Title: 13/14 PSI - Beyond the Standard Model - Lecture 9

Date: Mar 03, 2014 09:00 AM

URL: http://pirsa.org/14030010

Abstract:

$t \in [0,1]$

$\Omega_t = d\omega_t + \omega_t^2$

$\omega_t = \omega_0 + t(\omega_1 - \omega_0)$

$\frac{\partial}{\partial t} P_m(\Omega_t) = m\, d\Big[\underbrace{\mathrm{STr}\big[(\omega_1-\omega_0)\Omega_t^{m-1}\big]}_{Q_{2m-1}(\omega,\Omega_t)}\Big]$

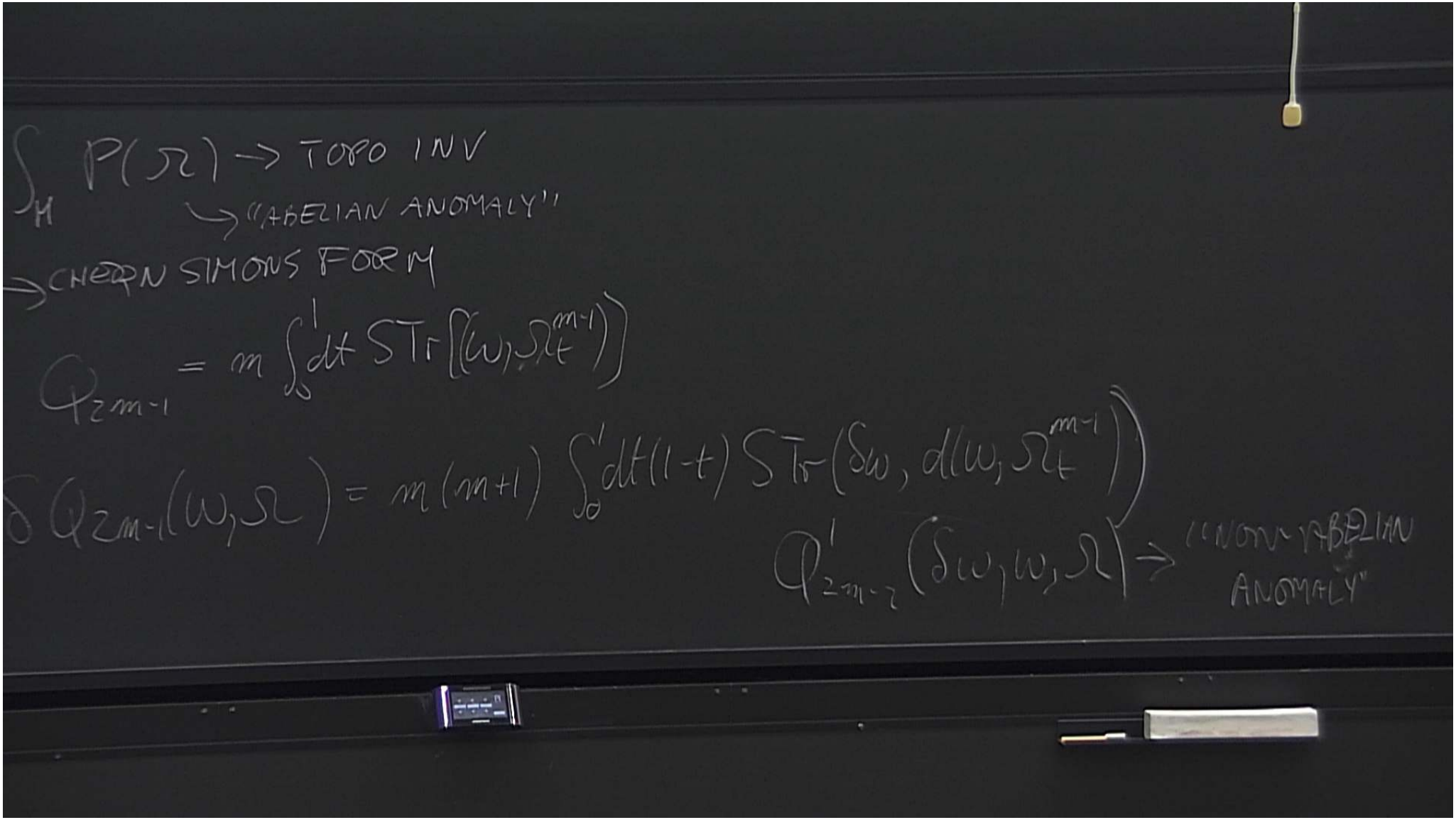
$\int_M P(\Omega_1) = \int_M P(\Omega_0)$

