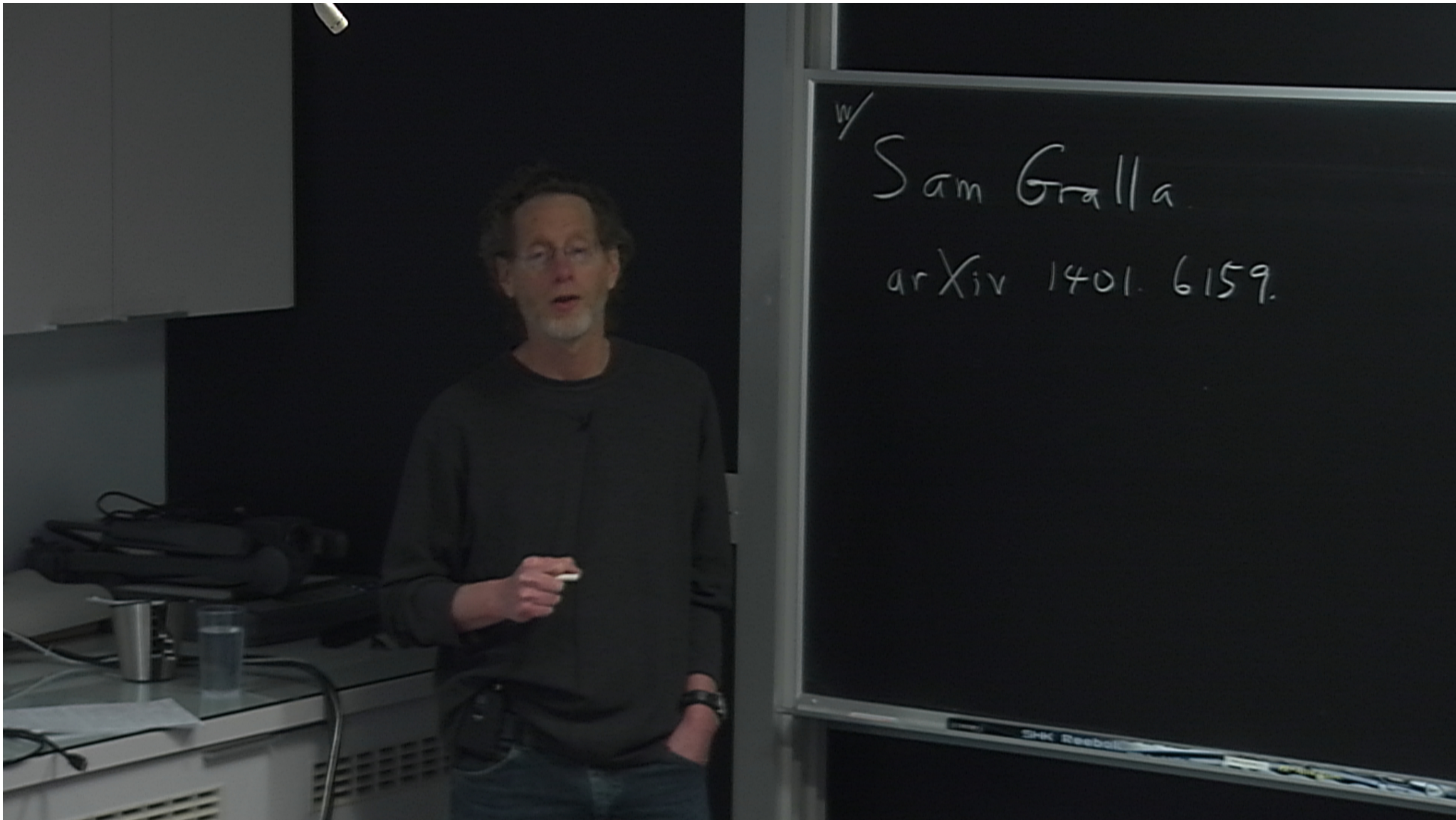


Title: Spacetime approach to force-free magnetospheres - Lecture 1

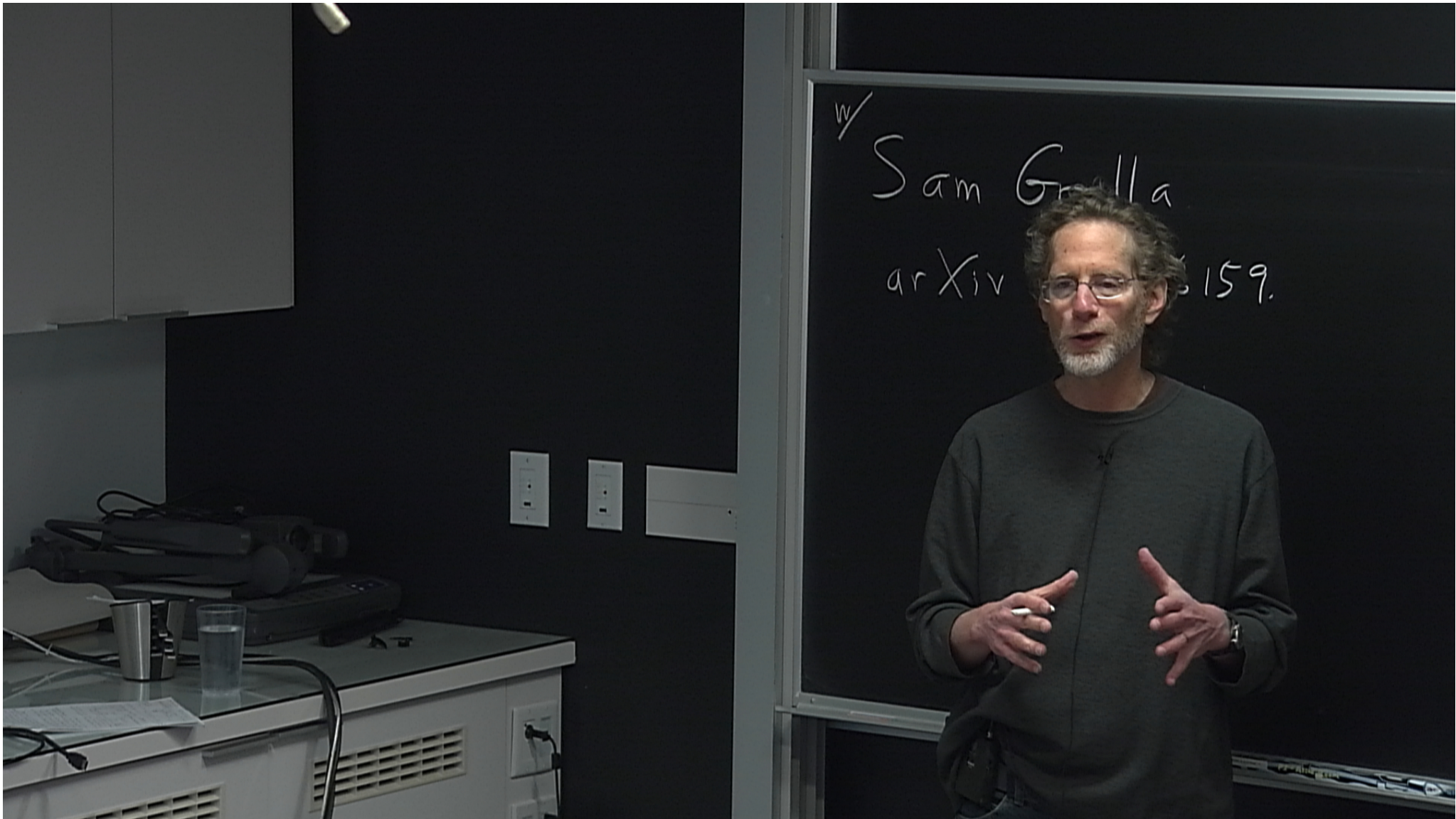
Date: Feb 25, 2014 01:00 PM

URL: <http://pirsa.org/14020154>

Abstract:



w/
Sam Gralla
arXiv 1401.6159.



w

Sam Gralla

arXiv 1401.6159.

force-free approximation

no net 4-force on charges locally.

force-free approximation

no net 4-force on charges locally.

field evolves autonomously via non-linear eq'n.

Rainich's "already unified theory"

$$G_{\mu\nu} = \kappa T_{\mu\nu}^{EM}$$

