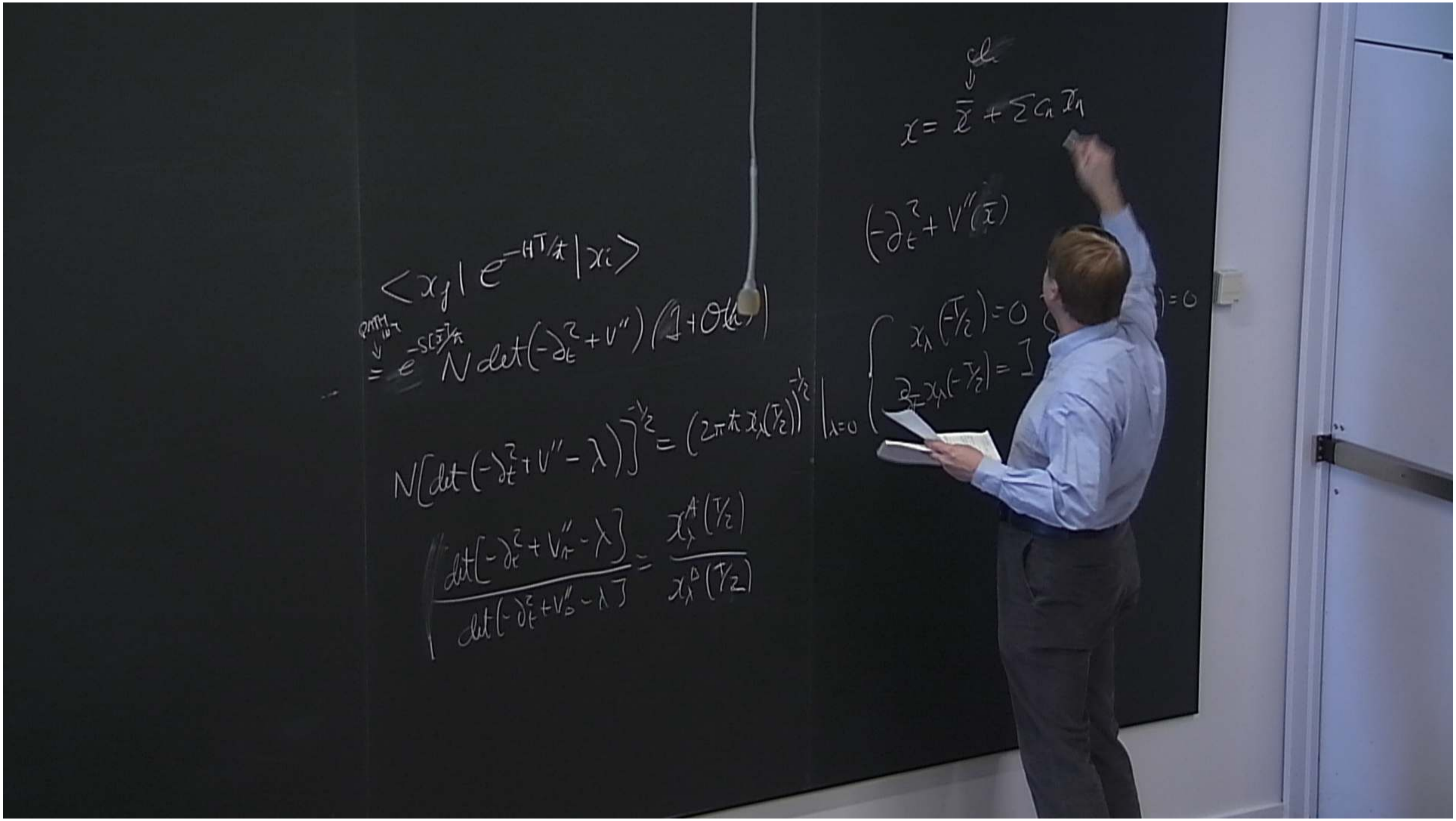
Title: 13/14 PSI - Beyond the Standard Model - Lecture 10

Date: Mar 04, 2014 03:15 PM

URL: http://pirsa.org/14030011

Abstract:



$$\langle x_f | e^{-HT/\hbar} | x_i \rangle$$

$$\overset{\text{QM TM}}{=} e^{-S[\bar{x}]/\hbar} N \det(-\partial_t^2 + V'')(1 + O(\hbar))$$

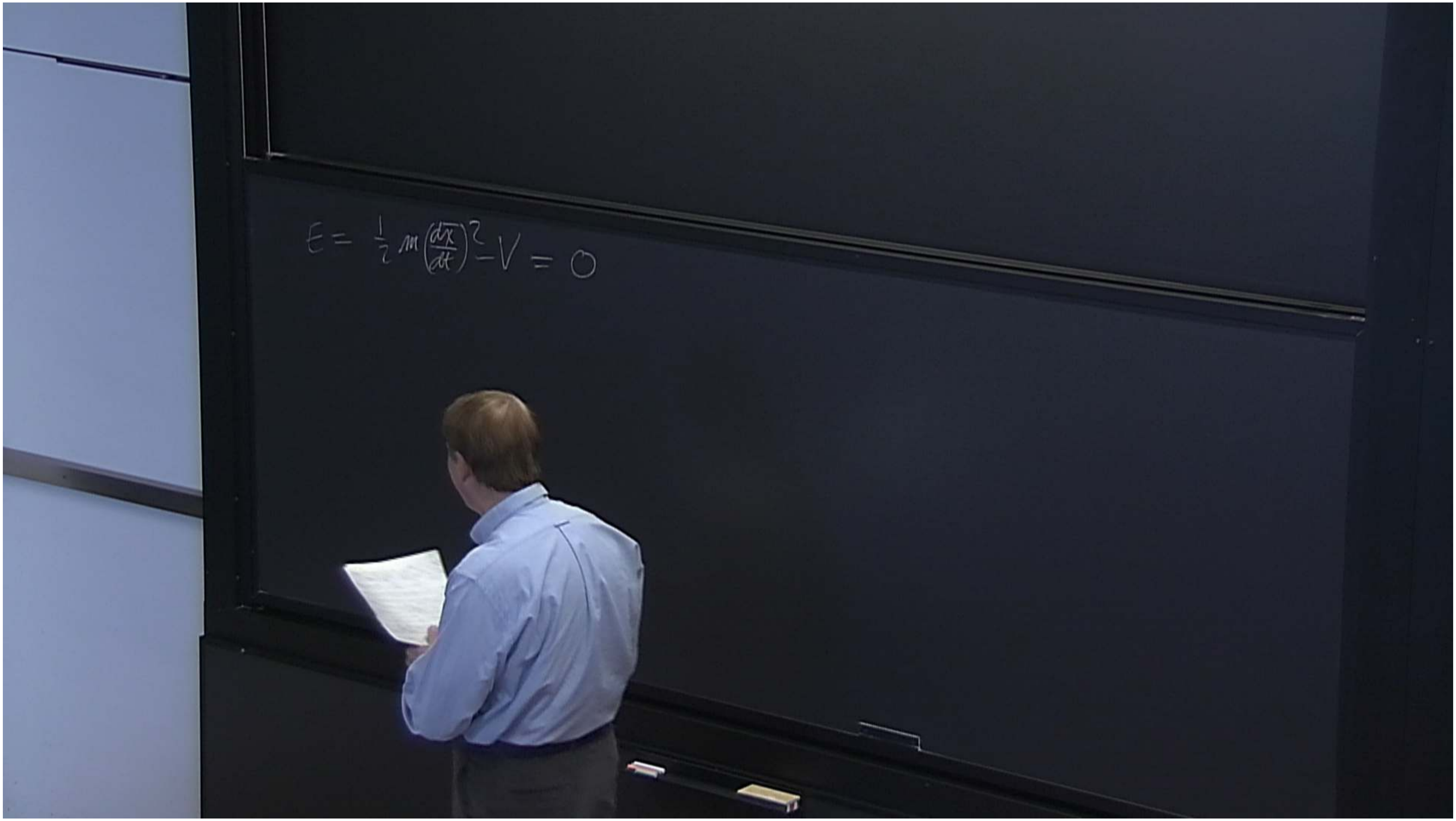
$$N[\det(-\partial_t^2 + V'' - \lambda)]^{-1/2} = (2\pi\hbar\, x_\lambda(T/2))^{-1/2} \Big|_{\lambda=0} \begin{cases} x_\lambda(-T/2) = 0 \\ \partial_t x_\lambda(-T/2) = 1 \end{cases}$$

$$\frac{\det[-\partial_t^2 + V_A'' - \lambda]}{\det[-\partial_t^2 + V_B'' - \lambda]} = \frac{x_\lambda^A(T/2)}{x_\lambda^B(T/2)}$$

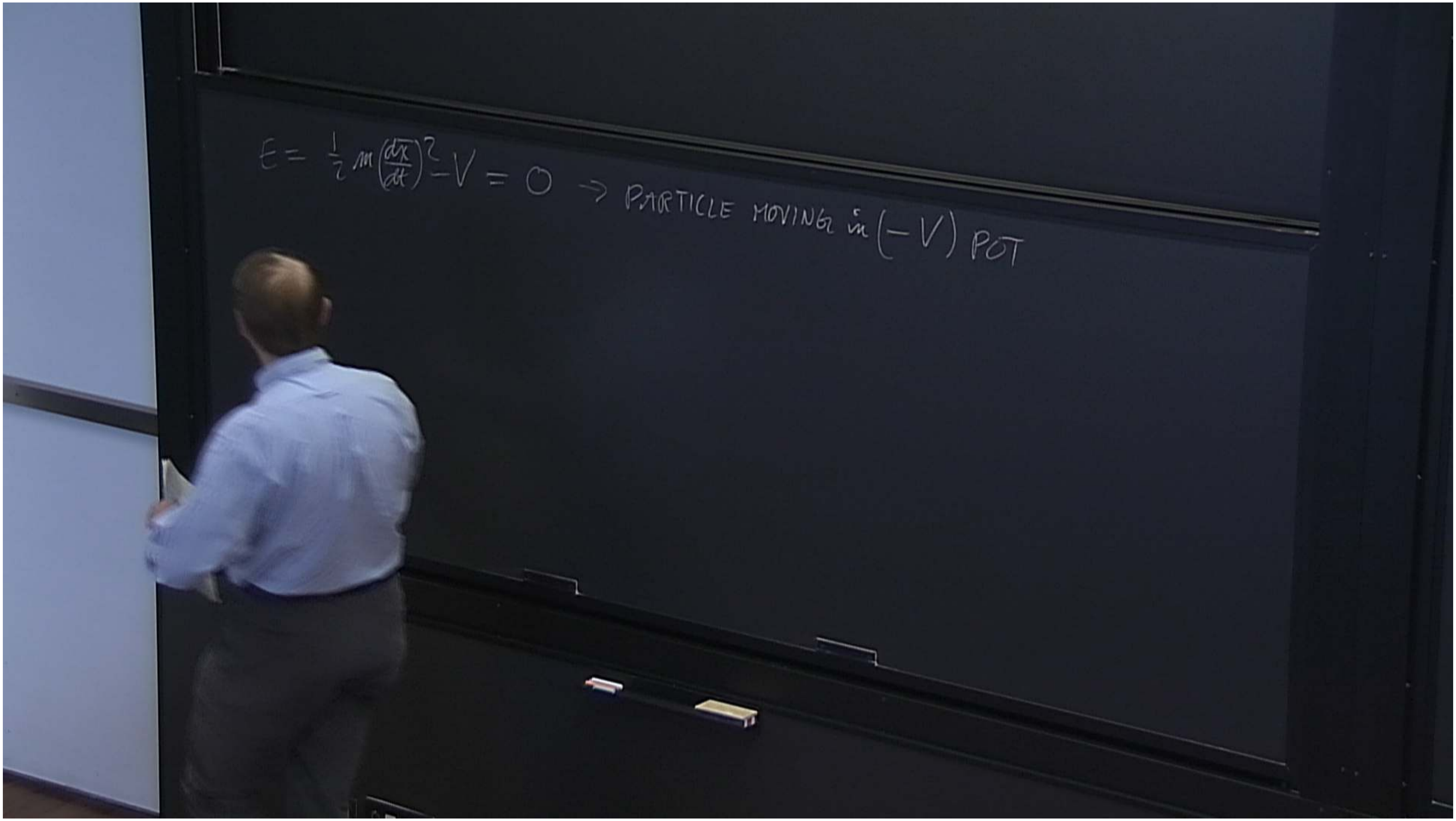
$$x = \overset{\text{cl}}{\bar{x}} + \sum c_n x_n$$

$$(-\partial_t^2 + V''(\bar{x})$$

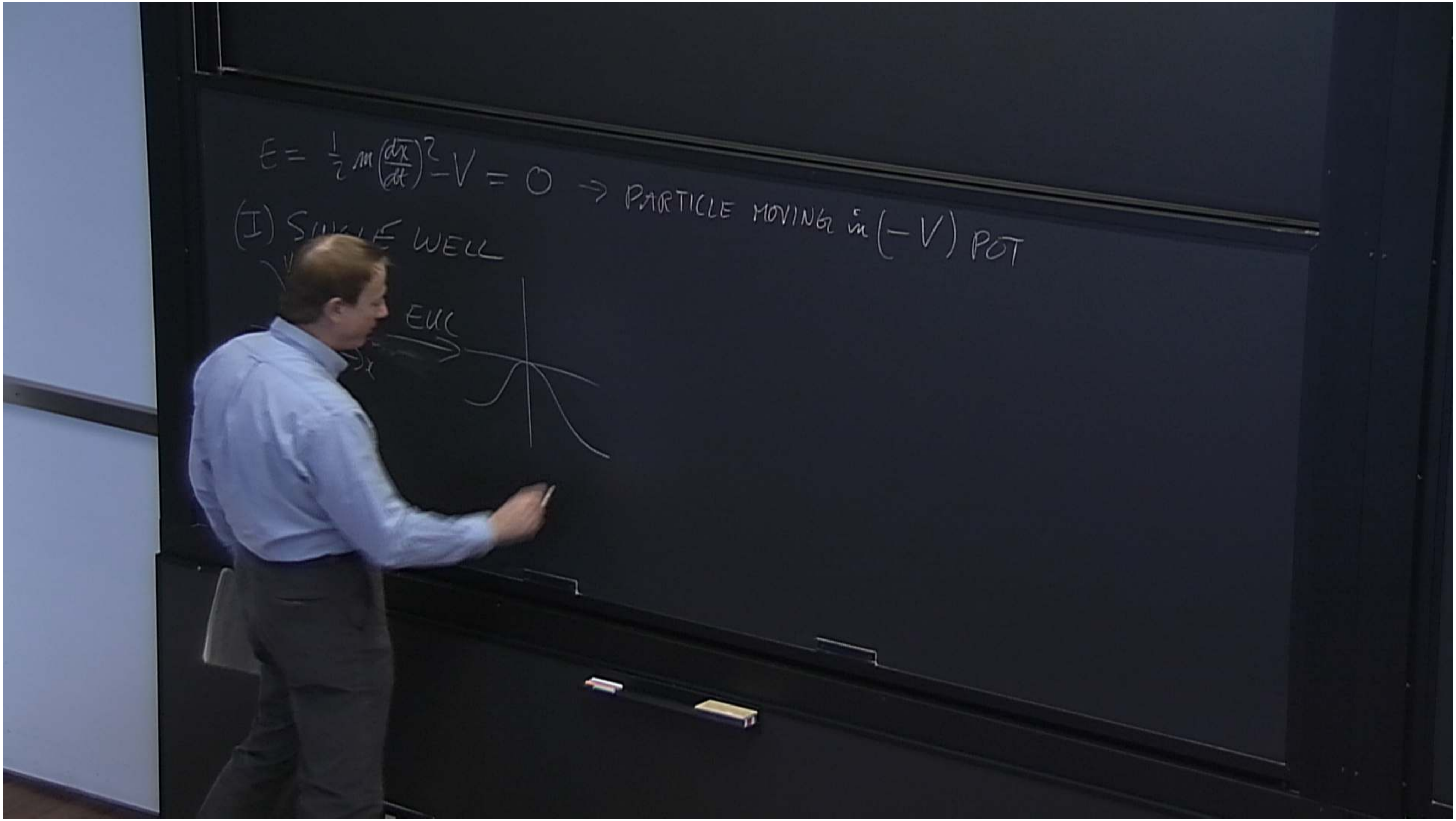
$$) = 0$$



$$E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 - V = 0$$

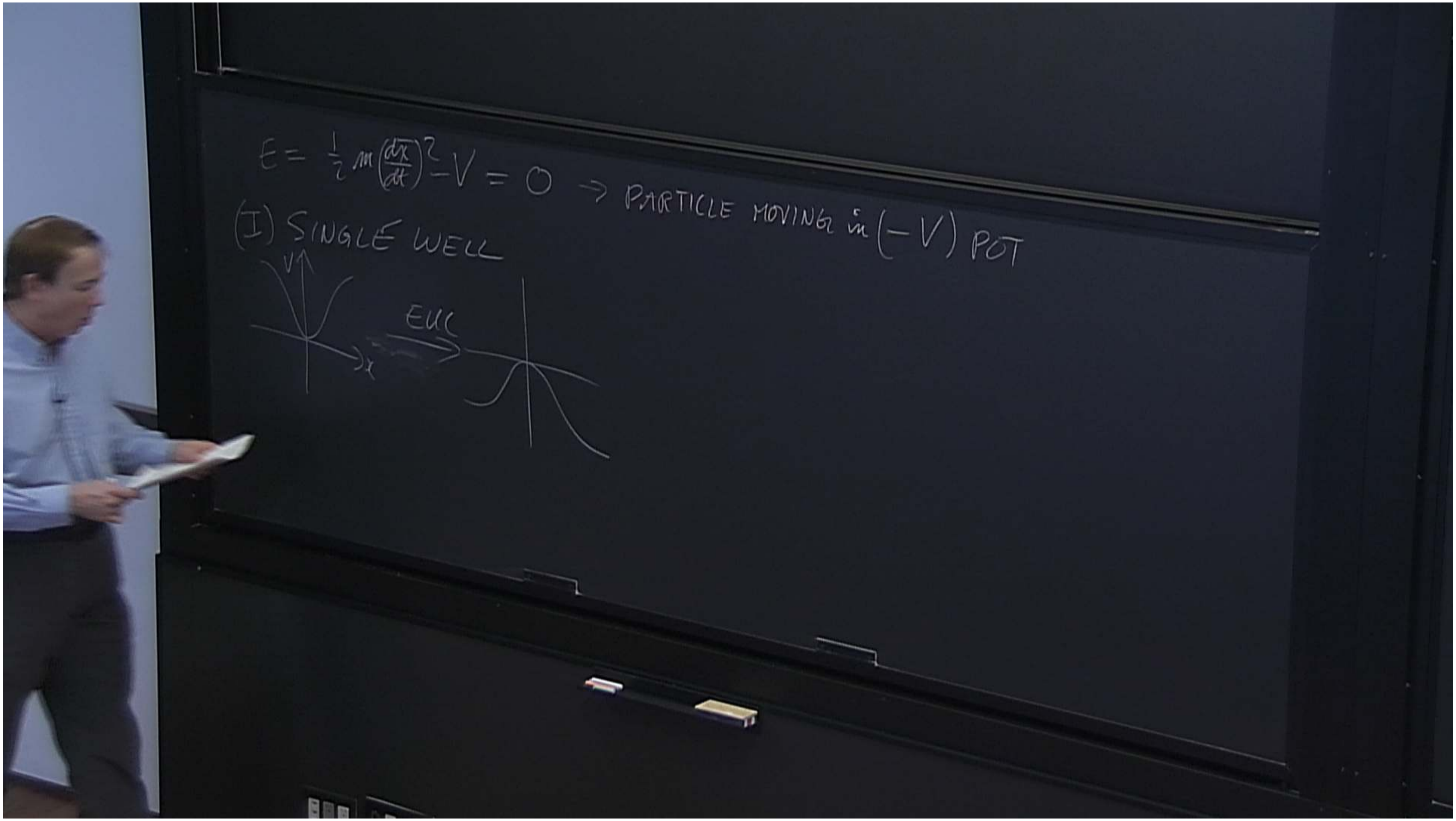


$E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 - V = 0 \rightarrow$ PARTICLE MOVING in $(-V)$ POT



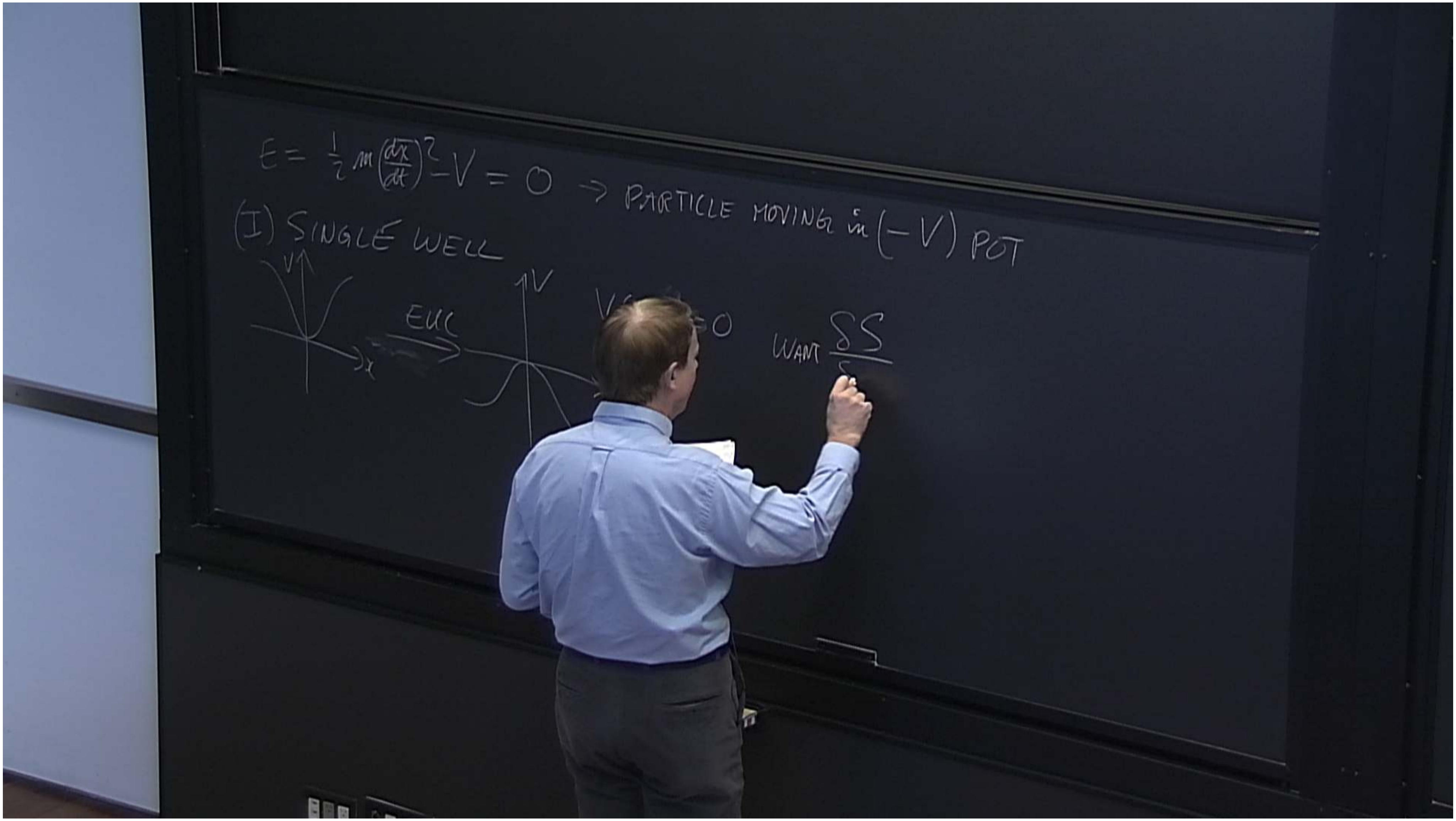
$E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 - V = 0 \rightarrow$ PARTICLE MOVING in $(-V)$ POT

(I) SINGLE WELL



$E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 - V = 0 \rightarrow$ PARTICLE MOVING in $(-V)$ POT

