

Title: Introduction to Quantum Field Theory for Cosmology - Lecture 18

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URL: <http://pirsa.org/14030004>

Abstract:

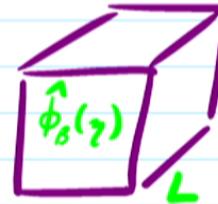
# QFT for Cosmology, Achim Kempf, Winter 14, Lecture 18

Note Title

Problem: How much do fields fluctuate depending on length scale?

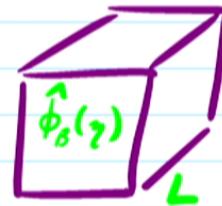
Recall: We considered the observable  $\hat{\phi}_B(\gamma)$  which is the average field amplitude in a spatial region  $B$ , of size  $L$  (where  $L$  is the region's length or radius):

$$\hat{\phi}_B(\gamma) := \int_B \hat{\phi}(x, \gamma) w(x) d^3x$$



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$$\hat{\phi}_B(\gamma) := \int_B \hat{\phi}(x, \gamma) w(x) d^3x$$



□ Mean value at some arbitrary time  $\gamma \geq \gamma_0$ :

$$\bar{\phi}_B(\gamma) = \langle \Omega | \hat{\phi}_B(\gamma) | \Omega \rangle = \langle \Omega | \cancel{a} + \cancel{a^\dagger} | \Omega \rangle = 0$$

□ Variance at arbitrary time  $\gamma \geq \gamma_0$ :

□ Mean value at some arbitrary time  $\gamma \geq \gamma_0$ :

$$\bar{\phi}_B(\gamma) = \langle \Omega | \hat{\phi}_B(\gamma) | \Omega \rangle = \langle \Omega | \alpha + \alpha^+ | \Omega \rangle = 0$$

□ Variance at arbitrary time  $\gamma \geq \gamma_0$ :

$$\Delta \phi_B(\gamma)^2 = \langle \Omega | \hat{\phi}_B^2(\gamma) | \Omega \rangle = \langle \Omega | \alpha^2 + \alpha^+ \alpha + \alpha \alpha^+ + \alpha^{++} | \Omega \rangle$$

= ... (some calculations and approximations)

$$\Delta \phi_B(\gamma)^2 \approx \alpha^{-2}(\gamma) k^3 |v_k(\gamma)|^2 \Big|_{\gamma=\pi}$$

(\*)

□ Variance at arbitrary time  $\eta \gg \eta_0$ :

$$\Delta \phi_B(\eta)^2 = \langle \Omega | \hat{\phi}_B^2(\eta) | \Omega \rangle = \langle 0 | \alpha^2 + \alpha^\dagger \alpha + \alpha \alpha^\dagger + \alpha^\dagger \alpha^2 | 0 \rangle$$

= ... (some calculations and approximations)

$$\Delta \phi_B(\eta)^2 \approx \alpha^{-2}(\eta) k^3 |v_k(\eta)|^2 \Big|_{k = \frac{2\pi}{L}}$$

(\*)

An alternative measure of how much the field amplitudes fluctuate depending on the length scale:

$$\Delta \phi_B(\gamma)^2 = \langle \Omega | \hat{\phi}_B^{\dagger}(\gamma) | \Omega \rangle = \langle 0 | \hat{a}^{\dagger} + \hat{a}^{\dagger}a + \hat{a}a^{\dagger} + \hat{a}^{\dagger}a^{\dagger} | 0 \rangle$$

$= \dots$  (some calculations and approximations)

$$\Delta \phi_B(\gamma)^2 \approx a^{-2}(\gamma) k^3 |v_k(\gamma)|^2 \Big|_{k = \frac{2\pi}{L}}$$

(\*)

An alternative measure of how much the field amplitudes fluctuate depending on the length scale:

How much are the fluctuations at points that are a distance  $L$  apart correlated?

## A primer on classical fluctuations:

all frequencies occur to same amount  
↓

□ Assume  $n(t)$  is a  $\Omega$ -bandlimited gaussian white noise signal, i.e., a random signal with gaussian distributed amplitudes, filtered to leave only frequencies in the interval  $[-\Omega, \Omega]$ .