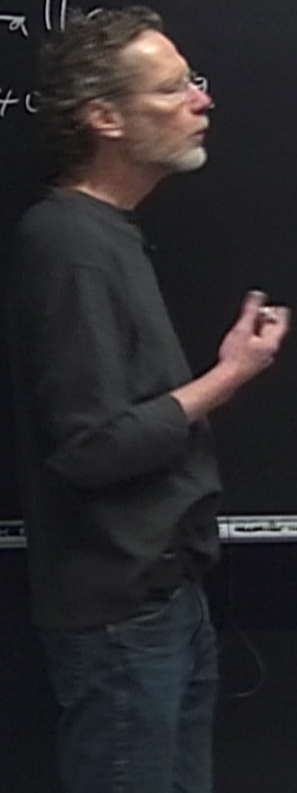
h. linear eq'n.

CAUTION
Do not touch the blackboard
as it is very hot and may
cause serious injury.

^{w/} Sam Gralla
arXiv 140

B. Carter

1979. Einstein Centenary Survey
eds Hawking & Israel.



CAUTION

^{w/} Sam Gralla
arXiv 1401.6159.

B. Carter

1979. Einstein Centenary Survey
eds Hawking & Israel.

Uchida ~ 1978 (five papers)

\vec{E}, \vec{B}

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = \vec{j}$$

\vec{E}, \vec{B}

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E} = -\partial_t \vec{A} + \vec{\nabla} \phi$$

