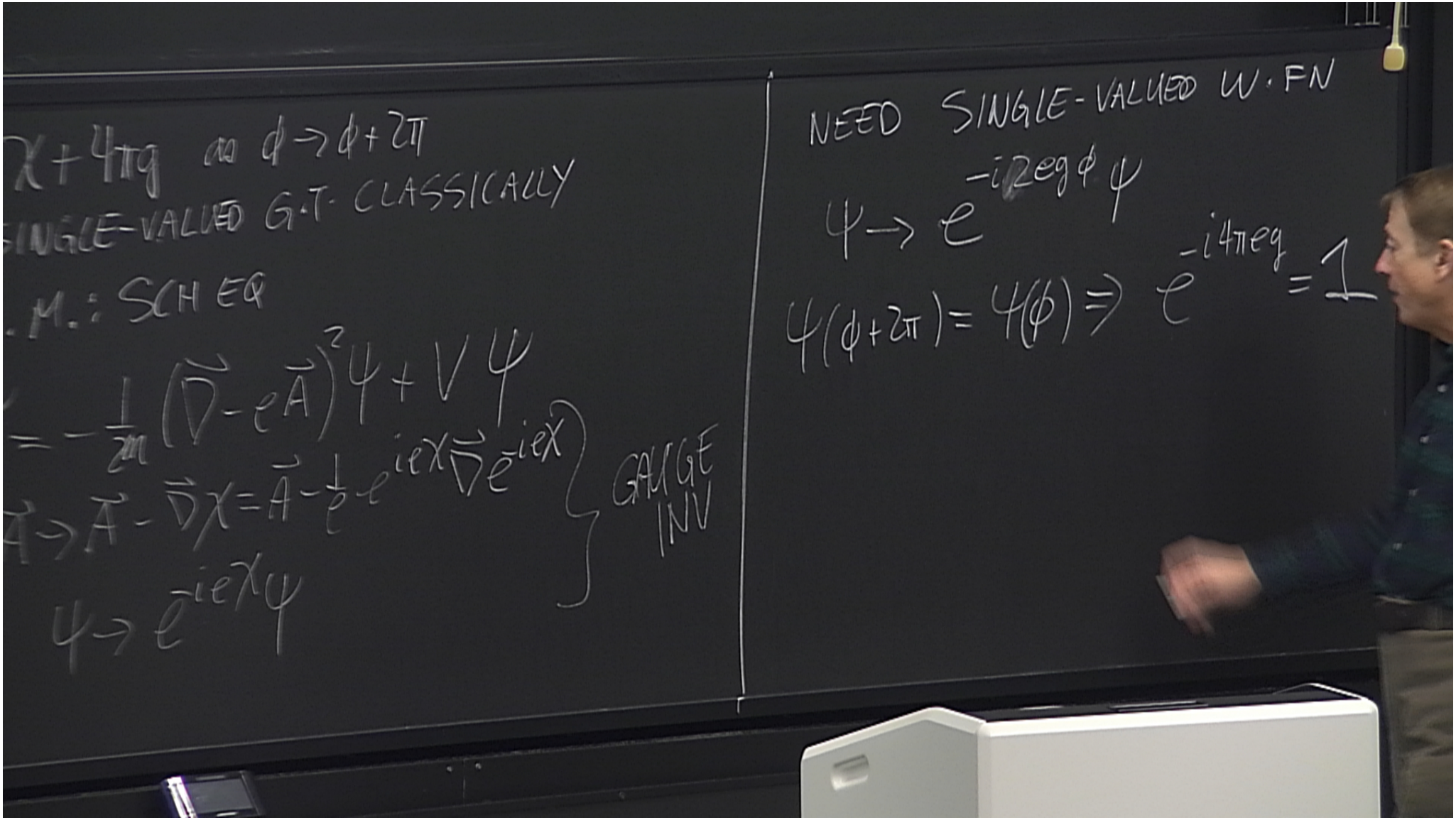
NOT SINGLE VALUED

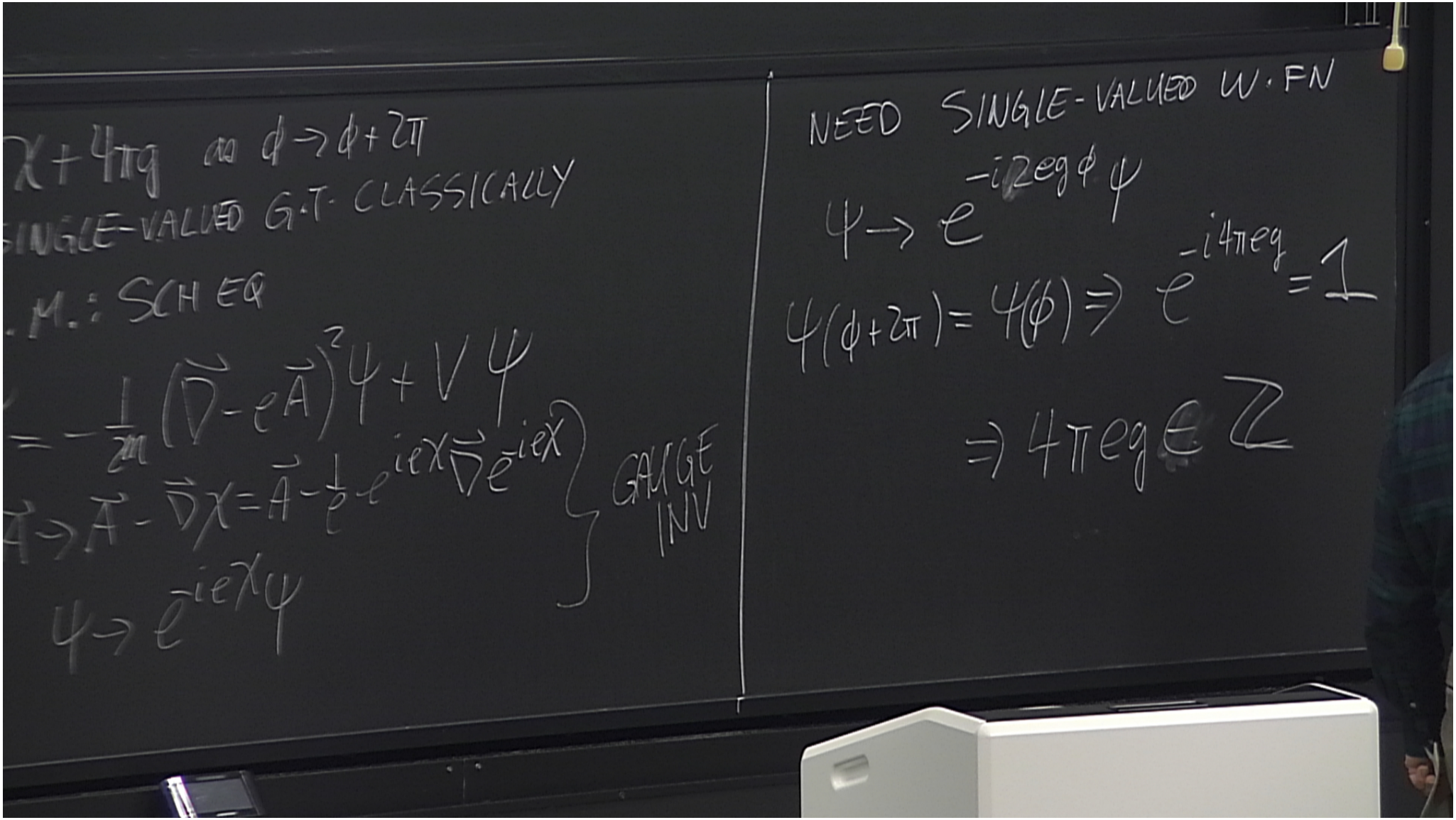
$\chi \to \chi + 4\pi g$ as $\phi \to$

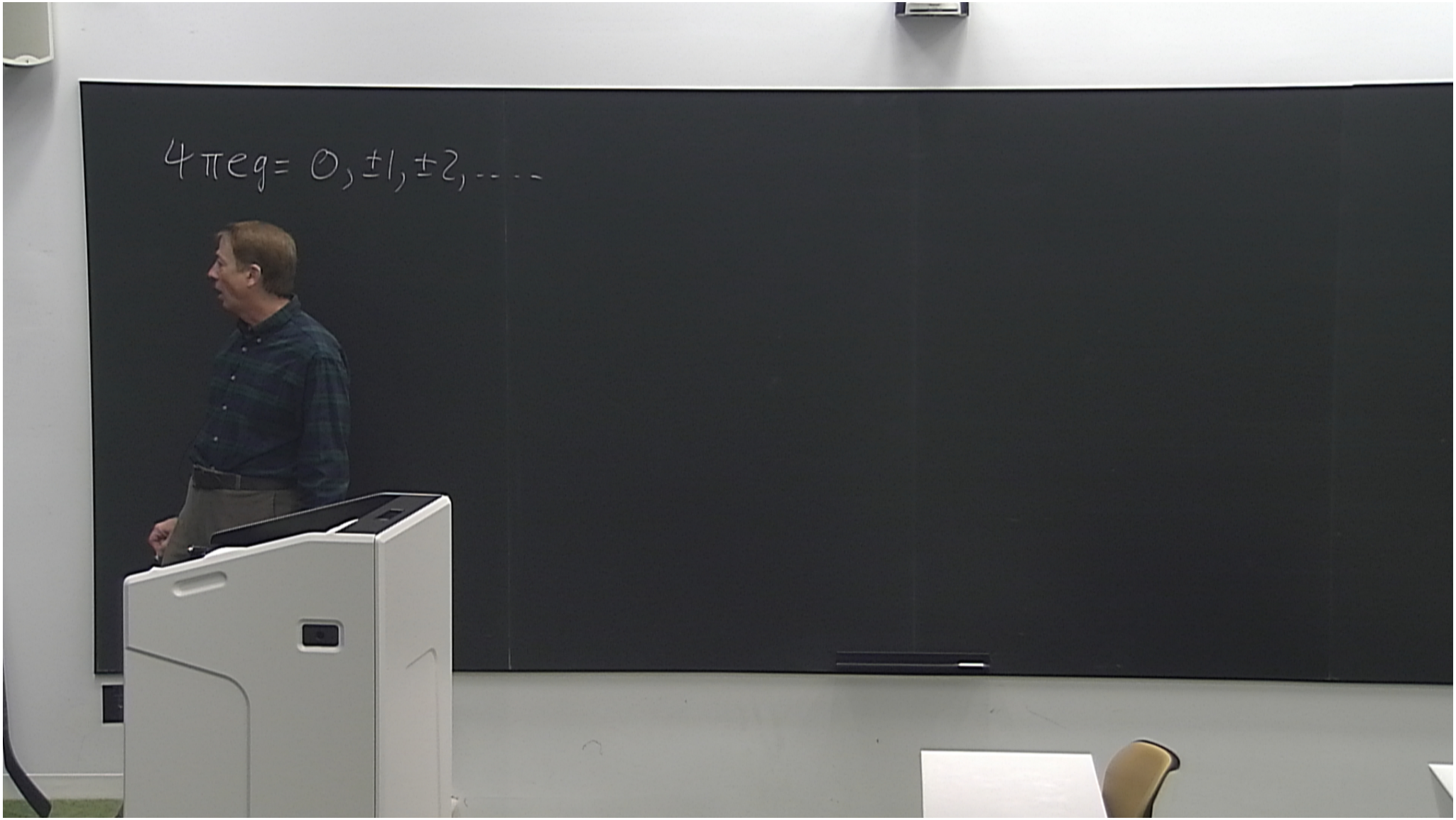
NO SINGLE-VALUED G.

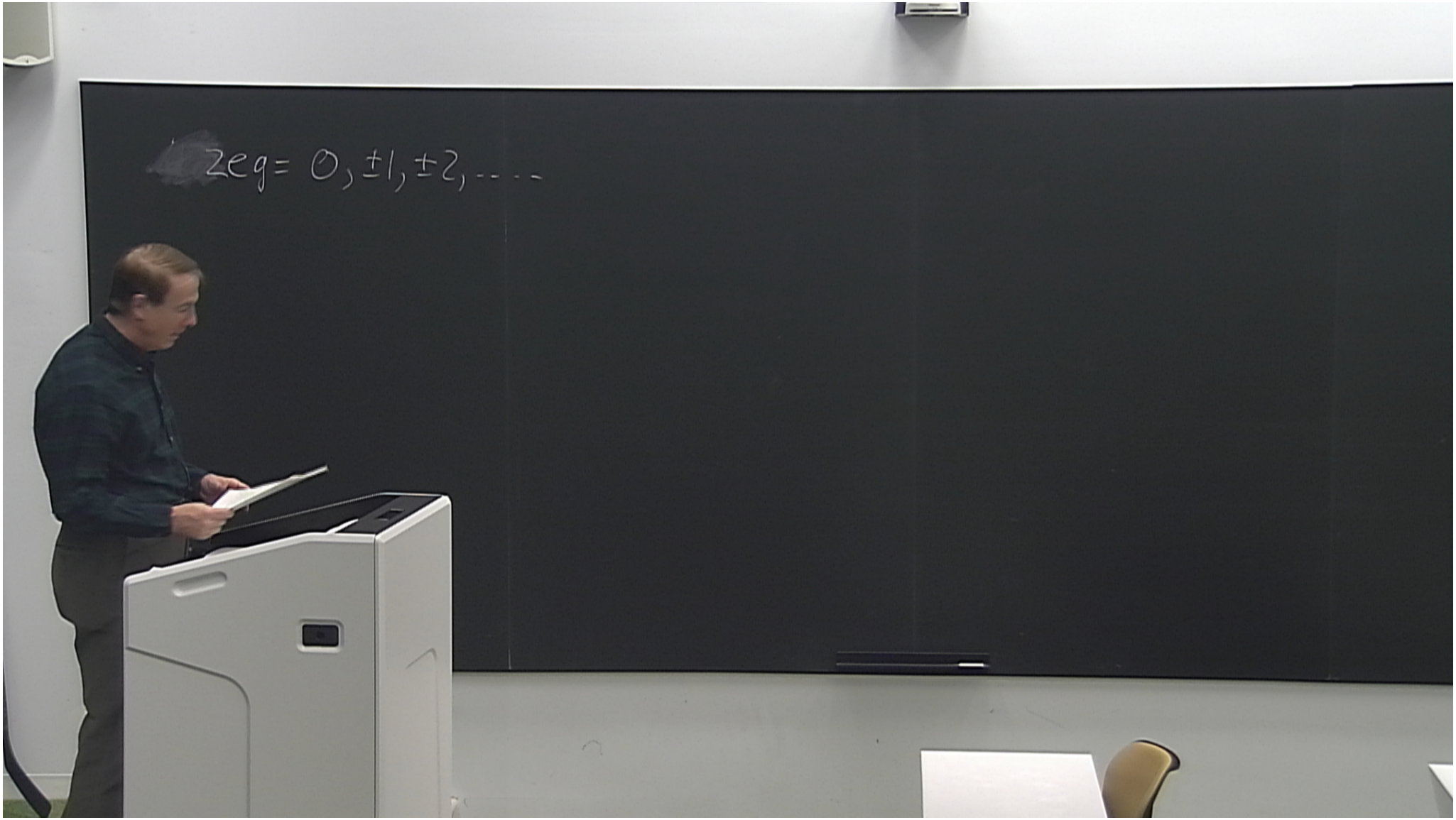

$\chi \to \chi + 4\pi q$ as $\phi \to \phi + 2\pi$
NO SINGLE-VALUED G.T. CLASSICALLY
IN Q.M.: SCH EQ
$i\frac{\partial \psi}{\partial t} = -\frac{1}{2m}(\vec{\nabla} - e\vec{A})^2\psi + V\psi$

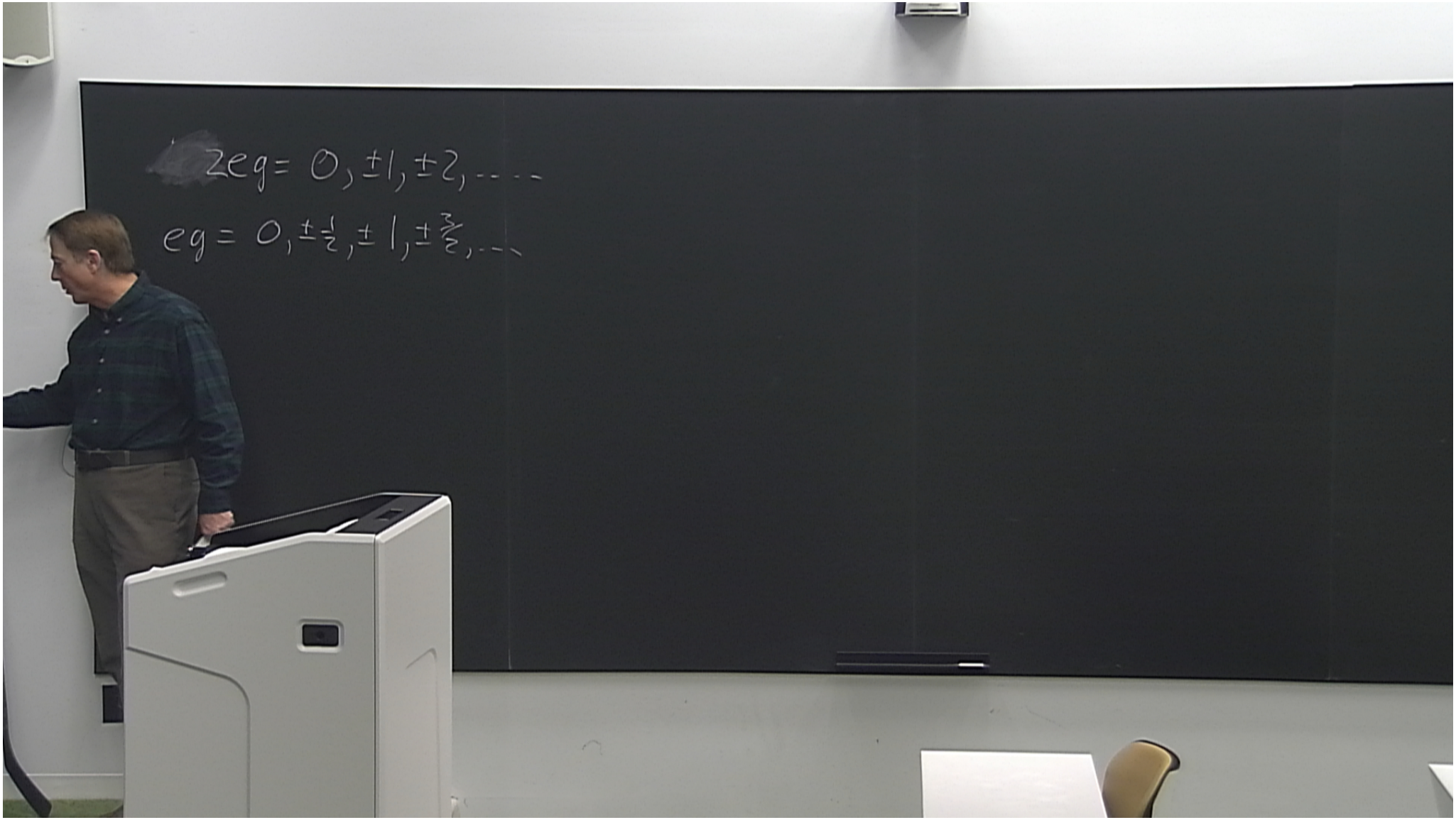

$\chi \to \chi + 4\pi q$ as $\phi \to \phi + 2\pi$
NO SINGLE-VALUED G.T. CLASSICALLY
IN Q.M.: SCH EQ
$i\frac{\partial \psi}{\partial t} = -\frac{1}{2m}(\vec{\nabla} - e\vec{A})^2\psi + V\psi$
IF $\vec{A} \to \vec{A} - \vec{\nabla}\chi = \vec{A} - \frac{1}{e}e^{ie\chi}\vec{\nabla}e^{-ie\chi}$
$\Rightarrow \psi \to e^{-ie\chi}\psi$
GAUGE INV

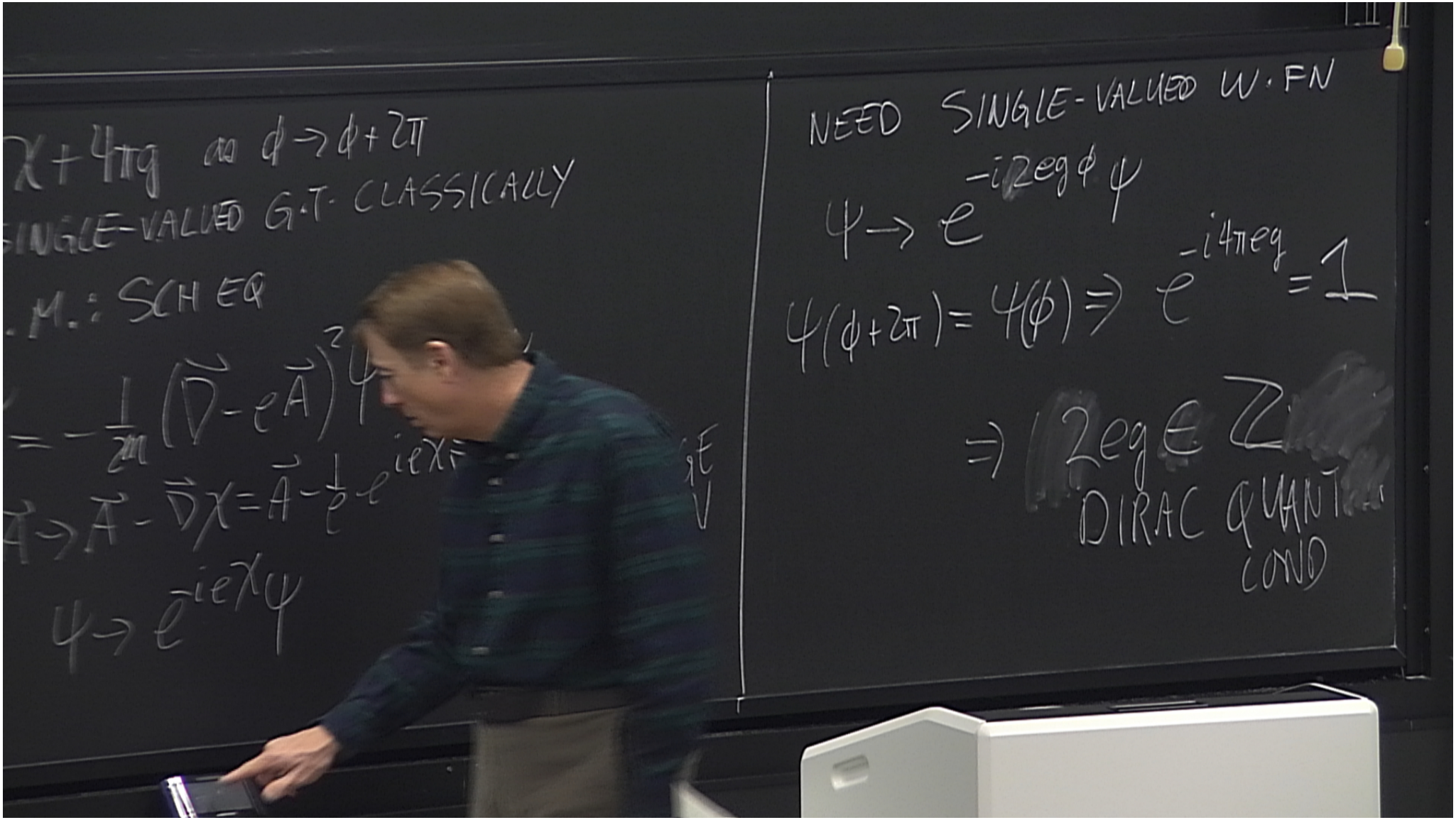

χ + 4πq ⇒ φ → φ + 2π
SINGLE-VALUED G.T. CLASSICALLY
.M.: SCH EQ
= −1/2m (∇ − eA)² ψ + V ψ
A → A − ∇χ = A − 1/e e^{ieχ} ∇ e^{−ieχ}
GAUGE INV
ψ → e^{−ieχ} ψ
NEED SINGLE-VALUED W.FN
ψ → e^{−i2eqφ} ψ
ψ(φ + 2π) = ψ(φ) ⇒ e^{−i4πeq} = 1

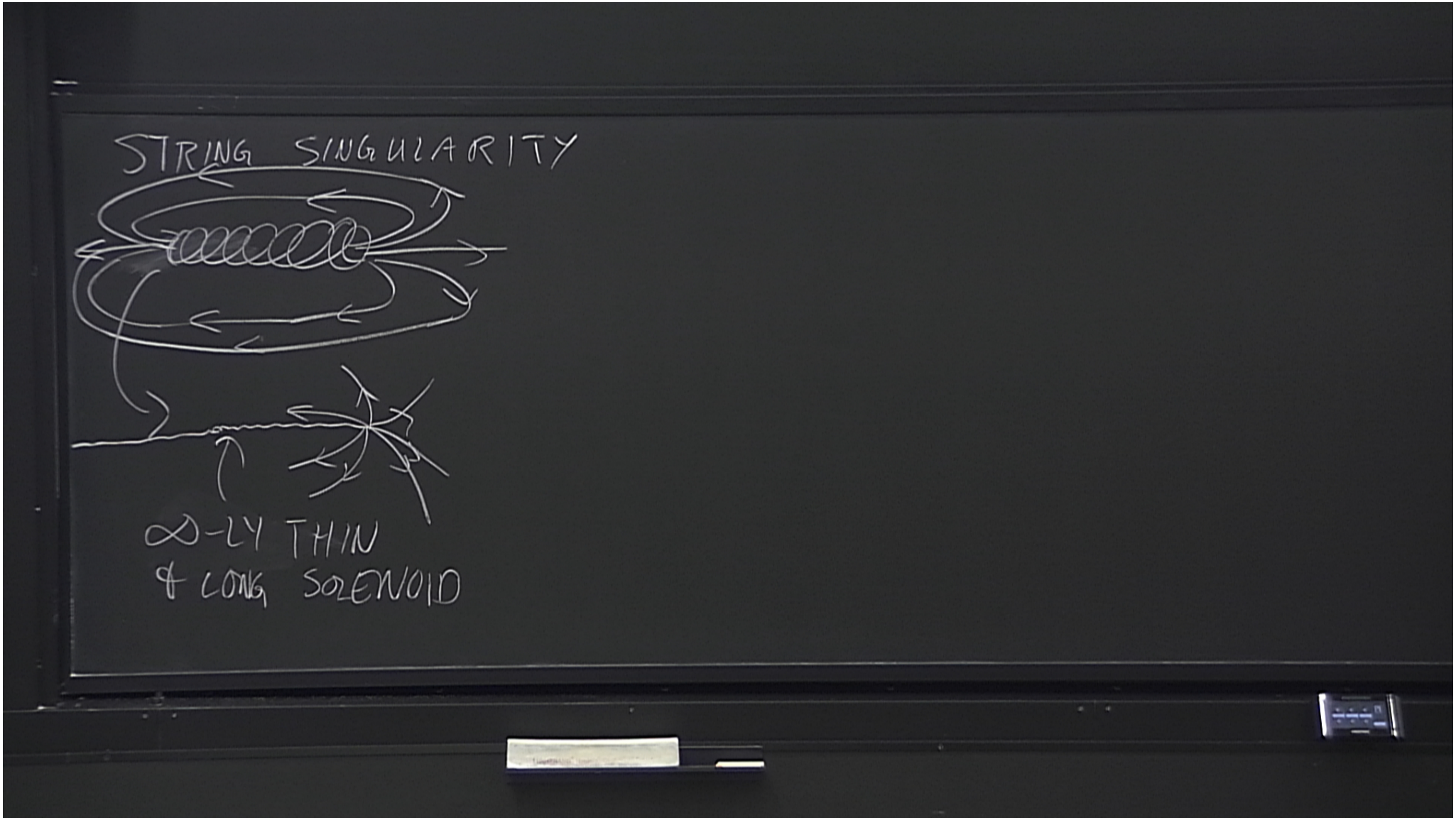

χ + 4πq ⇔ φ → φ + 2π
SINGLE-VALUED G.T. CLASSICALLY
.M.: SCH EQ
= −1/2m (∇ − eA)² ψ + V ψ
A → A − ∇χ = A − 1/e e^{ieχ} ∇ e^{−ieχ}
GAUGE INV
ψ → e^{−ieχ} ψ
NEED SINGLE-VALUED W.FN
ψ → e^{−i2eqφ} ψ
ψ(φ + 2π) = ψ(φ) ⇒ e^{−i4πeq} = 1
⇒ 4πeq ∈ Z


4πeq = 0, ±1, ±2, ...