□ Then, for an ensemble of such noise signals, one can show:

$$\overline{n(t)} = 0 \quad \forall t$$

"2-point correlator":  $\overline{n(t)n(t+L)} = c \frac{\sin(-\Omega L)}{\Omega L} \quad \forall t$

How can we see this?

This noise is ergodic, i.e. we could instead average over all  $t$ :

$$\overline{n(t)n(t+L)} = \int f(t)f(t+L)dt \quad \left( \begin{array}{l} \text{suitably regularize if non-normalizable,} \\ \text{e.g. } \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T f(t)f(t+L)dt \end{array} \right)$$

"Auto-correlator"

$$= \iiint \tilde{f}(\omega) \tilde{f}(\omega') e^{i\omega t} e^{i\omega'(t+L)} dt d\omega d\omega'$$

$$= \iiint e^{it(\omega+\omega')} dt \tilde{f}(\omega) \tilde{f}(\omega') e^{i\omega' L} d\omega d\omega'$$

$= (2\pi)^{-1} \delta(\omega + \omega')$

$$= \frac{1}{2\pi} \int \tilde{f}(\omega) \tilde{f}(-\omega) e^{i\omega L} d\omega$$

$$= \frac{1}{2} \underbrace{\left( |\tilde{f}(\omega)|^2 \right)}_{\text{"Spectral power function"}} e^{i\omega L} d\omega$$

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$$= \frac{1}{2\pi} \int \hat{f}(\omega) \tilde{f}(-\omega) e^{i\omega L} d\omega$$

$$= \frac{1}{2\pi} \int |\tilde{f}(\omega)|^2 e^{i\omega L} d\omega \quad \xrightarrow{\text{"Spectral power function"}}$$

$\Rightarrow$  Auto correlation and power spectrum are a Fourier pair.

Recall: flatness of spectrum means noise is "white"



$$= \iiint f(\omega) f(\omega') e^{-i\omega t} e^{-i\omega' L} dt d\omega d\omega'$$

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$\Rightarrow \square$  Autocorrelation and power spectrum are a Fourier pair.

Recall: flatness of spectrum means noise is "white"

$\square$  For white bandlimited noise:  $|\tilde{f}(\omega)|^2 = \boxed{\int_{-\Omega}^{\Omega} f(t) f(t) dt}$

Exercise: Show that its Fourier transform is indeed  $\sin(\Omega L)/\Omega L$ .

## The 2-point correlator in QFT:

$$\langle 0 | \hat{\phi}(\vec{x}, \gamma) \hat{\phi}(\vec{x} + \vec{l}, \gamma) | 0 \rangle = \frac{1}{a(\gamma)^2} \langle 0 | \hat{\chi}(\vec{x}, \gamma) \hat{\chi}(\vec{x} + \vec{l}, \gamma) | 0 \rangle$$

Exercise: use mode expansion of  $\hat{\chi}$  and spherical coordinates to derive:

$$= \frac{1}{a(\gamma)^2} \int_0^\infty \frac{k^2 dk}{4\pi^2} \frac{\sin(kl)}{kl} |V_k(\gamma)|^2 \quad \text{with } k = |\vec{k}|, l = |\vec{l}|.$$

Observe:  $\frac{\sin(kl)}{kl} \approx \begin{cases} 1 & \text{for } k \ll l \\ 0 & \text{for } k \gg l \end{cases}$

Observe:  $k^2 \frac{\sin(kl)}{kl}$  has largest amplitude around  $k \approx \frac{2\pi}{L}$

⇒ Estimate:

Exercise: use mode expansion of  $\hat{x}$  and spherical coordinates to derive:

$$= \frac{1}{a(\gamma)^2} \int_0^\infty \frac{k^2 dk}{4\pi^2} \frac{\sin(kL)}{kL} |V_k(\gamma)|^2 \quad \text{with } k = |\vec{k}|, L = |\vec{L}|.$$