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = \vec{j}$$

\vec{E}, \vec{B}

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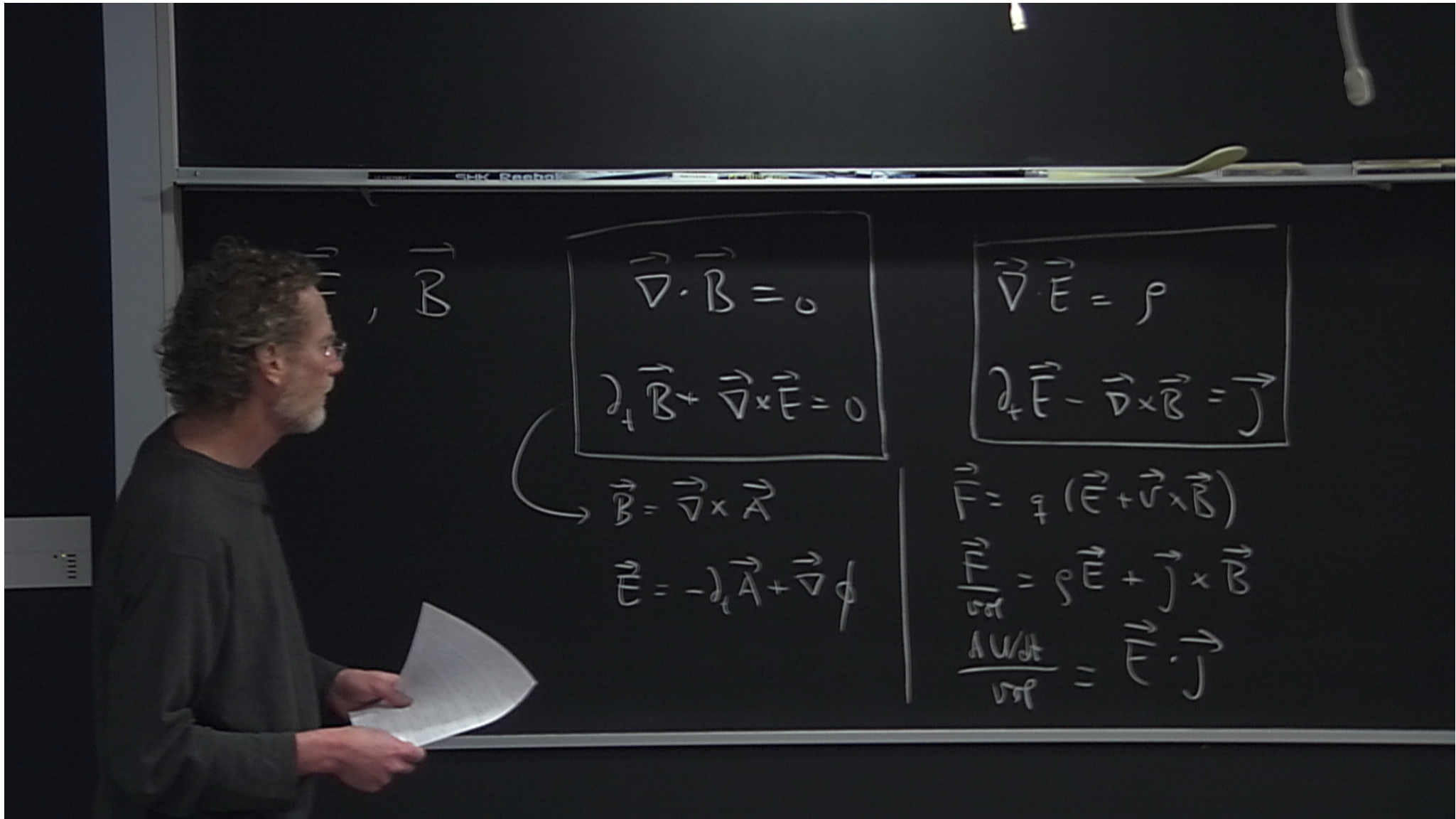
$$\vec{E} = -\partial_t \vec{A} + \vec{\nabla} \phi$$

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = \vec{j}$$

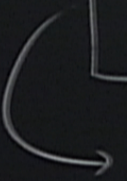
$$\vec{F} = q (\vec{E} + \vec{v} \times \vec{B})$$

$$\frac{d\vec{F}}{dt} = q \vec{E} + \vec{j} \times \vec{B}$$



\vec{E}, \vec{B}

$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$



$$\vec{B} = \vec{\nabla} \times \vec{A}$$
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$$\vec{F} = q (\vec{E} + \vec{v} \times \vec{B})$$
$$\frac{d\vec{F}}{dt} = \rho \vec{E} + \vec{j} \times \vec{B}$$
$$\frac{\Delta U/dt}{vq} = \vec{F} \cdot \vec{j}$$

$$F_{ab} = -F_{ba} = F_{[ab]} = \frac{1}{2}(F_{ab} - F_{ba}) \quad \text{field strength tensor}$$

observer 4-velocity u^a unit timelike vector

$$E_b = u^a F_{ab}, \quad u^b E_b = 0$$

$$F_{ab} = -F_{ba} = F_{[ab]} = \frac{1}{2}(F_{ab} - F_{ba}) \quad \text{field strength tensor}$$

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$$E_b = u^a F_{ab}, \quad u^b E_b = 0$$

$$B_b = u^a *F_{ab}, \quad *F_{ab} = \frac{1}{2} \epsilon_{ab}{}^{cd} F_{cd}, \quad \epsilon_{abcd} \text{ is "the" volume element}$$

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observer 4-velocity u^a unit timelike vector.

