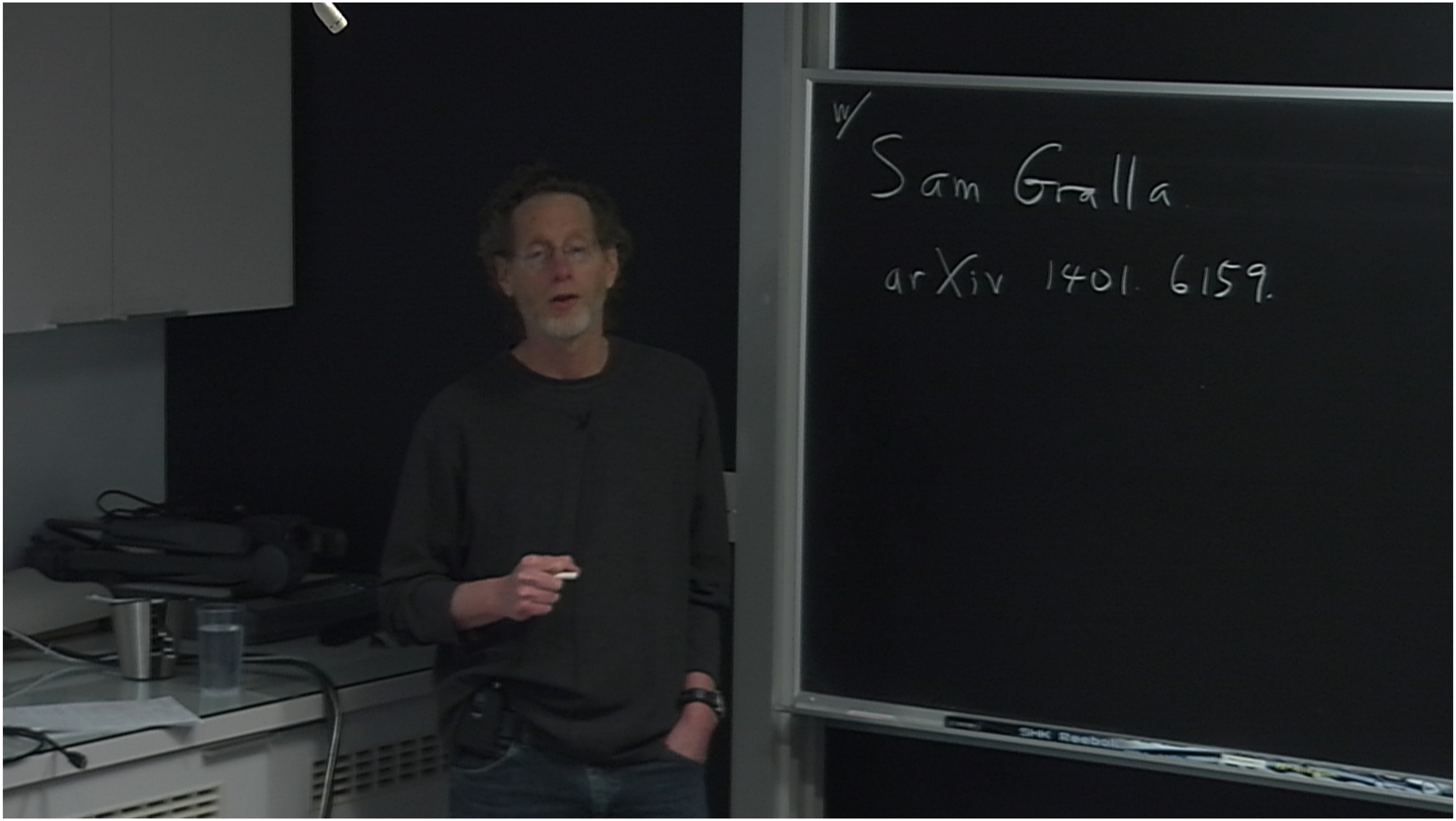
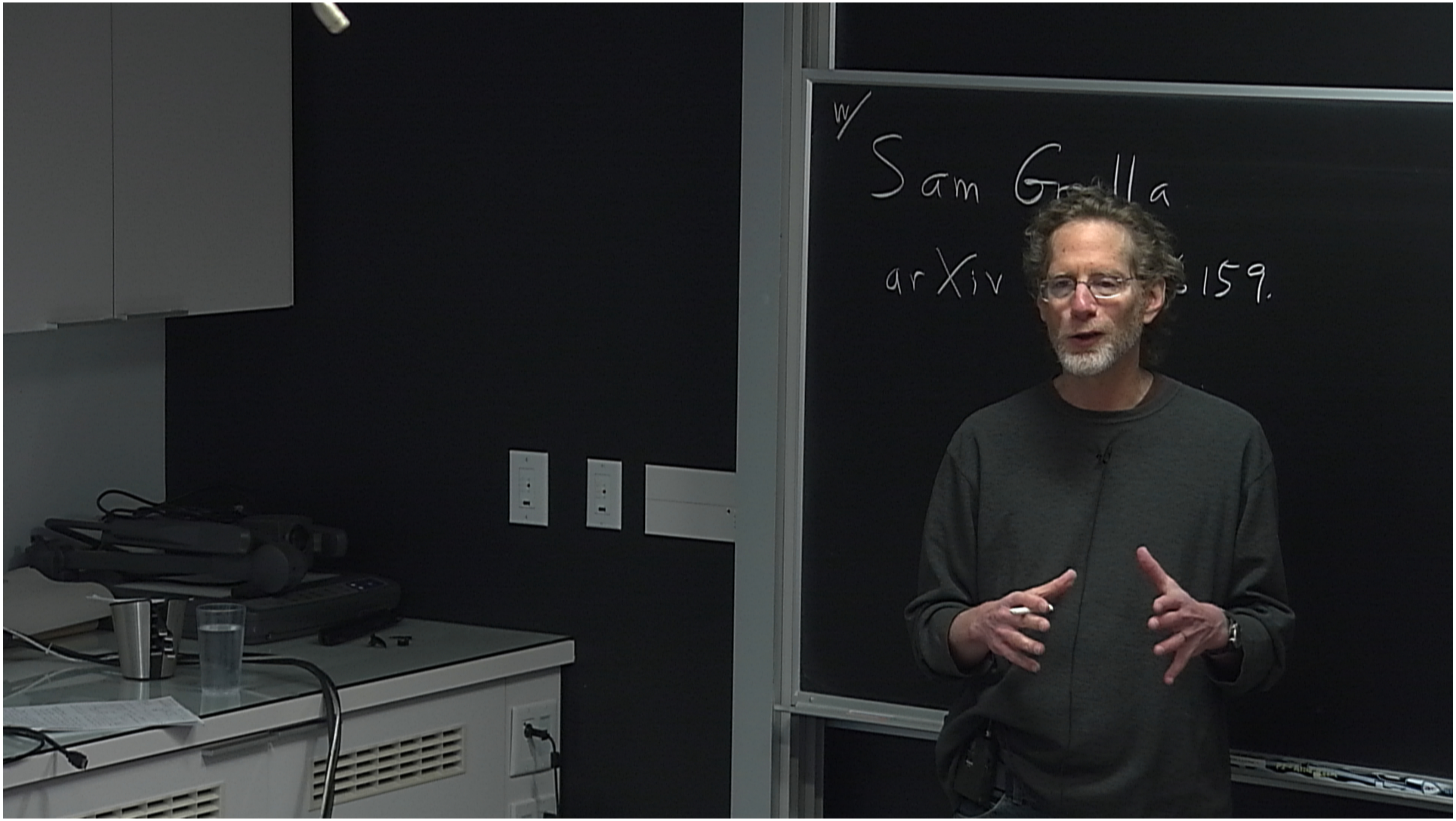
Title: Spacetime approach to force-free magnetospheres - Lecture 1

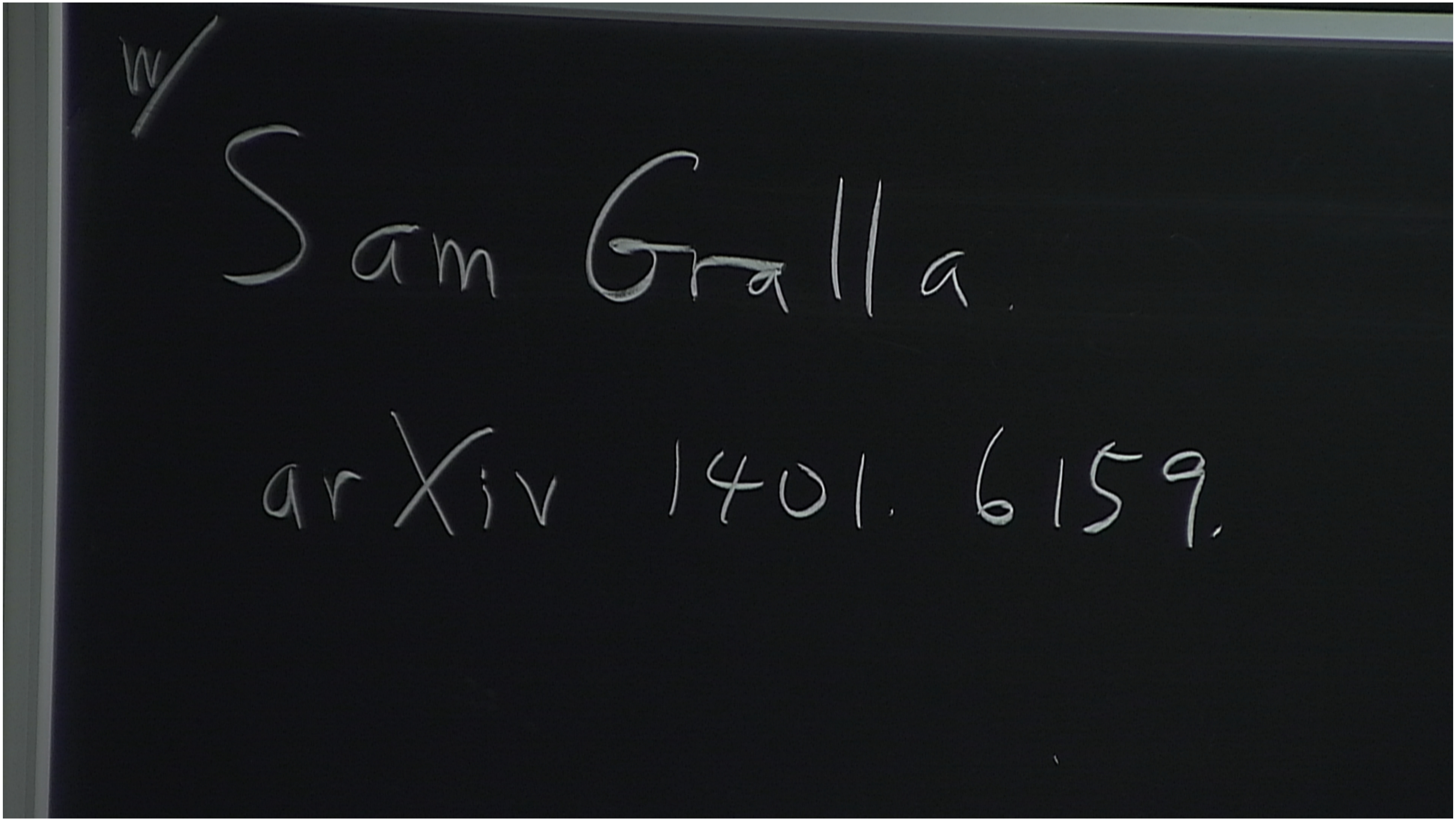
Date: Feb 25, 2014 01:00 PM

URL: http://pirsa.org/14020154

Abstract:

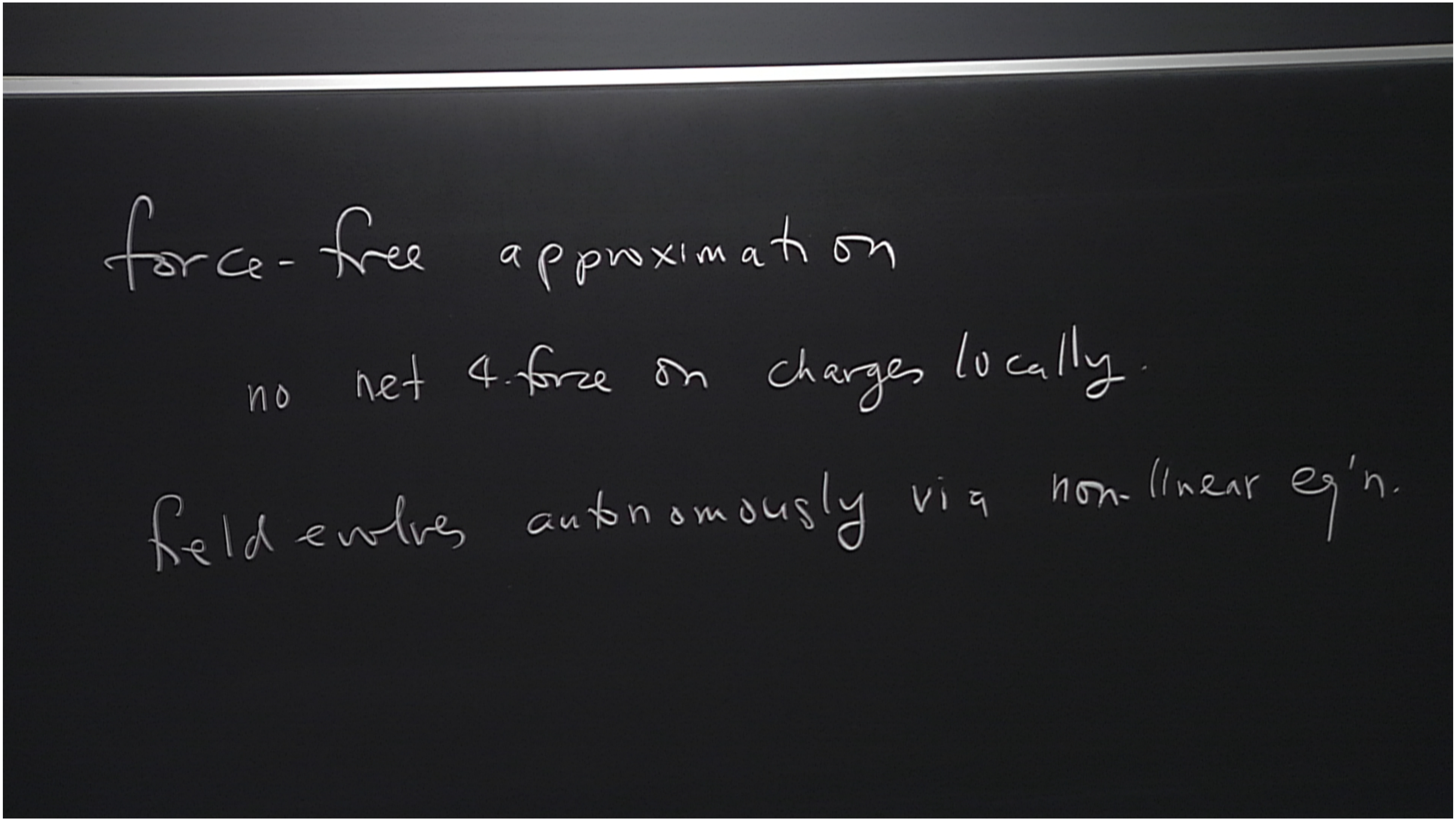

w/
Sam Gralla
arXiv 1401.6159.

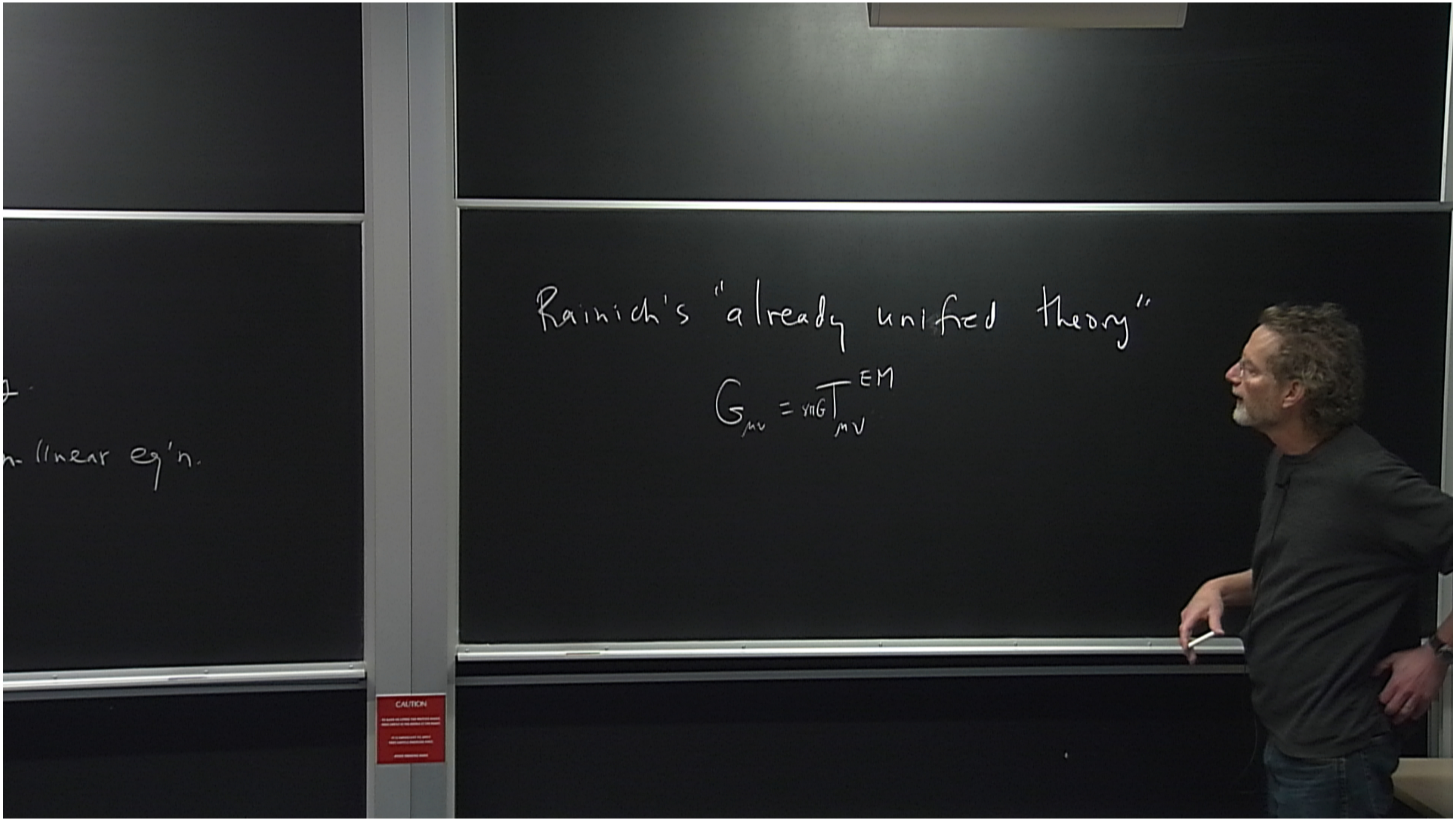

w/
Sam
arXiv
159.

