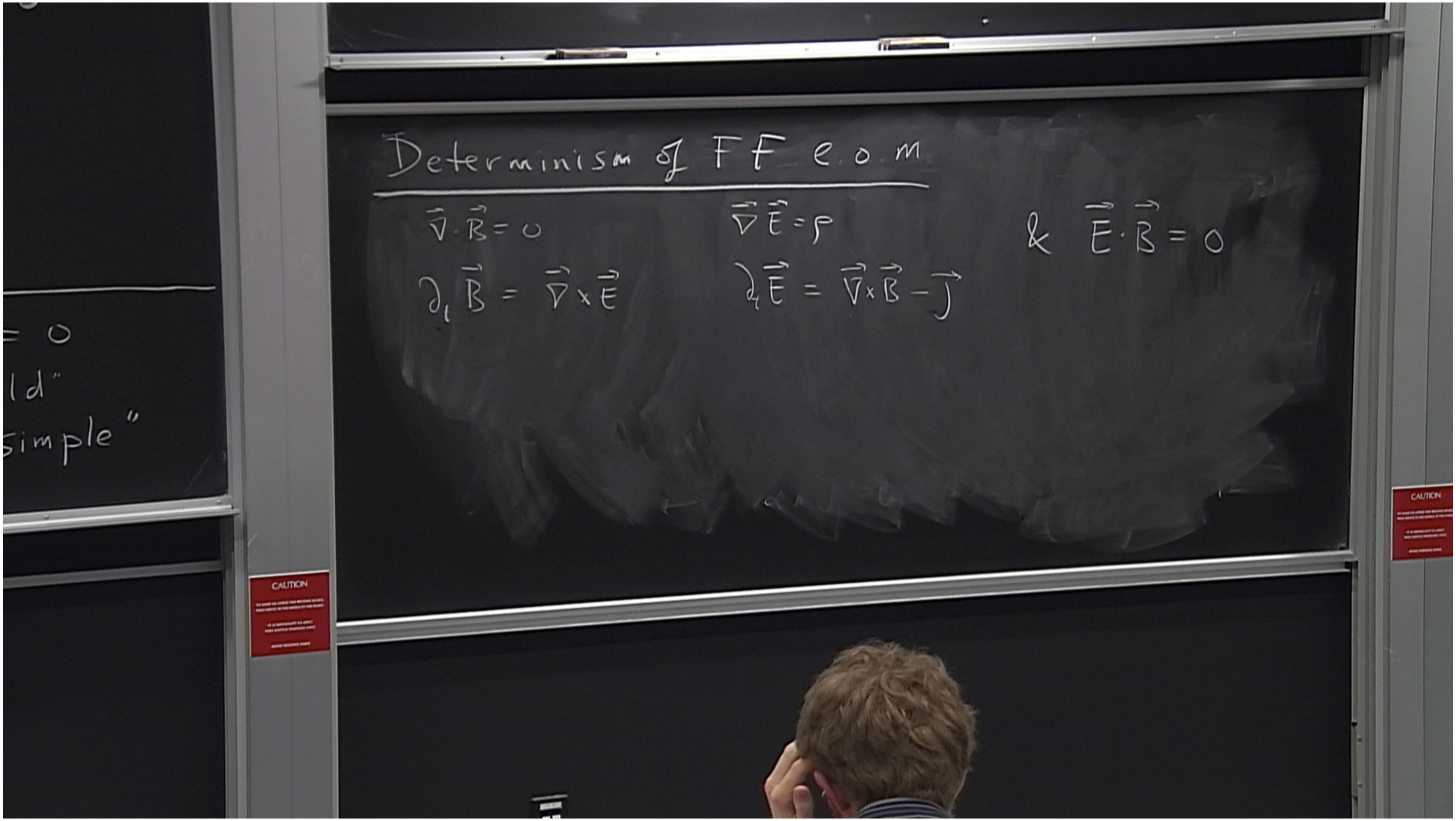
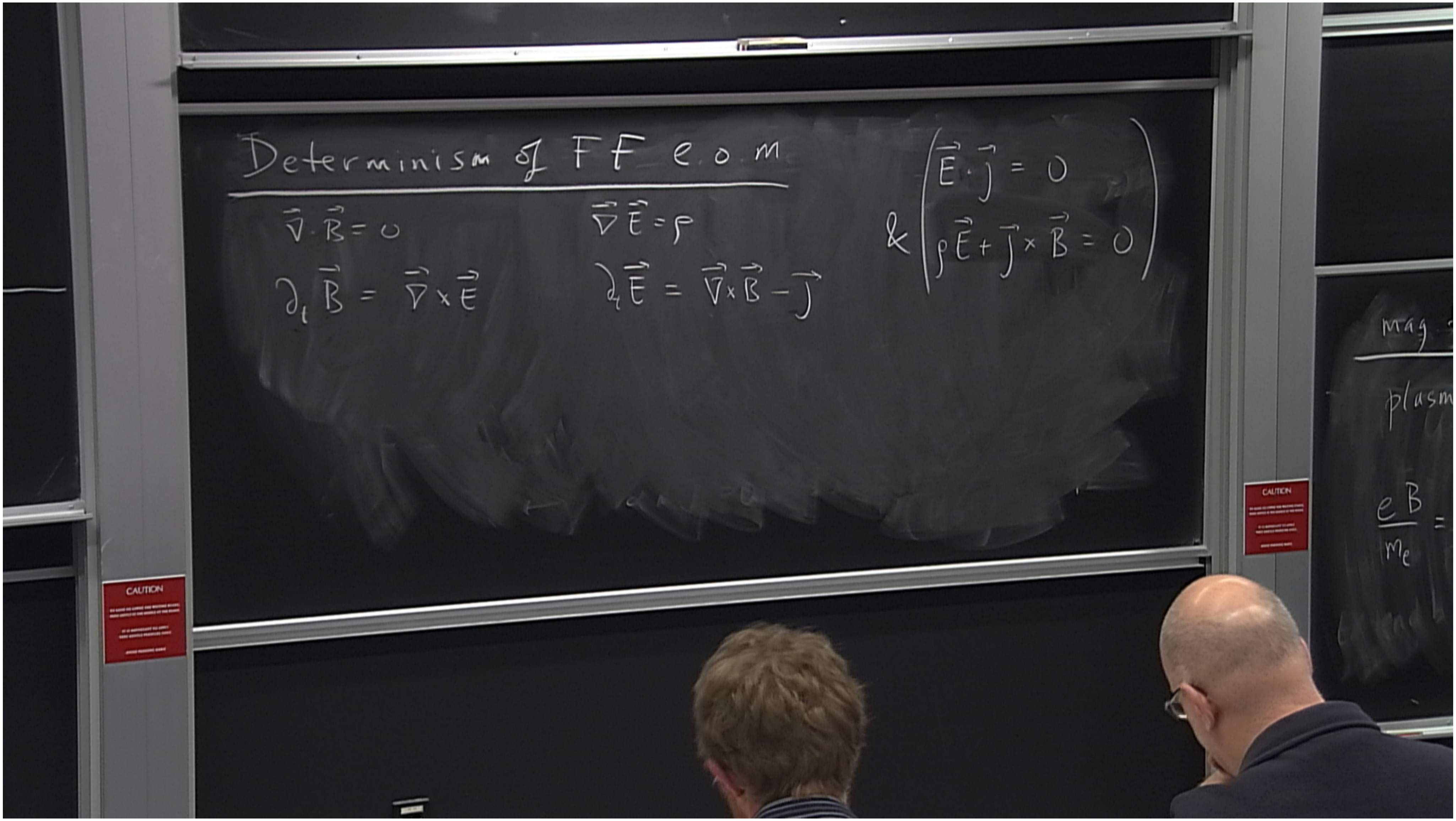
Title: Spacetime approach to force-free magnetospheres - Lecture 4