$$E_b = u^a F_{ab}, \quad u^b E_b = 0$$

$$\epsilon_{abcd} \epsilon^{abcd} = -4!$$

$$B_b = u^a *F_{ab}, \quad *F_{ab} = \frac{1}{2} \epsilon_{ab}{}^{cd} F_{cd}, \quad \epsilon_{abcd} \text{ is "the" volume element}$$

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Covariant
Faraday's law \Rightarrow

$$\nabla_{[a} F_{bc]} = 0 \quad \Rightarrow \quad F_{bc} = 2 \nabla_{[b} A_{c]} \quad \leftarrow \begin{array}{l} \text{4-vector} \\ \text{potential} \end{array}$$

$$\nabla_b F^{ab} = j^a = \text{4-current density} \quad \left(\text{Heaviside-Lorentz units} \right)$$

$$\nabla_b \times F^{ab} = 0$$

Covariant

Faraday's law \Rightarrow

$$F_{bc} = 2 \nabla_{[b} A_{c]}$$

4-vector potential

$$\nabla_{[a} F_{bc]} = 0$$
$$\nabla_b F^{ab} = j^a = 4\text{-current density}$$
$$\nabla_b \star F^{ab} = 0$$

(Heaviside-Lorentz units)

Conservation of energy-mom

$$\nabla^b T_{ab}^{EM} + F_{ab} j^b = 0, \quad T_{ab}^{EM} = F_{ac} F_b^c - \frac{1}{4} F_{cd} F^{cd} g_{ab}$$

Covariant

Faraday's law \Rightarrow

$$F_{bc} = 2 \nabla_{[b} A_{c]}$$

4-vector potential

$$\nabla_b F^{ab} = j^a = \text{4-current density}$$

(Heaviside-Lorentz units)

$$\nabla_b \star F^{ab} = 0$$

Conservation of
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$$\nabla^b T_{ab}^{EM} + F_{ab} j^b = 0,$$

$$T_{ab}^{EM} = F_{ac} F_b^c - \frac{1}{4} F_{cd} F^{cd} g_{ab}$$

$$\nabla_b * F^{ab} = 0$$

Conservation of
energy-mom

$$\nabla^b T_{ab}^{EM} + \boxed{F_{ab} j^b} = 0,$$

4-force density

$$T_{ab}^{EM} = F_{ac} F_b^c - \frac{1}{4} F_{cd} F^{cd} g_{ab}$$

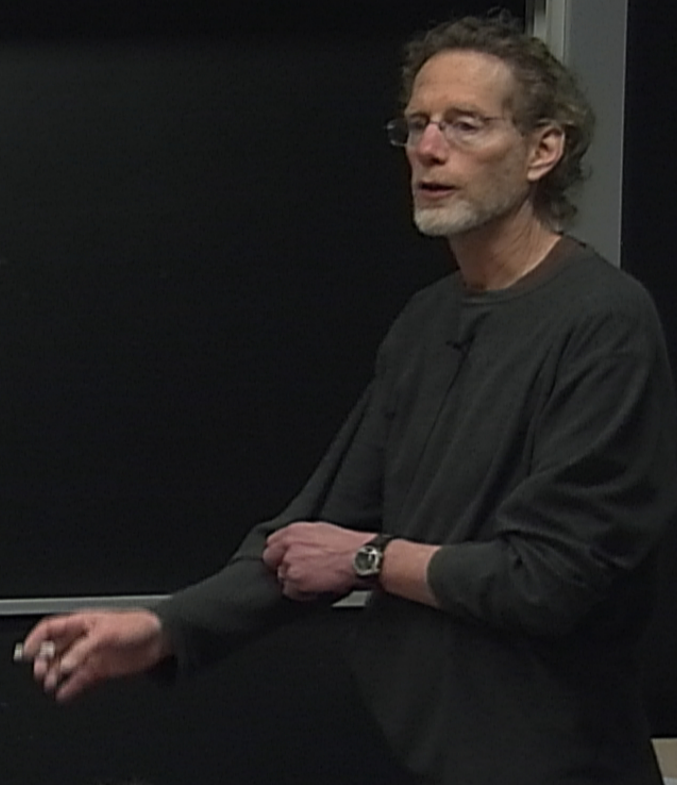
Conservation of energy-mom

4-force density

$$\nabla^b T_{ab}^{EM} + \boxed{F_{ab} j^b} = 0, \quad \left(T_{ab}^{EM} = F_{ac} F_b^c - \frac{1}{4} F_{cd} F^{cd} g_{ab} \right)$$

$$F_{ab} F^{ab} = \frac{1}{2} (B^2 - E^2)$$

$$F_{ab} \times F^{ab} = 4(\vec{E} \cdot \vec{B})$$



CAUTION
 DO NOT TOUCH THE BOARD OR THE CHALK
 WHEN THE BOARD IS HOT OR WHEN THE BOARD IS IN USE

Conservation of energy-mom

4-force density

$$\nabla^b T_{ab}^{EM} + \boxed{F_{ab} j^b} = 0, \quad \left(T_{ab}^{EM} = F_{ac} F_b^c - \frac{1}{4} F_{cd} F^{cd} g_{ab} \right)$$