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$\Rightarrow$  Estimate:

$$\langle 0 | \hat{\phi}(\vec{x}, \gamma) \hat{\phi}(\vec{x} + \vec{l}, \gamma) | 0 \rangle \approx a(\gamma)^{-2} \int_0^{2\pi/L} \frac{k^2}{4\pi^2} |V_{\frac{2\pi}{L}}(\gamma)|^2 \approx a(\gamma)^{-2} c k^3 |V_k(\gamma)|^2 \Big|_{k=\frac{2\pi}{L}}$$

a constant

Definition:

We define the so-called Fluctuation Spectrum at time  $\eta$  as a function of  $k$ :

$$\delta\phi_n(\eta) := a(\eta)^{-1} k^{3/2} |v_n(\eta)|$$

$$k = \frac{2\pi}{L}$$

Special case Minkowski space:

□ Scale factor:  $a(\eta) = 1$  for all  $\eta$

□ Mode functions:

$$v_n(\eta) = \frac{1}{\sqrt{\omega_n}} e^{i\eta\omega_n} \quad \text{with } \omega_n = \sqrt{k^2 + m^2}$$



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$\Rightarrow$  The fluctuation spectrum is: (recall:  $k = \frac{2\pi}{L}$ )

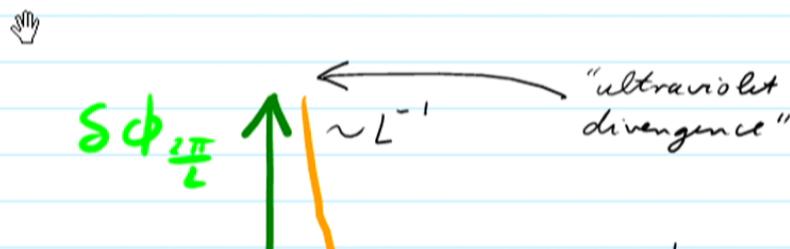
$$\delta\phi_k = \frac{k^{3/2}}{(m^2 + k^2)^{1/4}} \approx \begin{cases} k & \text{for } k \rightarrow \infty \\ \frac{k^{3/2}}{\sqrt{m}} & \text{for } k \rightarrow 0 \end{cases}$$

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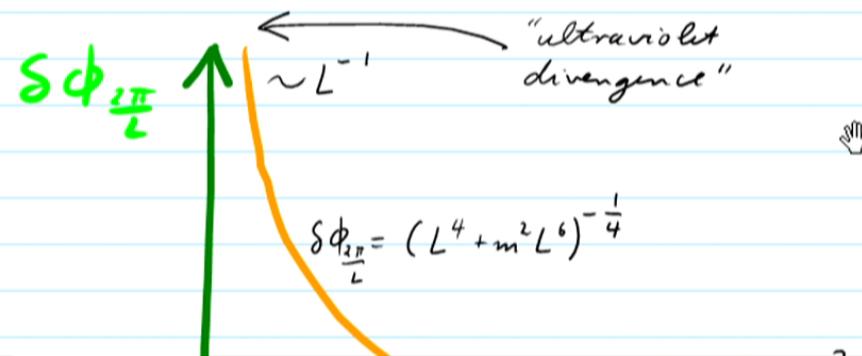
and as a function of  $L$  it is:  $\delta\phi_{\frac{2\pi}{L}} = \frac{(2\pi)^{3/2} L^{-3/2}}{\left(\frac{4\pi^2}{L^2} + m^2\right)^{1/4}}$



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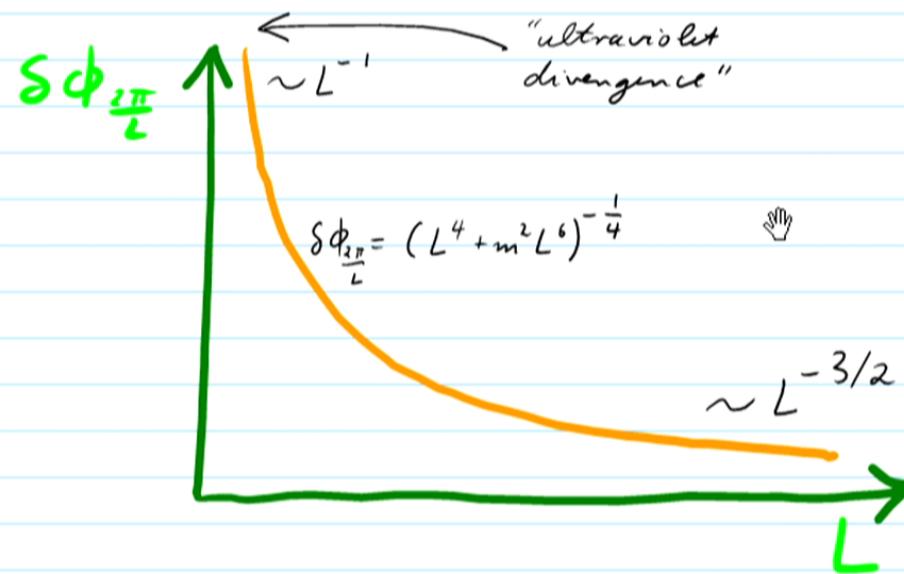
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Notice: The two different scaling behaviors are not clearly visible in this plot.