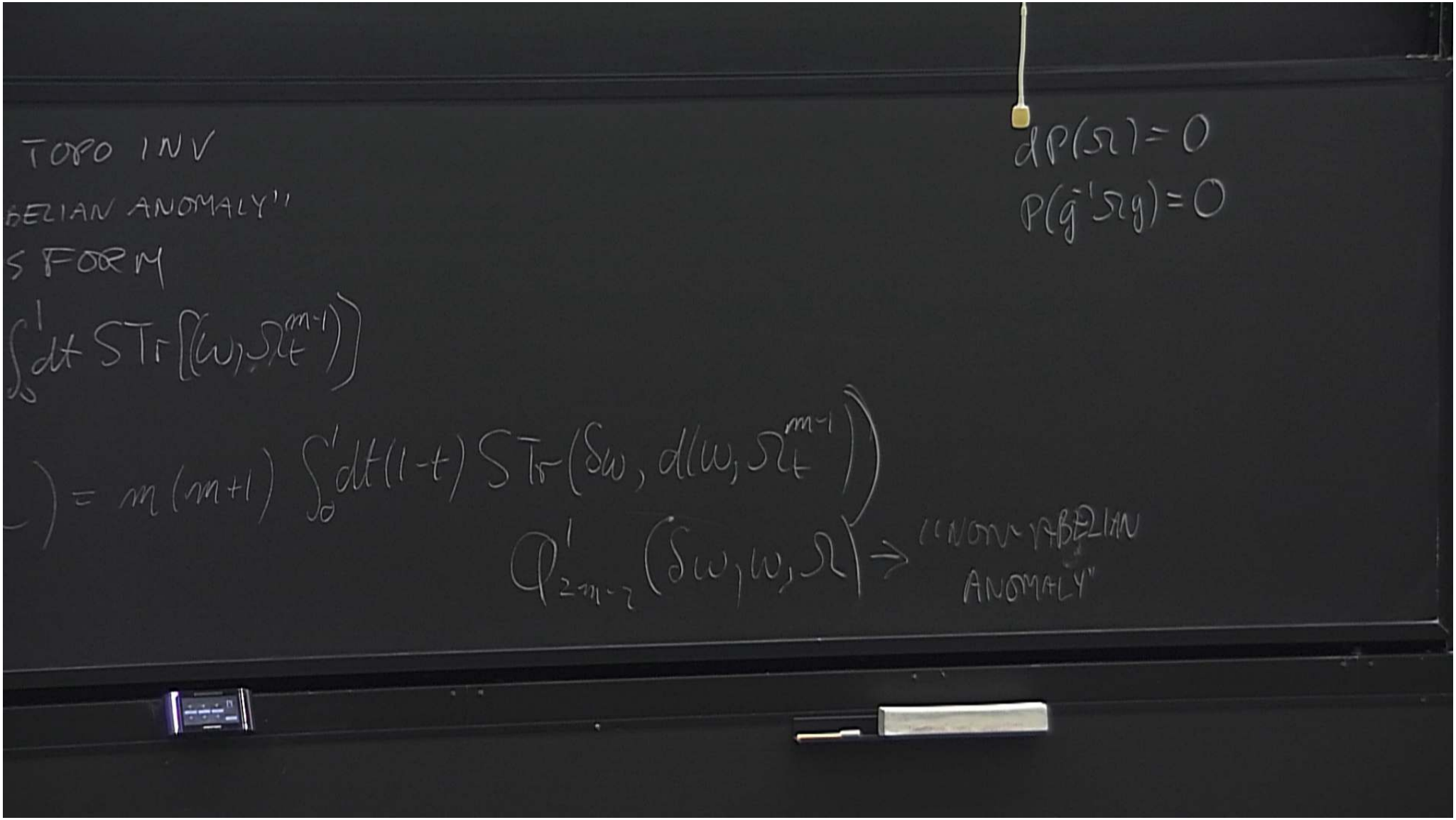
$\int_M P(\Omega) \to$ TOPO INV

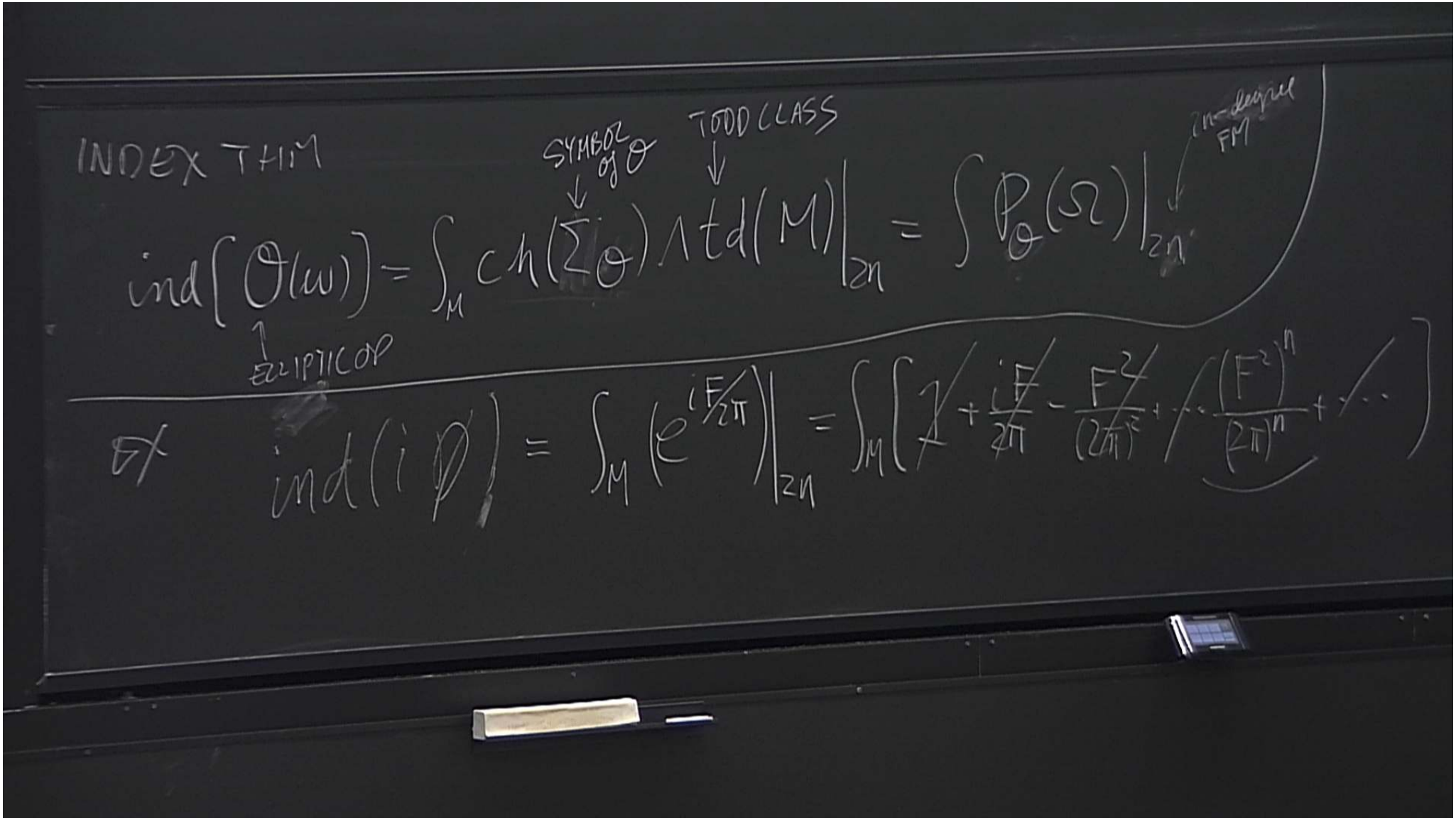
$\to$ CHERN SIMONS FORM

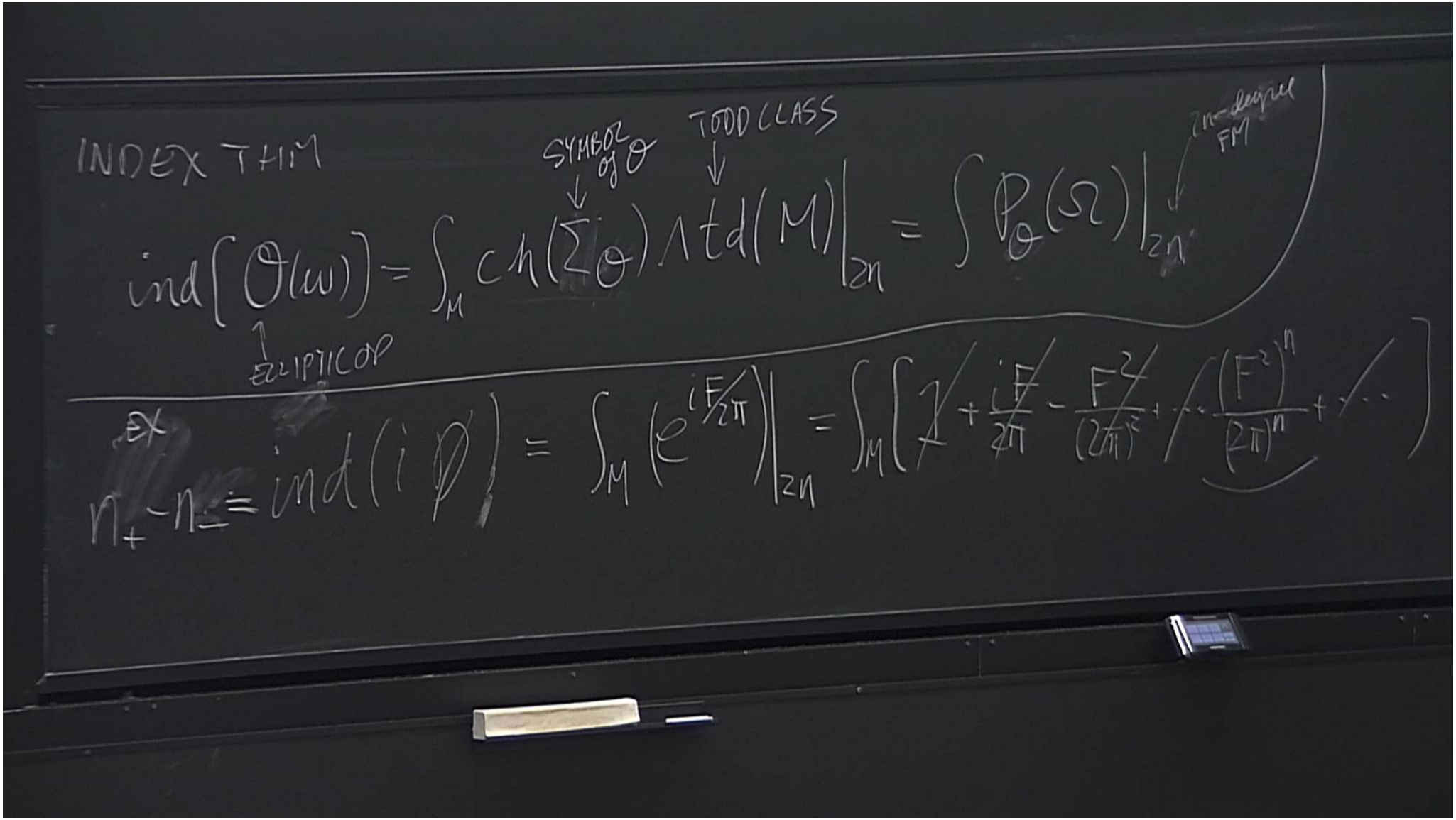
$Q_{2m-1} = m \int_0^1 dt\, \mathrm{STr}\big[(\omega,\Omega_t^{m-1})\big]$

$\delta Q_{2m-1}(\omega,\Omega) = m(m+1) \int_0^1 dt($


$\int_M P(\Omega) \to$ TOPO INV
$\to$ "ABELIAN ANOMALY"
$\to$ CHERN SIMONS FORM
$Q_{2m-1} = m \int_0^1 dt\, STr[(\omega, \Omega_t^{m-1})]$
$\delta Q_{2m-1}(\omega, \Omega) = m(m+1) \int_0^1 dt(1-t)\, STr(\delta\omega, d(\omega, \Omega_t^{m-1}))$
$Q'_{2m-2}(\delta\omega, \omega, \Omega) \to$ "NON-ABELIAN ANOMALY"


TOPO INV
BELIAN ANOMALY"
S FORM
$\int_0^1 dt\, STr[(\omega, \Omega_t^{m-1})]$
$dP(\Omega) = 0$
$P(g^{-1}\Omega g) = 0$
$= m(m+1) \int_0^1 dt(1-t)\, STr(\delta\omega, d(\omega, \Omega_t^{m-1}))$
$Q'_{2m-2}(\delta\omega, \omega, \Omega) \rightarrow$ "NON-ABELIAN ANOMALY"


INDEX THM
SYMBOL of $\mathcal{O}$
TODD CLASS
2n-dimensional FM
$\mathrm{ind}[\mathcal{O}(w)] = \int_M ch(\Sigma_{\mathcal{O}}) \wedge td(M)\big|_{2n} = \int P_{\mathcal{O}}(\Omega)\big|_{2n}$
1
ELLIPTIC OP
Ex
$\mathrm{ind}(iD) = \int_M \left(e^{(F/2\pi)}\right)\big|_{2n} = \int_M \left(1 + \frac{iF}{2\pi} - \frac{F^2}{(2\pi)^2} + \cdots + \frac{(F^2)^n}{(2\pi)^n} + \cdots\right)$