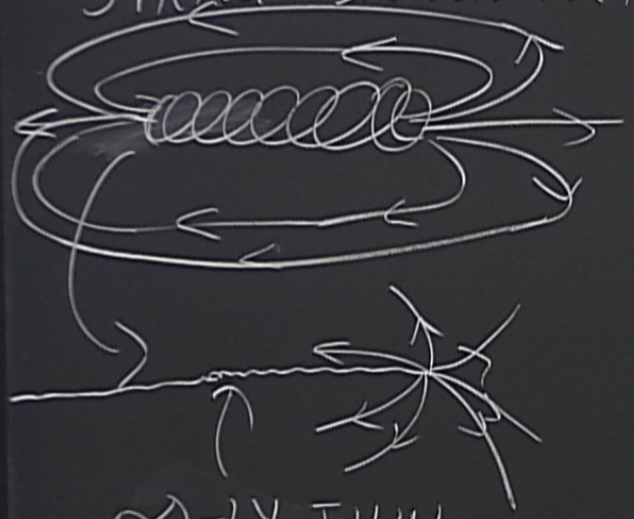

zeq = 0, ±1, ±2, ...


$2eq = 0, \pm 1, \pm 2, \ldots$
$eq = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \ldots$


χ + 4πq ⇒ φ → φ + 2π
SINGLE-VALUED G.T. CLASSICALLY
.M.: SCH EQ
= − 1/2m (∇ − eA)² ψ
A → A − ∇χ = A − 1/e
ψ → e^{ieχ} ψ
NEED SINGLE-VALUED W.FN
ψ → e^{−i2eqφ} ψ
ψ(φ+2π) = ψ(φ) ⇒ e^{−i4πeq} = 1
⇒ 2eq ∈ Z
DIRAC QUANT
COND


STRING SINGULARITY
∞-LY THIN
& LONG SOLENOID

STRING SINGULARITY

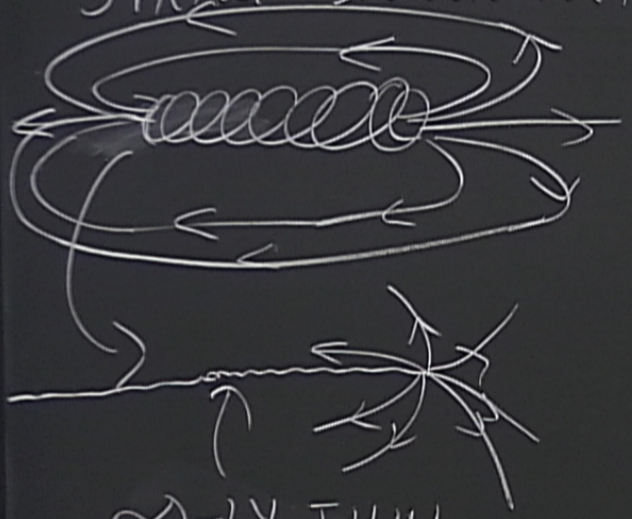


∞-LY THIN
& LONG SOLENOID

DYON

⇒ PARTICLE WITH
BOTH e & g NONZERO

STRING SINGULARITY



∞-LY THIN
& LONG SOLENOID

DYON

⇒ PARTICLE WITH
BOTH e & g NONZERO

PROPERTIES?

MAX EQNS IN VAC

$$\vec{\nabla}\cdot\vec{E}=0 \qquad -\frac{\partial\vec{E}}{\partial t}+\vec{\nabla}\times\vec{B}=0$$

$$\vec{\nabla}\cdot\vec{B}=0 \qquad \frac{\partial\vec{B}}{\partial t}+\vec{\nabla}\times\vec{E}=0$$

IF $\vec{E}' = \vec{E}\cos\alpha + \vec{B}\sin\alpha$
$\vec{B}' = -\vec{E}\sin\alpha + \vec{B}\cos\alpha$
$\Rightarrow (\vec{E}', \vec{B}')$ SATISFY MAX EQS

$\vec{\nabla} \times \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = 0$

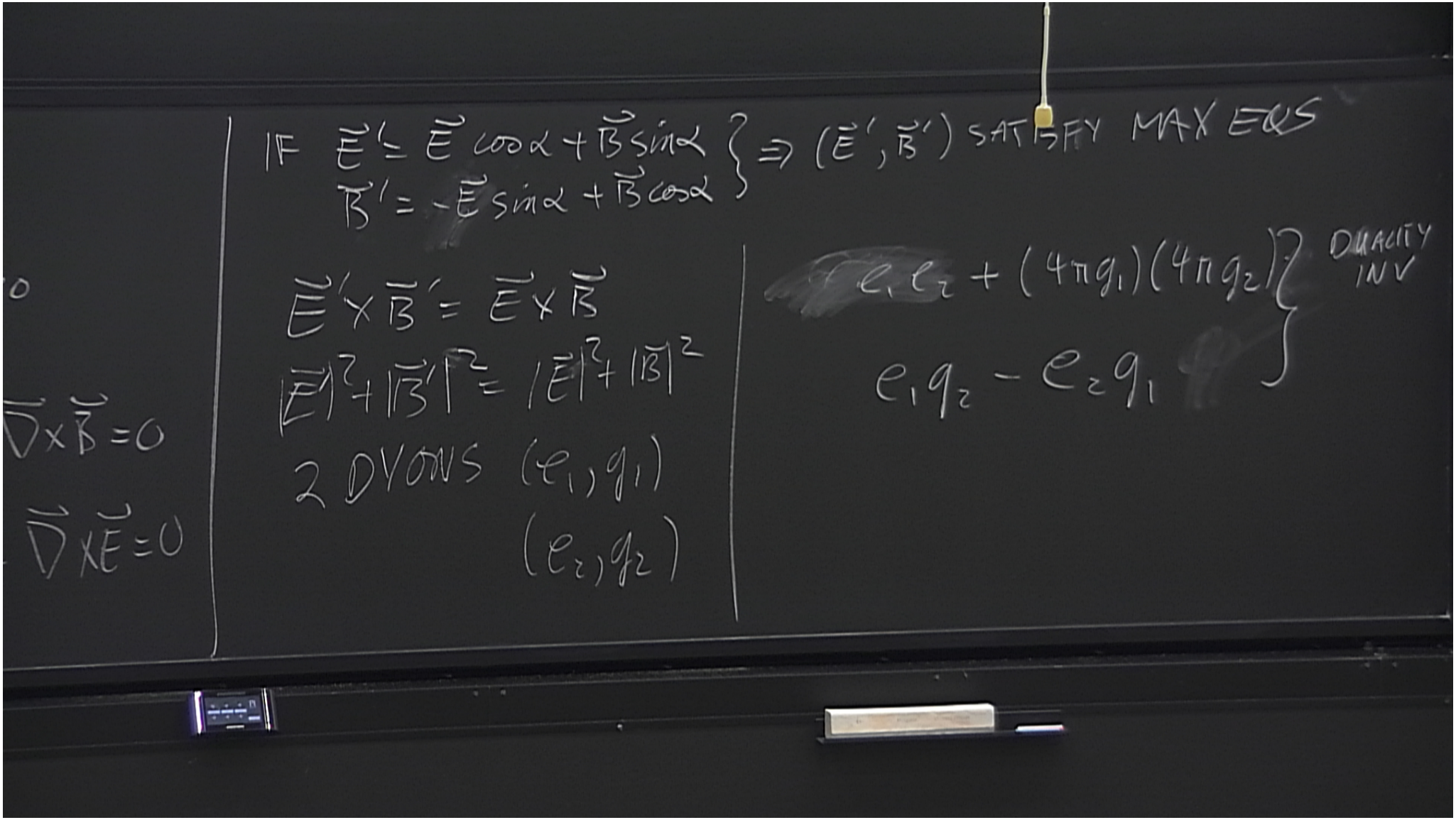
$\vec{\nabla} \times \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = 0$

IF $\left.\begin{aligned} \vec{E}' &= \vec{E}\cos\alpha + \vec{B}\sin\alpha \\ \vec{B}' &= -\vec{E}\sin\alpha + \vec{B}\cos\alpha \end{aligned}\right\} \Rightarrow (\vec{E}', \vec{B}')$ SATISFY MAX EQS

$\vec{E}' \times \vec{B}' = \vec{E} \times \vec{B}$

$|\vec{E}'|^2 + |\vec{B}'|^2 = |\vec{E}|^2 + |\vec{B}|^2$


IF $\vec{E}' = \vec{E}\cos\alpha + \vec{B}\sin\alpha$, $\vec{B}' = -\vec{E}\sin\alpha + \vec{B}\cos\alpha$ $\Rightarrow (\vec{E}', \vec{B}')$ SATISFY MAX EQS
$\vec{E}' \times \vec{B}' = \vec{E} \times \vec{B}$
$|\vec{E}'|^2 + |\vec{B}'|^2 = |\vec{E}|^2 + |\vec{B}|^2$
2 DYONS (e_1, g_1) (e_2, g_2)
$e_1 e_2 + (4\pi g_1)(4\pi g_2)$
$e_1 g_2 - e_2 g_1$
DUALITY INV
$\vec{\nabla} \times \vec{B} = 0$
$\vec{\nabla} \times \vec{E} = 0$

$\vec{\nabla} \times \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = 0$

IF $\left.\begin{aligned} \vec{E}' &= \vec{E}\cos\alpha + \vec{B}\sin\alpha \\ \vec{B}' &= -\vec{E}\sin\alpha + \vec{B}\cos\alpha \end{aligned}\right\} \Rightarrow (\vec{E}', \vec{B}')$ SATISFY MAX EQS

$\vec{E}' \times \vec{B}' = \vec{E} \times \vec{B}$

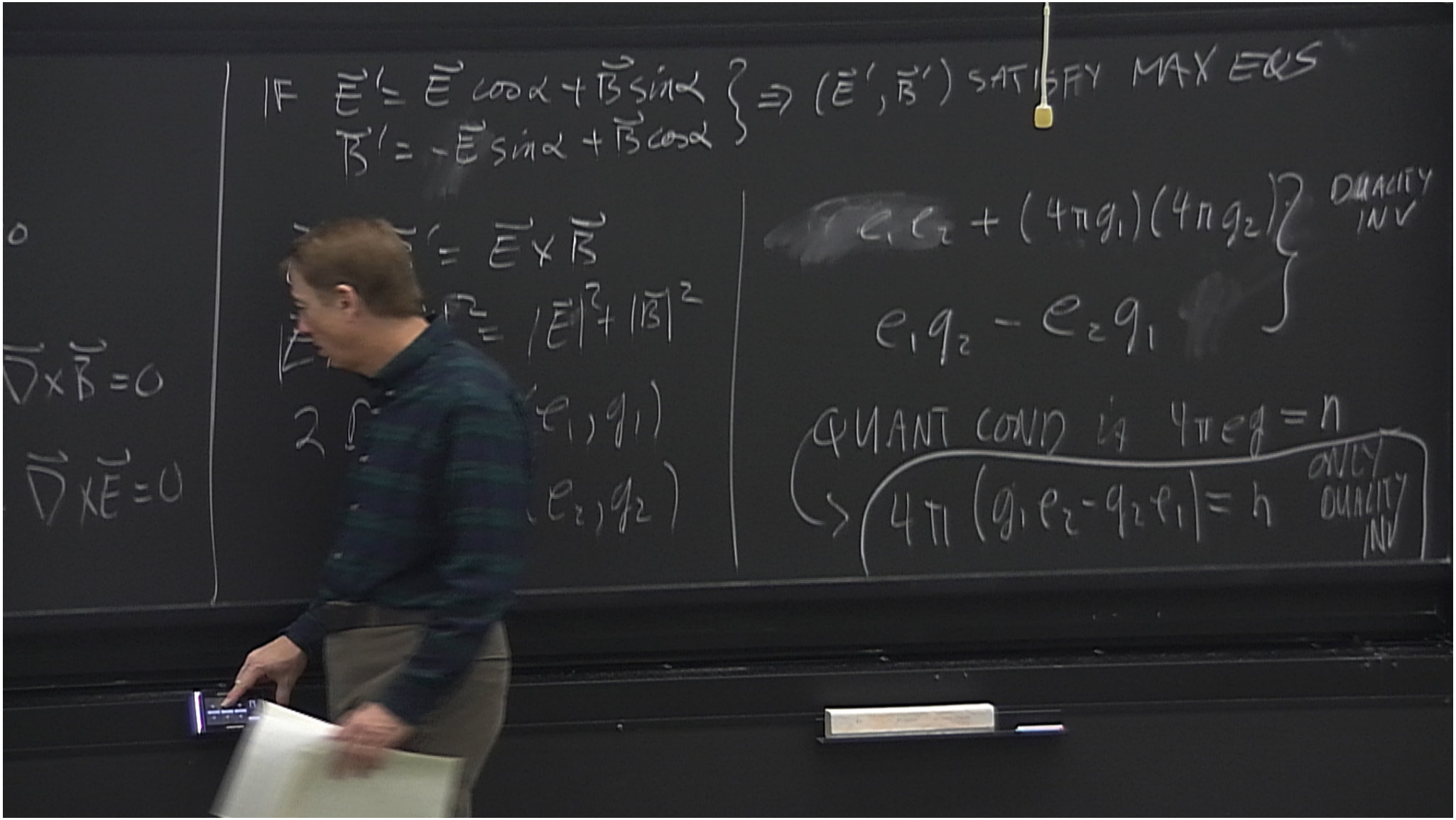
$|\vec{E}'|^2 + |\vec{B}'|^2 = |\vec{E}|^2 + |\vec{B}|^2$

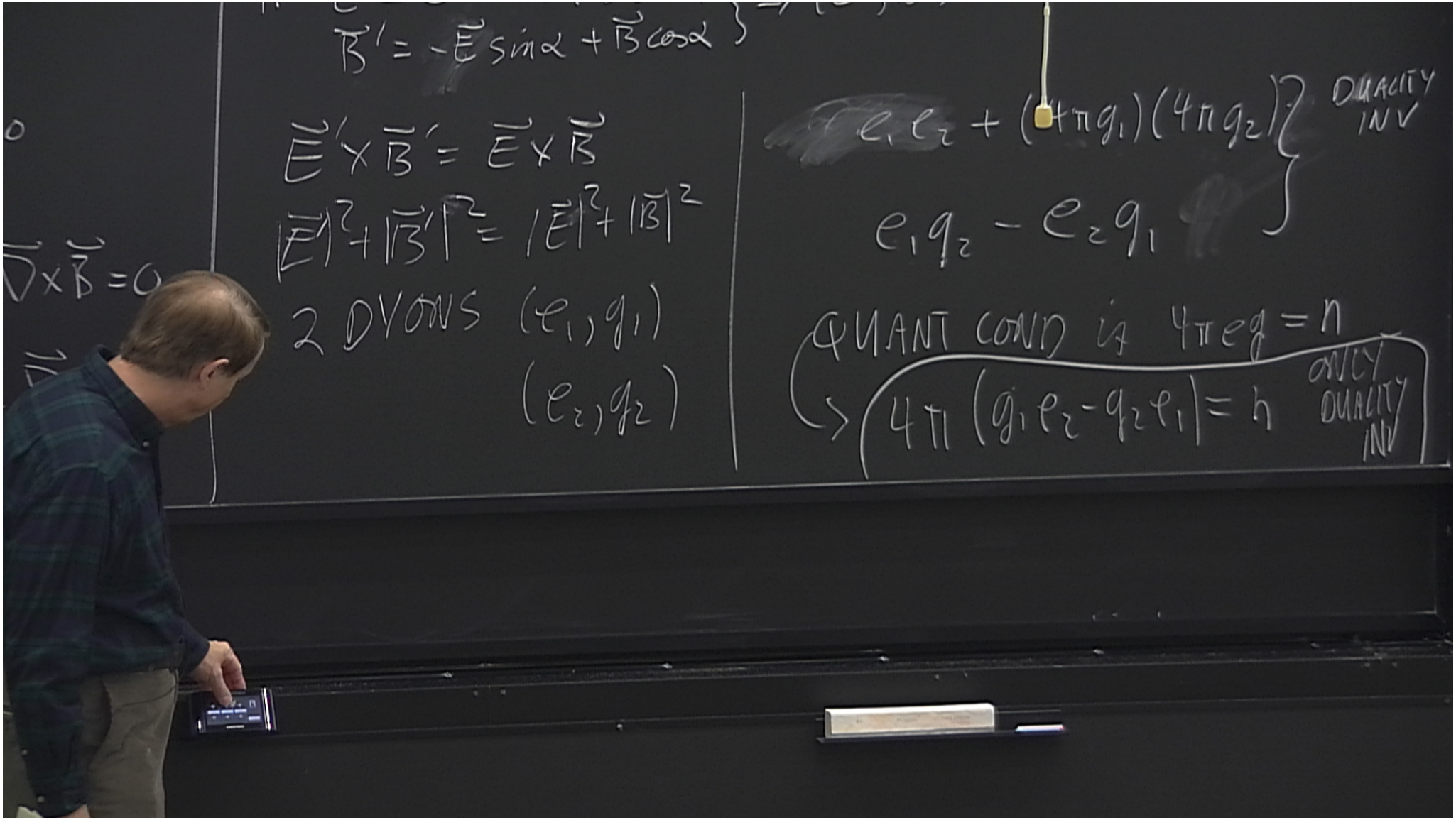
2 DYONS (e_1, g_1)

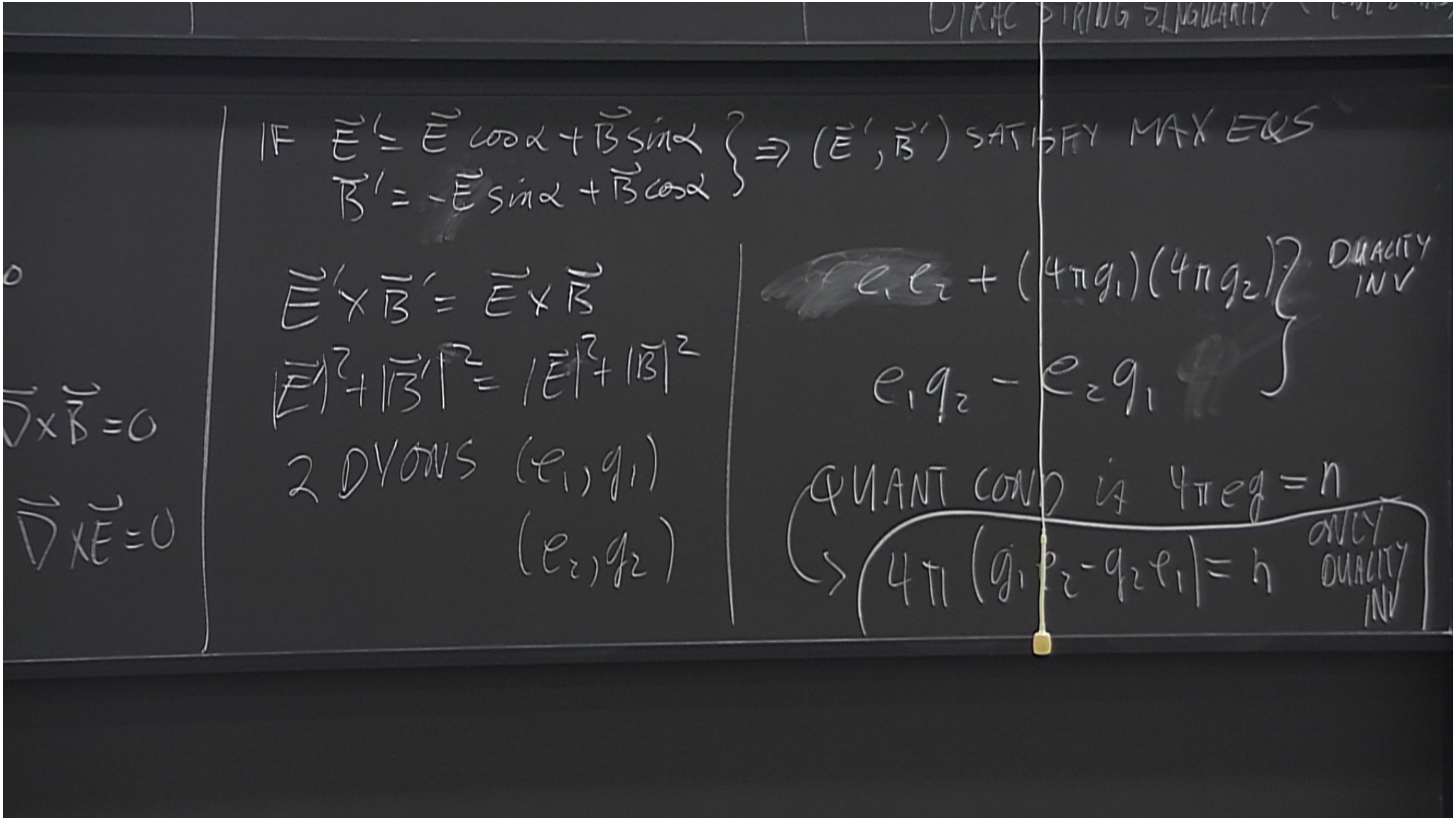
(e_2, g_2)