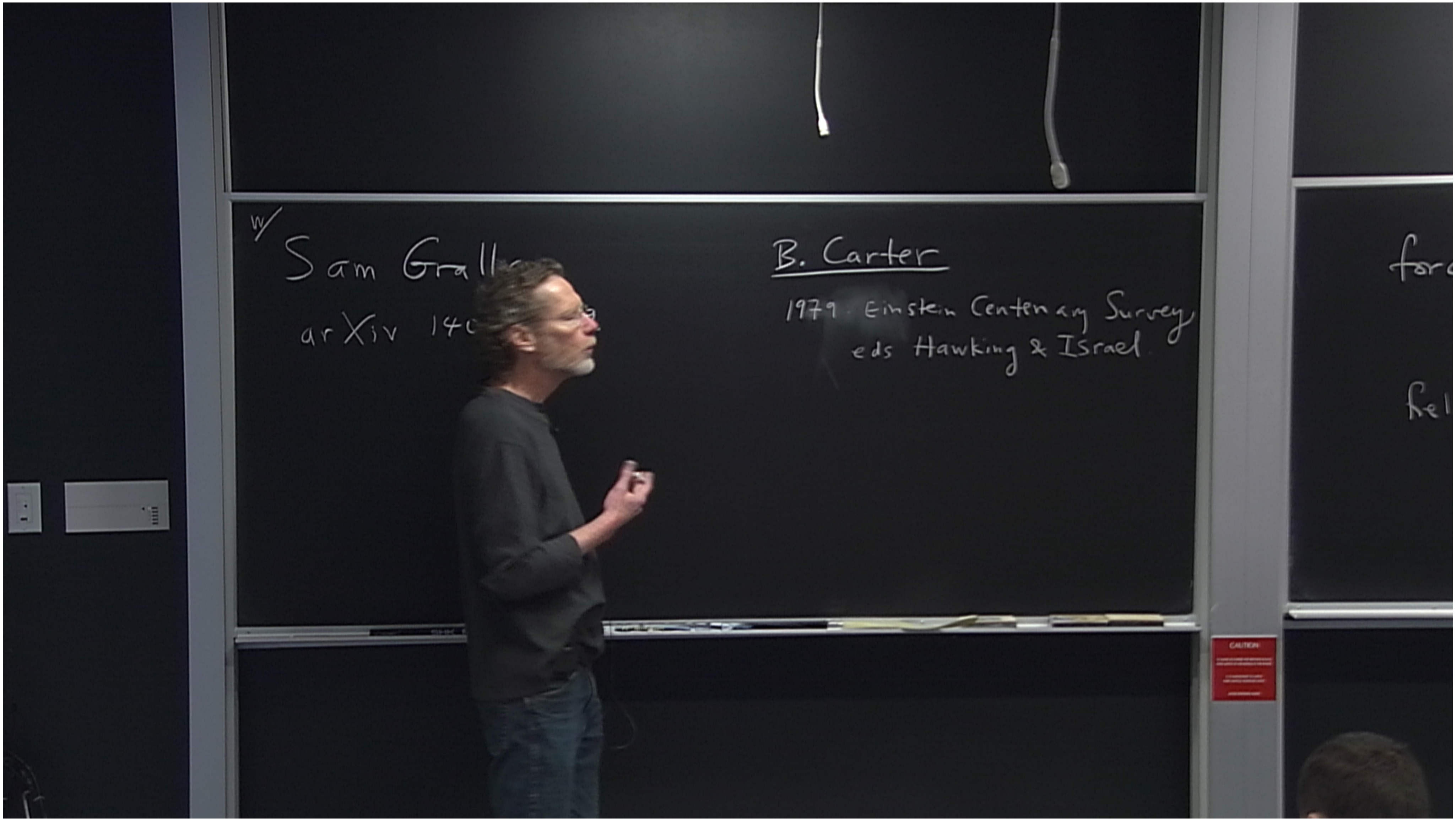

w/ Sam Gralla.
arXiv 1401.6159.

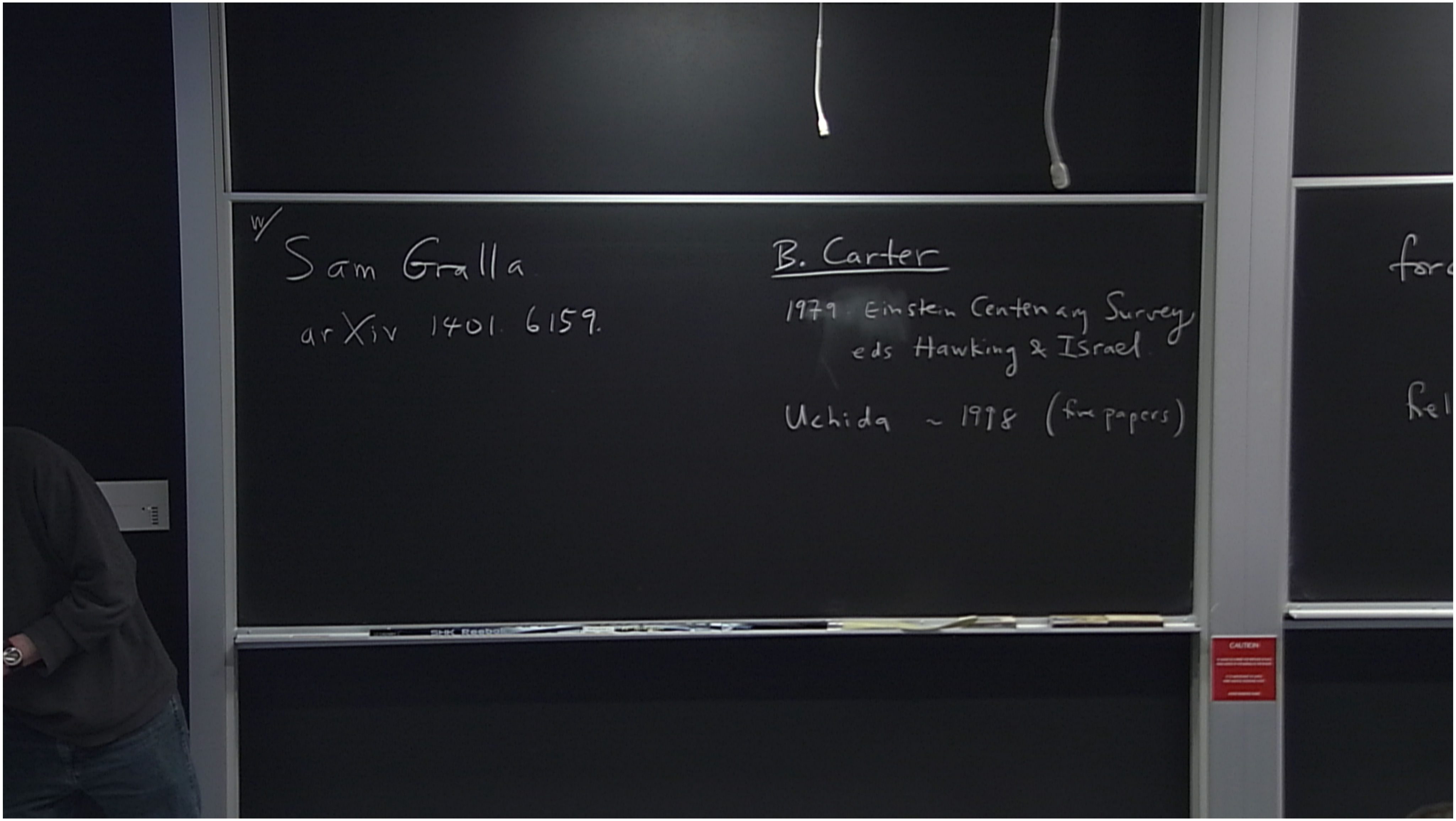
force-free approximation

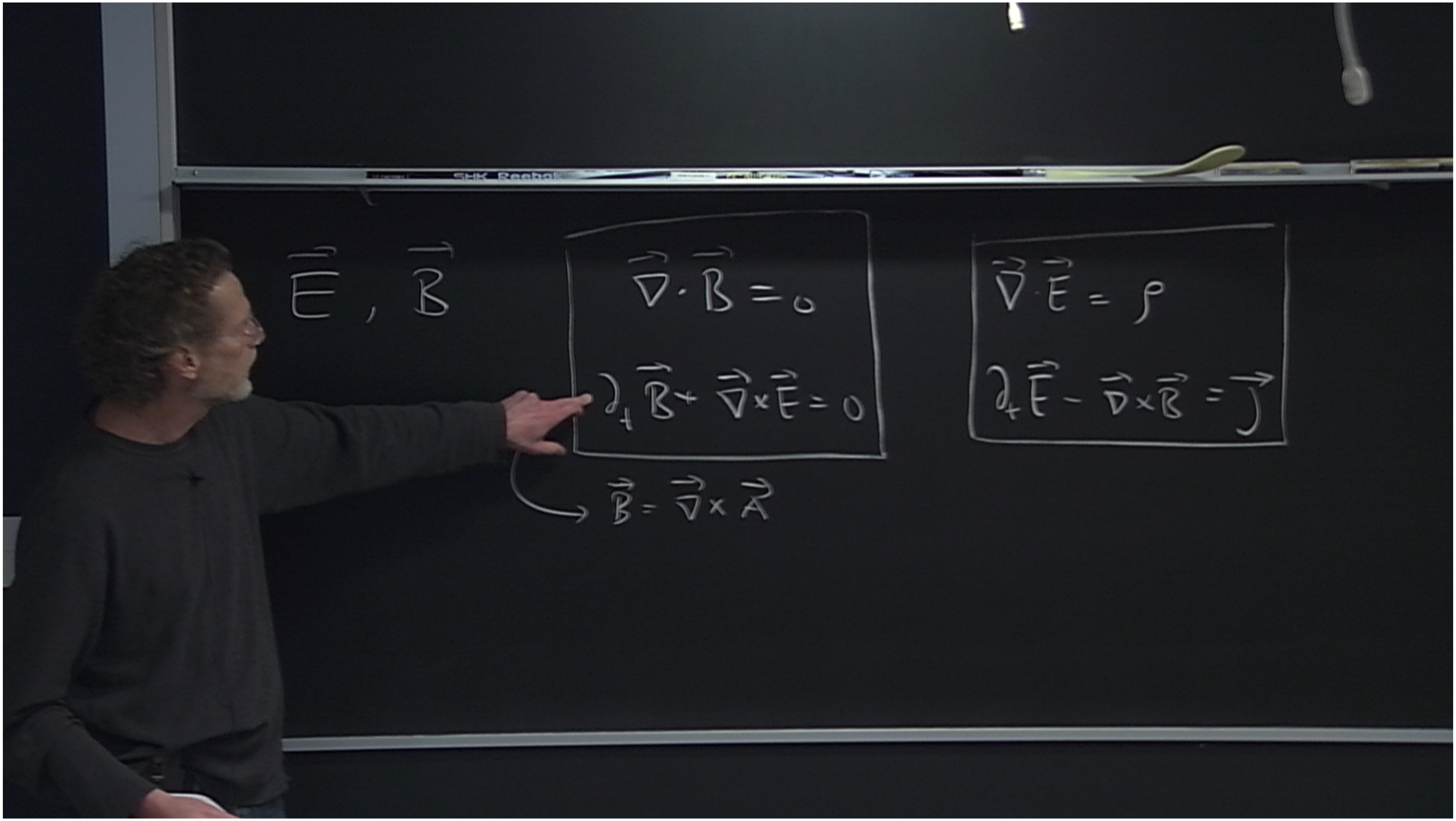
no net 4-force on charges locally.

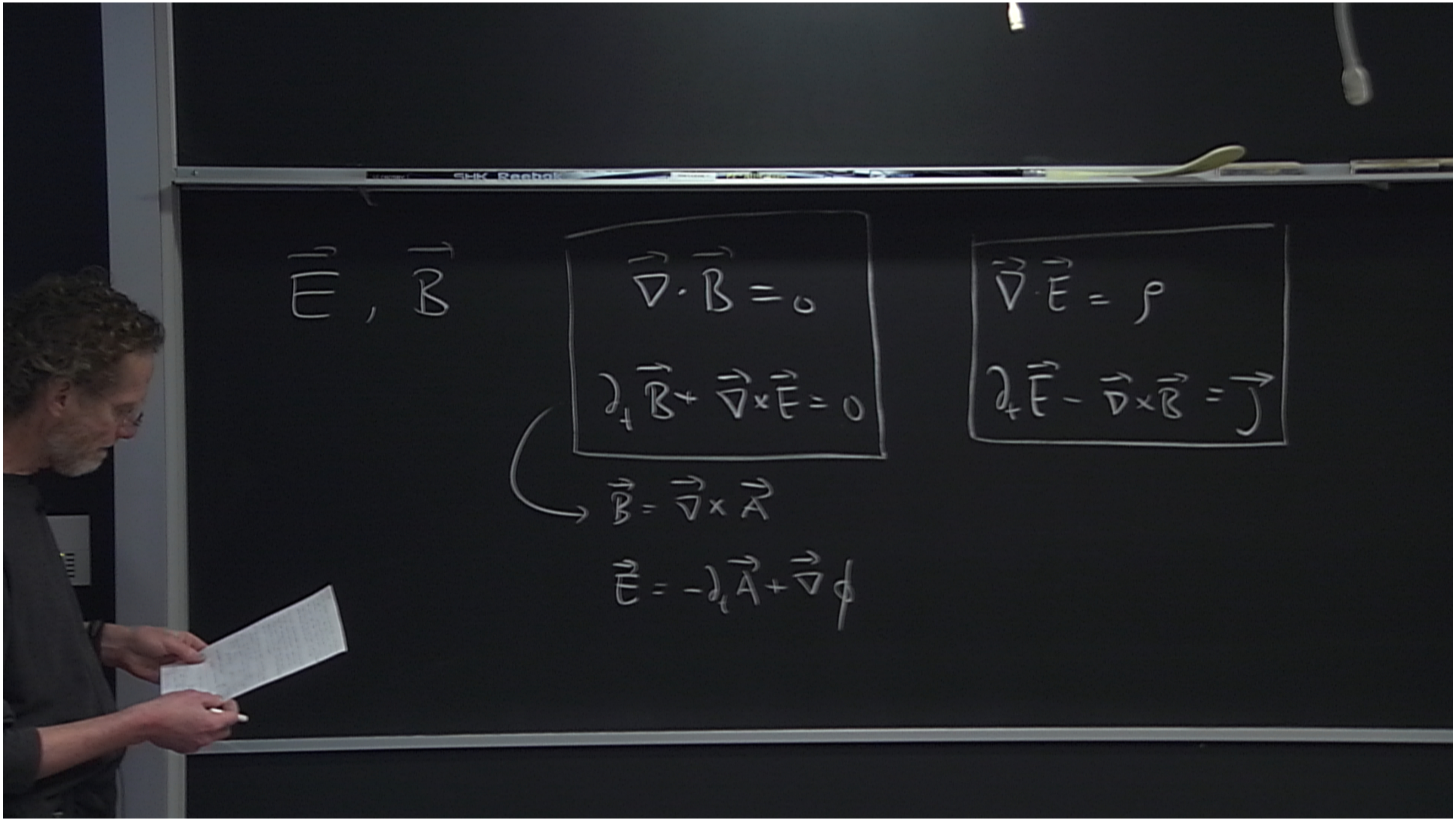

force-free approximation
no net 4-force on charges locally.
field evolves autonomously via non-linear eq'n.


Rainich's "already unified theory"
G_{μν} = 8πG T^{EM}_{μν}
linear eq'n.


w/ Sam Grall
arXiv 140
B. Carter
1979 Einstein Centenary Survey
eds Hawking & Israel


w/ Sam Gralla
arXiv 1401.6159.
B. Carter
1979 Einstein Centenary Survey
eds Hawking & Israel
Uchida ~ 1998 (five papers)


$\vec{E}$, $\vec{B}$
$\vec{\nabla}\cdot\vec{B} = 0$
$\partial_t \vec{B} + \vec{\nabla}\times\vec{E} = 0$
$\vec{B} = \vec{\nabla}\times\vec{A}$
$\vec{\nabla}\cdot\vec{E} = \rho$
$\partial_t \vec{E} - \vec{\nabla}\times\vec{B} = \vec{J}$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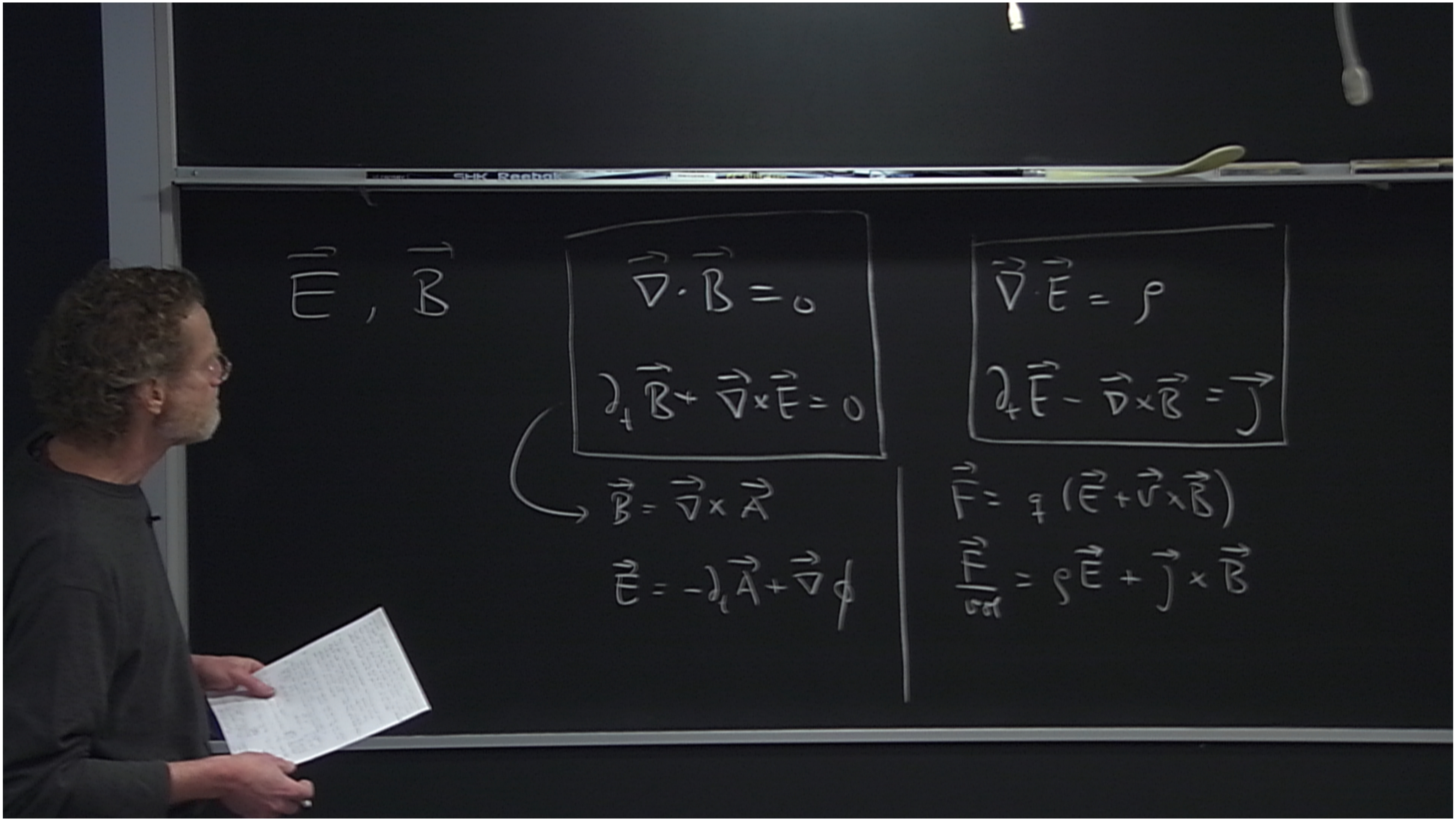
$\vec{B} = \vec{\nabla} \times \vec{A}$
$\vec{E} = -\partial_t \vec{A} + \vec{\nabla} \phi$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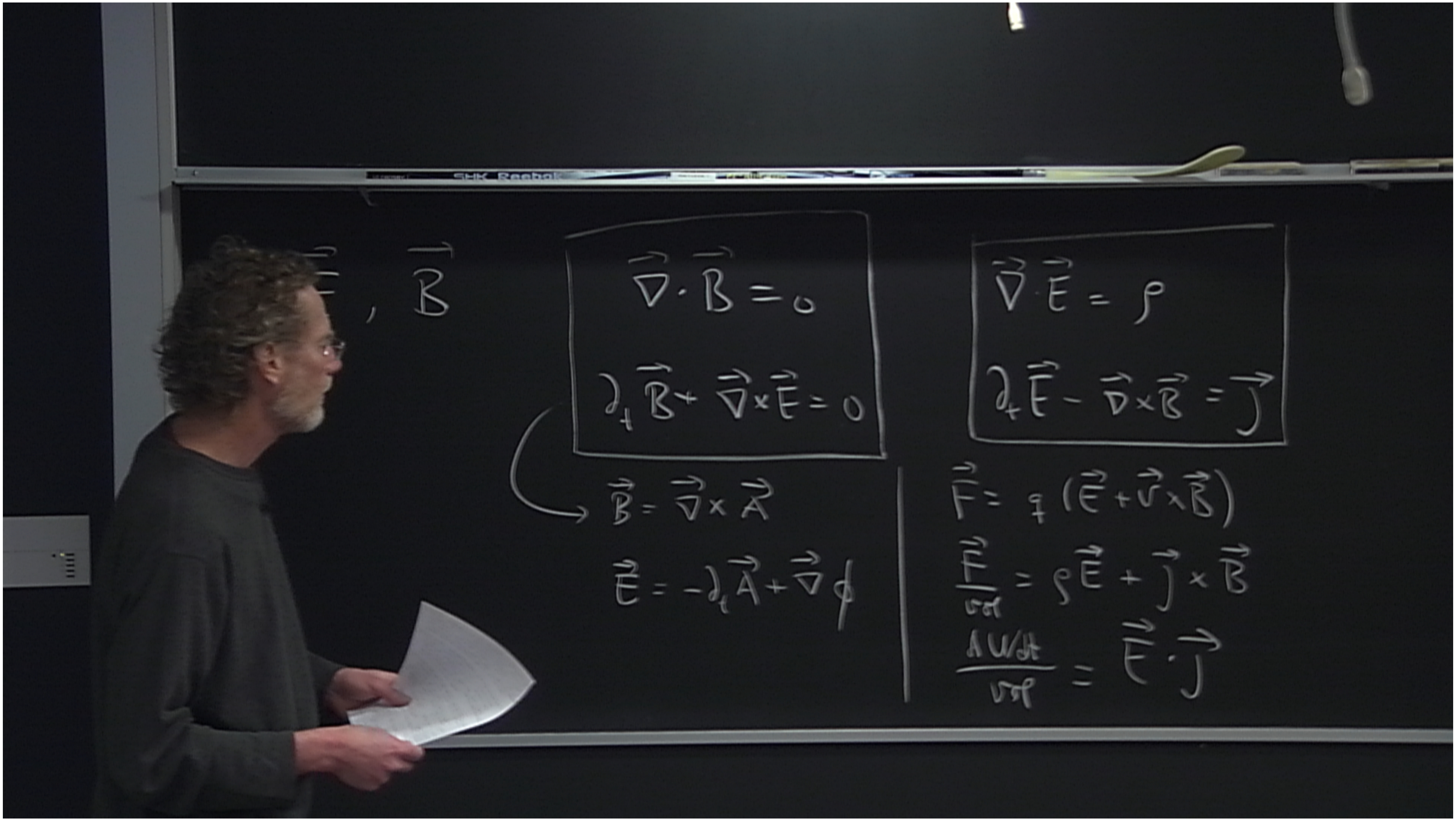
$\vec{B} = \vec{\nabla} \times \vec{A}$
$\vec{E} = -\partial_t \vec{A} + \vec{\nabla} \phi$
$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
$\frac{\vec{F}}{vol} = \rho \vec{E} + \vec{J} \times \vec{B}$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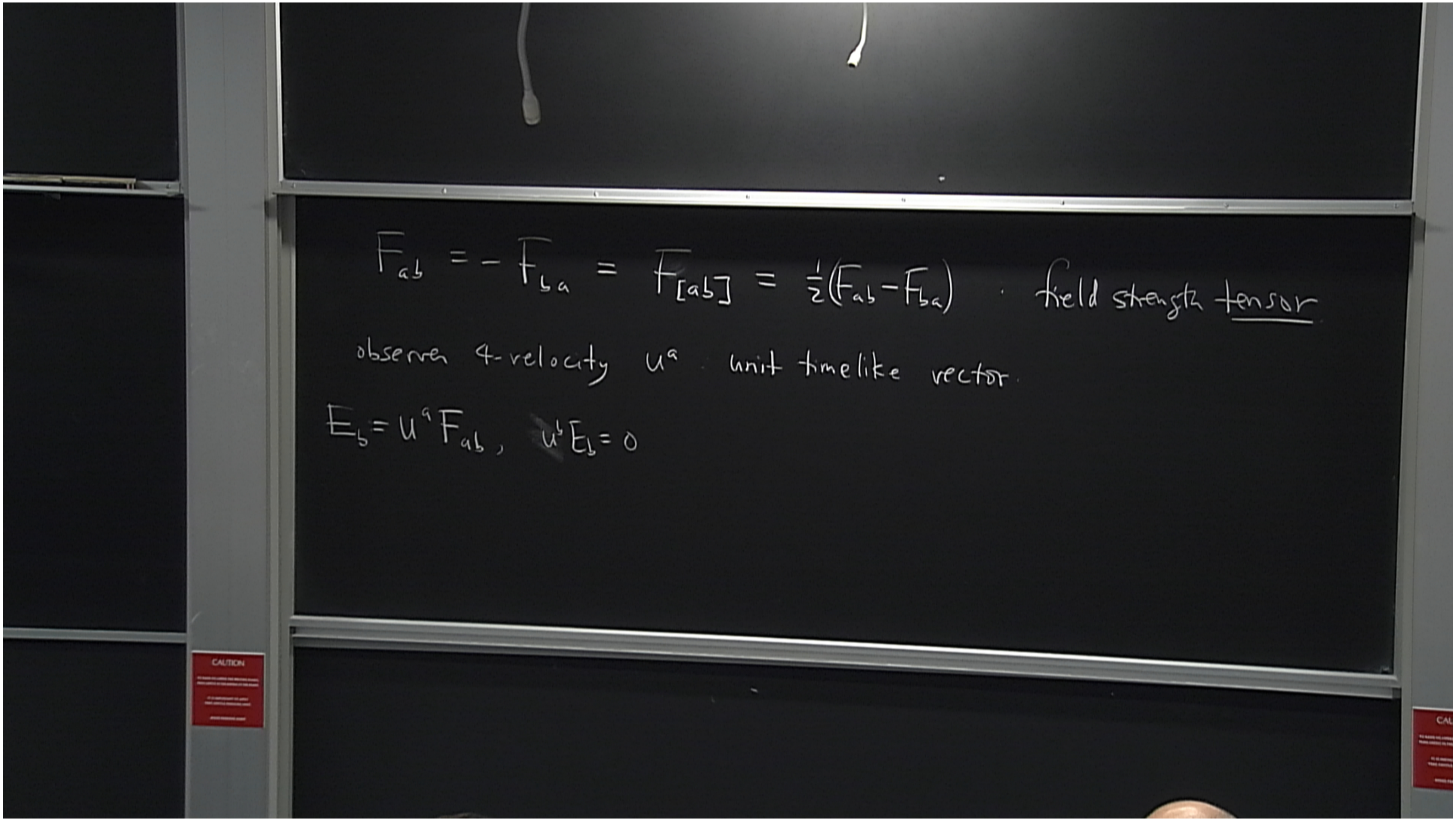
$\frac{dU/dt}{vol} = \vec{E}\cdot\vec{J}$