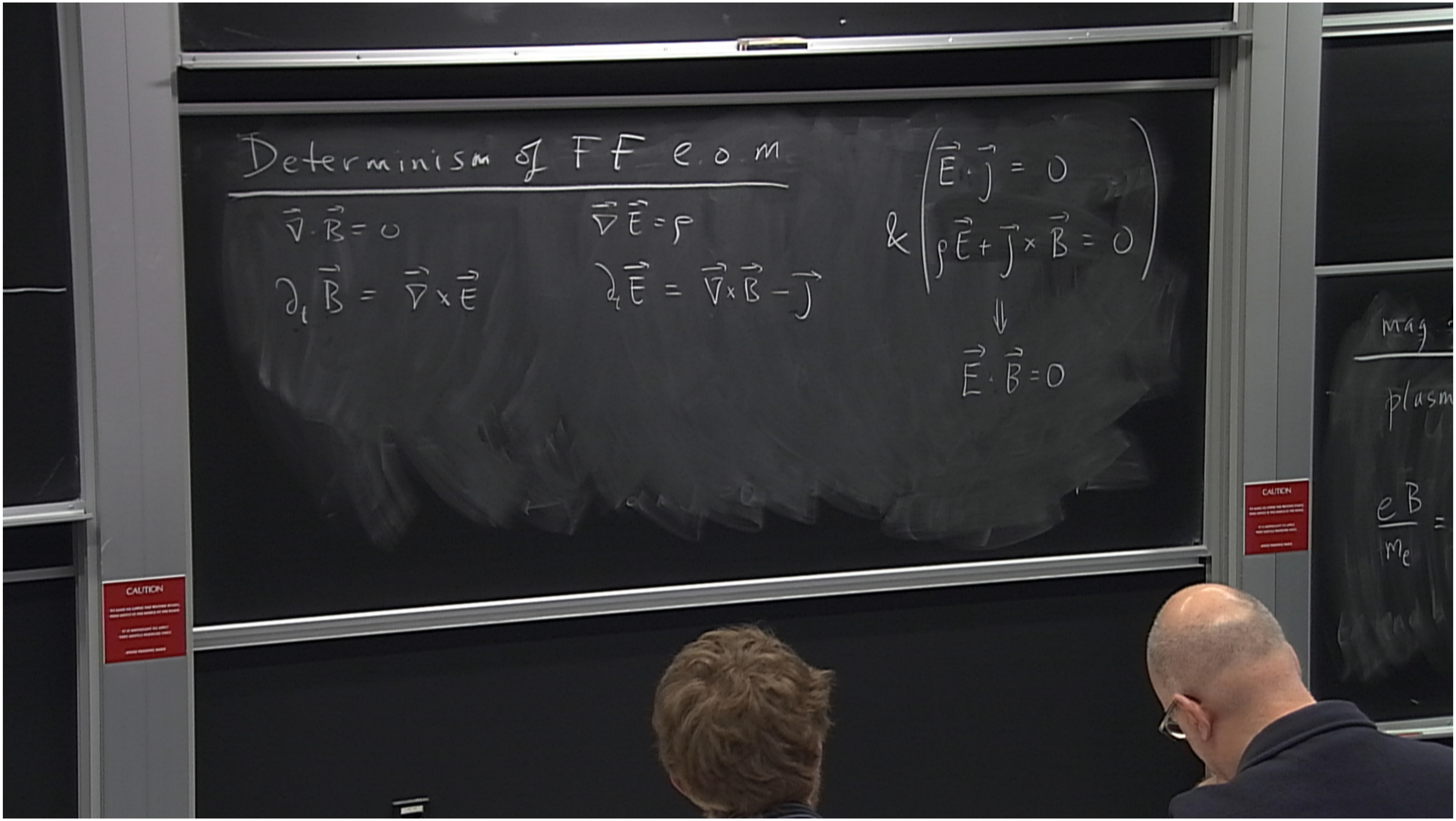
Date: Feb 26, 2014 10:45 AM

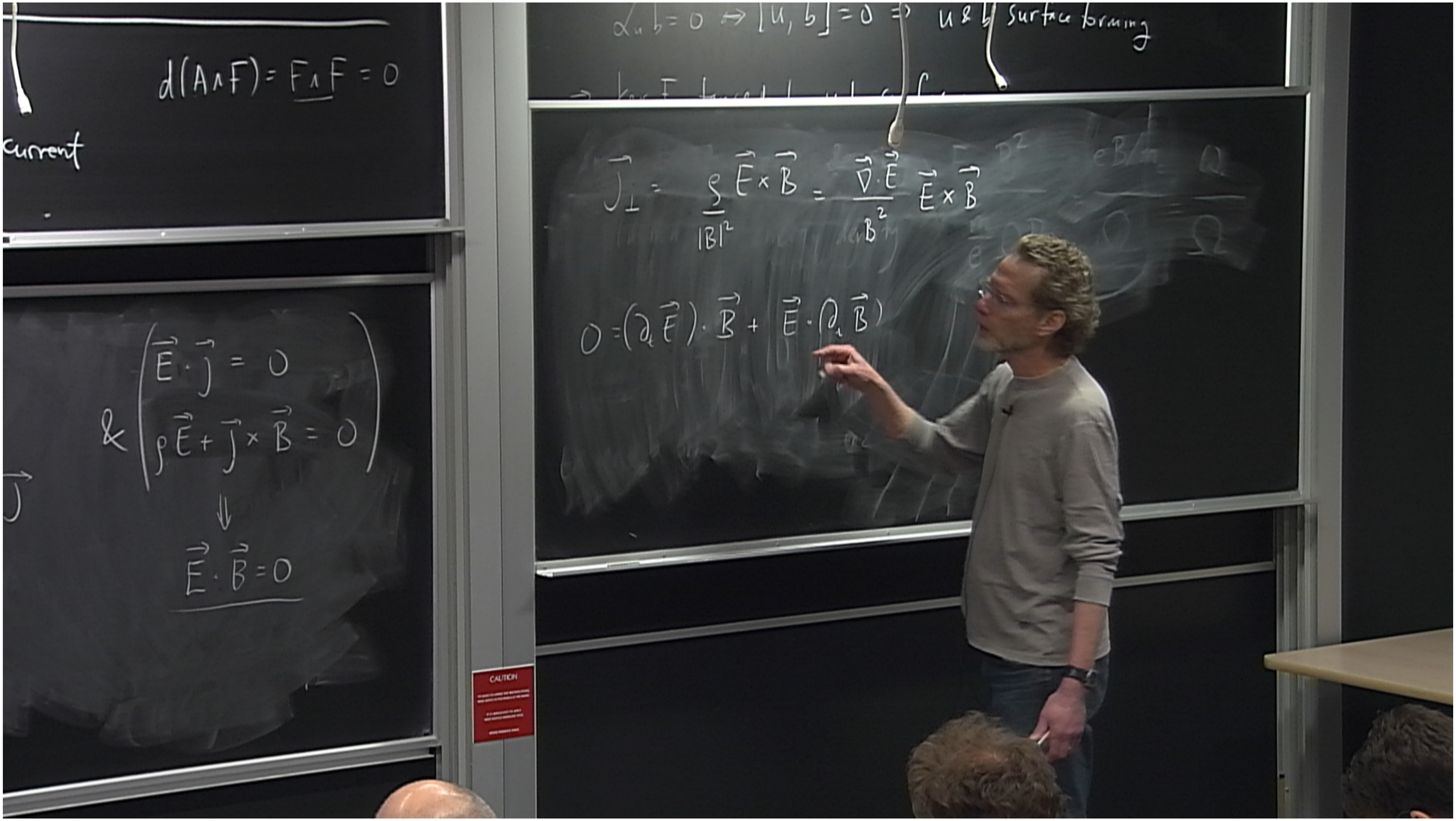
URL: http://pirsa.org/14020158

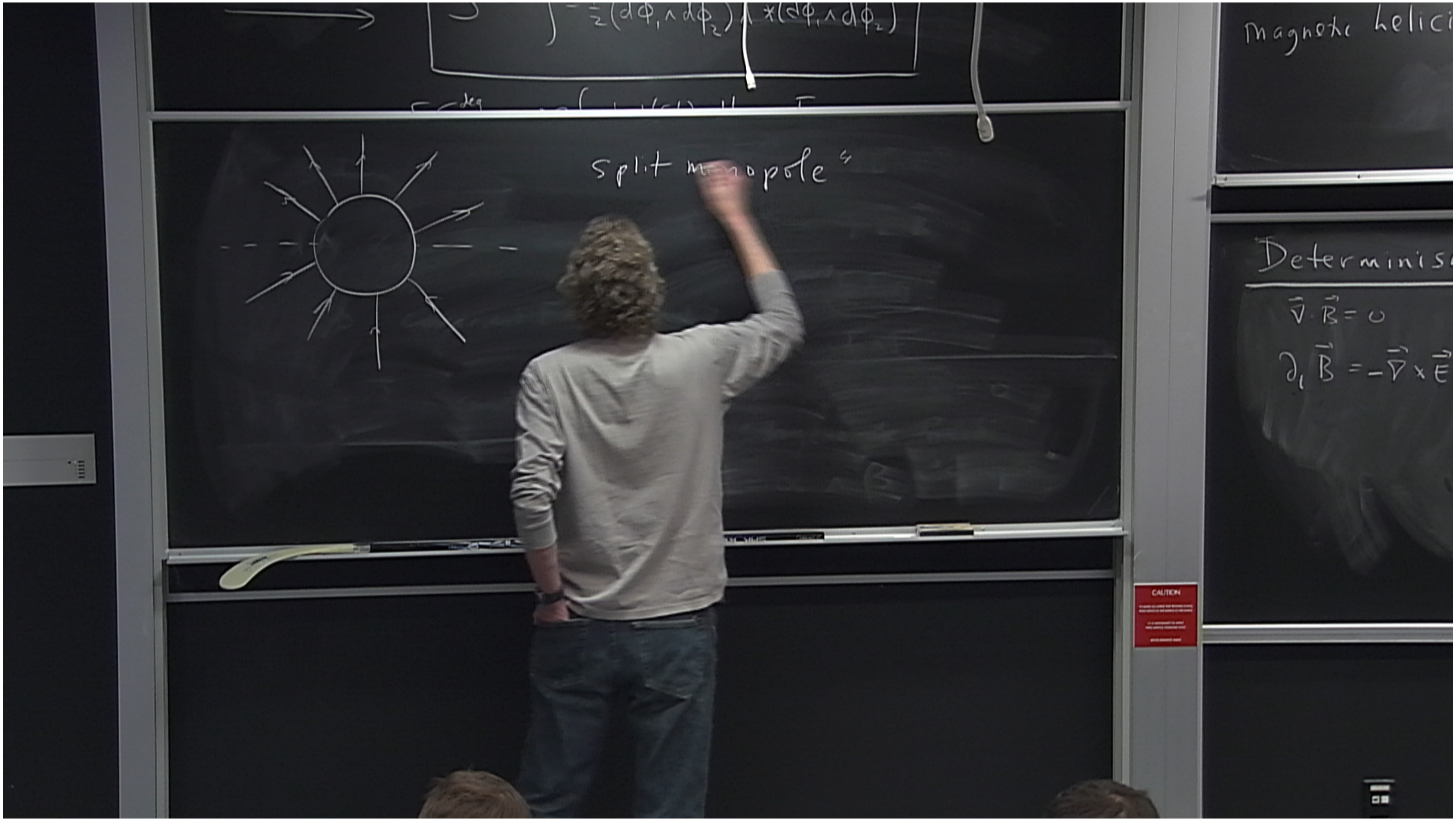
Abstract:

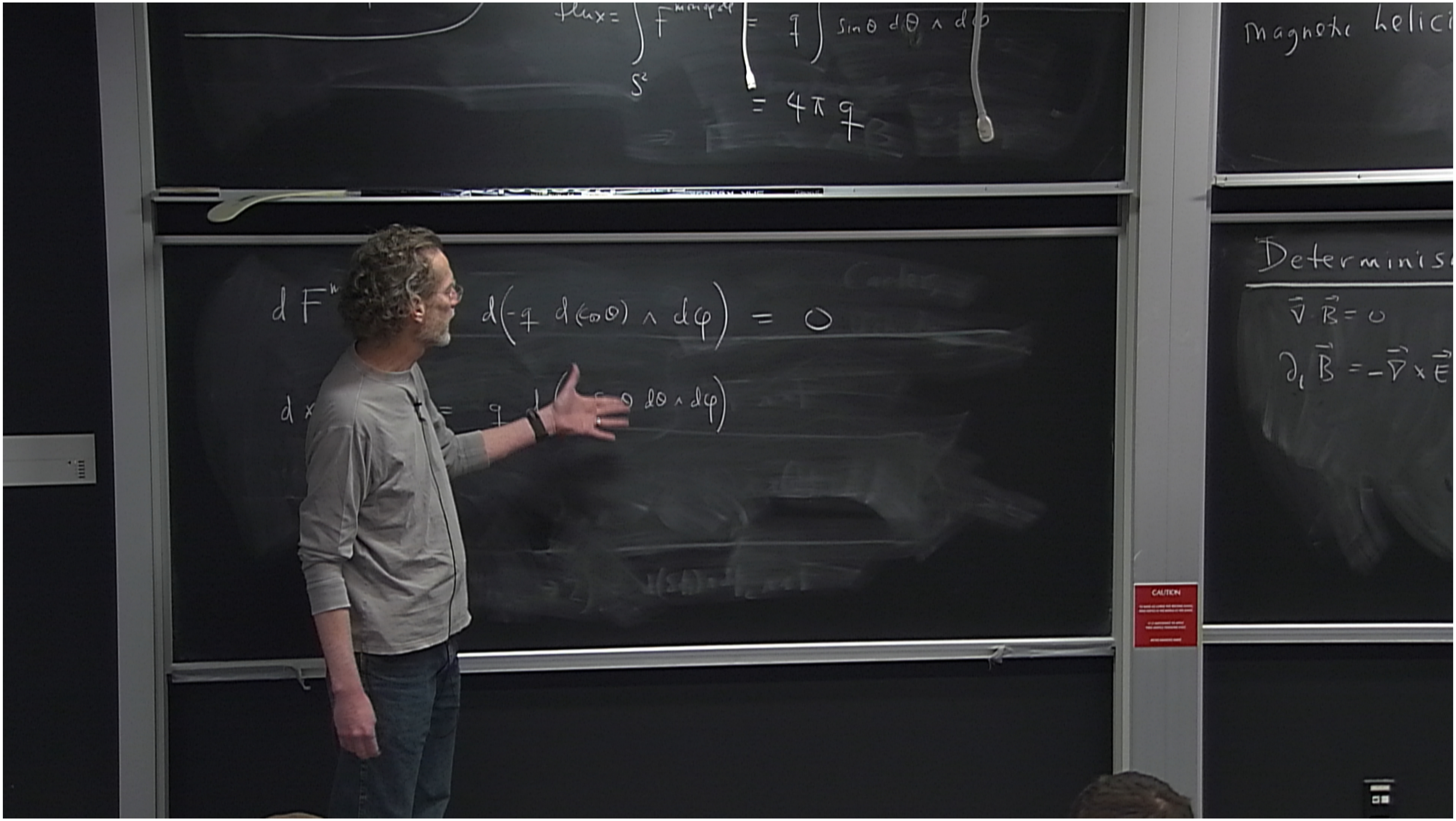

Determinism of FF e.o.m
$\vec{\nabla}\cdot\vec{B} = 0$
$\vec{\nabla}\vec{E} = \rho$
$\& \ \vec{E}\cdot\vec{B} = 0$
$\partial_t \vec{B} = \vec{\nabla}\times\vec{E}$
$\partial_t \vec{E} = \vec{\nabla}\times\vec{B} - \vec{J}$
CAUTION