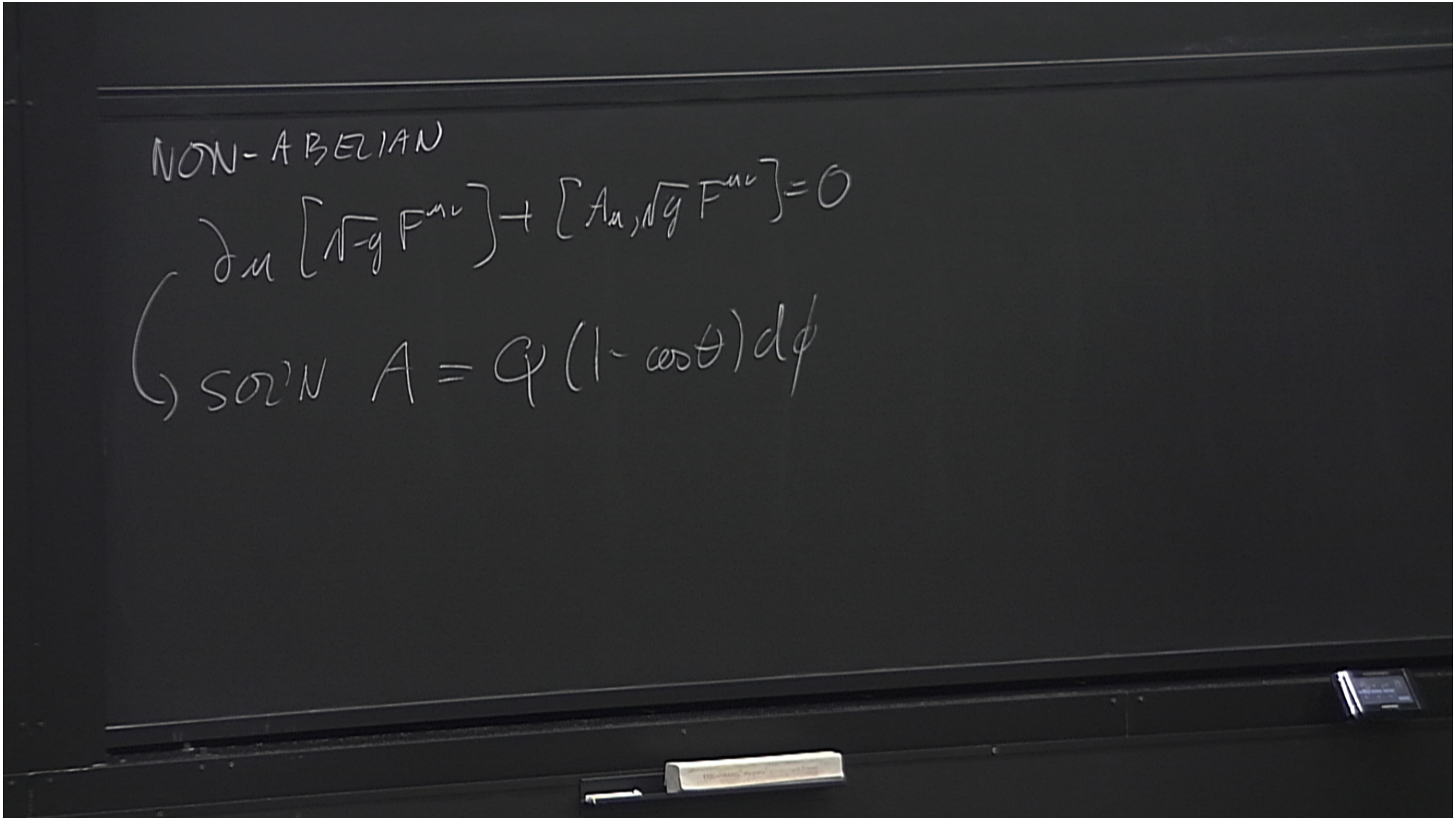
$\left.\begin{aligned} & e_1 e_2 + (4\pi g_1)(4\pi g_2) \\ & e_1 g_2 - e_2 g_1 \end{aligned}\right\}$ DUALITY INV

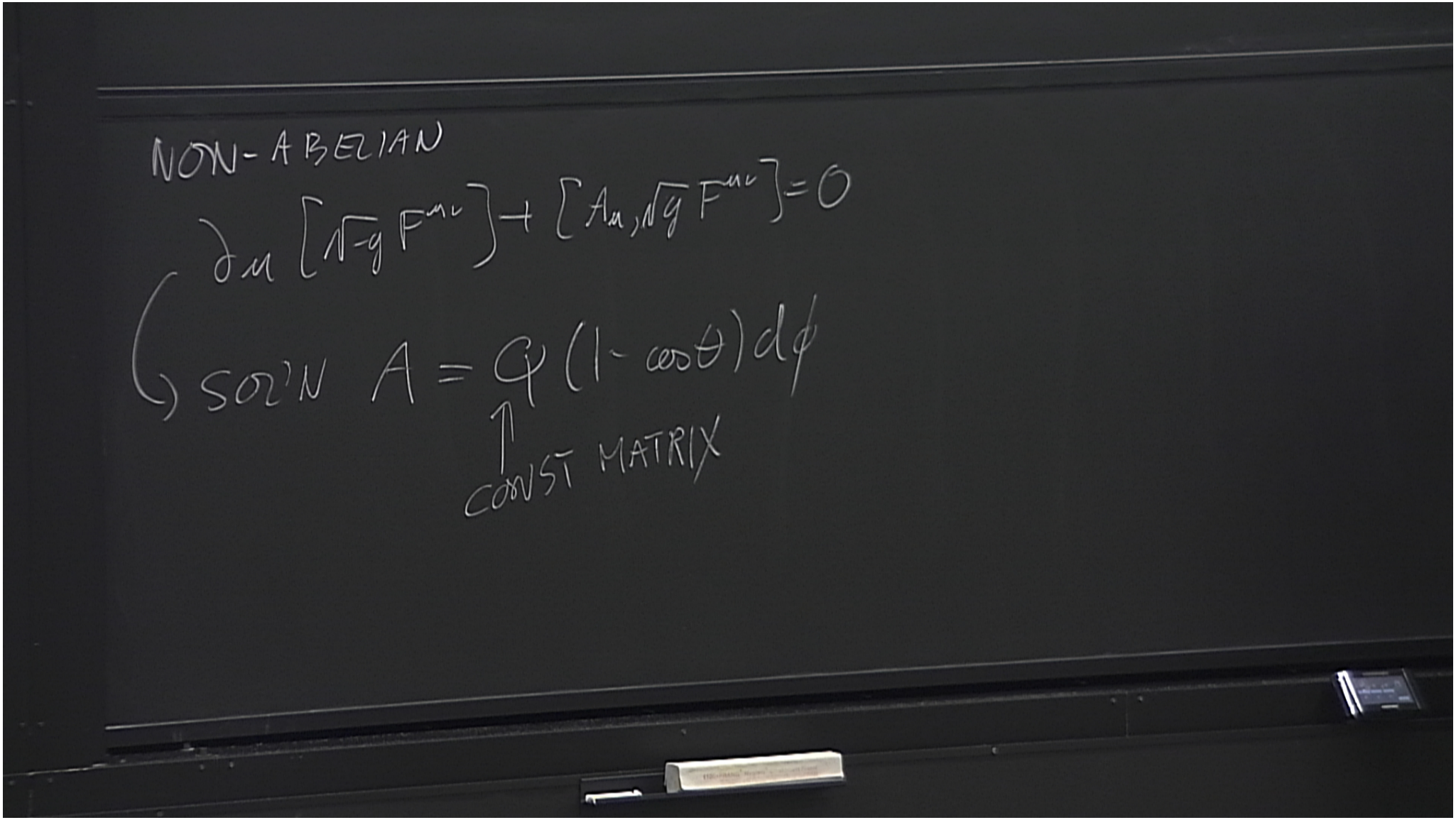
QUANT COND is $4\pi eg = n$

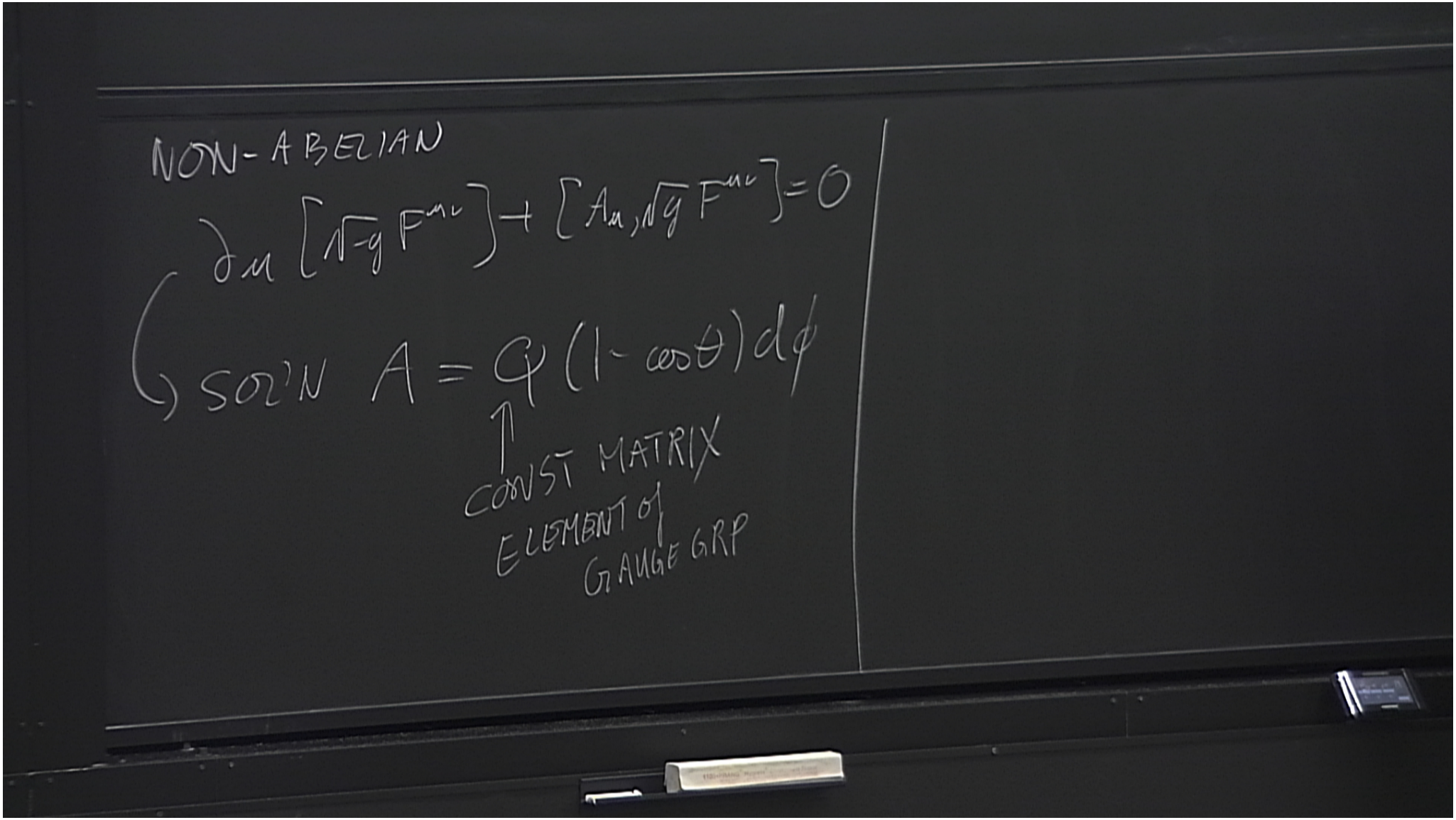

IF $\vec{E}' = \vec{E}\cos\alpha + \vec{B}\sin\alpha$, $\vec{B}' = -\vec{E}\sin\alpha + \vec{B}\cos\alpha$ $\Rightarrow (\vec{E}', \vec{B}')$ SATISFY MAX EQS
$\vec{\nabla}\times\vec{B} = 0$
$\vec{\nabla}\times\vec{E} = 0$
$e_1 e_2 + (4\pi g_1)(4\pi g_2)$ DUALITY INV
$e_1 g_2 - e_2 g_1$
QUANT COND is $4\pi eg = n$
$4\pi(g_1 e_2 - g_2 e_1) = n$ ONLY DUALITY INV

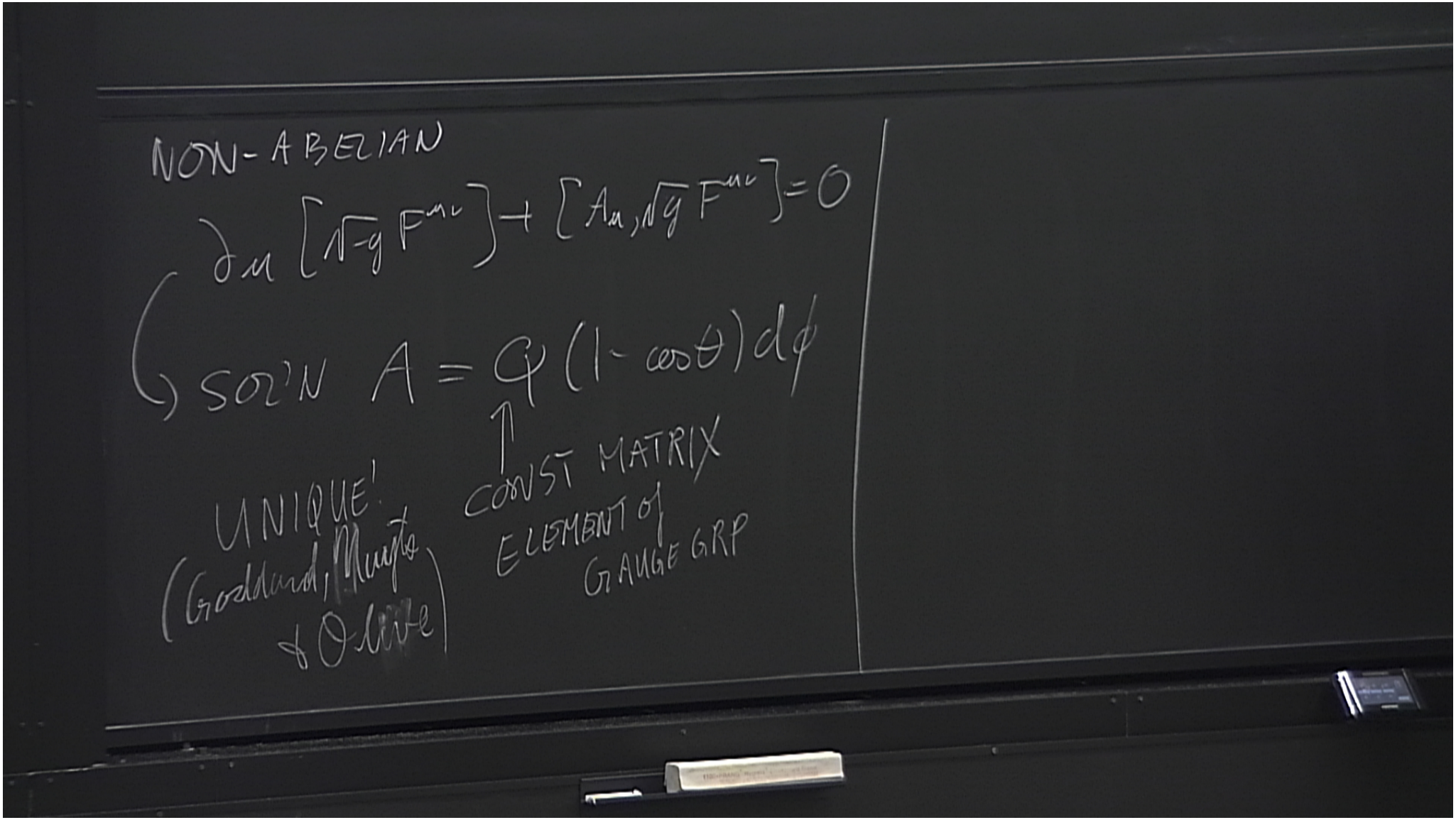

$\vec{B}' = -\vec{E}\sin\alpha + \vec{B}\cos\alpha$
$\vec{E}' \times \vec{B}' = \vec{E} \times \vec{B}$
$|\vec{E}'|^2 + |\vec{B}'|^2 = |\vec{E}|^2 + |\vec{B}|^2$
2 DYONS (e_1, g_1)
(e_2, g_2)
$e_1 e_2 + (4\pi g_1)(4\pi g_2)$ } DUALITY INV
$e_1 g_2 - e_2 g_1$
QUANT COND is $4\pi eg = n$
$4\pi (g_1 e_2 - g_2 e_1) = n$ ONLY DUALITY INV

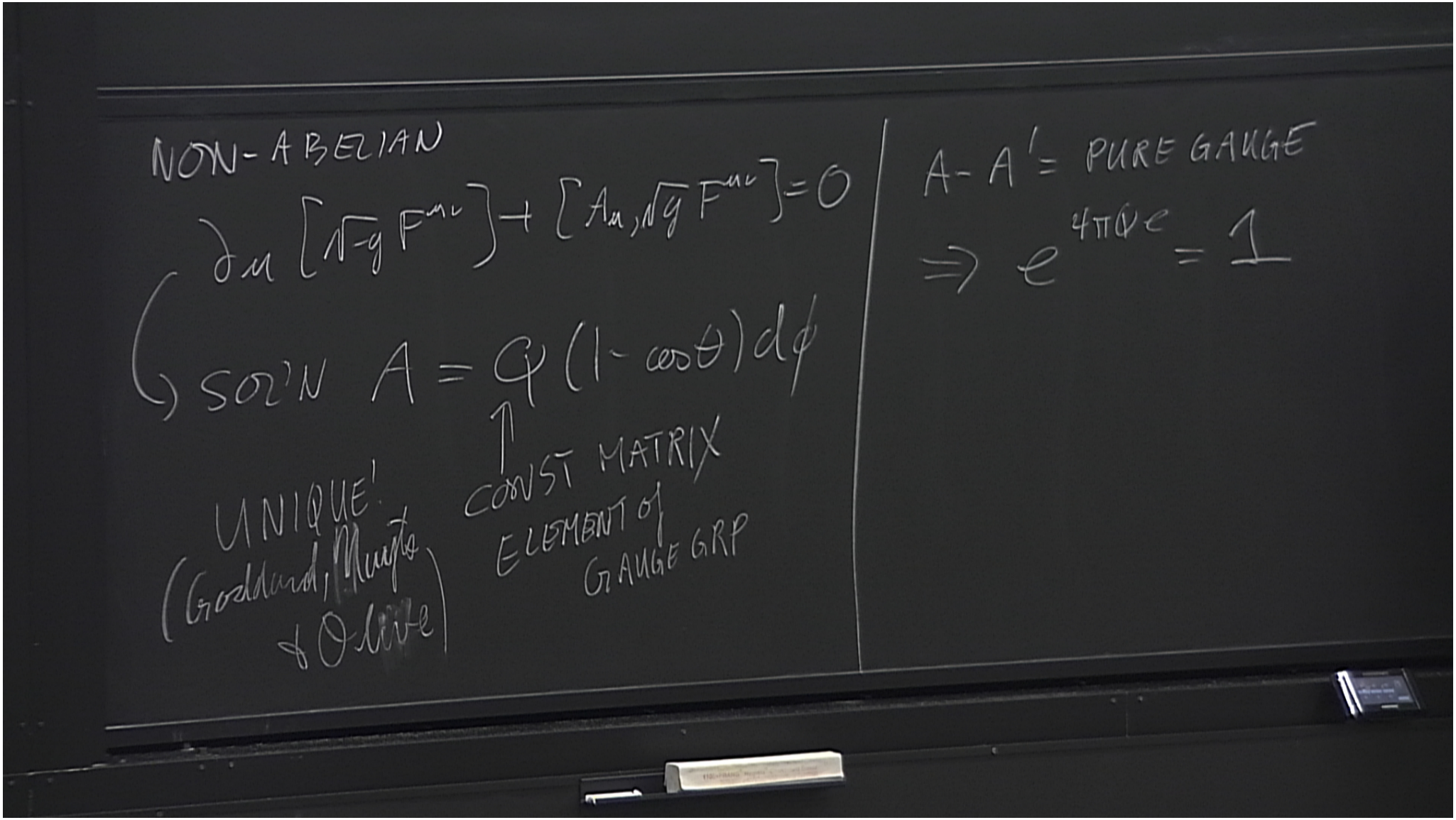

DIRAC STRING SINGULARITY
IF $\vec{E}' = \vec{E}\cos\alpha + \vec{B}\sin\alpha$, $\vec{B}' = -\vec{E}\sin\alpha + \vec{B}\cos\alpha$ $\Rightarrow (\vec{E}', \vec{B}')$ SATISFY MAX EQS
$\vec{E}' \times \vec{B}' = \vec{E} \times \vec{B}$
$|\vec{E}'|^2 + |\vec{B}'|^2 = |\vec{E}|^2 + |\vec{B}|^2$
2 DYONS (e_1, g_1) (e_2, g_2)
$e_1 e_2 + (4\pi g_1)(4\pi g_2)$ } DUALITY INV
$e_1 g_2 - e_2 g_1$
QUANT COND IS $4\pi eg = n$
$\Rightarrow 4\pi(g_1 e_2 - g_2 e_1) = n$ ONLY DUALITY INV
$\vec{\nabla} \times \vec{B} = 0$
$\vec{\nabla} \times \vec{E} = 0$

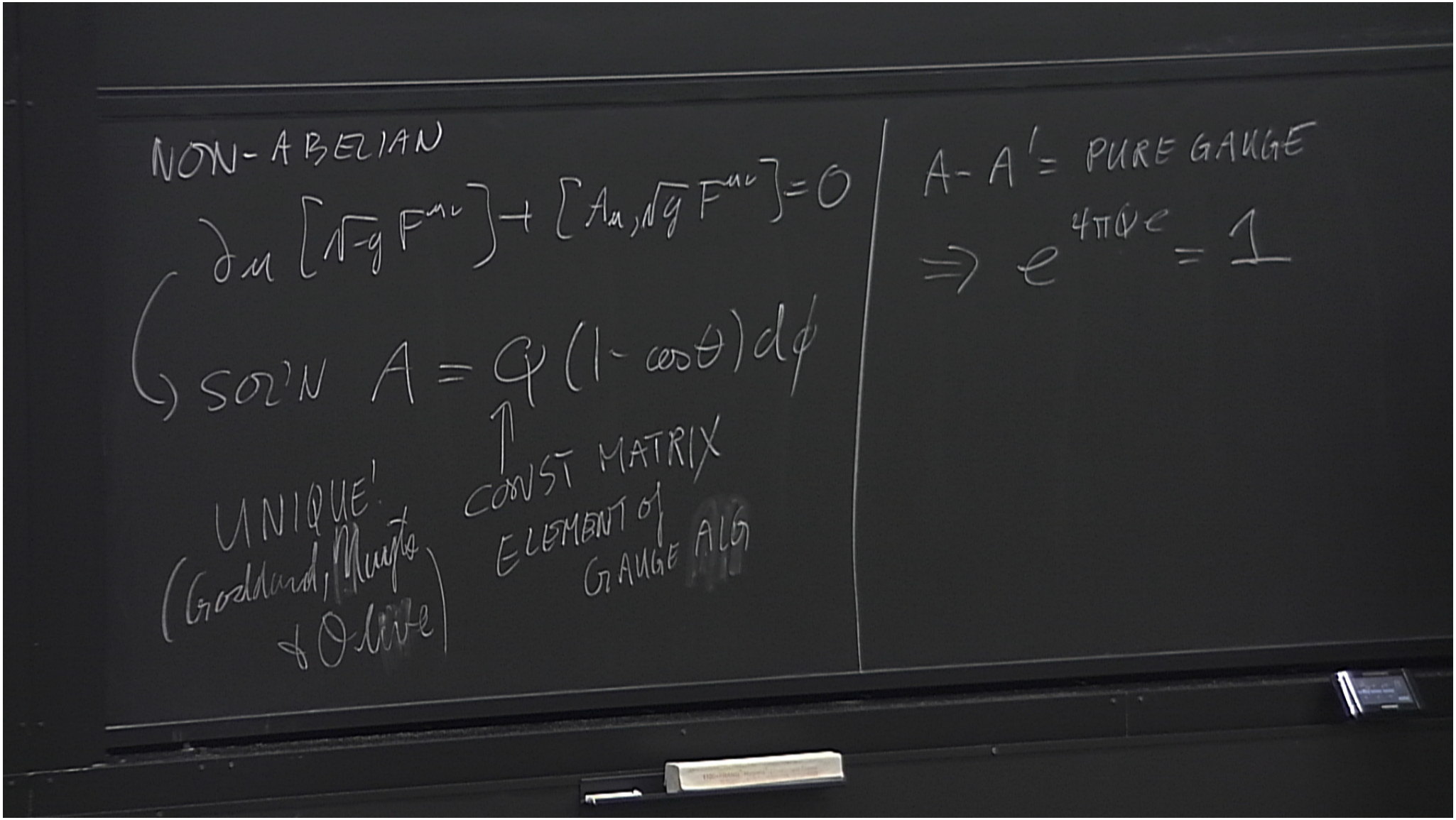

NON-ABELIAN
$\partial_\mu [\sqrt{-g} F^{\mu\nu}] + [A_\mu, \sqrt{g} F^{\mu\nu}] = 0$
SO'N $A = q(1-\cos\theta)d\phi$