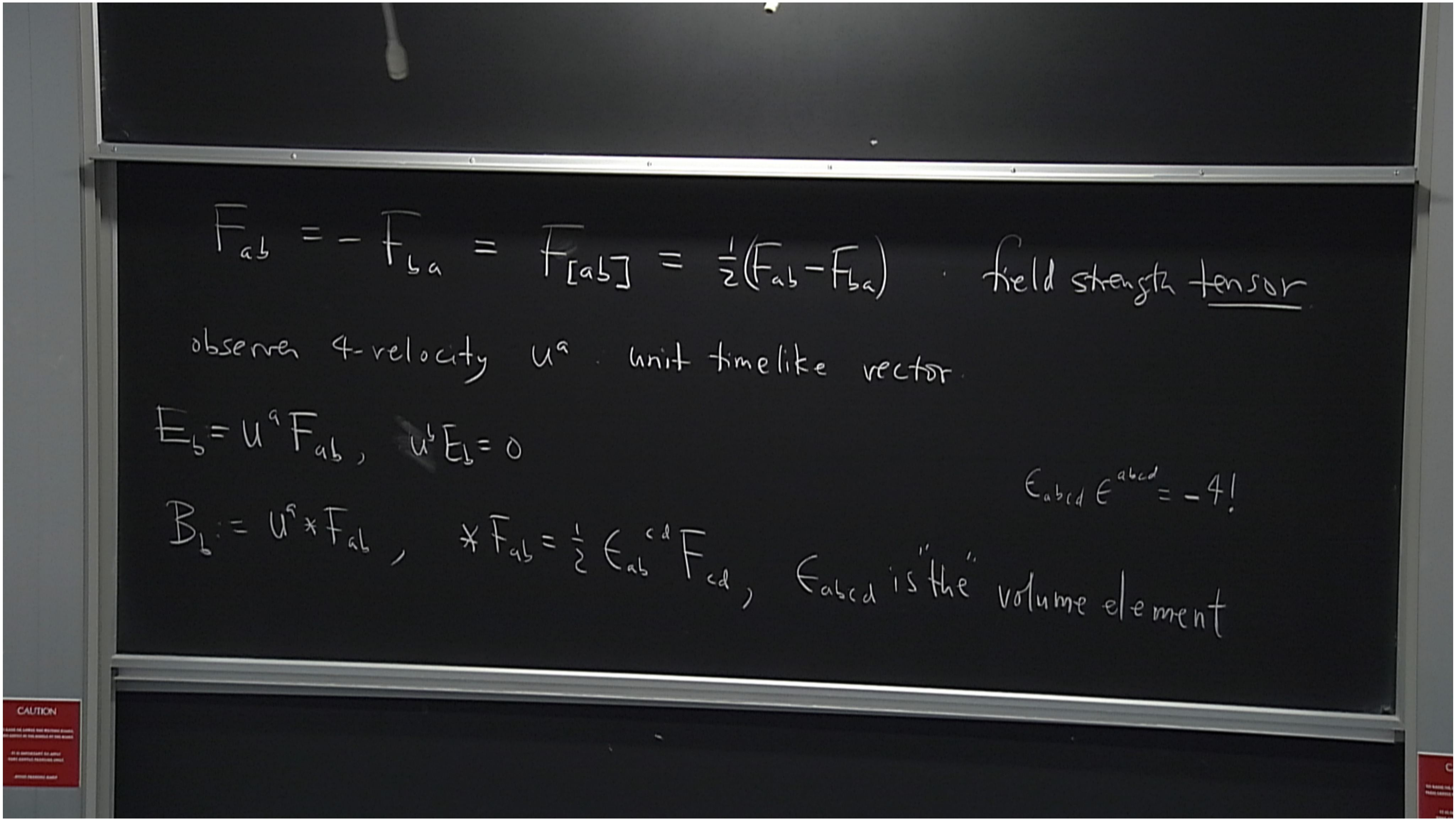

$F_{ab} = -F_{ba} = F_{[ab]} = \frac{1}{2}(F_{ab} - F_{ba})$. field strength tensor
observer 4-velocity u^a . unit timelike vector.
$E_b = u^a F_{ab}$, $u^b E_b = 0$

$F_{ab} = -F_{ba} = F_{[ab]} = \frac{1}{2}(F_{ab} - F_{ba})$ · field strength <u>tensor</u>

observer 4-velocity u^a · unit timelike vector

$E_b = u^a F_{ab}$, $u^b E_b = 0$

$B_b := u^a {*F}_{ab}$, ${*F}_{ab} = \frac{1}{2} \epsilon_{ab}{}^{cd} F_{cd}$, ϵ_{abcd} is "the" volume element


$F_{ab} = -F_{ba} = F_{[ab]} = \frac{1}{2}(F_{ab} - F_{ba})$. field strength tensor
observer 4-velocity u^a . unit timelike vector.
$E_b = u^a F_{ab}$, $u^b E_b = 0$
$\epsilon_{abcd}\,\epsilon^{abcd} = -4!$
$B_b := u^a \ast F_{ab}$, $\ast F_{ab} = \frac{1}{2}\epsilon_{ab}{}^{cd} F_{cd}$, ϵ_{abcd} is "the" volume element
CAUTION

$$F_{ab} = -F_{ba} = F_{[ab]} = \frac{1}{2}(F_{ab} - F_{ba})$$ field strength tensor

observer 4-velocity u^a unit timelike vector

$$E_b = u^a F_{ab}, \quad u^b E_b = 0$$

$$\epsilon_{abcd}\,\epsilon^{abcd} = -4!$$

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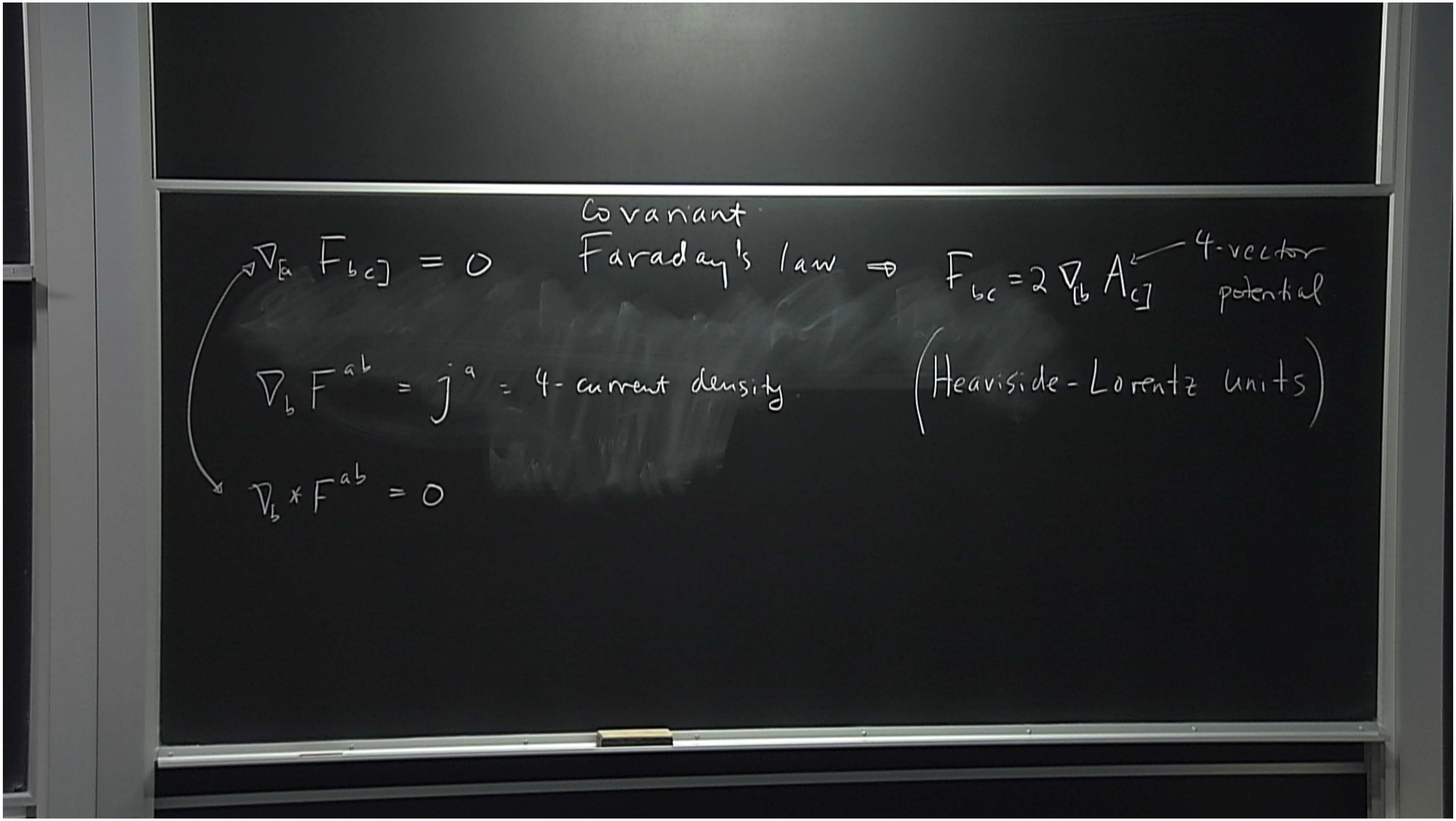
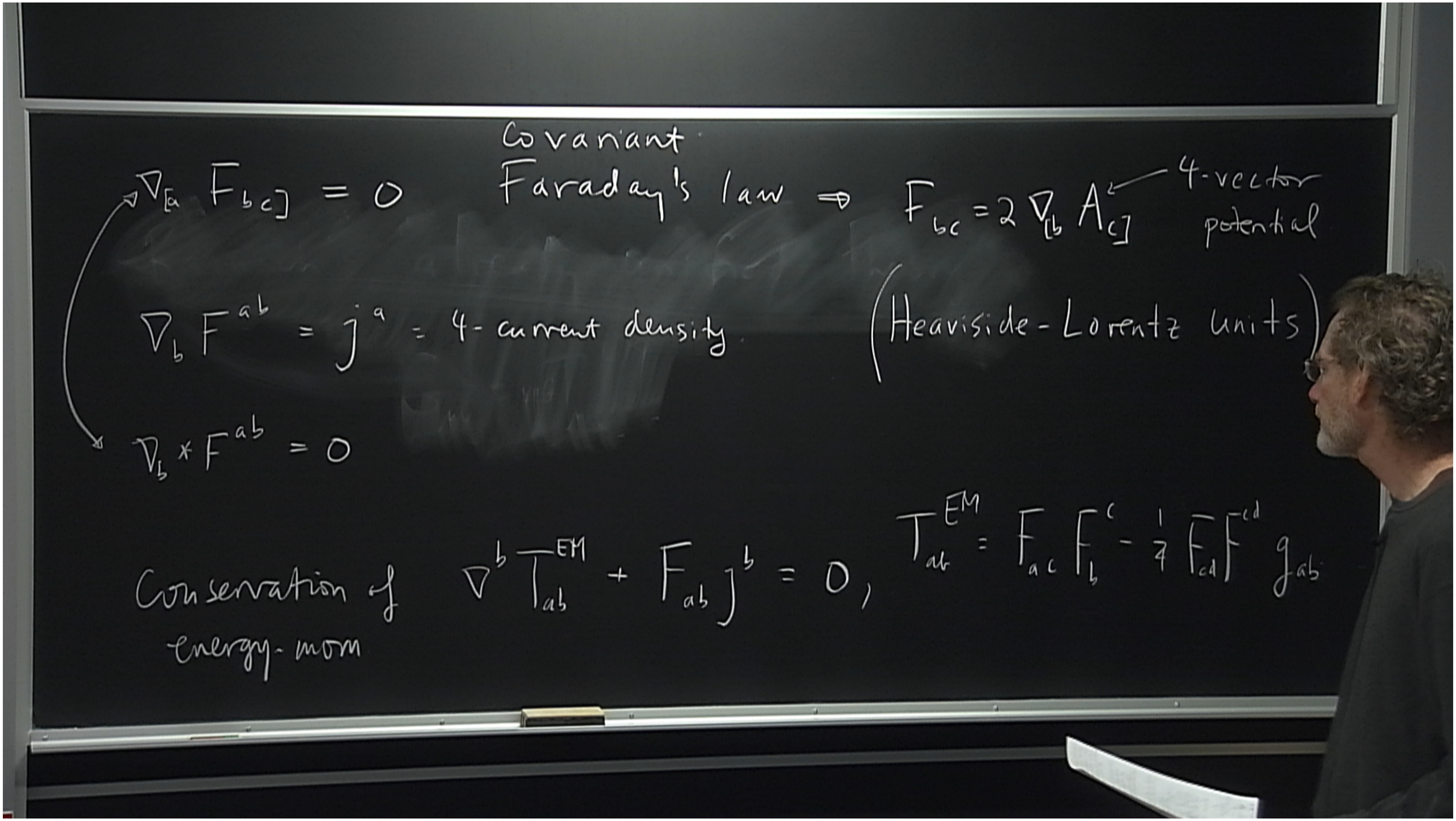

