

Title: 13/14 PSI - Beyond the Standard Model - Lecture 12

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URL: <http://pirsa.org/14030013>

Abstract:

IMPLICATIONS of INSTANTONS

$$\Theta = \sum_n e^{in\theta} |n\rangle$$

$U: S^3 \rightarrow$ IMPLICATIONS of INSTANTONS

$$\Theta = \sum_n e^{in\theta} |n\rangle$$

INST: $S^3 \rightarrow SU(N)$

IMPLICATIONS of INSTANTONS

$$\Theta = \sum_n e^{in\theta} |n\rangle$$

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_\Theta + \mathcal{L}_{GF}$$

$$= -\frac{1}{4g^2} F_{\mu\nu}^a F^{a\mu\nu} + \frac{\Theta}{32\pi^2} \tilde{F}_{\mu\nu}^a F^{a\mu\nu} + \mathcal{L}_{GF}$$

INST: $S^3 \rightarrow SU(N)$

IMPLICATIONS of INSTANTONS

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INST: $S^2 \rightarrow SU(N)$

IMPLICATIONS of INSTANTONS

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VAC $\rightarrow$ VAC TRANSITION $\sim e^{-S}$

$\Rightarrow$ OCCURS for $S = S_{MIN}$ $S \geq \frac{8\pi^2}{g^2} |V|$

INST: $S^3 \rightarrow SU(N)$

IMPLICATIONS of INSTANTONS

$$\Theta = \sum_n e^{in\theta} |n\rangle$$

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VAC $\rightarrow$ VAC TRANSITION $\sim e^{-S}$

$$\Rightarrow \text{OCCURS for } S = S_{\text{MIN}} \quad S \geq \frac{8\pi^2}{g^2} |\nu| = \frac{8\pi^2}{g} \left[\frac{1}{32\pi^2} \int d^4x F_{\mu\nu}^a F^{\mu\nu a} \right]$$

INST: $S^3 \rightarrow SU(N)$

IMPLICATIONS of INSTANTONS

$$\Theta = \sum_n e^{in\theta} |n\rangle$$

$$S = \frac{8\pi^2}{g^2} \text{ WHEN } F_{\mu\nu}^a = \pm F_{\mu\nu}^a$$

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_\theta + \mathcal{L}_{GF}$$

$$= -\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu a} + \frac{\Theta}{32\pi^2} \tilde{F}_{\mu\nu}^a F^{\mu\nu a} + \mathcal{L}_{GF}$$

VAC $\rightarrow$ VAC TRANSITION $\sim e^{-S}$

$\Rightarrow$ OCCURS for $S = S_{MIN}$ $S \geq \frac{8\pi^2}{g^2}$

SO $\Rightarrow A.$

$$\tilde{F}_{\mu\nu}^a$$

INST: $S^3 \rightarrow SU(2)$

IMPLICATIONS of INSTANTONS

$$\tau^a = \{\sigma^1, \sigma^2, \sigma^3, I\}$$

$$\Theta = \sum_n e^{in\theta} |n\rangle$$

$$S = \frac{8\pi^2}{g^2} \text{ WHEN } F_{\mu\nu}^a = \pm F_{\mu\nu}^a$$

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_\theta + \mathcal{L}_{GF}$$

$$= -\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu a} + \frac{\theta}{32\pi^2} \tilde{F}_{\mu\nu}^a F^{\mu\nu a} + \mathcal{L}_{GF}$$

$$SO(2N) \Rightarrow A_n = \frac{n^2}{n^2 + p^2} U \partial_\mu U^{-1}$$

[SU(2)]

$$x^a = \{x^1, x^2, x^3, x^4\} \quad U = \frac{x_a \tau^a}{r}$$

VAC $\rightarrow$ VAC TRANSITION $\sim e^{-S}$

$$\Rightarrow \text{OCCURS for } S = S_{\text{MIN}} \quad S \geq \frac{8\pi^2}{g^2} |V| = \frac{8\pi^2}{g} \left[\frac{1}{32\pi^2} \int d^4x F_{\mu\nu}^a F^{\mu\nu a} \right]$$

$$A_m = \frac{\rho^2}{\rho^2 + \rho_0^2} U \partial_m U^{-1} = U \partial_m U^{-1} \left(1 + \frac{\rho_0^2}{\rho^2} + \dots \right)$$

CONSTANT $\Rightarrow$ "SIZE" of THE INSTANTON
 COULD ALSO DISPLACE $x_\alpha \rightarrow (x - x_0)$