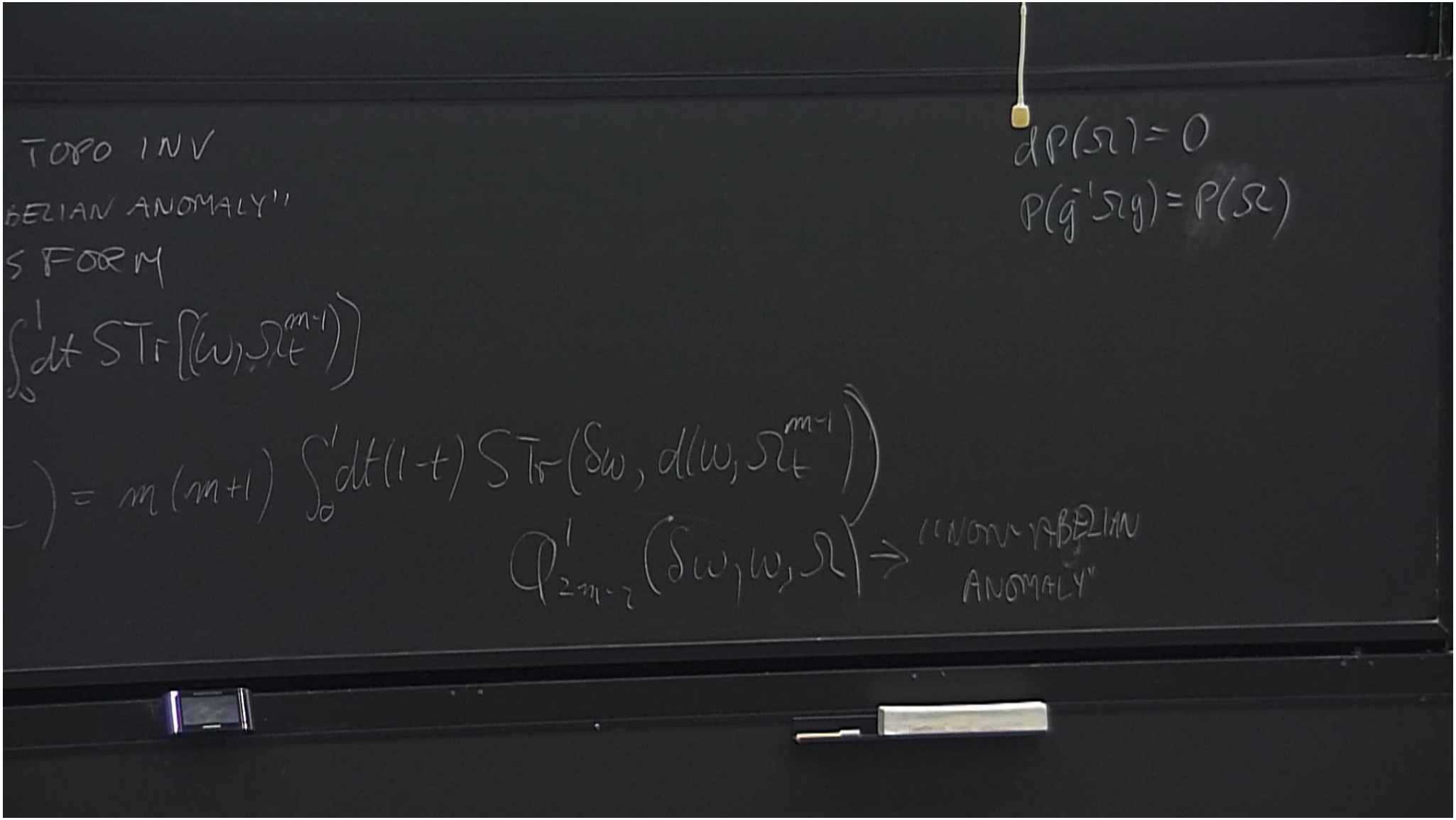

INDEX THM
SYMBOL of O
TODD CLASS
n-dimensional FM
ELLIPTIC OP
ex

RH or LH ALONE $i\not{D}_{1/2} = i\not{D}(1 \pm \gamma_5)$

$i\not{D}_{1/2}$: RH $\rightarrow$ LH

DEFINE $\mathrm{ind}(i\not{D}_{1/2}) = [(\#\text{ of 0-modes of } i\not{D}_{1/2}) - (\#\text{ of 0-modes of } (i\not{D}_{1/2})^{+})]$

$= \int_M \left(e^{iF/2\pi} \right)\Big|_{2n}$


TOPO INV
BELIAN ANOMALY"
S FORM
$\int_0^1 dt\, STr[(\omega, \Omega_t^{m-1})]$
$= m(m+1) \int_0^1 dt(1-t)\, STr(\delta\omega, d(\omega, \Omega_t^{m-1}))$
$Q'_{2m-2}(\delta\omega, \omega, \Omega)$ → "NON-ABELIAN ANOMALY"
$dP(\Omega) = 0$
$P(g^{-1}\Omega g) = P(\Omega)$

$$ind(i\not{D}_{\gamma_2}) = \int_M [\hat{A}(M)\, ch(F)]_{2n} \qquad \hat{A}(M) = \prod_a \frac{x_a/2}{\sinh(x_a/2)}$$

$$Td(M) = \prod_a \frac{x_a}{1-e^{-x_a}}$$

x_a's: $\rightarrow$ SKEW EVALUES of $R^a{}_b$ = CURVATURE

$$\begin{pmatrix} \begin{matrix} 0 & x_1 \\ -x_1 & 0 \end{matrix} & & \\ & \begin{matrix} 0 & x_2 \\ -x_2 & 0 \end{matrix} & \\ & & \ddots \end{pmatrix}$$

$$ch(\Sigma_{i\mathbb{D}_{\gamma_2}}) = \left(\prod_a e^{-x_a/2}\right) ch(F)$$

$$ch(\Sigma_{i\mathbb{D}_{\gamma_2}}) \wedge Td(M) = \prod_a \frac{x_a/2}{\sinh(x_a/2)} F$$

$$\prod_a \frac{x_a/2}{\sinh(x_a/2)} = 1 + \frac{1}{12}\frac{\mathrm{Tr}\, R^2}{(4\pi)^2} + \frac{1}{(4\pi)^4}\left[(\mathrm{Tr}\, R^2)^2 + \frac{1}{360}\mathrm{Tr}\, R^4\right] + \cdots$$

KEN EVALUES

$$= \begin{pmatrix} \begin{matrix} 0 & x_1 \\ -x_1 & 0 \end{matrix} & & \\ & \begin{matrix} 0 & x_2 \\ -x_2 & 0 \end{matrix} & \\ & & \ddots \end{pmatrix}$$

RE

RARITA-SCHWINGER $\rightarrow$ SPIN $3/2$

$$\mathrm{ind}(i\not{D}_{3/2}) = \int_M \hat{A}(M)\left[\mathrm{Tr}\left(e^{iR/2\pi}\right) - 1\right](\mathrm{ch}F)\Big]_{2n}$$

$$\hookrightarrow = 2\sum_a \cosh(x_a)$$

EX : ANTISYM TENSOR FIELD

$$A = A_{\mu_1 \cdots \mu_{n-1}} dx^{\mu_1} \cdots dx^{\mu_{n-1}}$$

SELF-DUAL FIELD STRENGTH

$$dA \underset{\text{DEF'N}}{=} F \overset{\downarrow}{=} \pm * F$$

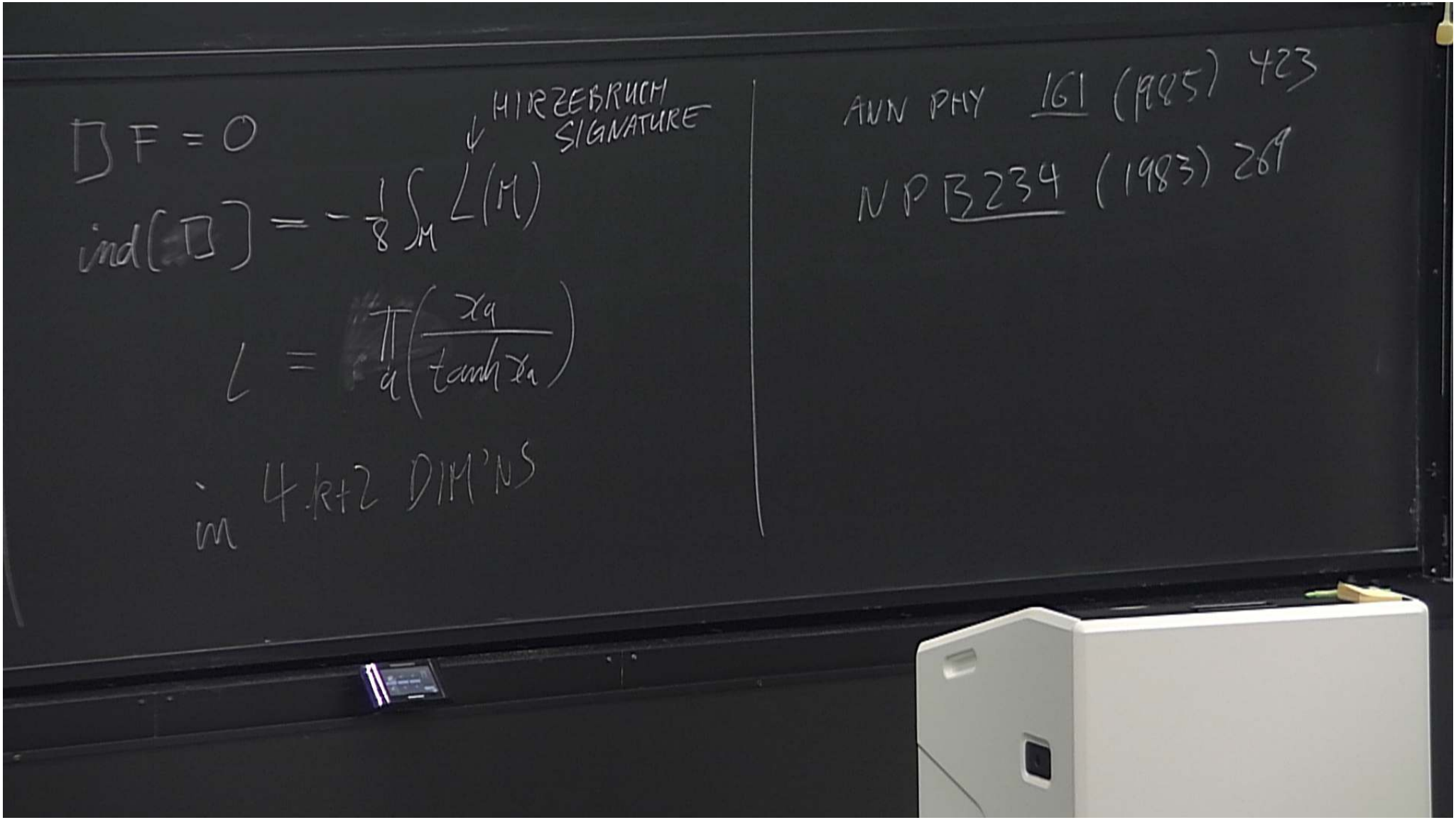
$$F_{\mu_1 \cdots \mu_n} = \frac{1}{n!} \epsilon_{\mu_1 \cdots \mu_n}{}^{\nu_1 \cdots \nu_n} F_{\nu_1 \cdots \nu_n} (\pm 1)$$