Covariant
$\nabla_{[a} F_{bc]} = 0$ Faraday's law $\Rightarrow$ $F_{bc} = 2 \nabla_{[b} A_{c]}$ ← 4-vector potential
$\nabla_b F^{ab} = j^a$ = 4-current density
(Heaviside-Lorentz units)
$\nabla_b {*F}^{ab} = 0$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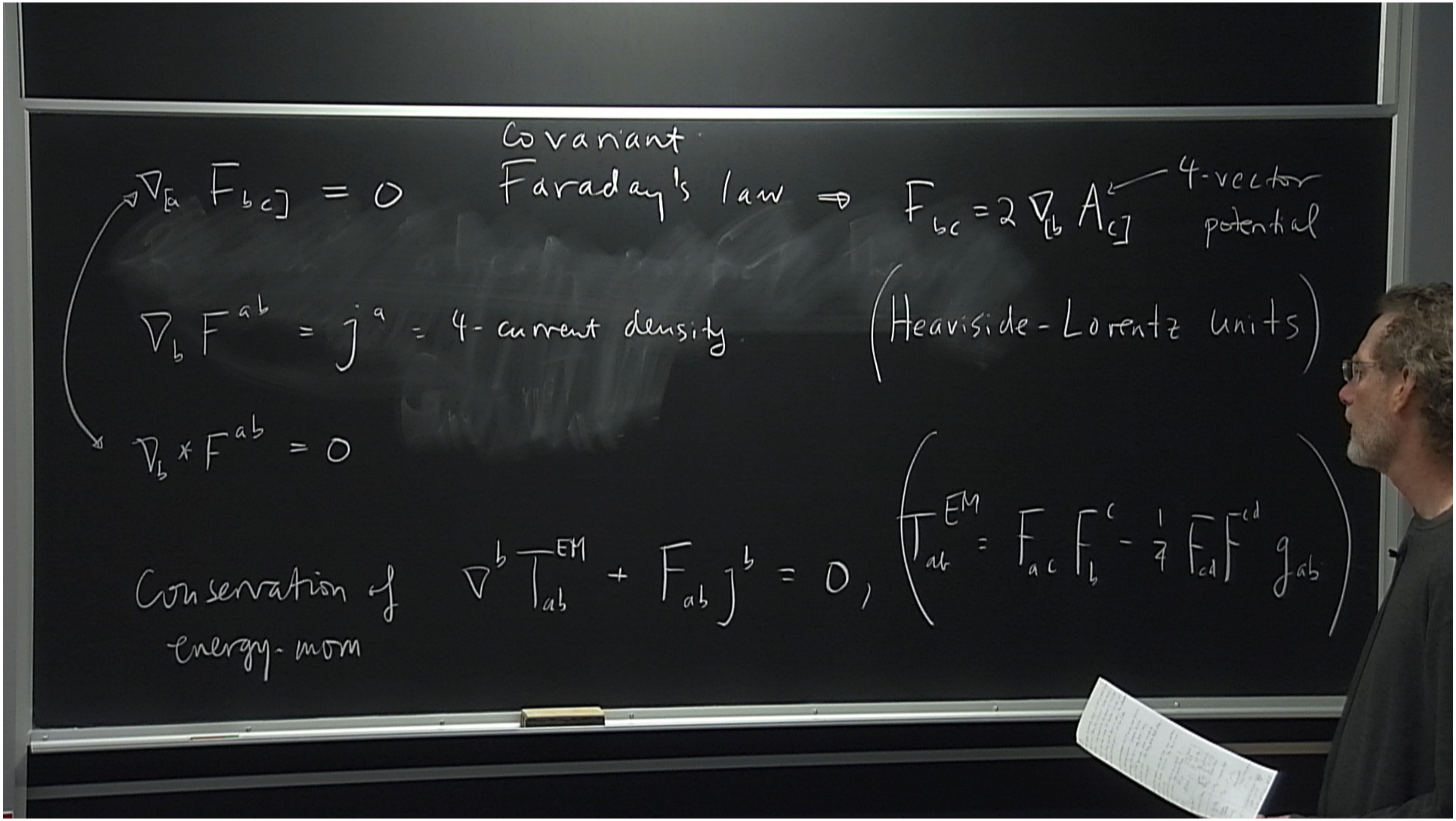
$\nabla_b F^{ab} = j^a$ = 4-current density
(Heaviside-Lorentz units)
$\nabla_b {}^*F^{ab} = 0$
Conservation of energy-mom
$\nabla^b T^{EM}_{ab} + F_{ab} j^b = 0,$
$T^{EM}_{ab} = F_{ac} F_b{}^c - \frac{1}{4} F_{cd} F^{cd} g_{ab}$



$\nabla_{[a} F_{bc]} = 0$
Covariant
Faraday's law
$\Rightarrow F_{bc} = 2 \nabla_{[b} A_{c]}$
4-vector
potential
$\nabla_b F^{ab} = j^a$ = 4-current density
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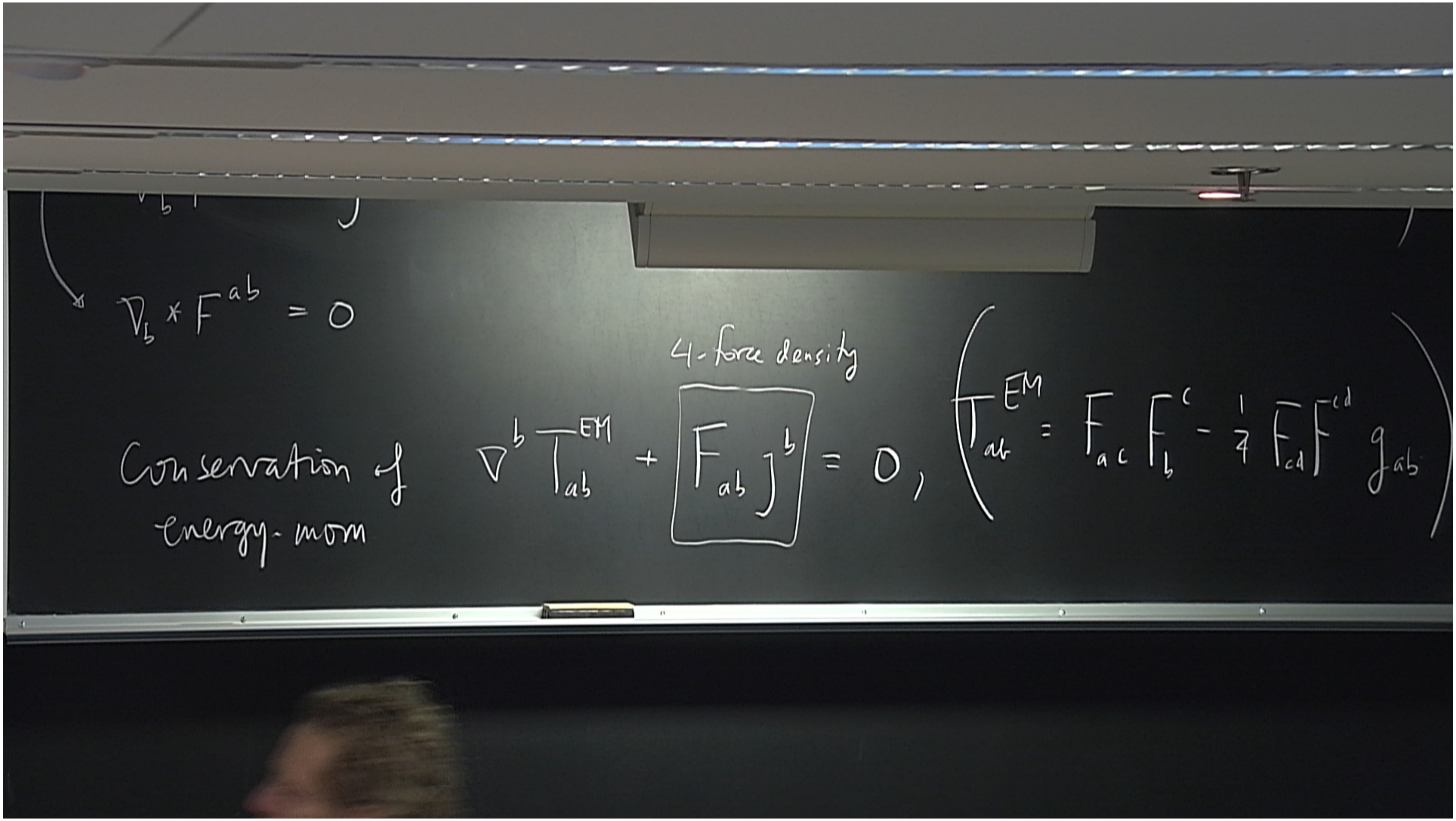
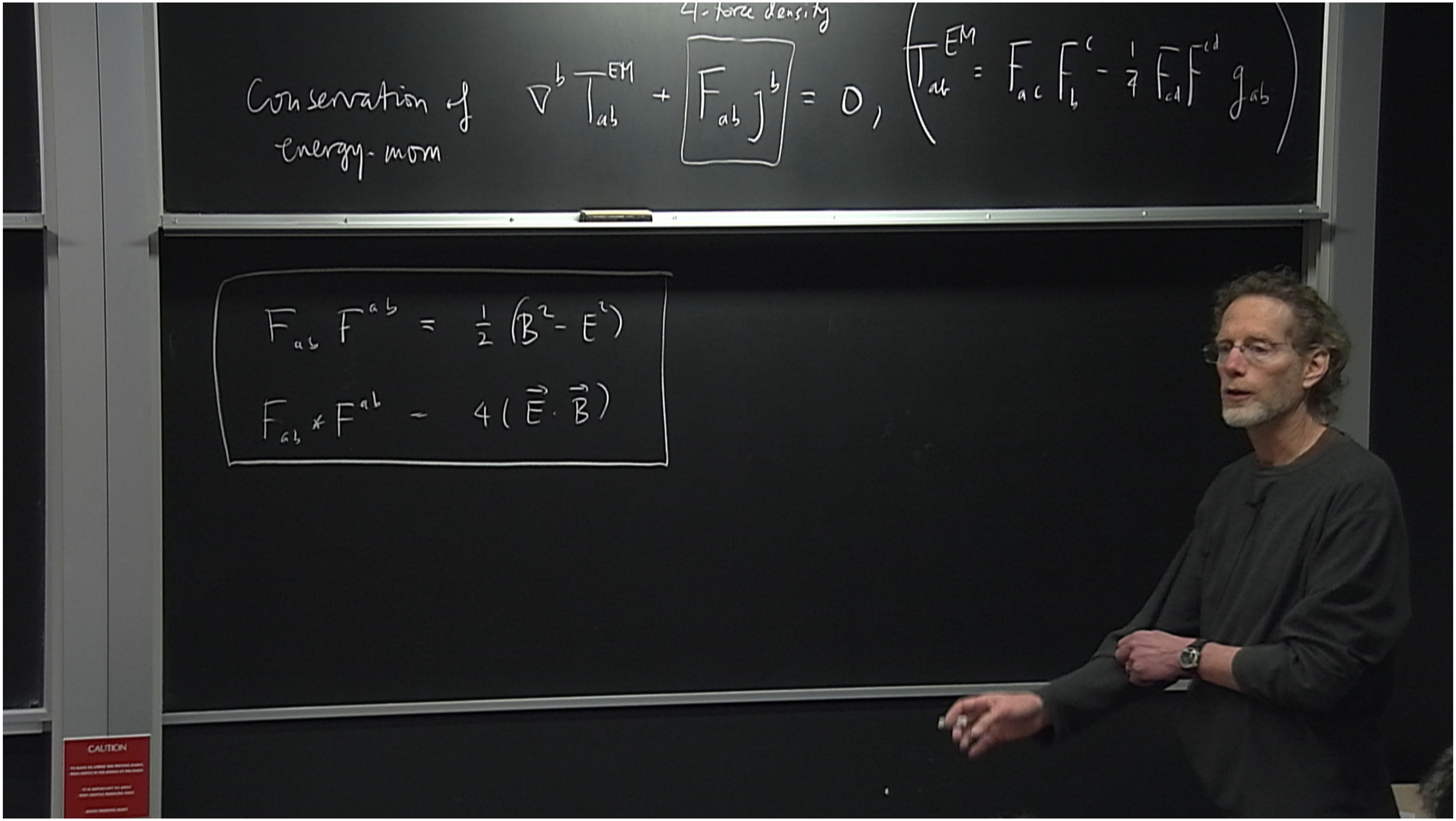
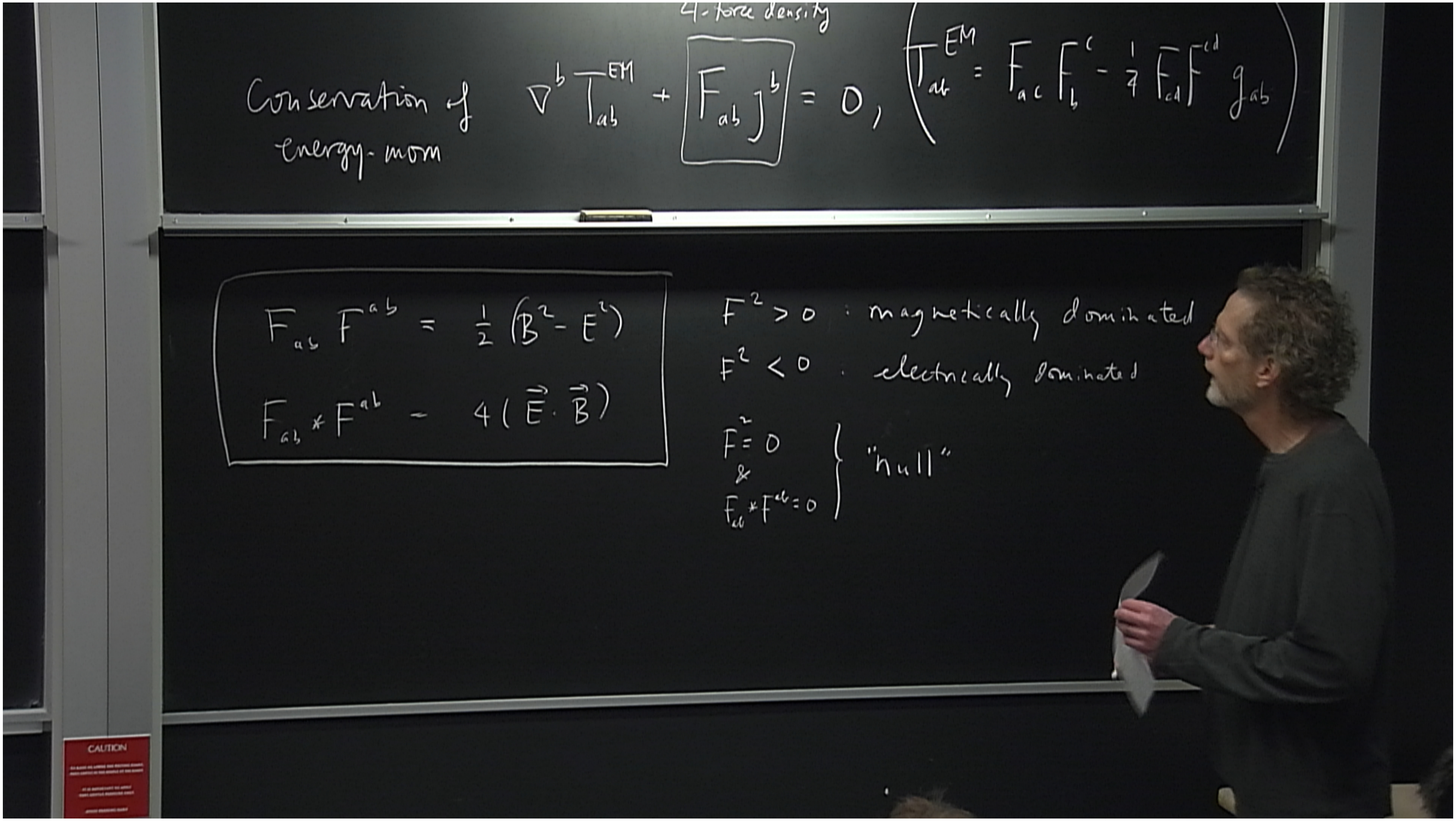
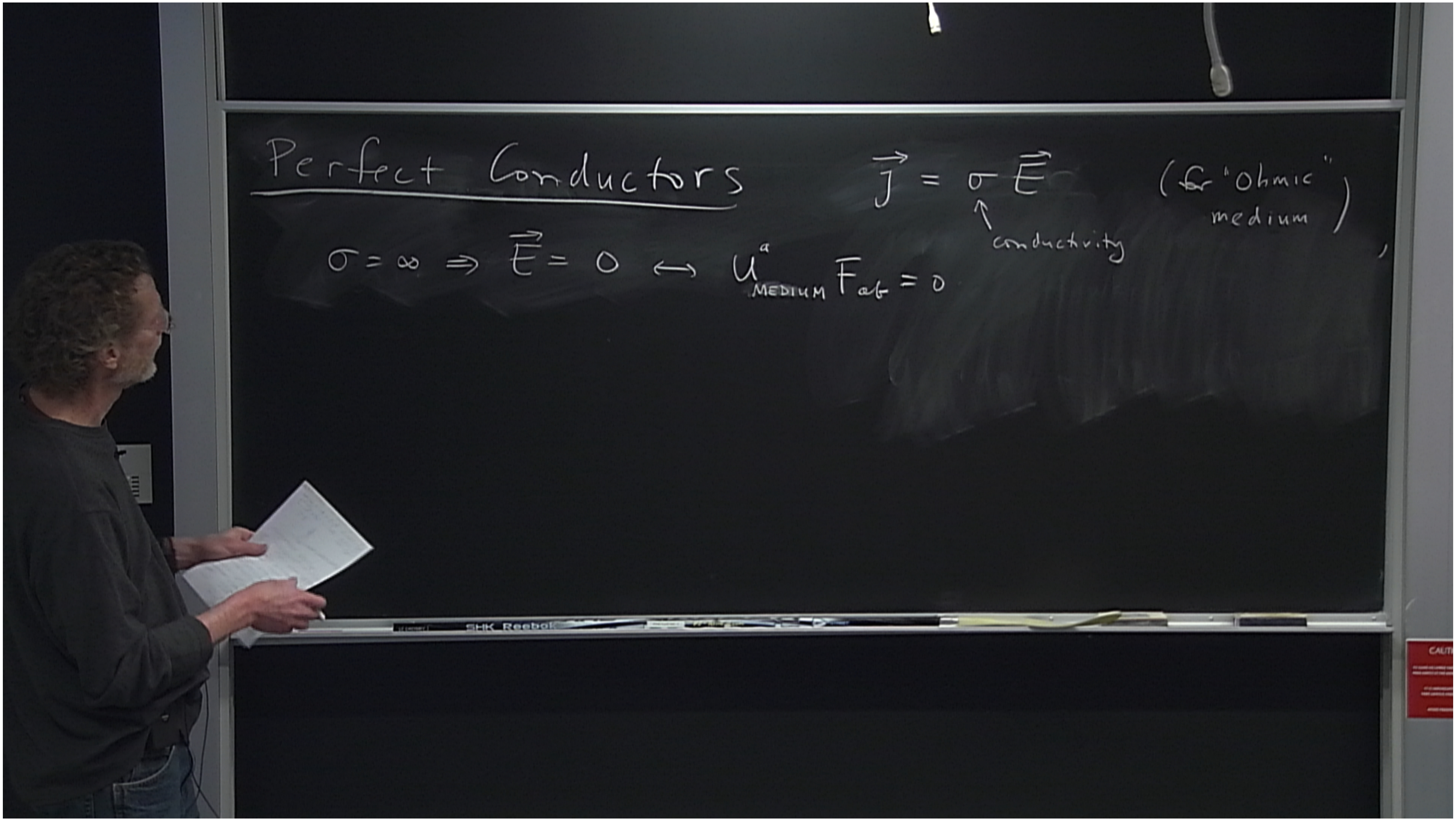
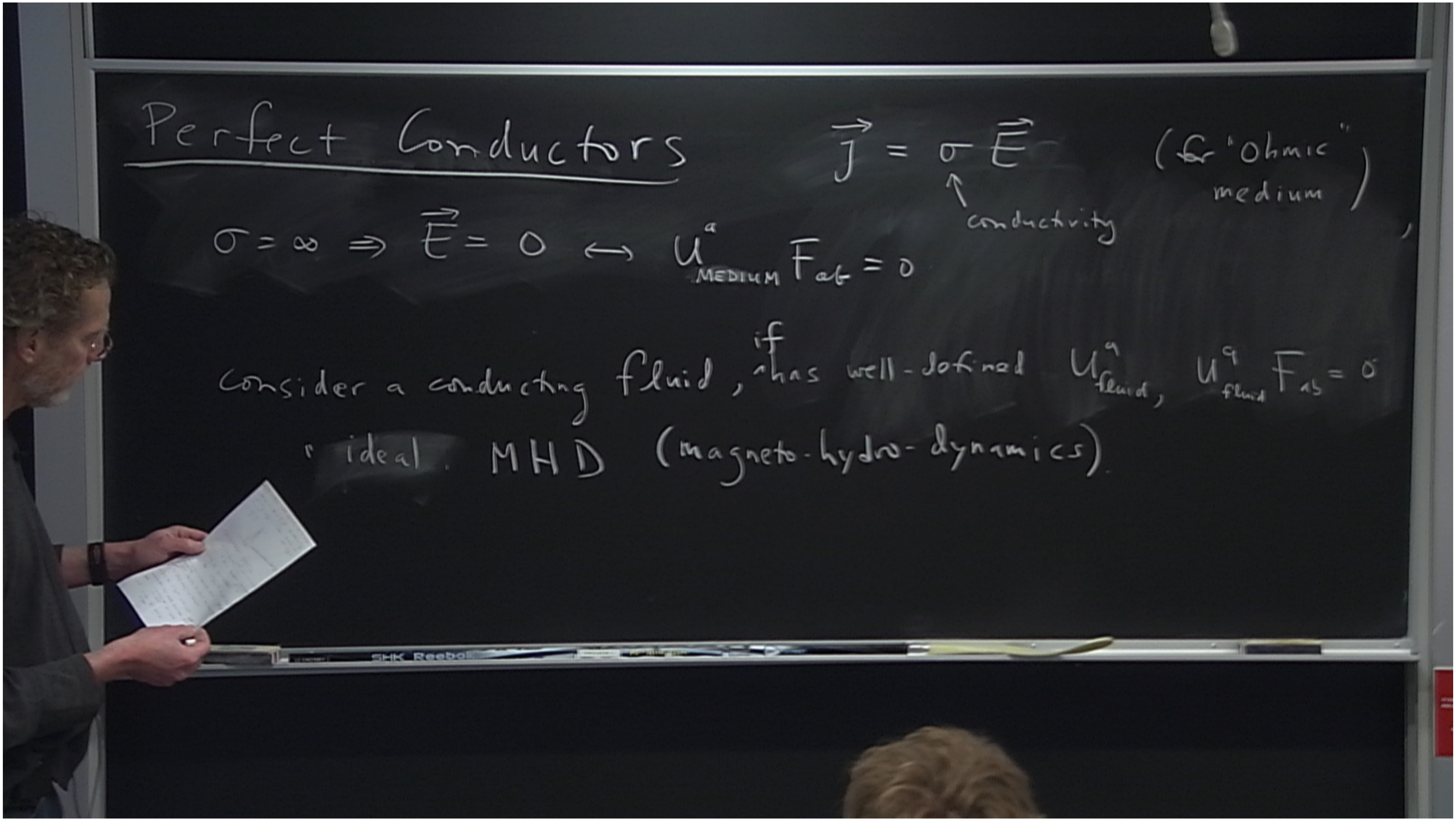
$\nabla^b T^{EM}_{ab} + F_{ab} j^b = 0,$
$\left(T^{EM}_{ab} = F_{ac} F_b{}^c - \frac{1}{4} F_{cd} F^{cd} g_{ab} \right)$

