Title: Introduction to Quantum Field Theory for Cosmology - Lecture 20

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Abstract:

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Note Title

QFT for Cosmology, Achim Kempf, Winter 14, Lecture 20

Recall:

- In Minkowski space, the fluctuation spectrum reads:

$$\delta\phi_\lambda = \frac{1}{\lambda} \qquad (\text{for } m = 0)$$

$\rightsquigarrow$ Fluctuations of large spatial extent λ are suppressed.

- We considered a period, $[\eta_i, \eta_f]$, of exponential expansion:

Ht

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- In Minkowski space, the fluctuation spectrum reads:

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- We considered a period, $[\eta_i, \eta_f]$, of exponential expansion:

$$a(t) = e^{Ht}$$

i.e.: $$a(\eta) = -\frac{1}{H\eta}$$

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- We said that a comoving mode k crosses the Hubble horizon when in its mode equation the sign of $\omega_k^2(\eta)$ changes:

$$v_k''(\eta) + \left(k^2 - \frac{2}{\eta^2}\right) v_k(\eta) = 0$$

(neglecting the mass: $m \ll H$)

This happens when $\eta = \eta_{hor}(k) = -\frac{\sqrt{2}}{k}$, if this time is in $[\eta_i, \eta_f]$.

- The case of very small modes: $\eta_{hor}(k) \gg \eta_f$

  * In K.G. eqn., k^2 keeps dominating over $-\frac{2}{\eta^2}$.
  * Mode functions behave as in Minkowski space.
  * The fluctuation spectrum was found to stay:

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* The fluctuation spectrum was found to stay:

$$\delta\phi_\lambda = \frac{1}{\lambda}$$

← proper wave length.

comoving wave length

Note: $\delta\phi_L$ for fixed L decreases over time, because $\lambda = a\,L$.

The case of medium size modes: $\eta_{hor}(k) \in [\eta_i, \eta_f]$

* In K.G. eqn., from horizon crossing onwards,

$-\frac{2}{\ldots}$ dominates over k^2. This can mean: ...

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The case of medium size modes: $\eta_{hor}(k) \in [\eta_i, \eta_f]$

* In K.G. eqn., from horizon crossing onwards, $-\frac{2}{\eta^2}$ dominates over k^2. This soon means:

$\Rightarrow$ Mode functions then no longer depend on k!

$\Rightarrow$ The fluctuation spectrum was found after Hubble horizon crossing to become constant:

$$\boxed{\delta\phi_k(\eta_f) \approx H \cdot 2^{3/2}\Gamma(3/2)/\pi}$$

Note: this behavior has been found only for scalar and scalar-derived fields. ⇒ Focus on scalar fields.

Conclusion:

- Normally, the size of a mode's fluctuations $\delta\phi_\lambda(\eta)$ decreases as its proper wavelength $\lambda(\eta)$ increases due to the expansi

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Conclusion:

- Normally, the size of a mode's fluctuations $\delta\phi_\lambda(\eta)$ decreases as its proper wavelength $\lambda(\eta)$ increases due to the expansion.
- But, after modes cross the horizon, $\frac{1}{H}$, they keep their fluctuation amplitudes even as their proper wavelength $\lambda(t)$ increases!

Conclusion:

- Assume that the expansion $a(t) = e^{Ht}$ is very fast.
- Thus, the Hubble horizon $\frac{1}{H}$ is very small.
- Thus, as modes cross the Hubble horizon, their

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Conclusion:

- Assume that the expansion $a(t) = e^{Ht}$ is very fast.
- Thus, the Hubble horizon $\frac{1}{H}$ is very small.
- Thus, as modes cross the Hubble horizon, their proper wavelength is very small $\lambda \sim \frac{1}{H}$.
- Thus, modes have large fluctuations $\delta\phi_\lambda(\eta) \sim H$ as they cross the horizon and they keep these large fluctuations as their proper wavelength further increases.
- If $[\eta_i, \eta_f]$ is long enough, modes with large fluctuations can reach even cosmological proper

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- Thus, the Hubble horizon $\frac{1}{H}$ is very small.

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- Thus, modes have large fluctuations $\delta\phi_\lambda(\eta) \sim H$ as they cross the horizon and they keep these large fluctuations as their proper wavelength further increases.

- If $[\eta_i, \eta_f]$ is long enough, modes with large fluctuations can reach even cosmological proper wavelengths – and can therefore cause the cosmic structure!