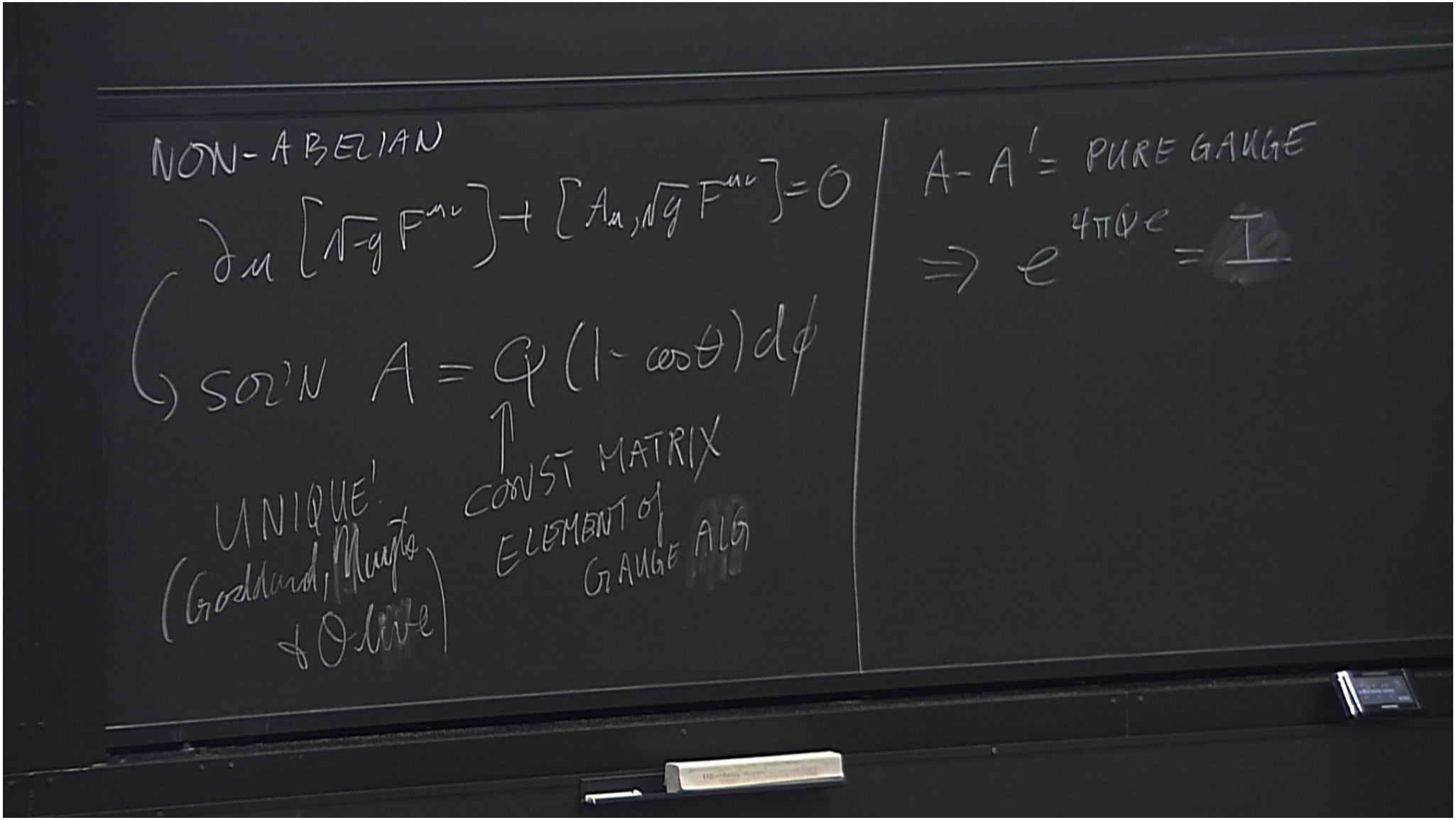

NON-ABELIAN
$\partial_\mu [\sqrt{-g} F^{\mu\nu}] + [A_\mu, \sqrt{g} F^{\mu\nu}] = 0$
SOLN $A = q(1 - \cos\theta) d\phi$
CONST MATRIX


NON-ABELIAN
$\partial_\mu [\sqrt{-g} F^{\mu\nu}] + [A_\mu, \sqrt{g} F^{\mu\nu}] = 0$
SOL'N $A = Q(1-\cos\theta)d\phi$
CONST MATRIX
ELEMENT of
GAUGE GRP


NON-ABELIAN
$\partial_\mu [\sqrt{-g} F^{\mu\nu}] + [A_\mu, \sqrt{g} F^{\mu\nu}] = 0$
$\Rightarrow$ SOL'N $A = Q(1-\cos\theta)\,d\phi$
CONST MATRIX
ELEMENT of
GAUGE GRP
UNIQUE!
(Goddard, Nuyts
& Olive)


NON-ABELIAN
$\partial_\mu[\sqrt{-g}F^{\mu\nu}] + [A_\mu, \sqrt{g}F^{\mu\nu}] = 0$
SO'N $A = Q(1-\cos\theta)d\phi$
UNIQUE!
(Goddard, Nuyts & Olive)
CONST MATRIX
ELEMENT of
GAUGE GRP
$A - A' =$ PURE GAUGE
$\Rightarrow e^{4\pi Q e} = 1$


NON-ABELIAN
$\partial_\mu[\sqrt{-g}F^{\mu\nu}] + [A_\mu, \sqrt{g}F^{\mu\nu}] = 0$
SO'N $A = q(1-\cos\theta)d\phi$
UNIQUE'
(Goddard, Nuyts & Olive)
CONST MATRIX
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NON-ABELIAN
$\partial_\mu[\sqrt{-g}F^{\mu\nu}] + [A_\mu, \sqrt{g}F^{\mu\nu}] = 0$
SO'N $A = Q(1-\cos\theta)d\phi$
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CONST MATRIX
ELEMENT of
GAUGE ALG
$A - A' =$ PURE GAUGE
$\Rightarrow e^{4\pi Q i} = I$

NON-ABELIAN

$$\partial_\mu [\sqrt{-g} F^{\mu\nu}] + [A_\mu, \sqrt{g} F^{\mu\nu}] = 0$$

$\hookrightarrow$ SO'N $A = q(1 - \cos\theta)\, d\phi$

q: CONST MATRIX ELEMENT of GAUGE ALG

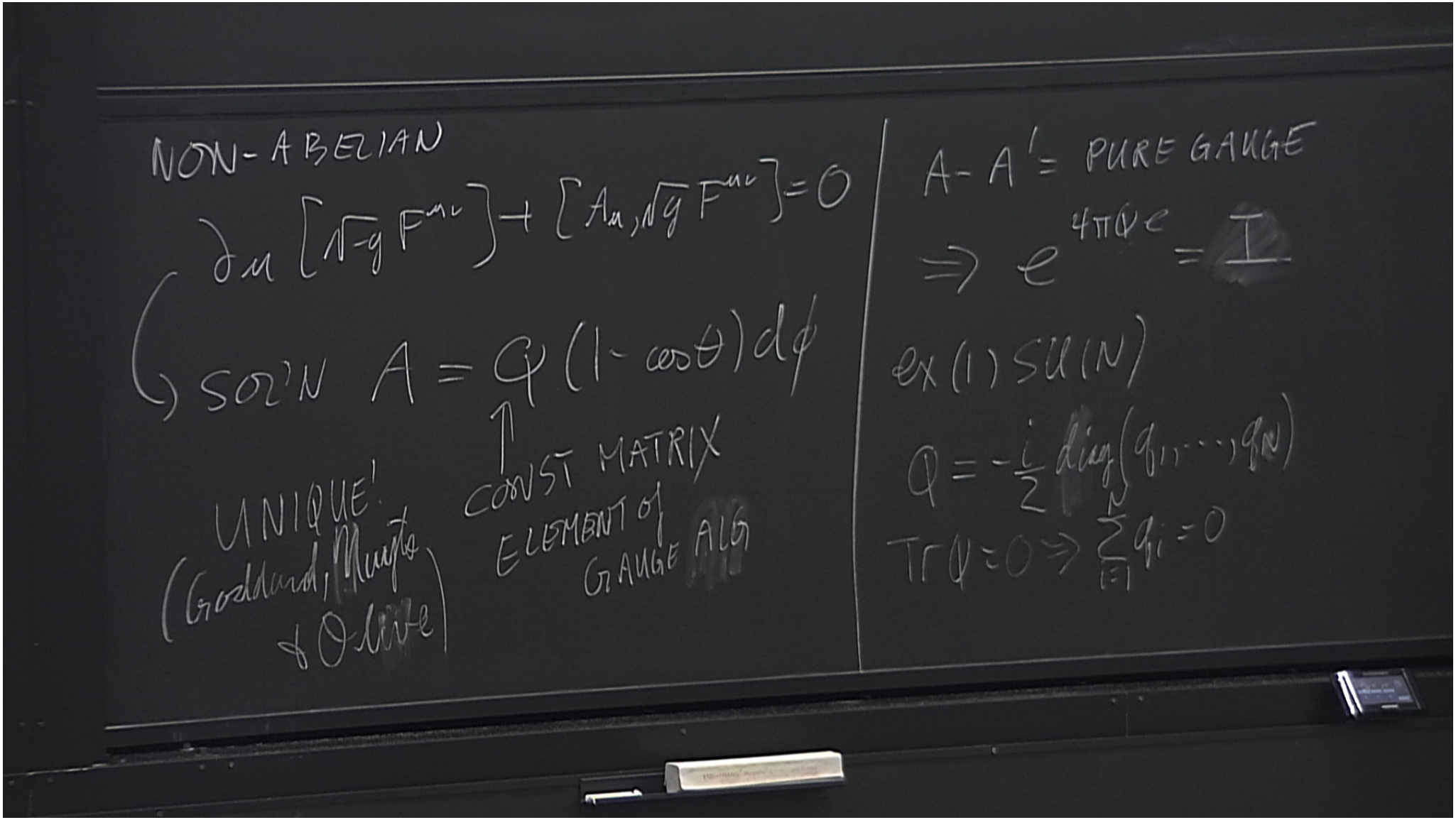
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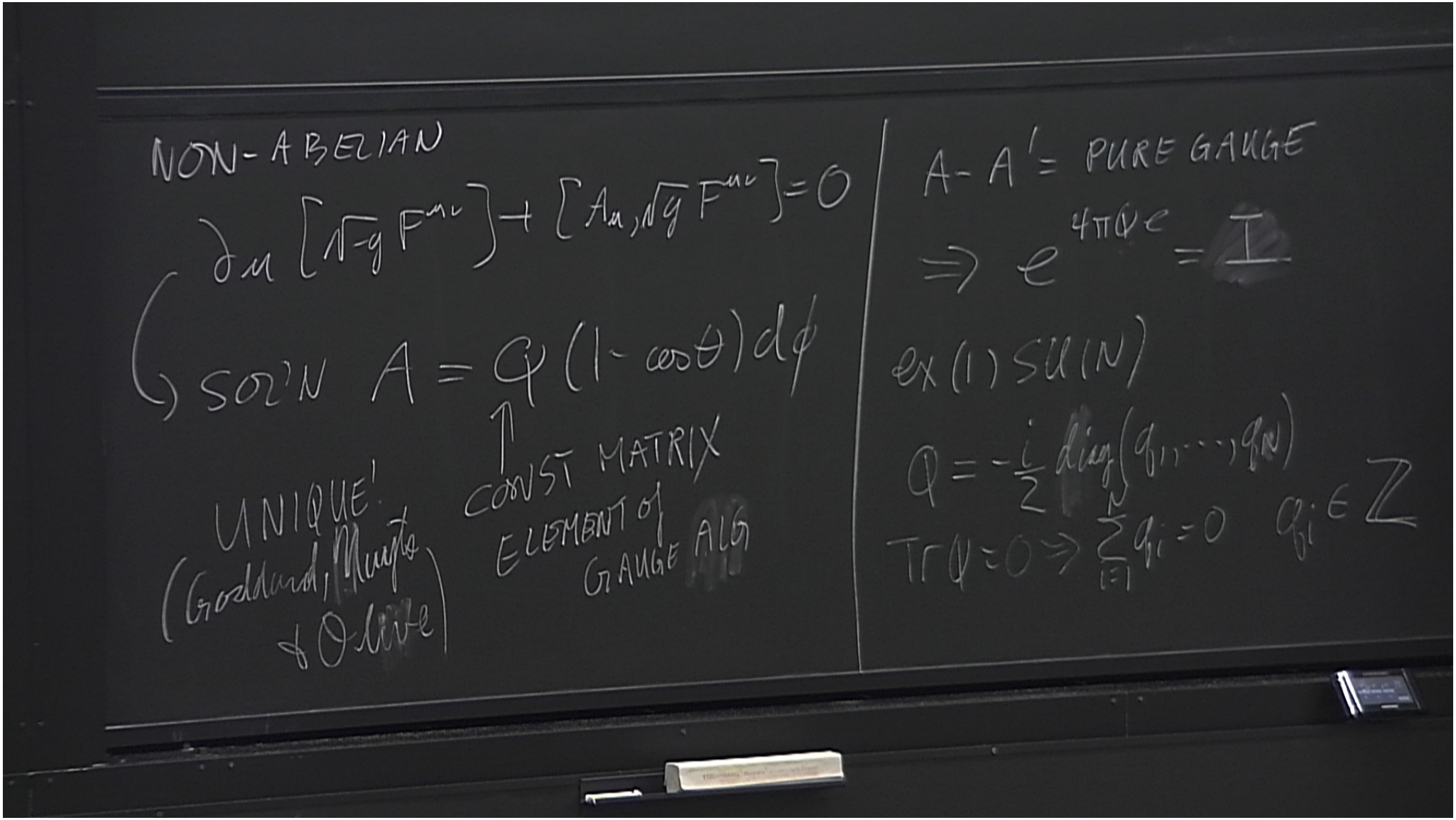
$A - A'$ = PURE GAUGE

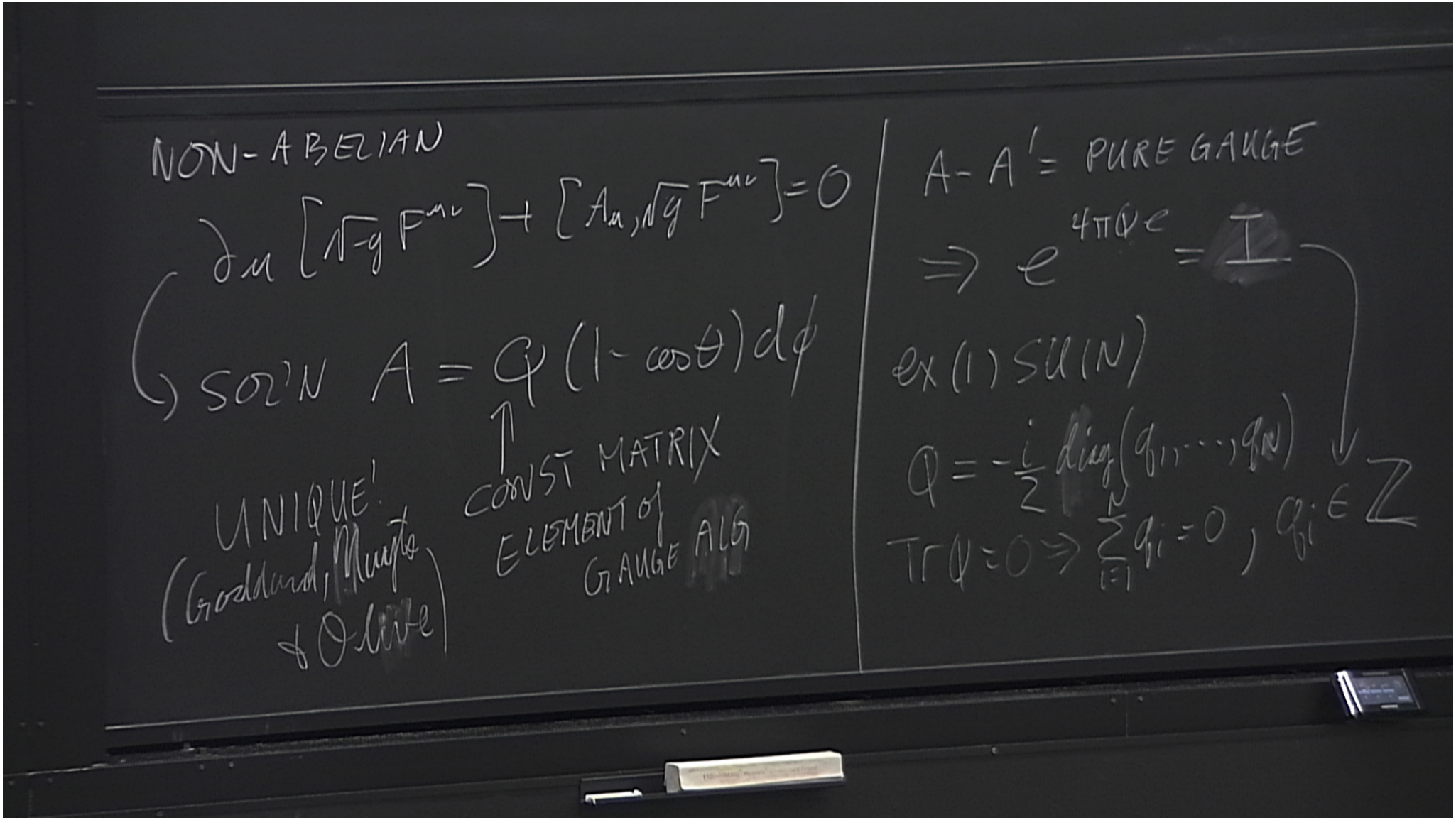
$\Rightarrow e^{4\pi q e} = I$

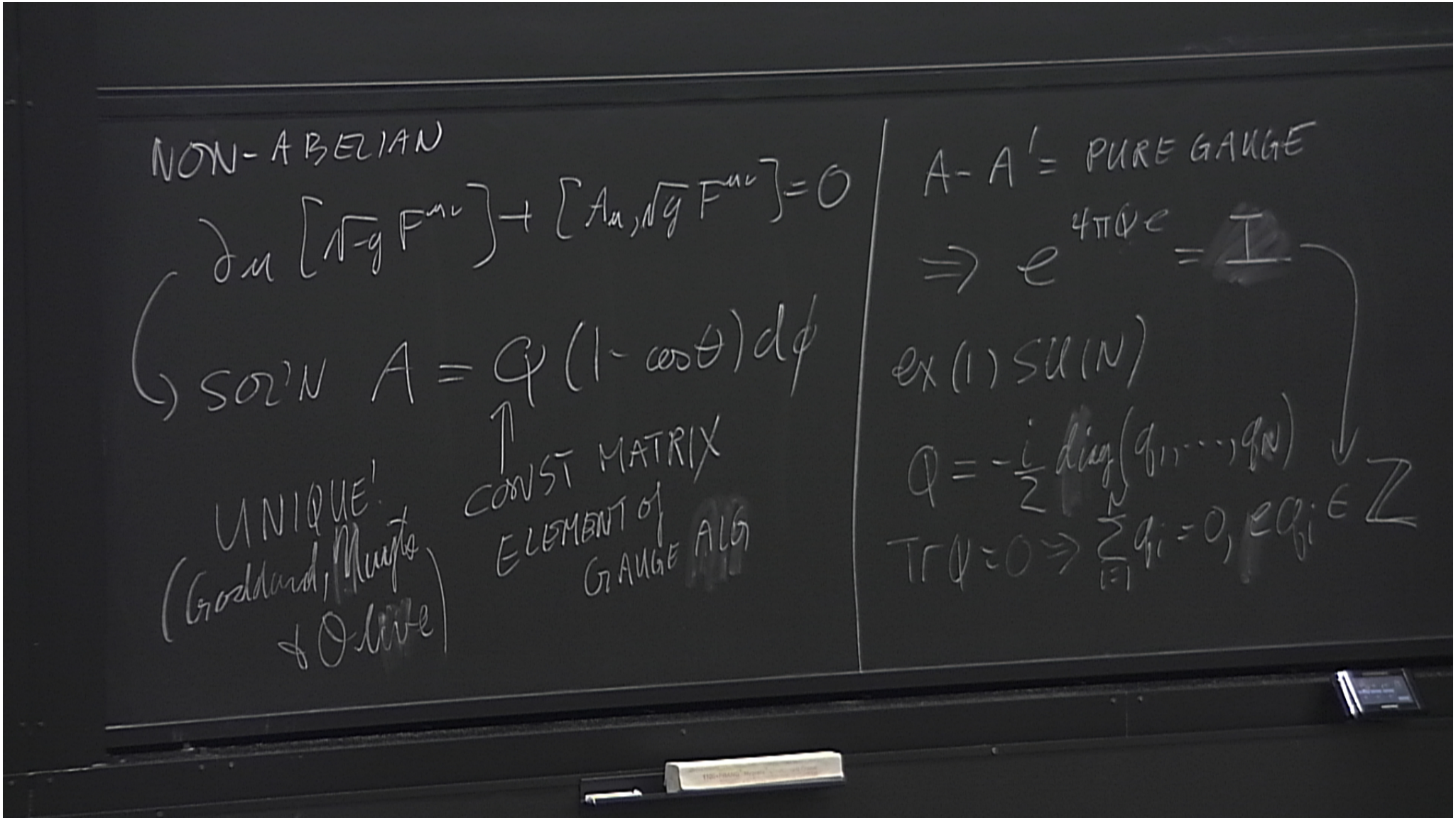
ex (1) SU(N)

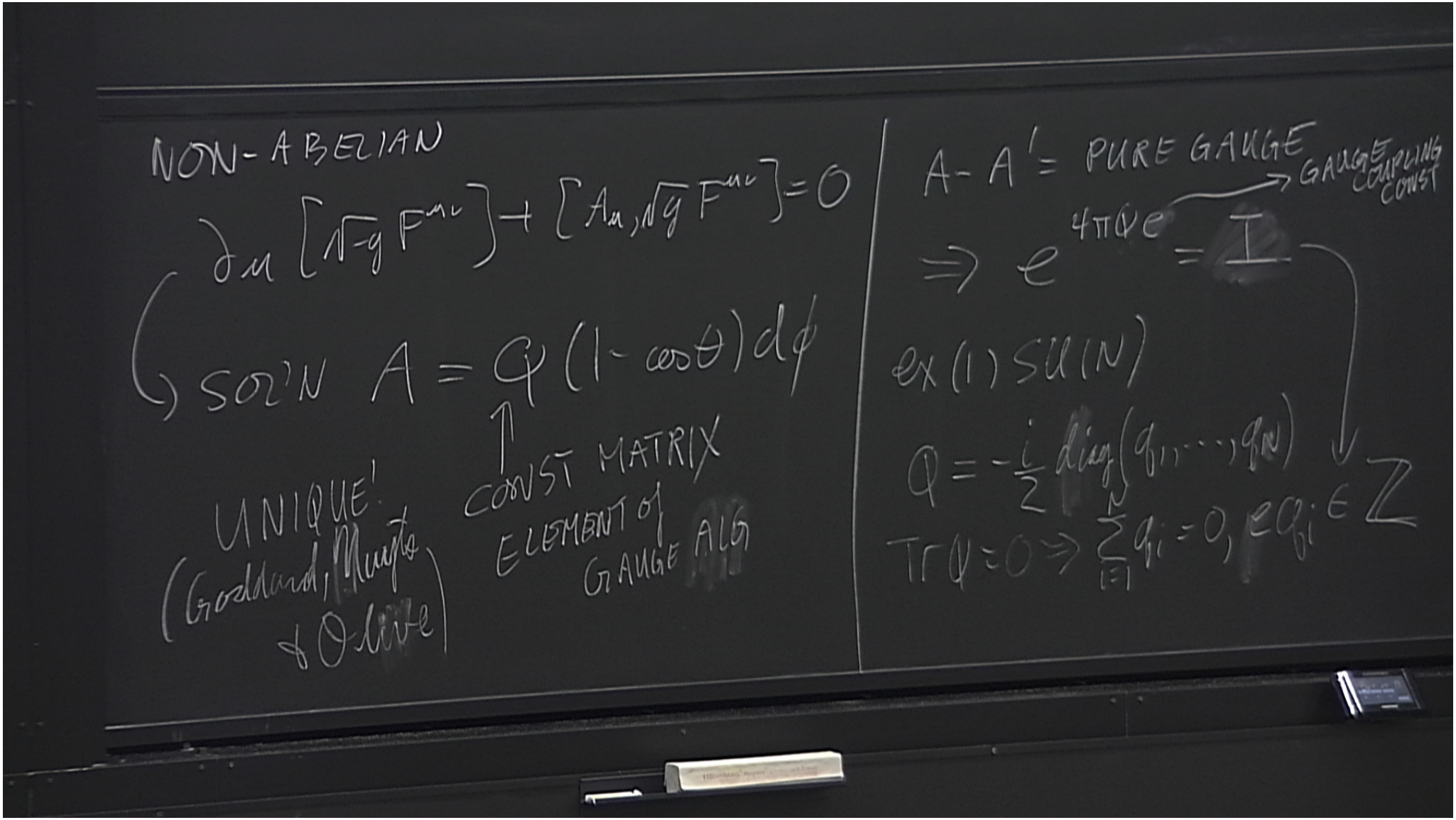
$q = -\frac{i}{2} \mathrm{diag}(q_1, \ldots, q_n)$