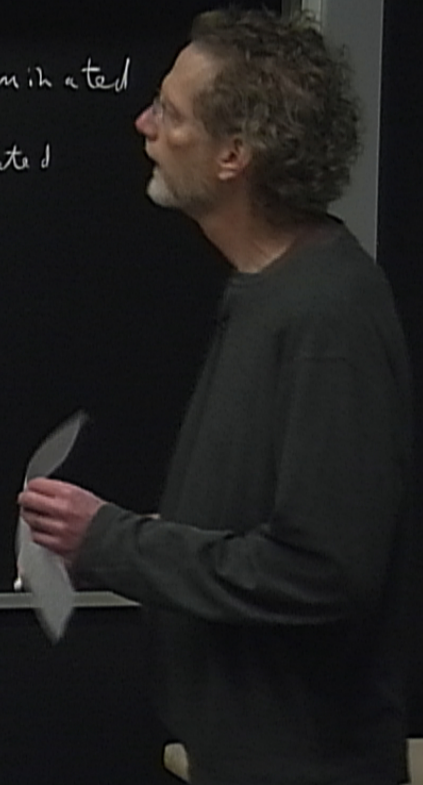
$$F_{ab} F^{ab} = \frac{1}{2} (B^2 - E^2)$$

$$F_{ab} *F^{ab} = 4(\vec{E} \cdot \vec{B})$$

$F^2 > 0$: magnetically dominated

$F^2 < 0$: electrically dominated

$F = 0$
 $\&$
 $F_{ab} *F^{ab} = 0$ } "null"



Perfect Conductors

$$\vec{j} = \sigma \vec{E} \quad (\text{for "ohmic" medium})$$

↑
conductivity

$$\sigma = \infty \Rightarrow \vec{E} = 0 \leftrightarrow \int_{\text{MEDIUM}} \vec{F}_{\text{ext}} = 0$$

Perfect Conductors

$$\vec{j} = \sigma \vec{E} \quad (\text{for "ohmic" medium})$$

↑
conductivity

$$\sigma = \infty \Rightarrow \vec{E} = 0 \Leftrightarrow U_{\text{MEDIUM}}^a F_{ab} = 0$$

consider a conducting fluid, if has well-defined U_{fluid}^a , $U_{\text{fluid}}^a F_{ab} = 0$
"ideal" MHD (magneto-hydro-dynamics)

\vec{E}, \vec{B}

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E} = -\partial_t \vec{A} + \vec{\nabla} \phi$$

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = \vec{j}$$

$$\vec{F} = q (\vec{E} + \vec{v} \times \vec{B})$$

$$\frac{d\vec{F}}{dt} = q \vec{E} + \vec{j} \times \vec{B}$$

$$\frac{\Delta U/dt}{v \cdot \rho} = \vec{E} \cdot \vec{j}$$

\vec{E}, \vec{B}

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

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$$\frac{d\vec{F}}{dt} = q \vec{E} + \vec{j} \times \vec{B}$$

$$\frac{\Delta U/dt}{v \mp} = \vec{E} \cdot \vec{j}$$

Suppose \exists vector W^a s.t. $W^a \bar{F}_a = 0$.

$\Rightarrow F_{ab}$ is called "degenerate"

$$j = W^a [F_{ab} F_{cd}]$$

Suppose \exists vector W^a s.t. $W^a \bar{F}_a = 0$.

$\Rightarrow F_{ab}$ is called "degenerate"

$$0 = W^a \bar{F}_{[ab} F_{cd]}$$

Suppose \exists vector W^a s.t. $W^a \bar{F}_a = 0$.

$\Rightarrow F_{ab}$ is called "degenerate"

$$\Rightarrow 0 = W^a \underbrace{\bar{F}_{[ab} F_{cd]}}_{= \beta \epsilon_{abcd}} \Rightarrow \beta W^a \epsilon_{abcd} = 0 \Rightarrow \beta = 0$$

Suppose \exists vector W^a s.t. $W^a F_{ab} = 0$.

$\Rightarrow F_{ab}$ is called "degenerate"

$$\Rightarrow 0 = W^a \underbrace{F_{[ab} F_{cd]}}_{=\beta \epsilon_{abcd}} \Rightarrow \beta W^a \epsilon_{abcd} = 0 \Rightarrow \beta = 0 \Rightarrow F_{[ab} F_{cd]} = 0 \Leftrightarrow F_a{}^b F_b{}^a = 0 \\ \Leftrightarrow \vec{E} \cdot \vec{B} = 0$$