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Preliminary estimates:

* If this is the seeding mechanism for cosmic structure formation, then:

* H determines the amplitude of the late-observed fluctuations and must be of the right size to conform with observations. Measurements of the CMB indicate:

$$H^{-1} \approx 10^{5}\, \ell_{planck} \approx 10^{-29}\, m$$

* The interval $[\eta_i, \eta_f]$ must be long enough so that such small modes have time to expand to

$\Rightarrow$ how much expansion?

$$\frac{a(t_f)}{a(t_i)} = e^{H(t_f - t_i)c}$$

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$$H^{-1} \approx 10^5 \ell_{planck} \approx 10^{-29} m$$

* The interval $[\eta_i, \eta_f]$ must be long enough so that such small modes have time to expand to cosmological size. For example this time period would do:

$$[10^{-34}s, 10^{-32}s]$$

$\Rightarrow$ how much expansion?

$$\frac{a(t_f)}{a(t_i)} = e^{H(t_f - t_i)c}$$

$$= e^{\frac{10^{-32} s \cdot 3 \cdot 10^8 m}{10^{-29} m \cdot s}}$$

$$= e^{3 \cdot 10^5}$$

Realistic cosmic inflation

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Realistic cosmic inflation

1. How can a period of near-exponential expansion be caused?

Recall the full action:

We neglect such terms by Occam's razor: there is no evidence for their existence as yet.

$$S = -\frac{1}{16\pi G}\int\Big[2\Lambda + R(x) + \overbrace{\cancel{\mathcal{O}(R\phi) + \mathcal{O}(R^2) + \ldots}}\Big]\sqrt{|g|}\,d^4x$$

$\uparrow$ cosm. constant

Note: ϕ is now called the "Inflaton" field.

$$+\int\left[\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\right]\sqrt{|g|}\,d^4x$$

$$+\cancel{S_{\text{other fields}}}$$

$\leftarrow$ We neglect this term because the contribution of the inflaton field ϕ and of $g_{\mu\nu}$ are assumed to have been dominant in the very early universe.

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Note: ϕ is now called the "Inflaton" field.

$$+ \int \left[\frac{1}{2} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\right]\sqrt{|g|}\, d^4x$$

$$+ \cancel{S_{\text{other fields}}}$$

$\leftarrow$ We neglect this term because the contribution of the inflaton field ϕ and of $g_{\mu\nu}$ are assumed to have been dominant in the very early universe.

Example choice of V: $V(\phi) = m\,\phi^2 + \lambda\,\phi^4$

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Equations of motion:

* $\frac{\delta S}{\delta \phi(x)} = 0$ yields the K.G. eqn.:

$$\frac{\partial}{\partial x^\mu}\left(g^{\mu\nu}(x)\,\phi_{,\nu}(x)\sqrt{|g(x)|}\right) + \frac{\partial V}{\partial \phi}(x)\sqrt{|g(x)|} = 0 \qquad (KG)$$

* $\frac{\delta S}{\delta g_{\mu\nu}(x)} = 0$ yields the Einstein eqn.:

$$R_{\mu\nu}(x) - g_{\mu\nu}(x)R(x) + \Lambda g_{\mu\nu}(x) = -8\pi G\, T_{\mu\nu}(x) \qquad (E)$$

where the energy-momentum tensor (for ϕ only) reads:

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$$\frac{\partial}{\partial x^\mu}\left(g^{\mu\nu}(x)\,\phi_{,\nu}(x)\sqrt{|g(x)|}\right)+\frac{\partial V}{\partial\phi}(x)\sqrt{|g(x)|}=0 \qquad \text{(KG)}$$

* $\frac{\delta S}{\delta g_{\mu\nu}(x)}=0$ yields the Einstein eqn.:

$$R_{\mu\nu}(x)-g_{\mu\nu}(x)R(x)+\Lambda g_{\mu\nu}(x)=-8\pi G\,T_{\mu\nu}(x) \qquad \text{(E)}$$

where the energy-momentum tensor (for ϕ only) reads:

$$T_{\mu\nu}=\phi_{,\mu}\phi_{,\nu}-g_{\mu\nu}\left(g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}-V(\phi)\right)+\underbrace{\cancel{T_{\mu\nu}^{(\text{other fields})}}}$$

We'll assume this small compared to the contribution of ϕ, during the very early universe.