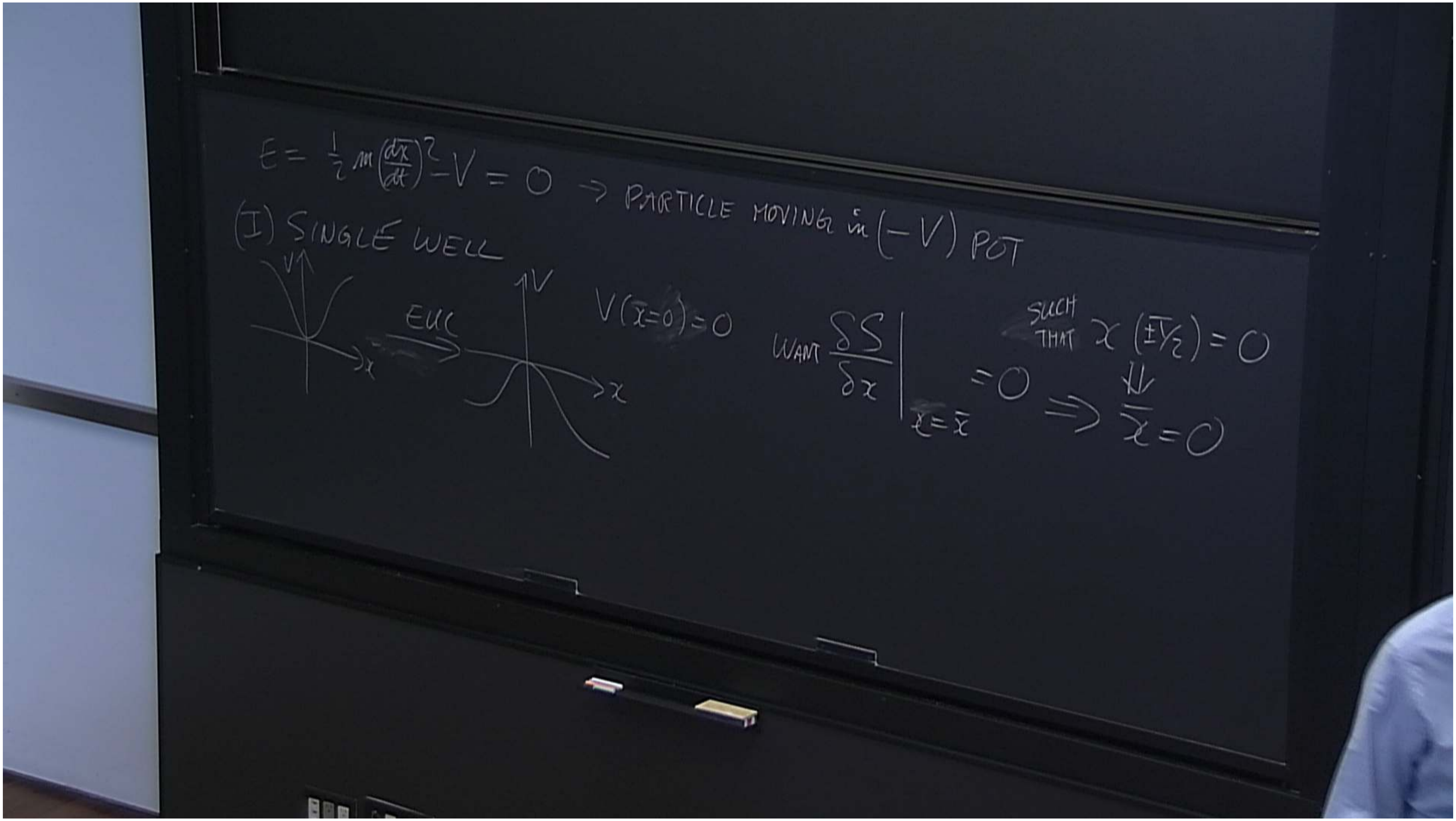
(I) SINGLE WELL



$E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 - V = 0$ → PARTICLE MOVING in $(-V)$ POT

(I) SINGLE WELL

WANT $\frac{\delta S}{\delta}$



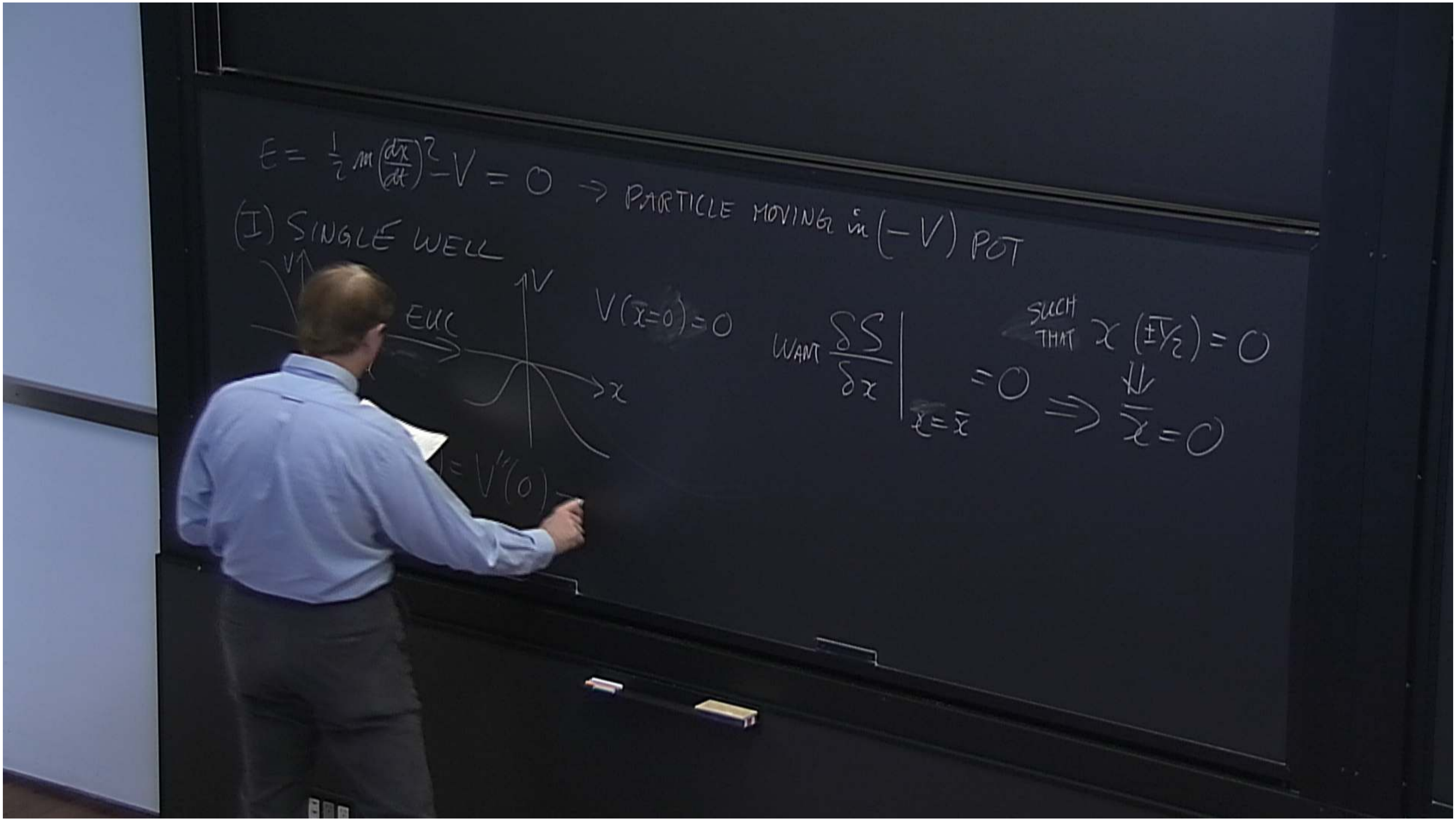
$E = \frac{1}{2} m \left(\frac{d\bar{x}}{dt}\right)^2 - V = 0 \rightarrow$ PARTICLE MOVING in $(-V)$ POT

(I) SINGLE WELL

$V(\bar{x}=0)=0$

WANT $\left.\frac{\delta S}{\delta x}\right|_{x=\bar{x}} = 0$ SUCH THAT $x(\pm T/2) = 0$

$\Rightarrow \Downarrow \bar{x} = 0$



$$E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 - V = 0 \rightarrow \text{PARTICLE MOVING in } (-V) \text{ POT}$$

(I) SINGLE WELL

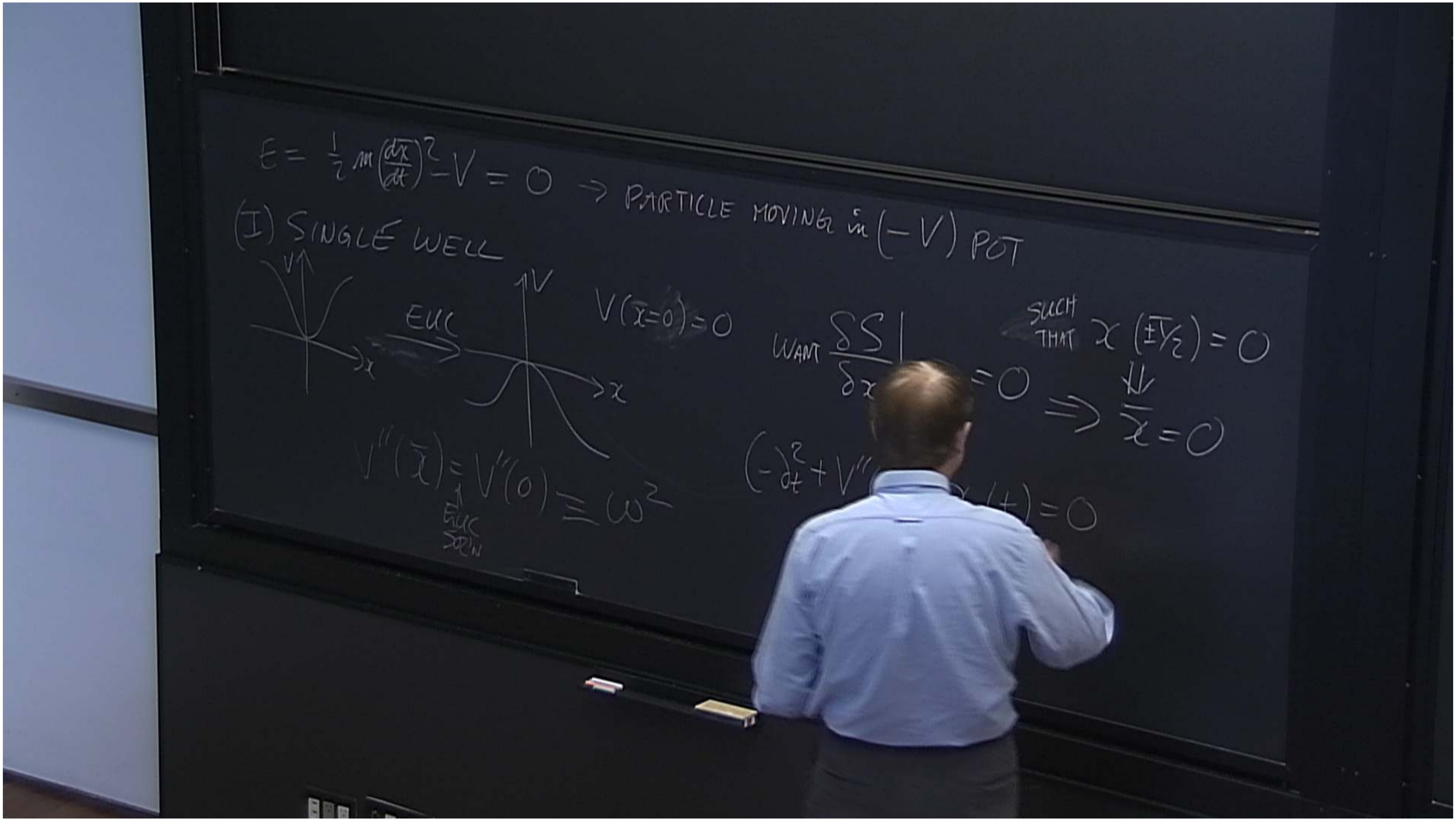
$V(x=0)=0$

$$\text{WANT } \left.\frac{\delta S}{\delta x}\right|_{x=\bar{x}} = 0 \quad \text{SUCH THAT } x(\pm T/2) = 0$$

$$\Downarrow$$

$$\Rightarrow \bar{x} = 0$$

$) = V''(0) -$



$E = \frac{1}{2} m \left(\frac{d\bar{x}}{dt}\right)^2 - V = 0$ → PARTICLE MOVING in $(-V)$ POT

(I) SINGLE WELL

$V(\bar{x}=0)=0$

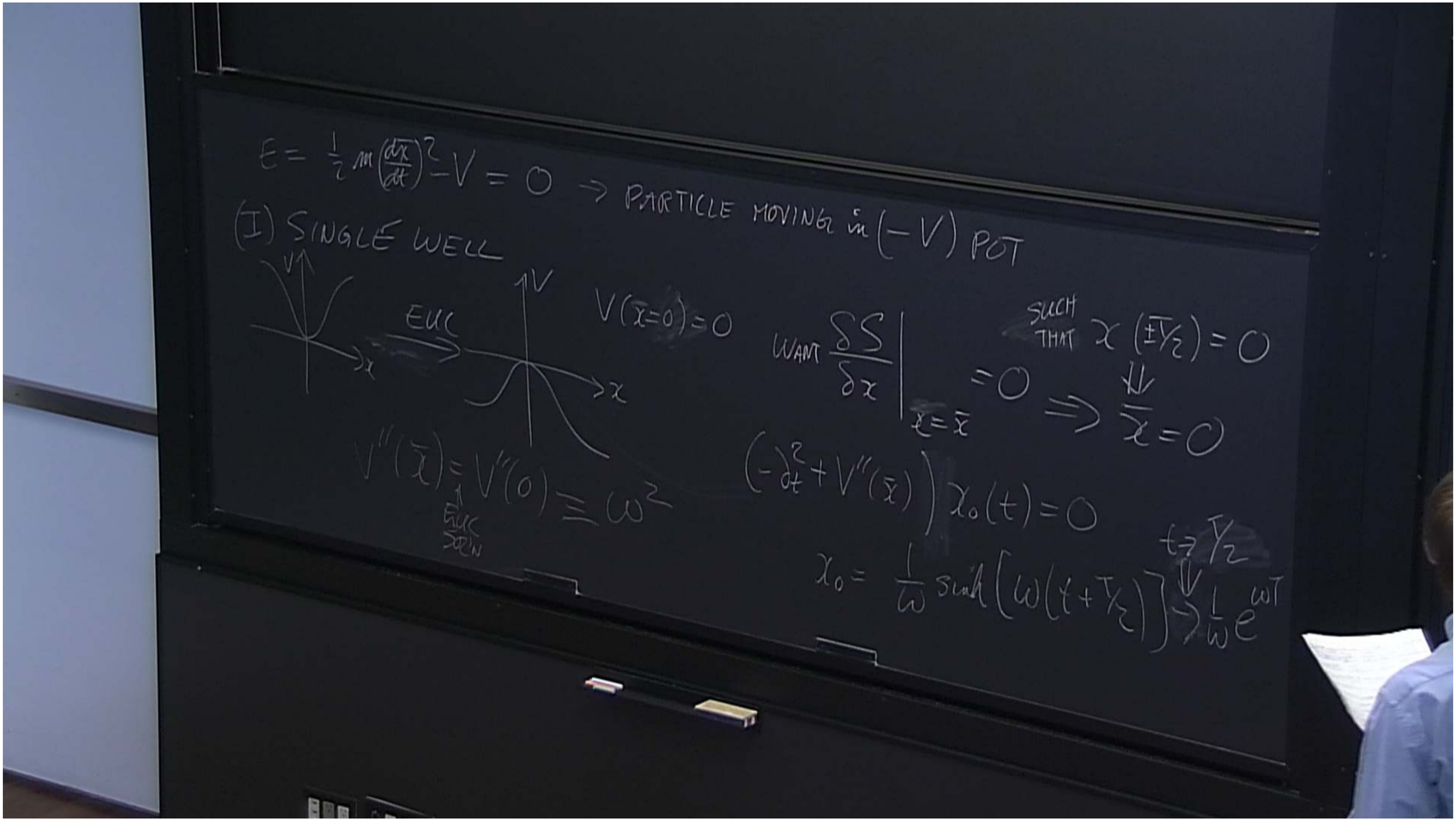
WANT $\frac{\delta S}{\delta x}\Big|$ $=0$ SUCH THAT $x(\pm T/2) = 0$

$\Downarrow$

$\Rightarrow \bar{x} = 0$

$V''(\bar{x}) \underset{\substack{\uparrow \\ \text{EUC} \\ \text{SOL'N}}}{=} V''(0) \equiv \omega^2$

$(-\partial_t^2 + V''$ $(t) = 0$



$E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 - V = 0 \rightarrow$ PARTICLE MOVING in $(-V)$ POT

(I) SINGLE WELL

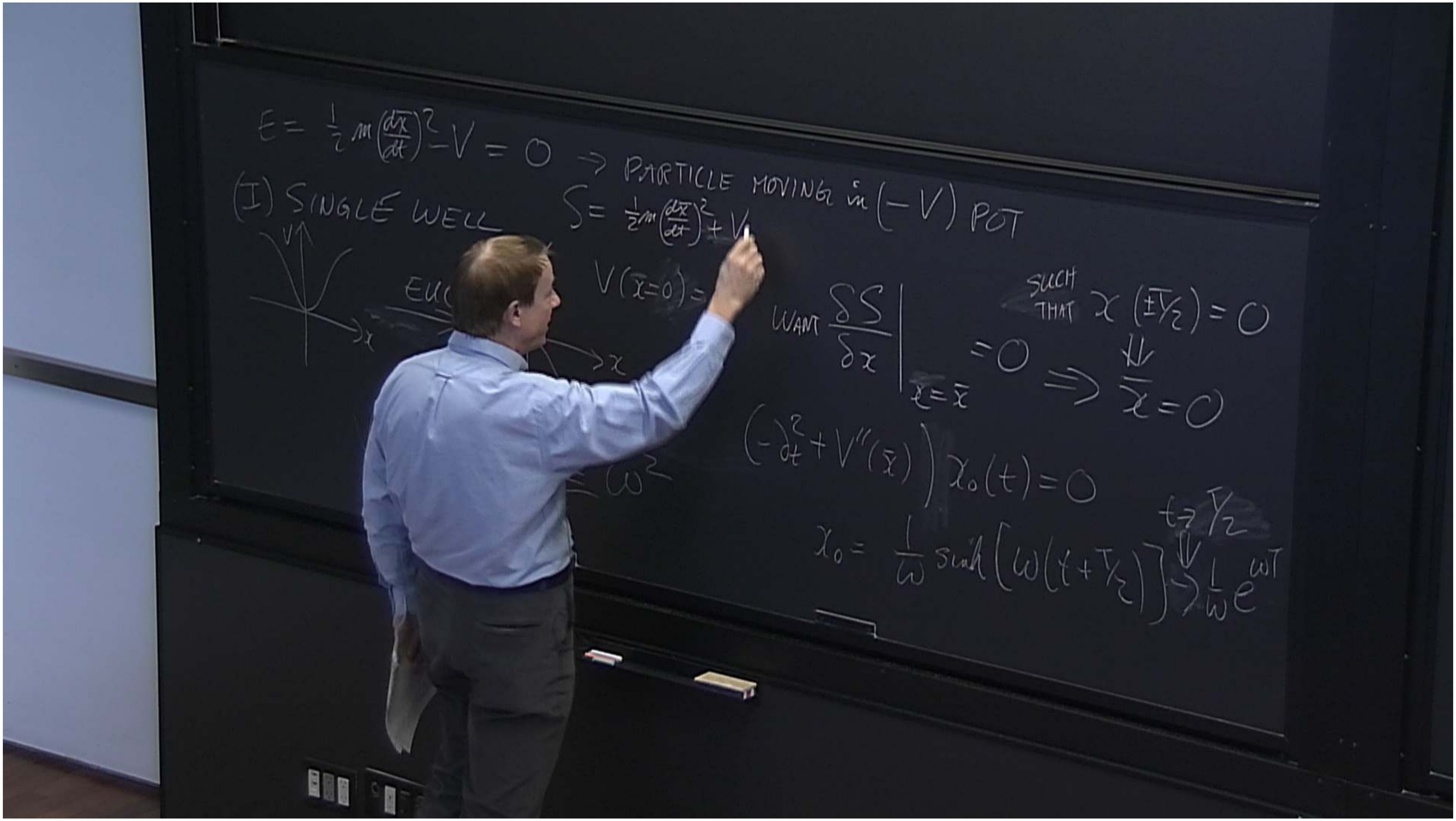
$V(\bar{x}=0) = 0$

WANT $\left.\frac{\delta S}{\delta x}\right|_{x=\bar{x}} = 0$ SUCH THAT $x(\pm T/2) = 0 \Rightarrow \bar{x} = 0$

$V''(\bar{x}) \underset{\substack{\uparrow \\ \text{EUC} \\ \text{SOLN}}}{=} V''(0) \equiv \omega^2$

$\left(-\partial_t^2 + V''(\bar{x})\right) x_0(t) = 0$

$x_0 = \frac{1}{\omega} \sinh\left[\omega\left(t + T/2\right)\right] \xrightarrow{t \to T/2} \frac{1}{\omega} e^{\omega T}$



$E = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 - V = 0 \rightarrow$ PARTICLE MOVING in $(-V)$ POT

(I) SINGLE WELL

$S = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + V$

$V(\bar{x}=0) =$

ω^2

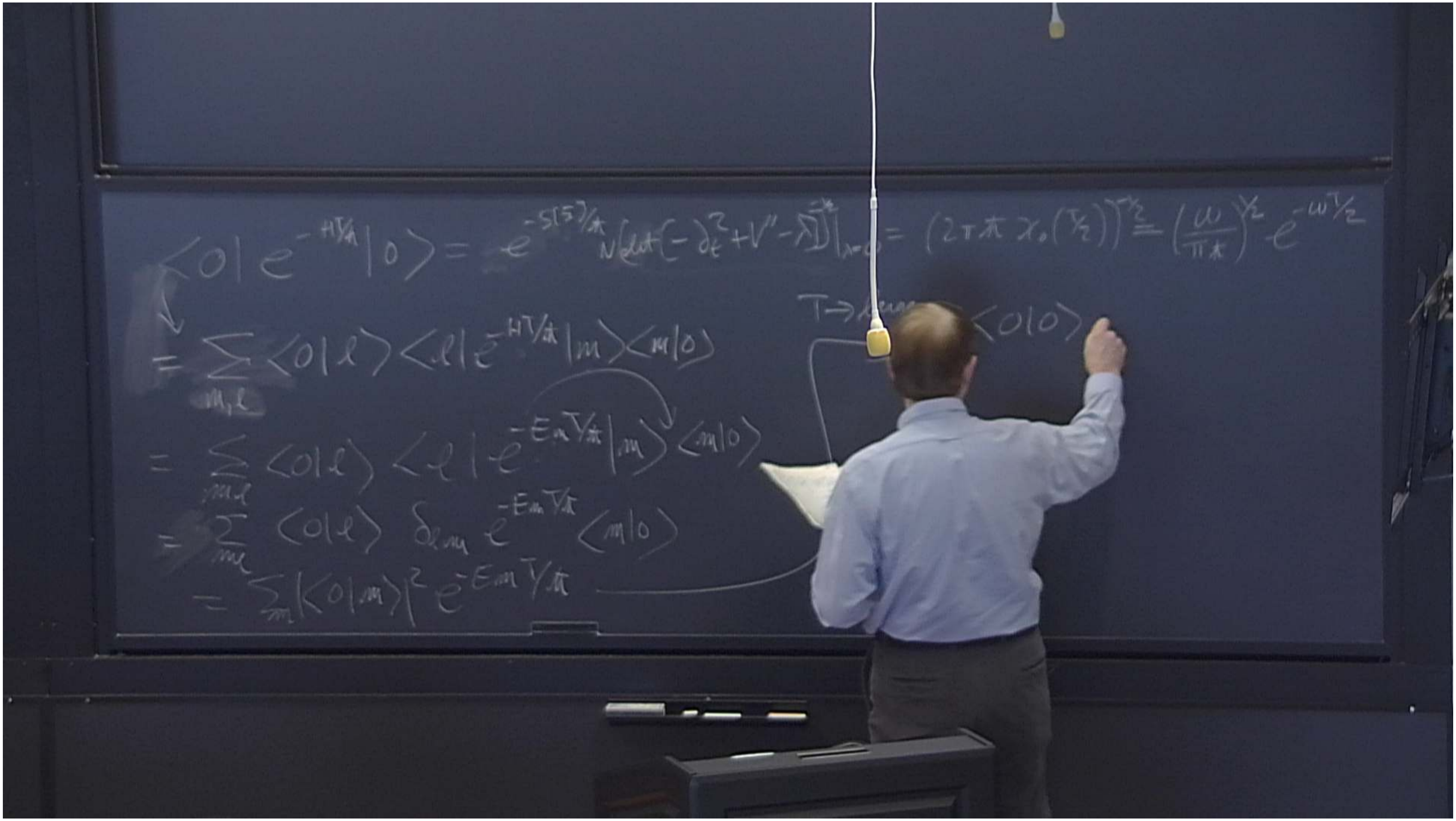
WANT $\left.\frac{\delta S}{\delta x}\right|_{x=\bar{x}} = 0$ SUCH THAT $x(\pm T/2) = 0$