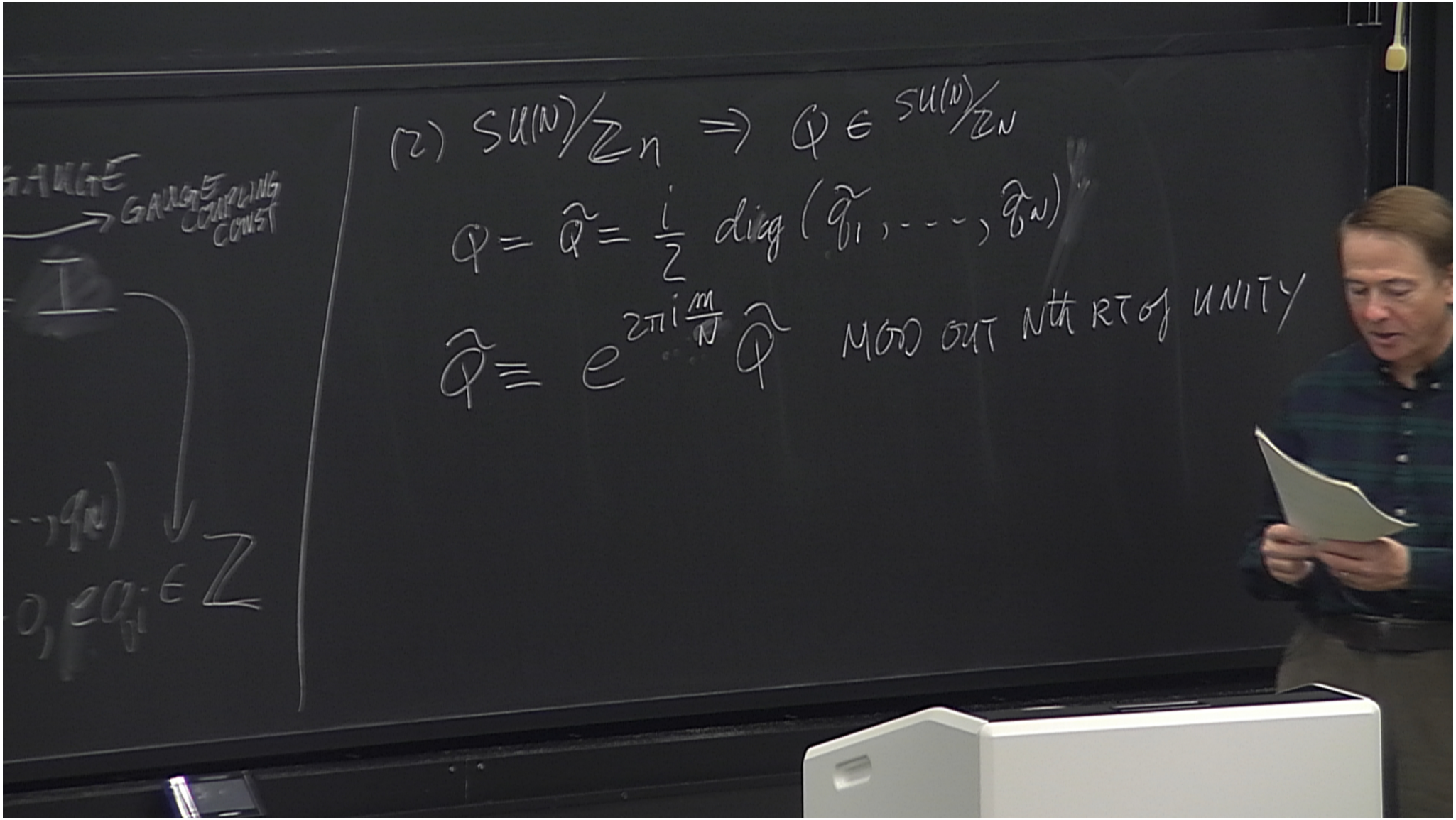

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CONST MATRIX
ELEMENT of
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A - A' = PURE GAUGE
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ex (1) SU(N)
$Q = -\frac{i}{2}\,\mathrm{diag}(q_1, \ldots, q_N)$
$\mathrm{Tr}\,Q = 0 \Rightarrow \sum_{i=1}^{N} q_i = 0$

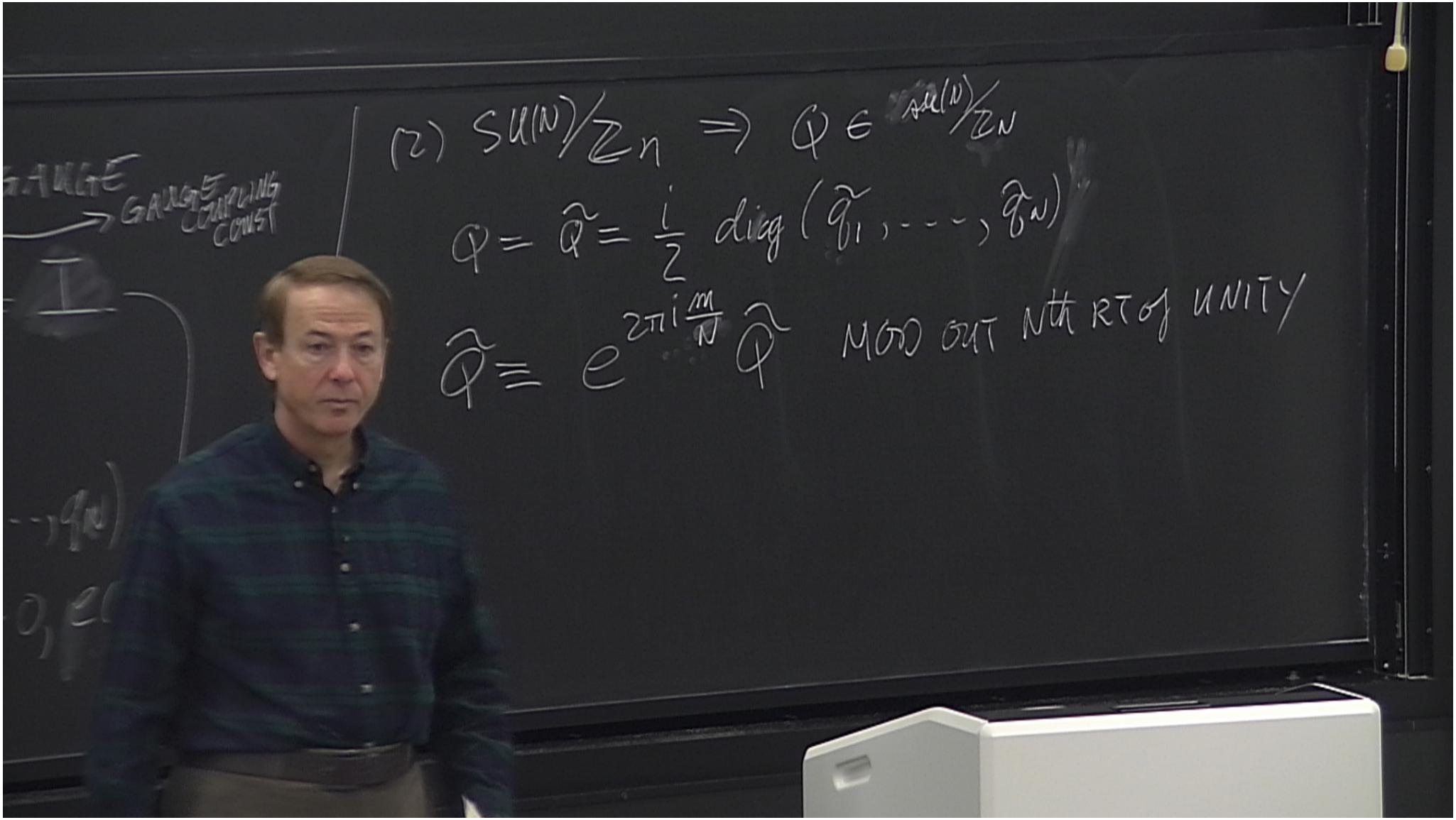

NON-ABELIAN
$\partial_\mu[\sqrt{-g}F^{\mu\nu}] + [A_\mu, \sqrt{g}F^{\mu\nu}] = 0$
SO'N $A = q(1-\cos\theta)d\phi$
CONST MATRIX
ELEMENT of
GAUGE ALG
UNIQUE'
(Goddard, Nuyts
& Olive)
$A - A' =$ PURE GAUGE
$\Rightarrow e^{4\pi q e} = I$
ex (1) SU(N)
$q = -\frac{i}{2}\,\mathrm{diag}(q_1, \ldots, q_N)$
$\mathrm{Tr}\,q = 0 \Rightarrow \sum_{i=1}^{N} q_i = 0 \quad q_i \in \mathbb{Z}$

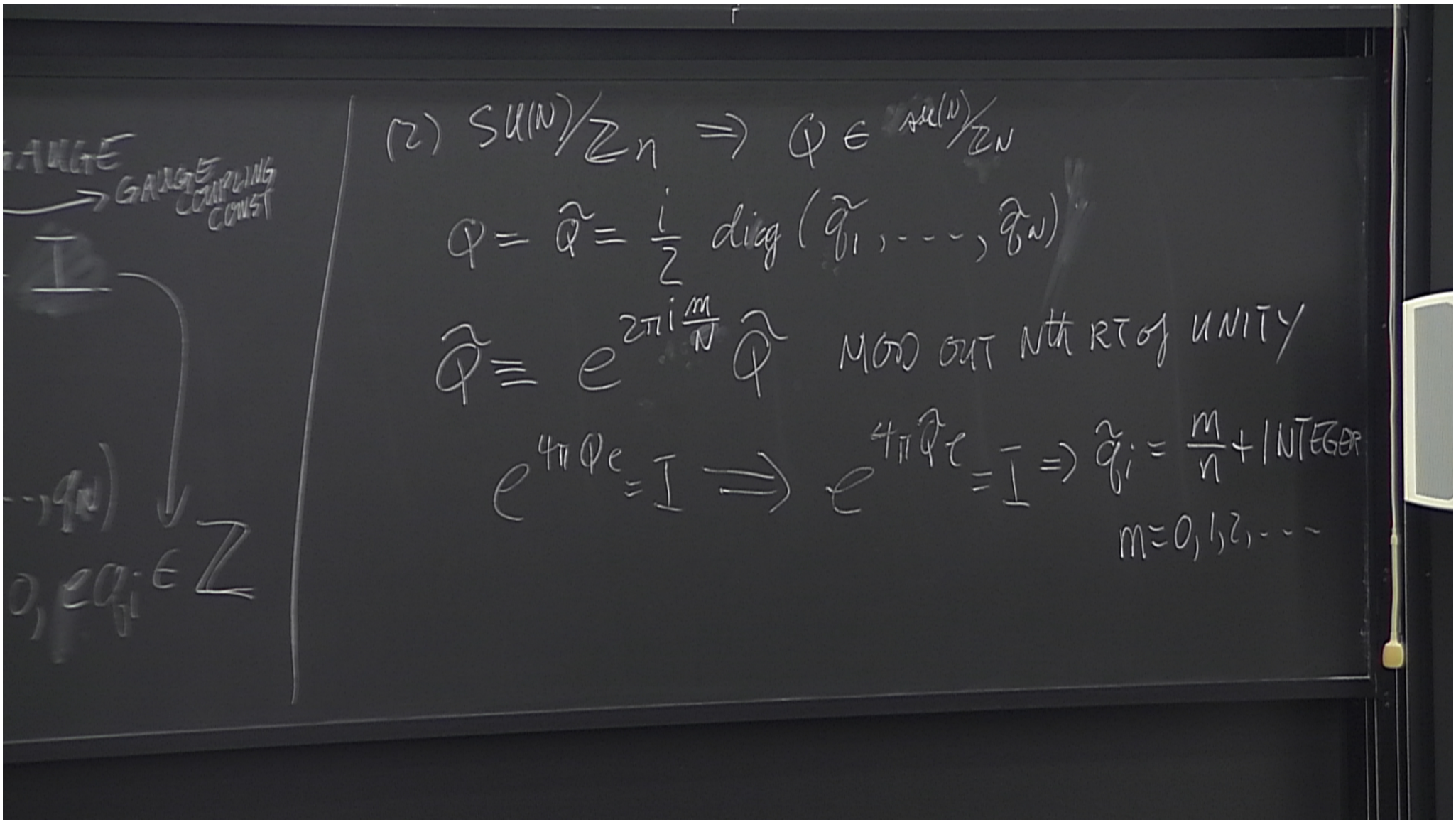

NON-ABELIAN
$\partial_\mu [\sqrt{-g} F^{\mu\nu}] + [A_\mu, \sqrt{g} F^{\mu\nu}] = 0$
SO'N $A = q(1-\cos\theta)d\phi$
UNIQUE!
(Goddard, Nuyts & Olive)
CONST MATRIX
ELEMENT of
GAUGE ALG
$A - A' =$ PURE GAUGE
$\Rightarrow e^{4\pi q e} = I$
ex (1) SU(N)
$q = -\frac{i}{2} \mathrm{diag}(q_1, \ldots, q_N)$
$\mathrm{Tr}\, q = 0 \Rightarrow \sum_{i=1}^{N} q_i = 0$, $q_i \in \mathbb{Z}$


NON-ABELIAN
$\partial_\mu [\sqrt{-g} F^{\mu\nu}] + [A_\mu, \sqrt{g} F^{\mu\nu}] = 0$
SO'N $A = q(1-\cos\theta)d\phi$
CONST MATRIX
ELEMENT of
GAUGE ALG
UNIQUE'
(Goddard, Mungo
& Olive)
$A - A' =$ PURE GAUGE
$\Rightarrow e^{4\pi q e} = I$
ex (1) SU(N)
$q = -\frac{i}{2} \mathrm{diag}(q_1, \ldots, q_N)$
$\mathrm{Tr}\, q = 0 \Rightarrow \sum_{i=1}^{N} q_i = 0, \; e q_i \in \mathbb{Z}$


NON-ABELIAN
$\partial_\mu[\sqrt{-g}F^{\mu\nu}] + [A_\mu, \sqrt{g}F^{\mu\nu}] = 0$
$\hookrightarrow SO^2N \quad A = Q(1-\cos\theta)d\phi$
CONST MATRIX
ELEMENT of
GAUGE ALG
UNIQUE!
(Goddard, Nuyts
& Olive)
$A - A' =$ PURE GAUGE
GAUGE COUPLING CONST
$\Rightarrow e^{4\pi Q e} = I$
ex (1) SU(N)
$Q = -\frac{i}{2}\,\mathrm{diag}(q_1, \ldots, q_N)$
$\mathrm{Tr}\,Q = 0 \Rightarrow \sum_{i=1}^{N} q_i = 0, \; e q_i \in Z$


(2) $SU(N)/\mathbb{Z}_n \Rightarrow Q \in SU(N)/\mathbb{Z}_N$
$Q = \tilde{Q} = \frac{i}{2}\,\mathrm{diag}(\tilde{q}_1, \ldots, \tilde{q}_N)$
$\tilde{Q} \equiv e^{2\pi i \frac{m}{N}} \tilde{Q}$ MOD OUT Nth RT of UNITY
GAUGE
GAUGE COUPLING CONST
$eq_i \in \mathbb{Z}$


GAUGE
GAUGE COUPLING CONST
(2) SU(N)/Z_n ⇒ Q ∈ su(N)/Z_N
Q = Q̃ = i/2 diag(q̃_1, ..., q̃_N)
Q̃ ≡ e^{2πi m/N} Q̃ MOD OUT Nth RT of UNITY


GAUGE
GAUGE COUPLING CONST
I
$q_N) \to$
$0, eq_i \in \mathbb{Z}$
(2) $SU(N)/\mathbb{Z}_n \Rightarrow Q \in SU(N)/\mathbb{Z}_N$
$Q = \tilde{Q} = \frac{i}{2}\,\mathrm{diag}(\tilde{q}_1, \ldots, \tilde{q}_N)$
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$e^{4\pi Q e} = I \Rightarrow e^{4\pi \tilde{Q} e} = I \Rightarrow \tilde{q}_i = \frac{m}{n} + \mathrm{INTEGER}$
$m = 0, 1, 2, \ldots$

& LONG SOLENOID

IN VAC

$\frac{\partial B}{\partial t} + \nabla \times E = 0$

(3) $SU(2)$ vs $SO(3) = SU(2)/\mathbb{Z}_2$

$\downarrow$

$Q = \frac{i}{2}\begin{pmatrix} q & 0 \\ 0 & -q \end{pmatrix} = -\frac{i}{2} q \sigma_3$

$\Rightarrow eq = 0, 1, 2, \ldots$

$\tilde{Q} = -\frac{i}{2}\begin{pmatrix} \tilde{q} & 0 \\ 0 & -\tilde{q} \end{pmatrix} = -\frac{i}{2}\tilde{q}\sigma_3$

$\Rightarrow e\tilde{q} = \frac{1}{2} + \text{INT}$

$= \frac{\text{INTEGER}}{2}$

& LONG SOLENOID

IN VAC

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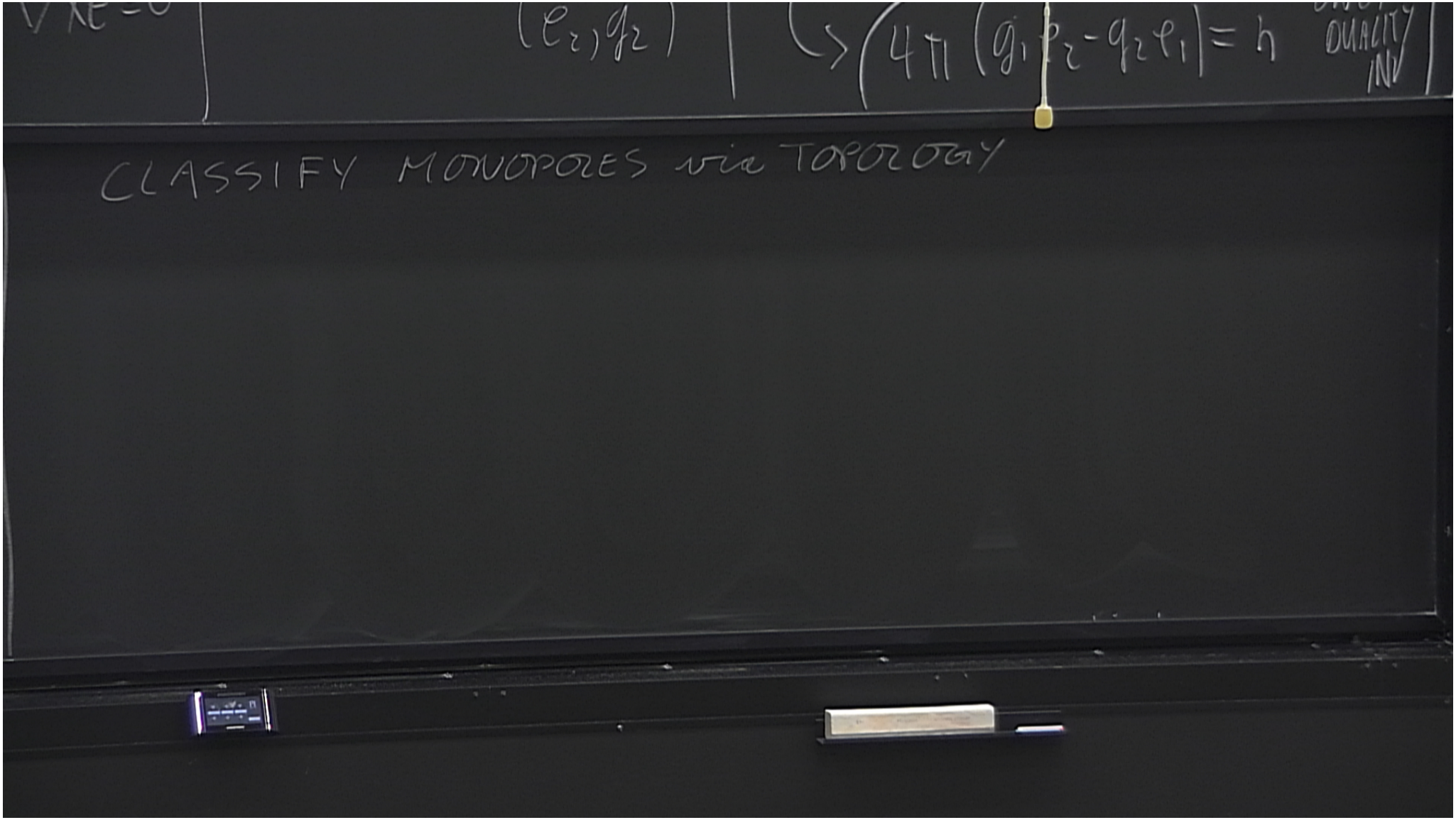
$\Rightarrow eq = 0, 1, 2, \cdots$

$\tilde{Q} = -\frac{i}{2}\begin{pmatrix} \tilde{q} & 0 \\ 0 & -\tilde{q} \end{pmatrix} = -\frac{i}{2}\tilde{q}\sigma_3$

$\Rightarrow e\tilde{q} = \frac{1}{2} + \text{INT}$

$= \frac{\text{INTEGER}}{2}$

CLASSIF


(e_2, q_2)
$\hookrightarrow (4\pi (g_1 p_2 - q_2 p_1) = n$
DUALITY
IN
CLASSIFY MONOPOLES via TOPOLOGY

(e_2, q_2) | $\hookrightarrow (4\pi (g_1 p_2 - q_2 p_1) = h$ DUALITY IN

CLASSIFY MONOPOLES via TOPOLOGY

CONSIDER: $g(\tau) = P\left(\exp\left(-e\int_0^\tau ds A_\mu \frac{dx^\mu}{ds}\right)\right] = P\left(e^{-e\int_{x_i}^{x_f} A_\mu dx^\mu}\right)$

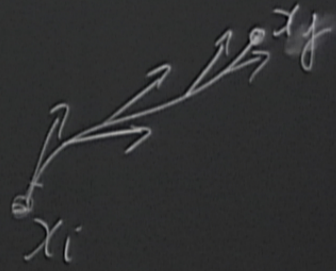
PATH ORDERING

(e_2, q_2) ↪ $4\pi(q_1 p_2 - q_2 p_1) = h$ DUALITY IN

CLASSIFY MONOPOLES via TOPOLOGY

CONSIDER: $g(\tau) = P\left[\exp\left(-e\int_0^\tau ds A_\mu \frac{dx^\mu}{ds}\right)\right] = P\left[e^{-e\int_{x_i}^{x_f} A_\mu dx^\mu}\right]$ PATH ORDERING

↪ $\in G$



(e_2, q_2) ↪ $4\pi(q_1 p_2 - q_2 p_1) = h$ DUALITY IN

CLASSIFY MONOPOLES via TOPOLOGY

CONSIDER: $g(\tau) = P\left[\exp\left(-e\int_0^\tau ds\, A_\mu \frac{dx^\mu}{ds}\right)\right] = P\left[e^{-e\int_{x_i}^{x_f} A_\mu dx^\mu}\right]$ (path ordering)

↪ $\in G$

MONOPOLE

$g(\tau) = P\left[e^{-e\int A_\mu dx^\mu}\right]$

$= P\left[\exp\left(-e\int_0^{2\pi} ds\, A_\phi \frac{d\phi}{ds}\right)\right.$

SPHERE → 

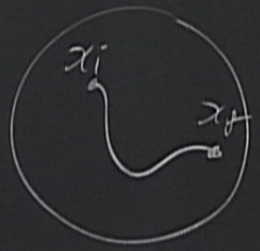
(e_2, q_2) ↪ $4\pi(g_1 p_2 - q_2 p_1) = h$ DUALITY IN

CLASSIFY MONOPOLES via TOPOLOGY

CONSIDER: $g(\tau) = P\left[\exp\left(-e\int_0^\tau ds A_\mu \frac{dx^\mu}{ds}\right)\right] = P\left[e^{-e\int_{x_i}^{x_f} A_\mu dx^\mu}\right]$ (path ordering)

↪ $\in G$

MONOPOLE

SPHERE → 

$g(\tau) = P\left[e^{-e\int A_\mu dx^\mu}\right]$

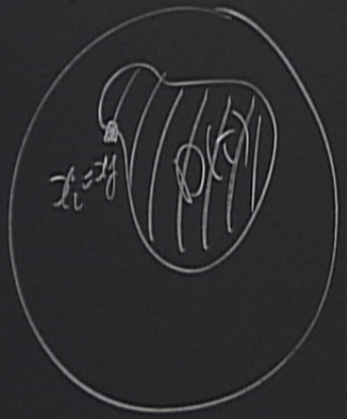
$= P\left[\exp\left(-e\int_0^{2\pi} ds A_\phi \frac{d\phi}{ds}\right)\right]$