SELF DUAL

ANTI SELF DUAL

$\square$

inc


$\not{D} F = 0$
HIRZEBRUCH SIGNATURE
$\text{ind}(\not{D}) = -\frac{1}{8}\int_M L(M)$
$L = \prod_a \left(\frac{x_a}{\tanh x_a}\right)$
$\therefore$ in $4k+2$ DIM'NS
ANN PHY 161 (1985) 423
NPB234 (1983) 269

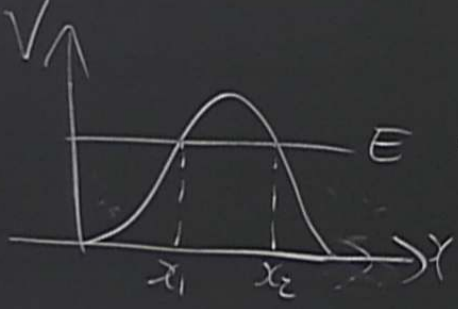
$$ch(\Sigma_{iD_{1/2}}) = \left(\prod_a e^{\ldots}\right) ch(F)$$

$$ch(\Sigma_{iD_{1/2}}) \wedge Td(M) = \prod_a \frac{x_a/2}{\sinh(x_a/2)} F$$

CURVATURE

INSTANTONS

→ TUNNELING



→ AMPLITUDE for TUNNELING $= A = e^{-\frac{1}{\hbar}\int_{x_1}^{x_2} dx \sqrt{2(V-E)}} \left(1 + O(\hbar)\right)$ WKB

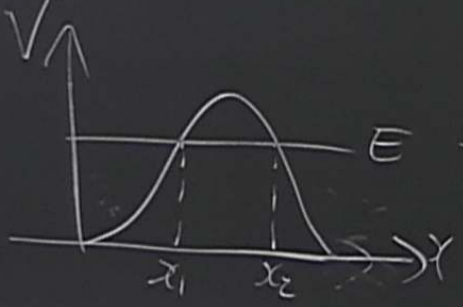
$$ch(\Sigma_{iD_{1/2}}) = \left(\prod_a e^{\ldots}\right) ch(F)$$

$$ch(\Sigma_{iD_{1/2}}) \wedge Td(M) = \prod_a \frac{x_a/2}{\sinh(x_a/2)} F$$

CURVATURE

INSTANTONS

→ TUNNELING



→ AMPLITUDE for TUNNELING $= A = e^{-\frac{1}{\hbar}\int_{x_i}^{x_f} dx \sqrt{2(V-E)}} \left(1 + O(\hbar)\right)$ WKB

CONSIDER

$$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N \int [dx]\, e^{-S/\hbar}$$

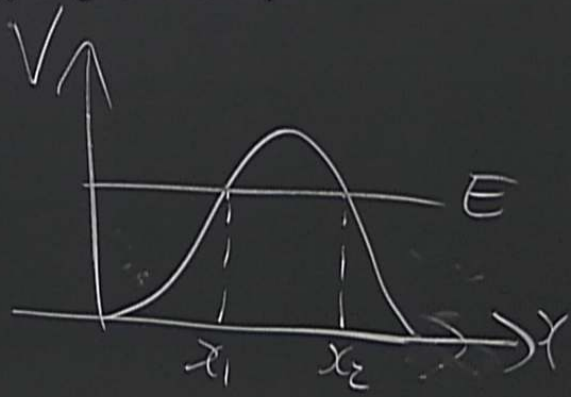
in EUC SPACE

$$\mathrm{Ch}(\Sigma_{iD_{1/2}})\,\mathrm{Td}(M) = \prod_a \frac{x_a/2}{\sinh(x_a/2)}\, F$$