$\Downarrow$

$\Rightarrow \bar{x} = 0$

$\left(-\partial_t^2 + V''(\bar{x})\right) x_0(t) = 0$

$x_0 = \frac{1}{\omega}\sinh\left[\omega(t + T/2)\right] \xrightarrow{t \to T/2} \frac{1}{\omega}e^{\omega T}$



$$\langle 0|e^{-HT/\hbar}|0\rangle = e^{-S[\bar{x}]/\hbar} N\left(\det[-\partial_t^2+V''-\lambda]\right)^{-1/2}\Big|_{\lambda=0} = \left(2\pi\hbar\, x_0(T/2)\right)^{-1/2} \underline{\underline{=}} \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2}$$

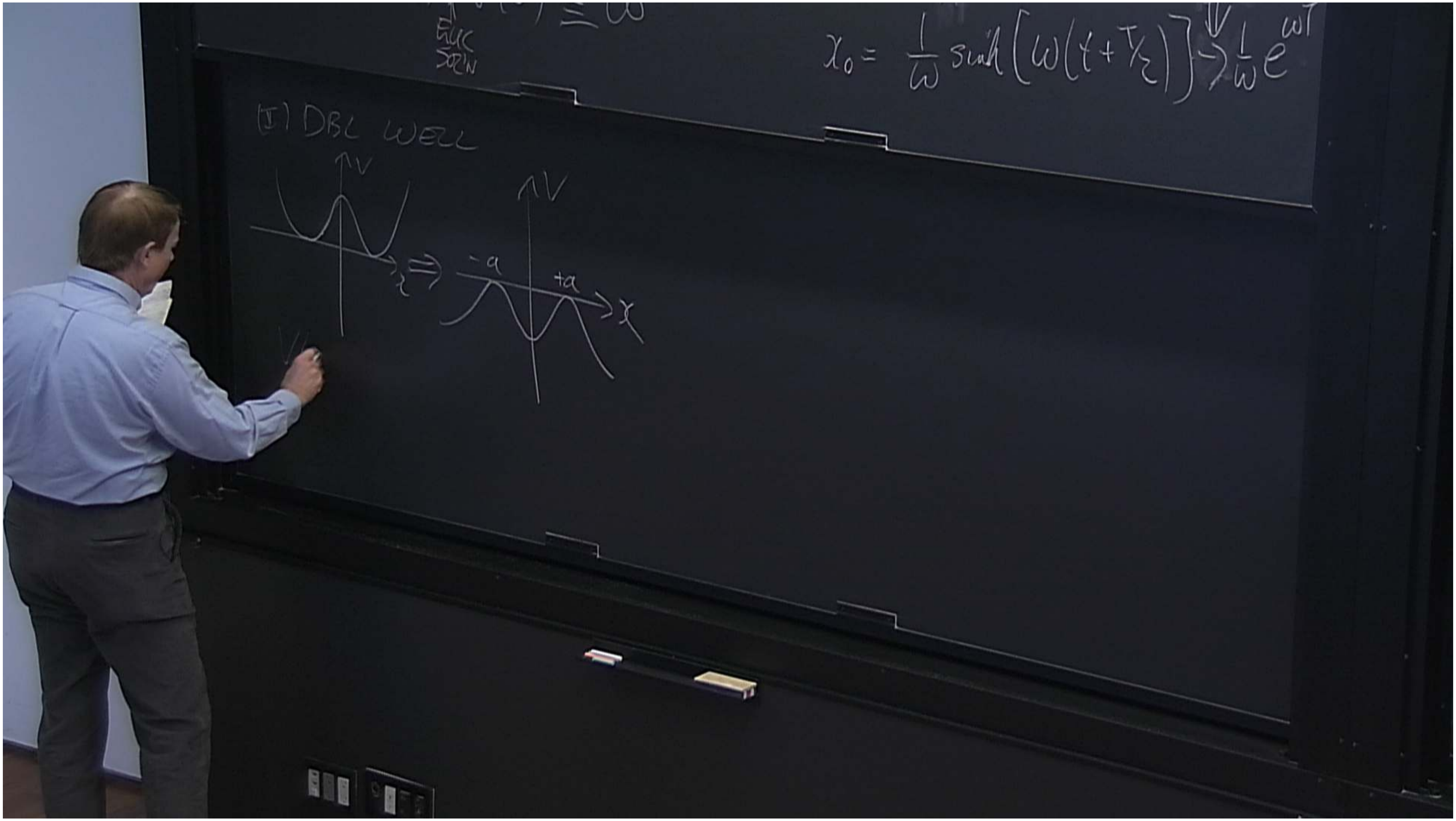
$$= \sum_{m,l} \langle 0|l\rangle\langle l|e^{-HT/\hbar}|m\rangle\langle m|0\rangle$$

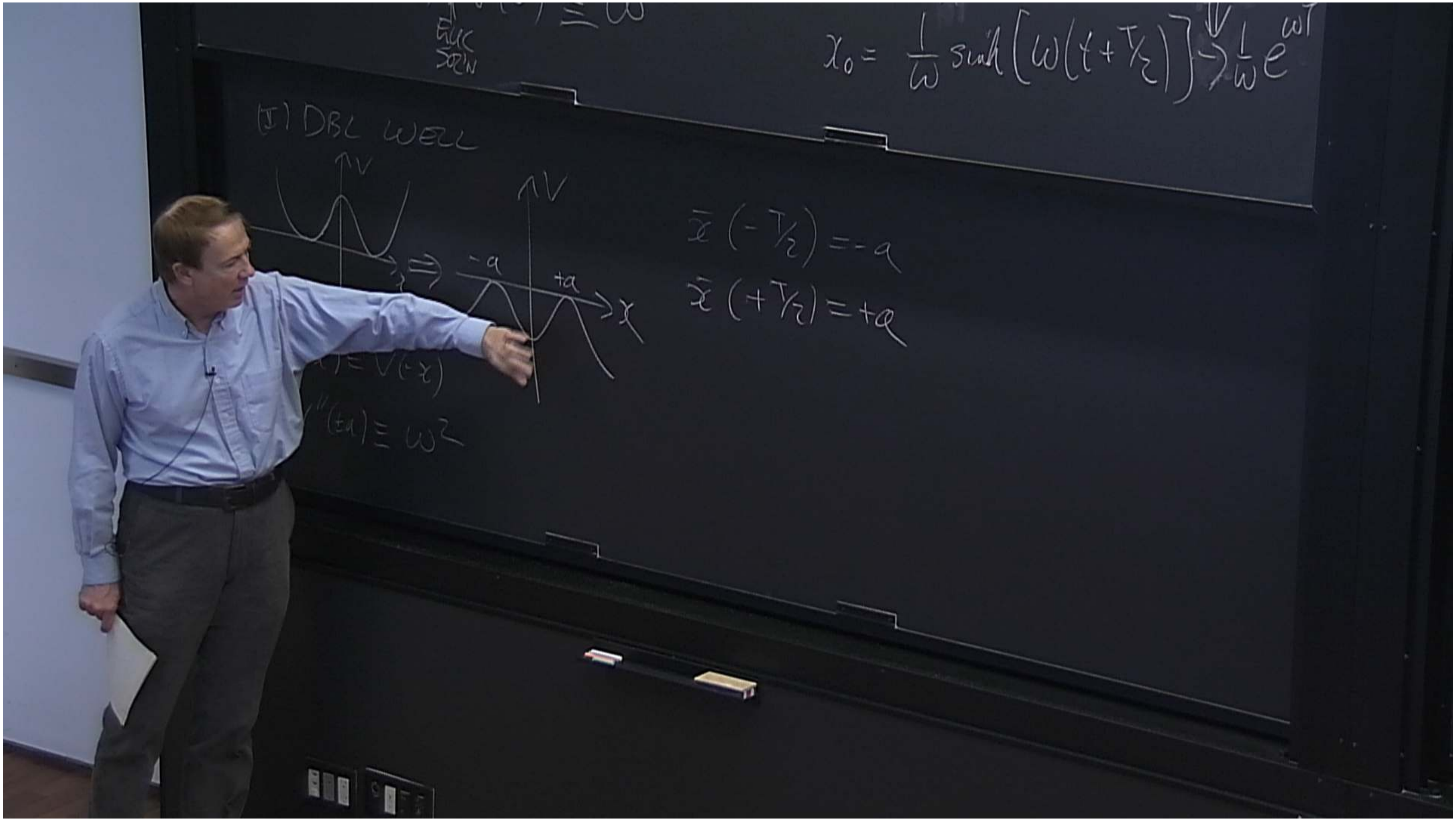
$$= \sum_{m,l} \langle 0|l\rangle\langle l|e^{-E_m T/\hbar}|m\rangle\langle m|0\rangle$$

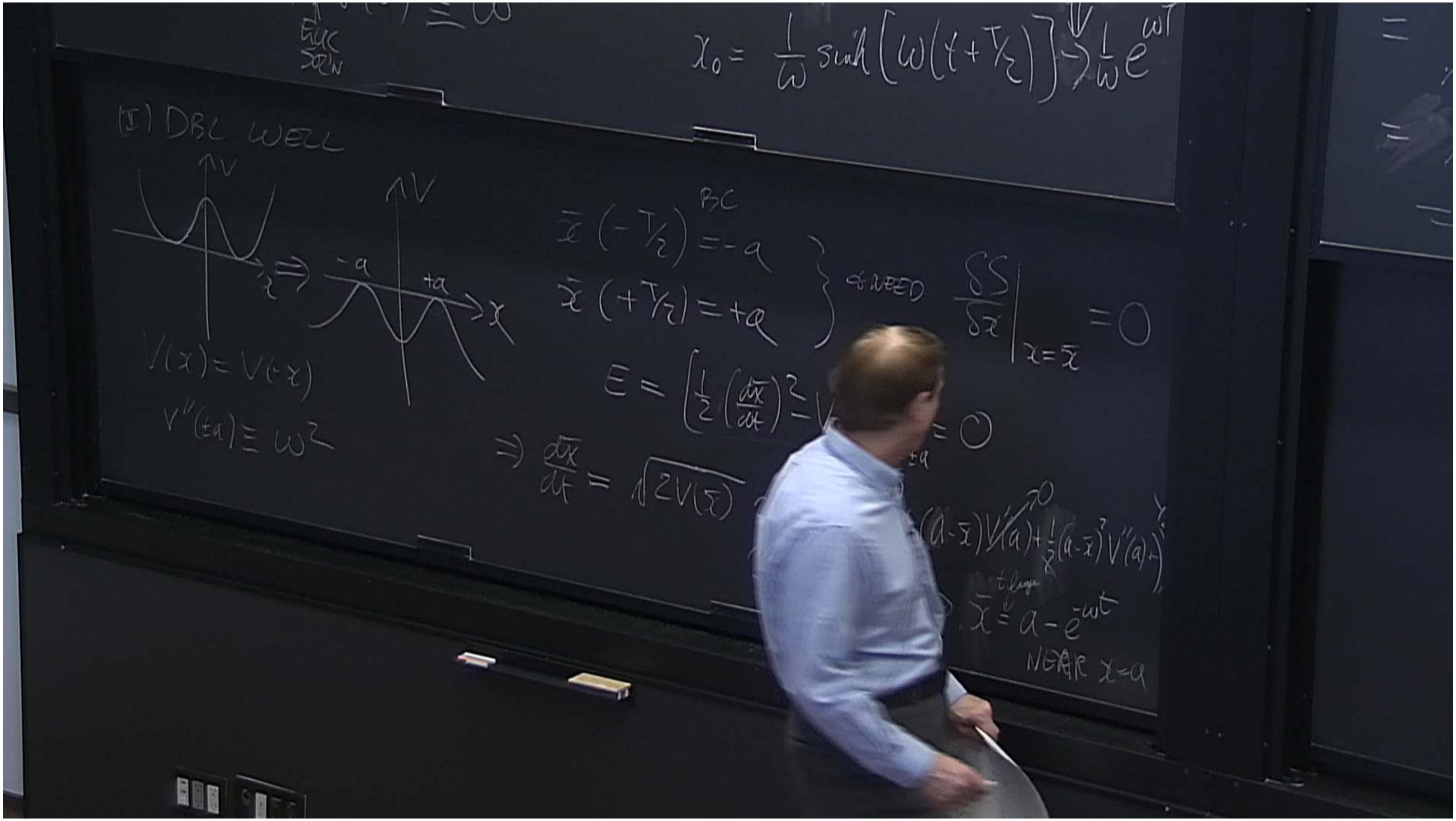
$$= \sum_{m,l} \langle 0|l\rangle\, \delta_{l,m}\, e^{-E_m T/\hbar}\langle m|0\rangle$$

$$= \sum_m |\langle 0|m\rangle|^2 e^{-E_m T/\hbar}$$

$T \Rightarrow$ [illegible] $\langle 0|0\rangle$


EUC SOL'N
$x_0 = \frac{1}{\omega}\sinh\left[\omega\left(t+T/2\right)\right] \to \frac{1}{\omega}e^{\omega T}$
(II) DBL WELL
V
x
V
-a
+a
x
V


EUC SOL'N
$x_0 = \frac{1}{\omega}\sinh[\omega(t+T/2)] \to \frac{1}{\omega}e^{\omega T}$
(I) DBL WELL
V
x
$\Rightarrow$
V
-a
+a
x
$V(x) = V(-x)$
$V''(\pm a) = \omega^2$
$\bar{x}(-T/2) = -a$
$\bar{x}(+T/2) = +a$



EUC SOL'N

$x_0 = \frac{1}{\omega}\sinh\left[\omega\left(t+\frac{T}{2}\right)\right] \to \frac{1}{\omega}e^{\omega T}$

(II) DBL WELL

$V(x) = V(-x)$

$V''(\pm a) \equiv \omega^2$

BC

$\bar{x}\left(-\frac{T}{2}\right) = -a$

$\bar{x}\left(+\frac{T}{2}\right) = +a$

& NEED $\left.\frac{\delta S}{\delta x}\right|_{x=\bar{x}} = 0$

$E = \left[\frac{1}{2}\left(\frac{dx}{dt}\right)^2 - V\right] = 0$

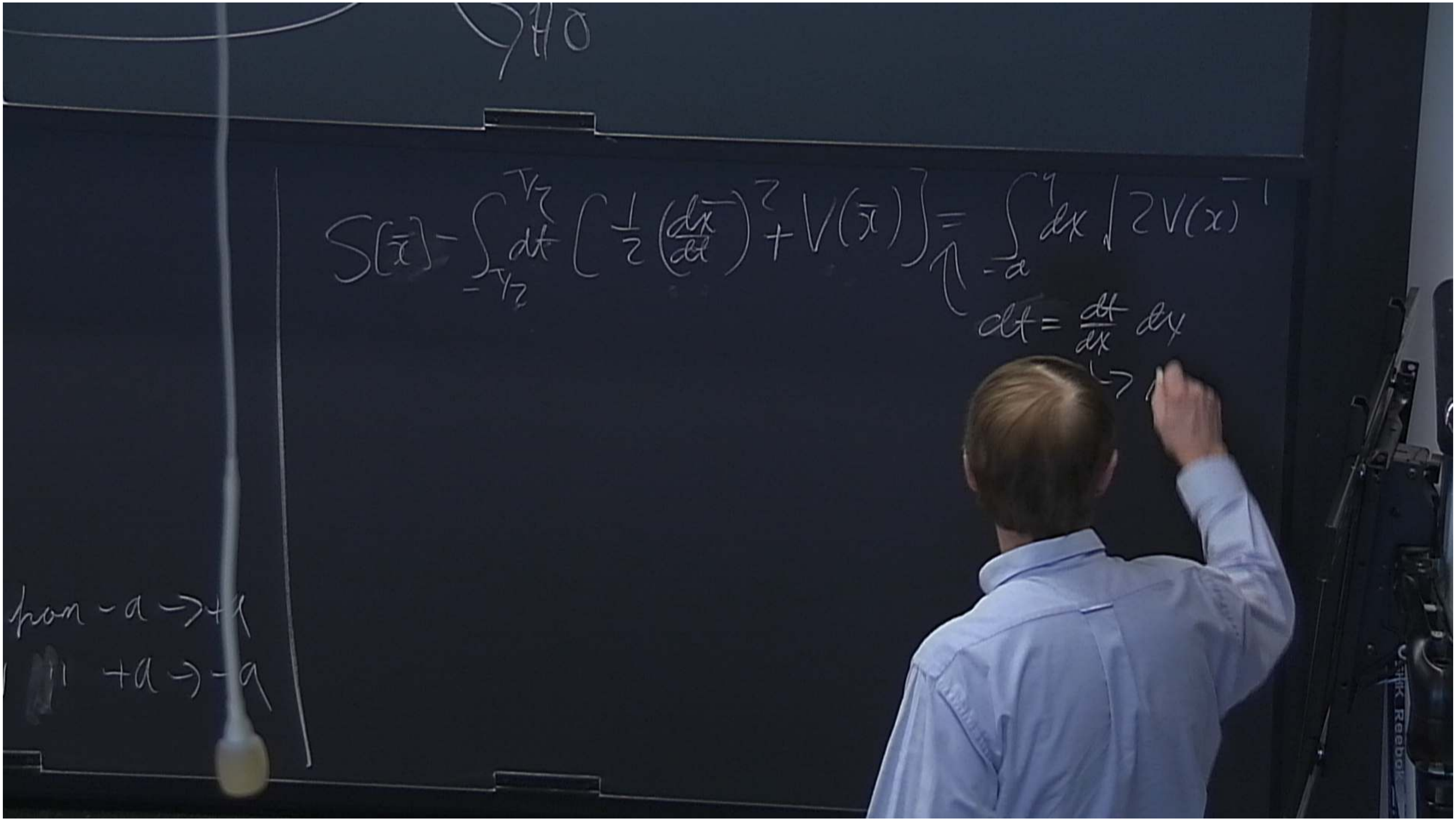
$\pm a$

$\Rightarrow \frac{d\bar{x}}{dt} = \sqrt{2V(\bar{x})}$

$(a-\bar{x})\cancel{V'(a)}^{\to 0} + \frac{1}{2}(a-\bar{x})^2 V''(a) + \ldots$

$\Rightarrow \bar{x} \overset{t\ \text{large}}{\cong} a - e^{-\omega t}$

NEAR $x=a$



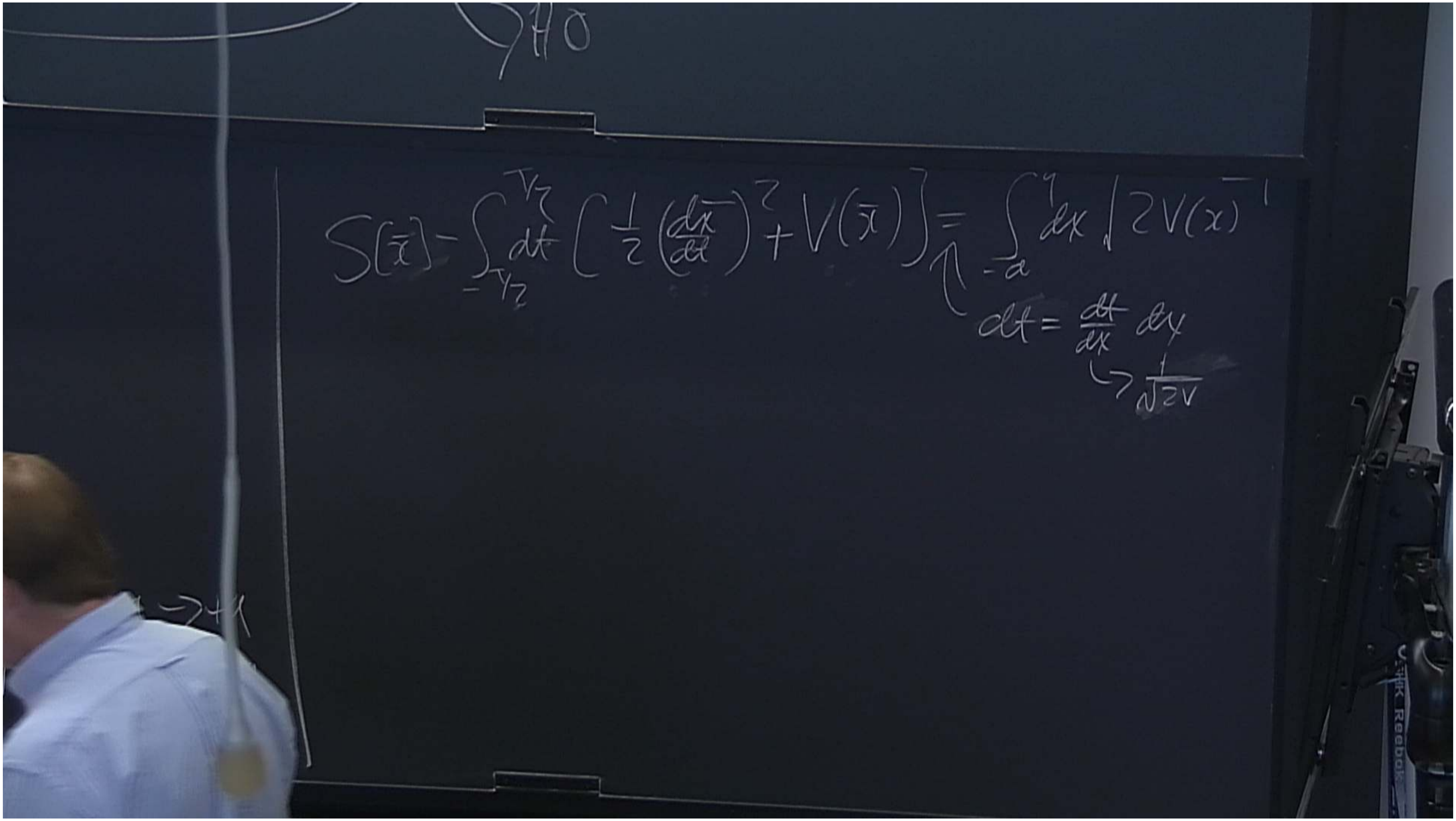
SHO

$$S[\bar{x}] = \int_{-T/2}^{T/2} dt \left[\frac{1}{2}\left(\frac{d\bar{x}}{dt}\right)^2 + V(\bar{x}) \right] = \int_{-a}^{a} dx \sqrt{2V(x)}$$