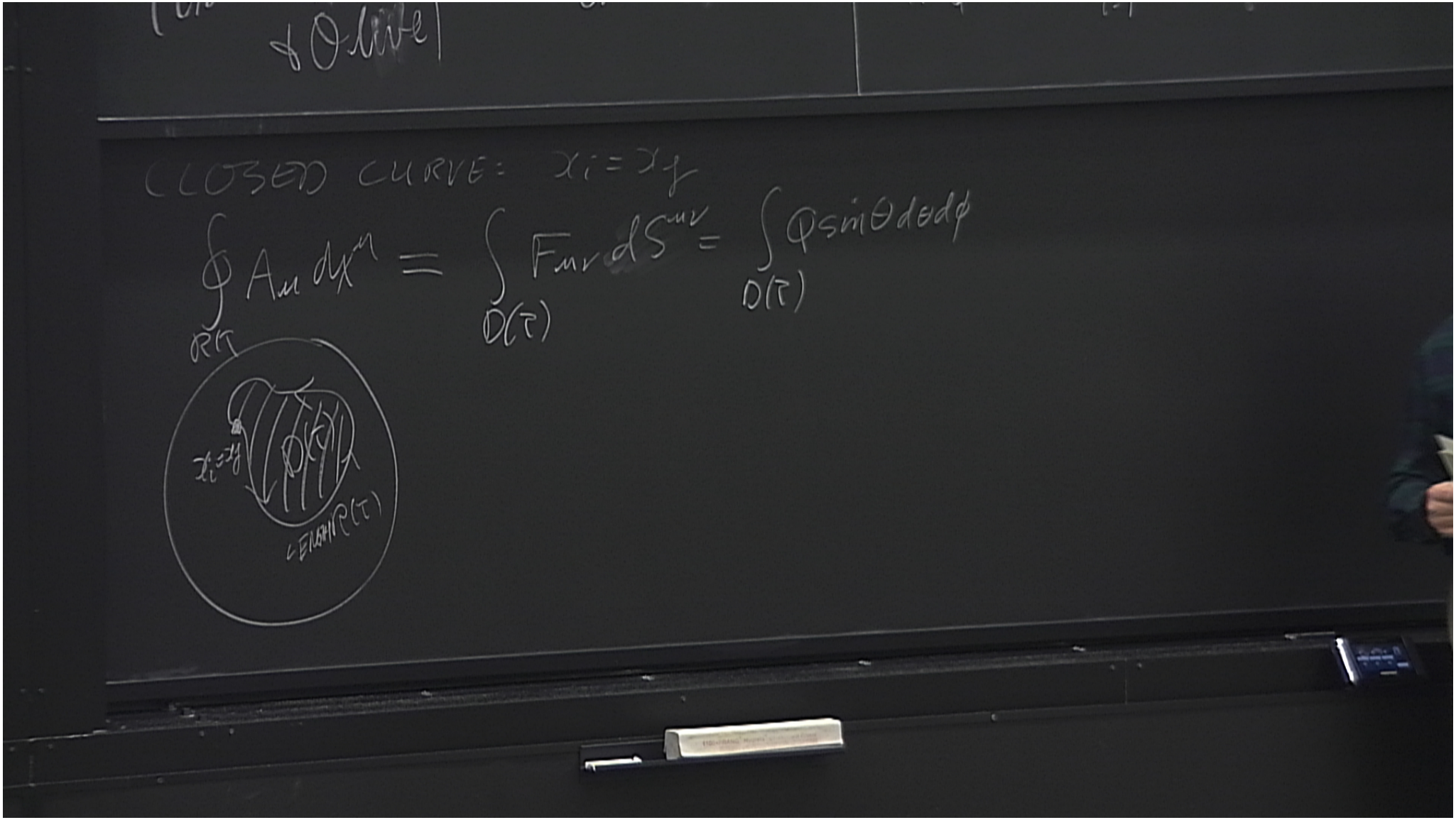


+ 0-like

CLOSED CURVE: $x_i = x_f$

$$\oint A_\mu dx^\mu = \int_{D(\tau)} F_{\mu\nu} dS^{\mu\nu} = \int_{D(\tau)} \varphi \sin\theta \, d\theta \, d\phi$$




CLOSED CURVE: $x_i = x_f$
$\oint_{R(\tau)} A_\mu dx^\mu = \int_{D(\tau)} F_{\mu\nu} dS^{\mu\nu} = \int_{D(\tau)} \varphi \sin\theta \, d\theta \, d\phi$
$x_i = x_f$
LENGTH $R(\tau)$

(e_2, q_2) ↪ $4\pi(g_1 p_2 - q_2 p_1) = n$ DUALITY IND

CLASSIFY MONOPOLES via TOPOLOGY

CONSIDER: $g(\tau) = P\left[\exp\left(-e\int_0^\tau ds A_\mu \frac{dx^\mu}{ds}\right)\right] = P\left[e^{-e\int_{x_i}^{x_f} A_\mu dx^\mu}\right]$ PATH ORDERING

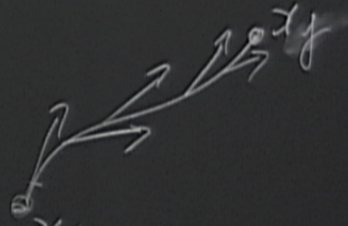
↪ $\in G$

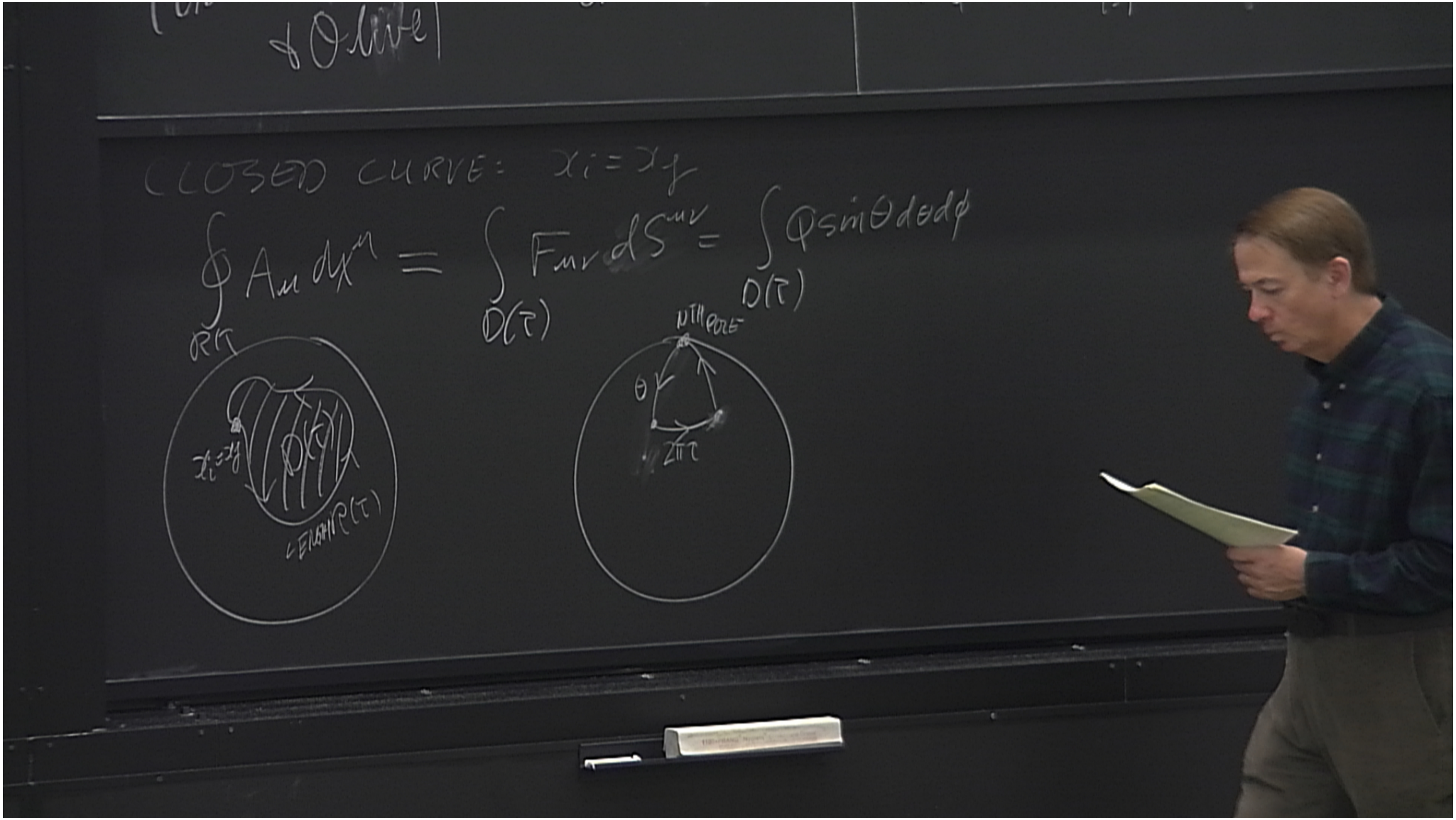
MONOPOLE

SPHERE → 

$g(\tau) = P\left[e^{-e\int A_\mu dx^\mu}\right]$

$= P\left[\exp\left(-e\int_0^\tau ds A_\phi \frac{d\phi}{ds}\right)\right]$

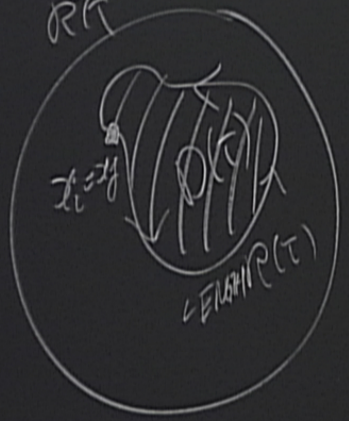


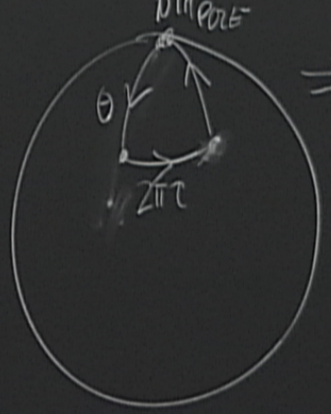

CLOSED CURVE: $x_i = x_f$
$\oint A_\mu dx^\mu = \int_{D(\tau)} F_{\mu\nu} dS^{\mu\nu} = \int_{D(\tau)} \varphi \sin\theta \, d\theta \, d\phi$
$\partial D(\tau)$
$x_i = x_f$
LENGTH $R(\tau)$
NTH POLE
θ
$2\pi\tau$

+ θ-line

CLOSED CURVE: $x_i = x_f$

$$\oint A_\mu dx^\mu = \int_{D(\tau)} F_{\mu\nu} dS^{\mu\nu} = \int_{D(\tau)} q \sin\theta \, d\theta \, d\phi$$

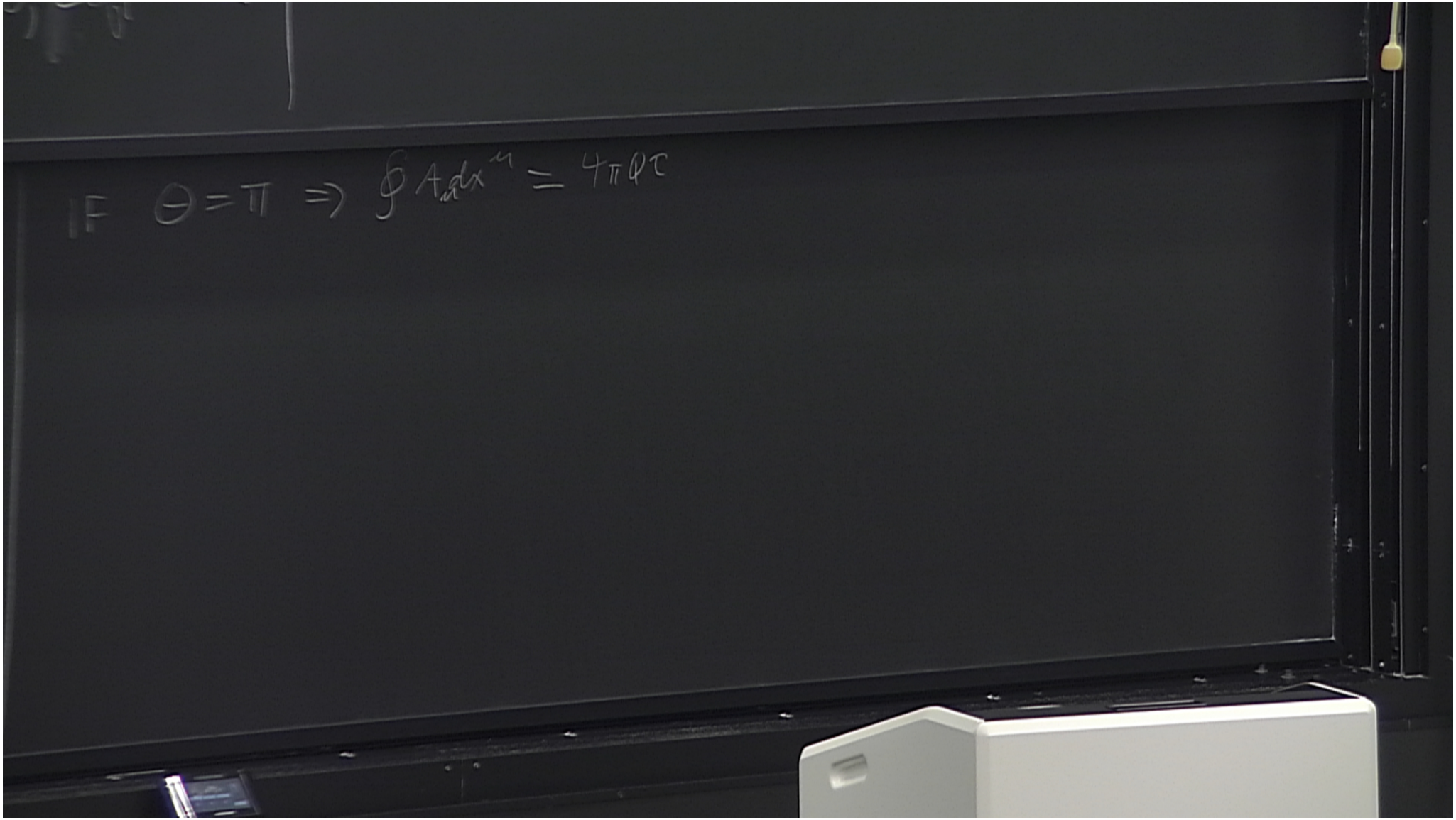


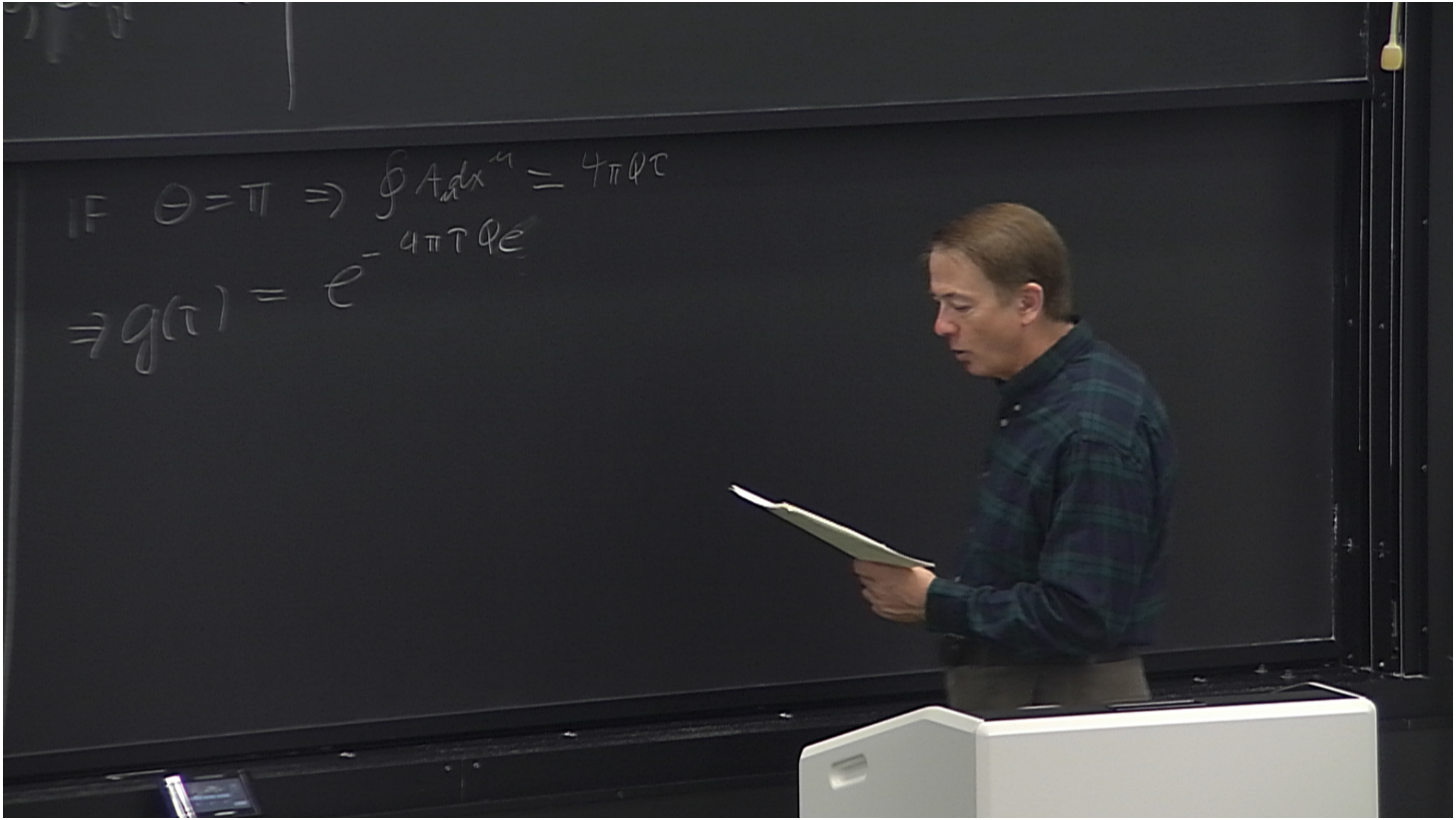


$$\Rightarrow \oint A_\mu dx^\mu$$

$$= q \int_0^\theta \sin\theta \, d\theta \int_0^{2\pi\tau} d\phi$$

$$= 2\pi q (1 - \cos\theta)\tau$$


IF $\theta = \pi \Rightarrow \oint A_\mu dx^\mu = 4\pi Q\tau$


IF $\theta = \pi \Rightarrow \oint A_\mu dx^\mu = 4\pi Q\tau$
$\Rightarrow g(\tau) = e^{-4\pi\tau Q\epsilon}$

(Gradient, Ricci + Orbit)

G Gauge

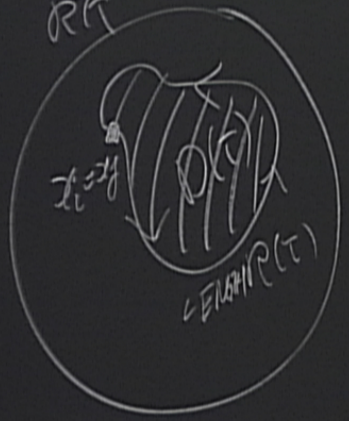
IF $\phi = 0 \Rightarrow \sum_{i=1}^{n}$

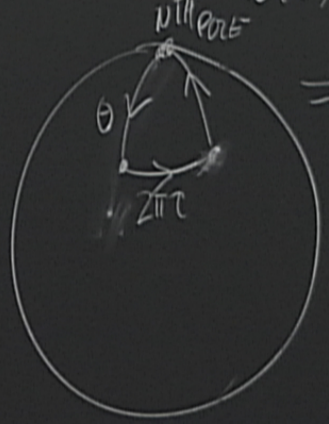
CLOSED CURVE: $x_i = x_f$

$$\oint_{R(\tau)} A_\mu dx^\mu = \int_{D(\tau)} F_{\mu\nu} dS^{\mu\nu} = \int_{D(\tau)} q \sin\theta \, d\theta \, d\phi$$

$\tau \in [0,1]$

$\phi \in [0, 2\pi)$



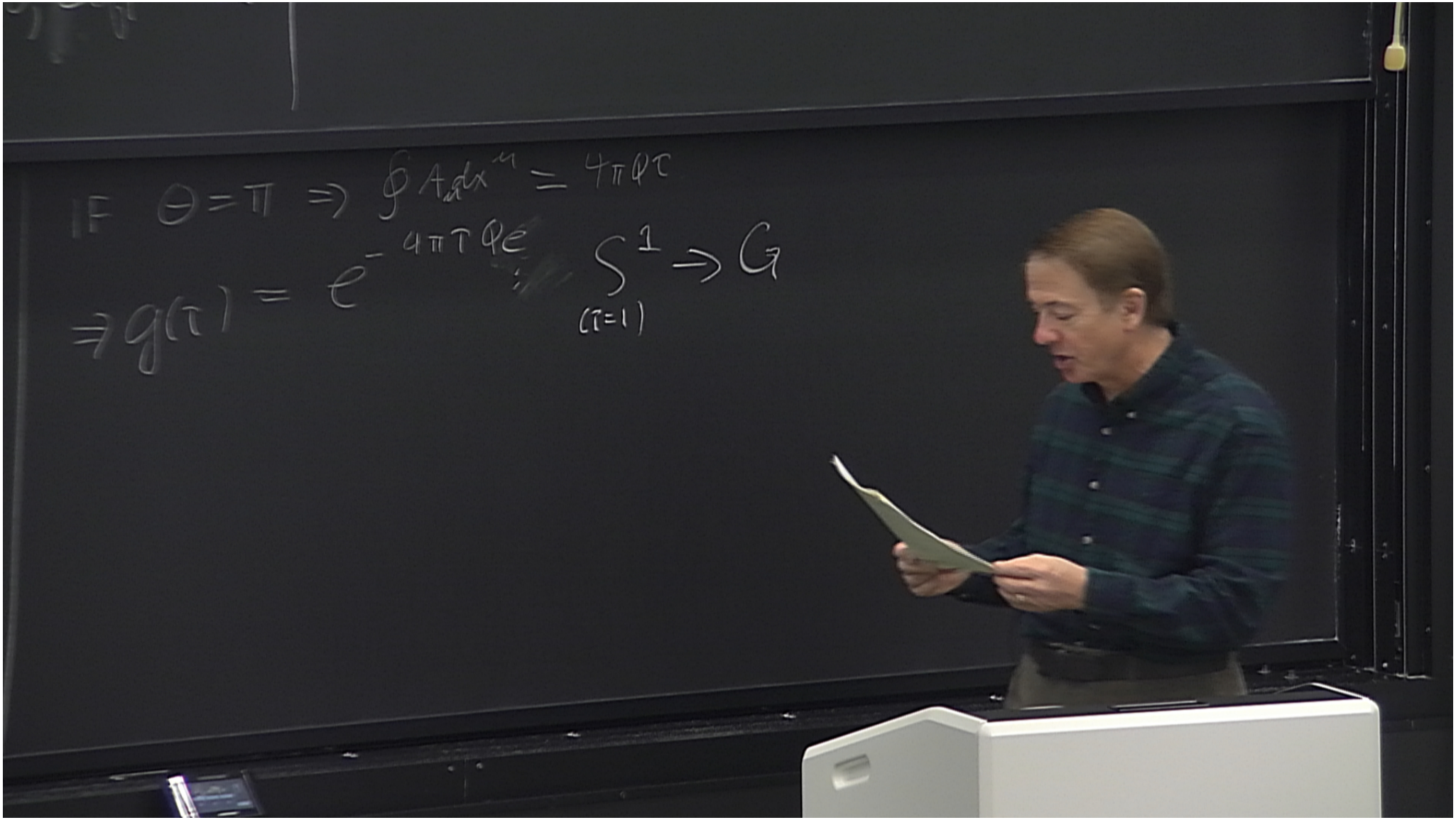


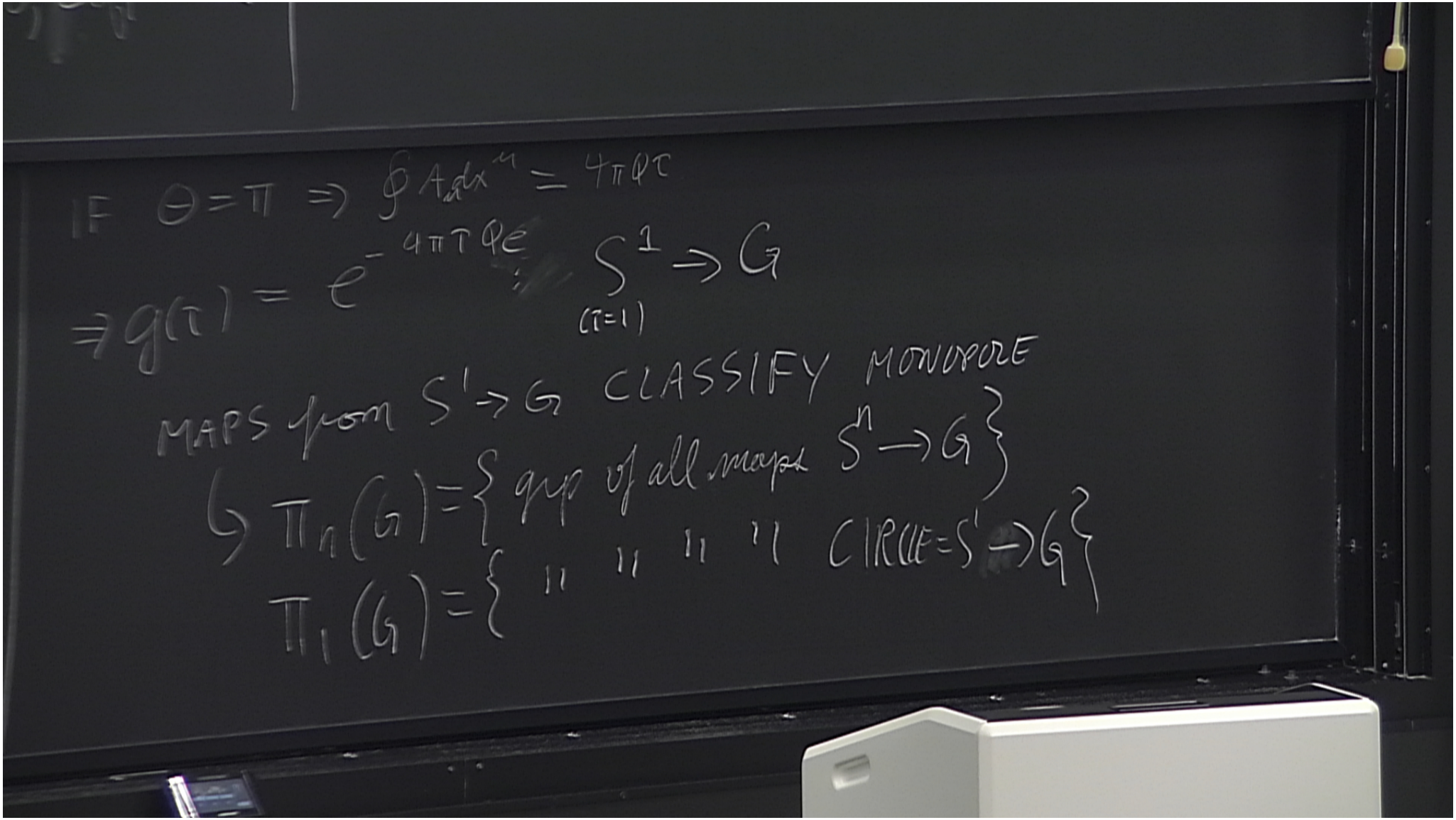
$$\Rightarrow \oint A_\mu dx^\mu = q \int_0^\theta \sin\theta \, d\theta \int_0^{2\pi\tau} d\phi$$

$$= 2\pi q (1 - \cos\theta)\tau$$

IF $\theta = \pi$

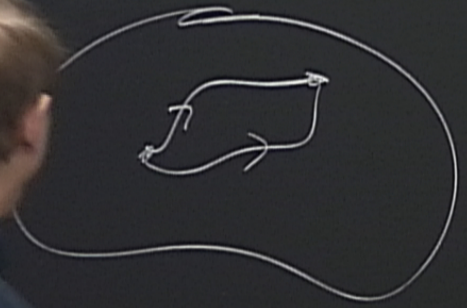
$\Rightarrow g(\tau) =$


IF $\theta = \pi \Rightarrow \oint A_\mu dx^\mu = 4\pi Q\tau$
$\Rightarrow g(\tau) = e^{-4\pi\tau Q e}$
$S^1 \to G$
$(\tau = 1)$