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The important special case of homogeneity & isotropy

Eqns. (KG) and (E) are a set of coupled nonlinear partial differential equations which are even classically very hard.

⇝ As a lowest order approximation we assume perfect homogeneity & isotropy:

$$\phi(x,t) = \phi(t)$$

$$g_{\mu\nu}(x,t) = g_{\mu\nu}(t)$$

Note:

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→ As a lowest order approximation we assume perfect homogeneity & isotropy:

$$\phi(x,t) = \phi(t)$$

$$g_{\mu\nu}(x,t) = g_{\mu\nu}(t)$$

Note:

This may also be viewed as considering only the $k=0$ modes, neglecting all other modes.

Thus, the eqns of motion simplify:

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Thus, the eqns of motion simplify:

$\left(\dot{} = \frac{\partial}{\partial t} \right)$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$ (K.G. eqn.)

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\, T^0_{\ 0} + \Lambda$$ (the 0,0 component of the Einstein equation)

no sum

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}}{a^2} = 8\pi G\, T^i_{\ i} - \Lambda$$ (the i,i components of the Einstein equation)

Here: $T^0_{\ 0} = \rho(t) = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ (the energy density ρ of ϕ)

$T^i_{\ i} = p(t) = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ (the pressure p of ϕ)

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Notice:

- The cosmological constant Λ contributes effectively a positive energy density ρ_Λ and effectively a negative pressure ρ_p.

- Vice versa, whenever $V(\phi) \gg \dot{\phi}^2/2$ then $V(\phi)$ temporarily plays the same rôle as Λ.

- How close we are to $V(\phi) \gg \dot{\phi}^2/2$ is described by the "Equation of state parameter"

$$w(t) := \frac{p(t)}{\rho(t)} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}$$

$-1 < w < 1$

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$$-2\frac{\ddot{a}}{a}-\frac{\dot{a}}{a^2}=8\pi G T^i_{\;i}-\Lambda$$ (the i,i components of the Einstein equation)

Here: $T^0_{\;0}=\rho(t)=\frac{1}{2}\dot{\phi}^2+V(\phi)$ (the energy density ρ of ϕ)

$T^i_{\;i}=p(t)=\frac{1}{2}\dot{\phi}^2-V(\phi)$ (the pressure p of ϕ)

Given any initial conditions and given any $V(\phi)$ one can now solve for $a(t)$, $\phi(t)$, at least numerically!

Notice:

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effectively a negative pressure $\mathcal{S}_p$.

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- How close we are to $V(\phi) \gg \dot{\phi}^2/2$ is described by the "Equation of state parameter":

$$w(t) := \frac{p(t)}{\rho(t)} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \qquad -1 < w < 1$$

$\Rightarrow$ Iff $w \approx -1$ then $V(\phi)$ acts like a cosm. constant.

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First attempt to get exponential expansion:

Assume that Λ dominates over $T_{\mu\nu}$ of all fields in nature.

Then, the 0,0 component of Einstein's equation,

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3} T^0_0 + \frac{1}{3}\Lambda \quad \text{becomes} \quad \left(\frac{\dot a}{a}\right)^2 = \frac{1}{3}\Lambda$$

whose solution has the desired behavior:

$$a(t) = a_0 e \qquad \text{with} \quad H = \sqrt{\Lambda/3} \ !$$

Problems: $\square$ Λ is too tiny! It is 122 orders of magnitude below the Planck scale. We'd need a Λ close to the Planck scale $10^{70}\,m^{-2}$.

Note:
Λ manifests itself as

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Problems: ☐ Λ is too tiny! It is 122 orders of magnitude below the Planck scale. We'd need a Λ close to the Planck scale $10^{70} m^{-2}$. (just a few orders of magnitude below)

☐ Since Λ is constant, such an inflation would never end!

Note:
Λ manifests itself as dark energy and has been measured:
$\Lambda \approx 10^{-52} m^{-2}$

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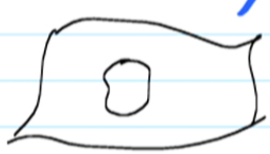
⇝ Realistic possibility:

$V(\phi)$ temporarily very large

⇝ One of the biggest ideas of science, ever:

- Consider a universe like ours.
  Everywhere, at all times, all fields quantum fluctuate.