INSTANTONS

→ TUNNELING

$$L = \frac{1}{2} m \dot{x}^2 - V$$

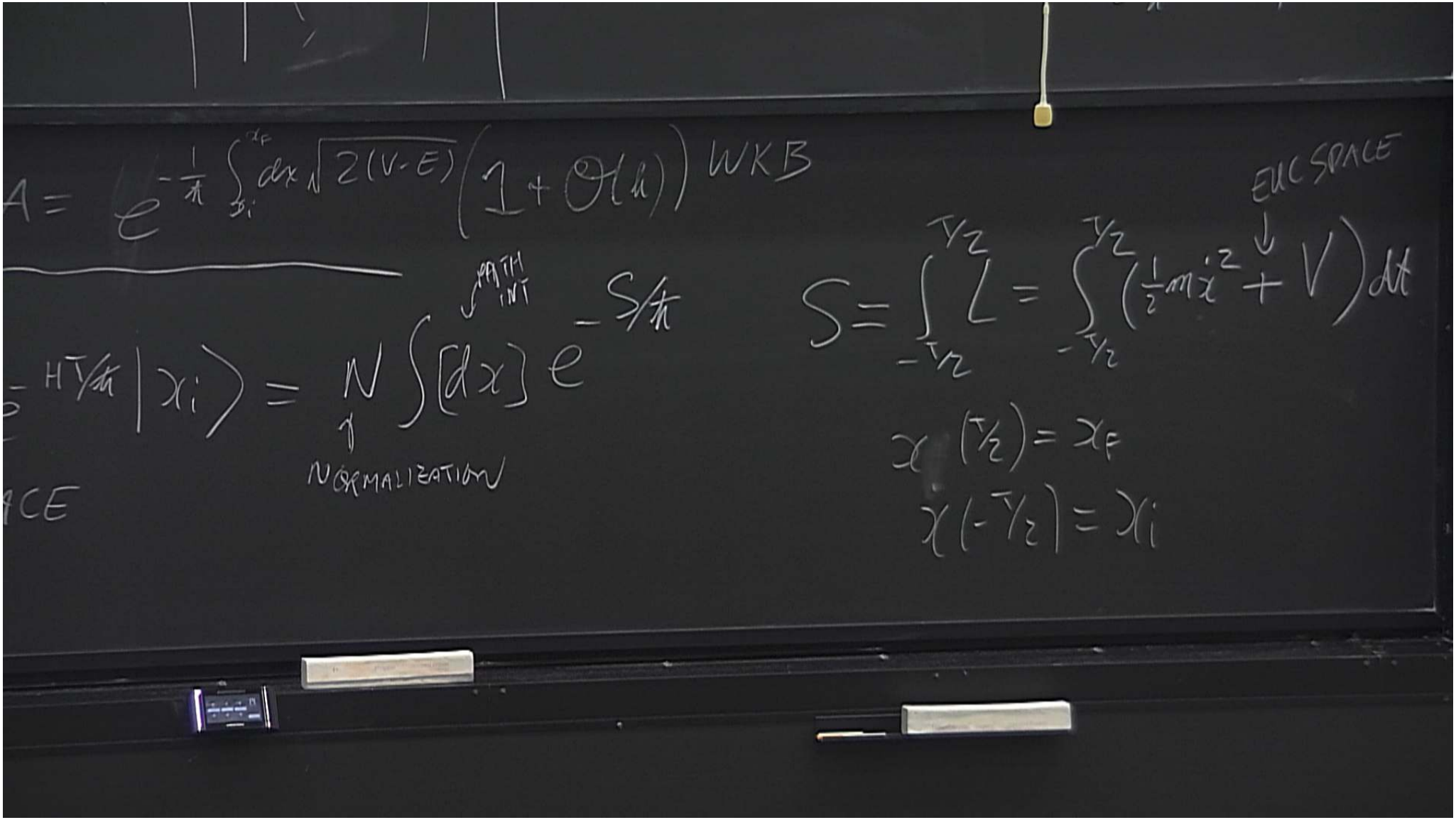


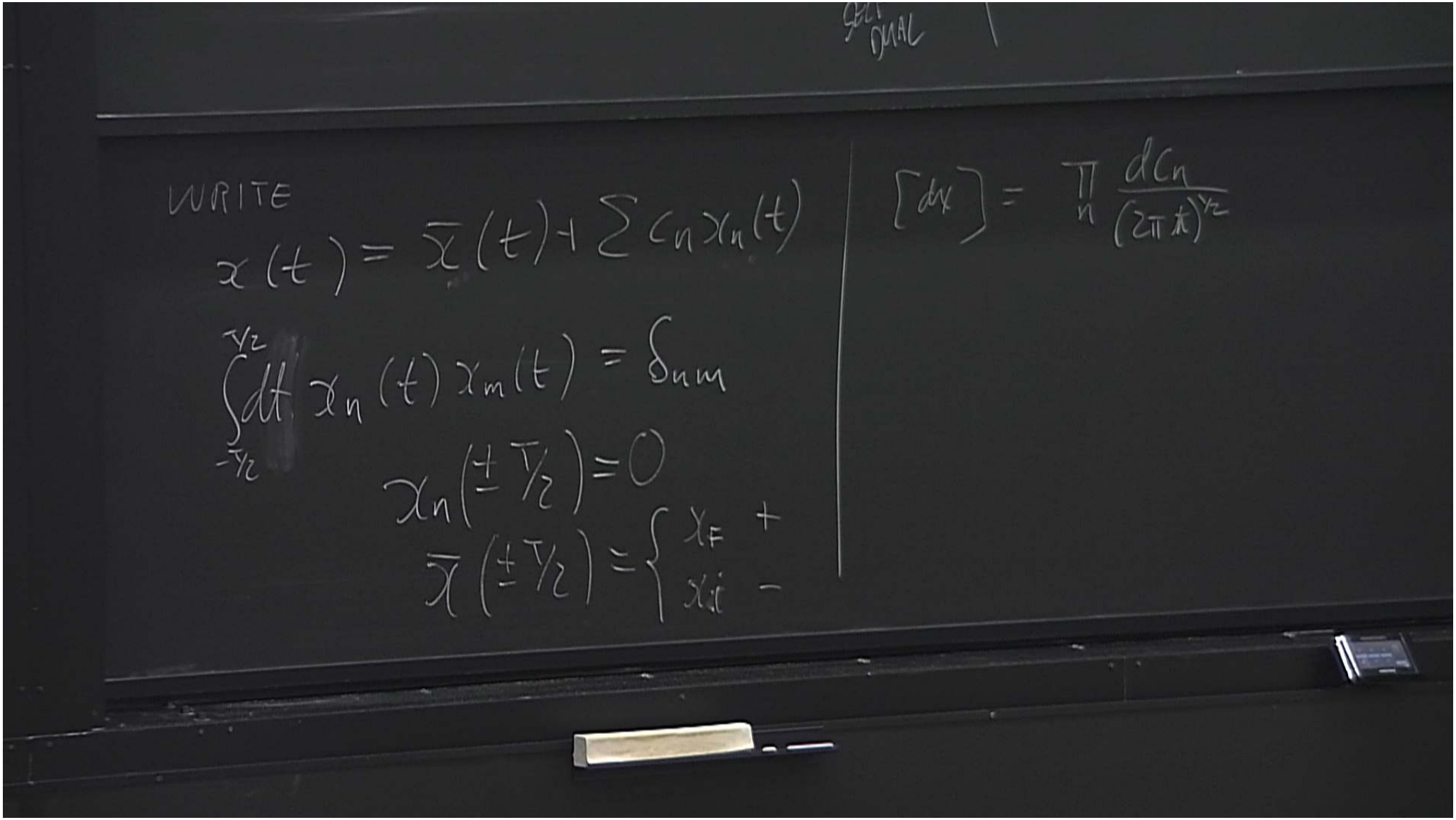
→ AMPLITUDE for TUNNELING $= A = e^{-\frac{1}{\hbar}\int_{x_i}^{x_f} dx\, \ldots}$

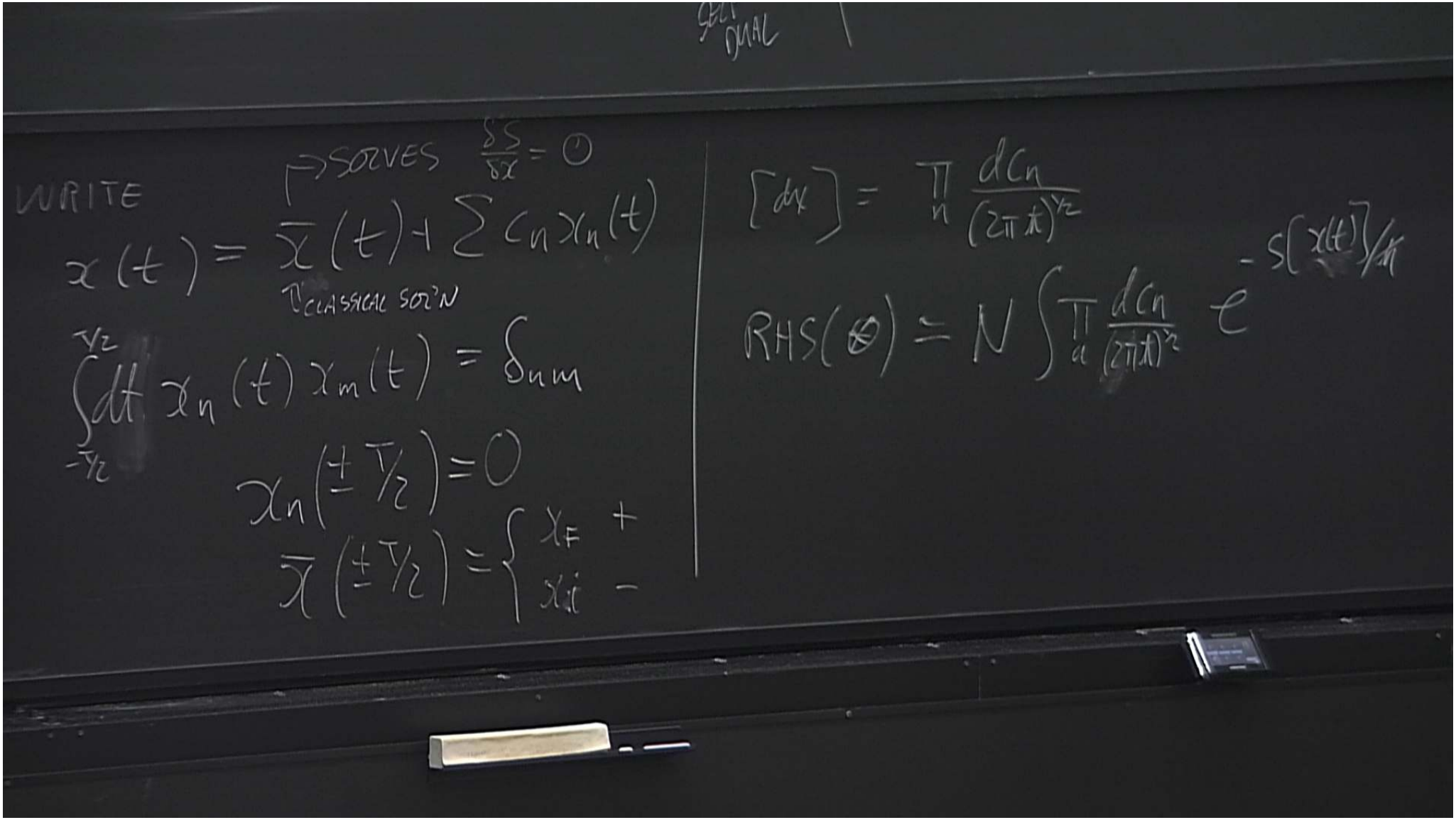
CONSIDER

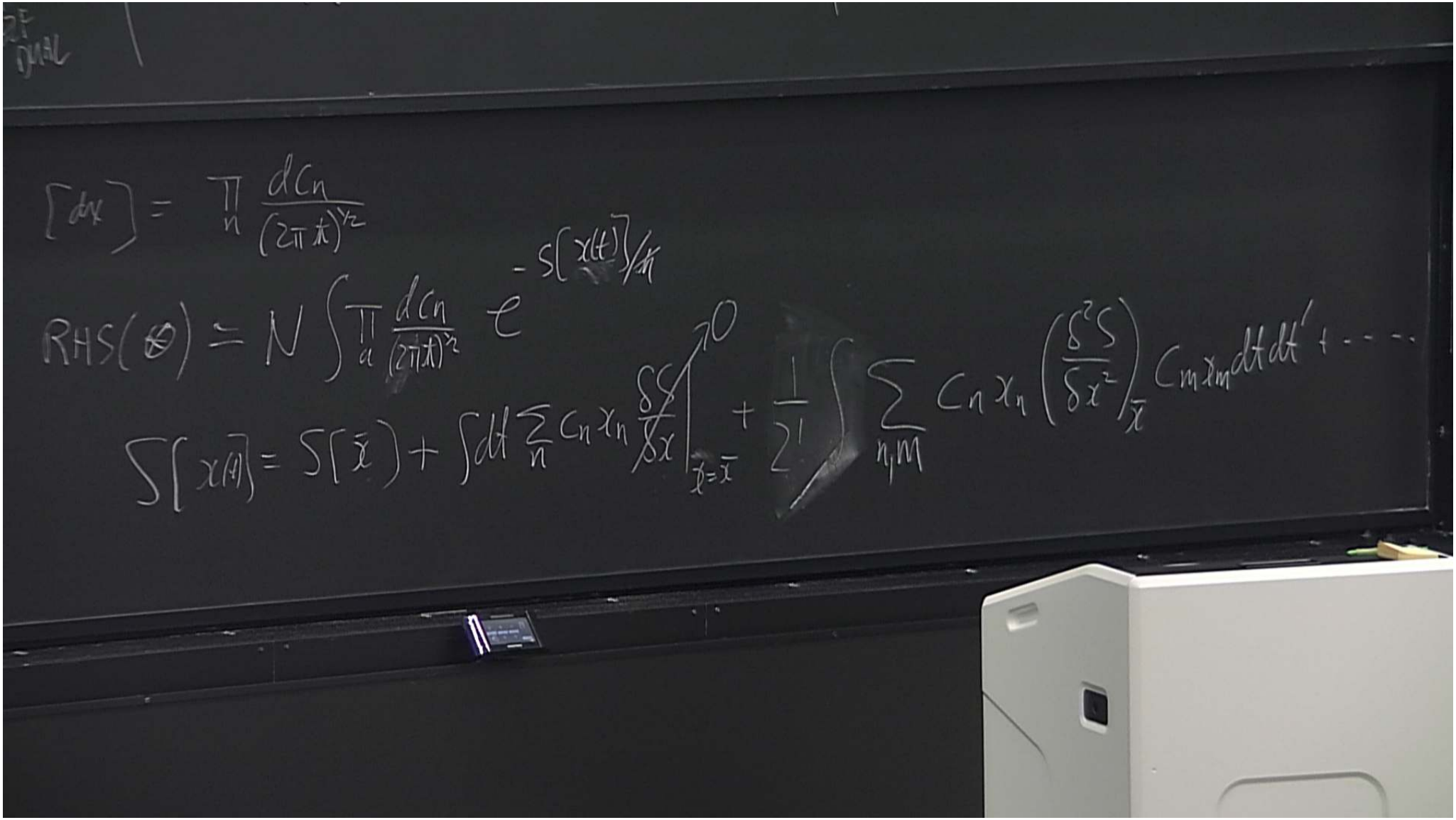
$$\langle x_f | e^{-HT/\hbar} | x_i \rangle =$$