BPST INSTANTON ($\nu=1$)

UNIQUE UP

(a) SCALE $\Rightarrow \rho$

(b) LOCATION $\Rightarrow x_0$

MULTI-INSTANTONS $|\nu| > 1$

$(8\nu-3)$ - PARAMETER FAMILY of SOLNS

$$\Rightarrow \text{occurs for } S = S_{\min} \quad S \gg \frac{8\pi^2}{g^2} |V| = \frac{8\pi^2}{g} \left[\frac{1}{32\pi^2} \int d^4x F_{\mu\nu}^a F^{\mu\nu a} \right] \quad F = \frac{1}{2} \frac{dx_1 + dx_2}{dx_3^2 + dx_4^2}$$

1) ENERGY of VACUUM

$$\langle \theta | e^{-HT} | \theta \rangle$$

$$= \sum_{n, \bar{n}} (K e^{-S_0})^{n+\bar{n}}$$

1 - INSTANTON
SOL'N

$\Rightarrow$ OCCURS for $S = S_{\min}$ $S > \frac{8\pi^2}{g^2} |v| = \frac{8\pi^2}{g} \left[\frac{1}{32\pi^2} \int d^3x F_{\mu\nu}^2 \right] = \frac{1}{4} \int d^3x F_{\mu\nu}^2$ $\Gamma = \frac{1}{4} \int d^3x F_{\mu\nu}^2 + 2\pi^2 + 2\pi^2$

1) ENERGY of VACUUM

$$S_0 = S[A]$$

$$\langle \theta | e^{-HT} | \theta \rangle$$

$$= \sum_{n, \bar{n}} (K e^{-S_0})^{n+\bar{n}} \left(\frac{VT}{n! \bar{n}!} \right)^{n+\bar{n}} e^{i(n-\bar{n})\theta} = \exp[2KV T \cos \theta e^{-S_0}]$$

DEI factor
for Path
Int

1- INSTANTON
SOL'N

integration
over location
of centre of
instanton

$$\Rightarrow \frac{E(\theta)}{V} = -$$

(b) LOCATION $\Rightarrow x_0 \Rightarrow$

MULTI-INSTANTIOUS $S_{\text{multi}} = |V| S_0$

$$\frac{E(\theta)}{V} = -2K \cos \theta e^{-S_0} - 2K' \cos(2\theta) e^{-2S_0} - \underbrace{2K'' \cos(3\theta) e^{-3S_0}}_{\text{PPRESSED}} + \dots$$

(b) LOCATION $\Rightarrow x_0 \neq 0$

MULTI-INSTANTONS $S_{\text{multi}} = |V| S_0$

$$E(\theta) = -2K \cos \theta e^{S_0} - \underbrace{2K' \cos(2\theta) e^{2S_0} + 2K'' \cos(4\theta) e^{4S_0} + \dots}_{\text{SUPPRESSED}}$$

ACTUAL E is IR-DIV

(b) LOCATION $\Rightarrow x_0 \neq 0$

$$\frac{E(\theta)}{V} = -2K \cos \theta e^{S_0} - 2K' \cos(2\theta) e^{2S_0} - 2K'' \cos^3 \theta e^{3S_0} + \dots$$

ACTUAL E is IR-DIV

$\Rightarrow$ I-INSTANTON $\Rightarrow$ S-PARAMETER of SO(2N)

$\Rightarrow$ 4 LOCATION INTEGRALS $\Rightarrow$ VT

$$\Rightarrow K \sim \frac{1}{[L]^4}$$

SUPPRESSED $\leftarrow$ HIGHER TO ORDER OF (μ, g)

$$\frac{E(\theta)}{V} = A - \cos \theta e^{-\frac{8\pi^2}{g^2} \left(\frac{4}{g^2 k} \right)^{5/2} \int_0^\infty \frac{dp}{p(1+p^2)}} \quad \text{RENORM SCALE} \downarrow$$