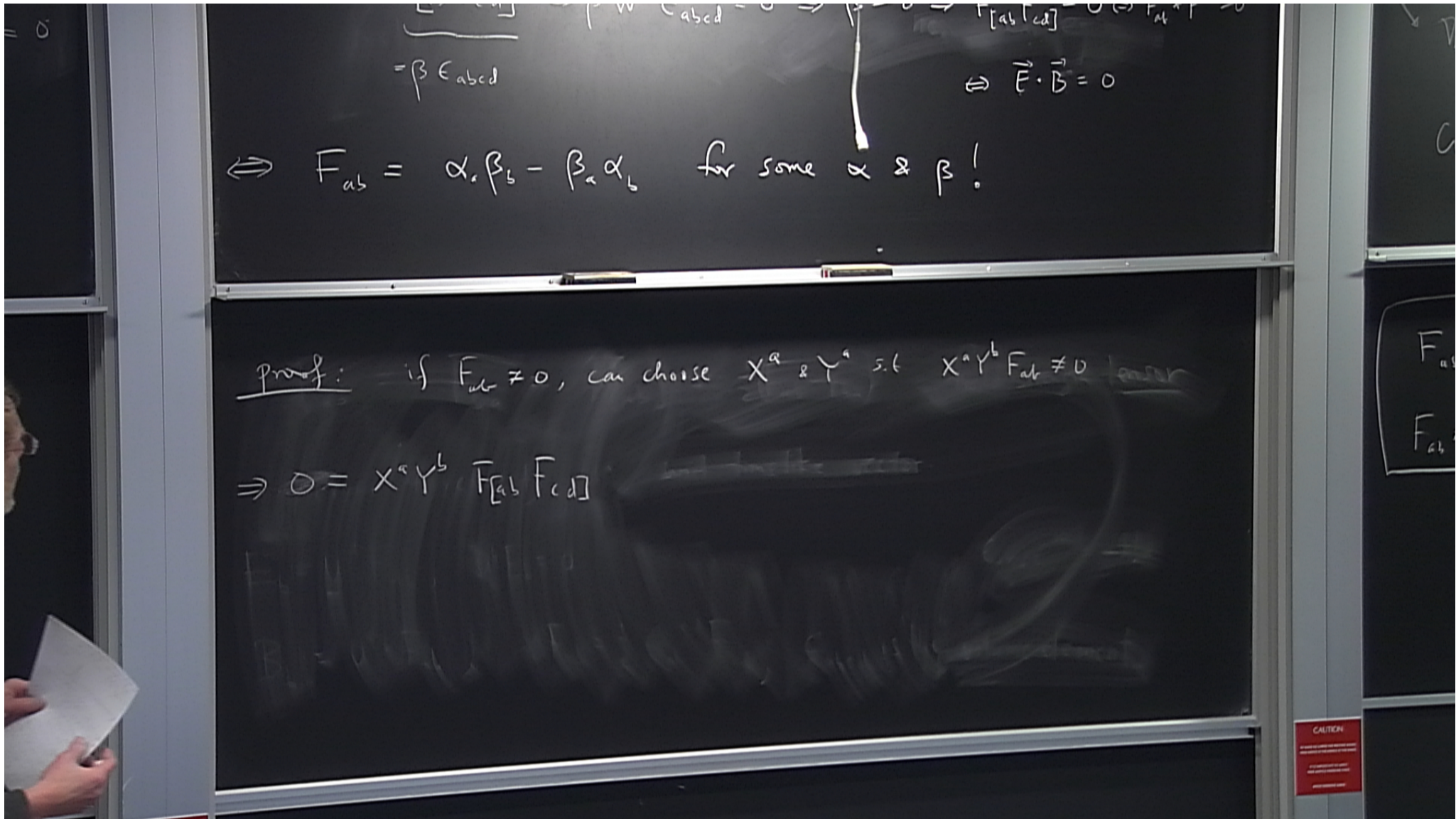
Suppose \exists vector W^a s.t. $W^a F_{at} = 0$.

$\Rightarrow F_{ab}$ is called "degenerate"

$$\Rightarrow 0 = W^a \underbrace{F_{[ab} F_{cd]}}_{= \beta \epsilon_{abcd}} \Rightarrow \beta W^a \epsilon_{abcd} = 0 \Rightarrow \beta = 0 \Rightarrow F_{[ab} F_{cd]} = 0 \Leftrightarrow F_a + F^{ab} = 0$$
$$\Leftrightarrow \vec{E} \cdot \vec{B} = 0$$

$$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b \quad \text{for some } \alpha \text{ \& } \beta!$$

$$B_b = U^a * F_{ab}, \quad * F_{ab} = \frac{1}{2} \epsilon_{ab}{}^{cd} F_{cd}, \quad \epsilon_{abcd} \text{ is the "volume element"}$$



$$= \beta \epsilon_{abcd} \quad \Rightarrow \quad \vec{E} \cdot \vec{B} = 0$$

$$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b \quad \text{for some } \alpha \text{ \& } \beta!$$

proof: if $F_{ab} \neq 0$, can choose X^a & Y^a s.t. $X^a Y^b F_{ab} \neq 0$

$$\Rightarrow 0 = X^a Y^b F_{[ab} F_{cd]}$$

$$= \beta \epsilon_{abcd}$$

$$\Leftrightarrow \vec{E} \cdot \vec{B} = 0$$

$$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b \text{ for some } \alpha \text{ \& } \beta!$$

proof: if $F_{ab} \neq 0$, can choose X^a & Y^a s.t. $X^a Y^b F_{ab} \neq 0$

$$\Rightarrow 0 = X^a Y^b F_{[ab} F_{cd]}$$

$$(X^a Y^b F_{ab}) F_{cd}$$

Proof: if $F_{ab} \neq 0$, can choose X^a & Y^a s.t. $X^a Y^b F_{ab} \neq 0$

$$\Rightarrow 0 = X^a Y^b [F_{ab} F_{cd}]$$

$$(X^a Y^b F_{ab}) F_{cd} + \# (X^a F_{ac})(Y^b F_{bd})$$

well-defined U_{fluid}^a , $\int_{\text{fluid}} F_{ab} = 0$
 hydro-dynamics)