- As a rare fluke, the field ϕ quantum fluctuates in a patch a few Planck lengths in size



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One of the biggest ideas of science, ever:

- Consider a universe like ours.

  Everywhere, at all times, all fields quantum fluctuate.

- As a rare fluke, the field ϕ quantum fluctuates in a patch a few Planck lengths in size

  to a ϕ value that makes $V(\phi)$ close to the Planck scale.

  (Assume homogeneity in that patch, so that the $\partial_i \phi$ are small)

- In this patch, the equations above hold, with $V(\phi)$

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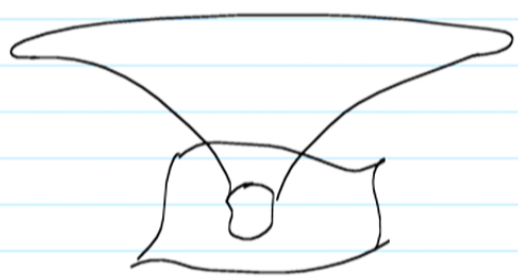
- In this patch, the equations above hold, with $V(\phi)$ dominant and impacting $a(t)$ like a large Λ would

$\Rightarrow$ Before the fluctuation can "snap back", general relativity will quasi-exponentially inflate this patch (potentially e.g., by 10^5 orders of magnitude).



$\Rightarrow$ The mother universe spawns a daughter universe!

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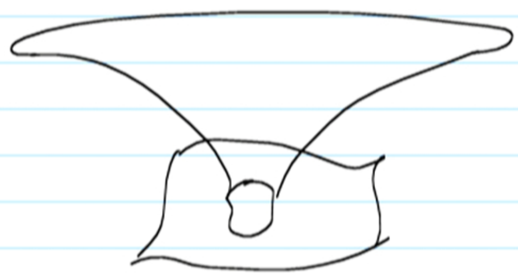

⇒ The mother universe spawns a daughter universe!

- $V(\phi)$ in the patch starts out high but will dynamically fall eventually to low value → Inflation ends.
- The energy in $V(\phi)$ turns into hot matter.

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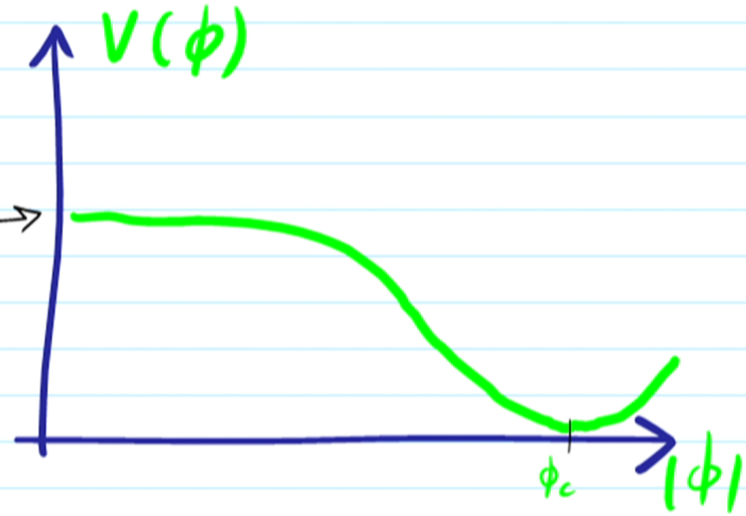
- The daughter universe can spawn new universes and so on ...

Example potential:



- Then, inflation starts when, in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)
- Then, after ϕ starts out at $\phi=0$ and large $V(\phi)$, it will slowly evolve

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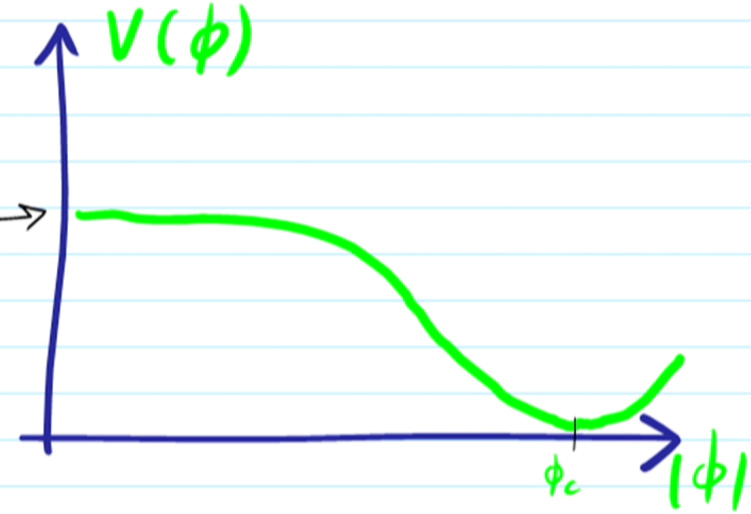
Example potential:

very large value so that expansion will be fast.