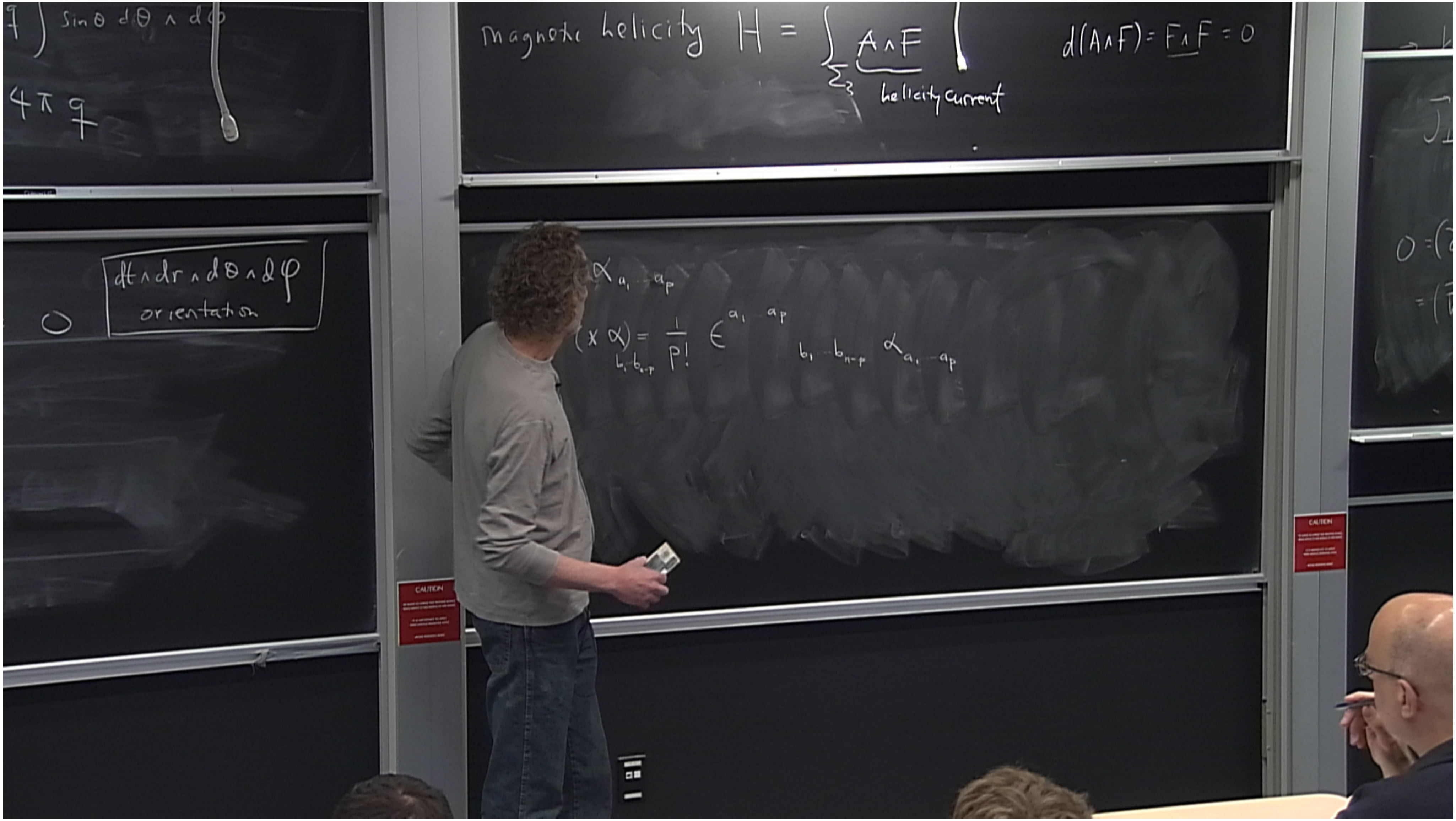

Determinism of FF e.o.m
$\vec\nabla \cdot \vec B = 0$
$\vec\nabla \vec E = \rho$
$\partial_t \vec B = \vec\nabla \times \vec E$
$\partial_t \vec E = \vec\nabla \times \vec B - \vec J$
& $\vec E \cdot \vec J = 0$
$\rho \vec E + \vec J \times \vec B = 0$
CAUTION
CAUTION
mag
plasm
$\frac{eB}{m_e} =$

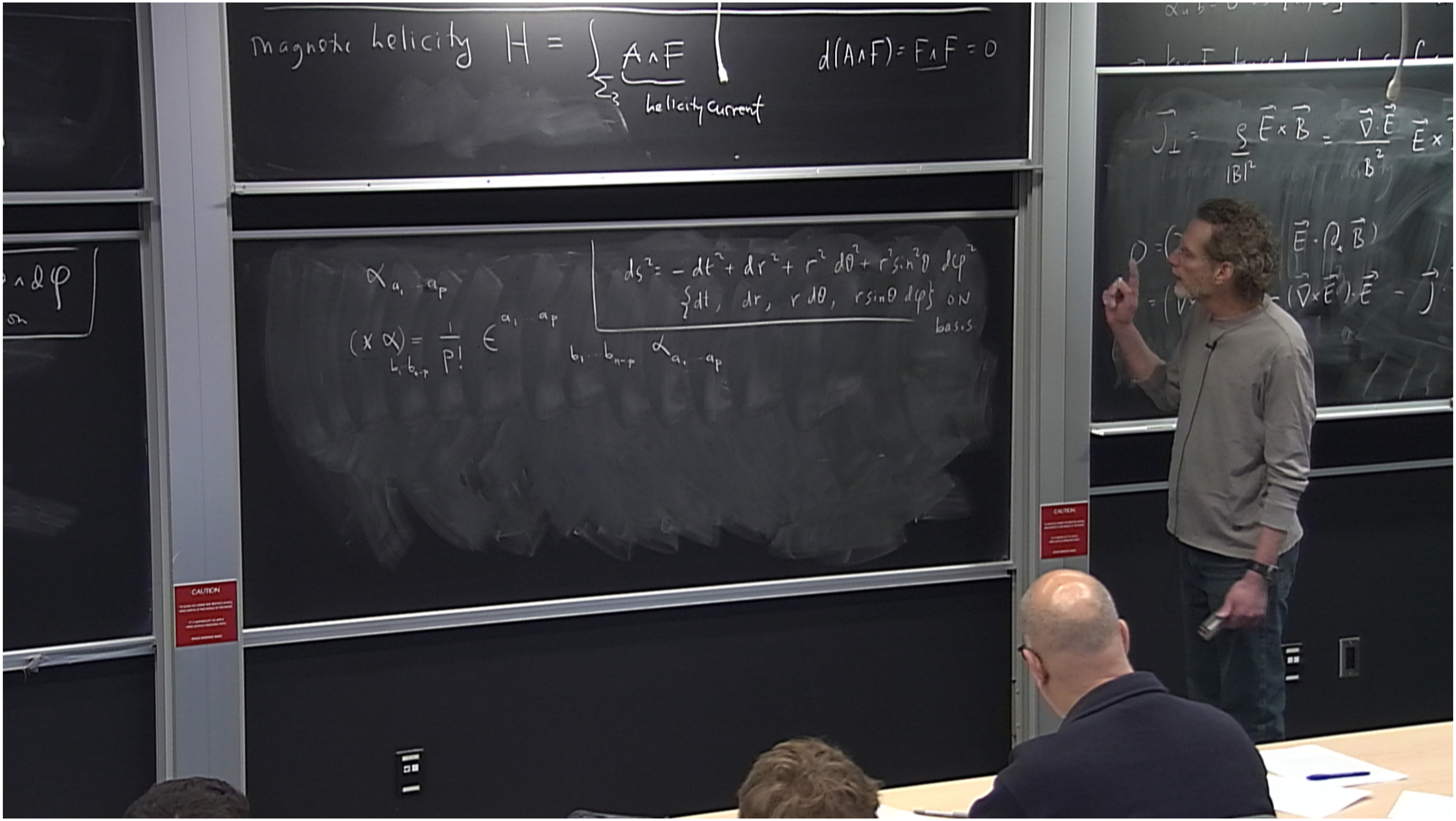

Determinism of FF e.o.m
$\vec{\nabla}\cdot\vec{B}=0$
$\vec{\nabla}\vec{E}=\rho$
$\partial_t\vec{B}=\vec{\nabla}\times\vec{E}$
$\partial_t\vec{E}=\vec{\nabla}\times\vec{B}-\vec{J}$
$\&\left(\begin{array}{l}\vec{E}\cdot\vec{J}=0\\ \rho\vec{E}+\vec{J}\times\vec{B}=0\end{array}\right)$
$\Downarrow$
$\vec{E}\cdot\vec{B}=0$
mag
plasm
$\frac{eB}{m_e}=$

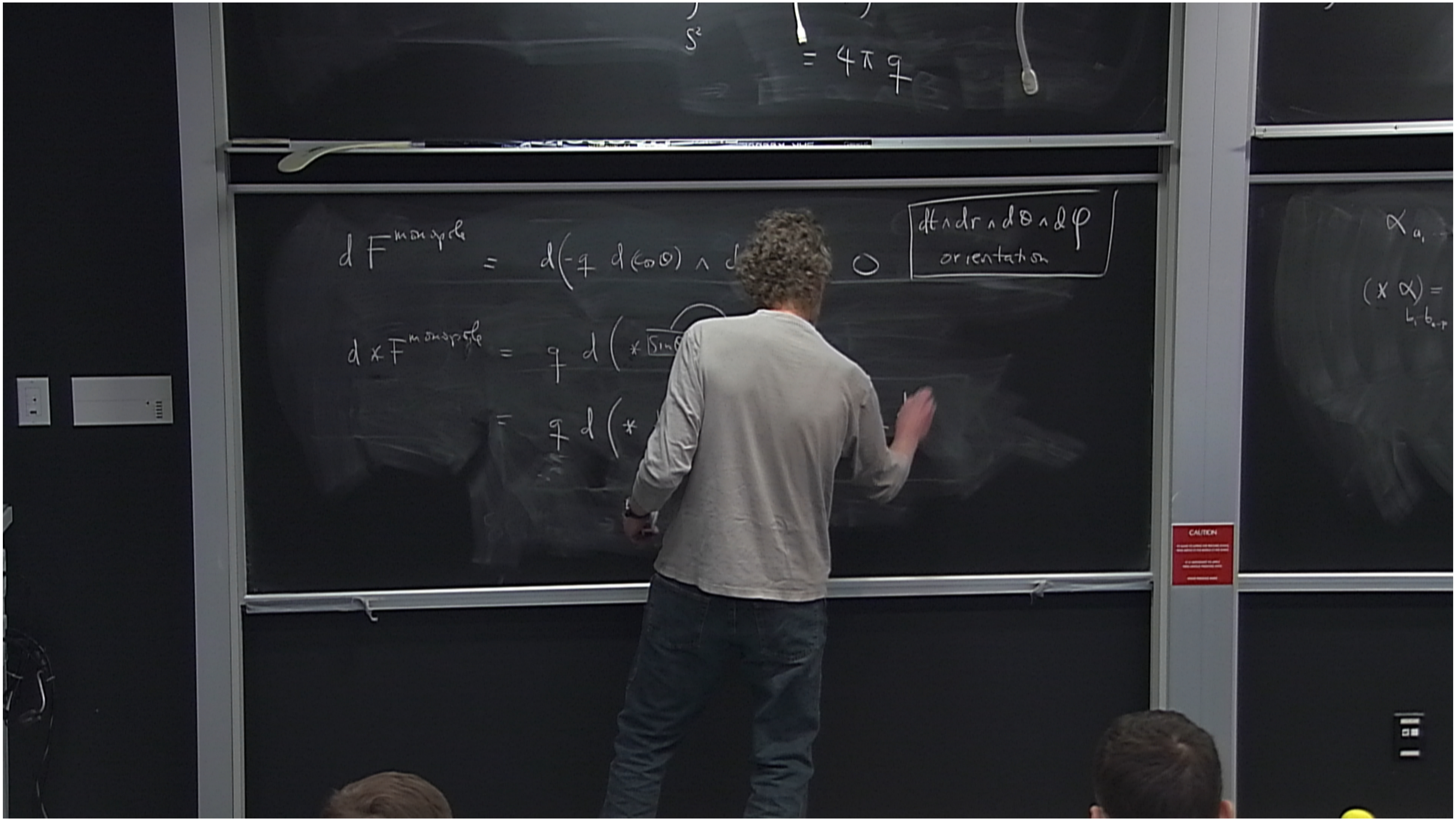

u & b surface forming
d(A∧F) = F∧F = 0
current
J⊥ = ρ/|B|² E × B = ∇·E/B² E × B
0 = (∂t E)·B + E·(∂t B)
E·j = 0
& ρE + j × B = 0
E·B = 0

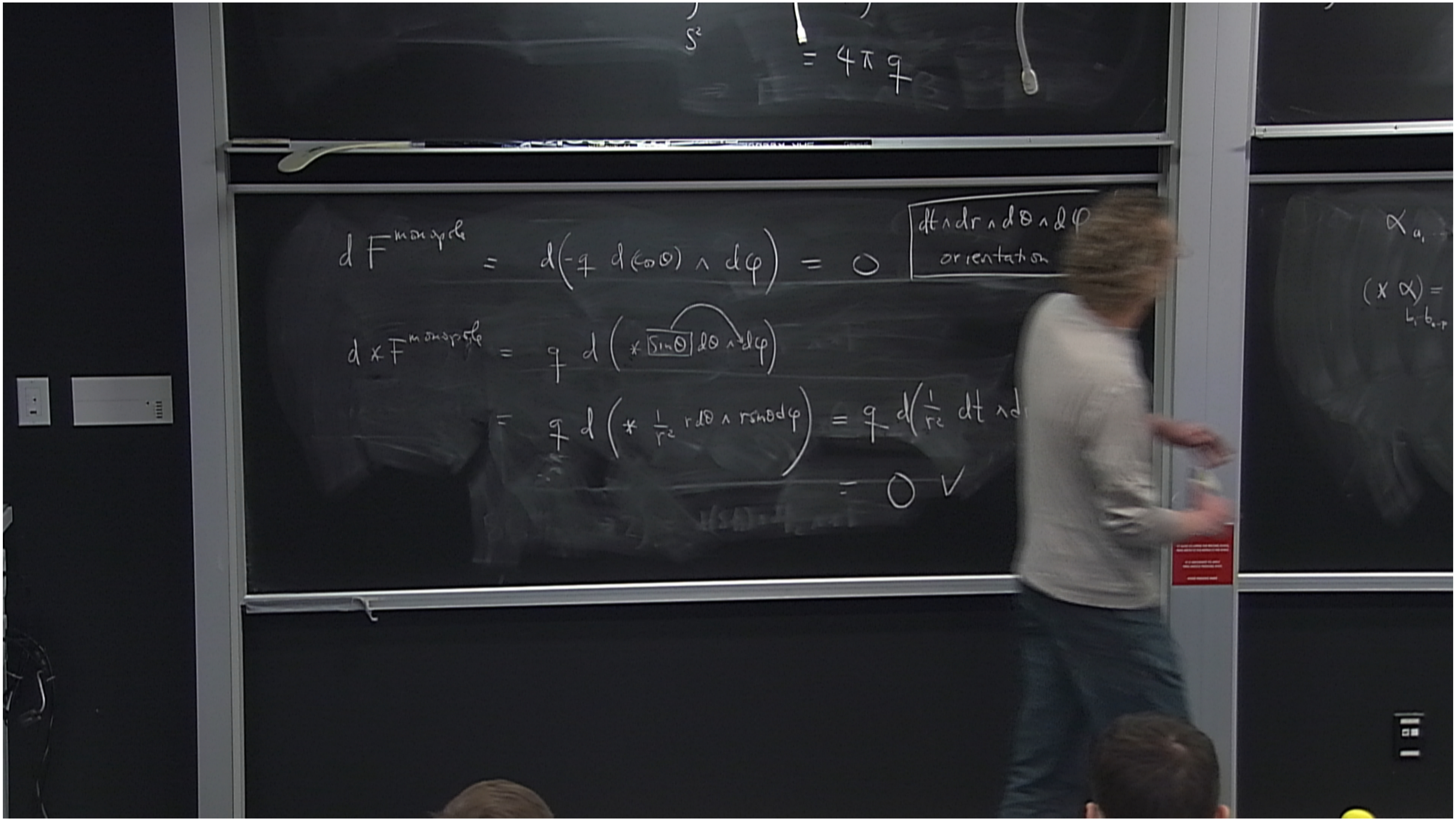

$\vec{\nabla} \cdot \vec{B} = 0$
$\partial_t \vec{B} = -\vec{\nabla} \times \vec{E}$