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = \vec{j}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\frac{\vec{F}}{q} = \rho \vec{E} + \vec{j} \times \vec{B}$$

$$\frac{\Delta U/dt}{\rho q} = \vec{F} \cdot \vec{j}$$

Suppose \exists vector W^a st $W^a F_{ab} = 0$
 $\Rightarrow F_{ab}$ is called 'degenerate'
 $\Rightarrow 0 = W^a \underbrace{F_{[ab]} F_{c]d}}_{-\beta \epsilon_{abcd}} \Rightarrow \beta W^a \epsilon_{abcd} = 0 \Rightarrow \beta = 0 \Rightarrow F_{[ab]} = 0$
 $\Rightarrow \vec{E} \cdot \vec{B} = 0$
 $\Leftrightarrow F_{ab} = \alpha \beta_c - \beta_a \alpha_c$ for some α & β !

$\nabla_a F^{ab} = 0$
 Conservation of energy-mom $\nabla^b T_{ab}^{\text{EM}} + \underbrace{(-T_{ab})^b}_4 = 0$,
 4 force density

$$F_{ab} F^{ab} = \frac{1}{2} (B^2 - E^2)$$

$$F_{ab} F^{cd} = 4(\vec{E} \cdot \vec{B})$$

$$F^2 > 0 \text{ - magnetic}$$

$$F^2 < 0 \text{ - electric}$$

$$F^2 = 0 \text{ - null}$$

$$F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b \quad \text{for some } \alpha \text{ \& } \beta!$$

Proof: If $F_{ab} \neq 0$, can choose X^a & Y^a s.t. $X^a Y^b F_{ab} \neq 0$

$$\Rightarrow 0 = X^a Y^b F_{[ab} F_{cd]}$$

$$= \underbrace{(X^a Y^b F_{ab})}_{\neq 0} F_{cd} + \neq \left[(X^a F_{ac})(Y^b F_{bd}) - (X^a F_{ad})(Y^b F_{bc}) \right]$$

$a_5 = \alpha_1 \beta_5 - \beta_1 \alpha_5$ for some α & β !

Can choose α & β orthogonal w.r.t. g

$$(\alpha + \lambda \beta) \cdot \beta = \alpha \cdot \beta + \lambda \beta \cdot \beta$$

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It is recommended to use
proper electrical safety procedures.
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$F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !

Can choose α & β orthogonal w.r.t. g

$$\begin{aligned} \text{Then } F_{ab} F^{ab} &= (\alpha_a \beta_b - \beta_a \alpha_b)(\alpha^a \beta^b - \beta^a \alpha^b) \\ &= 2 \alpha^2 \beta^2 \end{aligned} \quad \left(\begin{array}{cccc} - & + & + & + \end{array} \right)$$

if $F^2 > 0$, both α & β spacelike
 $F^2 < 0$,

$F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !

$$\begin{aligned} \text{Then } F_{ab} F^{ab} &= (\alpha_a \beta_b - \beta_a \alpha_b)(\alpha^a \beta^b - \beta^a \alpha^b) \\ &= 2 \alpha^2 \beta^2 \end{aligned} \quad \left(\begin{array}{cccc} - & + & + & + \end{array} \right)$$

if $F^2 > 0$, both α & β spacelike
 $F^2 < 0$, one timelike
 $F = 0$,

CAUTION
DO NOT REVERSE THE DIRECTION OF THE
BLACKBOARD OR WHITEBOARD
IF AN OBSTRUCTION IS MET
PLEASE CONTACT THE STAFF
PLEASE REPORT DAMAGE

conducting fluid, thus well-defined U_{fluid}^a , $F_{ab} = 0$
MHD (magneto hydro-dynamics)

$$\begin{aligned} \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{B} - \nabla \times \vec{E} &= 0 \\ \vec{B} &= \nabla \times \vec{A} \\ \vec{E} &= -\nabla \phi - \dot{\vec{A}} \end{aligned}$$

$$\begin{aligned} \nabla \cdot \vec{E} &= \rho \\ \nabla \times \vec{E} - \nabla \times \vec{B} &= \vec{j} \\ \vec{F} &= \gamma(\vec{E} + \vec{v} \times \vec{B}) \\ \vec{E} &= \rho \vec{E} + \vec{j} \times \vec{B} \\ \frac{\Delta U}{\Delta t} &= \vec{E} \cdot \vec{j} \end{aligned}$$

$$\Rightarrow 0 = W^a \frac{F_{[a} F_{b]}]}{-\beta \epsilon_{abcd}} \Rightarrow \rho W^a \epsilon_{abcd} = 0 \Rightarrow \rho = 0 \Rightarrow F_{[ab]} = 0 \Rightarrow F_{ab} F^{ab} = 0$$

$$\Rightarrow F_{ab} = \alpha_i \beta_i - \beta_i \alpha_i \text{ for some } \alpha \text{ \& } \beta!$$

Can choose α & β orthogonal w.l.o.g.