$$\phi(x) \sim \phi_0 x^{8\pi^2 \beta_1} [1 - \beta_1 g \ln x]^{3/2}$$

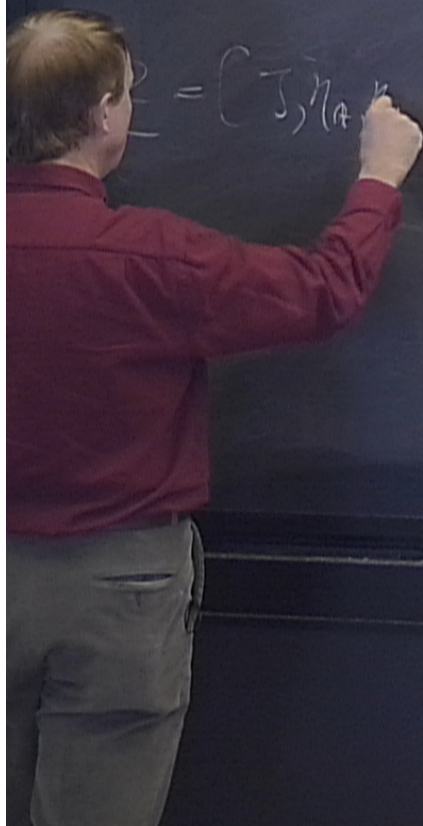
$$\text{YM THY } \beta_1 = \frac{11}{3} \left(\frac{1}{(4\pi)^2} \right) = \frac{11}{12\pi^2}, \quad C = \frac{4}{(I^2)}$$

for Path
Int

only location
of centre of
instantiation

$(M \cap Y) \theta = 0$

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_{A=1}^{N_F} \bar{q}_A (i \not{D}) q_A + \frac{1}{32\pi^2} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a + \mathcal{L}_{GF} + \sum_{A,B} \bar{q}_A M_{AB} q_B \text{ H.c.}$$



$$P = [J, \eta_A]$$

for path
Int

over location
of centre of
instantiation

$(M \cap Y) \cap \theta = 0$

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_{A=1}^{N_F} \bar{q}_A (i \not{D}) q_A + \frac{g}{32\pi^2} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a + \mathcal{L}_{GF} + \sum_{A,B} \bar{q}_L^A M_{AB} q_R^B + \text{H.c.}$$

$$Z = [J, \eta_A, \bar{\eta}_A] = \prod_{A=1}^{N_F} \int [d\bar{q}_A] [dq_A] [DA_\mu] e^{\frac{i}{\pi} \int d^4x (\mathcal{L}_{QCD} + \sum_{A=1}^{N_F} (\bar{\eta}_A q_A + \bar{q}_A \eta_A) + J \cdot A)}$$

$$-i \gamma_5 / N_F q_A$$

$$\prod_{A=1}^{N_F} [\bar{\eta}_A] [dq_A]$$

for path
Int

only location
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instanton

$(M, N) \neq 0$

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_{A=1}^{N_F} \bar{q}_A (i \not{D}) q_A + \frac{\theta}{32\pi^2} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a + \mathcal{L}_{GF} + \sum_{A,B} \bar{q}_L^A M_{AB} q_R^B + \text{H.c.}$$

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$$q_A \rightarrow e^{-i\theta \gamma_5 / N_F} q_A$$

$$\Rightarrow \prod_{A=1}^{N_F} [d\bar{q}_A] [dq_A] \rightarrow \prod_{A=1}^{N_F} [d\bar{q}_A] [dq_A] e^{\sum_{A=1}^{N_F} -\frac{\theta}{32\pi^2 N_F} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a}$$

for Path
Int

only location
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$(M \ll Y_0 \theta = 0)$

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_{A=1}^{N_F} \bar{q}_A (i \not{D}) q_A + \frac{\theta}{32\pi^2} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a + \mathcal{L}_{GF} + \sum_{A,B} \bar{q}_{LA} M_{AB} q_{RB} + H.c.$$

$$Z = [J, \eta_A, \bar{\eta}_A] = \prod_{A=1}^{N_F} \int [d\bar{q}_A] [dq_A] [DA_\mu] e^{\frac{i}{\pi} \int d^4x \left(\mathcal{L}_{QCD} + \sum_{A=1}^{N_F} (\bar{\eta}_A q_A + \bar{q}_A \eta_A) + J \cdot A \right)}$$

$$\begin{aligned} q_A &\rightarrow e^{-i\theta/5/N_F} q_A \\ \Rightarrow \prod_{A=1}^{N_F} [d\bar{q}_A] [dq_A] &\Rightarrow \prod_{A=1}^{N_F} [d\bar{q}_A] [dq_A] e^{\sum_{A=1}^{N_F} -\frac{\theta}{32\pi^2 N_F} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a} \end{aligned} \Rightarrow \begin{aligned} \mathcal{L} &= -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_{A=1}^{N_F} \bar{q}_A (i \not{D}) q_A \\ &+ \mathcal{L}_{GF} \\ &+ [\bar{q}_L (e^{-i\frac{\theta}{N_F}} M) q_R + H.c.] \end{aligned}$$