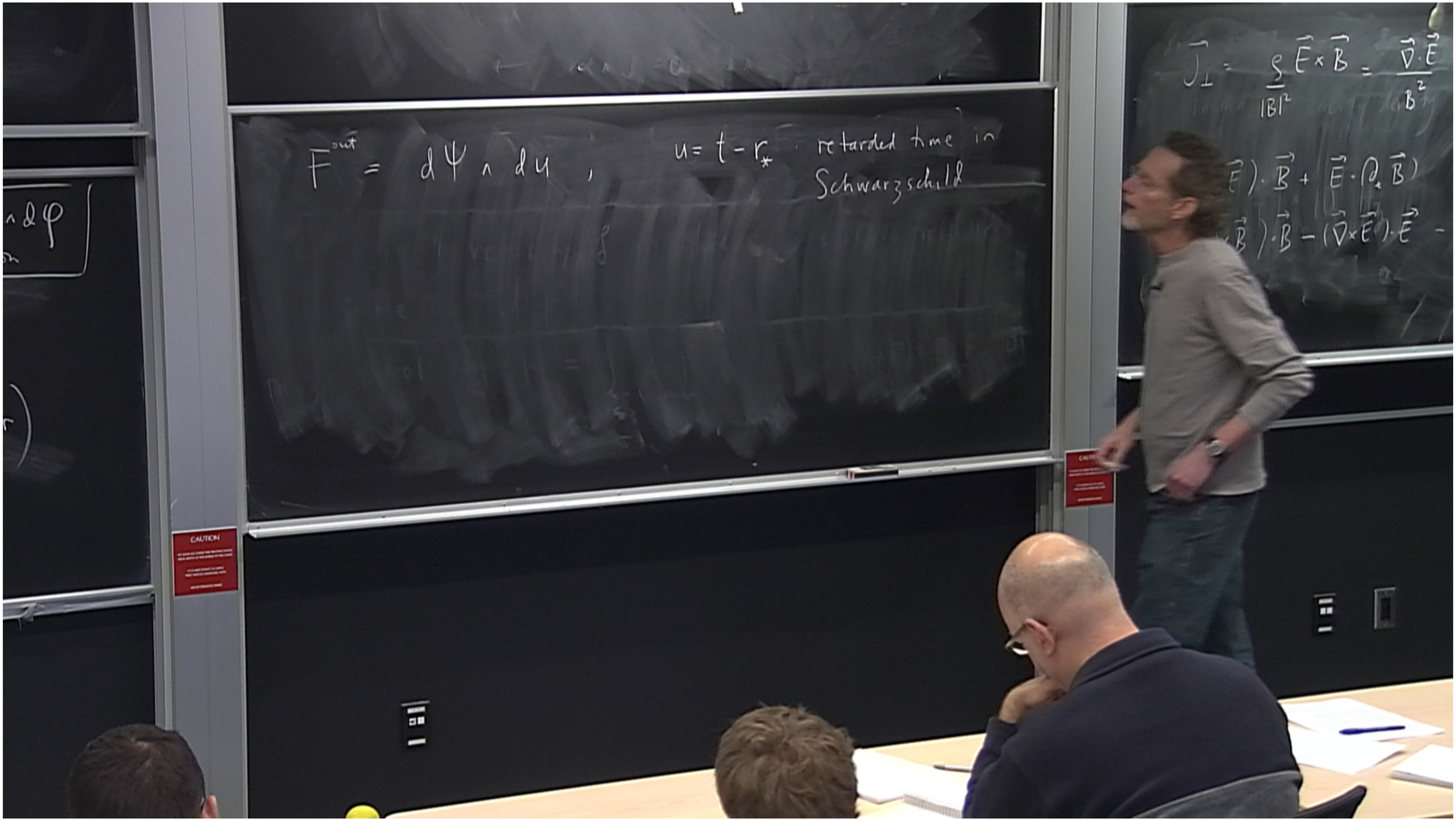

flux =
$= 4\pi q$
magnetic helici
$d(-q\, d(\cos\theta) \wedge d\varphi) = 0$
Determinis
$\vec{\nabla}\cdot\vec{B} = 0$
$\partial_t \vec{B} = -\vec{\nabla}\times\vec{E}$
CAUTION

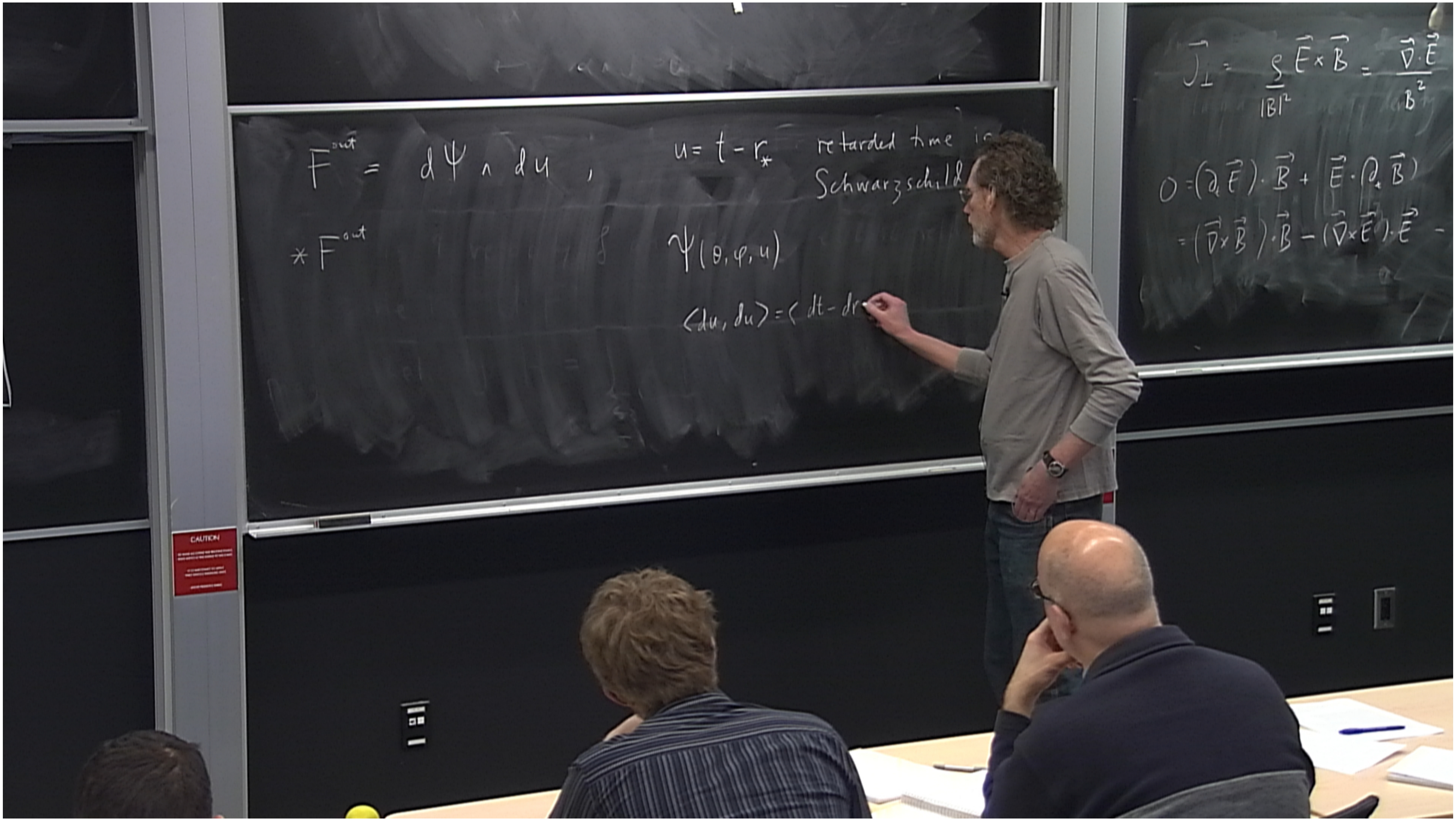

sin θ dθ ∧ dφ
4π q
magnetic helicity H = ∫ A∧F
helicity current
d(A∧F) = F∧F = 0
dt∧dr∧dθ∧dφ
orientation
CAUTION
CAUTION

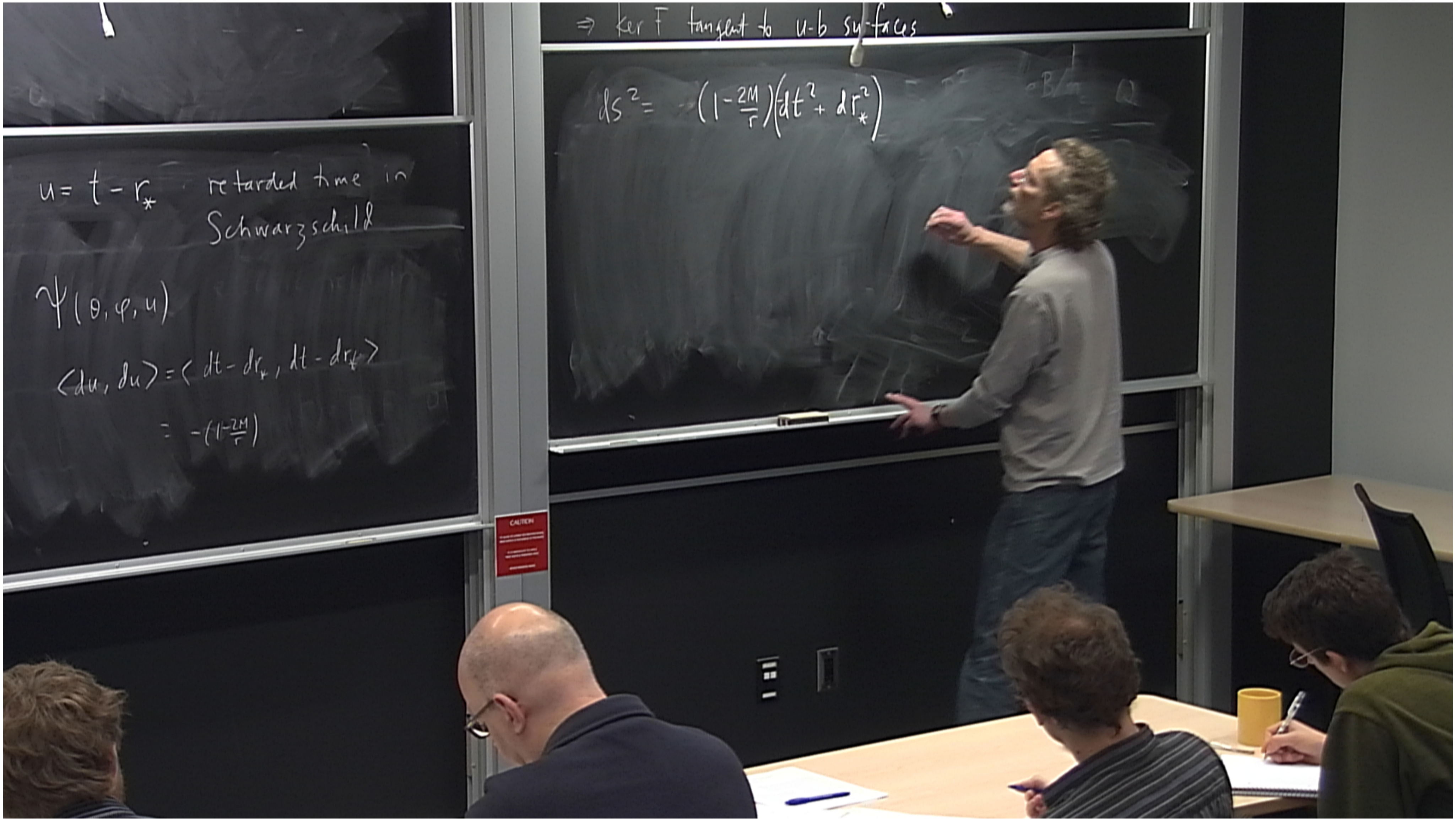

magnetic helicity $H = \int_{\Sigma_3} \underbrace{A \wedge F}_{\text{helicity current}}$
$d(A \wedge F) = F \wedge F = 0$
$\alpha_{a_1 \cdots a_p}$
$(\star\alpha)_{b_1 \cdots b_{n-p}} = \frac{1}{p!} \epsilon^{a_1 \cdots a_p}{}_{b_1 \cdots b_{n-p}} \alpha_{a_1 \cdots a_p}$
$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2$
$\{dt, dr, r\,d\theta, r\sin\theta\, d\varphi\}$ ON basis
$\vec{J}_\perp = \frac{\rho}{|B|^2} \vec{E} \times \vec{B} = \frac{\vec{\nabla}\cdot\vec{E}}{B^2} \vec{E} \times$