$$dt = \frac{dt}{dx}\, dx$$

from $-a \to +a$

$+a \to -a$



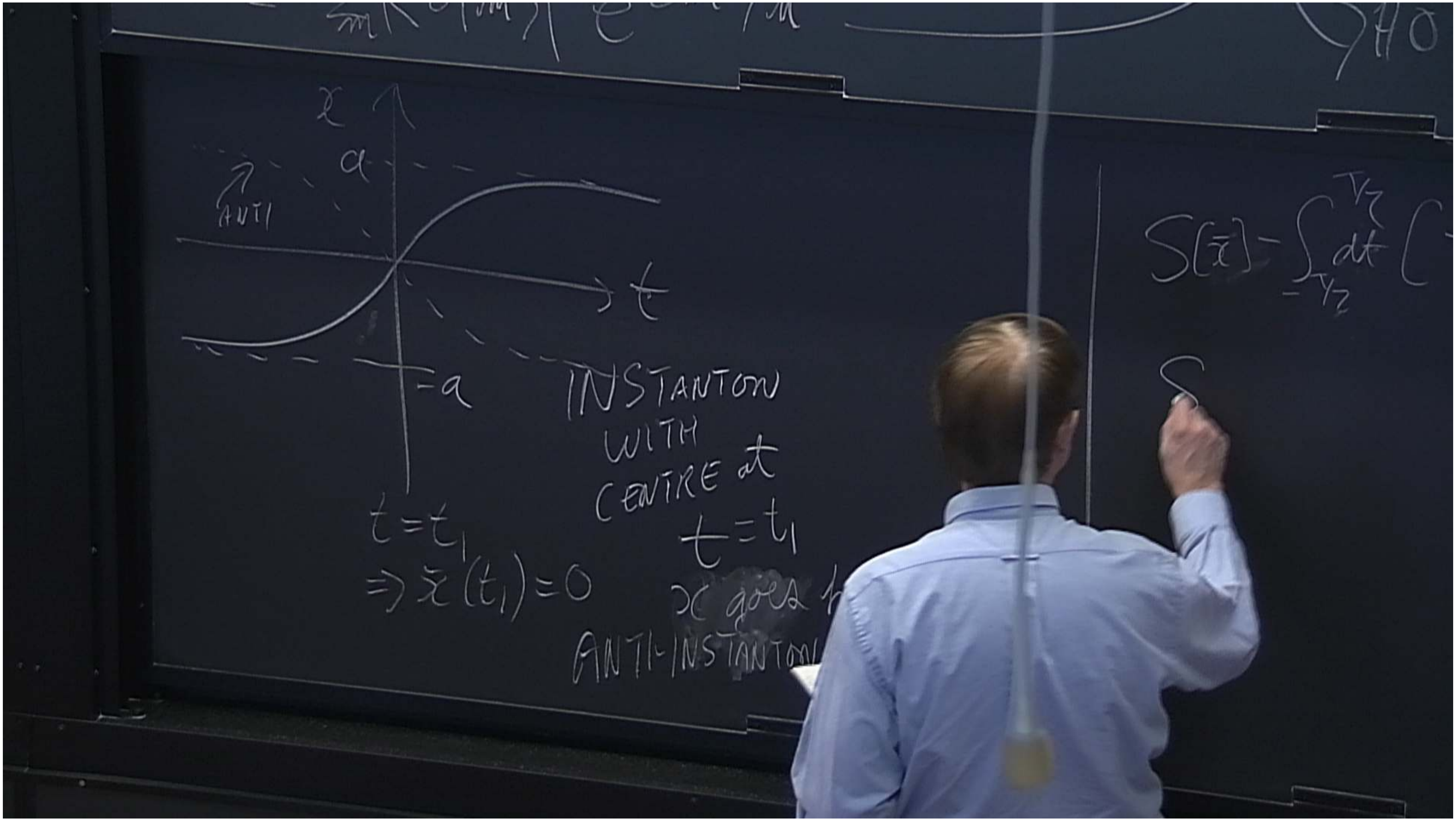
SHO

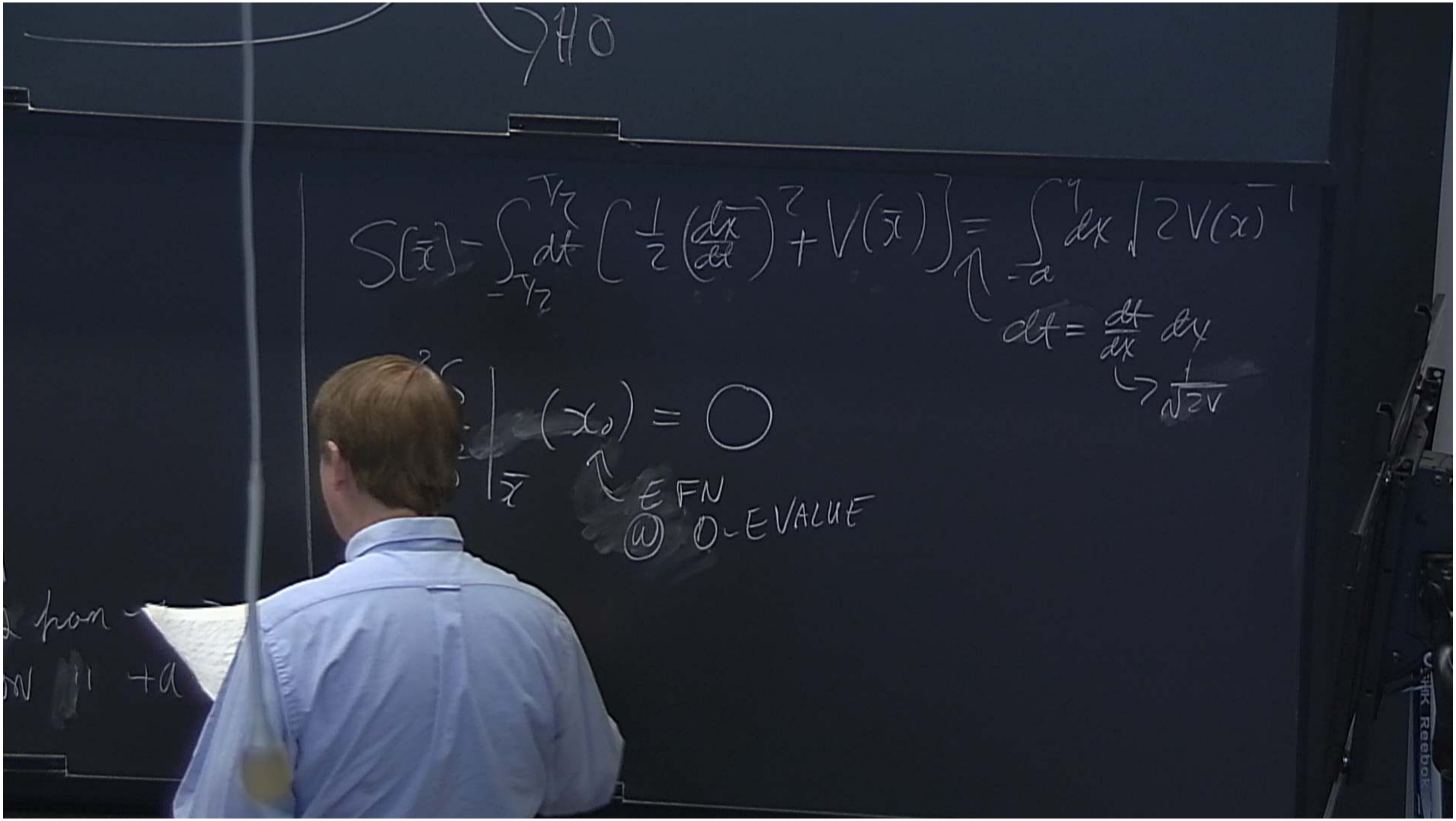
$\to +1$

$$S[\bar{x}] = \int_{-T/2}^{T/2} dt \left[\frac{1}{2}\left(\frac{d\bar{x}}{dt}\right)^2 + V(\bar{x}) \right] = \int_{-a}^{a} dx \sqrt{2V(x)}$$

$$dt = \frac{dt}{dx}\, dx$$

$$\frac{dt}{dx} \hookrightarrow \frac{1}{\sqrt{2V}}$$


SHO
x
a
ANTI
t
-a
INSTANTON
WITH
CENTRE at
t = t1
t = t1
⇒ ẋ(t1) = 0
x goes to
ANTI-INSTANTON
S[x̄] = ∫ dt (
T/2
-T/2
S

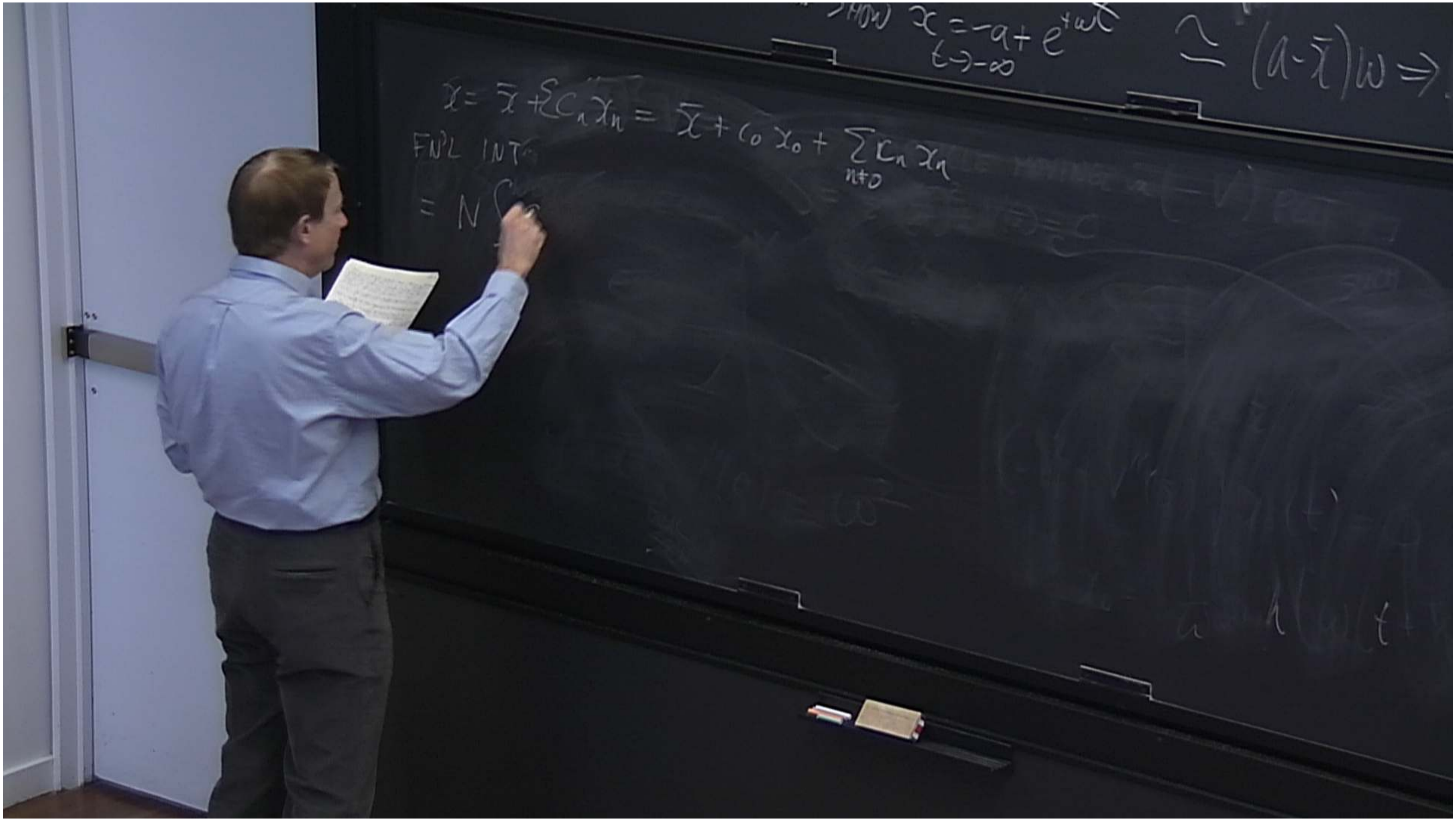


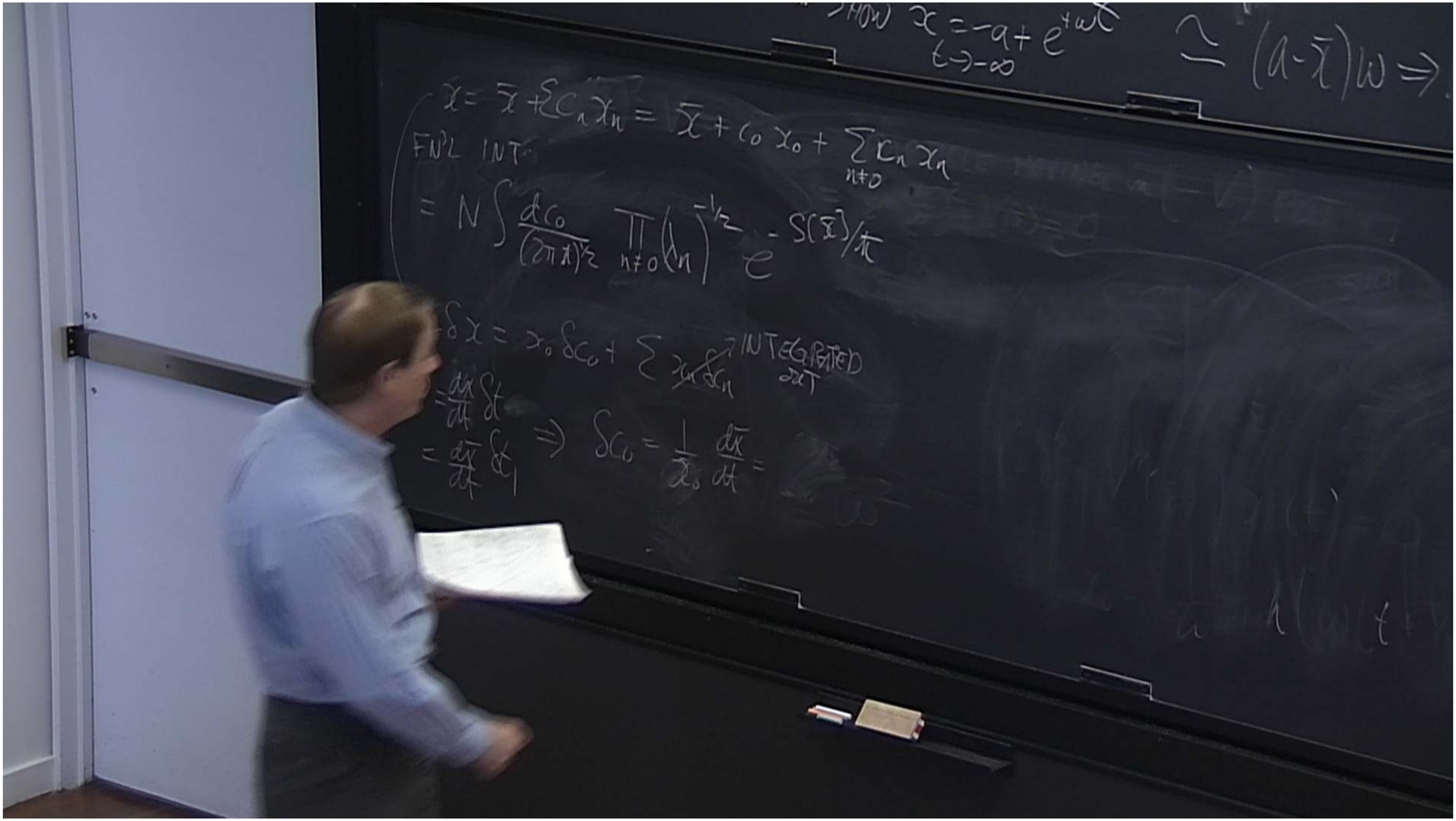
$$S[\bar{x}] = \int_{-T/2}^{T/2} dt \left[\frac{1}{2}\left(\frac{d\bar{x}}{dt}\right)^2 + V(\bar{x}) \right] = \int_{-a}^{a} dx \sqrt{2V(x)}$$

$$dt = \frac{dt}{dx}\, dx \qquad \frac{dt}{dx} \hookrightarrow \frac{1}{\sqrt{2V}}$$

$$\frac{\delta^2 S}{\delta x^2}\Big|_{\bar{x}} (x_0) = 0$$

EFN
@ 0-EVALUE


SHOW $x = -a + e^{+\omega t}$
$t \to -\infty$
$\simeq (a-\bar{x})\omega \Rightarrow$
$x = \bar{x} + \sum c_n x_n = \bar{x} + c_0 x_0 + \sum_{n \neq 0} c_n x_n$
FN'L INT
$= N \int$



SHOW $x \underset{t\to-\infty}{=} -a + e^{+\omega t}$ $\simeq (a-\bar{x})\omega \Rightarrow$

$$x = \bar{x} + \sum c_n x_n = \bar{x} + c_0 x_0 + \sum_{n\neq 0} c_n x_n$$

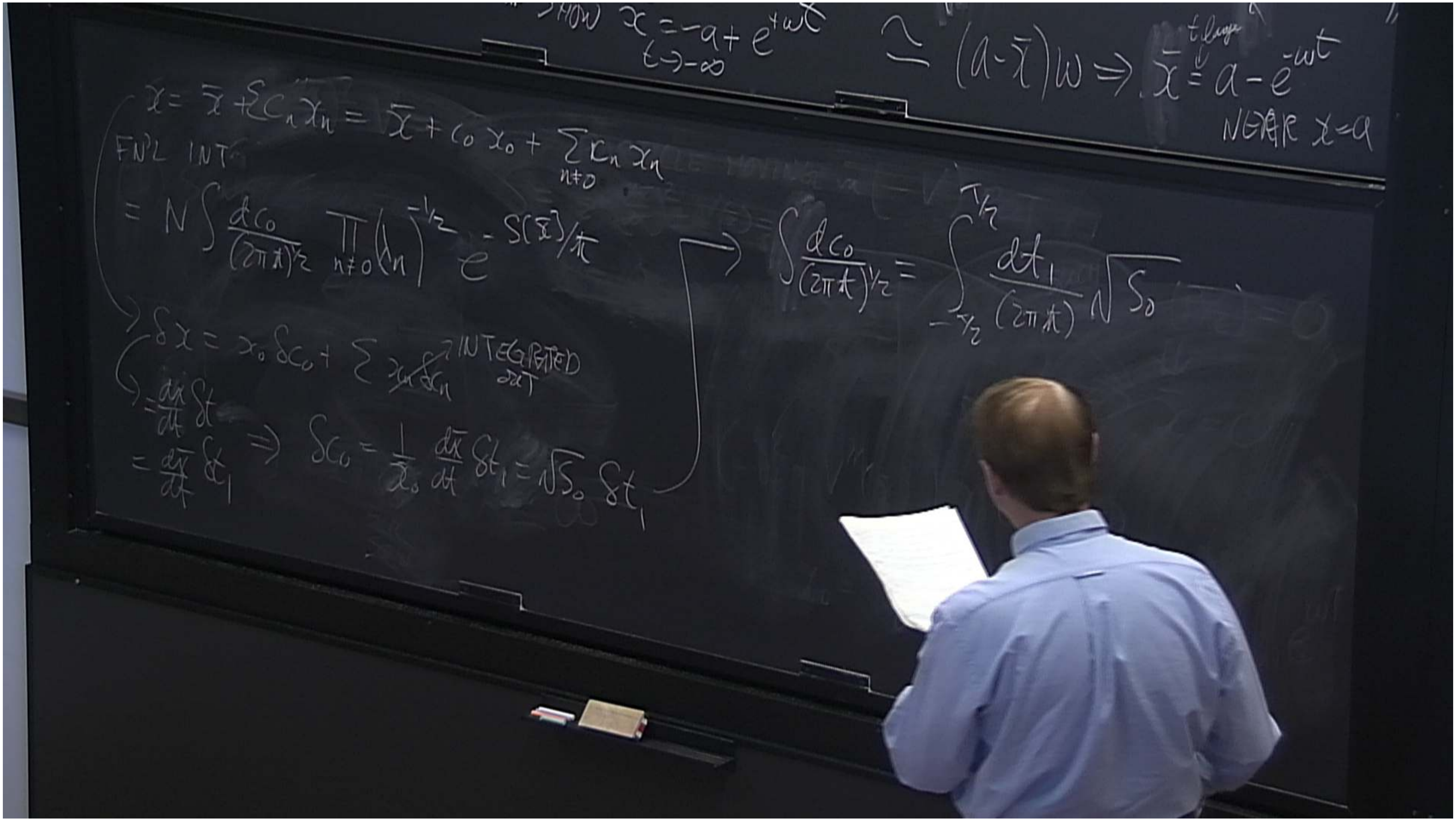
FN'L INT

$$= N \int \frac{dc_0}{(2\pi\hbar)^{1/2}} \prod_{n\neq 0} (\lambda_n)^{-1/2} e^{-S[\bar{x}]/\hbar}$$

$$\delta x = x_0 \delta c_0 + \sum \cancel{x_n} \delta c_n \rightarrow \text{INTEGRATED } \delta x T$$

$$= \frac{dx}{dt}\delta t$$

$$= \frac{d\bar{x}}{dt}\delta t_1 \Rightarrow \delta c_0 = \frac{1}{\bar{x}_0}\frac{d\bar{x}}{dt} =$$

</br>
SHOW $x = -a + e^{+\omega t}$ $\quad t \to -\infty$

$\simeq (a-\bar{x})\omega \Rightarrow \bar{x} \overset{t \text{ large}}{=} a - e^{-\omega t}$

NEAR $x = a$

$$x = \bar{x} + \sum c_n x_n = \bar{x} + c_0 x_0 + \sum_{n\neq 0} c_n x_n$$

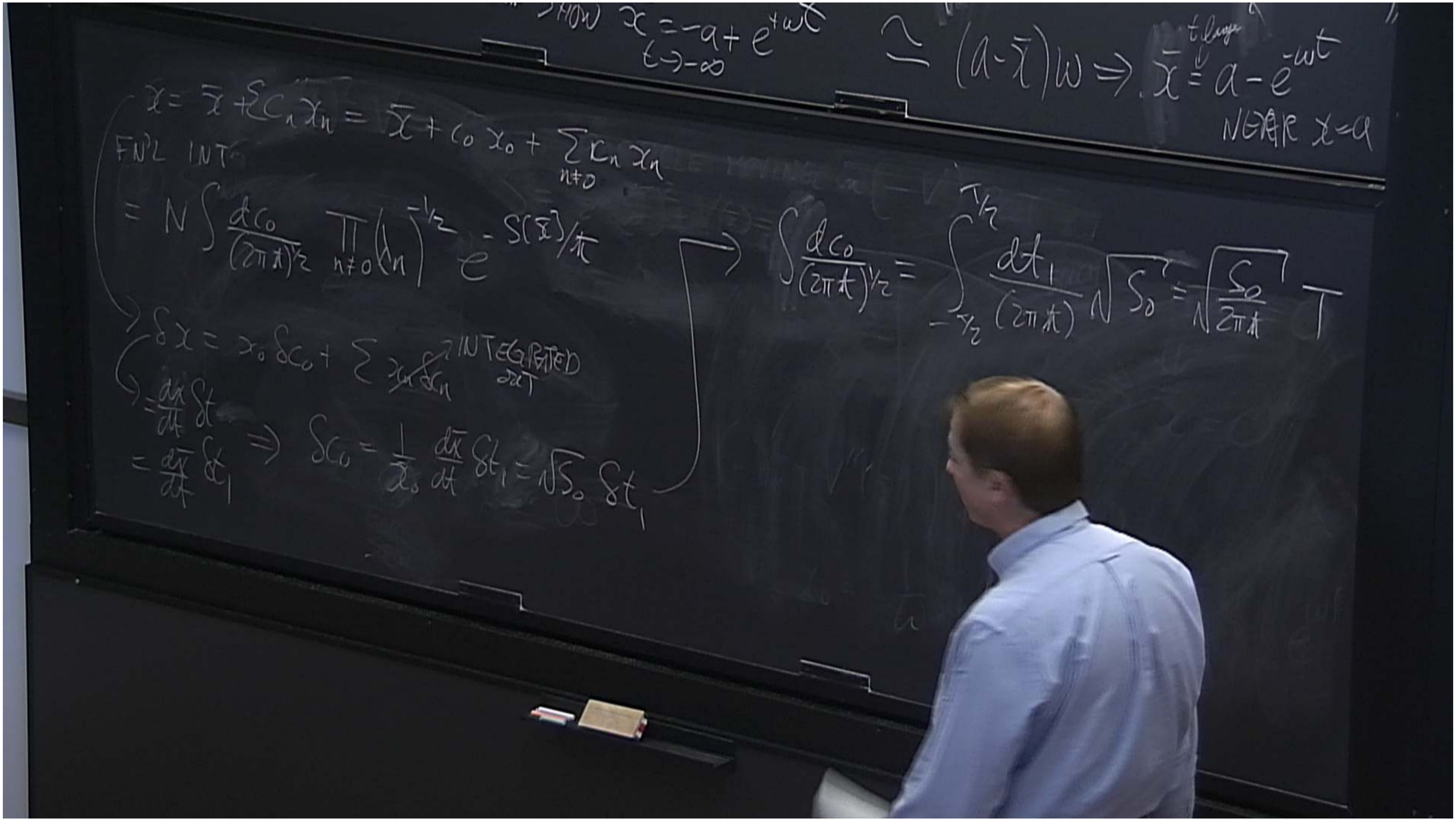
FN'L INT

$$= N \int \frac{dc_0}{(2\pi\hbar)^{1/2}} \prod_{n\neq 0} (\lambda_n)^{-1/2} e^{-S[\bar{x}]/\hbar}$$

$$\delta x = x_0 \delta c_0 + \sum \cancel{x_n \delta c_n}$$ INTEGRATED

$$= \frac{dx}{dt}\delta t = \frac{d\bar{x}}{dt}\delta t_1 \Rightarrow \delta c_0 = \frac{1}{x_0}\frac{d\bar{x}}{dt}\delta t_1 = \sqrt{S_0}\,\delta t_1$$

$$\int \frac{dc_0}{(2\pi\hbar)^{1/2}} = \int_{-T/2}^{T/2} \frac{dt_1}{(2\pi\hbar)} \sqrt{S_0}$$



SHOW $x = -a + e^{+\omega t}$, $t \to -\infty$

$\simeq (a-\bar{x})\omega \Rightarrow \bar{x} \overset{t\ \text{large}}{=} a - e^{-\omega t}$