Recall "Log-Log plots":

$$x := \ln(k), \quad y = \ln(\delta\phi_k)$$

Here:

$$\ln \delta\phi_k = \ln \left( \frac{k^{3/2}}{(m^2 + k^2)^{1/4}} \right) \approx \begin{cases} \ln k \text{ for } k \rightarrow \infty \\ \underbrace{\ln \left( \frac{k^{3/2}}{\sqrt{m}} \right)}_{\Rightarrow -\frac{1}{2}\ln(m) + \frac{3}{2}\ln k} \text{ for } k \rightarrow 0 \end{cases}$$

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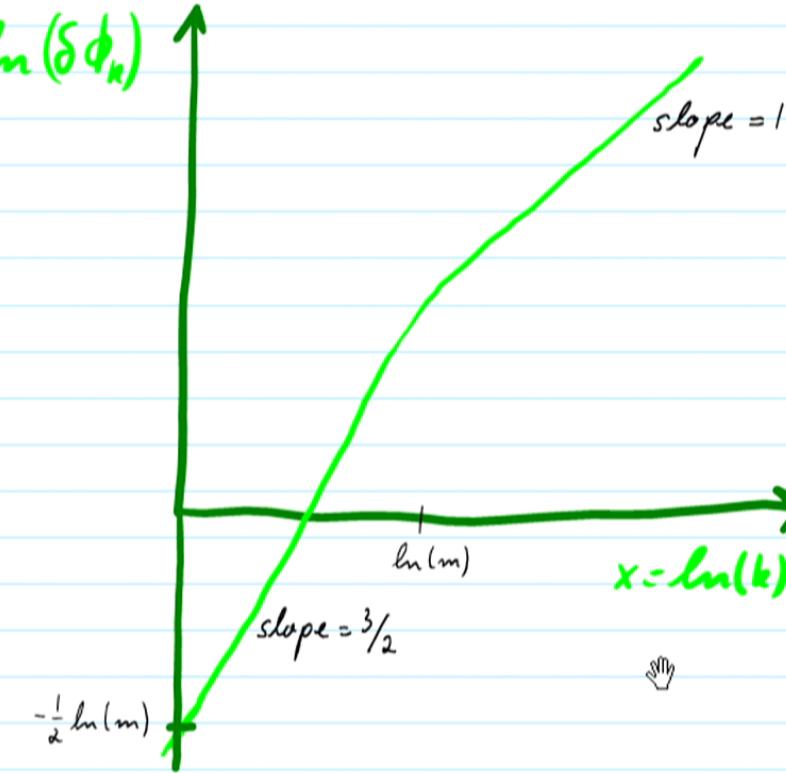
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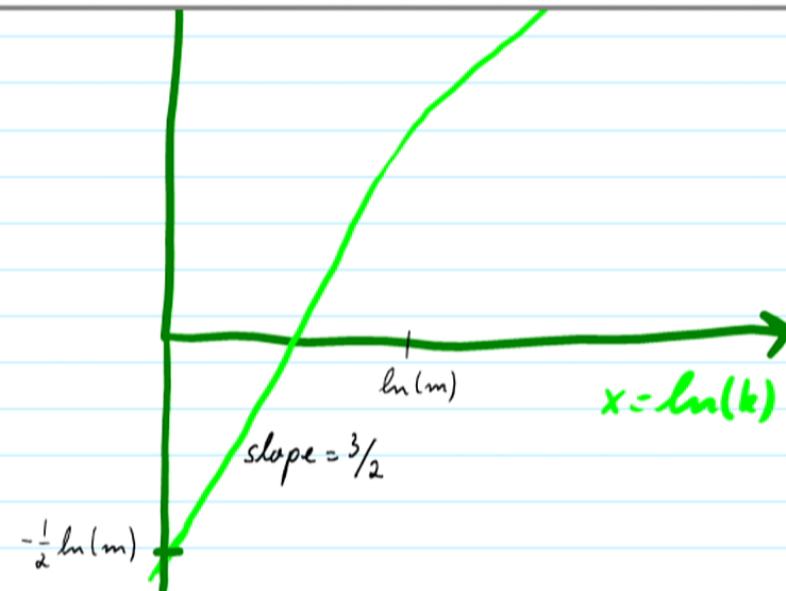
Thus:

$$y \approx \begin{cases} x \text{ for } x \rightarrow \infty \\ -\frac{1}{2} \ln(m) + \frac{3}{2} x \text{ for } x \rightarrow -\infty \end{cases}$$

Plot:

$$y = \ln(\delta\phi_n)$$





□ We notice that, in Minkowski space, large scale (i.e., large  $L$ , small  $k$ ) fluctuations are strongly suppressed, especially if mass  $m \neq 0$ .

## D Regarding the Infrared (IR):

- \* The mass  $m$  does not matter at very short distances, i.e., in the ultraviolet.
- \* But, for large  $L$  the mass term does help to suppress the quantum fluctuations. ( $\delta\phi \sim L^{-3/2}$  vs.  $\sim L^{-1}$ )
- \* Generally, in phenomena of QFT, the mass of particles tends to play a rôle only in the infrared, but not in the ultraviolet, which is why in studies of UV phenomena the mass can often be neglected (e.g. for "perturbative

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- \* We see that QFT predicts divergingly large quantum fluctuations to be found in measurements that resolve smaller and smaller regions.
- \* For small enough regions  $B$  the fluctuations  $\Delta\phi_B^1$  in  $\phi_B^1$  would lead to fluctuations in the Klein Gordon energy momentum tensor that are large enough to cause black holes.  
 $\Rightarrow$  At this scale,  $\approx 10^{-35}$  m, the Planck scale, the notion of distance is assumed to break down

## Time evolution and the fluctuation spectrum:

Recall: We assumed here that the system is (of course always) in the state  $|0\rangle$  which comes along with  $v_k(\gamma)$ ,  $a_k$ ,  $a_k^\dagger$ .

$\Rightarrow$  Our result,

$$\delta \phi_k(\gamma) = \tilde{a}^\dagger(\gamma) k^{3/2} |v_k(\gamma)|$$

then holds for all times  $\gamma > \gamma_0$ , even if evolution at  $\gamma$  highly nonadiabatic!

A Clear: Larger  $v_k$  means larger fluctuations.

B When does  $v_k$  grow most?

$\Rightarrow$  Our result,

$$\delta \phi_k(\gamma) = \tilde{a}'(\gamma) k^{3/2} |v_k(\gamma)|$$

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A Clear: Larger  $v_k$  means larger fluctuations.

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## B When does $v_k$ grow most?

\* Obviously,  $v_k$  may grow adiabatically or also nonadiabatically.

\* But the most efficient mechanism to enlarge  $v_k$  occurs when the mode is nonadiabatically evolving in the sense that its frequency is imaginary:

$$\omega_k(\gamma) = \sqrt{k^2 + m^2 a^2(\gamma) - \frac{a''(\gamma)}{a(\gamma)}}$$

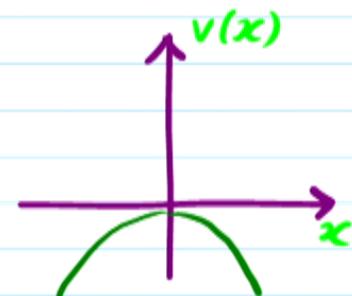
☞ This term may be large enough to make the discriminant negative.