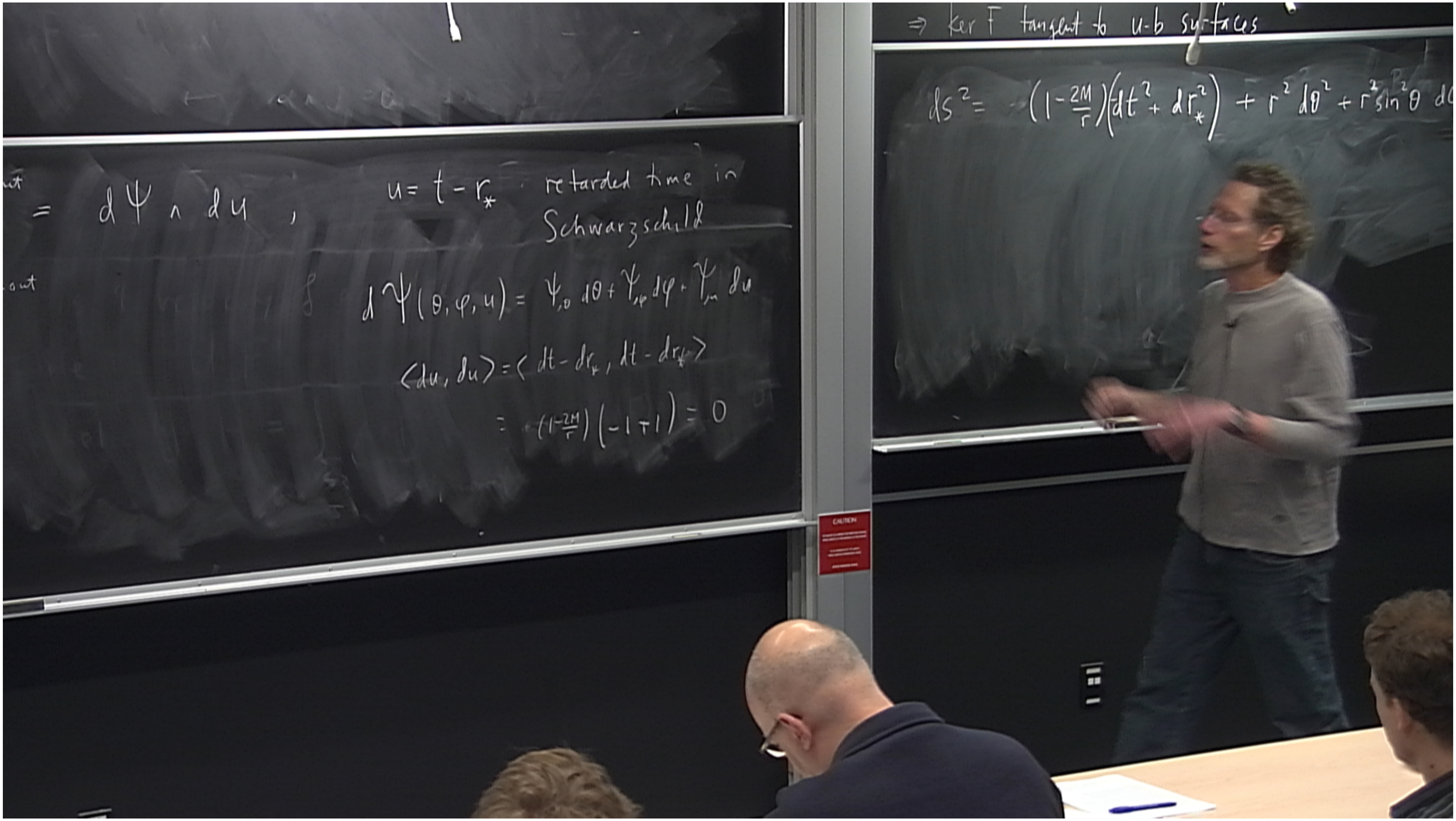

= 4π q
dt ∧ dr ∧ dθ ∧ dφ
orientation
d F^monopole = d(-q d(cos θ) ∧ d
d ∗ F^monopole = q d(∗
= q d(∗

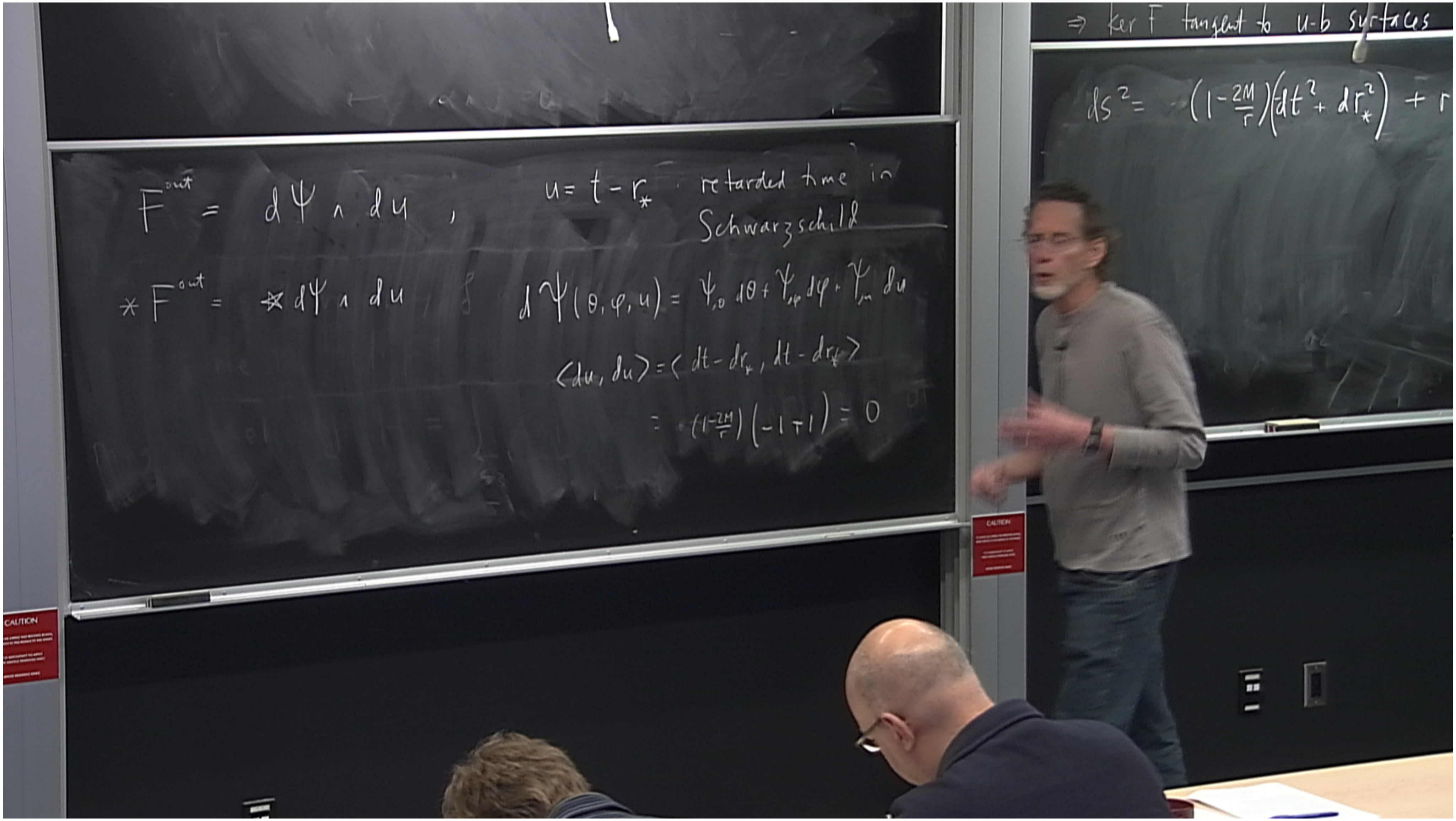

S²
= 4π q
dt ∧ dr ∧ dθ ∧ dφ
orientation
d F^monopole = d(-q d(cos θ) ∧ dφ) = 0
d * F^monopole = q d(* sin θ dθ ∧ dφ)
= q d(* 1/r² r dθ ∧ r sin θ dφ) = q d(1/r² dt ∧ d
= 0 ✓
(* α) =

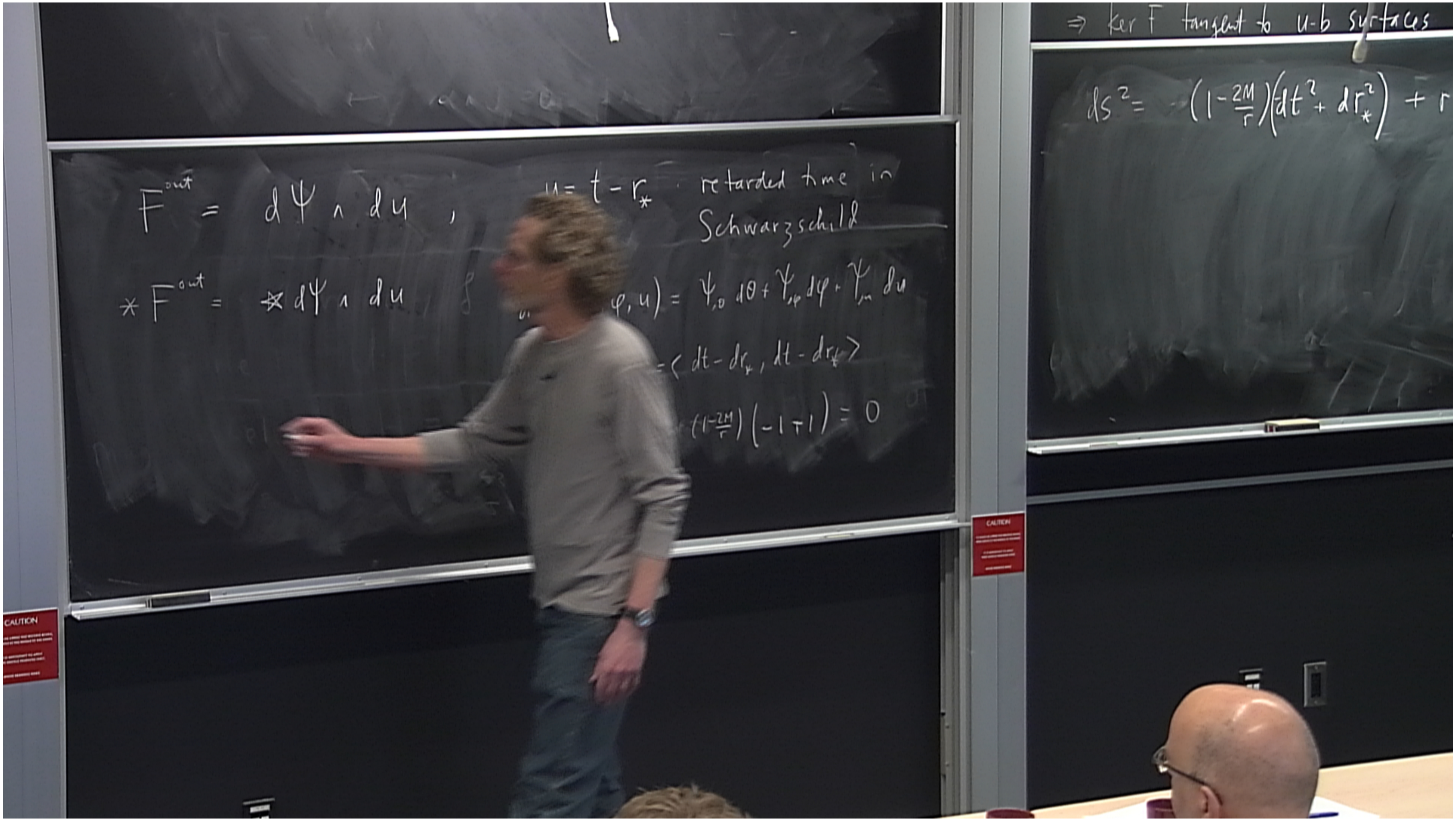

F^out = dΨ ∧ du ,
u = t - r_*
retarded time in Schwarzschild
J_⊥ = ρ E × B / |B|^2 = ∇·E / B^2