- Then, inflation starts when, in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)
- Then, after ϕ starts out at $\phi=0$ and large $V(\phi)$, it will slowly evolve towards ϕ_c while the universe inflates, thus flattens, and the matter dilutes.
- Once $\phi=\phi_c$ is reached, $V(\phi)=0$, and inflation has ended.

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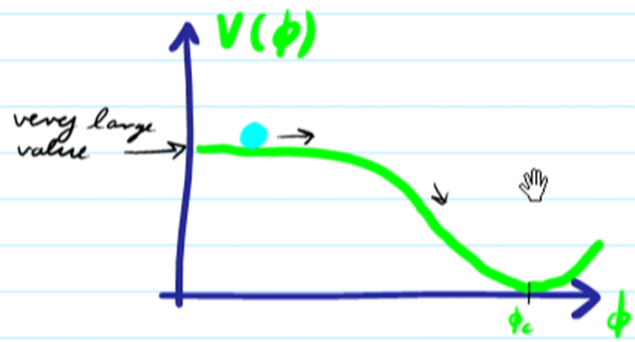
* Concretely:

The Klein Gordon equation reads:

friction term

$$\ddot{\phi} = -3\frac{\dot{a}}{a}\dot{\phi} - \frac{dV}{d\phi}$$

This is like the equation of motion of a ball rolling down a hill, with friction:



$\left(-\frac{dV}{d\phi}\right)$ acts to pull ϕ down the potential hill.

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* Definition:

If the initial value of $V(\phi)$ is very large and if the initial slope is very flat, i.e., if the ball for a period rolls slowly, with approximately constant $V(\phi)$, we call this a period of "Slow Roll Inflation".

* Observation:

During the slow roll period, we have, in particular, that

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* **Observation:**

During the slow roll period, we have, in particular, that $V(\phi)$ dominates over $\frac{1}{2}\dot{\phi}^2$.

$$\Rightarrow \qquad w(t) = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \simeq -1 \qquad (\text{temporarily})$$

* **But, do we also get temporary exponential inflation?**

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$

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is during the slow roll period:

$$\left(\frac{\dot a}{a}\right)^2 \approx \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \approx a_0\, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a

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(ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

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Definition:

The function $H(t) := \frac{\dot{a}(t)}{a(t)}$ is called the Hubble parameter function.

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$.

* In the case of slow roll inflation, we have

$$H(t) = \sqrt{\frac{8\pi G}{3} V(\phi(t))}$$

Remarks:

- The BICEP2 results indicate: $\frac{1}{H(t)} = L^{(t)}_{Hubble} \approx 10^{+3} L_{Planck}$

- As $V(t)$ decreases, also $H(t)$ decreases.

$\Rightarrow$ inflation predicts that $\delta\phi$ decreases for later and later

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- The BICEP2 results indicate: $\frac{1}{H(t)} = L_{Hubble}^{(t)} \approx 10^{+3} L_{Planck}$

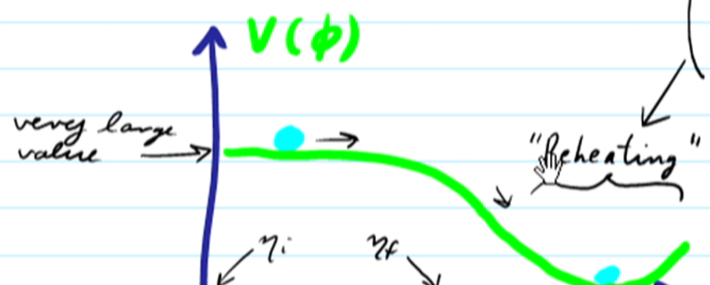
- As $V(t)$ decreases, also $H(t)$ decreases.

  $\Rightarrow$ inflation predicts that $\delta\phi_k$ decreases for later and later horizon crossing modes, i.e., for smaller and smaller wavelength modes.