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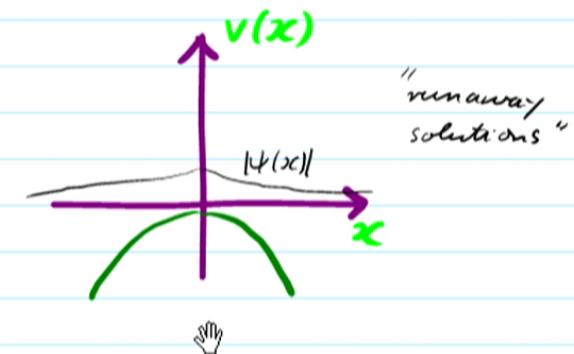
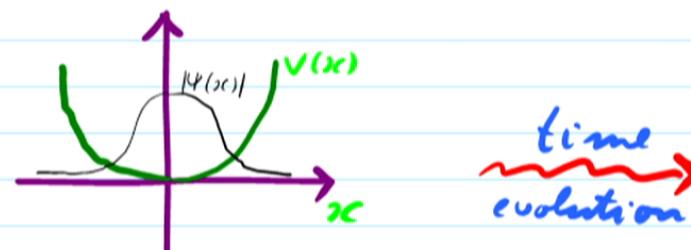
- \* In such a time period, the Klein Gordon equation's solutions are not oscillatory:

$$\hat{\chi}_k''(\gamma) + \omega_k^2(\gamma) \chi_k(\gamma) = 0$$



- \* Instead, there will be one exponentially decaying,

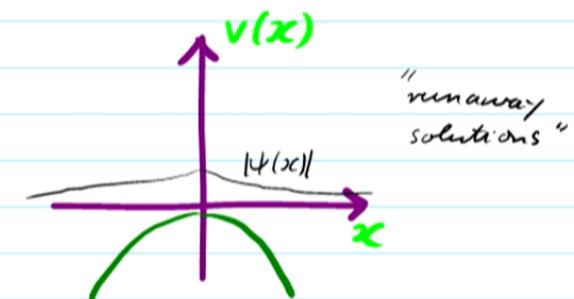
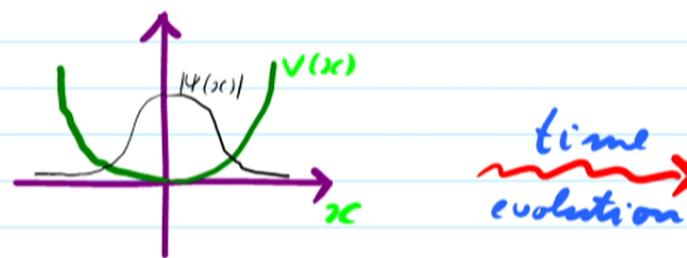
\* Instead, there will be one exponentially decaying and one exponentially growing solution. Inverting a harmonic oscillator is an efficient way to increase  $\Delta\phi$ :



### □ Caveat:

Notice that this argument applies to  $x$  but  $\phi = \frac{1}{a} x$

increase  $\Delta\phi$ :



### □ Caveat:

Notice that this argument applies to  $X$  but  $\phi = \frac{1}{a} X$ .  
Thus,  $\phi$ 's growth is slower than that of  $X$ .

Recall: The equation of motion of  $\phi$  has a friction-type term.

Before we calculate the fluctuation amplitudes explicitly (next lecture).

Before we calculate the fluctuation amplification explicitly (next lecture):

## Relationship of fluctuation amplification to particle creation

□ Assume that at a later time,  $\eta_1$ , the evolution is adiabatic for mode  $k$  (i.e. its  $\omega_k$  changes slowly).

$\Rightarrow$  We can identify  $|\text{vac}_{\eta_1}\rangle$ :



Using the adiabatic vacuum identification criterion, we find the mode function  $\tilde{\psi}_k$  for which:

$$|\tilde{0}\rangle = |\text{vac}_{\eta_1}\rangle$$

⇒ We can identify  $|vac_{\gamma_1}\rangle$ :

Using the adiabatic vacuum identification criterion, we find the mode function  $\tilde{v}_k$  for which:

$$|\tilde{o}\rangle = |vac_{\gamma_1}\rangle$$

□ Case 1: The evolution of mode  $k$  was adiabatic from  $\gamma_0$  to  $\gamma_1$ .