F^out = dψ ∧ du ,
u = t − r_*   retarded time in
Schwarzschild
*F^out
ψ(θ, φ, u)
⟨du, du⟩ = ⟨ dt − dr
J_⊥ = ρ/|B|² E × B = ∇·E/B²
0 = (∂_t E)·B + E·∂_t B
= (∇×B)·B − (∇×E)·E
CAUTION

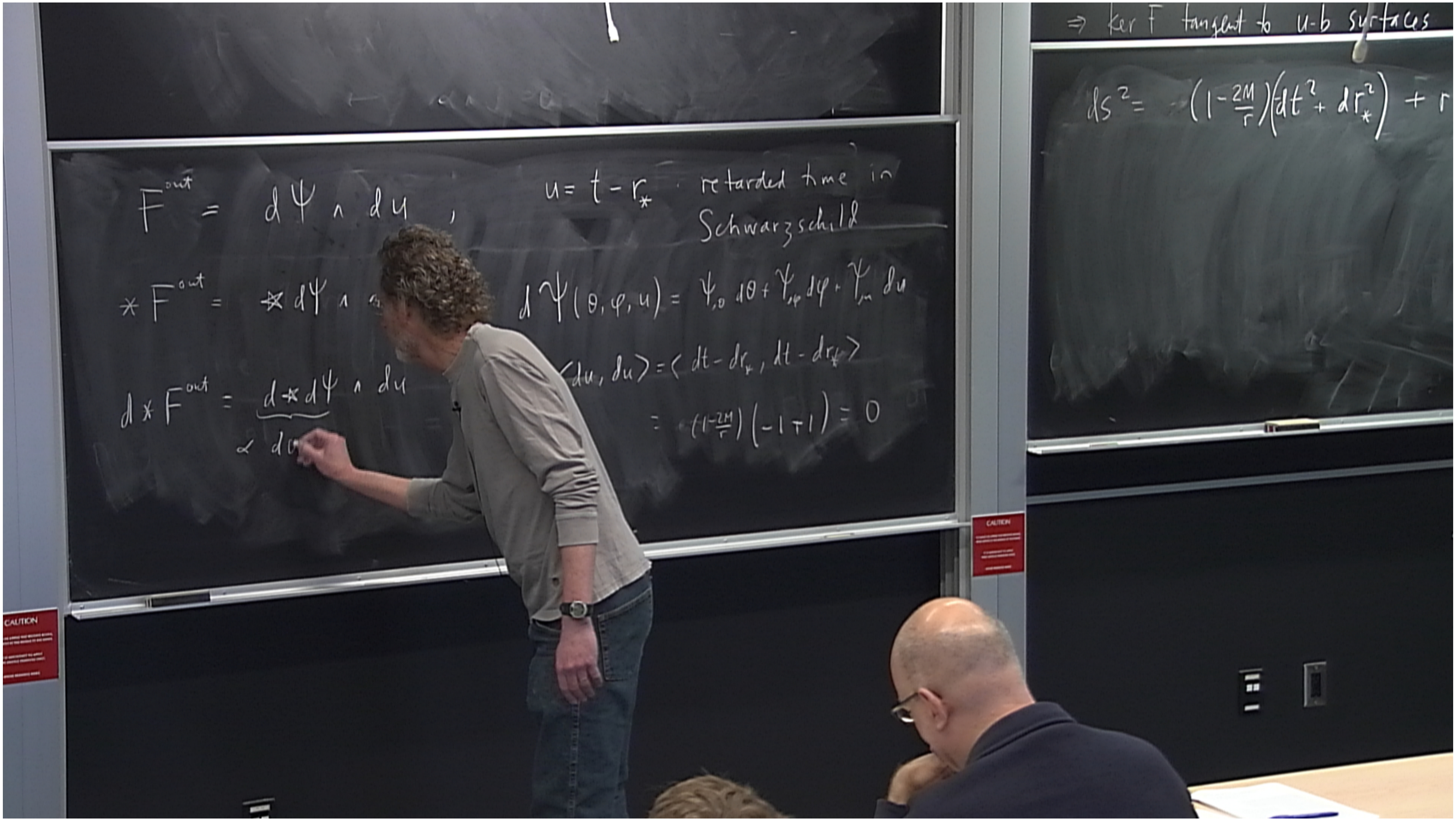

⇒ ker F tangent to u-b surfaces
ds² = (1 - 2M/r)(dt² + dr*²)
u = t - r* retarded time in Schwarzschild
ψ(θ, φ, u)
<du, du> = <dt - dr*, dt - dr*>
= -(1 - 2M/r)

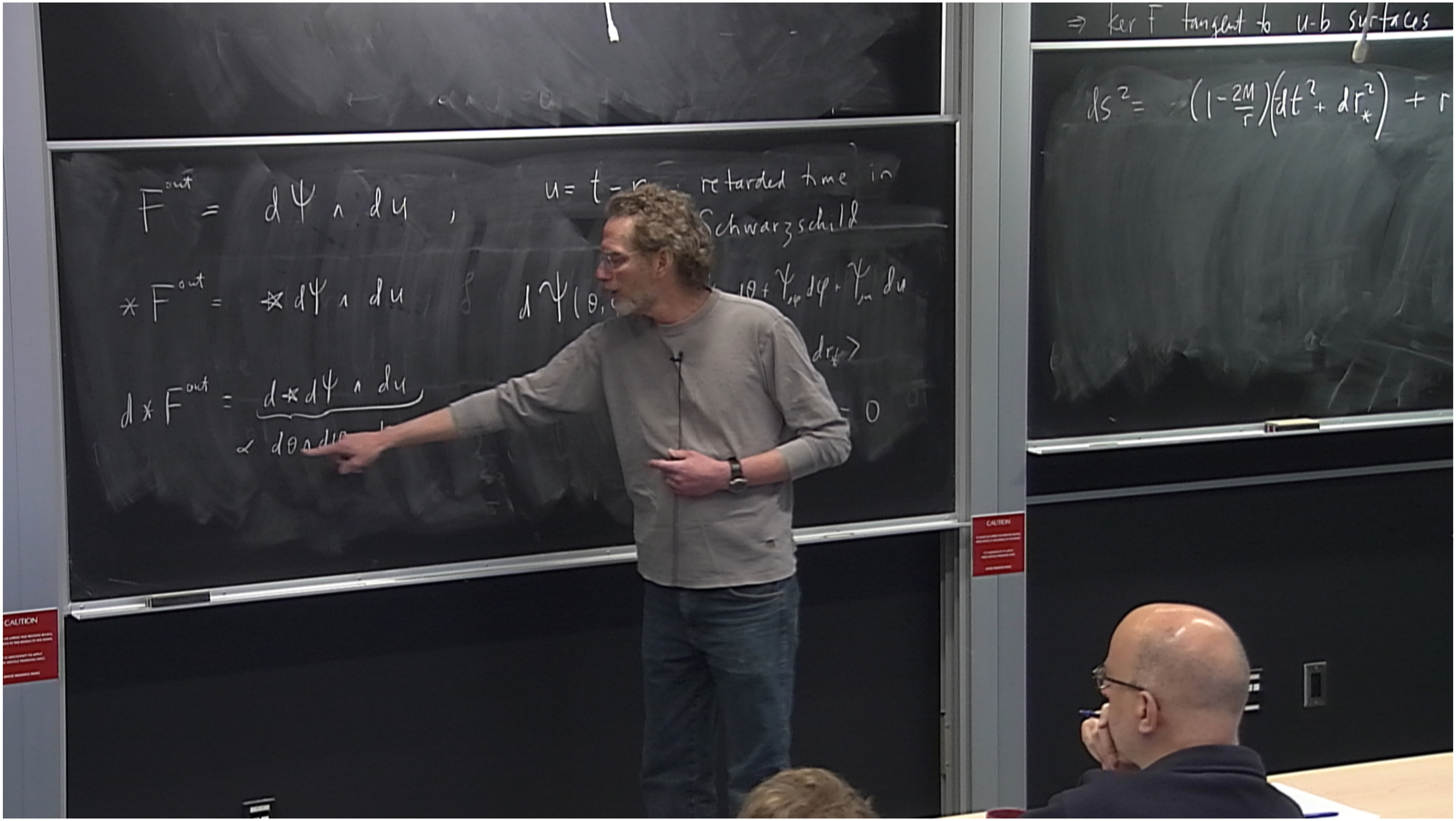

⇒ ker F tangent to u-b surfaces
$ds^2 = (1-\frac{2M}{r})(-dt^2 + dr_*^2) + r^2 d\theta^2 + r^2 \sin^2\theta \, d$
$= d\psi \wedge du$,
$u = t - r_*$ retarded time in Schwarzschild
$d\psi(\theta, \varphi, u) = \psi_{,\theta} d\theta + \psi_{,\varphi} d\varphi + \psi_{,u} du$
$\langle du, du \rangle = \langle dt - dr_*, dt - dr_* \rangle$
$= (1-\frac{2M}{r})(-1+1) = 0$

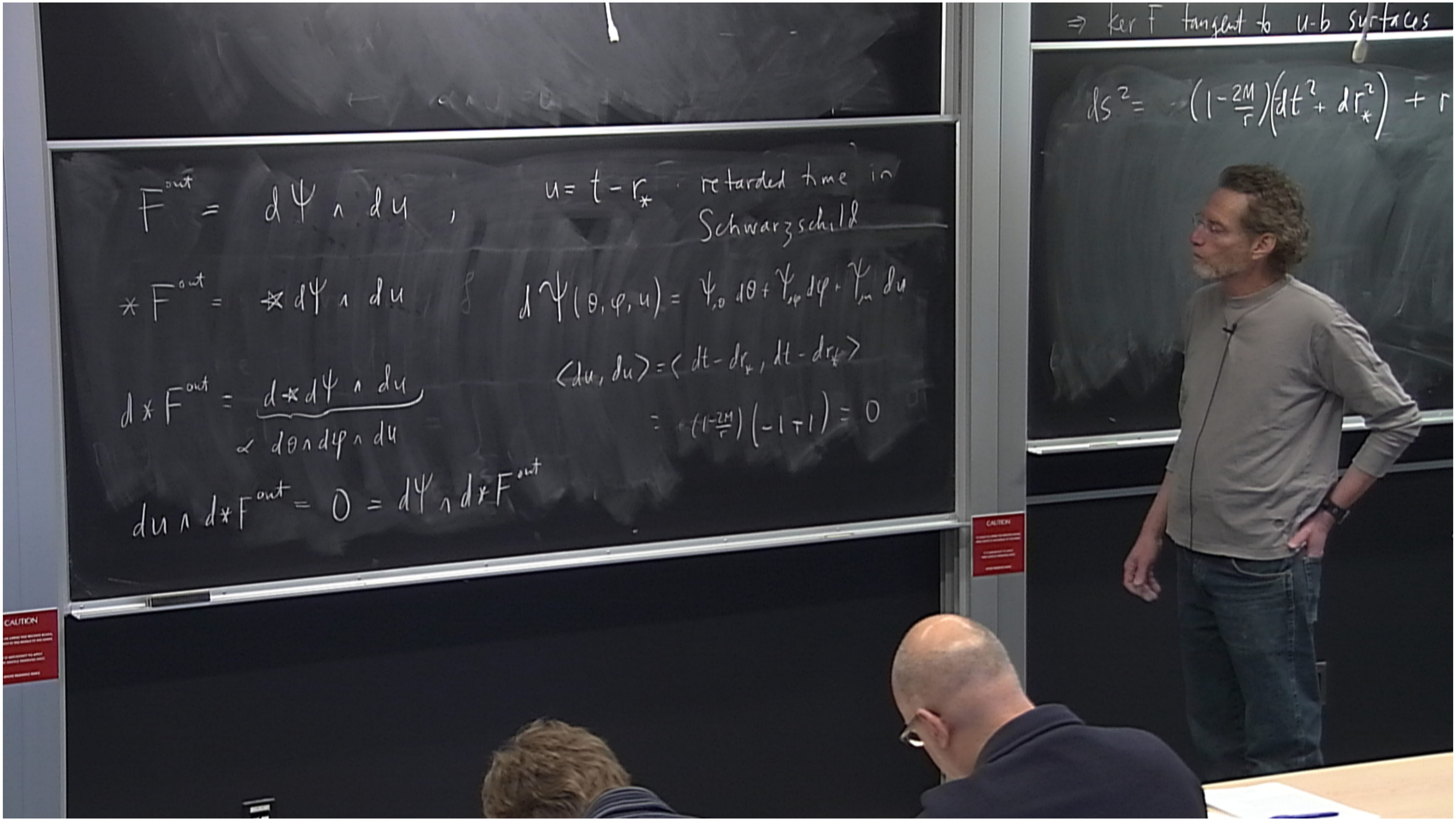

⇒ ker F tangent to u-b surfaces
ds² = (1 - 2M/r)(dt² + dr*²) + r
F^out = dΨ ∧ du , u = t - r* retarded time in Schwarzschild
*F^out = *dΨ ∧ du
dΨ(θ, φ, u) = Ψ,θ dθ + Ψ,φ dφ + Ψ,u du
<du, du> = < dt - dr*, dt - dr* >
= (1 - 2M/r)(-1 + 1) = 0
CAUTION