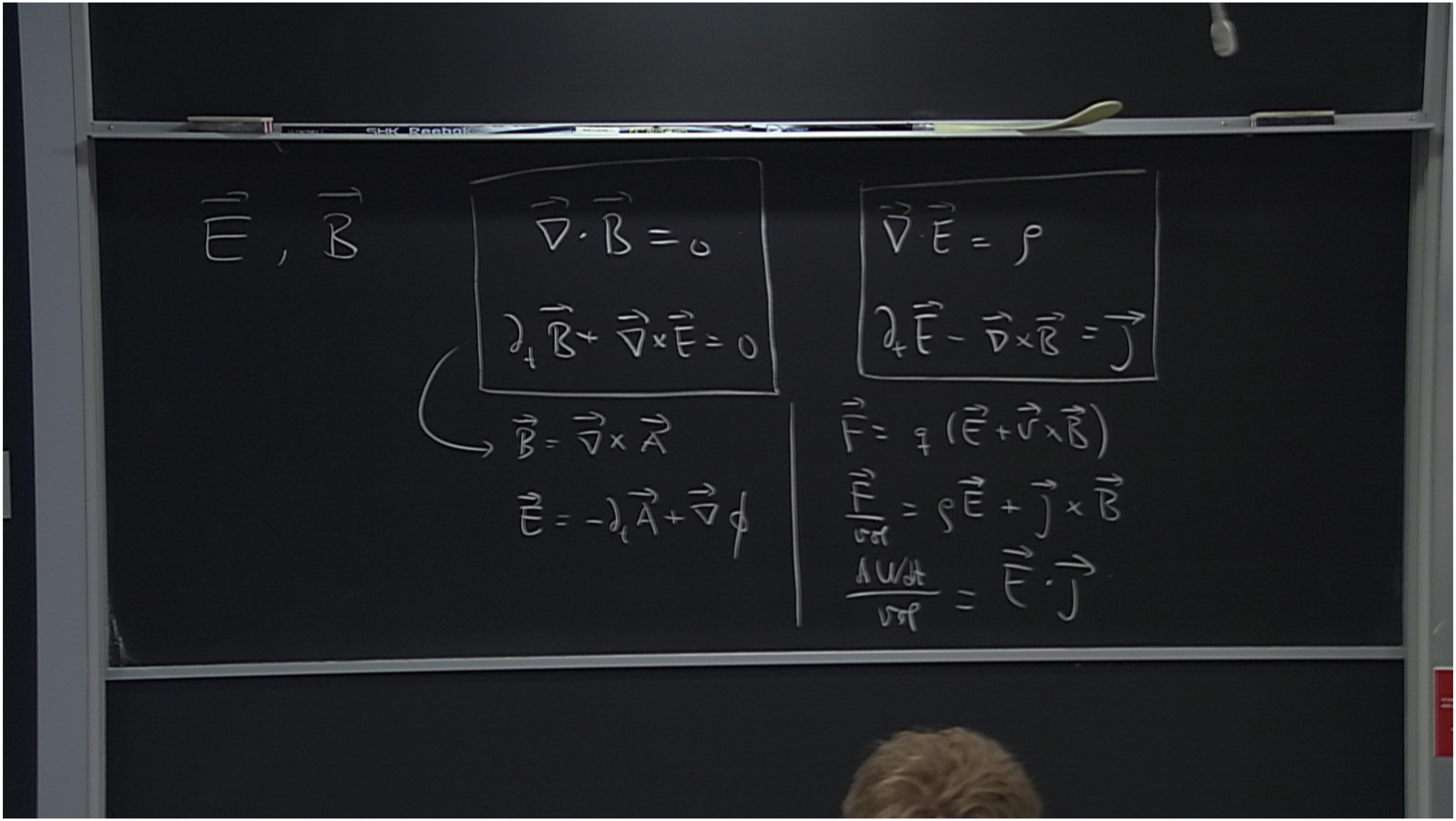

$\nabla_b {*F}^{ab} = 0$
4-force density
Conservation of energy-mom
$\nabla^b T^{EM}_{ab} + F_{ab} J^b = 0, \quad \left(T^{EM}_{ab} = F_{ac} F_b{}^c - \frac{1}{4} F_{cd} F^{cd} g_{ab} \right)$

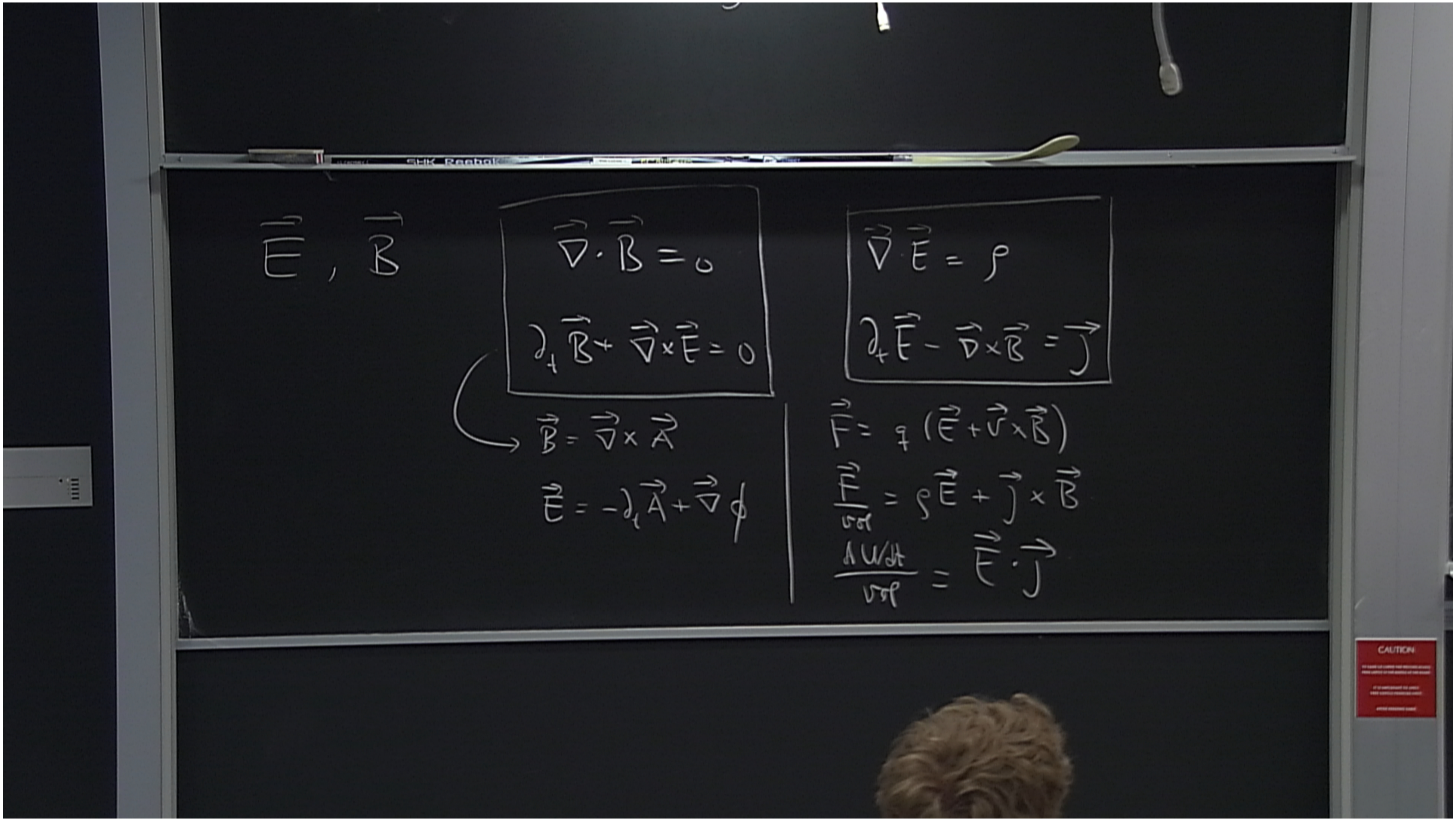

4-force density
Conservation of energy-mom
∇^b T_ab^EM + F_ab j^b = 0, (T_ab^EM = F_ac F_b^c - 1/4 F_cd F^cd g_ab)
F_ab F^ab = 1/2 (B^2 - E^2)
F_ab *F^ab = 4 (E · B)
CAUTION


4-force density
Conservation of energy-mom
$\nabla^b T^{EM}_{ab} + F_{ab} j^b = 0$, $\left(T^{EM}_{ab} = F_{ac} F_b{}^c - \frac{1}{4} F_{cd} F^{cd} g_{ab} \right)$
$F_{ab} F^{ab} = \frac{1}{2}(B^2 - E^2)$
$F_{ab} {}^*F^{ab} - 4(\vec{E} \cdot \vec{B})$
$F^2 > 0$: magnetically dominated
$F^2 < 0$: electrically dominated
$F^2 = 0$ & $F_{ab} {}^*F^{ab} = 0$ } "null"
CAUTION


Perfect Conductors
$\vec{J} = \sigma \vec{E}$
(for "Ohmic" medium)
conductivity
$\sigma = \infty \Rightarrow \vec{E} = 0 \leftrightarrow U^a_{MEDIUM} F_{ab} = 0$


Perfect Conductors
J = σ E (for "Ohmic" medium)
conductivity
σ = ∞ ⇒ E = 0 ⟷ U^a_MEDIUM F_ab = 0
consider a conducting fluid, if has well-defined U^a_fluid, U^a_fluid F_ab = 0
"ideal" MHD (magneto-hydro-dynamics)


E , B
∇ · B = 0
∂t B + ∇ × E = 0
∇ · E = ρ
∂t E − ∇ × B = J
B = ∇ × A
E = −∂t A + ∇ φ
F = q (E + v × B)
F / vol = ρ E + J × B
dU/dt / vol = E · J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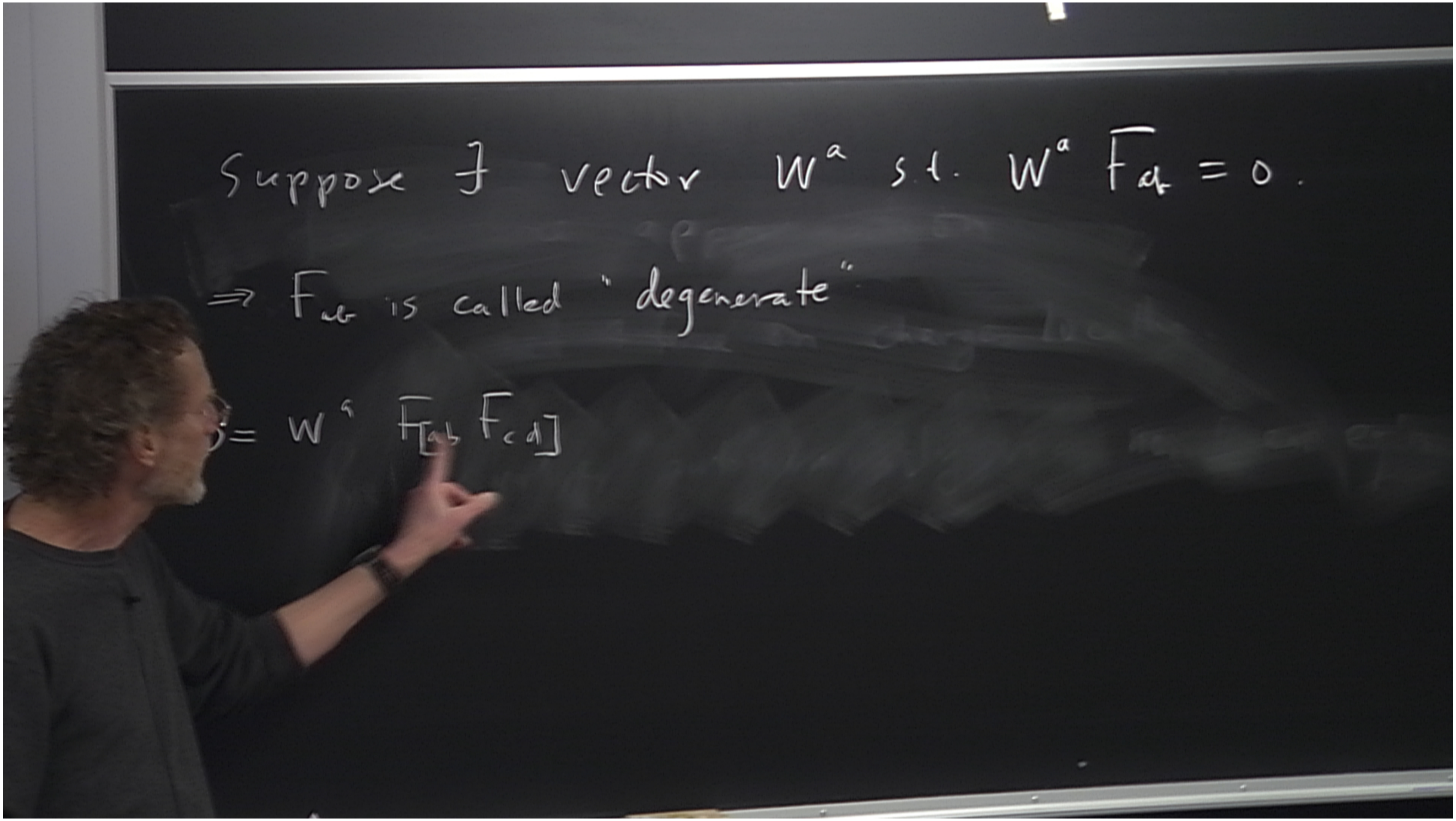
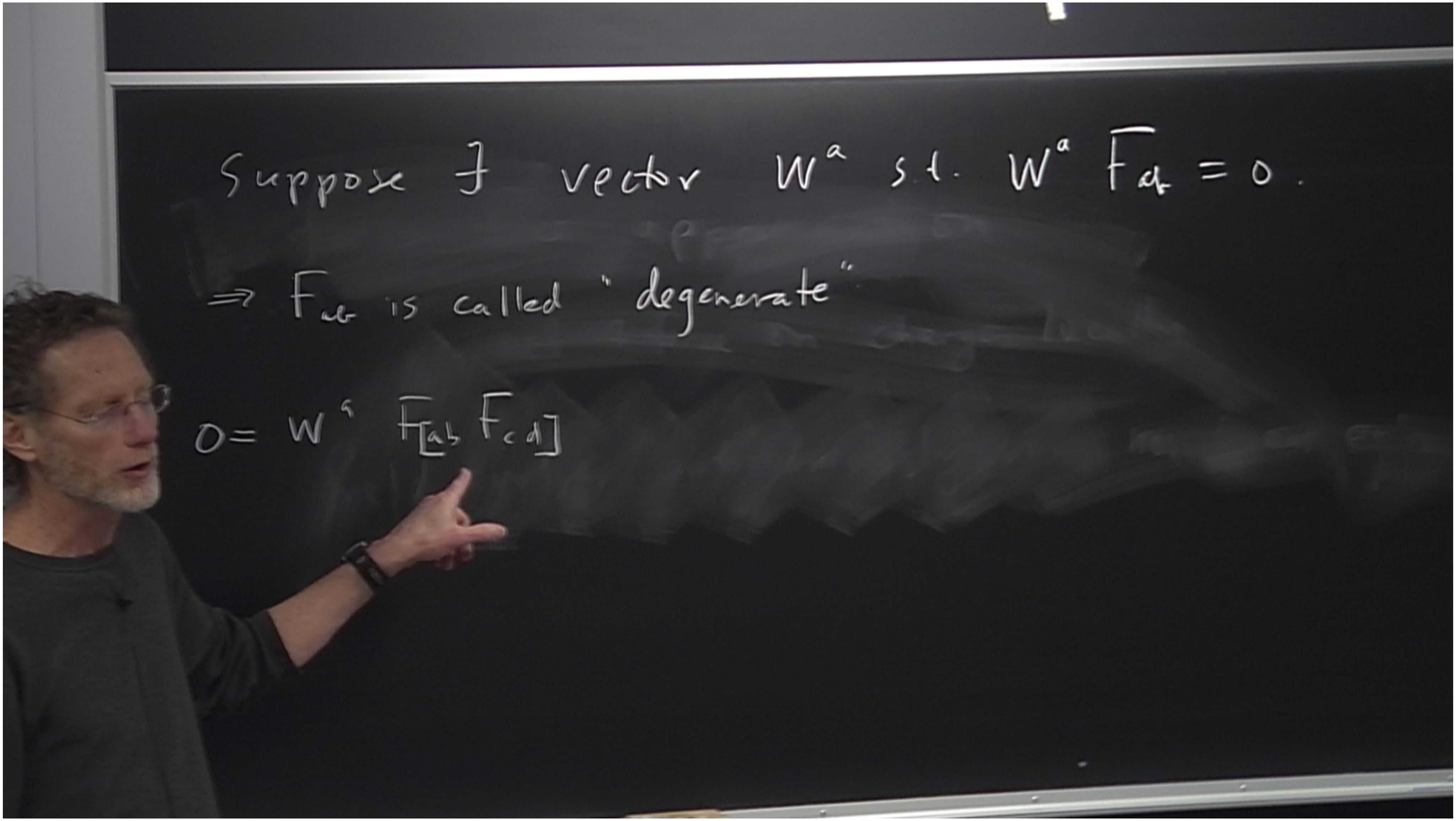
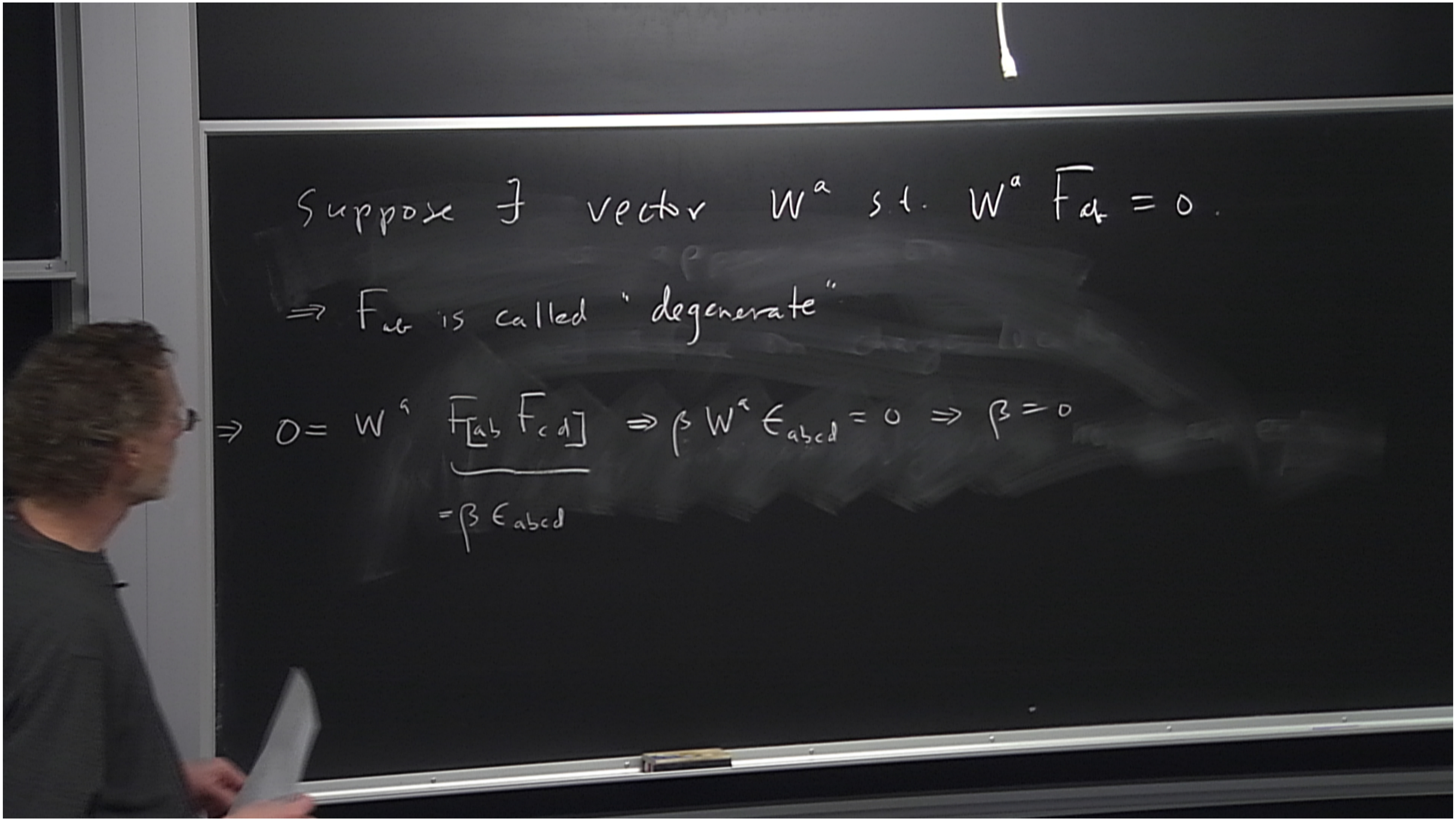
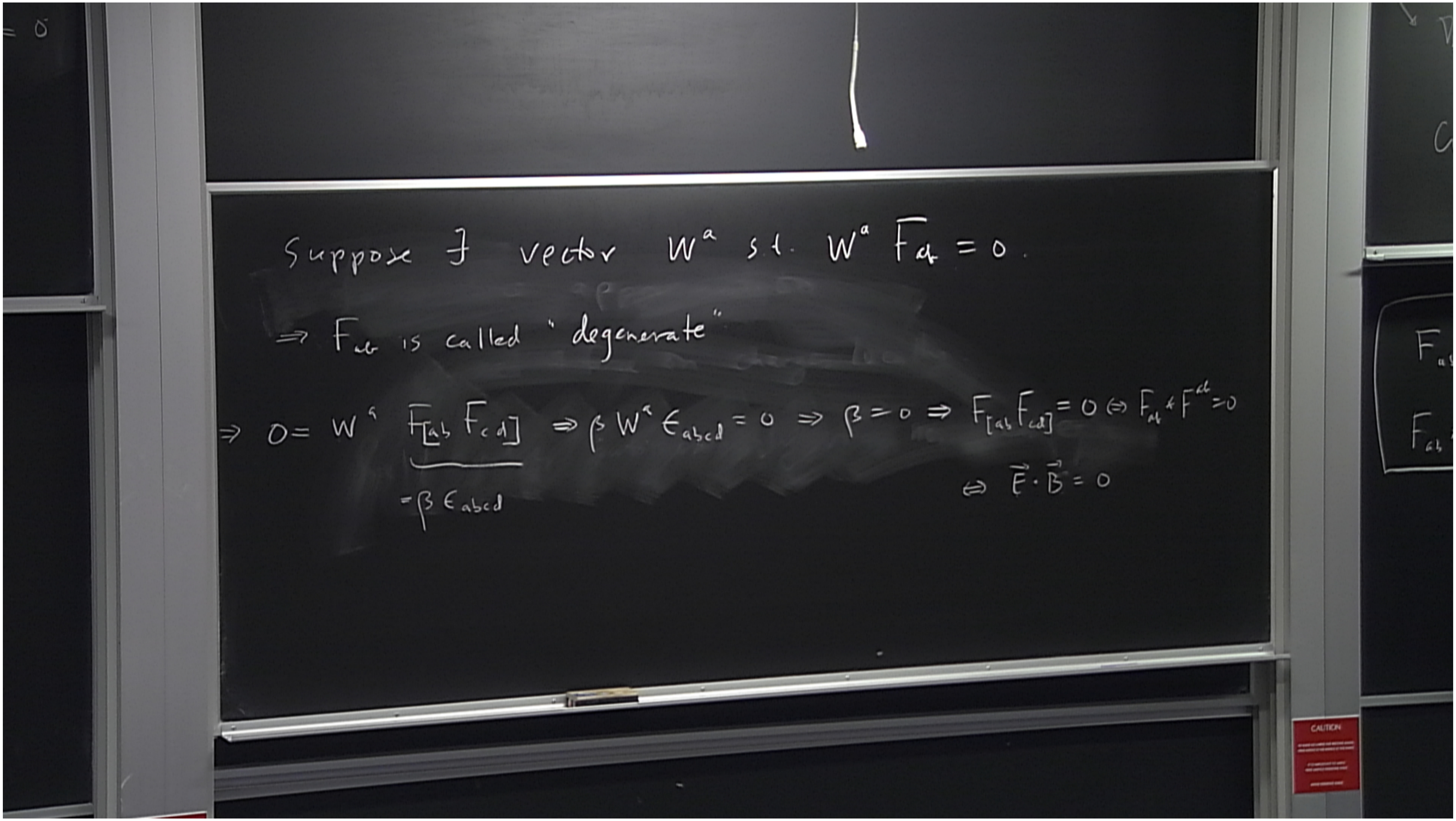
$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$
$\frac{\vec{F}}{vol} = \rho\vec{E} + \vec{J}\times\vec{B}$
$\frac{dW/dt}{vol} = \vec{E}\cdot\vec{J}$
CAUTION