NEAR $x=a$

$x = \bar{x} + \sum c_n x_n = \bar{x} + c_0 x_0 + \sum_{n\neq 0} c_n x_n$

FN'L INT

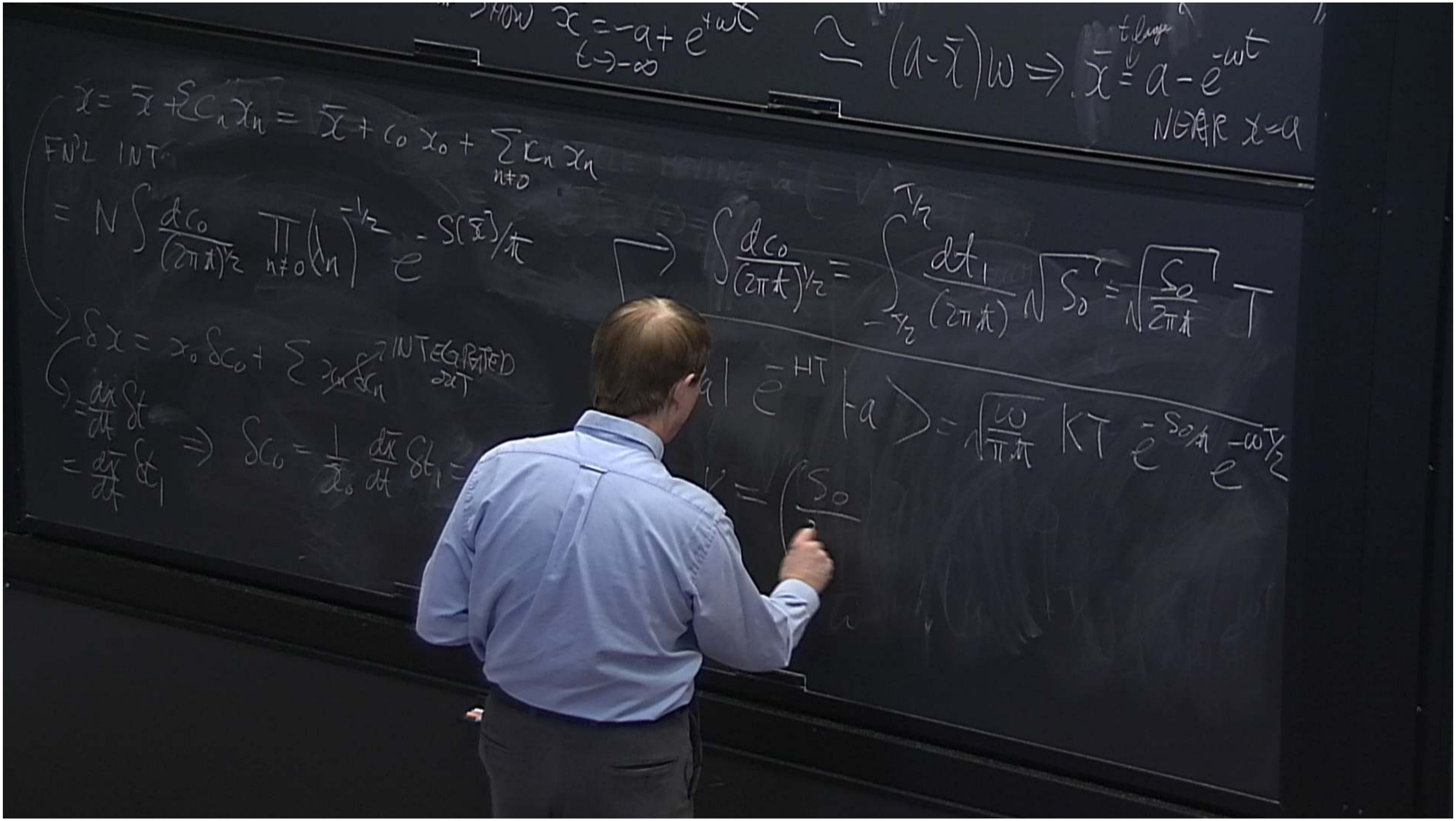
$$= N \int \frac{dc_0}{(2\pi\hbar)^{1/2}} \prod_{n\neq 0} (\lambda_n)^{-1/2} e^{-S[\bar{x}]/\hbar}$$

$\delta x = x_0 \delta c_0 + \sum \cancel{x_n \delta c_n}$ INTEGRATED $\partial x T$

$= \frac{d\bar{x}}{dt}\delta t$

$= \frac{d\bar{x}}{dt}\delta t_1 \Rightarrow \delta c_0 = \frac{1}{x_0}\frac{d\bar{x}}{dt}\delta t_1 = \sqrt{S_0}\,\delta t_1$

$$\int \frac{dc_0}{(2\pi\hbar)^{1/2}} = \int_{-T/2}^{T/2} \frac{dt_1}{(2\pi\hbar)}\sqrt{S_0} = \sqrt{\frac{S_0}{2\pi\hbar}}\,T$$



SHOW $x = -a + e^{+\omega\tau}$, $t \to -\infty$

$\simeq (a-\bar{x})\omega \Rightarrow \bar{x} \overset{t\ \text{large}}{\approx} a - e^{-\omega t}$

NEAR $x=a$

$x = \bar{x} + \sum c_n x_n = \bar{x} + c_0 x_0 + \sum_{n\neq 0} c_n x_n$

FN'L INT

$= N \int \frac{dc_0}{(2\pi\hbar)^{1/2}} \prod_{n\neq 0} (\lambda_n)^{-1/2} e^{-S(\bar{x})/\hbar}$

$\int \frac{dc_0}{(2\pi\hbar)^{1/2}} = \int_{-T/2}^{T/2} \frac{dt_1}{(2\pi\hbar)} \sqrt{S_0} = \sqrt{\frac{S_0}{2\pi\hbar}}\, T$

$\delta x = x_0 \delta c_0 + \sum \cancel{x_n \delta c_n}$ → INTEGRATED OUT

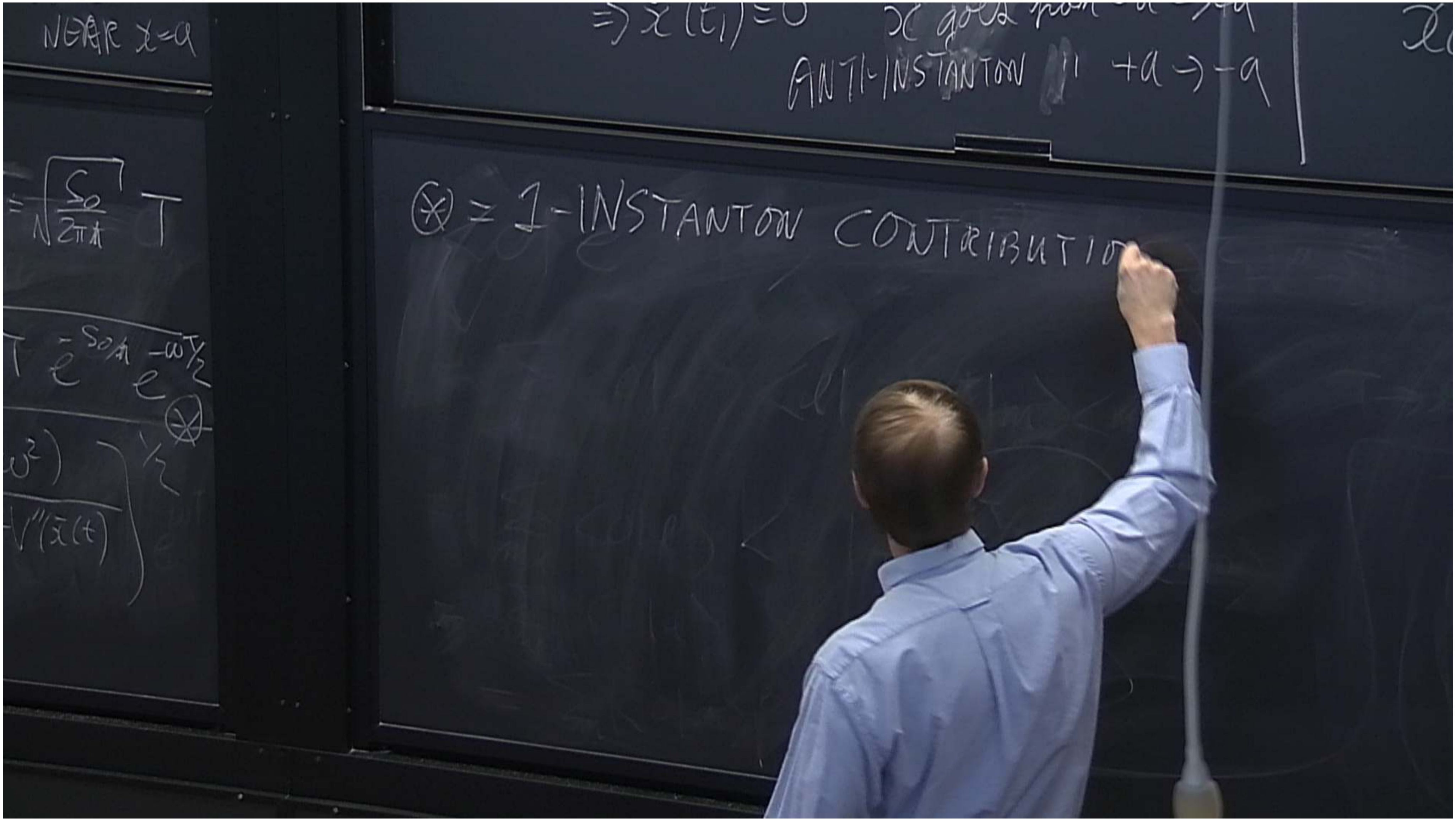
$= \frac{dx}{dt}\delta t$

$= \frac{d\bar{x}}{dt}\delta t_1$

$\Rightarrow \delta c_0 = \frac{1}{x_0}\frac{d\bar{x}}{dt}\delta t_1 =$

$\langle a | e^{-HT} | -a \rangle = \sqrt{\frac{\omega}{\pi\hbar}}\, K T\, e^{-S_0/\hbar} e^{-\omega T/2}$

$K = \Big(\frac{S_0}{\ }$



NEAR $x=a$

$\Rightarrow \bar{x}(t_1) = 0$ $\quad$ x goes from $-a \to +a$

ANTI-INSTANTON " $+a \to -a$

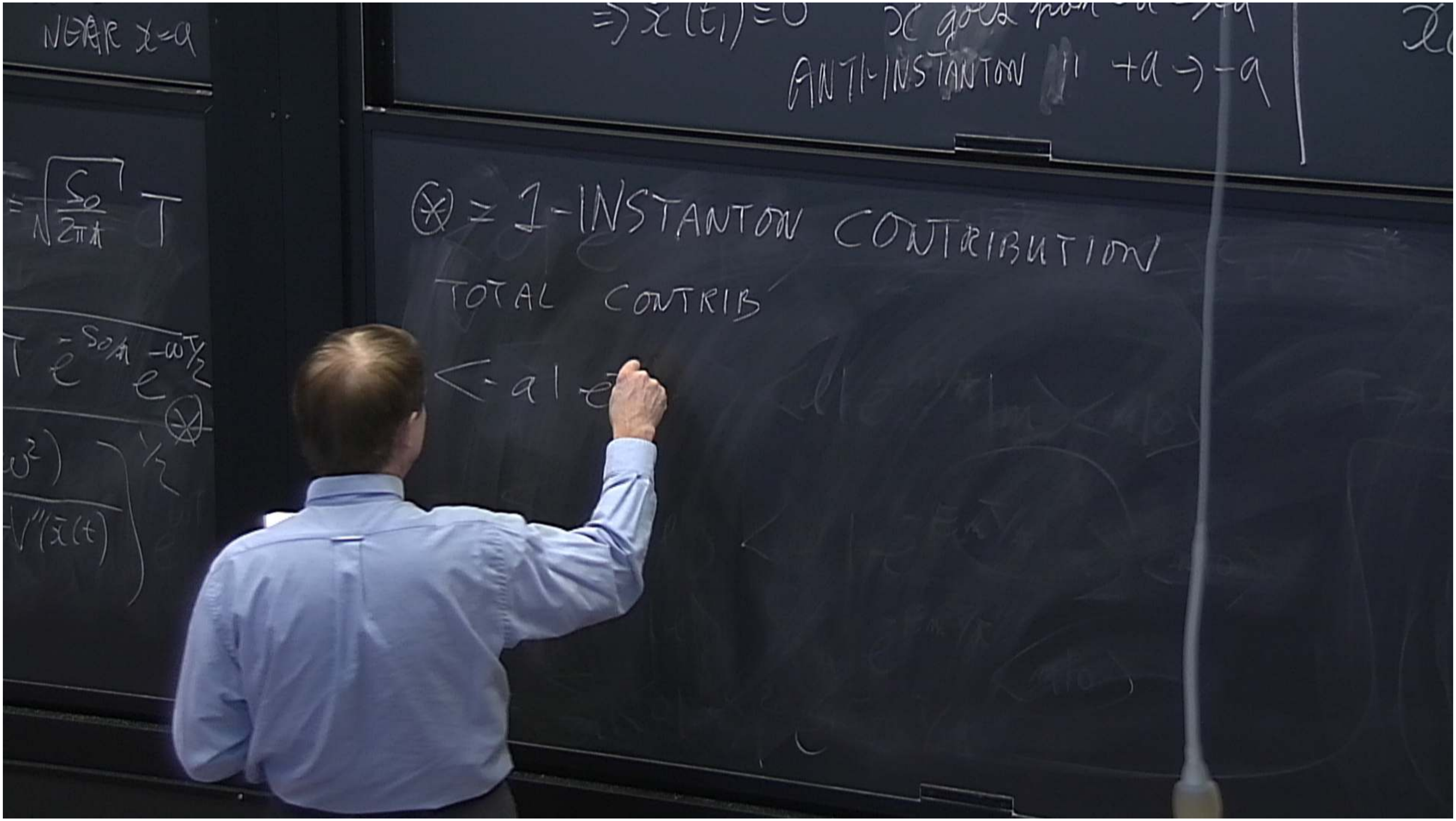
$= \sqrt{\frac{S_0}{2\pi\hbar}}\, T$

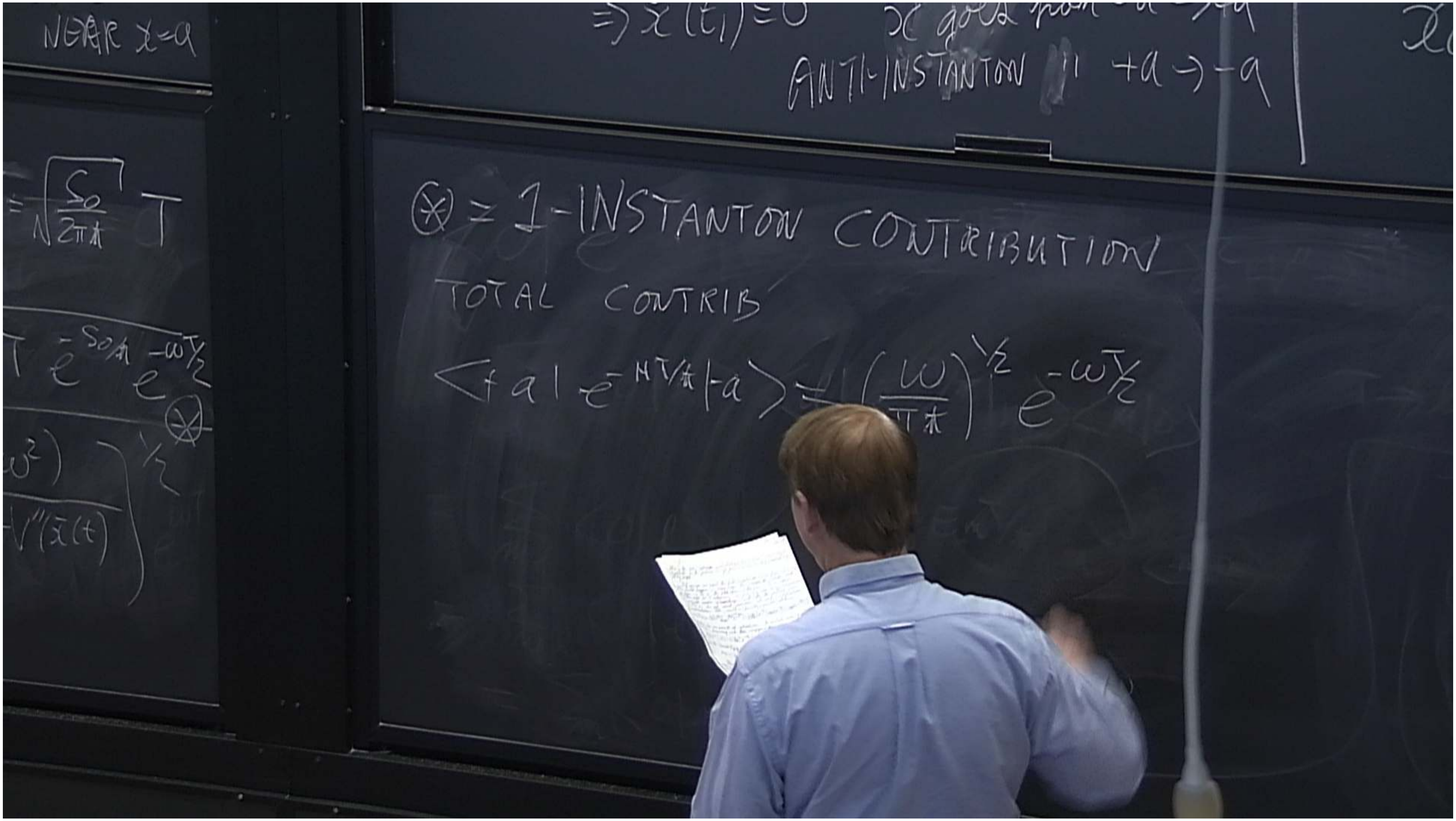
$T\, e^{-S_0/\hbar}\, e^{-\omega T/2}$ ⊛

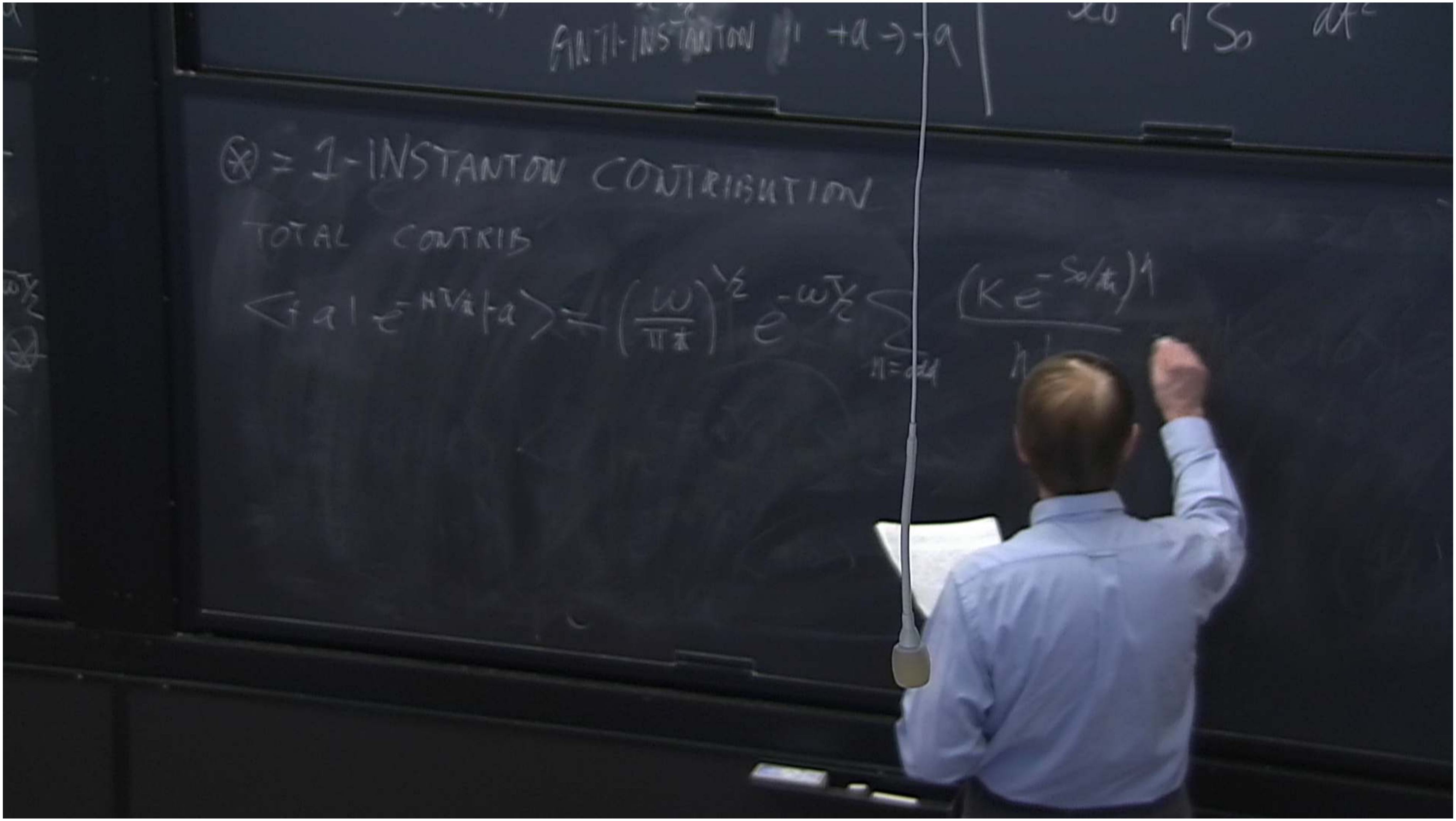
$\omega^2)$ $\Big]^{1/2}$

$V''(\bar{x}(t))$

⊛ = 1-INSTANTON CONTRIBUTIO


NEAR x=a
⇒ ẋ(t₁)=0
x goes from −a → +a
ANTI-INSTANTON +a → −a
√(S₀/2πħ) T
T e^{−S₀/ħ} e^{−ωT/2}
⊛ = 1-INSTANTON CONTRIBUTION
TOTAL CONTRIB
<−a| e


NEAR $x=a$
$\Rightarrow \dot{x}(t_1)=0$ x goes from $-a \to +a$
ANTI-INSTANTON " $+a \to -a$
$= \sqrt{\frac{S_0}{2\pi\hbar}}\, T$
$T\, e^{-S_0/\hbar}\, e^{-\omega T/2}$
$\circledast$
$\omega^2)$ $\frac{1}{2}$
$V''(\bar{x}(t))$
$\circledast$ = 1-INSTANTON CONTRIBUTION
TOTAL CONTRIB
$\langle +a | e^{-HT/\hbar} | -a \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2}$