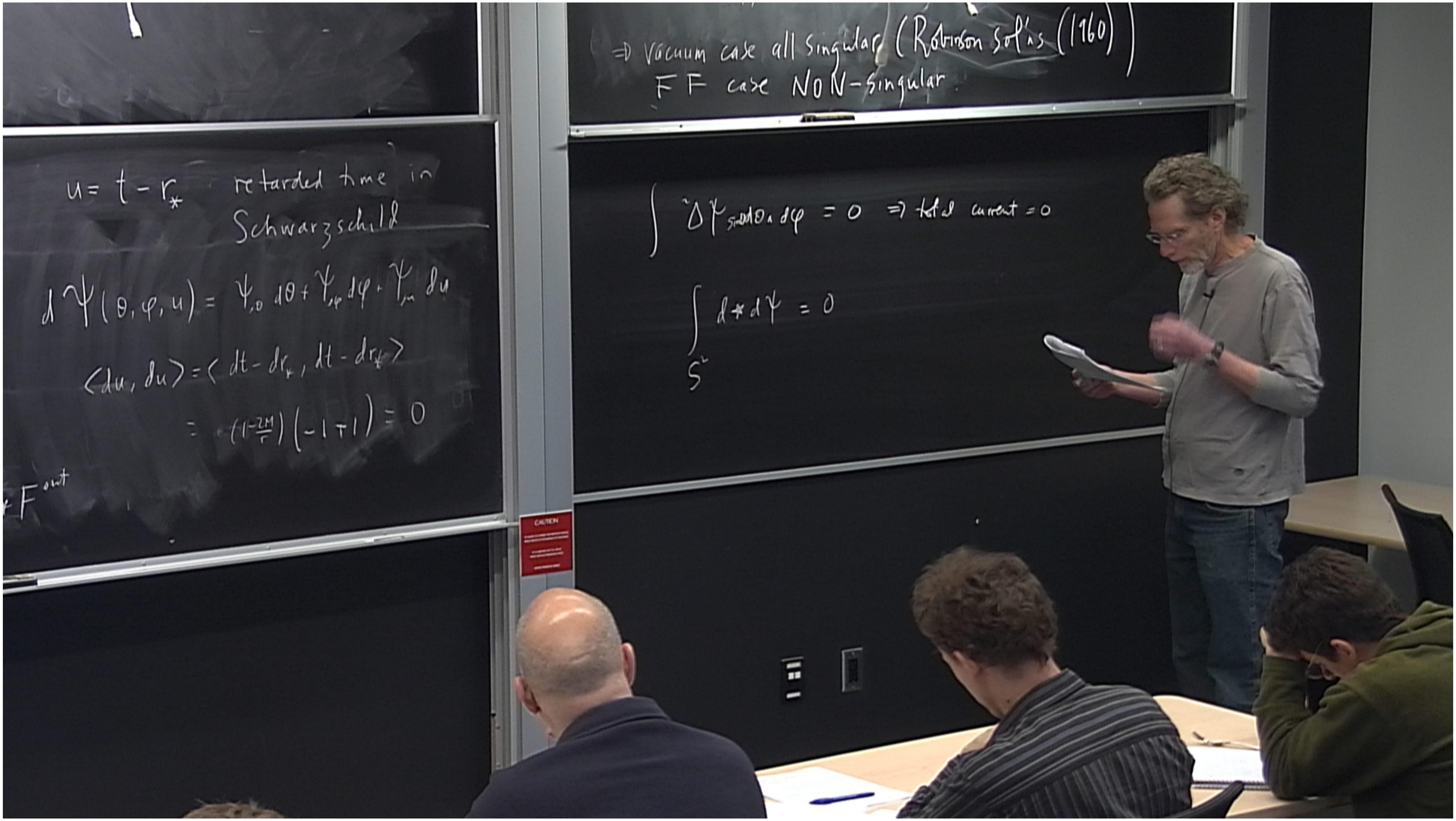


$\Rightarrow$ ker F tangent to u-b surfaces
$ds^2 = -\left(1-\frac{2M}{r}\right)\left(dt^2 + dr_*^2\right) + r$
$F^{out} = d\Psi \wedge du$,
$u = t - r_*$ retarded time in Schwarzschild
$*F^{out} = \star d\Psi \wedge du$
$\varphi, u) = \Psi_{,\theta}\, d\theta + \Psi_{,\varphi}\, d\varphi + \Psi_{,u}\, du$
$= \langle dt - dr_*, dt - dr_* \rangle$
$\left(1-\frac{2M}{r}\right)(-1 + 1) = 0$

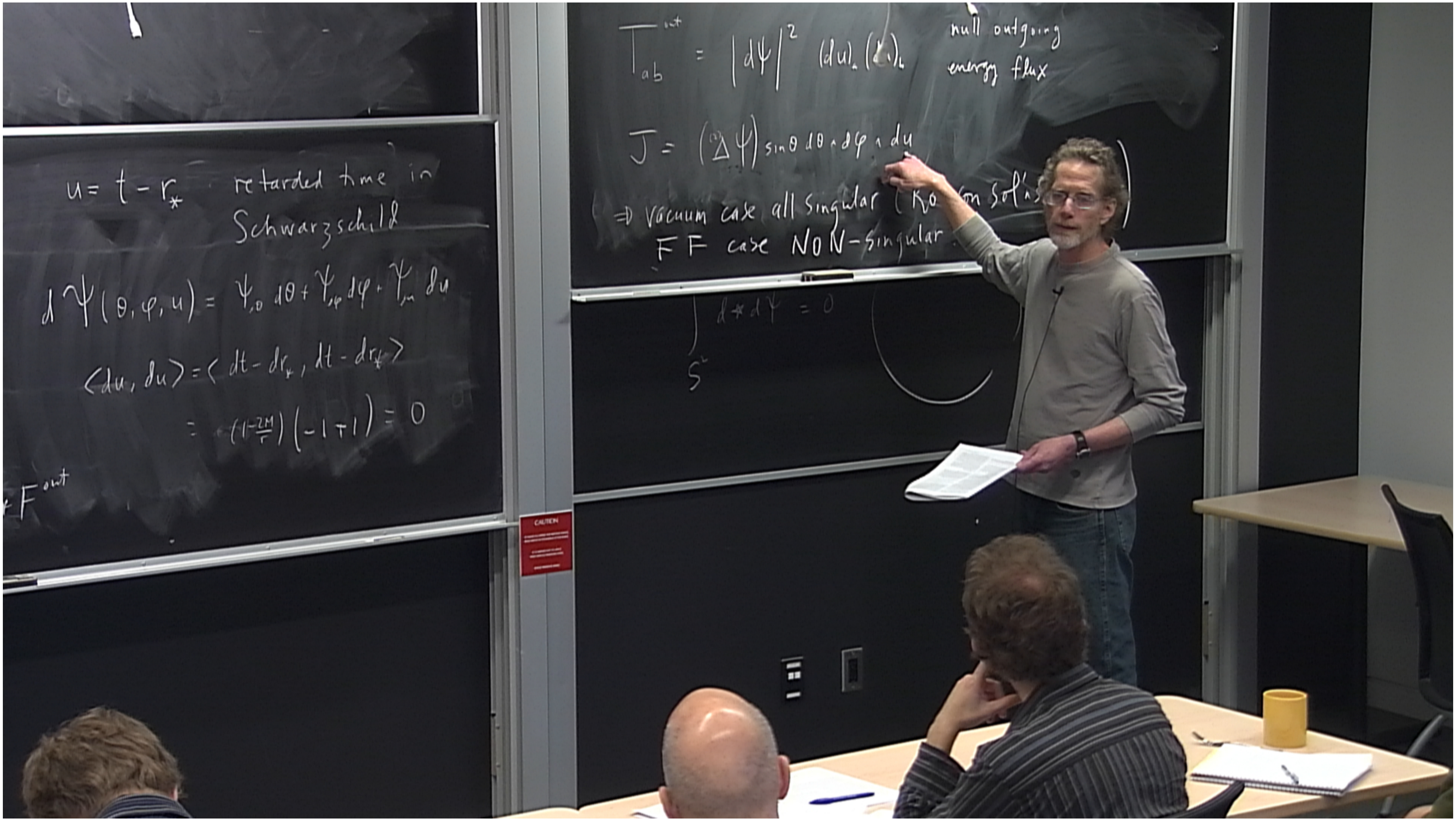

⇒ ker F tangent to u-b surfaces
$ds^2 = -(1-\frac{2M}{r})(dt^2 + dr_*^2) + r$
$F^{out} = d\Psi \wedge du$,
$u = t - r_*$: retarded time in Schwarzschild
$*F^{out} = *d\Psi \wedge$
$d\Psi(\theta, \varphi, u) = \Psi_{,\theta}\, d\theta + \Psi_{,\varphi}\, d\varphi + \Psi_{,u}\, du$
$\langle du, du \rangle = \langle dt - dr_*, dt - dr_* \rangle$
$= (1-\frac{2M}{r})(-1+1) = 0$
$d*F^{out} = d*d\Psi \wedge du$
$\propto d$

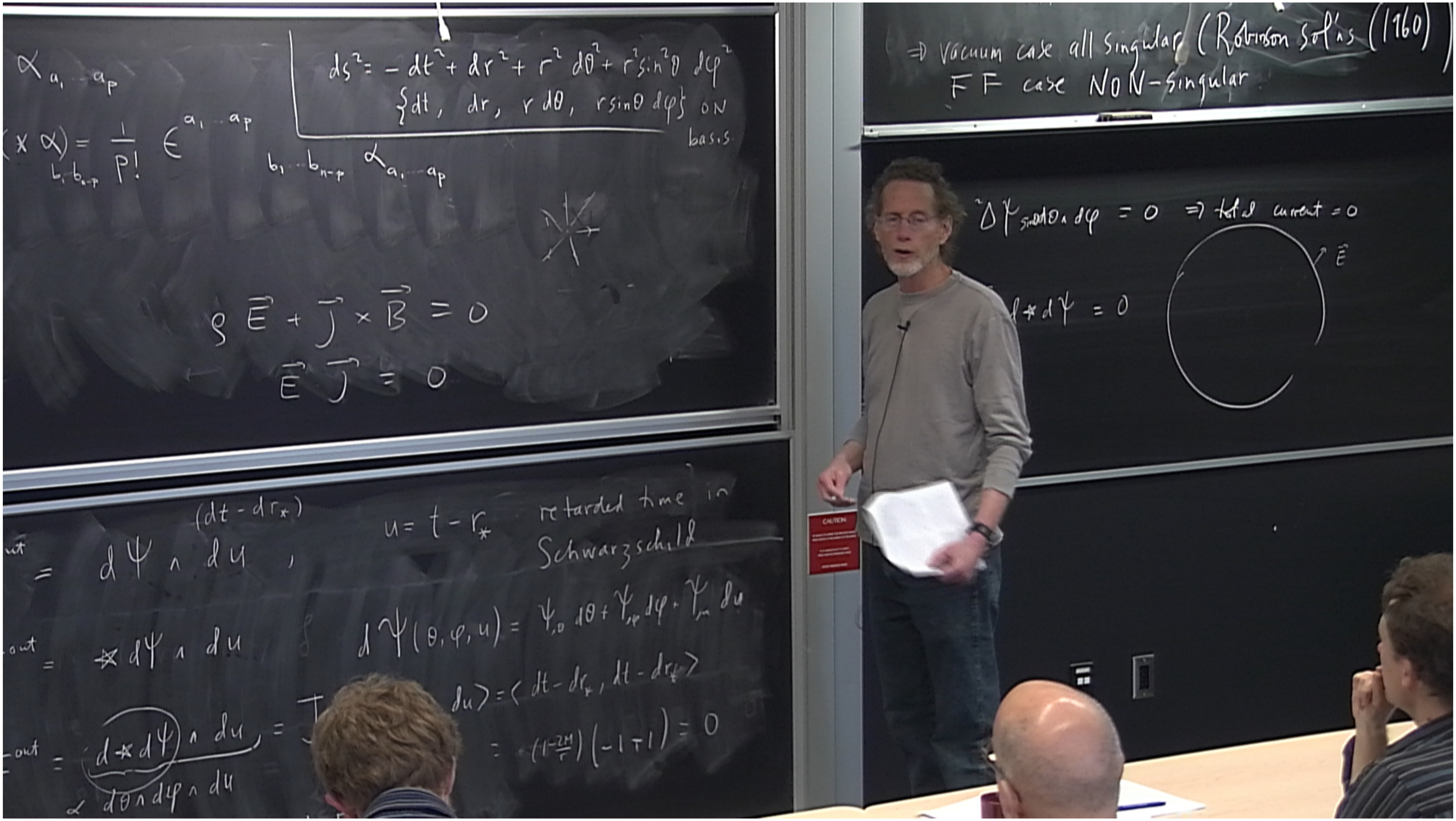

⇒ ker F tangent to u-b surfaces
F^out = dΨ ∧ du ,
u = t − r : retarded time in
Schwarzschild
*F^out = *dΨ ∧ du
d*F^out = d*dΨ ∧ du
ds² = (1 − 2M/r)(dt² + dr*²) + r