Suppose ∃ vector W^a s.t. $W^a F_{ab} = 0$.
⇒ F_{ab} is called "degenerate"
$= W^a F_{[ab} F_{cd]}$


Suppose ∃ vector W^a s.t. $W^a F_{ab} = 0$.
⇒ F_{ab} is called "degenerate"
$0 = W^a F_{[ab} F_{cd]}$


Suppose ∃ vector W^a s.t. $W^a F_{ab} = 0$.
⇒ F_{ab} is called "degenerate"
⇒ $0 = W^a F_{[ab} F_{cd]}$ ⇒ $\beta W^a \epsilon_{abcd} = 0$ ⇒ $\beta = 0$
$= \beta \epsilon_{abcd}$


Suppose $\exists$ vector W^a s.t. $W^a F_{ab} = 0$.
$\Rightarrow F_{ab}$ is called "degenerate"
$\Rightarrow 0 = W^a F_{[ab} F_{cd]} \Rightarrow \beta W^a \epsilon_{abcd} = 0 \Rightarrow \beta = 0 \Rightarrow F_{[ab} F_{cd]} = 0 \Leftrightarrow F_{ab} {}^*F^{ab} = 0$
$= \beta \epsilon_{abcd}$
$\Leftrightarrow \vec{E} \cdot \vec{B} = 0$

Suppose $\exists$ vector W^a s.t. $W^a F_{ab} = 0$.

$\Rightarrow F_{ab}$ is called "degenerate"

$\Rightarrow 0 = W^a \underbrace{F_{[ab} F_{cd]}}_{= \beta \epsilon_{abcd}} \Rightarrow \beta W^a \epsilon_{abcd} = 0 \Rightarrow \beta = 0 \Rightarrow F_{[ab} F_{cd]} = 0 \Leftrightarrow F_{ab} {*F}^{ab} = 0$

$\Leftrightarrow \vec{E} \cdot \vec{B} = 0$

$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β!

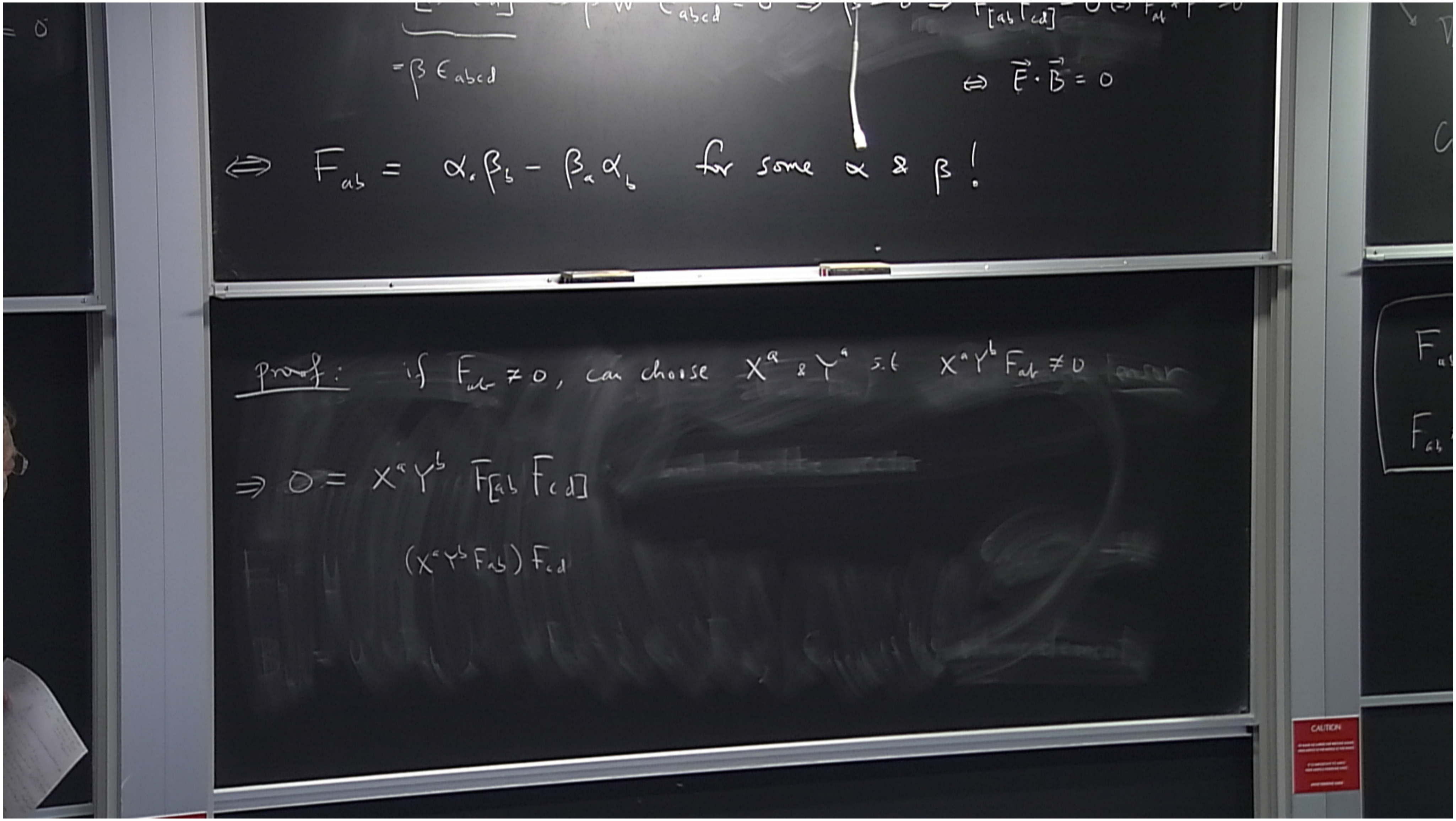
$B_b := U^a {*F}_{ab}$, ${*F}_{ab} = \frac{1}{2} \epsilon_{ab}{}^{cd} F_{cd}$, ϵ_{abcd} is "the" volume element

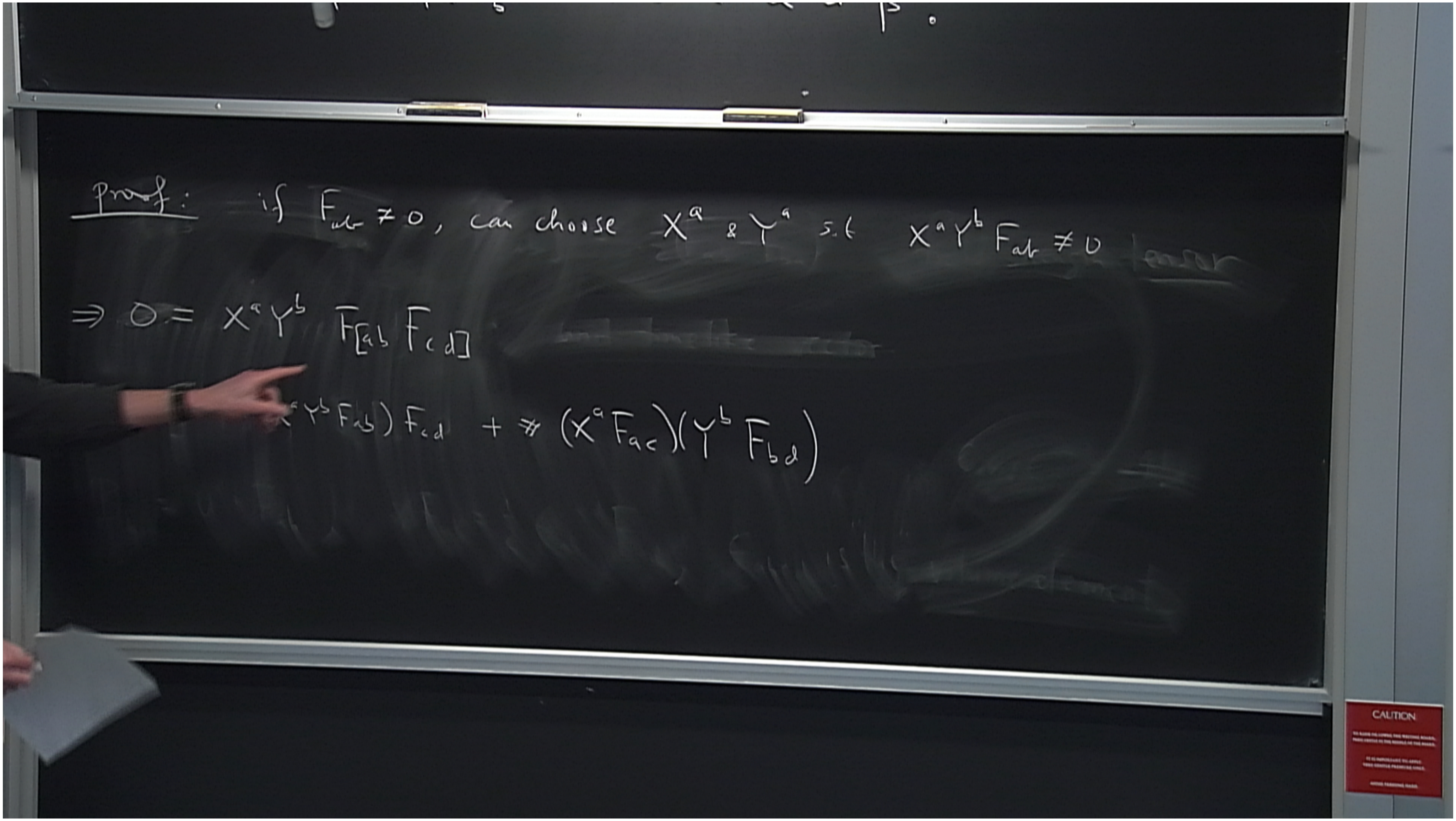
$$= \beta\, \epsilon_{abcd} \qquad\qquad \Leftrightarrow \vec{E}\cdot\vec{B} = 0$$

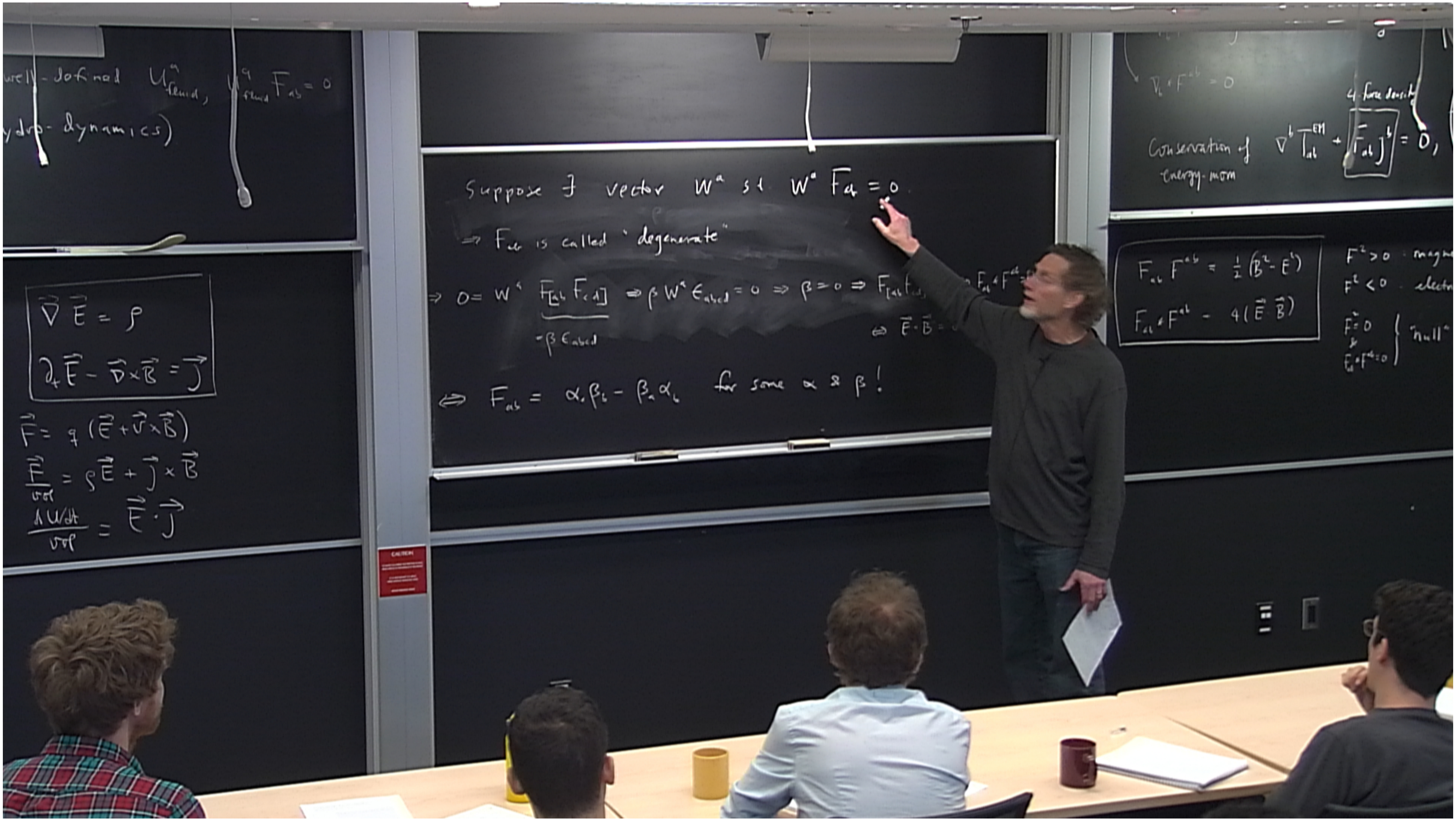
$$\Leftrightarrow \quad F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b \quad \text{for some } \alpha \ \& \ \beta \, !$$