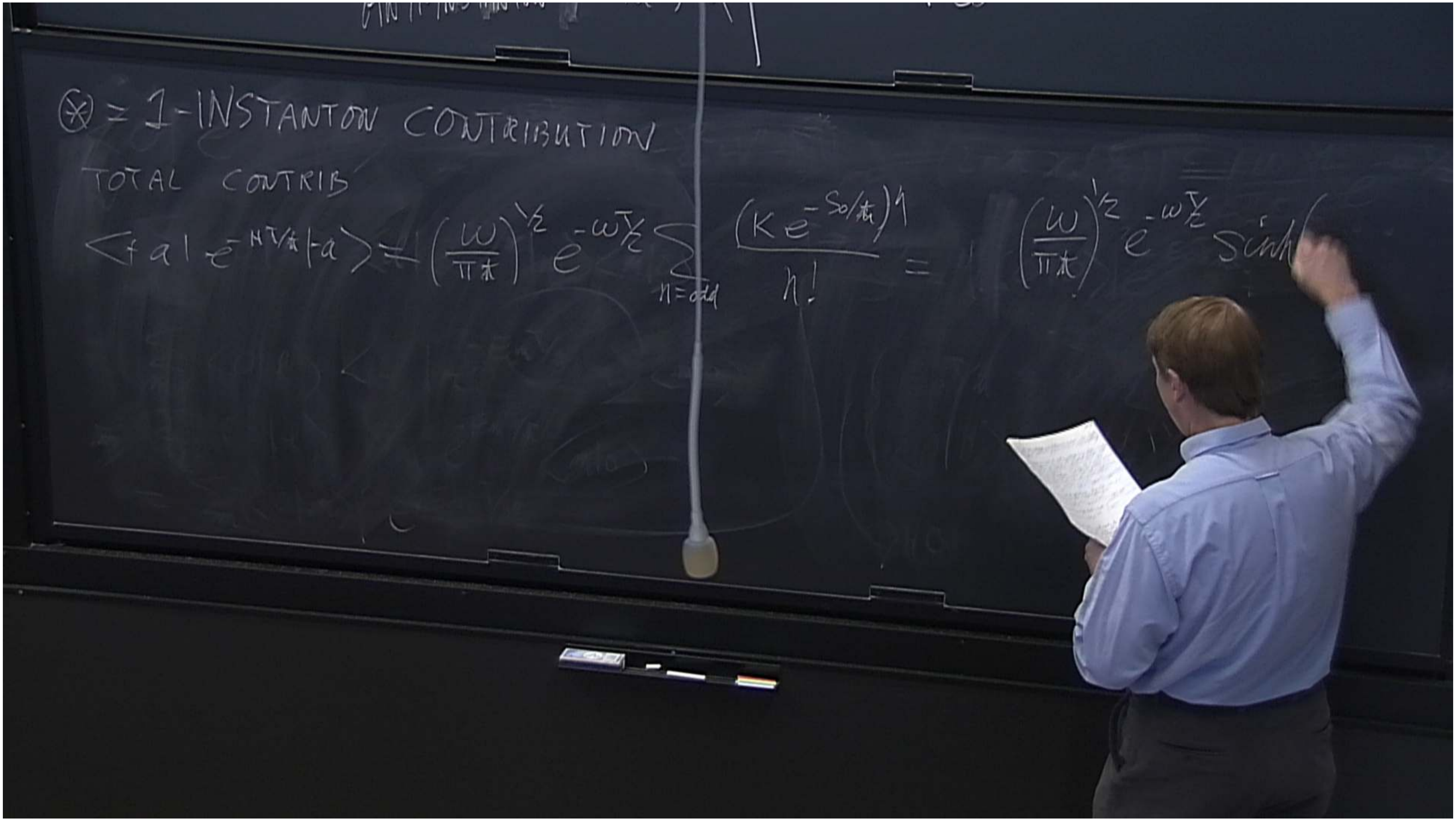


ANTI-INSTANTON : $+a \to -a$

$\otimes$ = 1-INSTANTON CONTRIBUTION

TOTAL CONTRIB

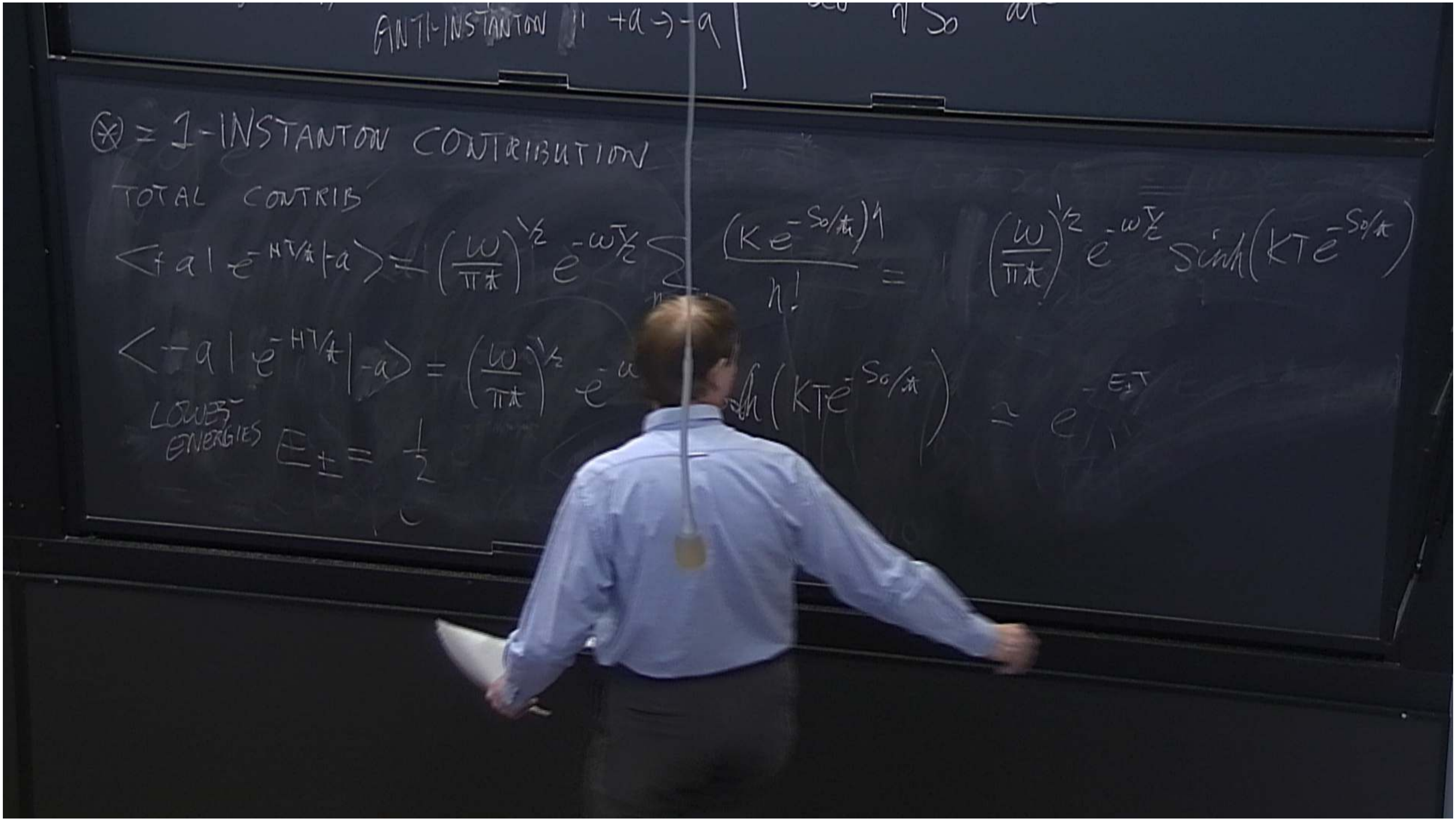
$$\langle +a | e^{-HT/\hbar} | -a \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sum_{n=\text{odd}} \frac{\left(K e^{-S_0/\hbar}\right)^n}{n!}$$



$\circledast$ = 1-INSTANTON CONTRIBUTION

TOTAL CONTRIB

$$\langle +a | e^{-HT/\hbar} | -a \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sum_{n=\text{odd}} \frac{\left(K e^{-S_0/\hbar}\right)^n}{n!} = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sinh($$



ANTI-INSTANTON n $+a \to -a$

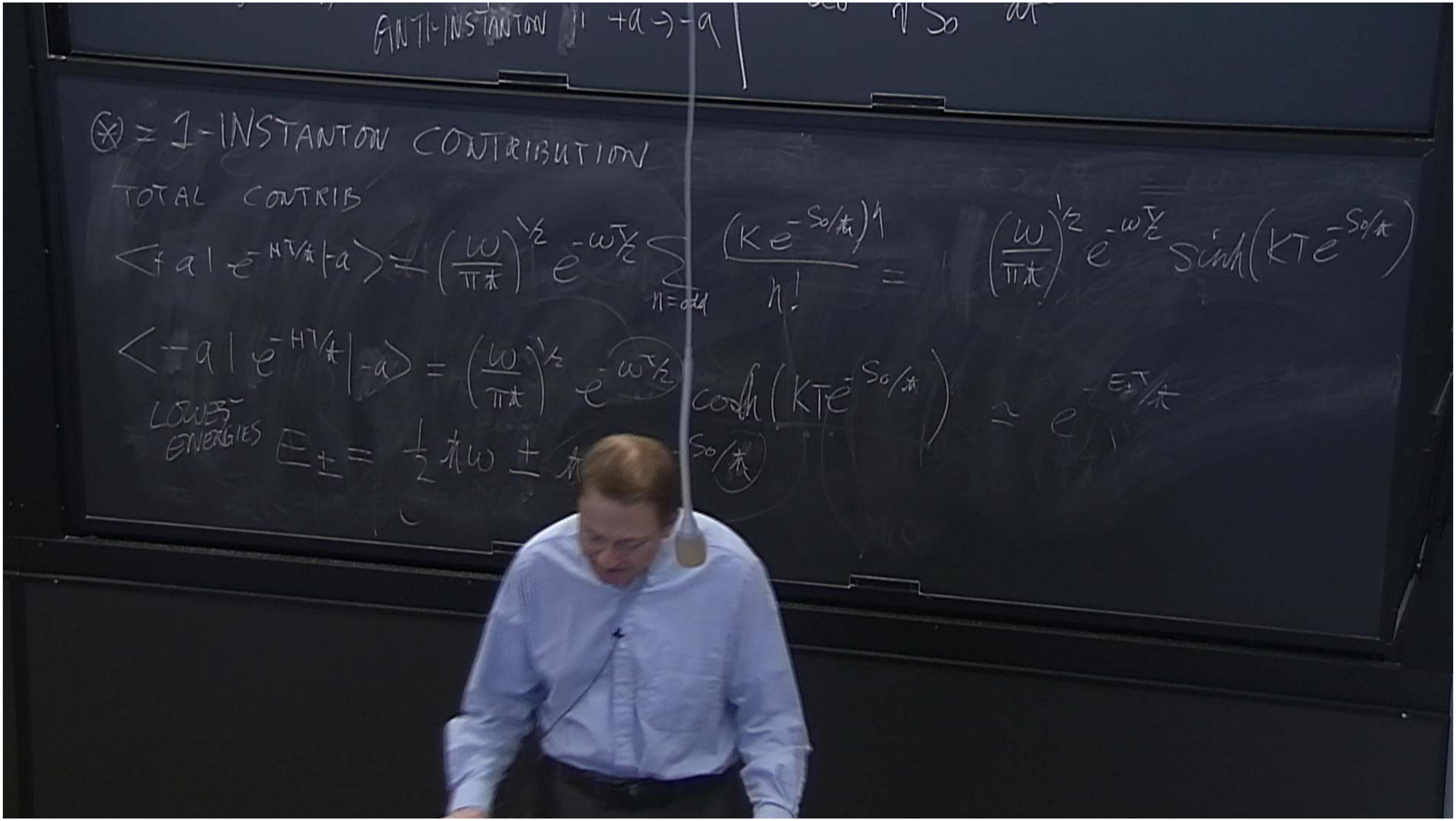
⊛ = 1-INSTANTON CONTRIBUTION

TOTAL CONTRIB

$$\langle +a | e^{-HT/\hbar} | -a \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sum_n \frac{\left(K e^{-S_0/\hbar}\right)^n}{n!} = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sinh\left(KT e^{-S_0/\hbar}\right)$$

$$\langle -a | e^{-HT/\hbar} | -a \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega} \quad h\left(KT e^{-S_0/\hbar}\right) \simeq e^{-E_\pm T}$$

LOWEST ENERGIES $E_\pm = \frac{1}{2}$



ANTI-INSTANTON $+a \to -a$

$\sqrt{S_0}$

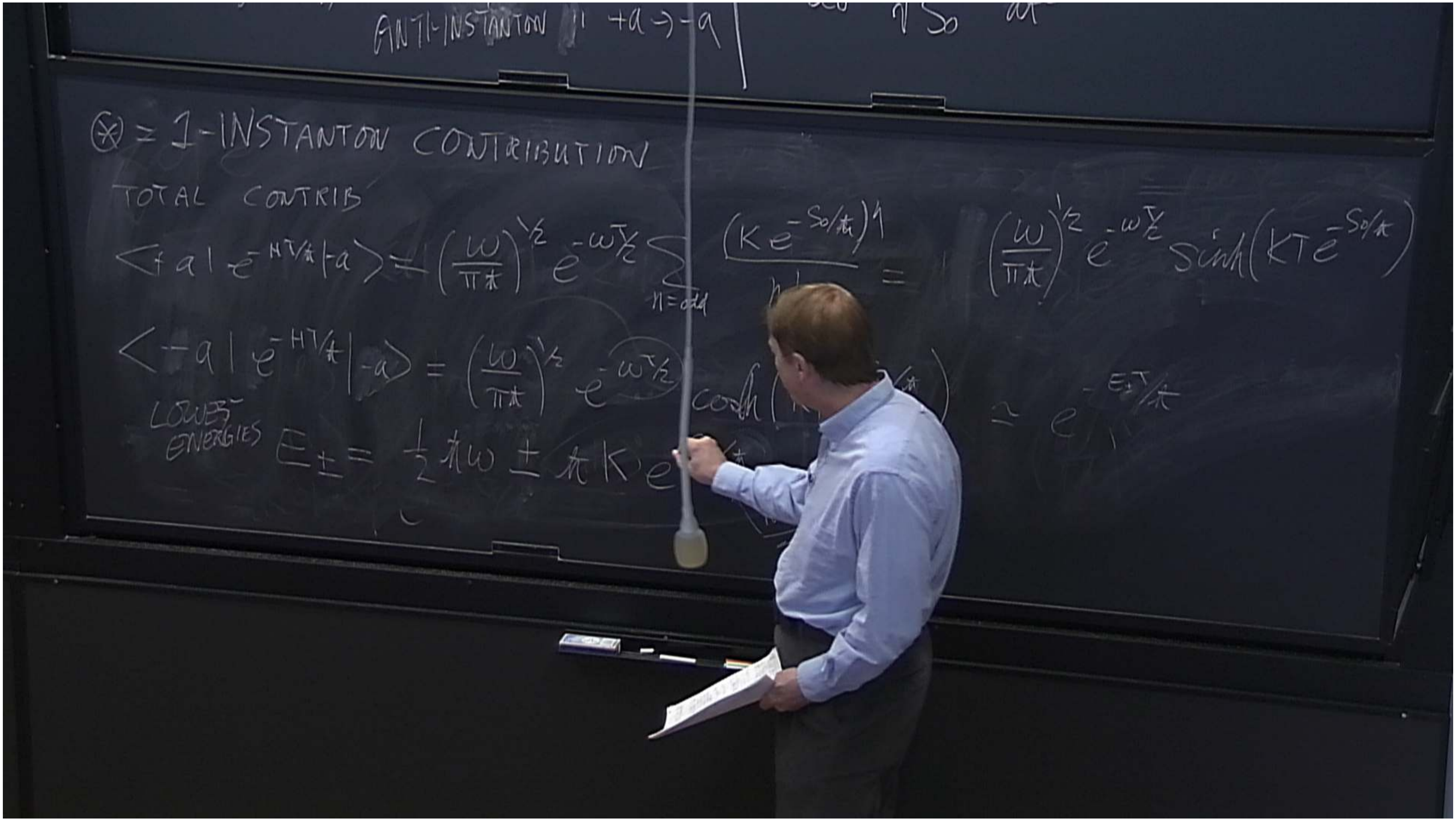
$\circledast$ = 1-INSTANTON CONTRIBUTION

TOTAL CONTRIB

$$\langle +a | e^{-HT/\hbar} | -a \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sum_{n=\text{odd}} \frac{(K e^{-S_0/\hbar})^n}{n!} = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sinh\left(K T e^{-S_0/\hbar}\right)$$

$$\langle -a | e^{-HT/\hbar} | -a \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \cosh\left(K T e^{-S_0/\hbar}\right) \simeq e^{-E_\pm T/\hbar}$$

LOWEST ENERGIES $E_\pm = \frac{1}{2}\hbar\omega \pm \hbar$ $^{-S_0/\hbar}$



ANTI-INSTANTON $+a \to -a$ $\sqrt{S_0}$

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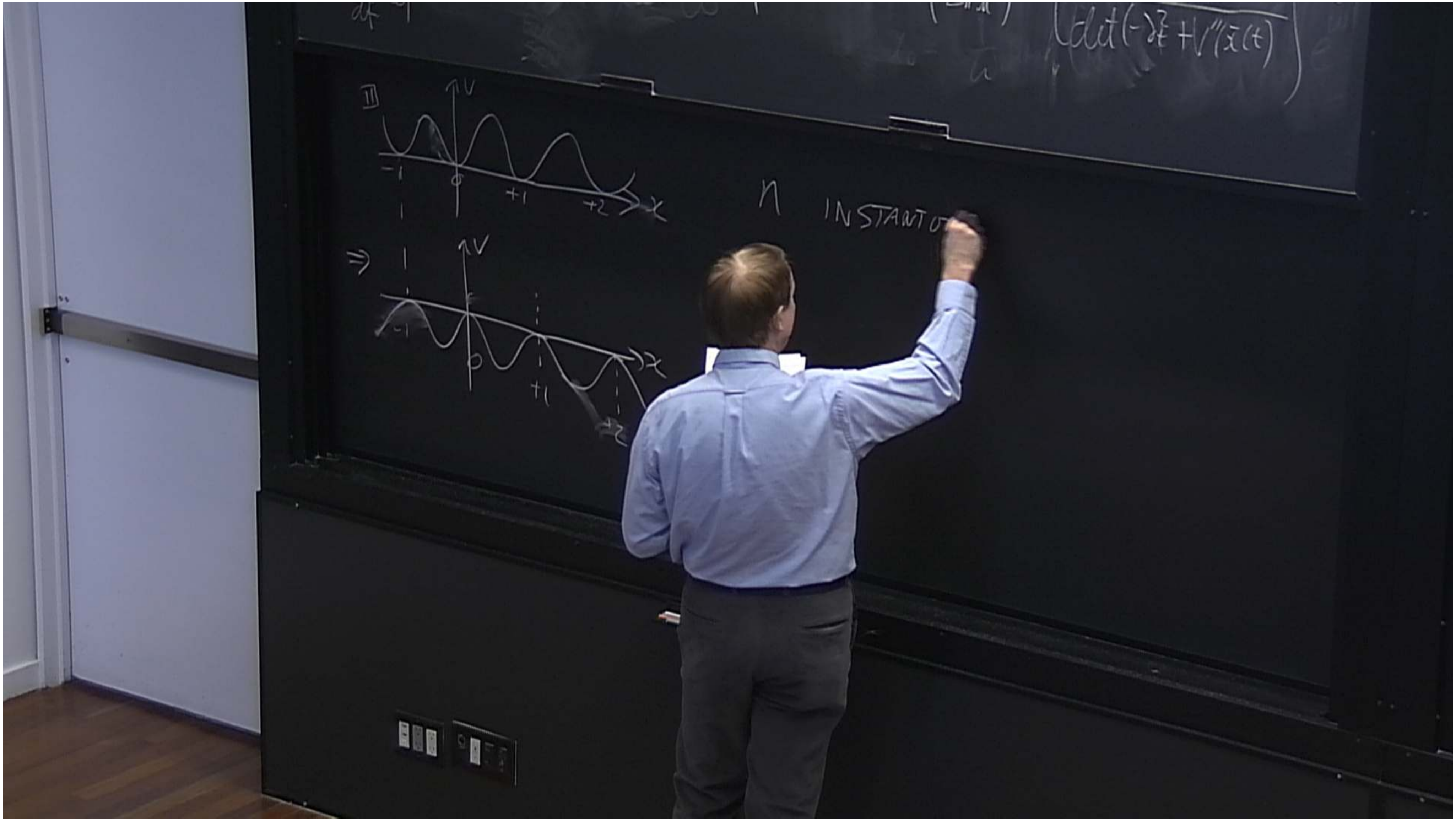
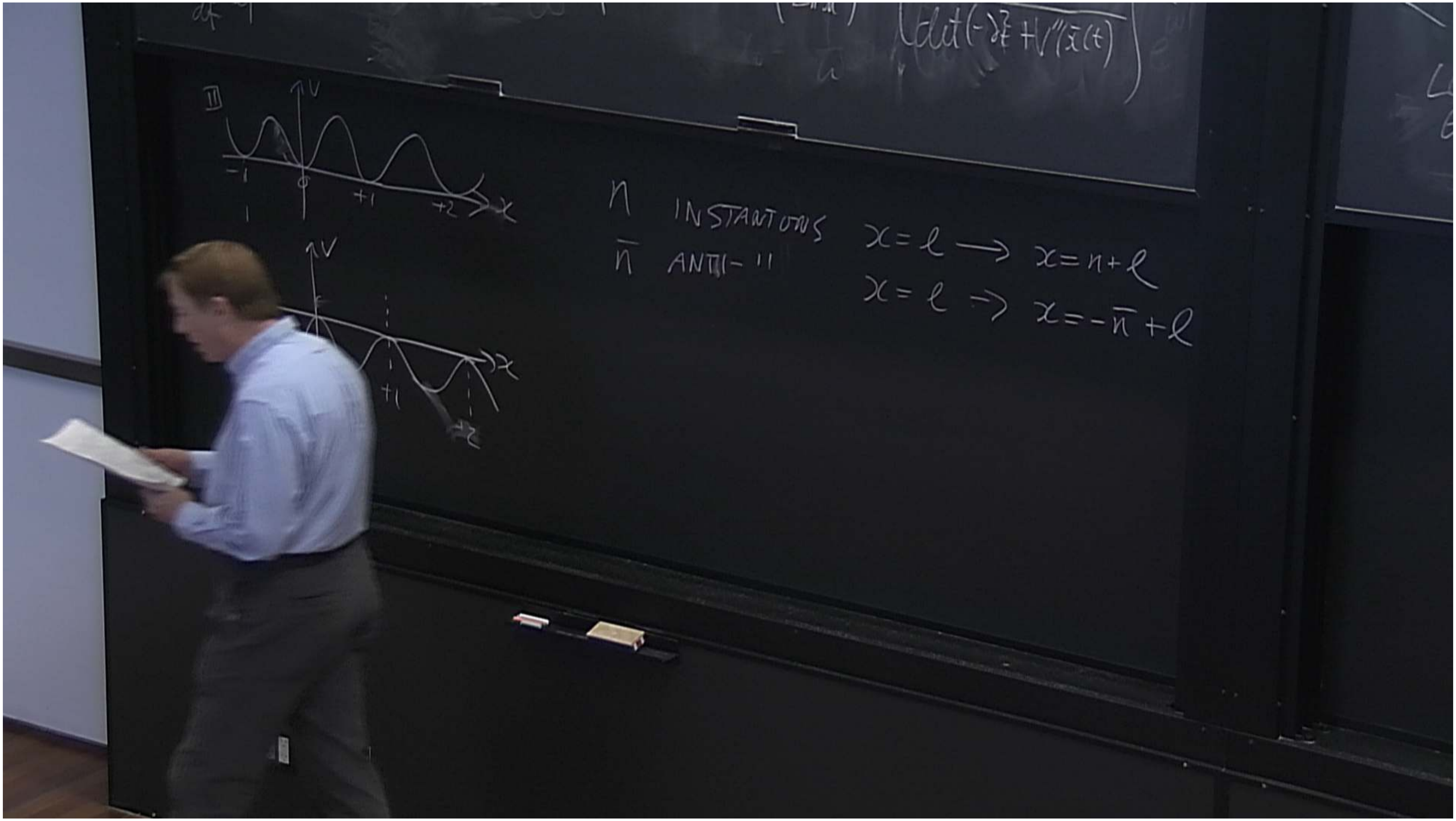
TOTAL CONTRIB

$$\langle +a | e^{-HT/\hbar} | -a \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sum_{n=odd} \frac{(K e^{-S_0/\hbar})^n}{n!} = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sinh\left(KT e^{-S_0/\hbar}\right)$$

$$\langle -a | e^{-HT/\hbar} | -a \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \cosh\left(\quad /\hbar\right) \simeq e^{-E_\pm T/\hbar}$$

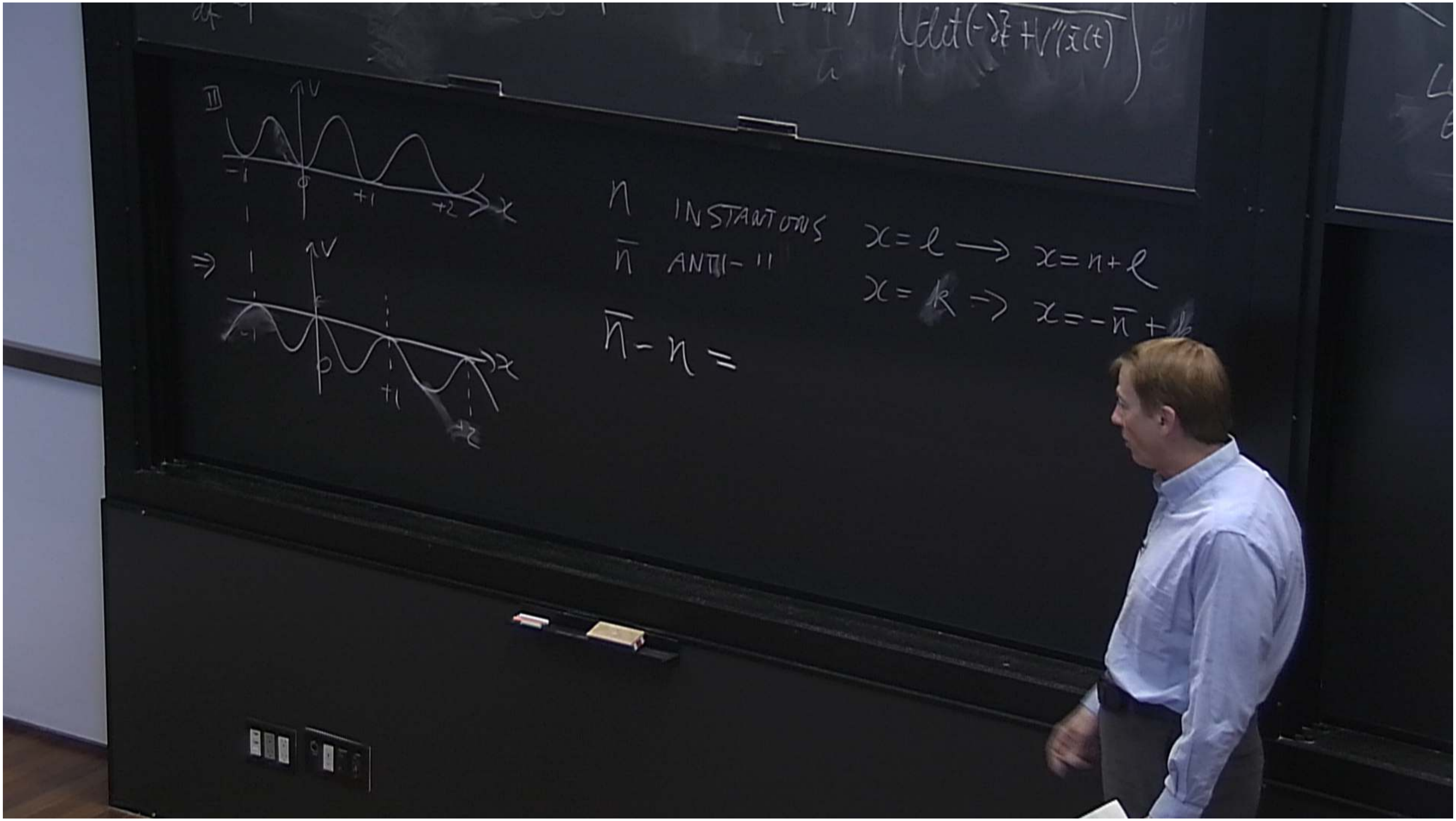
LOWEST ENERGIES $E_\pm = \frac{1}{2}\hbar\omega \pm \hbar K e^{\ /\hbar}$