$\Rightarrow$ 4 LOCATION INTEGRALS $\Rightarrow VT$

$$\Rightarrow K \sim \frac{1}{1.74}$$

$$\frac{YM}{4\pi} \beta_1 = \frac{1}{8} \left(\frac{1}{(4\pi)^2} = \frac{11}{12\pi^2} \right), \quad C=4 \text{ for FREE YM}$$

FURTHER GLOBAL TRMP ($SU(N) \otimes SU(N)$)

$$q_R \rightarrow V_R q_R \quad \bar{q}_L \rightarrow \bar{q}_L V_L^\dagger$$

$$e^{-\frac{2\theta i}{NF} M} \rightarrow V_L^\dagger M V_R e^{i\theta/NF}$$

MULTI-INSTANTONS $|N| > 1$

$(8N-3)$ - PARAMETER FAMILY of SOLNS

$\Rightarrow$ 4 LOCATION INTEGRALS $\Rightarrow VT$

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$$\beta_1 = \frac{1}{5} \left(\frac{1}{(4\pi)^2} = \frac{11}{12\pi^2} \right) \quad C=4 \text{ for FREE YM}$$

FURTHER GLOBAL TRMP ($SU(N) \otimes SU(N)$)

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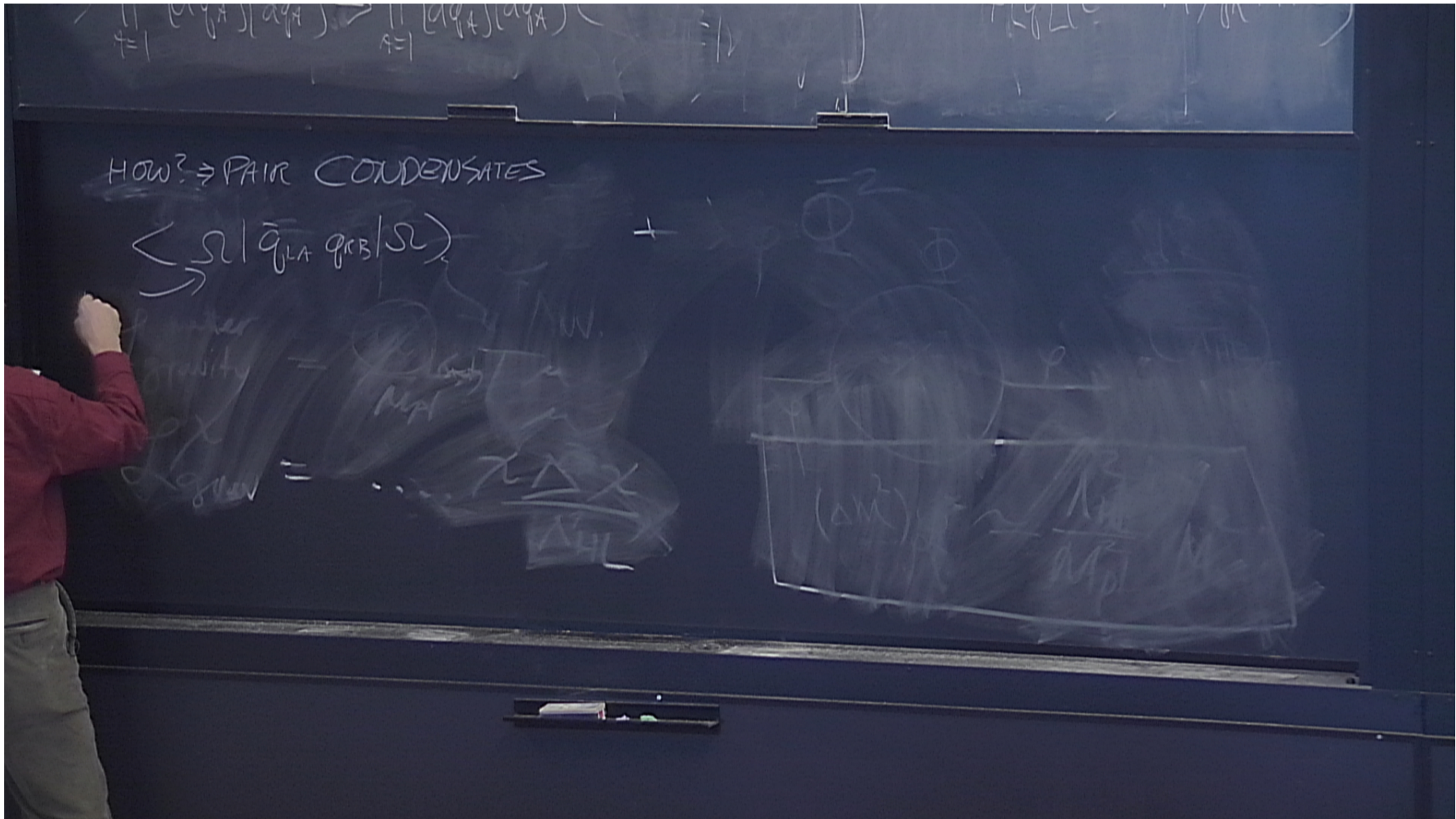
$$e^{-\frac{2\theta i}{N_F} M} \rightarrow V_L^\dagger M V_R e^{i\theta/N_F}$$