  The WMAP satellite's CMB data show evidence for this!

The end of inflation:

(Note that this is a misnomer: We do not necessarily assume the existence of a prior hot phase)



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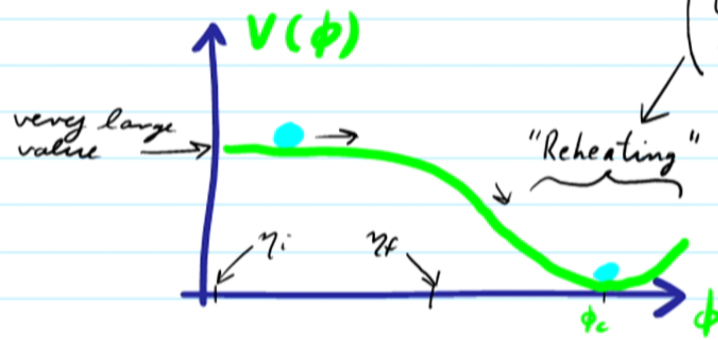

* In the period called "Re-heating", the energy of ϕ is transferred into the mode oscillators of the usual (low mass) particles, i.e., the inflaton particles decay and thereby create a high energetic, i.e. hot, plasma of literally all sorts of particles.

* From thereon, the usual big bang cosmology is

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assumed to have followed.

Quantum fluctuations

Strategy:

- We assume some $V(\phi)$ and suitable initial conditions which yield classical solutions $\phi_0(t)$ and $a_0(t)$ which exhibit slow roll inflation for a suitable finite time interval.

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- We allow small inhomogeneities in the inflaton field:

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Strategy:

- We assume some $V(\phi)$ and suitable initial conditions which yield classical solutions $\phi_0(t)$ and $a_0(t)$ which exhibit slow roll inflation for a suitable finite time interval.

- We allow small inhomogeneities in the inflaton field:

$$\phi(x,\eta) = \phi_0(\eta) + \varphi(x,\eta) \quad \text{with} \quad |\varphi(x,\eta)| \ll |\phi_0(\eta)|$$

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- We allow small inhomogeneities in the inflaton field:

$$\phi(x,\eta) = \phi_0(\eta) + \varphi(x,\eta) \quad \text{with} \quad |\varphi(x,\eta)| \ll |\phi_0(\eta)|$$

- We allow also small inhomogeneities in the metric: $g_{\mu\nu}(x,\eta) = a(\eta)\,\eta_{\mu\nu} + \gamma_{\mu\nu}(x,\eta)$

$$\sum |\gamma_{\mu\nu}(x,\eta)| \ll 1$$

- The perturbations $\gamma_{\mu\nu}$ of the metric tensor can be decomposed into three types:

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- We allow also small inhomogeneities in the metric : $g_{\mu\nu}(x,\eta) = a(\eta)\,\eta_{\mu\nu} + \gamma_{\mu\nu}(x,\eta)$

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- The perturbations $\gamma_{\mu\nu}$ of the metric tensor can be decomposed into three types:

  a) The part of $\gamma_{\mu\nu}$ which can be derived from scalar functions

  b) The part of $\gamma_{\mu\nu}$ which can be derived from vector fields

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- The perturbations $\gamma_{\mu\nu}$ of the metric tensor can be decomposed into three types:

a) The part of $\gamma_{\mu\nu}$ which can be derived from scalar functions

b) The part of $\gamma_{\mu\nu}$ which can be derived from vector fields

c) The part of $\gamma_{\mu\nu}$ which is purely tensor.

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- This decomposition is convenient because the three types obey independent equations of motion.

Remark:

The scalar part of $\gamma_{\mu\nu}$ and the inflaton perturbation φ will yield one combined scalar equation of motion (which is similar but not identical to the K.G. eqn.

- We quantize these perturbation fields $\hat{\gamma}_{\mu\nu}(x,\eta)$ and $\hat{\varphi}(x,\eta)$ but we keep the $g_{0\mu\nu} = a(\eta)\,\eta_{\mu\nu}$ and $\phi_0(x,\eta)$ as fixed classical background fields.

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- We calculate the 3 quantum fluctuation spectra.

"The backreaction problem" In principle, need to check that $|\langle\gamma_{\mu\nu}\rangle|, |\langle\varphi\rangle|$ stay small.

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