II
V
−1
0
+1
+2
x
⇒
V
−1
0
+1
+2
x
n INSTANTO
det(−∂t² + V″(x̄(t))



n INSTANTONS $x = \ell \longrightarrow x = n + \ell$

$\bar{n}$ ANTI- " $x = \ell \rightarrow x = -\bar{n} + \ell$

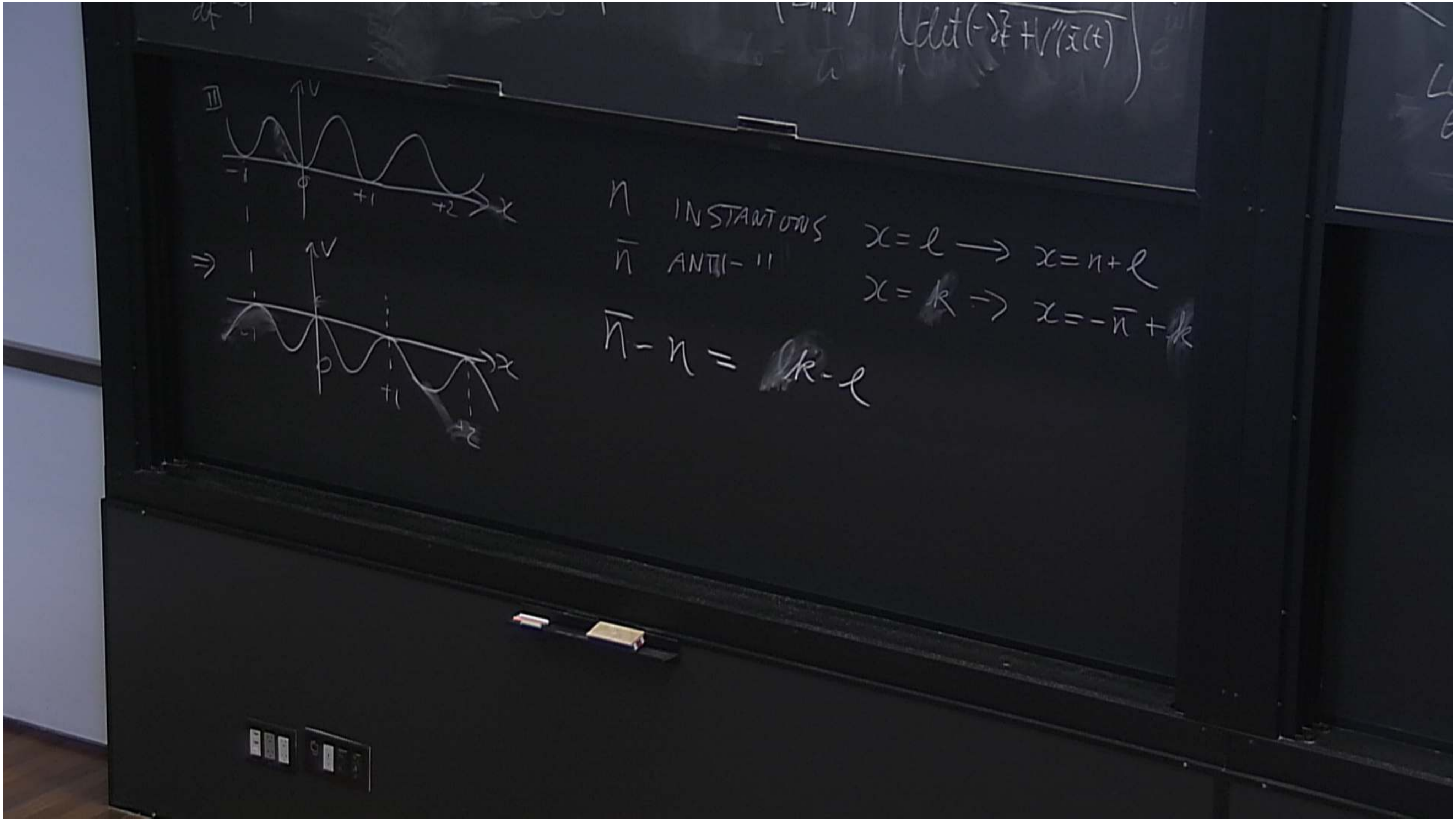


$dt\left(-\partial_t^2 + V''(\bar{x}(t)\right)$

n INSTANTONS $\quad x = \ell \longrightarrow x = n + \ell$

$\bar{n}$ ANTI- '' $\quad x = k \Rightarrow x = -\bar{n} + k$

$\bar{n} - n =$

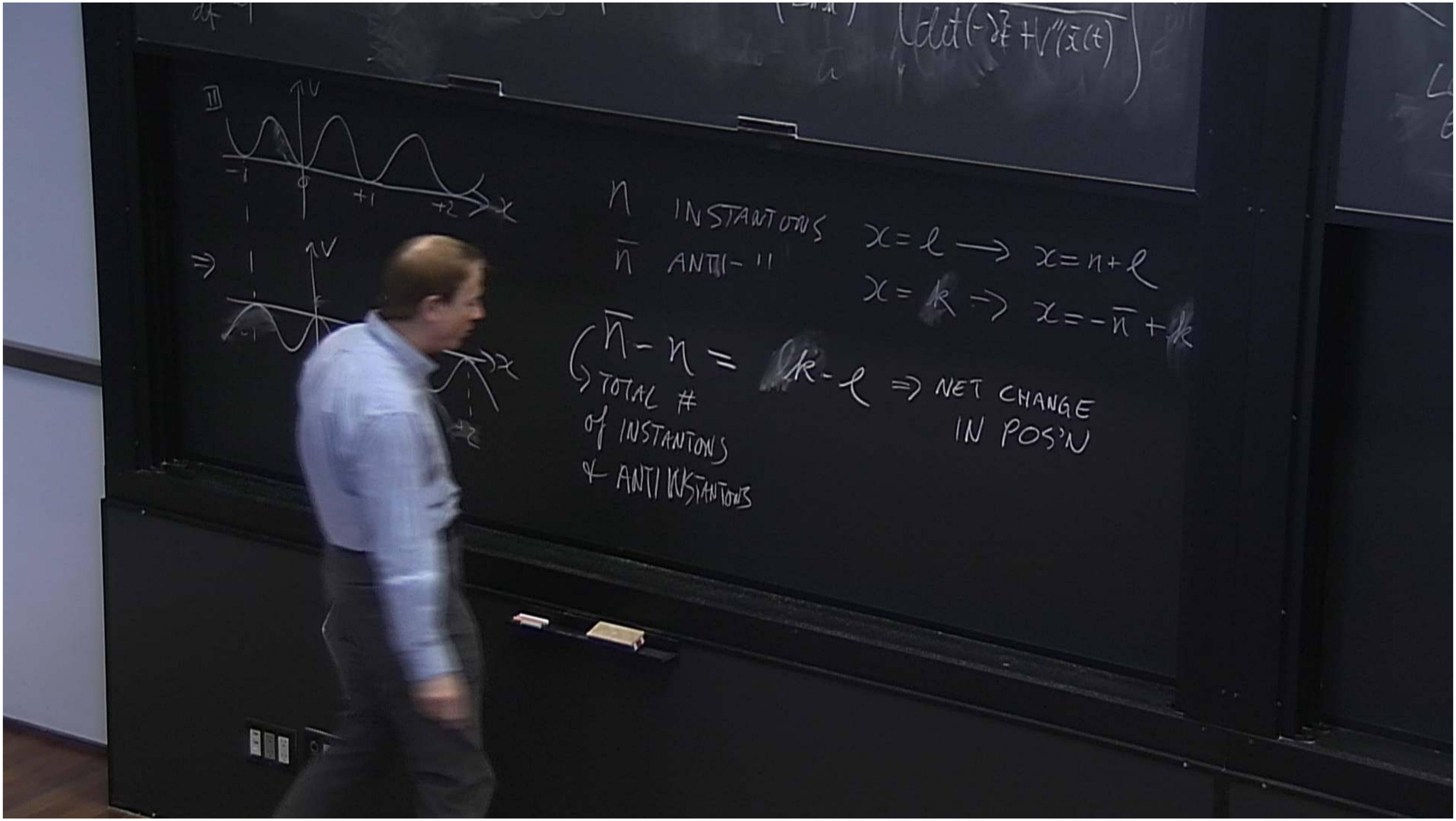


$dt\left(-\partial_t^2+V''(\bar{x}(t)\right)$

n INSTANTONS $\quad x=\ell \longrightarrow x=n+\ell$

$\bar{n}$ ANTI- " $\quad x=k \rightarrow x=-\bar{n}+k$

$\bar{n}-n=k-\ell$



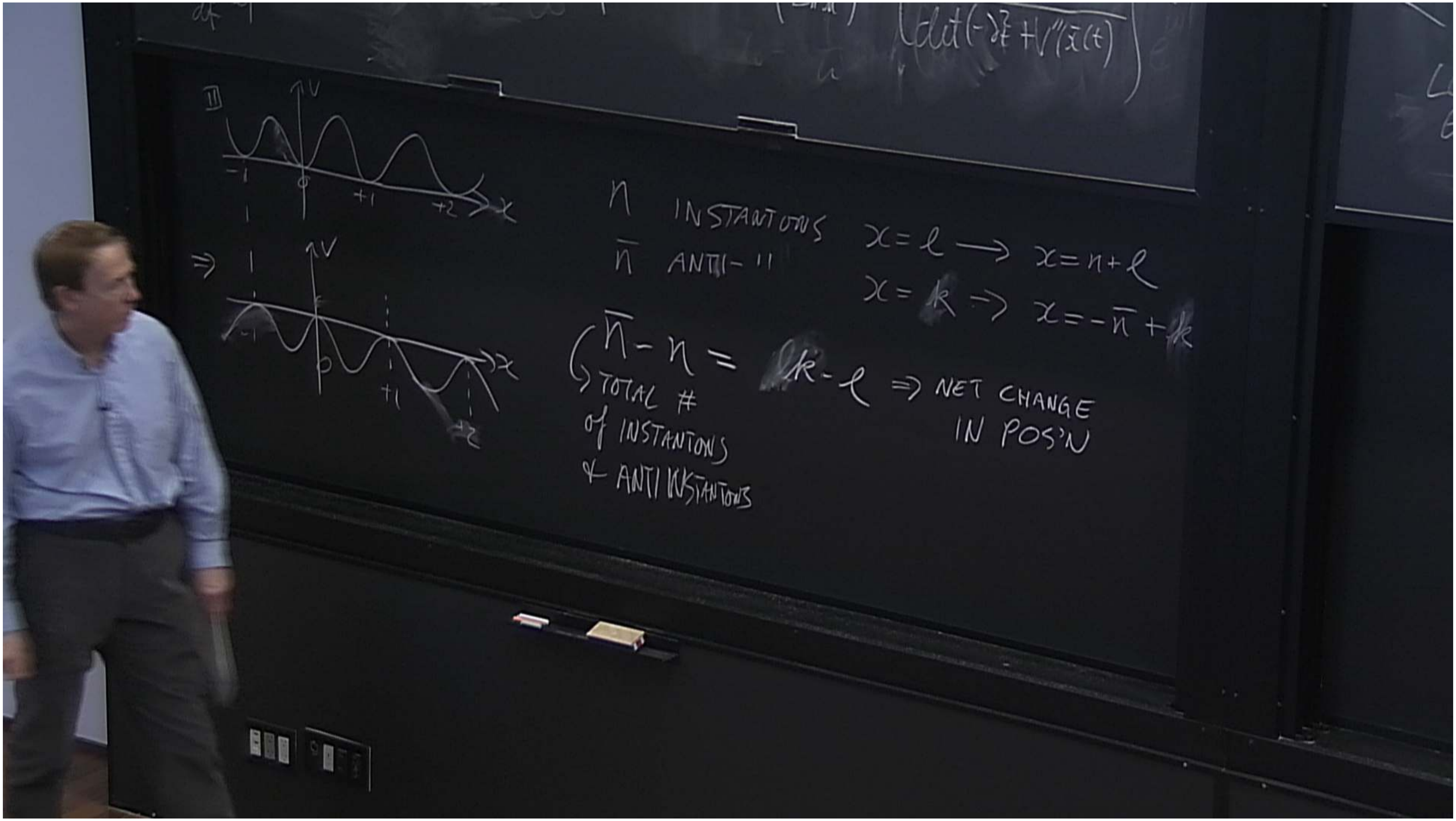
$\det(-\partial_\tau^2 + V''(\bar{x}(t))$

n INSTANTONS $x = \ell \longrightarrow x = n + \ell$

$\bar{n}$ ANTI- '' $x = k \Rightarrow x = -\bar{n} + k$

$\bar{n} - n = k - \ell$ ⇒ NET CHANGE IN POS'N

↳ TOTAL # of INSTANTONS & ANTI INSTANTONS



$\det(-\partial_t^2 + V''(\bar{x}(t))$

n INSTANTONS

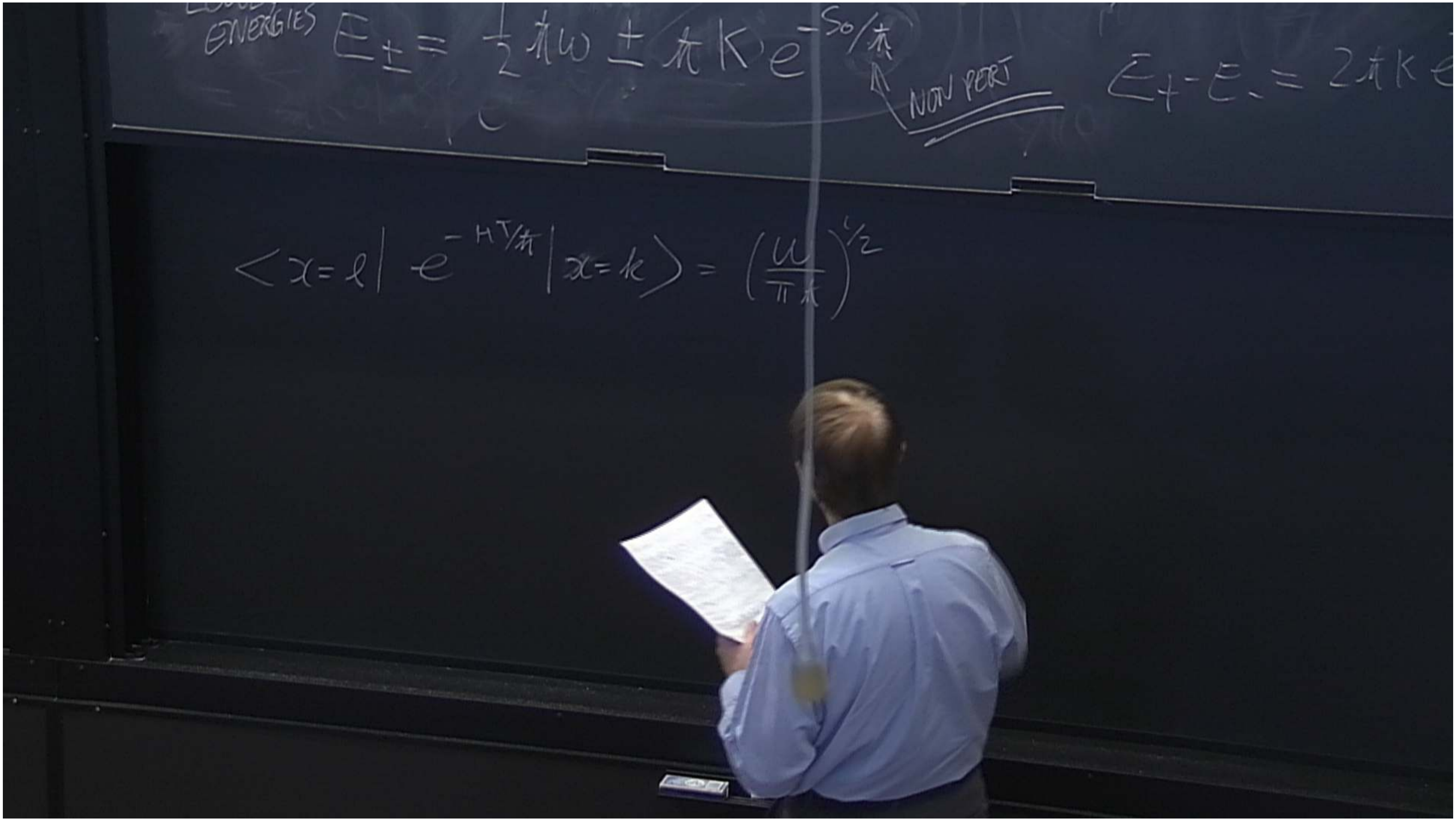
$\bar{n}$ ANTI- ''

$x = l \longrightarrow x = n + l$

$x = k \Rightarrow x = -\bar{n} + k$

$\bar{n} - n = k - l \Rightarrow$ NET CHANGE IN POS'N

↳ TOTAL # of INSTANTONS & ANTIINSTANTONS

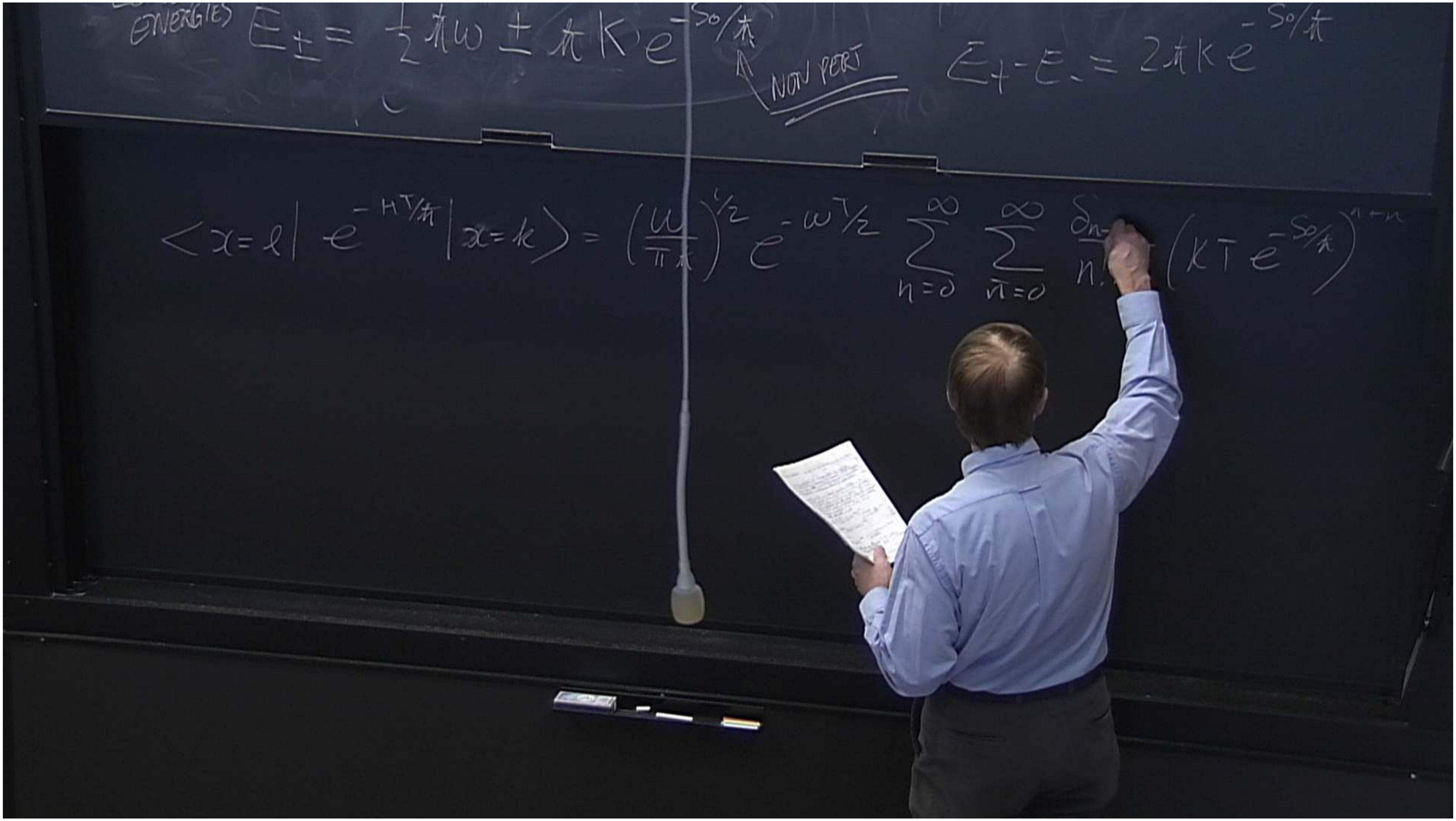


ENERGIES $E_\pm = \frac{1}{2}\hbar\omega \pm \hbar K e^{-S_0/\hbar}$

NON PERT

$E_+ - E_- = 2\hbar K e$

$$\langle x=\ell |\, e^{-HT/\hbar}\, | x=k \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2}$$