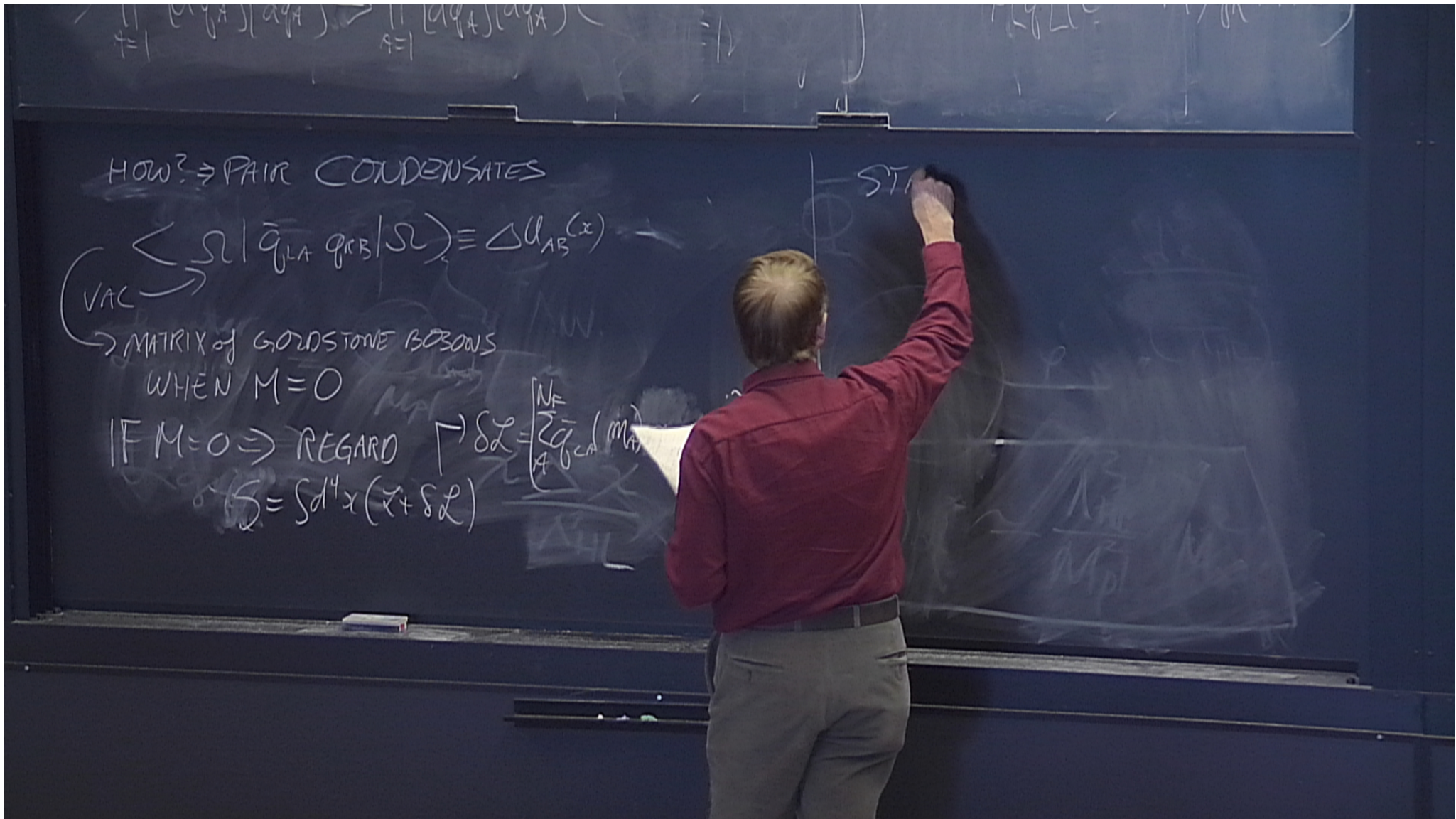
$$= (m_A S_{AB}) e^{i(1-\theta)/N_F}$$

$$= m_A S_{AB} e^{i\tilde{\theta}} \Rightarrow \text{CP-VIOLATING}$$

$$\frac{1}{2} M V_R = e^{ip} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 & \\ & & & \ddots & \\ & & & & m_{N_F} \end{pmatrix}$$

IS $\tilde{\theta} = 0$ OR NOT?





HOW? => PAIR CONDENSATES

$$\langle \Omega | \bar{q}_{LA} q_{RB} | \Omega \rangle \equiv \Delta u_{AB}(x)$$

VAC

MATRIX of GOLDSTONE BOSONS
WHEN $M=0$

IF $M=0 \Rightarrow$ REGARD

$$\mathcal{L} = S d^4x (\bar{\psi} + \delta \mathcal{L})$$

$$\delta \mathcal{L} = \sum_{A=1}^{N_F} \bar{q}_{cA} (m_A)$$

ST

HOW? $\Rightarrow$ PAIR CONDENSATES

$$\langle \Omega | \bar{q}_{LA} q_{RB} | \Omega \rangle = \Delta u_{AB}(x)$$

(VAC $\rightarrow$)

$\rightarrow$ MATRIX of GOLDSTONE BOSONS
WHEN $M=0$

IF $M=0 \Rightarrow$ REGARD

$$\delta \mathcal{L} = \sum_A \bar{q}_{LA} (m_A) q_{RA} e^{i\hat{\theta}} + \text{H.C.}$$

$$\delta S = \int d^4x (\bar{\psi} + \delta \mathcal{L})$$

STABILITY of Ω :

MINIMIZE

$$\langle \Omega | \delta \mathcal{L} | \Omega \rangle$$