in EUC SPACE


$A = e^{-\frac{1}{\hbar}\int_{x_i}^{x_f} dx \sqrt{2(V-E)}} \left(1 + O(\hbar)\right)$ WKB
PATH INT
$e^{-HT/\hbar}|x_i\rangle = N \int [dx]\, e^{-S/\hbar}$
NORMALIZATION
$S = \int_{-T/2}^{T/2} L = \int_{-T/2}^{T/2} \left(\frac{1}{2}m\dot{x}^2 + V\right) dt$
EUC SPACE
$x(T/2) = x_f$
$x(-T/2) = x_i$


WRITE
$x(t) = \bar{x}(t) + \sum c_n x_n(t)$
$\int_{-T/2}^{T/2} dt\, x_n(t) x_m(t) = \delta_{nm}$
$x_n(\pm T/2) = 0$
$\bar{x}(\pm T/2) = \begin{cases} x_F & + \\ x_i & - \end{cases}$
$[dx] = \prod_n \frac{dc_n}{(2\pi\hbar)^{1/2}}$


DUAL
WRITE
→ SOLVES $\frac{\delta S}{\delta x} = 0$
$x(t) = \bar{x}(t) + \sum c_n x_n(t)$
$\bar{x}$ CLASSICAL SOL'N
$\int_{-T/2}^{T/2} dt\, x_n(t)\, x_m(t) = \delta_{nm}$
$x_n(\pm T/2) = 0$
$\bar{x}(\pm T/2) = \begin{cases} x_F & + \\ x_i & - \end{cases}$
$[dx] = \prod_n \frac{dc_n}{(2\pi\hbar)^{1/2}}$
$RHS(\otimes) = N \int \prod_n \frac{dc_n}{(2\pi\hbar)^{1/2}}\, e^{-S[x(t)]/\hbar}$


$[dx] = \prod_n \frac{dc_n}{(2\pi\hbar)^{1/2}}$
$RHS(\hbar) = N \int \prod_n \frac{dc_n}{(2\pi\hbar)^{1/2}} e^{-S[x(t)]/\hbar}$
$S[x(t)] = S[\bar{x}] + \int dt \sum_n c_n x_n \frac{\delta S}{\delta x}\Big|_{\bar{x}} + \frac{1}{2!} \int \sum_{n,m} c_n x_n \left(\frac{\delta^2 S}{\delta x^2}\right)_{\bar{x}} c_m x_m dt dt' + \cdots$

$$0 = \left(\frac{\delta S}{\delta x}\right)_{\bar{x}} = -m\frac{d^2\bar{x}}{dt^2} + V'(\bar{x})$$