IF $\theta = \pi \Rightarrow \oint A_\mu dx^\mu = 4\pi QC$
$\Rightarrow g(\tau) = e^{-4\pi T Q \epsilon}$
$S^1 \to G$
$(\tau = 1)$
MAPS from $S^1 \to G$ CLASSIFY MONOPOLE
$\hookrightarrow \pi_n(G) = \{$ grp of all maps $S^n \to G \}$
$\pi_1(G) = \{$ " " " " CIRCLE $= S^1 \to G \}$

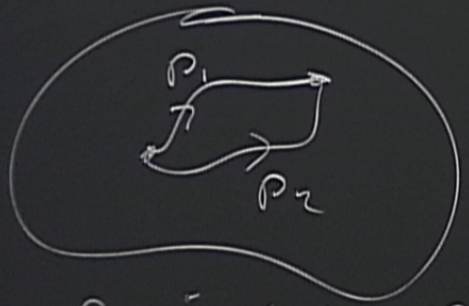
$\pi_1(X)$ IS A MEASURE of the CONNECTEDNESS of X

$\pi_1(X) = 1 \rightarrow$ SIMPLY CONNECTED



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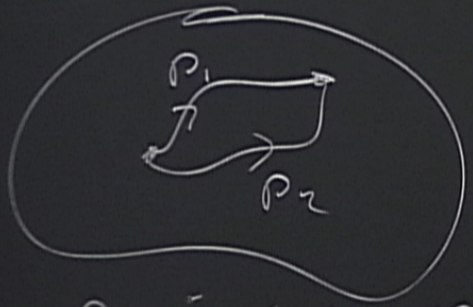
SIMPLY CONNECTED

p_1 is CTSLY DEFORMABLE TO p_2

MULTIPL[illegible]NEC[illegible]

$\pi_1(X)$ IS A MEASURE of the CONNECTEDNESS of X

$\pi_1(X) = 1 \rightarrow$ SIMPLY CONNECTED



SIMPLY CONNE-

p_1 IS CTSLY DEFORM TO p_2

MULTIPLY CONNECTED

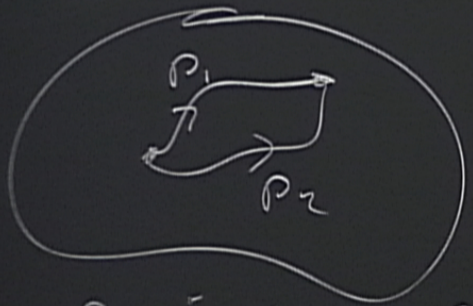
$\pi_1(X) \neq 1$



p_1 IS NOT CTSLY DEFORMABLE TO p_2

$\pi_1(X)$ IS A MEASURE of the CONNECTEDNESS of X

$\pi_1(X) = 1 \rightarrow$ SIMPLY CONNECTED



SIMPLY CONNECTED

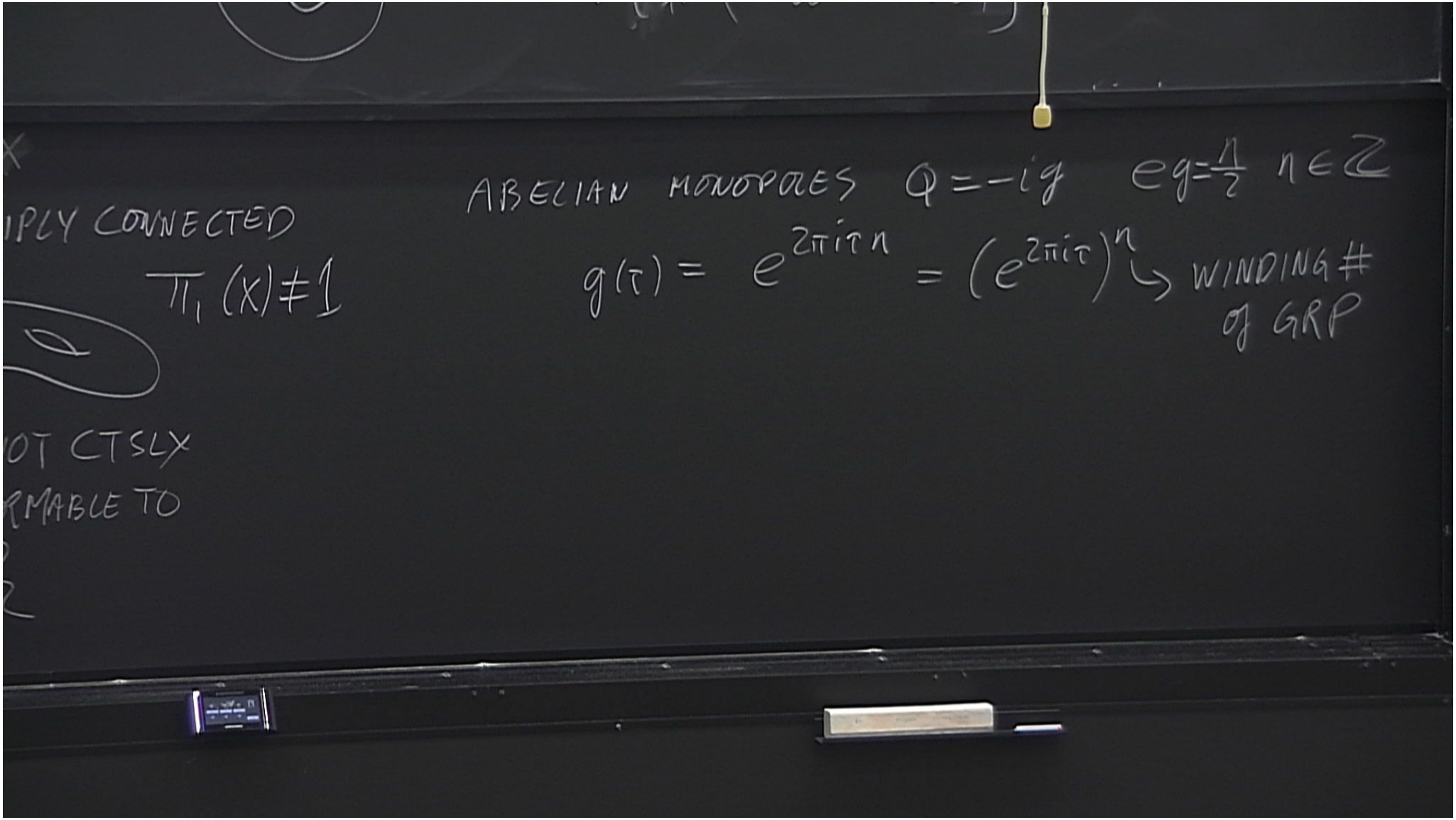
p_1 is CTSLY DEFORMABLE TO p_2

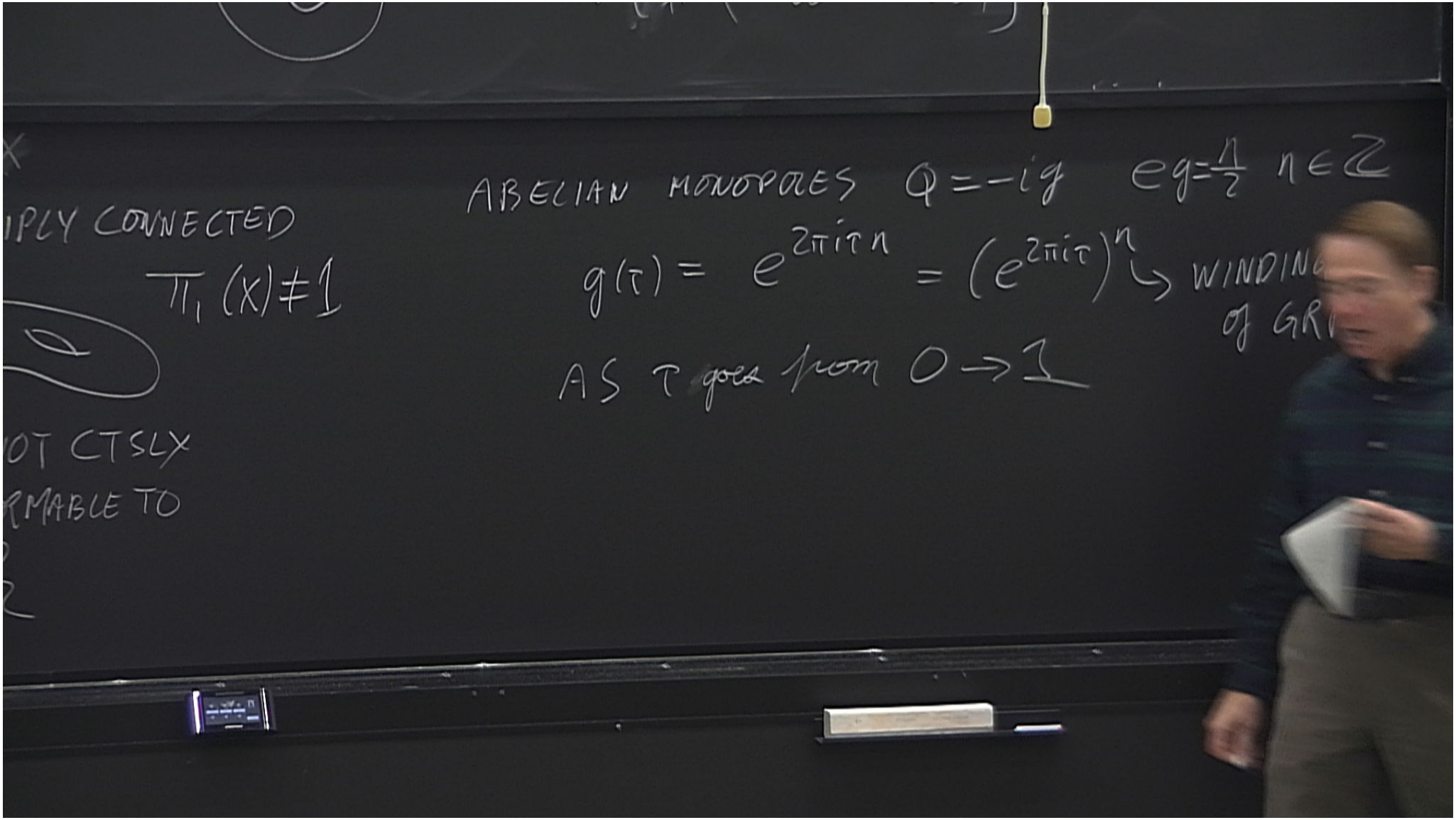
MULTIPLY CONNECTED

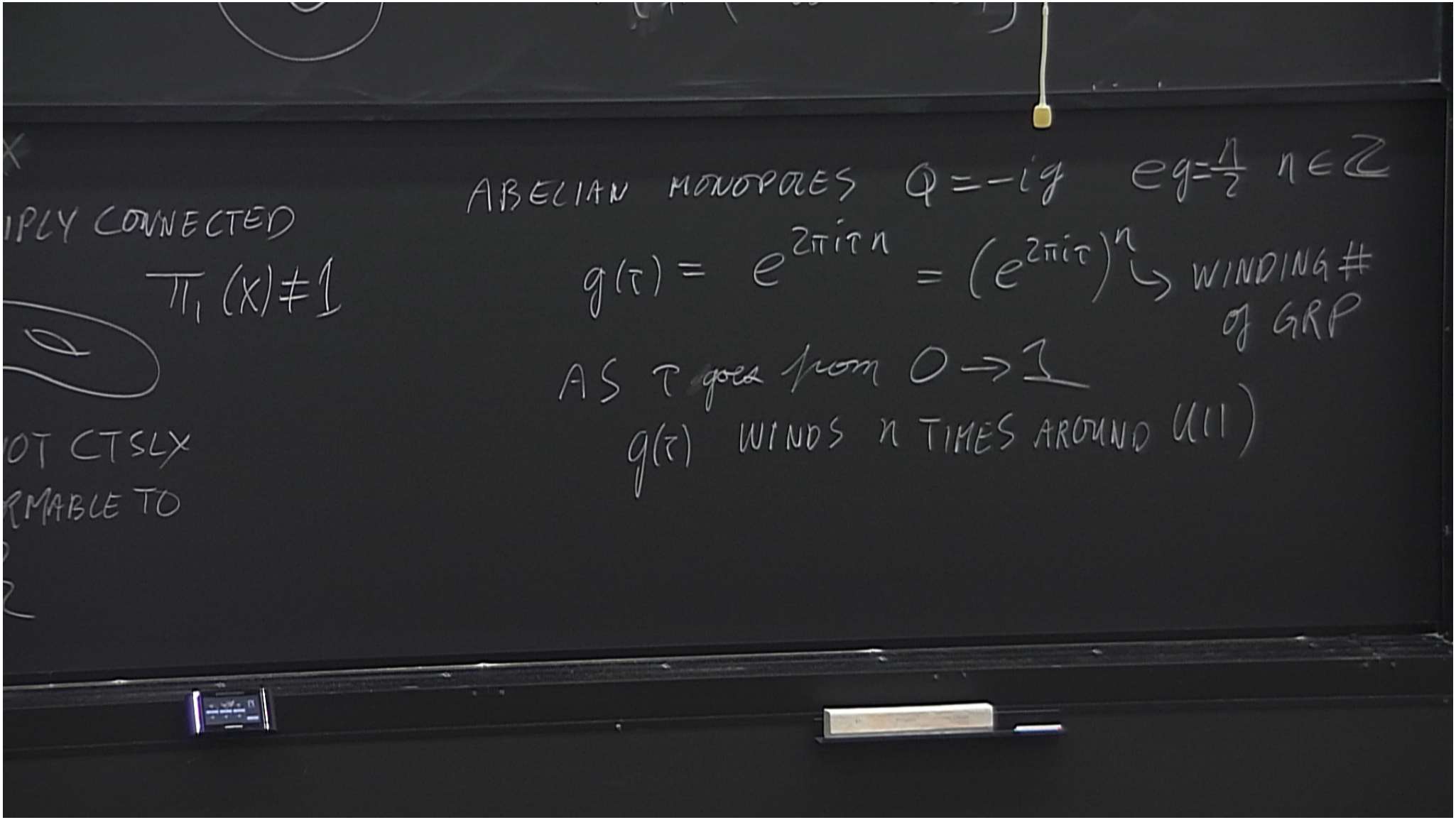
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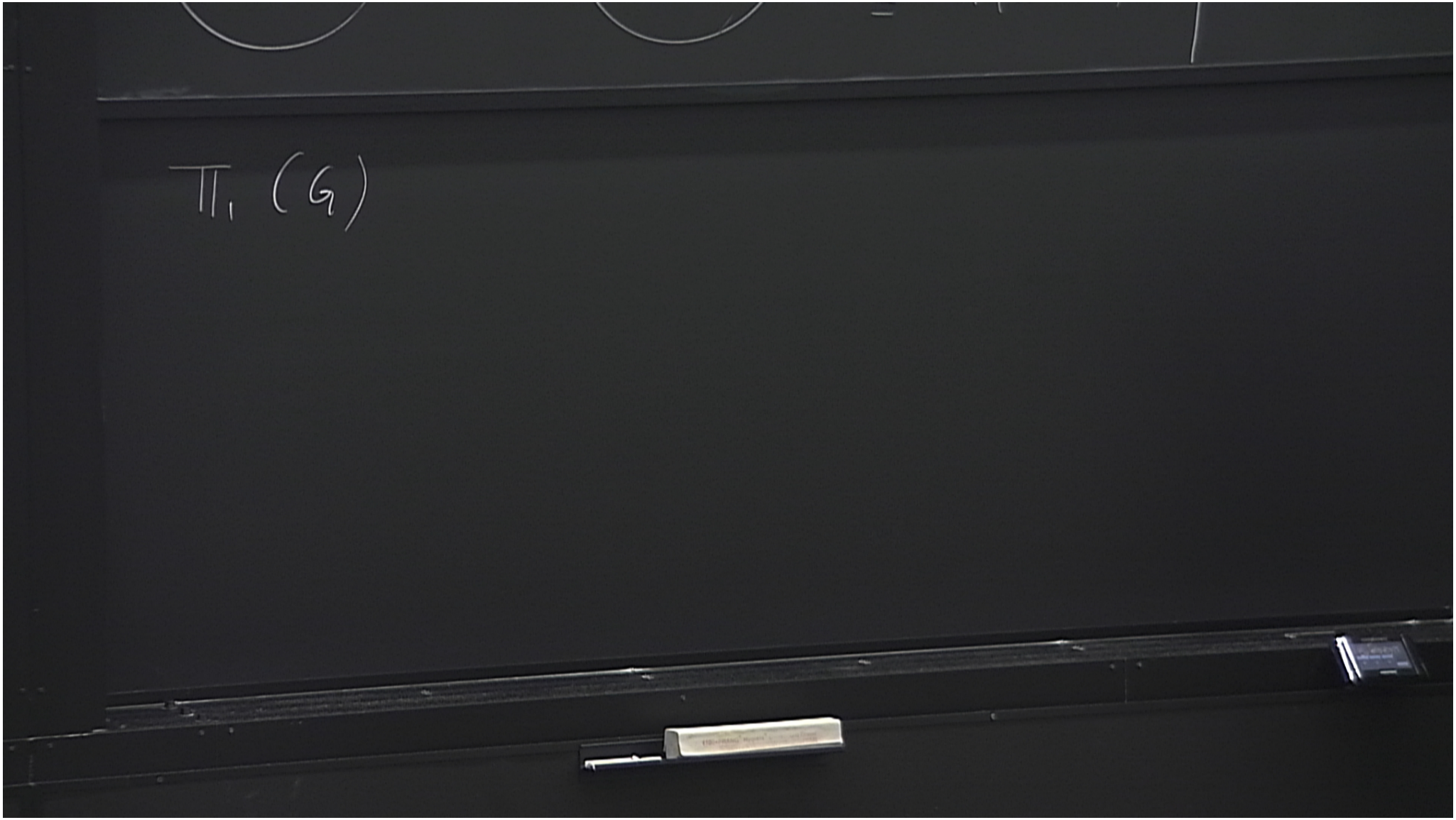


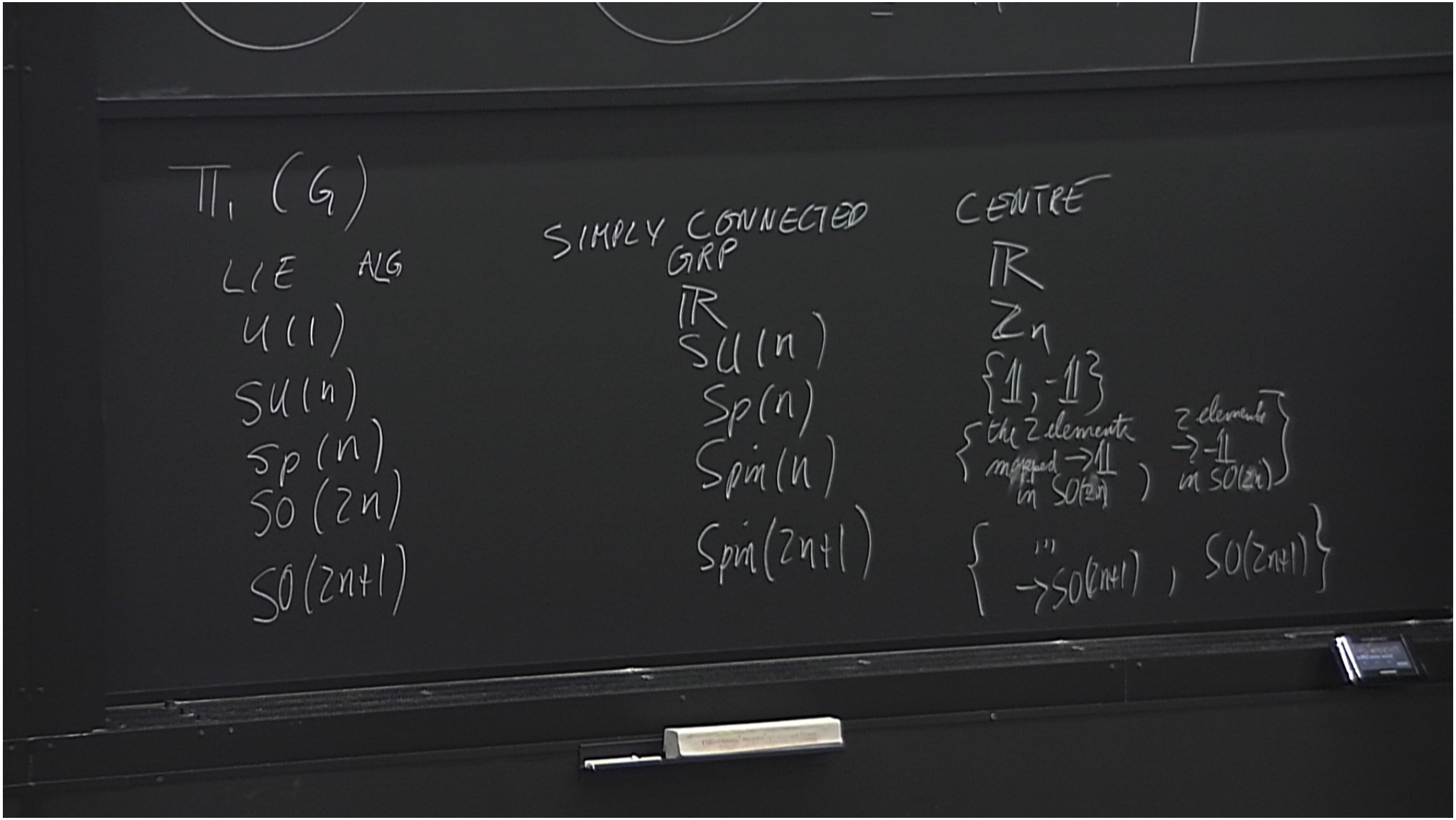
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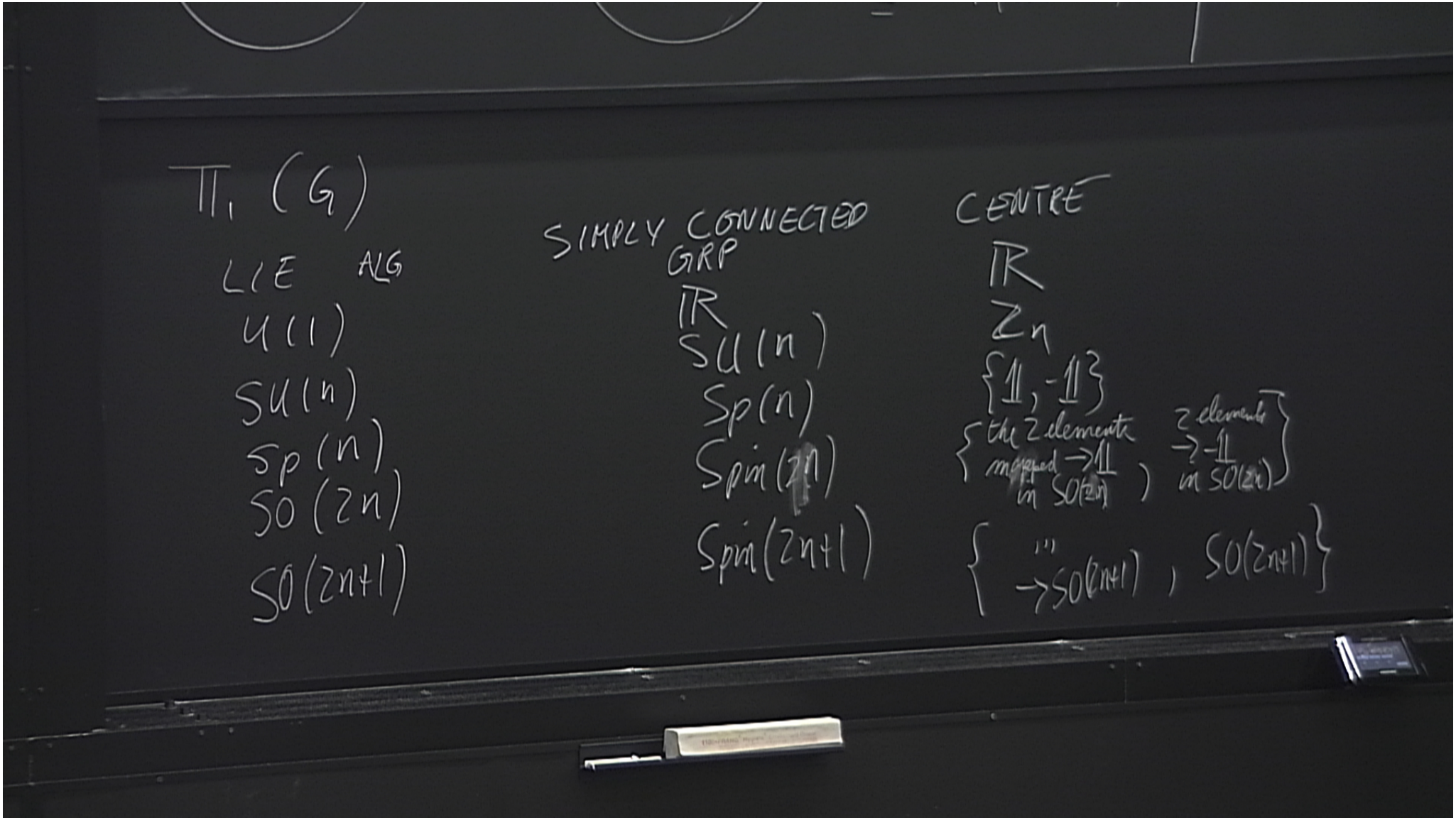