$$= \text{Tr} [m e^{i\hat{\theta}} U]$$

$$3 \text{ F.I. } \Rightarrow U = \begin{pmatrix} e^{i\alpha_1/s} & 0 \\ 0 & e^{i\alpha_2/s} \\ 0 & e^{i\alpha_3/s} \end{pmatrix}$$

$= m_A \delta_{15} \rightarrow$ - VIOLATING

SOLVE Θ

$$\alpha_1 = \frac{\Theta (2m_s m_d - m_u (m_d + m_s))}{m_u m_d + m_u m_s + m_d m_s}$$

$$\alpha_2 = \alpha_1 (m_u \leftrightarrow m_d) \quad \text{IF } \Theta \ll 1$$

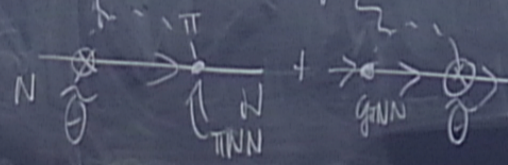
$$\text{EXPT } |\vec{d}_n| \leq 5 \times 10^{-25} \text{ e-cm}$$

$$\Rightarrow \Theta < 10^{-9}$$

$$\Rightarrow \delta \mathcal{L} = m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s + i \Theta \left(\frac{m_s m_u m_d}{m_u m_d + m_u m_s + m_d m_s} \right) [\bar{u} \gamma_5 u + \bar{d} \gamma_5 d + \bar{s} \gamma_5 s]$$

REC DIPOLE MOMENT OF NEUTRON

$$\Rightarrow \vec{d}_n = \langle N | \vec{d} | N \rangle = \int d^3 x \langle N | \vec{x} \Phi(x) | N \rangle$$



$$= e \frac{\Theta m_u m_d}{(m_u + m_d) f_\pi} g_{\pi NN} \frac{\ln(m_u/m_d)}{4\pi^2 m_\pi} = 5 \times 10^{-16} \Theta \text{ e-cm}$$

LOOP

elec CH16 MATRIX = 48.64