Then $F_{ab} F^{ab} = (\alpha_i \beta_i - \beta_i \alpha_i)(\alpha^i \beta^i - \beta^i \alpha^i) \quad (-+++)$
 $= 2 \alpha^2 \beta^2$

if $F^2 > 0$, both α & β spacelike
 $F^2 < 0$, one timelike
 $F = 0$, one null, one spacelike

Conservation of energy-mom

$$\nabla^a T_{ab} = 0, \quad T_{ab} = F_{[a} F_{b]} - \frac{1}{2} \epsilon_{abcd} F^c F^d$$

$$F_{ab} F^{ab} = \frac{1}{2} (\vec{B}^2 - \vec{E}^2)$$

$$F_{ab} F^{ab} = 4(\vec{E} \cdot \vec{B})$$

$F^2 > 0$ magnetically dominated
 $F^2 < 0$ electrically dominated
 $F^2 = 0$ "null"

$F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !

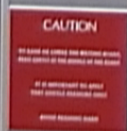
Can choose α & β orthogonal w.r.t. g

$$\begin{aligned} \text{Then } F_{ab} F^{ab} &= (\alpha_a \beta_b - \beta_a \alpha_b)(\alpha^a \beta^b - \beta^a \alpha^b) \\ &= 2 \alpha^2 \beta^2 \end{aligned} \quad \left(\begin{array}{cccc} - & + & + & + \end{array} \right)$$

if $F^2 > 0$, both α & β spacelike

$F^2 < 0$, one timelike

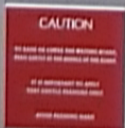
$F = 0$, one null, one spacelike



$$F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b \quad \text{for some } \alpha \text{ \& } \beta!$$

$$\ker F = \left\{ W^a : W^a F_{ab} = 0 \right\} = (\ker \alpha) \cap (\ker \beta)$$

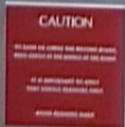
$$W^a F_{ab} = (W^a \alpha_a) \beta_b - (W^a \beta_a) \alpha_b = 0$$



$F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !

$$\ker F = \left\{ W^a : W^a F_{ab} = 0 \right\} = (\ker \alpha) \cap (\ker \beta)$$

$$W^a F_{ab} = \underbrace{(W^a \alpha_a)}_{=0} \beta_b - (W^a \beta_a) \alpha_b = 0$$



$$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b \quad \text{for some } \alpha \text{ \& } \beta !$$

$$\begin{aligned} \text{Then } F_{ab} F^{ab} &= (\alpha_a \beta_b - \beta_a \alpha_b)(\alpha^a \beta^b - \beta^a \alpha^b) \\ &= 2 \alpha^2 \beta^2 \end{aligned} \quad (-+++)$$

if $F^2 > 0$, both α & β spacelike, ker F timelike

$F^2 < 0$, one timelike,

$F = 0$, one null, one spacelike

CAUTION

Do not touch the screen when
the screen is not open to the room.
If a warning is off
the screen is not open.
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$$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b \quad \text{for some } \alpha \text{ \& } \beta!$$

$$\begin{aligned} \text{Then } F_{ab} F^{ab} &= (\alpha_a \beta_b - \beta_a \alpha_b)(\alpha^a \beta^b - \beta^a \alpha^b) \\ &= 2 \alpha^2 \beta^2 \end{aligned} \quad (-+++)$$

if $F^2 > 0$, both α & β spacelike, ker F timelike

$F^2 < 0$, one timelike,

$F = 0$, one null, one spacelike

CAUTION

Do not lean on the blackboard
 when it is in use
 Do not touch the board
 when it is being used
 Thank you

$$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b \quad \text{for some } \alpha \text{ \& } \beta!$$

$$\begin{aligned} \text{Then } F_{ab} F^{ab} &= (\alpha_a \beta_b - \beta_a \alpha_b)(\alpha^a \beta^b - \beta^a \alpha^b) \\ &= 2 \alpha^2 \beta^2 \end{aligned} \quad (-+++)$$

if $F^2 > 0$, both α & β spacelike, ker F timelike

$F^2 < 0$, one timelike,

$F = 0$, one null, one spacelike

CAUTION

Do not touch the screen when
the screen is hot or the screen is
off. If necessary, do not
touch the screen when it is
off.

$$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b \quad \text{for some } \alpha \text{ \& } \beta !$$

example of null case

$\{t^a, x^a, y^a, z^a\}$ ON Basis

$t^a + z^a$. null

$$\text{Null } F_{ab} = (t_a + z_a)x_b - x_a(t_b + z_b)$$

$$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b \quad \text{for some } \alpha \text{ \& } \beta !$$

example of null case

$\{t^a, x^a, y^a, z^a\}$ ON Basis

$t^a + z^a$ null

$$\text{Null } F_{ab} = (t_a + z_a)x_b - x_a(t_b + z_b)$$

$$\ker F = \{t^a + z^a, y^a\}$$

CAUTION