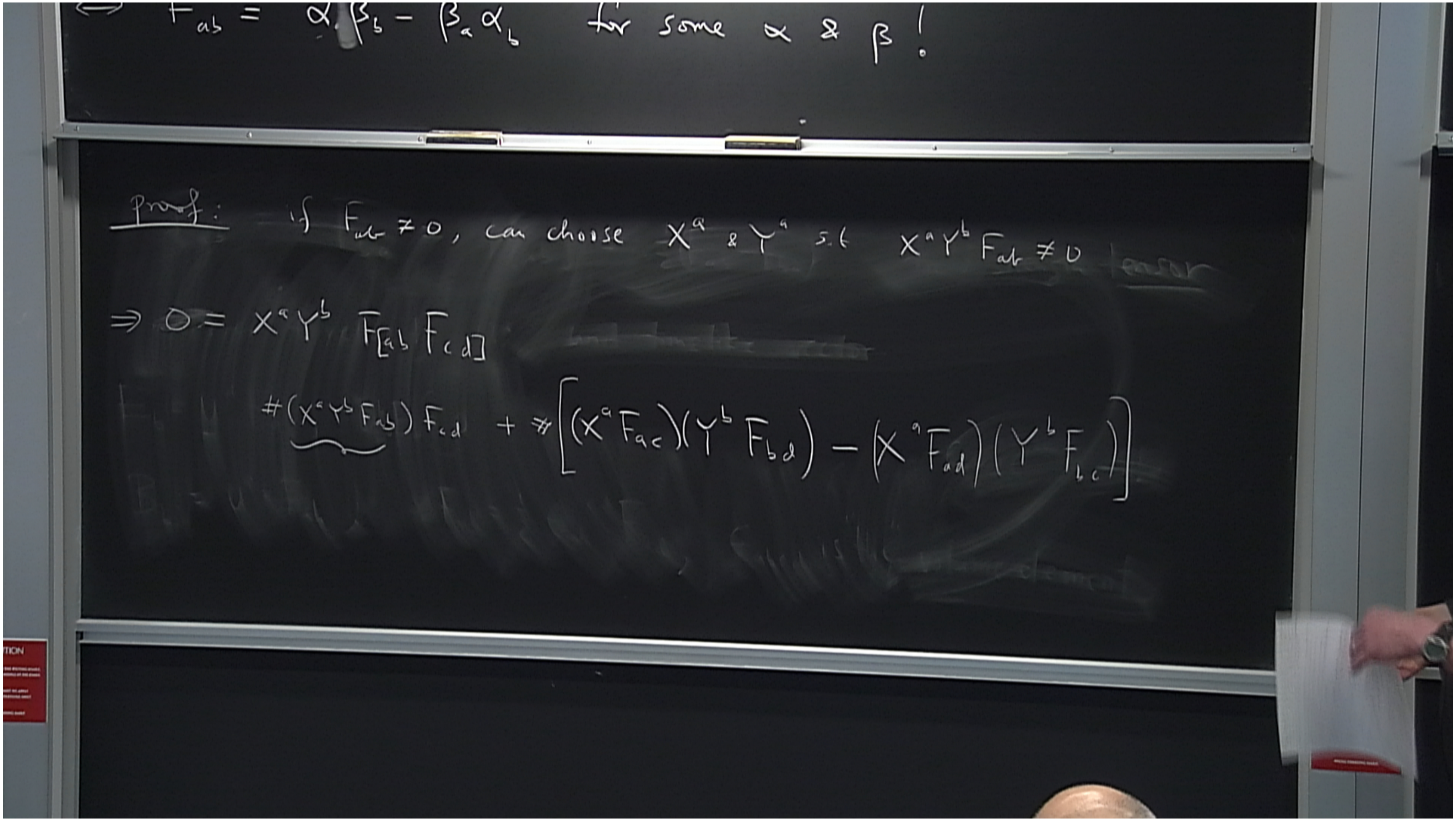
<u>proof:</u> if $F_{ab} \neq 0$, can choose X^a & Y^a s.t. $X^a Y^b F_{ab} \neq 0$

$$\Rightarrow 0 = X^a Y^b F_{[ab} F_{cd]}$$

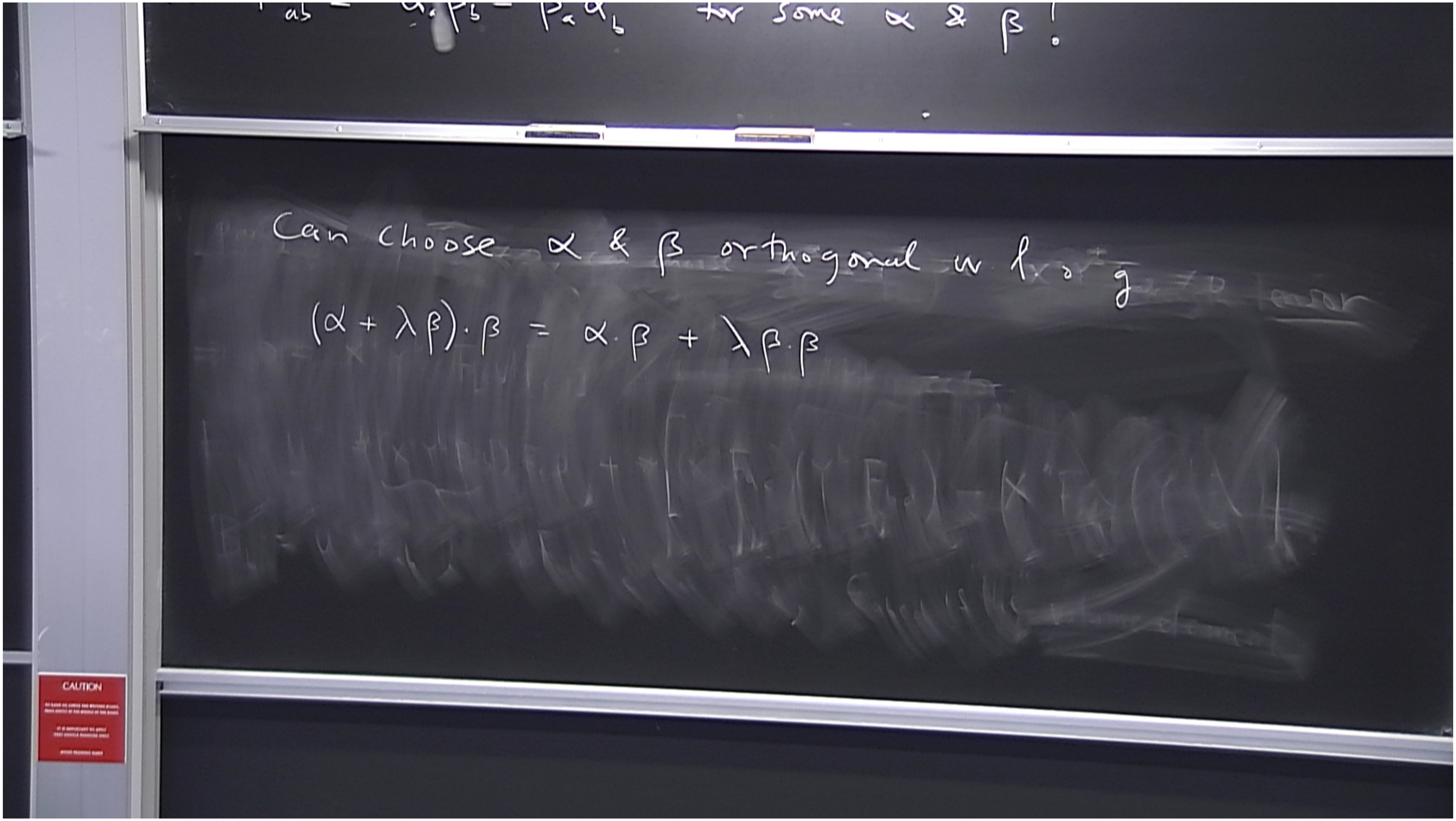

= β ε abcd
⟺ E · B = 0
⟺ F ab = α a β b − β a α b for some α & β !
proof: if F ab ≠ 0, can choose X a & Y a s.t. X a Y b F ab ≠ 0
⟹ 0 = X a Y b F [ab F cd]
(X a Y b F ab) F cd

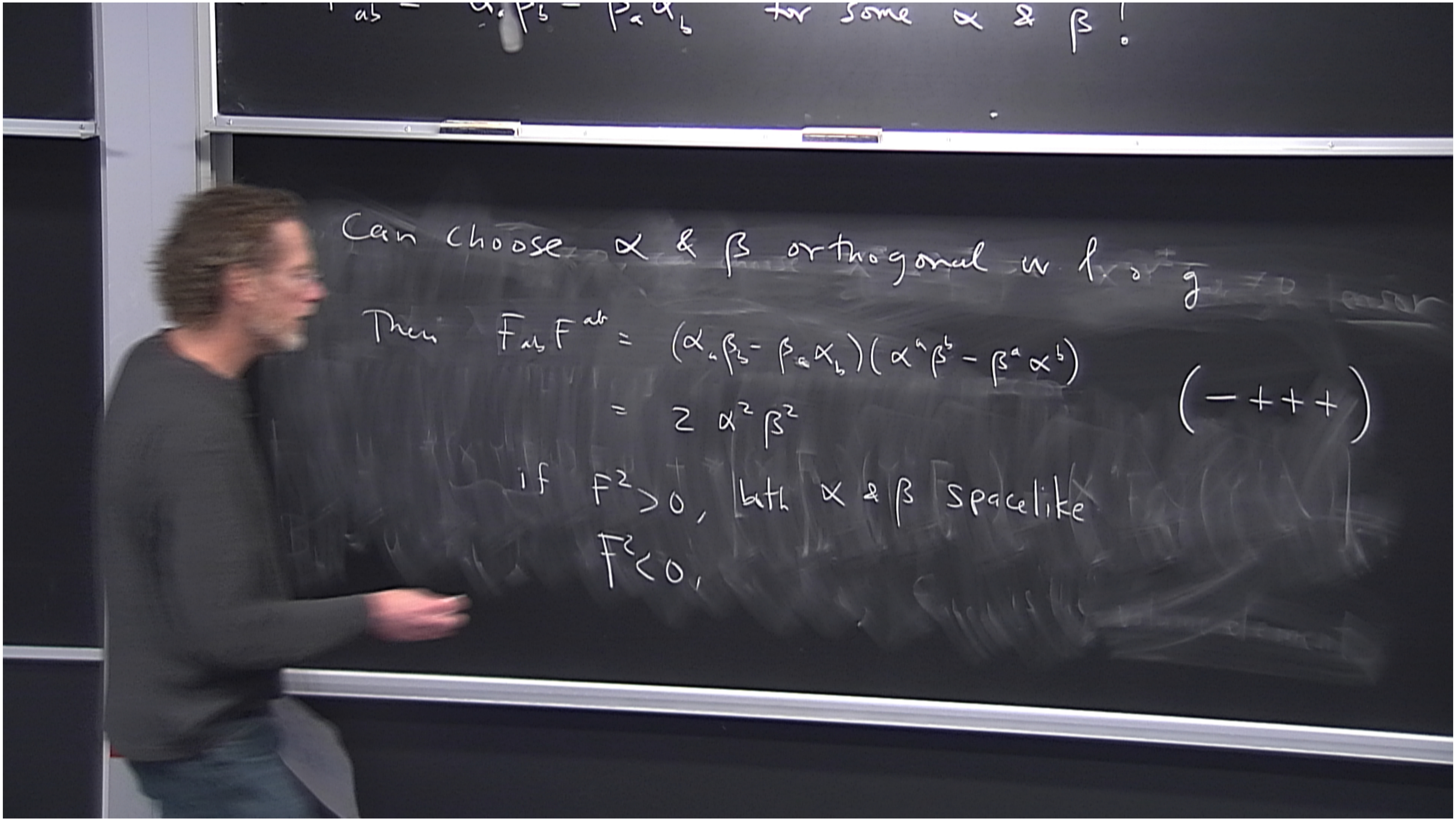

proof: if $F_{ab} \neq 0$, can choose X^a & Y^a s.t. $X^a Y^b F_{ab} \neq 0$
$\Rightarrow 0 = X^a Y^b F_{[ab} F_{cd]}$
$(X^a Y^b F_{ab}) F_{cd} + \# (X^a F_{ac})(Y^b F_{bd})$
CAUTION

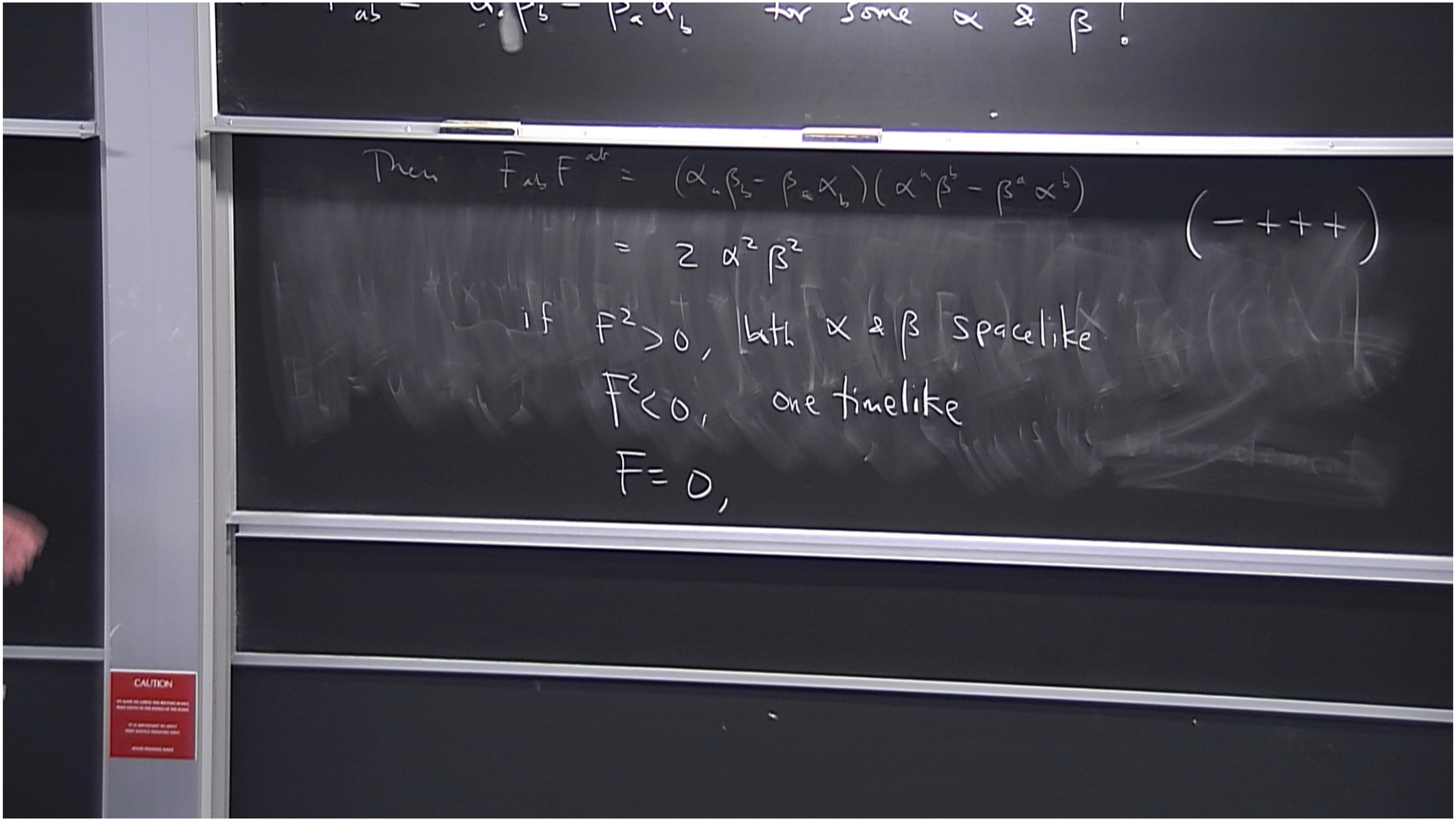

Suppose ∃ vector W^a s.t. W^a F_ab = 0
⇒ F_ab is called 'degenerate'
⇔ F_ab = α_a β_b − β_a α_b for some α & β!
Conservation of energy-mom
4-force density
F^2 > 0 magnetic
F^2 < 0 electric
"null"

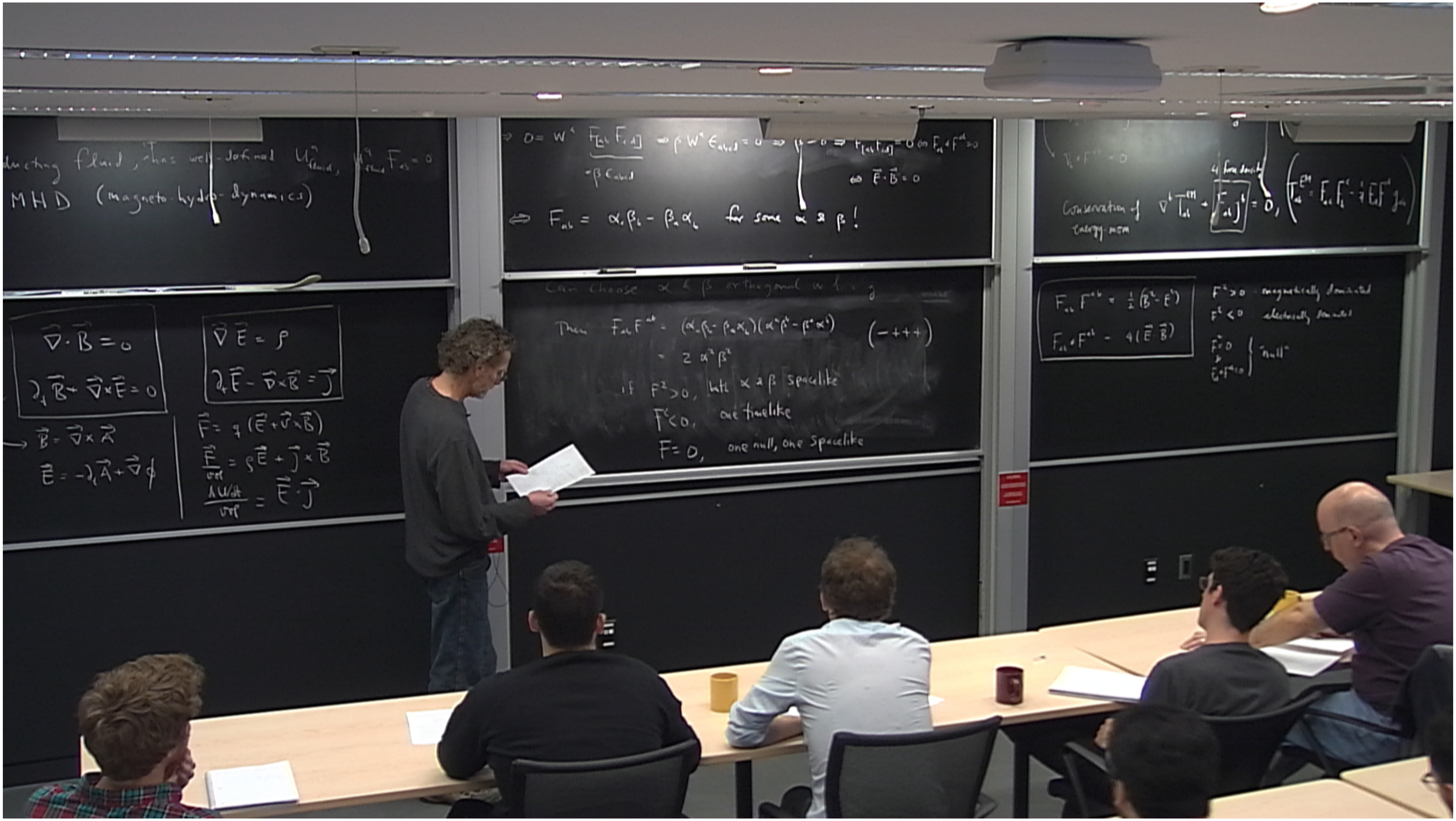


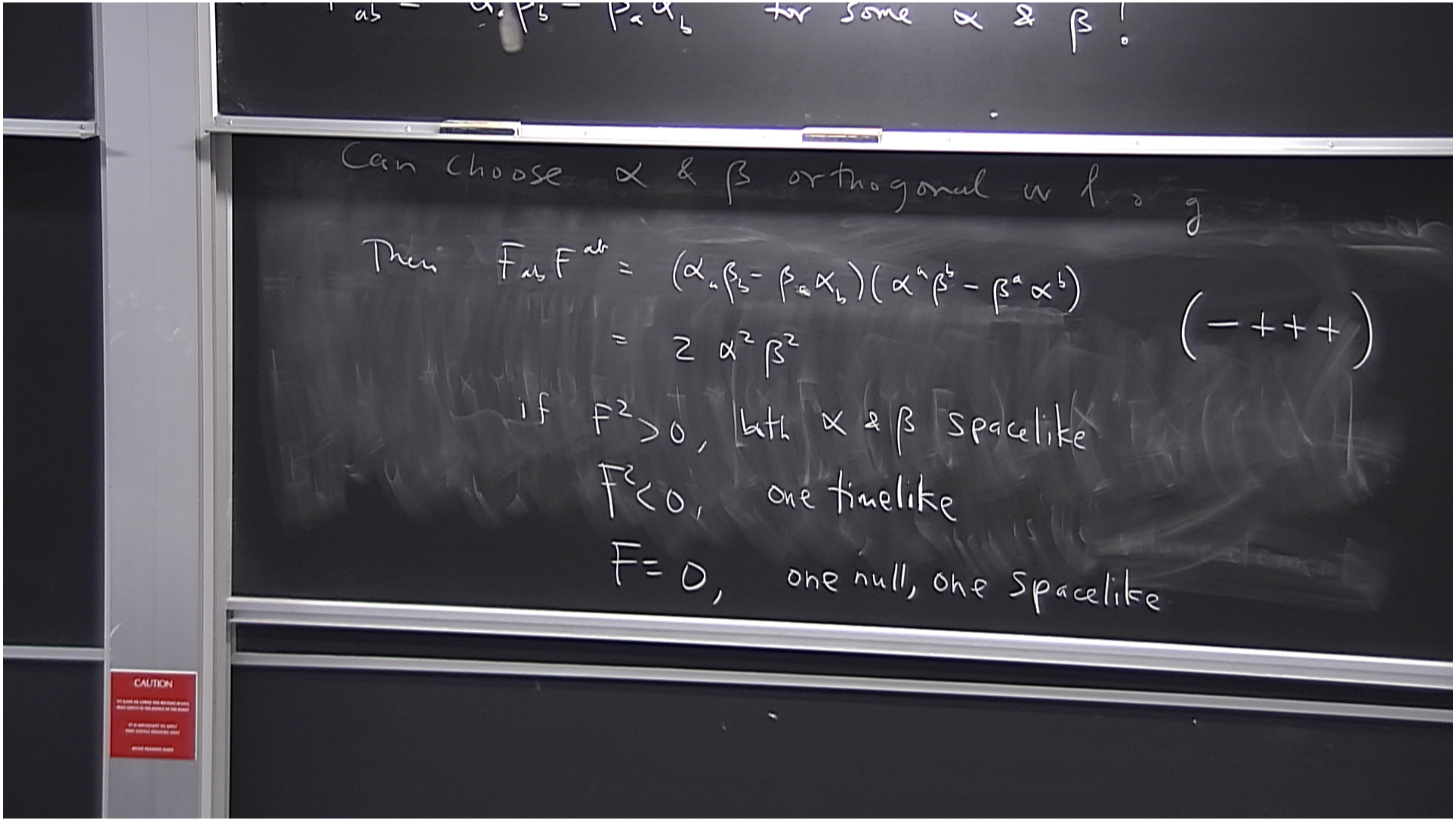
$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β!
proof: if $F_{ab} \neq 0$, can choose X^a & Y^a s.t. $X^a Y^b F_{ab} \neq 0$
$\Rightarrow 0 = X^a Y^b F_{[ab} F_{cd]}$
$\# (X^a Y^b F_{ab}) F_{cd} + \# \left[(X^a F_{ac})(Y^b F_{bd}) - (X^a F_{ad})(Y^b F_{bc}) \right]$