\* Therefore:

$$v_k = \tilde{v}_k \quad \text{and} \quad |o\rangle = |\tilde{o}\rangle$$

\* Therefore:

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\* Therefore:

The state of the system,  $|\Omega\rangle = |\tilde{0}\rangle$ , is still the vacuum state at time  $\eta_1$ :

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•  $\langle \hat{O} \rangle = \langle \hat{O} \rangle_0$ 

The state of the system,  $|\Omega\rangle = |\tilde{\Omega}\rangle$ , is still the vacuum state at time  $\gamma_1$ :

$$|\Omega\rangle = |\tilde{\Omega}\rangle$$

- \* There is no particle creation.

- \* But since  $V_k = \frac{1}{\sqrt{\omega_k(\gamma)}} e^{i \int_{\gamma_0}^{\gamma} w(\gamma') d\gamma'}$ , in general:

$$|V_k(\gamma_1)| \neq |V_k(\gamma_0)|$$

(namely  $|V_k(\gamma)| = \omega_k(\gamma) \cdot \frac{1}{2}$ )

$\Rightarrow$  the fluctuations can be affected even if there is no particle creation.

II Case 2: The evolution was not always adiabatic between  $\eta_0$  and  $\eta_1$ .

\* Then,

$$v_k \neq \tilde{v}_k$$

\* But since both are in the same 2 dimensional solution space to the K.G. equation, there exist  $\alpha_k, \beta_k$ :

Recall: When particle concept applies,  $|\beta_k|$  yields nonadiabatic particle production

$$v_k(\eta) = \alpha_k \tilde{v}_k(\eta) + \beta_k^* \tilde{v}_k^*(\eta)$$

\* Substitute in the fluctuations equation:

$$\delta \phi_k(\eta)^2 = \dot{\alpha}^{-2}(\eta) k^3 |v_k(\eta)|^2$$

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$$\begin{aligned} \delta \phi_k(\gamma)^2 &= \dot{\alpha}^{-2}(\gamma) k^3 |v_k(\gamma)|^2 \\ &= \dot{\alpha}^{-2}(\gamma) k^3 |\alpha_k \tilde{v}_k(\gamma) + \beta_k^* \tilde{v}_k^*(\gamma)|^2 \end{aligned}$$



particle production

\* Substitute in the fluctuations equation:

$$\delta \phi_k(\eta)^2 = \dot{\alpha}^2(\eta) k^3 |v_k(\eta)|^2$$

$$= \dot{\alpha}^2(\eta) k^3 |\alpha_k \tilde{v}_k(\eta) + \beta_k' \tilde{v}_k'(\eta)|^2$$

\* For clarity, assume that the nonadiabatic period is over by  $\eta_1$ .

\* Also, assume that spacetime is again Minkowski around  $\eta_1$ . (Thus, we focus on nonadiabatic effects only)

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\* In this case :

$$\tilde{v}_n(\gamma) = \frac{1}{\Gamma \omega_n(\gamma_1)} e^{i \omega_n(\gamma_1) \gamma} \quad \text{for all } \gamma \approx \gamma_1$$

$$\Rightarrow \delta \phi_k^2(\gamma) = \tilde{\alpha}^2(\gamma) \frac{k^3}{\omega_n(\gamma_1)} \left( |\alpha_n|^2 + |\beta_n|^2 - 2 \operatorname{Re} (\alpha_n \beta_n e^{2i \omega_n(\gamma_1) \gamma}) \right)$$

Over a long enough time period this term averages

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*Over a long enough time period this term averages 0.*

\* We use:  $|\alpha_k|^2 - |\beta_k|^2 = 1$  (W)

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$$\Rightarrow \delta\phi_k^2(\eta) = \tilde{\alpha}^{-2}(\eta) \frac{k^3}{\sqrt{k^2 + m^2(\eta)}} \left( 1 + \underbrace{2|\beta_k|^2}_{\uparrow} \right) \quad (\text{F})$$

This term is the same, whether or not the expansion of spacetime has been adiabatic.