$$\frac{\delta^2 S}{\delta(x(t_1))\delta(x(t_2))} = -m\frac{d^2}{dt^2}$$

$$\Rightarrow E = \frac{m}{2}\left(\frac{d\bar{x}}{dt}\right)^2 - V(\bar{x})$$

↑ CONST of the MOTION

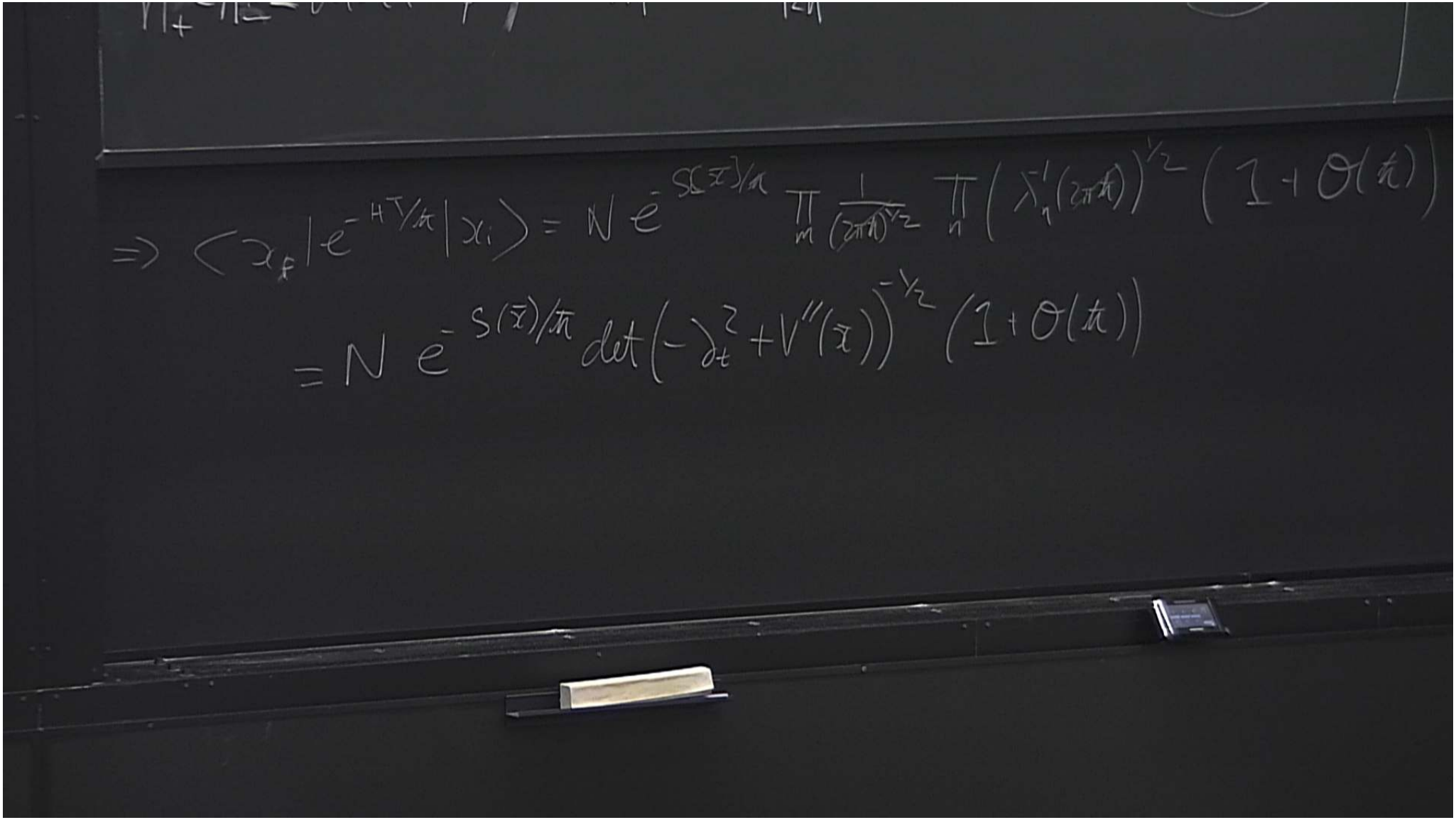
↑ EUC SPACE

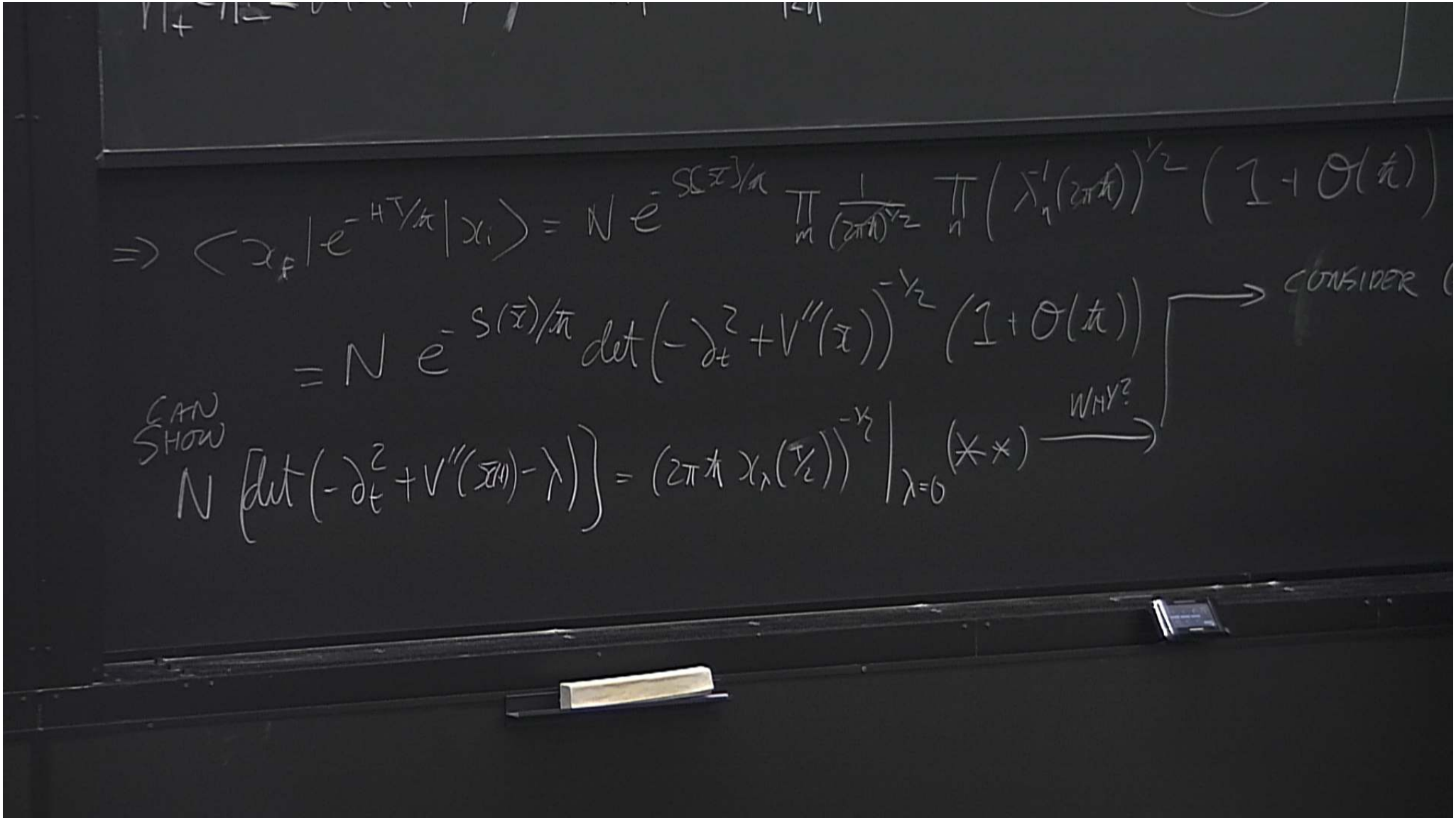
$Q^1_{2n-2}(\delta\omega, \omega, \Omega) \Rightarrow$ "NON-ABELIAN ANOMALY"

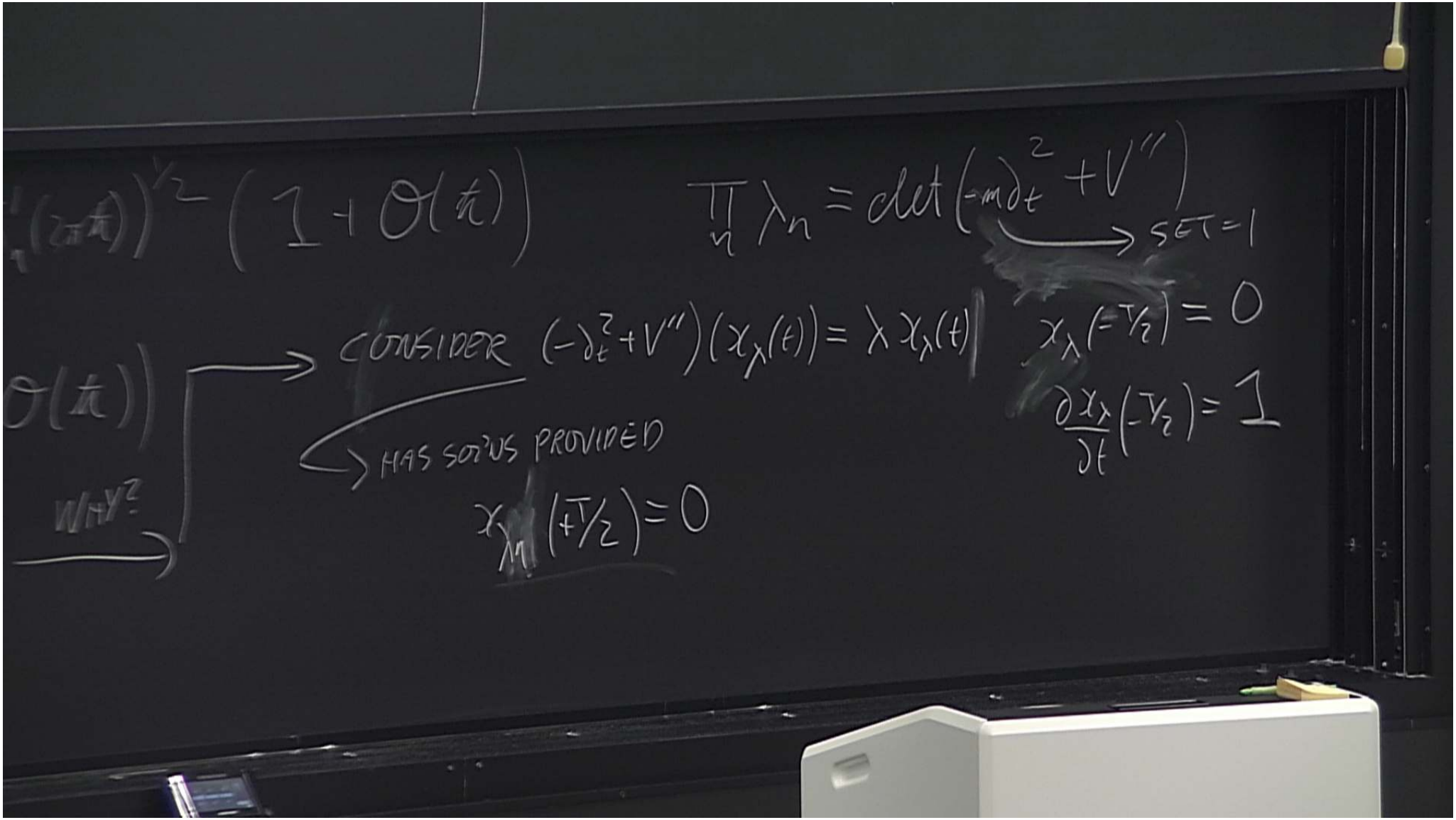
$$\frac{\delta^2 S}{\delta x(t_1)\delta x(t_2)} = \left(-m\frac{d^2}{dt^2} + V''(\bar{x})\right)\delta(t-t')$$

CHOOSE $\left.\frac{\delta^2 S}{\delta x^2}\right|_{\bar{x}} (x_n) = \lambda_n x_n$

$$\Rightarrow S = S[\bar{x}] + \frac{1}{2!}\int \sum_{nm} c_n c_m \lambda_m x_n(t) x_m(t)\,dt + \mathcal{O}(c^3) = S(\bar{x}) + \tfrac{1}{2}\sum c_n c_m \lambda_m + \cdots$$


$\Rightarrow \langle x_f | e^{-HT/\hbar} | x_i \rangle = N e^{-S[\bar{x}]/\hbar} \prod_m \frac{1}{(2\pi\hbar)^{1/2}} \prod_n \left(\lambda_n' (2\pi\hbar) \right)^{-1/2} \left(1 + O(\hbar) \right)$
$= N e^{-S(\bar{x})/\hbar} \det\left(-\partial_t^2 + V''(\bar{x}) \right)^{-1/2} \left(1 + O(\hbar) \right)$


$\Rightarrow \langle x_f | e^{-HT/\hbar} | x_i \rangle = N e^{-S[\bar{x}]/\hbar} \prod_m \frac{1}{(2\pi\hbar)^{1/2}} \prod_n \left(\lambda_n^{-1} (2\pi\hbar) \right)^{1/2} \left(1 + O(\hbar) \right)$
$= N e^{-S(\bar{x})/\hbar} \det\left(-\partial_t^2 + V''(\bar{x}) \right)^{-1/2} \left(1 + O(\hbar) \right)$
CONSIDER
CAN SHOW
$N \left[\det\left(-\partial_t^2 + V''(\bar{x}(t)) - \lambda \right) \right] = (2\pi\hbar)^{-1/2} \chi_\lambda(T/2) \Big|_{\lambda=0}^{-1/2}$ (**)
WHY?