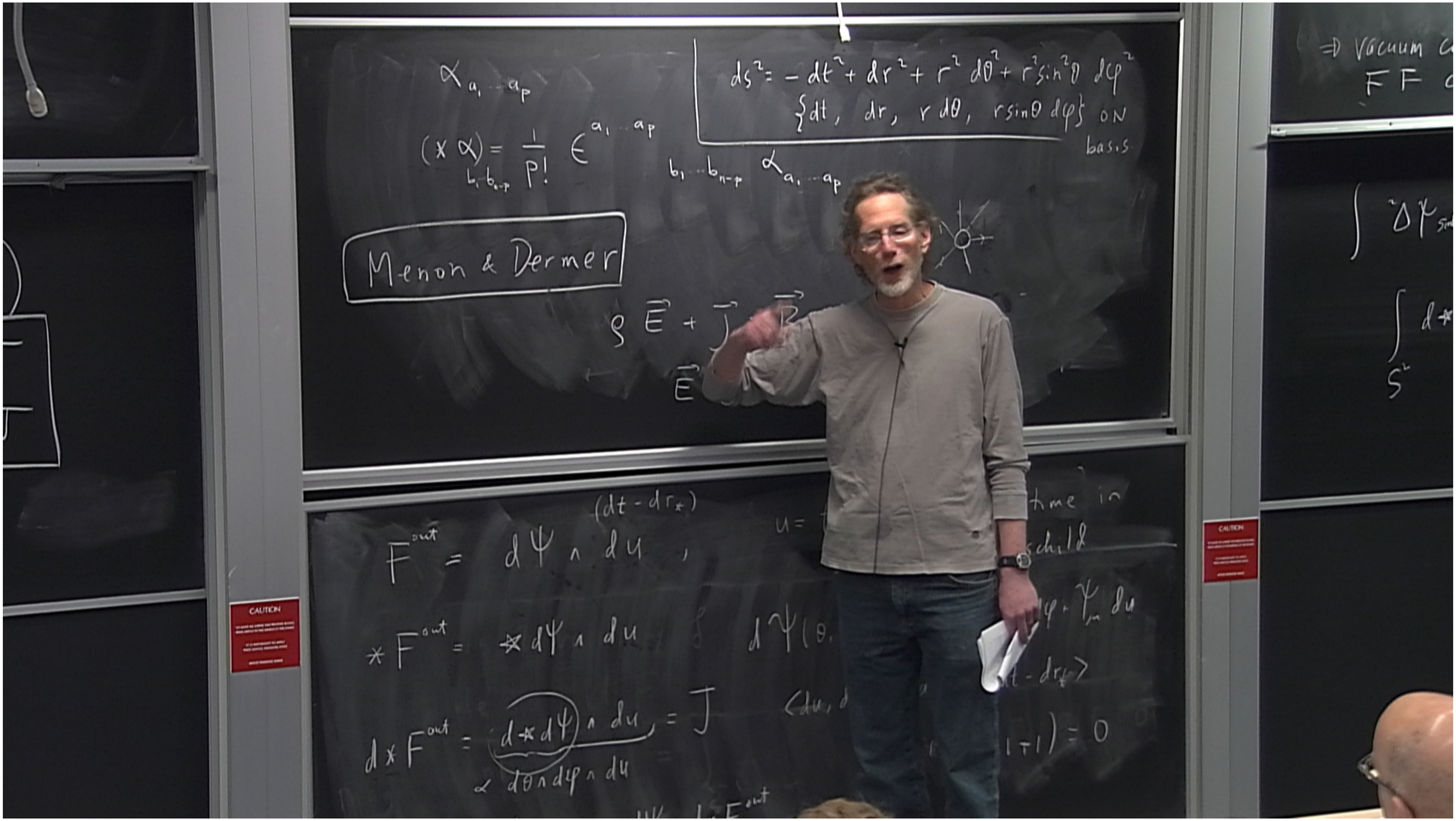

⇒ ker F tangent to u-b surfaces
ds² = (1 - 2M/r)(dt² + dr*²) + r
F^out = dΨ ∧ du ,
u = t - r* retarded time in Schwarzschild
*F^out = *dΨ ∧ du
dΨ(θ, φ, u) = Ψ,θ dθ + Ψ,φ dφ + Ψ,u du
<du, du> = <dt - dr*, dt - dr*>
= (1 - 2M/r)(-1 + 1) = 0
d*F^out = d*dΨ ∧ du
∝ dθ ∧ dφ ∧ du
du ∧ d*F^out = 0 = dΨ ∧ d*F^out
CAUTION



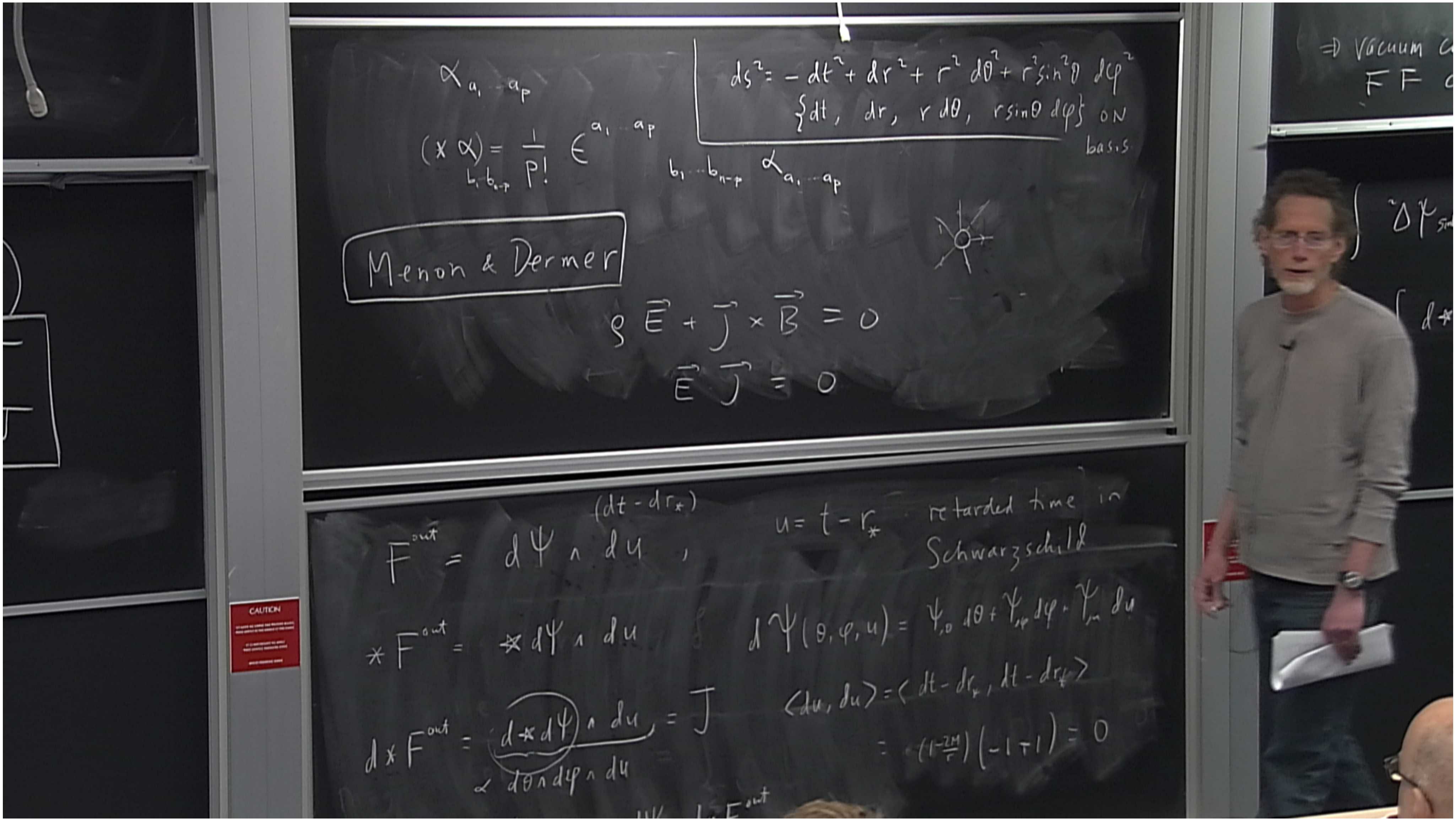
⇒ vacuum case all singular (Robinson sol'ns (1960))
FF case NON-singular
u = t - r* retarded time in Schwarzschild
dψ(θ, φ, u) = ψ,θ dθ + ψ,φ dφ + ψ,u du
<du, du> = <dt - dr*, dt - dr*>
= (1-2M/r)(-1+1) = 0
F out
∫ ∆ψ sinθ dθ ∧ dφ = 0 ⇒ total current = 0
∫ S² d*dψ = 0
CAUTION


u = t - r* retarded time in Schwarzschild
d ψ(θ, φ, u) = ψ,θ dθ + ψ,φ dφ + ψ,u du
<du, du> = < dt - dr*, dt - dr* >
= (1 - 2M/r)(-1 + 1) = 0
F out
T out ab = |dψ|² (du)a (...)b
null outgoing energy flux
J = (Δψ) sin θ dθ ∧ dφ ∧ du
⇒ vacuum case all singular
FF case NON-singular
d * d ψ = 0
S²


ds^2 = - dt^2 + dr^2 + r^2 dθ^2 + r^2 sin^2θ dφ^2
{dt, dr, r dθ, r sinθ dφ} ON basis
ρ E + J × B = 0
E · J = 0
⇒ vacuum case all singular (Robinson sol'ns (1960))
FF case NON-singular
⇒ total current = 0
u = t - r_*   retarded time in Schwarzschild



$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\varphi^2$
$\{dt,\ dr,\ r\,d\theta,\ r\sin\theta\, d\varphi\}$ ON basis
$\alpha_{a_1 \cdots a_p}$
$(\ast\alpha)_{b_1 \cdots b_{n-p}} = \frac{1}{p!}\,\epsilon^{a_1 \cdots a_p}{}_{b_1 \cdots b_{n-p}}\,\alpha_{a_1 \cdots a_p}$
Menon & Dermer
$F^{out} = d\Psi \wedge du$
$\ast F^{out} = \ast d\Psi \wedge du$
$d\ast F^{out} = d\ast d\Psi \wedge du = J$
$\propto d\theta \wedge d\varphi \wedge du$
$(dt - dr_*)$


$\alpha_{a_1 \cdots a_p}$
$(*\alpha)_{b_1 \cdots b_{n-p}} = \frac{1}{p!} \epsilon^{a_1 \cdots a_p}{}_{b_1 \cdots b_{n-p}} \alpha_{a_1 \cdots a_p}$
$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2$
$\{dt, dr, r\,d\theta, r\sin\theta\, d\varphi\}$ ON basis
Menon & Dermer
$\rho \vec{E} + \vec{J} \times \vec{B} = 0$
$\vec{E} \cdot \vec{J} = 0$
$F^{out} = d\psi \wedge du$, $(dt - dr_*)$
$u = t - r_*$ retarded time in Schwarzschild
$*F^{out} = *d\psi \wedge du$
$d\psi(\theta, \varphi, u) = \psi_{,\theta} d\theta + \psi_{,\varphi} d\varphi + \psi_{,u} du$
$\langle du, du \rangle = \langle dt - dr_*, dt - dr_* \rangle$
$= (1 - \frac{2M}{r})(-1 + 1) = 0$
$d*F^{out} = d*d\psi \wedge du = J$
$\propto d\theta \wedge d\varphi \wedge du$
⇒ vacuum
FF