This term is only non zero if the evolution was non-adiabatic.

\* Notice:  $|\beta_k|$  and  $|\alpha_k|$  can both become very large. This is consistent with

if the evolution of the source, environment or noise is adiabatic, then the expansion of spacetime has been adiabatic.

the evolution was non-adiabatic.

\* Notice:  $|f_{\text{pl}}|$  and  $|d_{\text{pl}}|$  can both become very large. This is consistent with the Wronskian condition, (W).

### \* Particle production:

Recall that the expected number of created particles is also given by  $|f_{\text{pl}}|^2$ :

$$\begin{aligned}\bar{N}_k(z_i) &= \langle \Omega | \tilde{N}_k | \Omega \rangle = \dots \\ &= |f_{\text{pl}}|^2\end{aligned}$$

$\tilde{N}_k = \tilde{a}_k^+ \tilde{a}_k$

\* Remark:

Bert (F) holds even if  $\omega^2 \ll 0$  at  $\eta_1$ ! We know what we mean by field fluctuations even when we do not have a concept of vacuum and particles.

$\Rightarrow$  'Quantum fields more fundamental than quantum particles.'



## □ Fluctuations in proper coordinates as opposed to comoving coordinates

\* We have:  $d = a(\eta) L$ ,  $p = \frac{1}{a(\eta)} k$

$\uparrow$ proper length	$\uparrow$ comoving length	$\uparrow$ proper momentum
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comoving		momentum
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## □ Fluctuations in proper coordinates as opposed to comoving coordinates

\* We have:  $d = a(\gamma) L$ ,  $p = \frac{1}{a(\gamma)} k$

$\uparrow$   
 proper length       $\uparrow$   
 comoving length       $\uparrow$   
 proper momentum       $\nwarrow$   
 comoving momentum

\* Therefore,  $\delta\phi_p^2(\gamma) = a^{-2}(\gamma) \frac{k^3}{\sqrt{k^2 + m^2}} (1 + 2|\beta_{ap}|^2)$  becomes:

$$\delta\phi_p^2(\gamma) = a^{-2} \frac{a^3 p^3}{\sqrt{a^2 p^2 + m^2}} (1 + 2|\beta_{ap}|^2)$$



$$= \underbrace{\frac{p^3}{\sqrt{p^2 + m^2}}}_{\uparrow} (1 + 2|\beta_{ap}|^2)$$

\* Therefore,  $\delta\phi_r^2(\gamma) = \tilde{a}^{-2}(\gamma) \frac{k^3}{\sqrt{k^2 + \omega^2 m^2}} (1 + 2|\beta_{ap}|^2)$  becomes:

$$\delta\phi_r^2(\gamma) = \tilde{a}^{-2} \frac{a^3 p^3}{\sqrt{a^2 p^2 + a^2 m^2}} (1 + 2|\beta_{ap}|^2)$$

$$= \frac{p^3}{\sqrt{p^2 + m^2}} (1 + 2|\beta_{ap}|^2)$$

Same as Minkowski



\* Note: The nonadiabatic term depends on  $p$ .

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\* Note: This was the case when we end in a Minkowski

$$\Omega(p^2) = n \frac{1}{\sqrt{p^2 + m^2}} (1 - \alpha_{\text{Pap}})$$

$$= \frac{p^3}{\sqrt{p^2 + m^2}} (1 + 2|\beta_{\text{Pap}}|^2)$$

Same as Minkowski



\* Note: The nonadiabatic term depends on  $p$ .

\* Note: This was the case where we end in a Minkowski space. We see that in this case we must get back the original Minkowski spectrum if the evolution from  $\gamma_0$  to  $\gamma_1$  was adiabatic.



$$= \underbrace{\frac{p^3}{\sqrt{p^2 + m^2}}}_{\text{Same as Minkowski}} (1 + 2|\beta_{ap}|^2)$$

\* Note: The nonadiabatic term depends on  $p$ .

\* Note: This was the case where we end in a Minkowski space. We see that in this case we must get back the original Minkowski spectrum if the evolution from  $\eta_0$  to  $\eta_1$  was adiabatic.



\* Note: Also in thermodynamics, "adiabatic" processes are reversible processes.