IPLY CONNECTED
$\pi_1(X) \neq 1$
OT CTSLY
RMABLE TO
ABELIAN MONOPOLES $Q = -ig$ $eg = \frac{n}{2}$ $n \in \mathbb{Z}$
$g(\tau) = e^{2\pi i \tau n} = (e^{2\pi i \tau})^n$
WINDING # of GRP

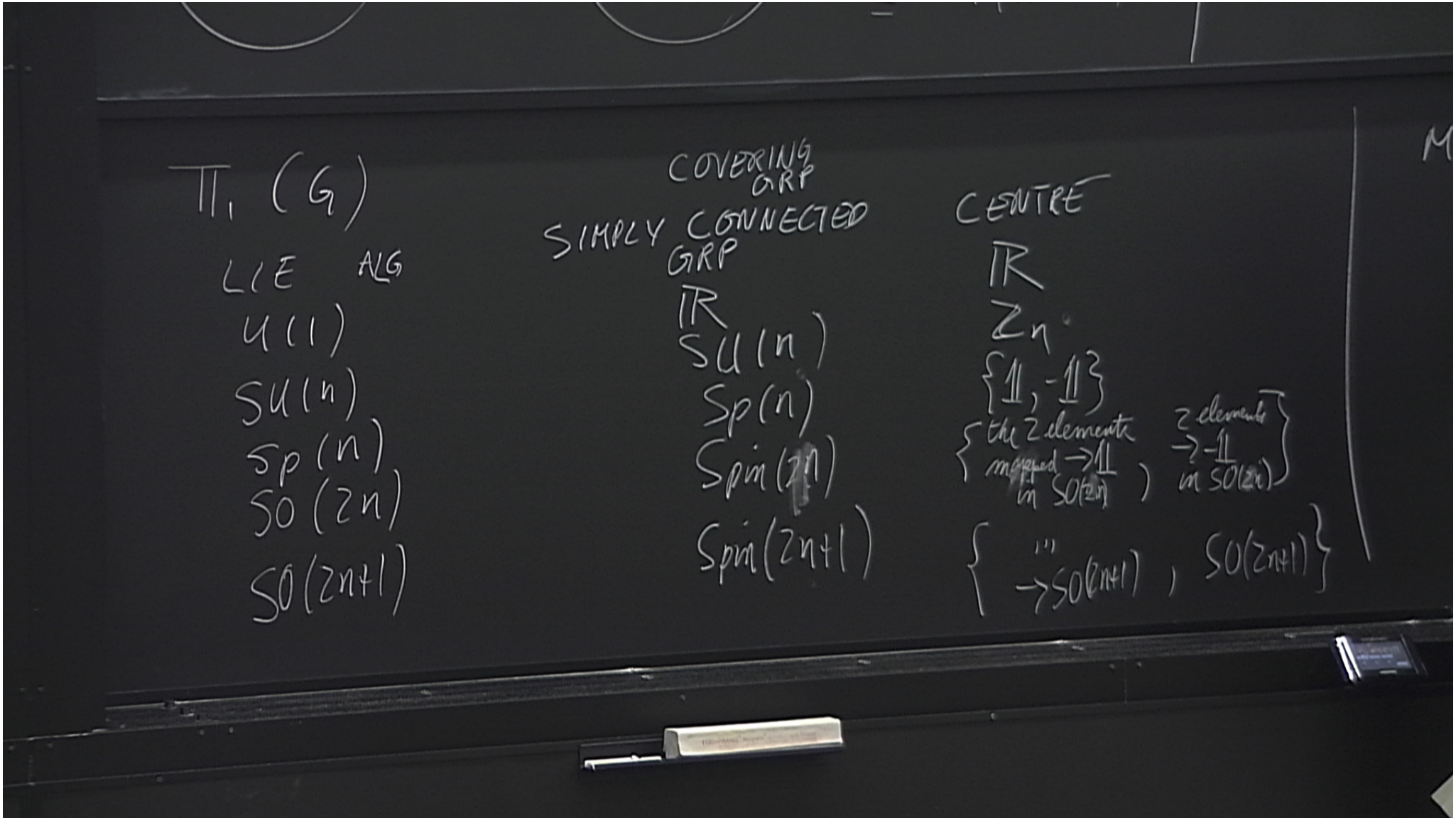

ABELIAN MONOPOLES $Q = -ig$ $eg = \frac{n}{2}$ $n \in \mathbb{Z}$
$g(\tau) = e^{2\pi i \tau n} = (e^{2\pi i \tau})^n$
AS τ goes from $0 \to 1$
$\pi_1(X) \neq 1$

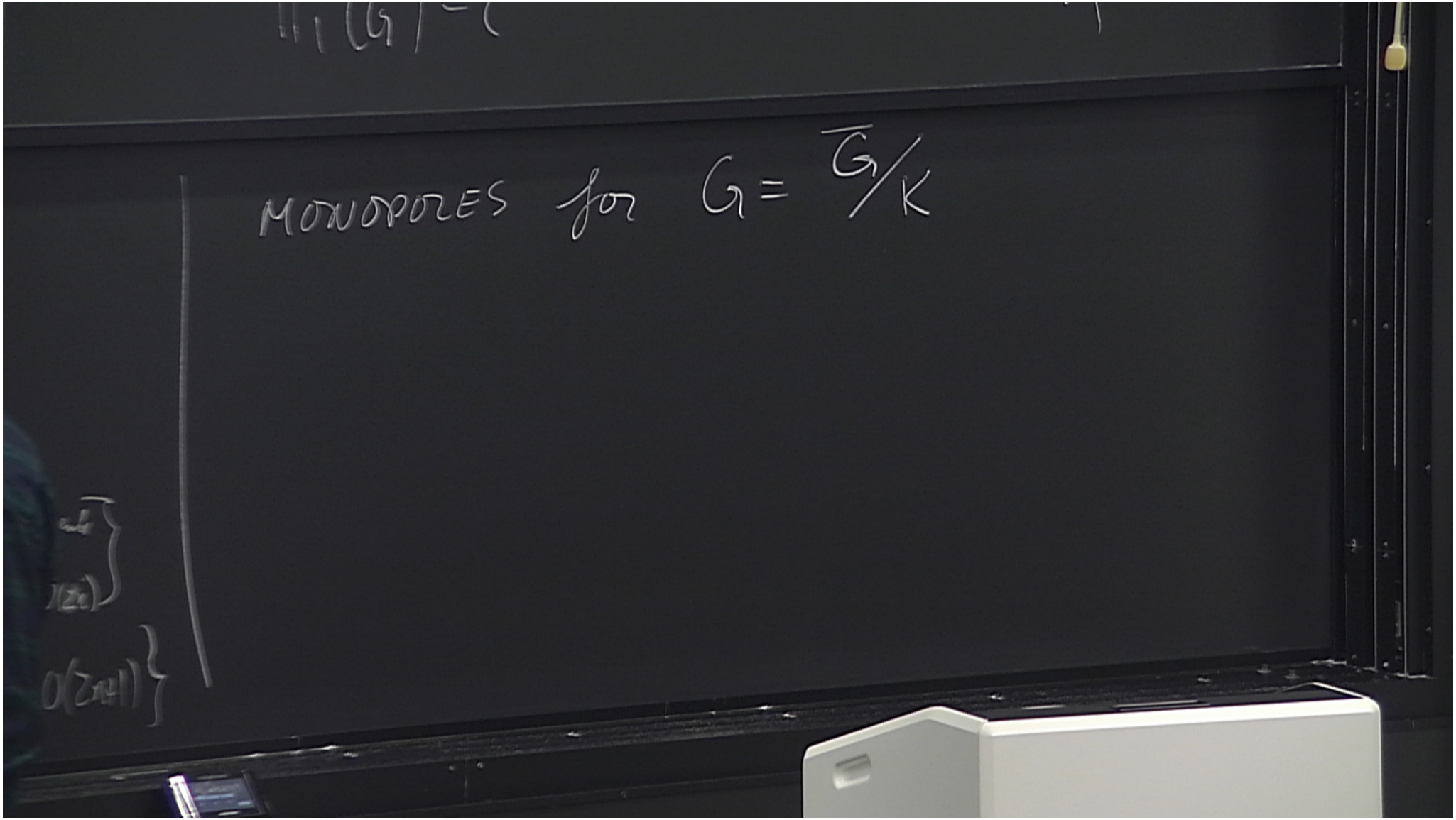

IPLY CONNECTED
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ABELIAN MONOPOLES $Q = -ig$ $eg = \frac{n}{2}$ $n \in \mathbb{Z}$
$g(\tau) = e^{2\pi i \tau n} = (e^{2\pi i \tau})^n$
WINDING # of GRP
AS τ goes from $0 \to 1$
$g(\tau)$ WINDS n TIMES AROUND $U(1)$

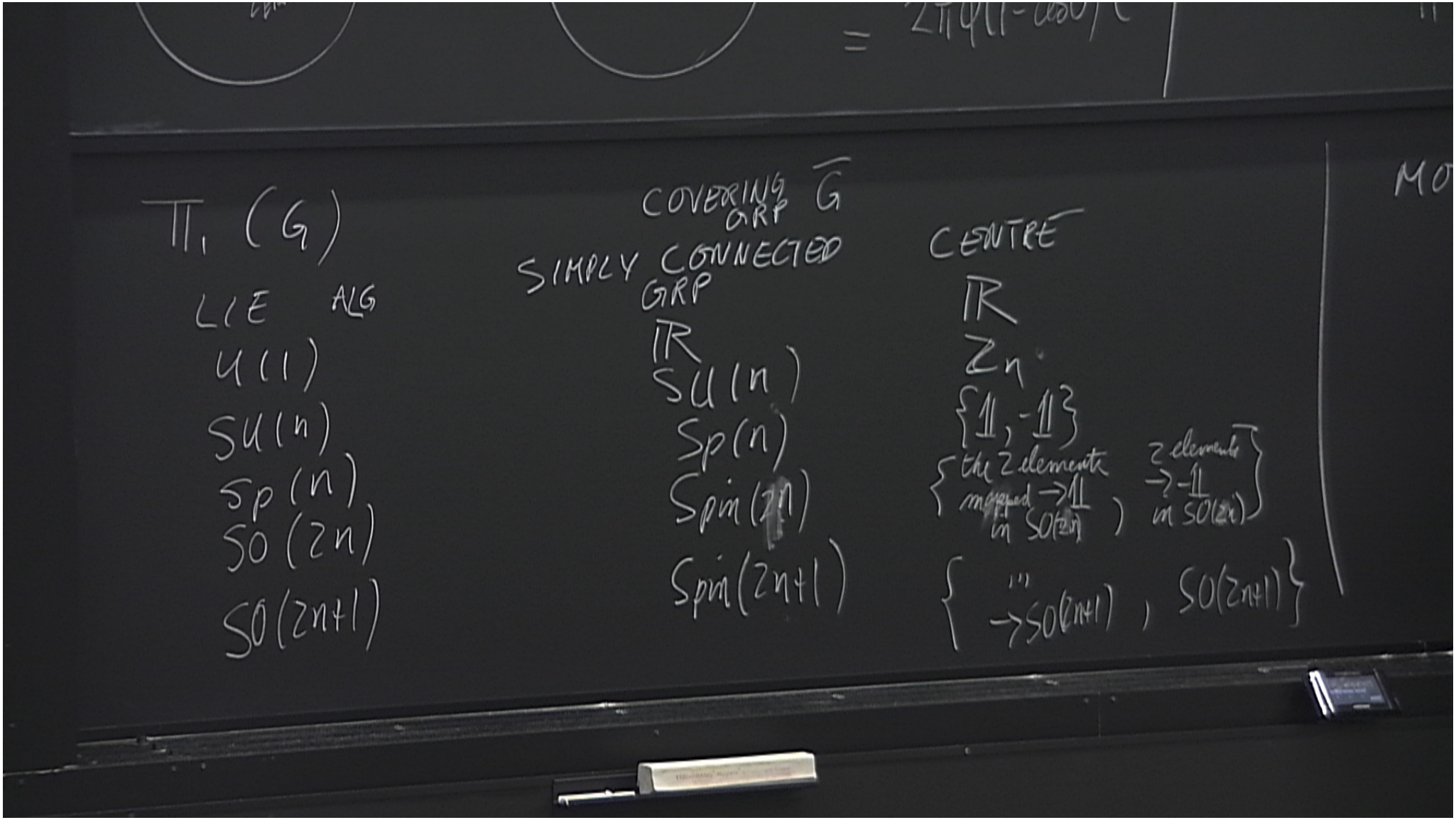

$\pi_1(G)$

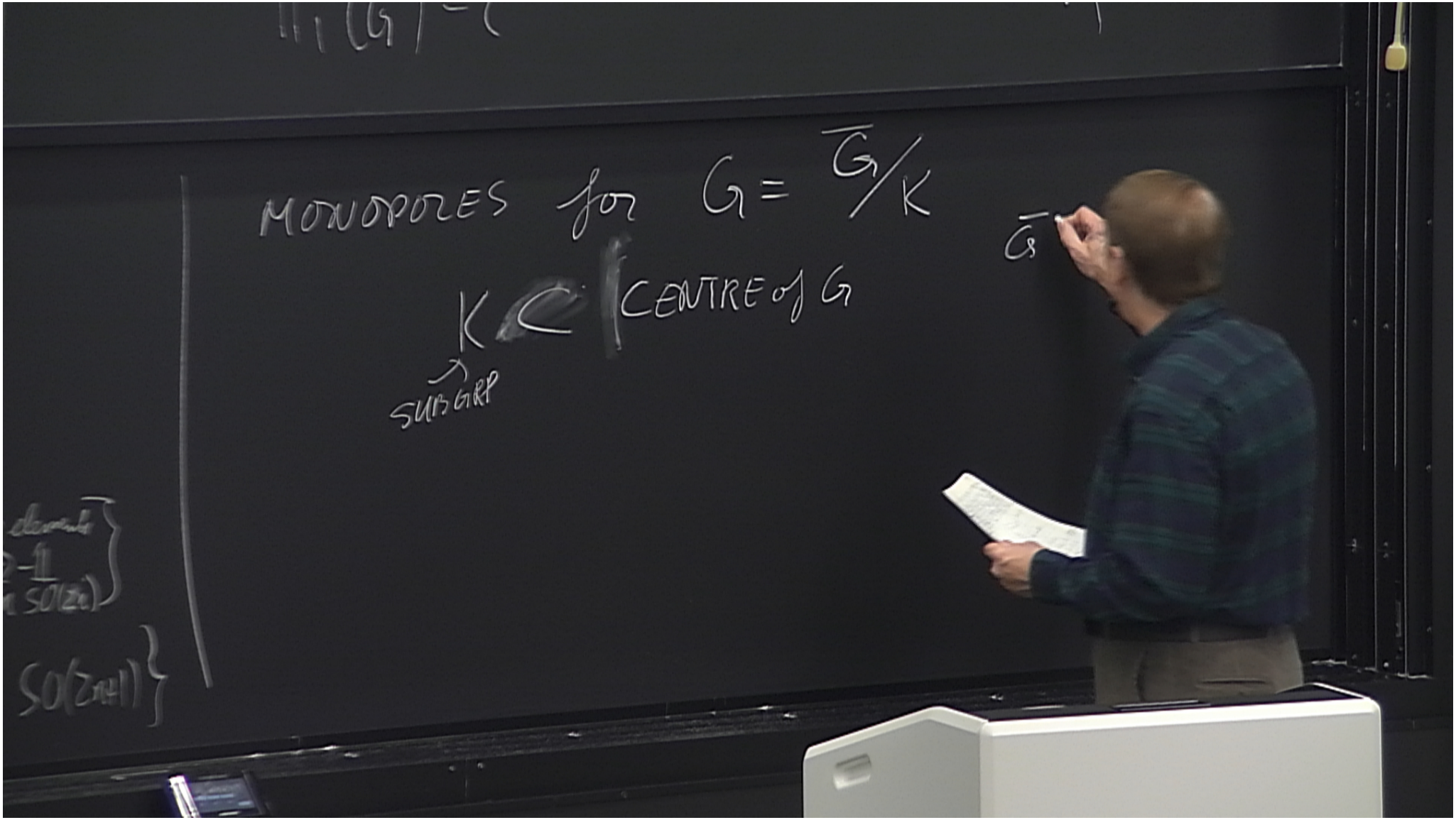

$\Pi_1(G)$
LIE ALG
$u(1)$
$su(n)$
$sp(n)$
$so(2n)$
$so(2n+1)$
SIMPLY CONNECTED GRP
$\mathbb{R}$
$su(n)$
$sp(n)$
$Spin(n)$
$Spin(2n+1)$
CENTRE
$\mathbb{R}$
$\mathbb{Z}_n$
$\{\mathbb{1}, -\mathbb{1}\}$
{the 2 elements mapped $\to \mathbb{1}$ in $SO(2n)$, 2 elements $\to -\mathbb{1}$ in $SO(2n)$}
{'' $\to SO(2n+1)$, $SO(2n+1)$}

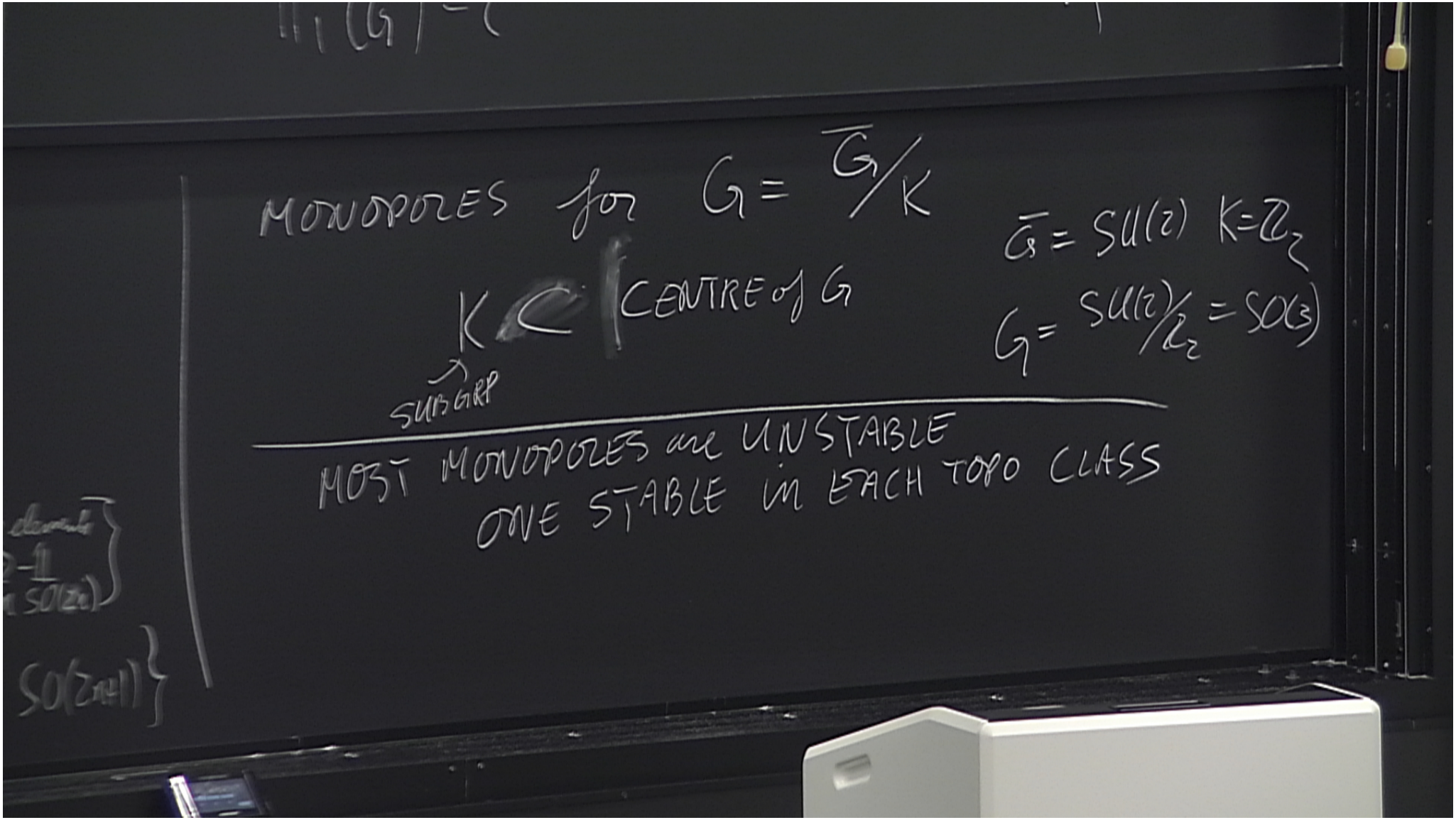

$\pi_1(G)$
LIE ALG
$u(1)$
$su(n)$
$sp(n)$
$so(2n)$
$so(2n+1)$
SIMPLY CONNECTED GRP
$\mathbb{R}$
$su(n)$
$sp(n)$
$Spin(2n)$
$Spin(2n+1)$
CENTRE
$\mathbb{R}$
$\mathbb{Z}_n$
$\{\mathbb{1}, -\mathbb{1}\}$
{the 2 elements mapped $\to \mathbb{1}$ in $SO(2n)$, 2 elements $\to -\mathbb{1}$ in $SO(2n)$}
{" $\to SO(2n+1)$, $SO(2n+1)$}


$\pi_1(G)$
LIE ALG
$u(1)$
$su(n)$
$sp(n)$
$so(2n)$
$so(2n+1)$
COVERING GRP
SIMPLY CONNECTED GRP
$\mathbb{R}$
$su(n)$
$sp(n)$
$Spin(2n)$
$Spin(2n+1)$
CENTRE
$\mathbb{R}$
$\mathbb{Z}_n$
$\{\mathbb{1}, -\mathbb{1}\}$
$\{$ the 2 elements mapped $\to \mathbb{1}$ in $SO(2n)$, 2 elements $\to -\mathbb{1}$ in $SO(2n)\}$
$\{$ " $\to SO(2n+1)$, $SO(2n+1)\}$


MONOPOLES for $G = \tilde{G}/K$


$\pi_1(G)$
COVERING GRP $\tilde{G}$
SIMPLY CONNECTED GRP
CENTRE
LIE ALG
$u(1)$ $\mathbb{R}$ $\mathbb{R}$
$su(n)$ $su(n)$ $\mathbb{Z}_n$
$sp(n)$ $sp(n)$ $\{\mathbb{1}, -\mathbb{1}\}$
$so(2n)$ $Spin(2n)$ {the 2 elements mapped $\to \mathbb{1}$ in $SO(2n)$, 2 elements $\to -\mathbb{1}$ in $SO(2n)$}
$so(2n+1)$ $Spin(2n+1)$ {" $\to SO(2n+1)$, $SO(2n+1)$}


MONOPOLES for $G = \bar{G}/K$
$K \subset$ CENTRE of G
SUBGRP
$\bar{G}$


MONOPOLES for $G = \bar{G}/K$
$K \subset$ CENTRE of G
SUBGRP
$\bar{G} = SU(2)$ $K = \mathbb{Z}_2$
$G = SU(2)/\mathbb{Z}_2 = SO(3)$
MOST MONOPOLES are UNSTABLE
ONE STABLE in EACH TOPO CLASS