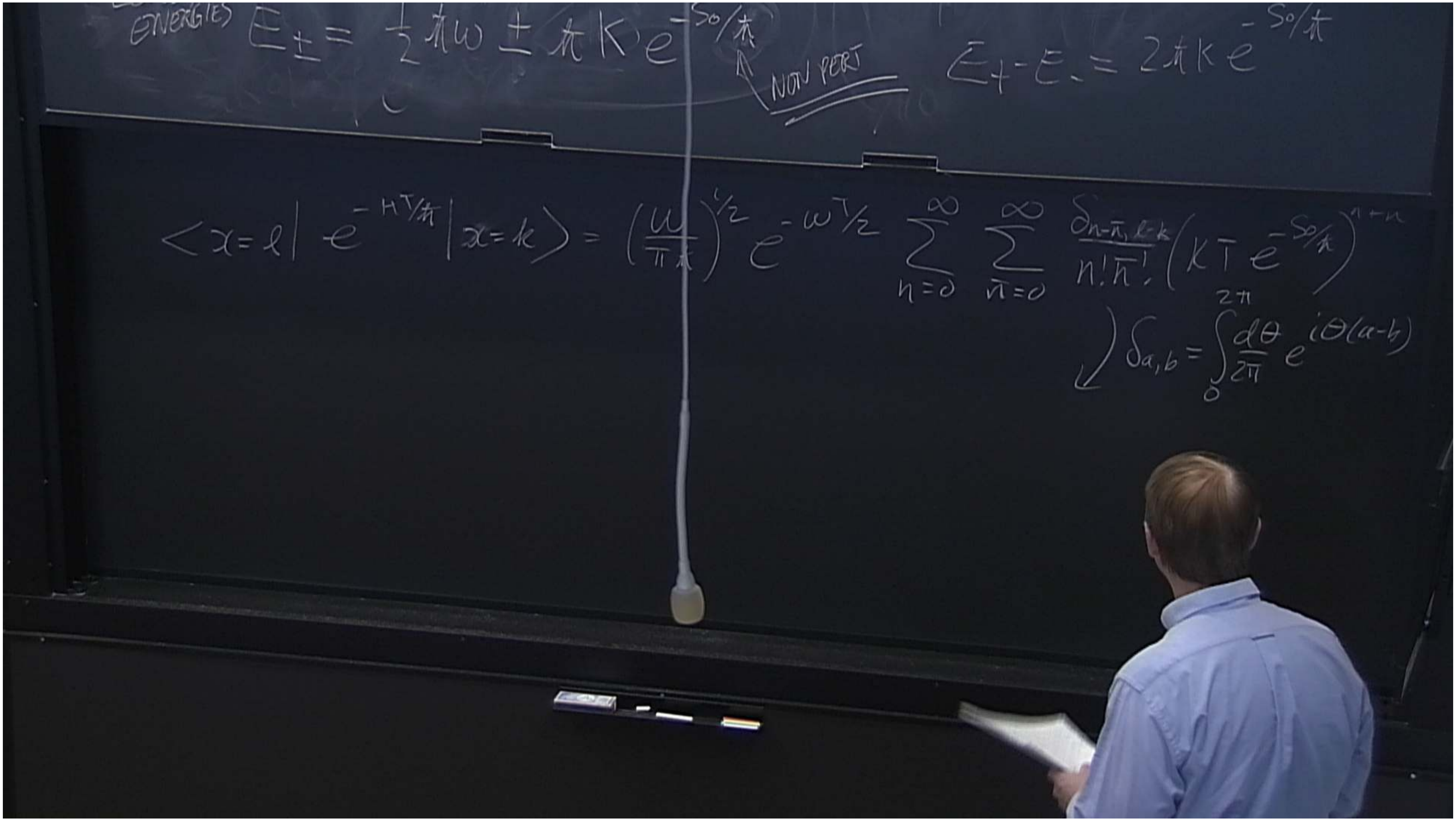


ENERGIES $E_\pm = \frac{1}{2}\hbar\omega \pm \hbar K e^{-S_0/\hbar}$

NON PERT

$E_+ - E_- = 2\hbar K e^{-S_0/\hbar}$

$$\langle x=\ell | e^{-HT/\hbar} | x=k \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sum_{n=0}^{\infty} \sum_{\bar{n}=0}^{\infty} \frac{\delta_{n-}}{n!} \left(K T e^{-S_0/\hbar}\right)^{n+\bar{n}}$$



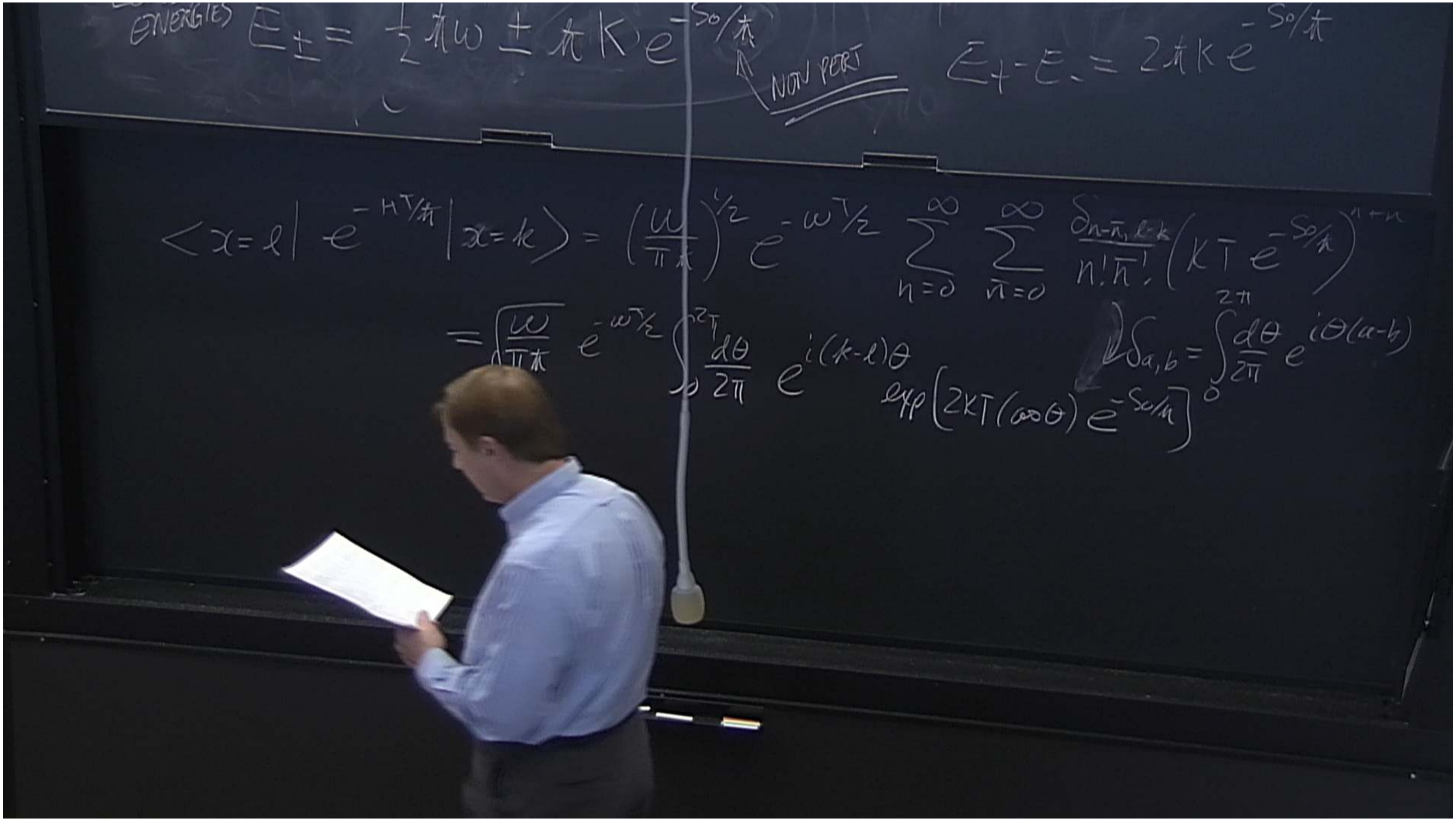
ENERGIES $E_\pm = \frac{1}{2}\hbar\omega \pm \hbar K e^{-S_0/\hbar}$

NON PERT

$E_+ - E_- = 2\hbar K e^{-S_0/\hbar}$

$$\langle x=l |\, e^{-HT/\hbar} \,| x=k \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sum_{n=0}^{\infty} \sum_{\bar n=0}^{\infty} \frac{\delta_{n-\bar n,\, l-k}}{n!\,\bar n!} \left(K T e^{-S_0/\hbar}\right)^{n+\bar n}$$

$$\delta_{a,b} = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{i\theta(a-b)}$$



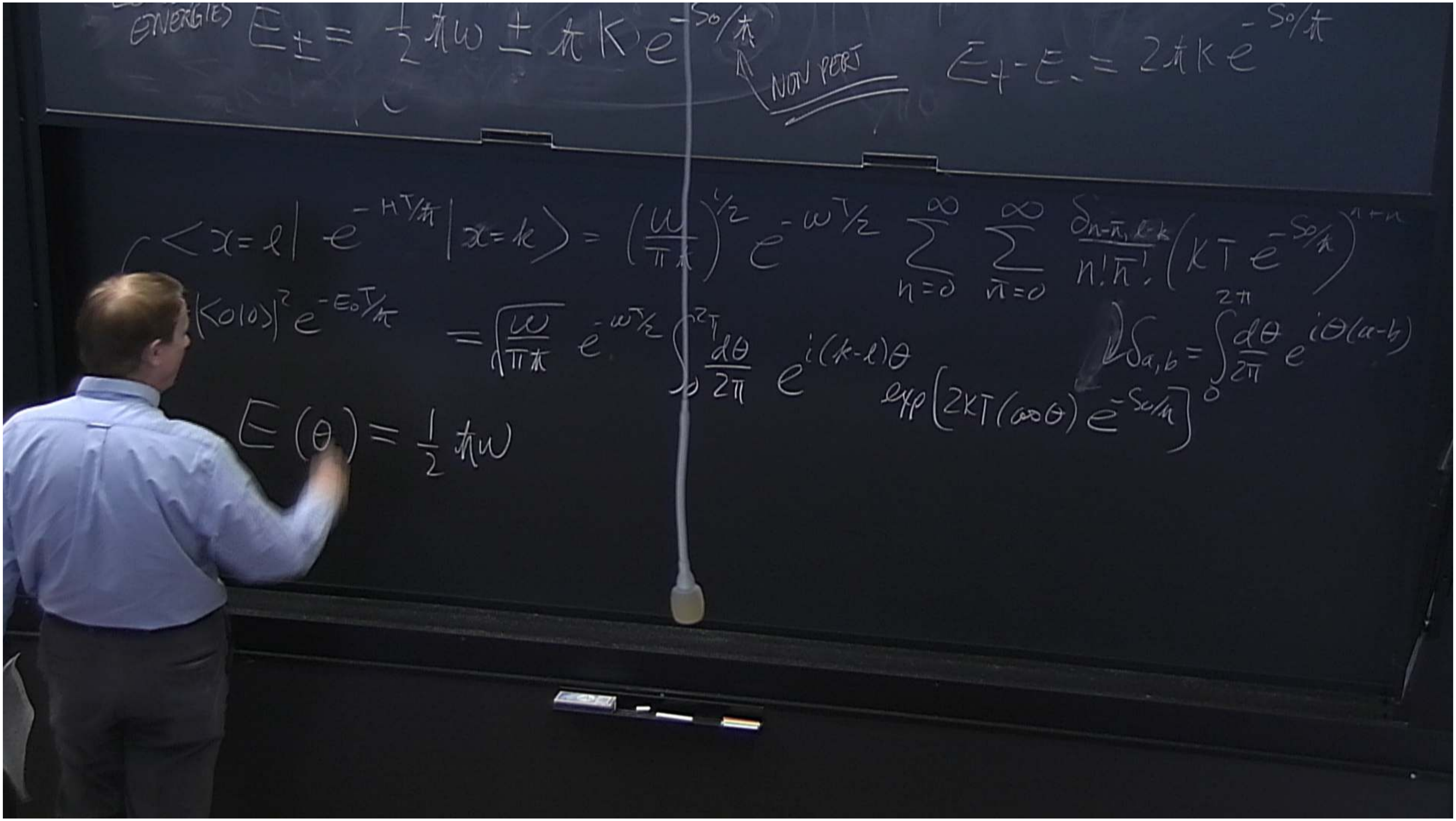
ENERGIES $E_\pm = \frac{1}{2}\hbar\omega \pm \hbar K e^{-S_0/\hbar}$ NON PERT

$$E_+ - E_- = 2\hbar K e^{-S_0/\hbar}$$

$$< x=\ell | e^{-HT/\hbar} | x=k > = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sum_{n=0}^{\infty} \sum_{\bar{n}=0}^{\infty} \frac{\delta_{n-\bar{n},\ell-k}}{n!\,\bar{n}!} \left(KT e^{-S_0/\hbar}\right)^{n+\bar{n}}$$

$$= \sqrt{\frac{\omega}{\pi\hbar}}\, e^{-\omega T/2} \int_0^{2\pi} \frac{d\theta}{2\pi}\, e^{i(k-\ell)\theta} \exp\left[2KT(\cos\theta)\, e^{-S_0/\hbar}\right]$$

$$\delta_{a,b} = \int_0^{2\pi} \frac{d\theta}{2\pi}\, e^{i\theta(a-b)}$$



ENERGIES $E_\pm = \frac{1}{2}\hbar\omega \pm \hbar K e^{-S_0/\hbar}$

NON PERT

$E_+ - E_- = 2\hbar K e^{-S_0/\hbar}$

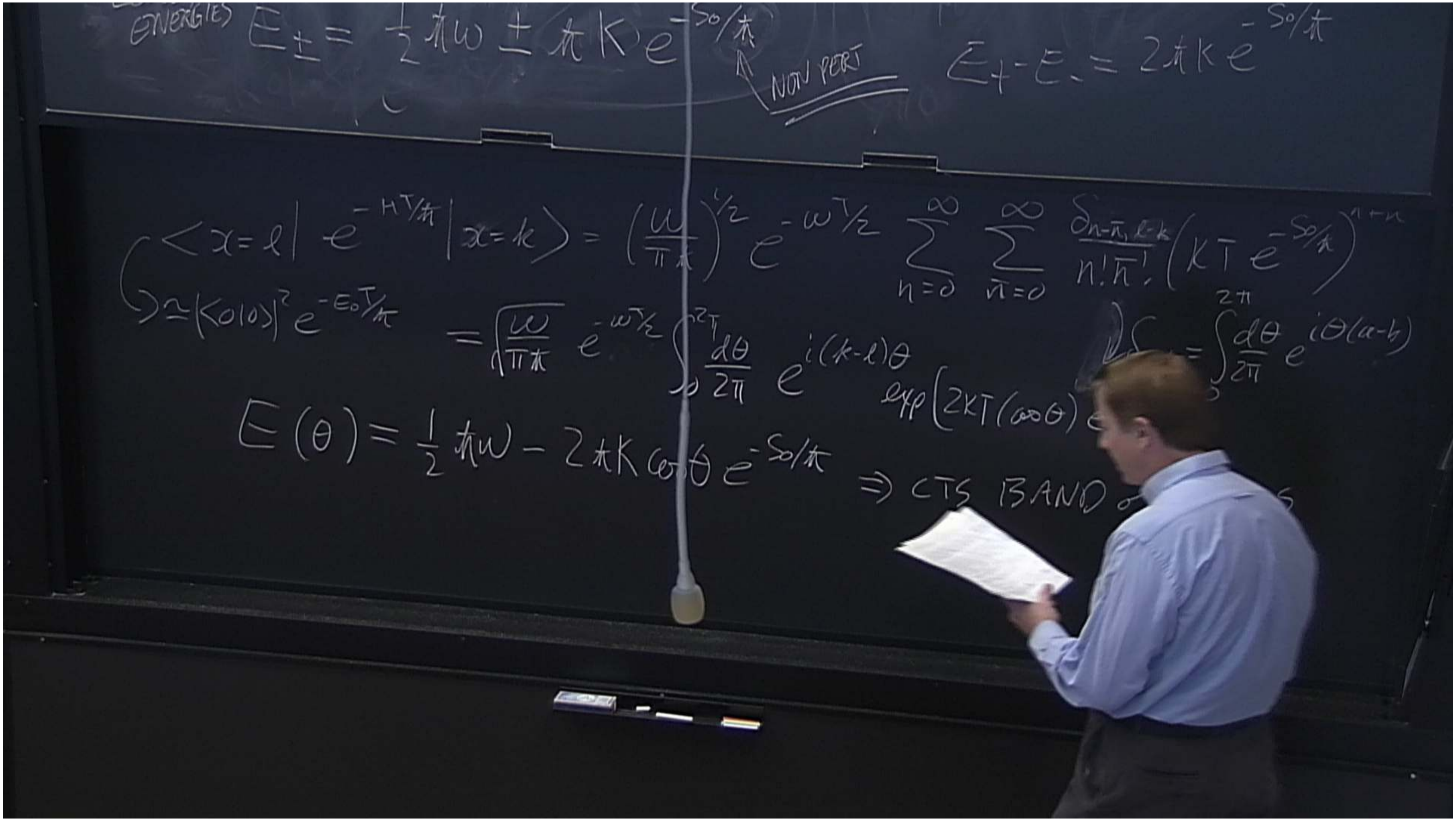
$$\langle x=\ell | e^{-HT/\hbar} | x=k \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sum_{n=0}^{\infty} \sum_{\bar{n}=0}^{\infty} \frac{\delta_{n-\bar{n},\,\ell-k}}{n!\,\bar{n}!} \left(K T e^{-S_0/\hbar}\right)^{n+\bar{n}}$$

$|\langle 0|0\rangle|^2 e^{-E_0 T/\hbar}$

$$= \sqrt{\frac{\omega}{\pi\hbar}}\, e^{-\omega T/2} \int_0^{2\pi} \frac{d\theta}{2\pi}\, e^{i(k-\ell)\theta} \exp\left[2KT(\cos\theta)\, e^{-S_0/\hbar}\right]$$

$$\delta_{a,b} = \int_0^{2\pi} \frac{d\theta}{2\pi}\, e^{i\theta(a-b)}$$

$$E(\theta) = \frac{1}{2}\hbar\omega$$



ENERGIES $E_\pm = \frac{1}{2}\hbar\omega \pm \hbar K e^{-S_0/\hbar}$

NON PERT

$E_+ - E_- = 2\hbar K e^{-S_0/\hbar}$

$$\langle x=\ell|\, e^{-HT/\hbar}\, |x=k\rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} \sum_{n=0}^{\infty} \sum_{\bar{n}=0}^{\infty} \frac{\delta_{n-\bar{n},\, \ell-k}}{n!\,\bar{n}!} \left(KT e^{-S_0/\hbar}\right)^{n+n}$$

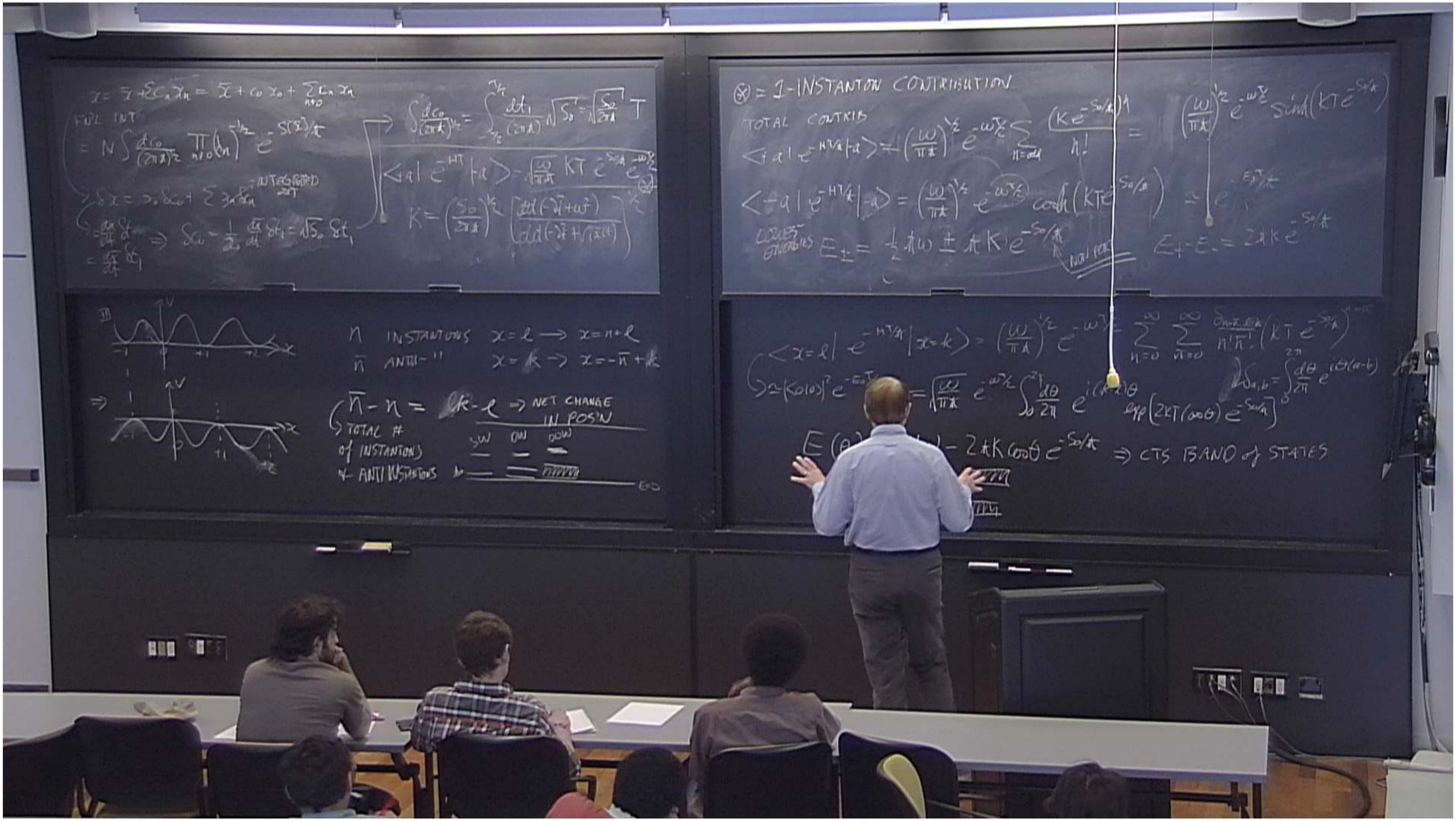
$$\simeq |\langle 0|0\rangle|^2 e^{-E_0 T/\hbar}$$

$$= \sqrt{\frac{\omega}{\pi\hbar}}\, e^{-\omega T/2} \int_0^{2\pi} \frac{d\theta}{2\pi}\, e^{i(k-\ell)\theta} \exp\left(2KT(\cos\theta)\, e\right.$$

$$\delta = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{i\theta(a-b)}$$

$$E(\theta) = \frac{1}{2}\hbar\omega - 2\hbar K \cos\theta\, e^{-S_0/\hbar}$$

$\Rightarrow$ CTS BAND o s



$x = \bar{x} + \sum c_n x_n = \bar{x} + c_0 x_0 + \sum_{n\neq 0} c_n x_n$

FN'L INT

$= N \int \frac{dc_0}{(2\pi\hbar)^{1/2}} \prod_{n\neq 0} (\lambda_n)^{-1/2} e^{-S(\bar{x})/\hbar}$

INTEGRATED

$\delta x = x_0 \delta c_0 + \sum x_n \delta c_n$

$= \frac{d\bar{x}}{dt}\delta t = \frac{d\bar{x}}{dt}\delta t_1 \Rightarrow \delta c_0 = \frac{1}{x_0}\frac{d\bar{x}}{dt}\delta t_1 = \sqrt{S_0}\,\delta t_1$

$\int \frac{dc_0}{(2\pi\hbar)^{1/2}} = \int_{-T/2}^{T/2} \frac{dt_1}{(2\pi\hbar)}\sqrt{S_0} = \sqrt{\frac{S_0}{2\pi\hbar}}\,T$

$\langle a| e^{-HT}|-a\rangle = \sqrt{\frac{\omega}{\pi\hbar}}\, KT\, e^{-S_0/\hbar}\, e^{-\omega T/2}$ ⊗

$K = \left(\frac{S_0}{2\pi\hbar}\right)^{1/2}\left[\frac{\det(-\partial_t^2+\omega^2)}{\det(-\partial_t^2+V''(\bar{x}(t)))}\right]^{1/2}$

⊗ = 1-INSTANTON CONTRIBUTION

TOTAL CONTRIB

$\langle + a| e^{-HT/\hbar}|-a\rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2}\sum_{n=\text{odd}} \frac{(K e^{-S_0/\hbar})^n}{n!} = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2}\sinh(KTe^{-S_0/\hbar})$

$\langle -a|e^{-HT/\hbar}|-a\rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2}\cosh(KTe^{-S_0/\hbar}) \simeq e^{-E_\pm T/\hbar}$

LOWEST ENERGIES $E_\pm = \frac{1}{2}\hbar\omega \pm \hbar K e^{-S_0/\hbar}$

NON PERT

$E_+ - E_- = 2\hbar K e^{-S_0/\hbar}$

III

n INSTANTONS $x = \ell \rightarrow x = n + \ell$

$\bar{n}$ ANTI- " $x = k \rightarrow x = -\bar{n} + k$

$\bar{n} - n = k - \ell \Rightarrow$ NET CHANGE IN POS'N

TOTAL # of INSTANTONS + ANTI INSTANTONS

SW OW OOW

E=0

$\langle x=\ell| e^{-HT/\hbar}|x=k\rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2}\sum_{n=0}^{\infty}\sum_{\bar{n}=0}^{\infty} \frac{\delta_{n-\bar{n},\ell-k}}{n!\,\bar{n}!}\left(KTe^{-S_0/\hbar}\right)^{n+\bar{n}}$

$\simeq |\langle 0|0\rangle|^2 e^{-E_0 T}$ $= \sqrt{\frac{\omega}{\pi\hbar}}\, e^{-\omega T/2}\int_0^{2\pi}\frac{d\theta}{2\pi}\, e^{i(k-\ell)\theta}\exp\left[2KT(\cos\theta)e^{-S_0/\hbar}\right]$

$\delta_{a,b} = \int_0^{2\pi}\frac{d\theta}{2\pi}\, e^{i\theta(a-b)}$

$E(\theta)$ $- 2\hbar K\cos\theta\, e^{-S_0/\hbar}$ ⇒ CTS BAND of STATES