$\left(1+O(\hbar)\right)$
$\prod_n \lambda_n = \det\left(-m\partial_t^2 + V''\right)$
SET = 1
CONSIDER $\left(-\partial_t^2 + V''\right)\chi_\lambda(t) = \lambda\chi_\lambda(t)$
HAS SOLNS PROVIDED
$\chi_\lambda(-T/2) = 0$
$\frac{\partial\chi_\lambda}{\partial t}(-T/2) = 1$
$O(\hbar)$
WHY?
$\chi_\lambda(+T/2) = 0$

$$1 \xleftarrow{\lambda\to\infty} \frac{\det(-\partial_t^2 + V_A'' - \lambda)}{\det(-\partial_t^2 + V_B'' - \lambda)} = \frac{\chi_\lambda^A(T/2)}{\chi_\lambda^B(T/2)} \xrightarrow[\lambda\to\infty]{} 1$$

FOR TWO POTENTIALS V_A & V_B

WHY? EACH SIDE IS A MEROMORPHIC FN of λ HAVING SAME POLES & SAME ZEROS

$= m(m+1)$

$$\frac{\chi_\lambda^A(T/2)}{\chi_\lambda^B(T/2)} \xrightarrow{\lambda \to \infty} 1$$

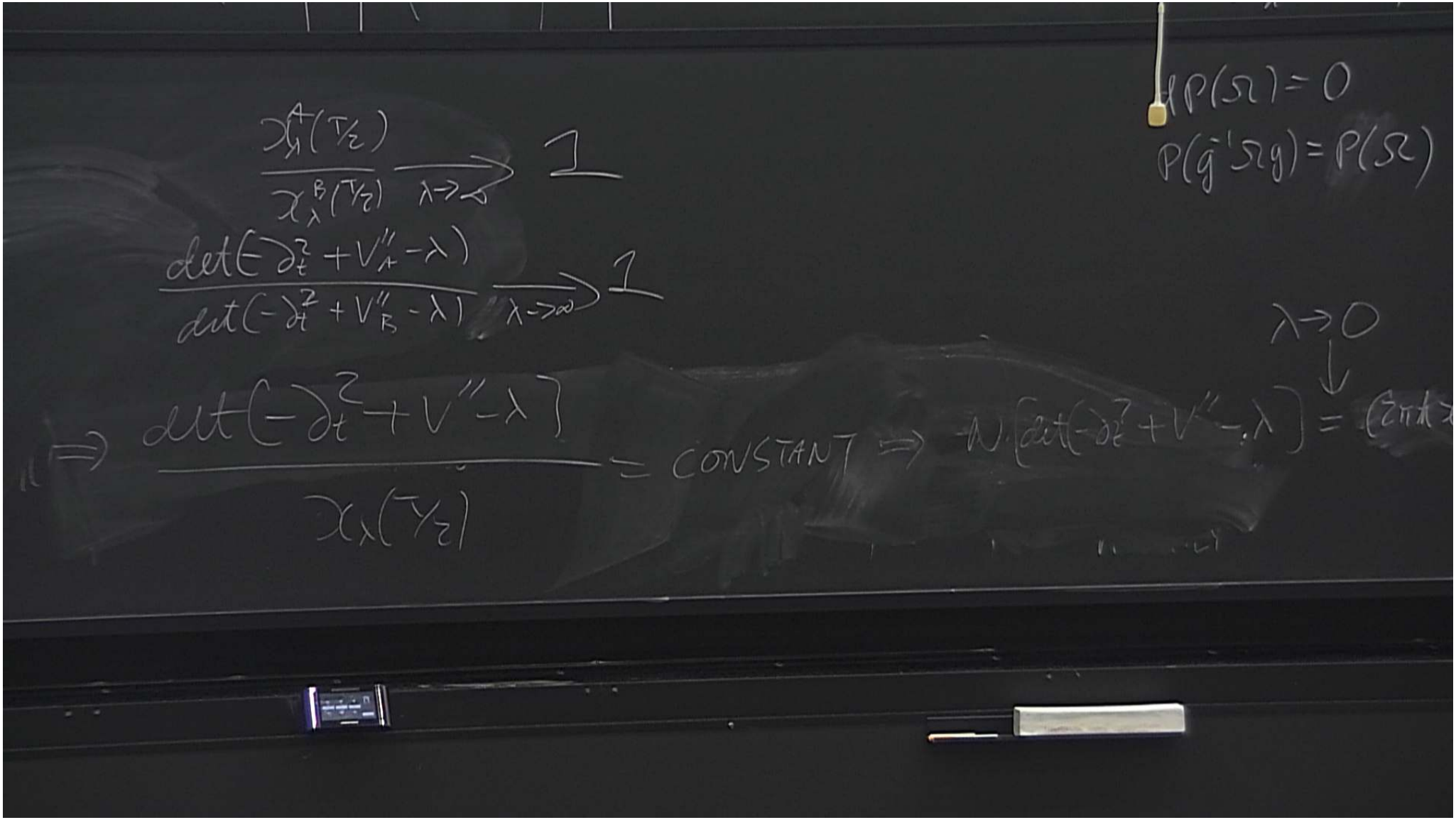
$$\frac{\det(-\partial_t^2 + V_A'' - \lambda)}{\det(-\partial_t^2 + V_B'' - \lambda)} \xrightarrow{\lambda \to \infty} 1$$

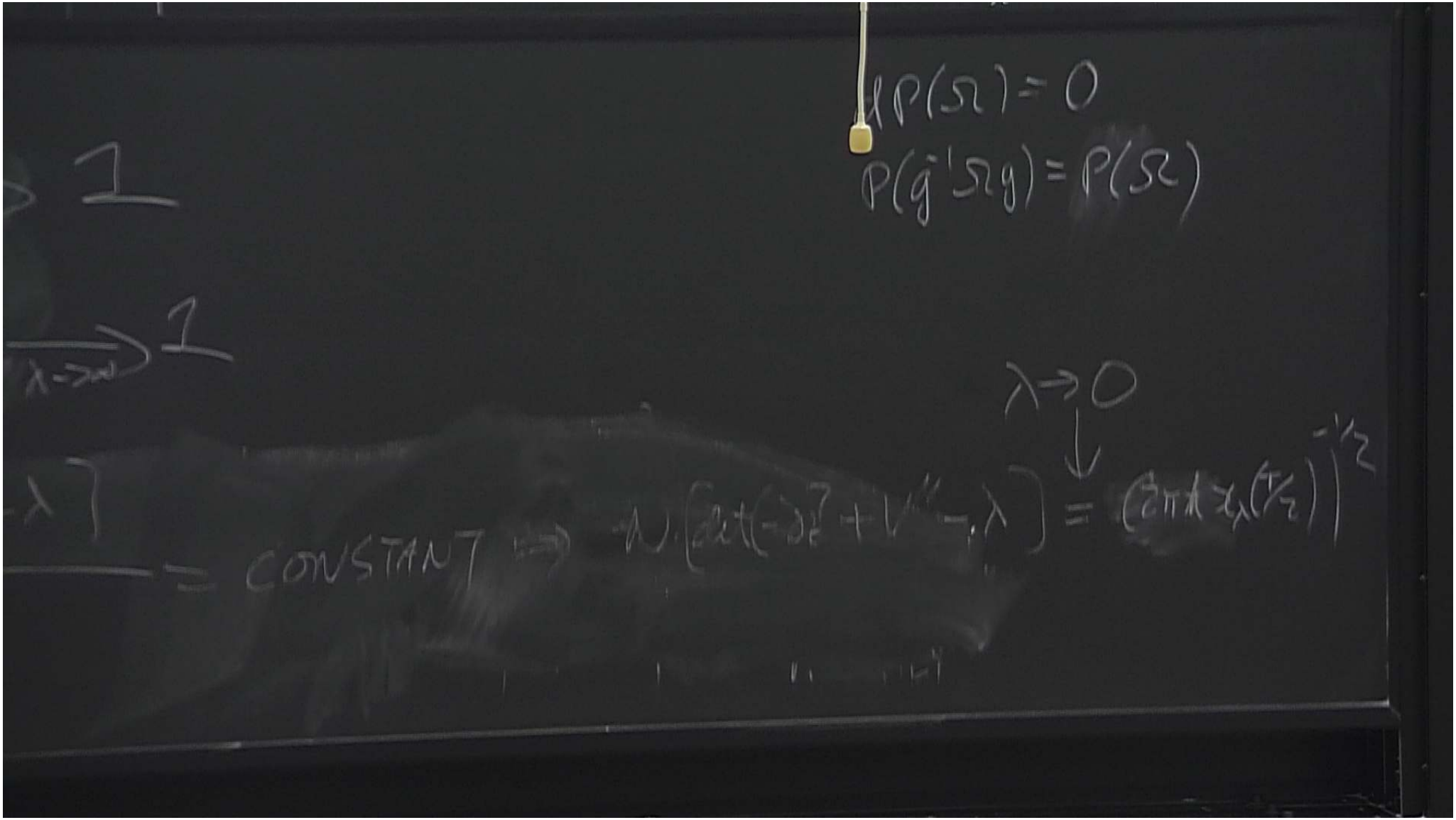
$$= m(m+1) \int_0^1 dt\,(1-t)\, STr\left(\delta\omega, d(\omega, \Omega_t^{m-1})\right)$$

$$Q^1_{2m-2}(\delta\omega, \omega, \Omega) \to \text{"NON-ABELIAN ANOMALY"}$$

$$HP(\Omega) = 0$$

$$P(g^{-1}\Omega g) = P(\Omega)$$


$\frac{\chi_\lambda^A(T/2)}{\chi_\lambda^B(T/2)} \xrightarrow{\lambda \to -\infty} 1$
$\frac{\det(-\partial_t^2 + V_A'' - \lambda)}{\det(-\partial_t^2 + V_B'' - \lambda)} \xrightarrow{\lambda \to \infty} 1$
$\Rightarrow \frac{\det[-\partial_t^2 + V'' - \lambda]}{\chi_\lambda(T/2)} = \text{CONSTANT} \Rightarrow N\cdot[\det(-\partial_t^2 + V'' - \lambda] =$
$\lambda \to 0$
$dP(\Omega) = 0$
$P(g^{-1}\Omega g) = P(\Omega)$


P(Ω) = 0
P(g⁻¹Ωg) = P(Ω)
1
1
λ → 0
= CONSTANT