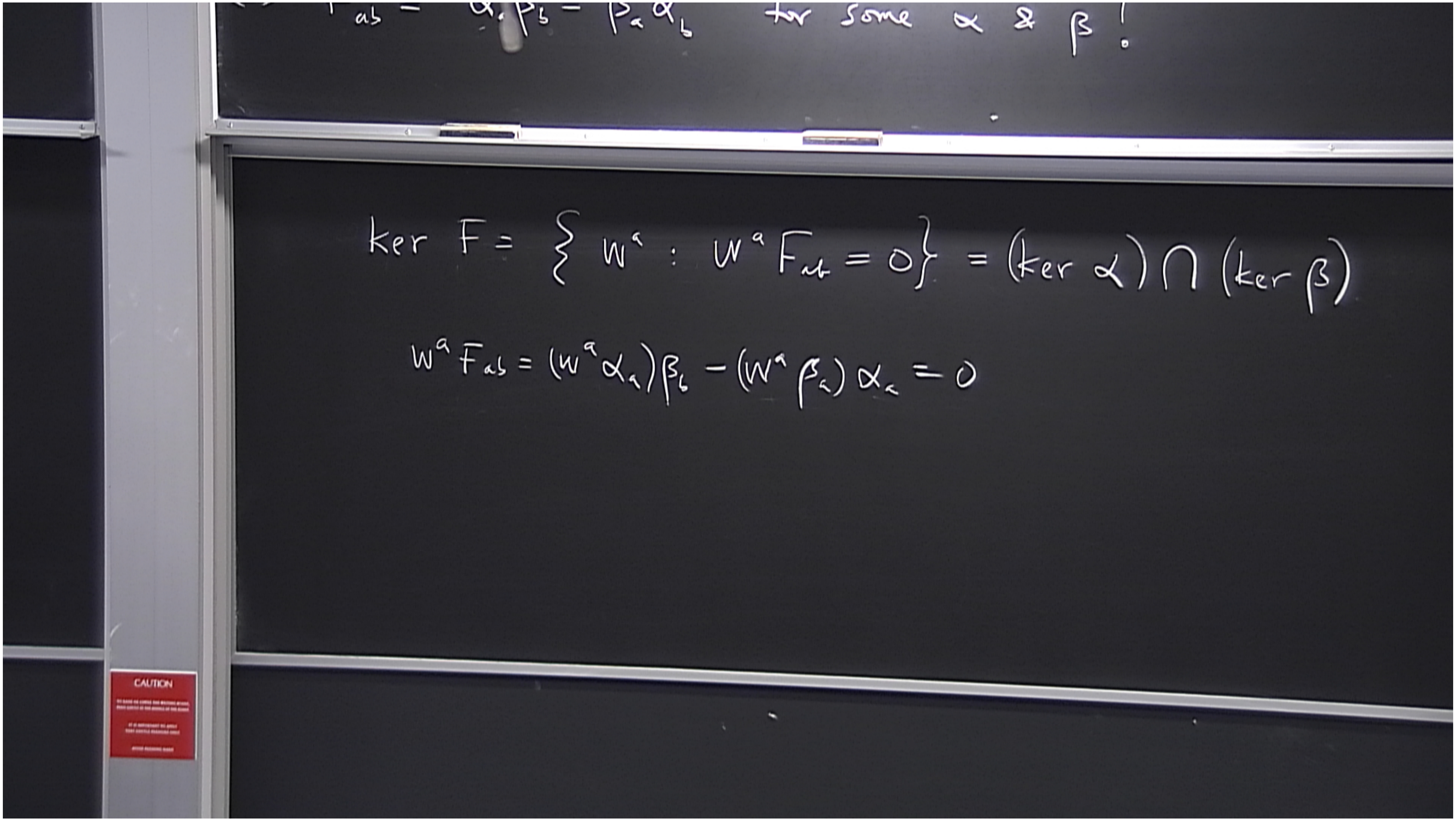

ab = α_a β_b − β_a α_b for some α & β !
Can choose α & β orthogonal w.r.t. g
(α + λβ)·β = α·β + λβ·β
CAUTION

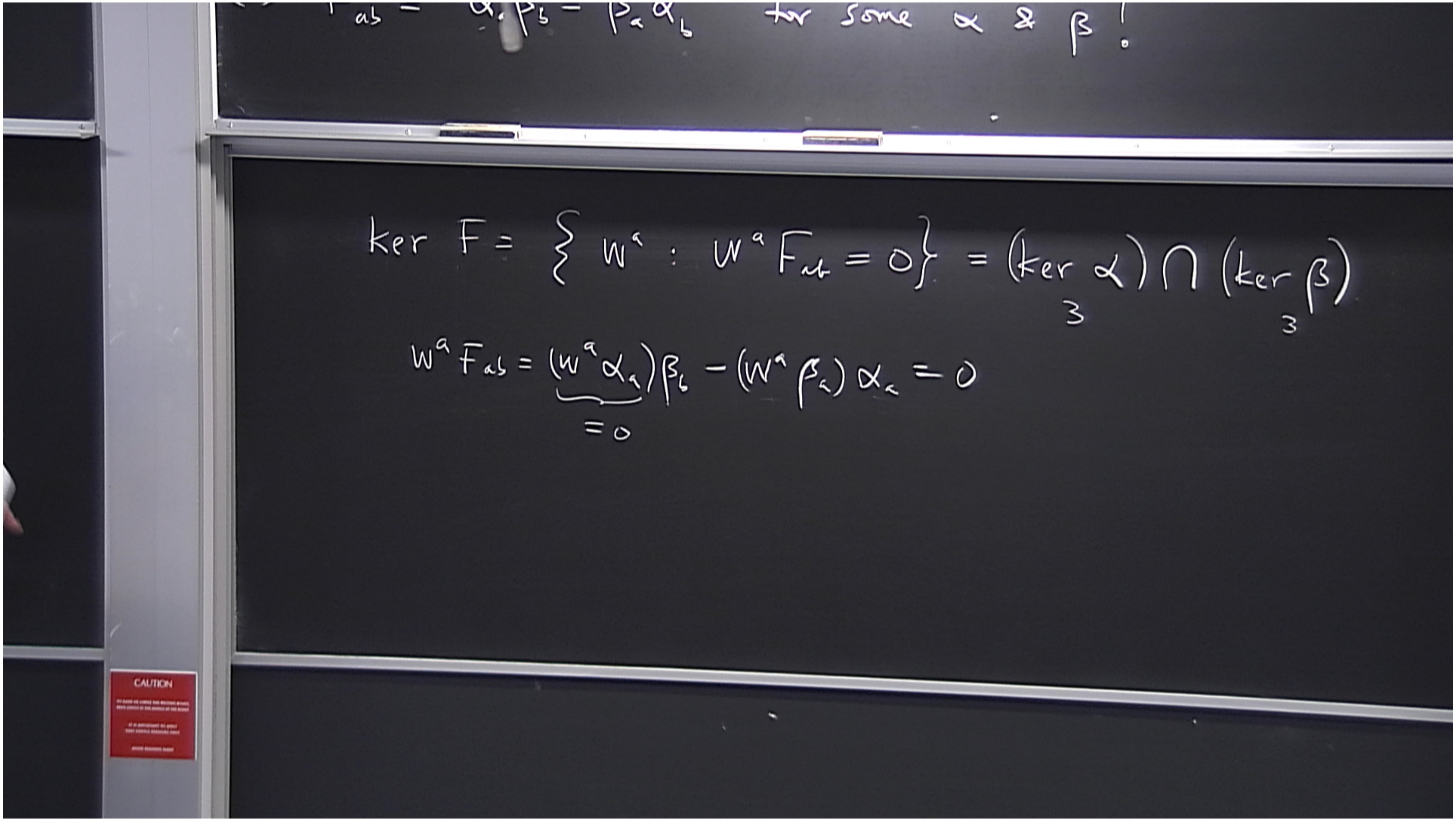

for some α & β !
Can choose α & β orthogonal w.r.t. g
Then $F_{ab}F^{ab} = (\alpha_a\beta_b - \beta_a\alpha_b)(\alpha^a\beta^b - \beta^a\alpha^b)$
$= 2\alpha^2\beta^2$
(− + + +)
if $F^2 > 0$, both α & β spacelike
$F^2 < 0$,

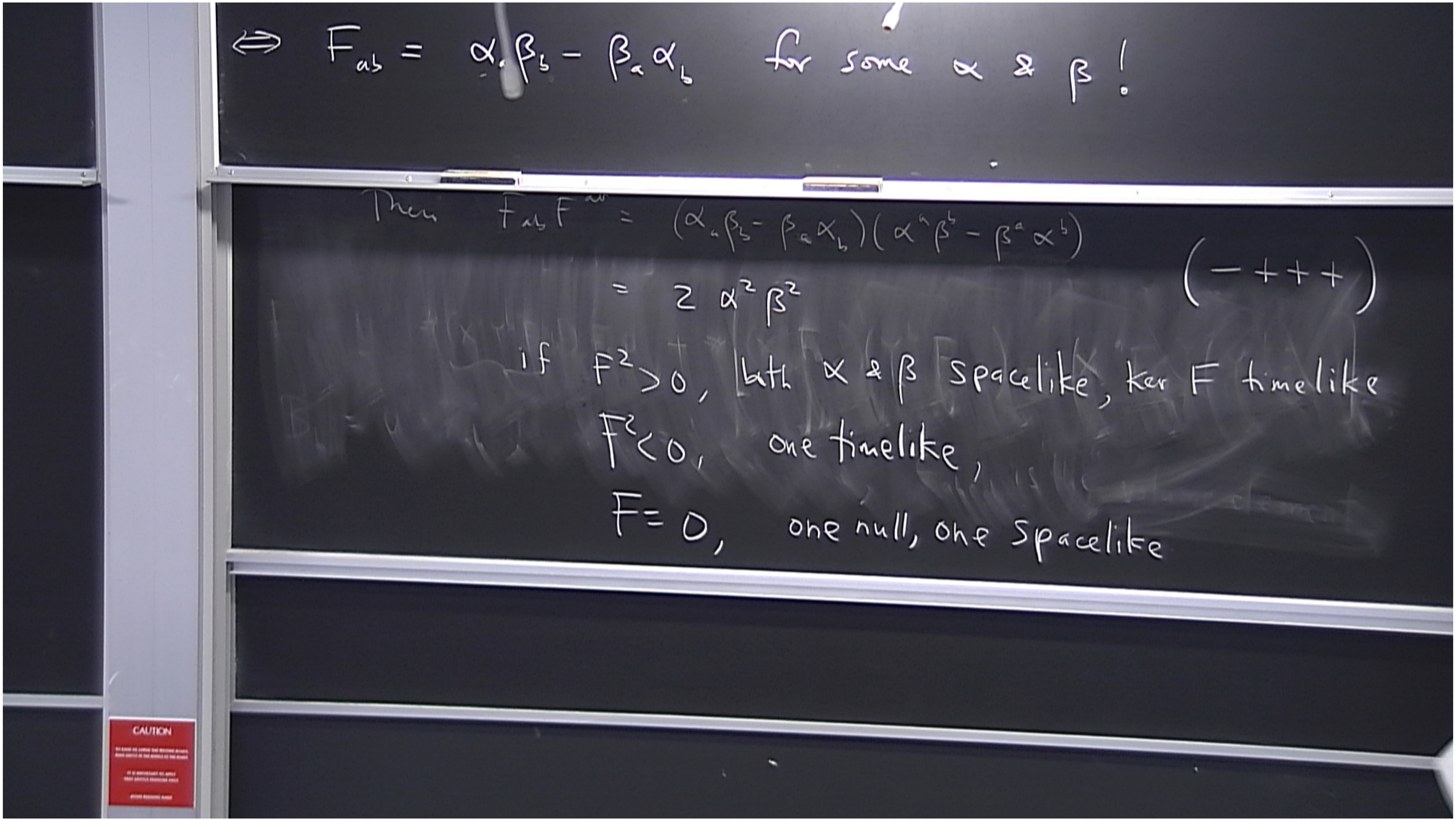

F_ab = α_a β_b − β_a α_b for some α & β !
Then F_ab F^ab = (α_a β_b − β_a α_b)(α^a β^b − β^a α^b)
= 2 α² β²
(− + + +)
if F² > 0, both α & β spacelike
F² < 0, one timelike
F = 0,
CAUTION


MHD (magneto-hydro-dynamics)
for some α & β!
Conservation of energy-mom
(−+++)
if F²>0, both α & β spacelike
F²<0, one timelike
F=0, one null, one spacelike
magnetically dominated
electrically dominated
"null"


$F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !
Can choose α & β orthogonal w.r.t. g
Then $F_{ab}F^{ab} = (\alpha_a\beta_b - \beta_a\alpha_b)(\alpha^a\beta^b - \beta^a\alpha^b)$
$= 2\,\alpha^2\beta^2$
$(-+++)$
if $F^2 > 0$, both α & β spacelike
$F^2 < 0$, one timelike
$F = 0$, one null, one spacelike
CAUTION


$_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !
$\ker F = \{ w^a : w^a F_{ab} = 0 \} = (\ker \alpha) \cap (\ker \beta)$
$w^a F_{ab} = (w^a \alpha_a) \beta_b - (w^a \beta_a) \alpha_a = 0$
CAUTION


$_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !
$\ker F = \{ W^a : W^a F_{ab} = 0 \} = (\ker_3 \alpha) \cap (\ker_3 \beta)$
$W^a F_{ab} = \underbrace{(W^a \alpha_a)}_{=0} \beta_b - (W^a \beta_a) \alpha_a = 0$


$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !
Then $F_{ab}F^{ab} = (\alpha_a\beta_b - \beta_a\alpha_b)(\alpha^a\beta^b - \beta^a\alpha^b)$
$= 2\,\alpha^2\beta^2$
$(-+++)$
if $F^2 > 0$, both α & β spacelike, ker F timelike
$F^2 < 0$, one timelike,
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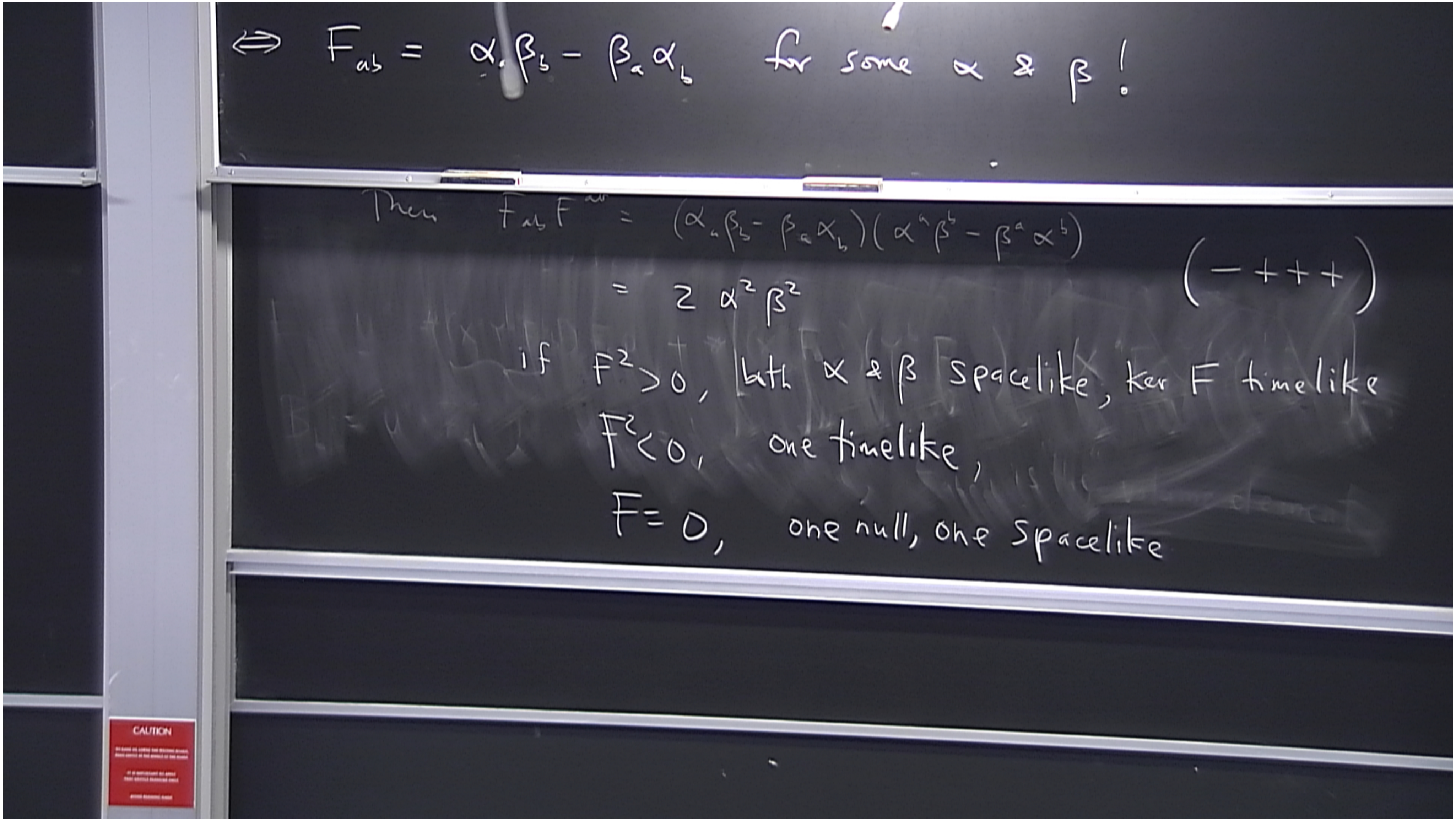
$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !
Then $F_{ab}F^{ab} = (\alpha_a\beta_b - \beta_a\alpha_b)(\alpha^a\beta^b - \beta^a\alpha^b)$
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if $F^2 > 0$, both α & β spacelike, ker F timelike
$F^2 < 0$, one timelike,
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$\Longleftrightarrow \quad F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !

Then $F_{ab}F^{ab} = (\alpha_a\beta_b - \beta_a\alpha_b)(\alpha^a\beta^b - \beta^a\alpha^b)$

$= 2\,\alpha^2\beta^2$ $\quad (-+++)$

if $F^2 > 0$, both α & β spacelike, ker F timelike

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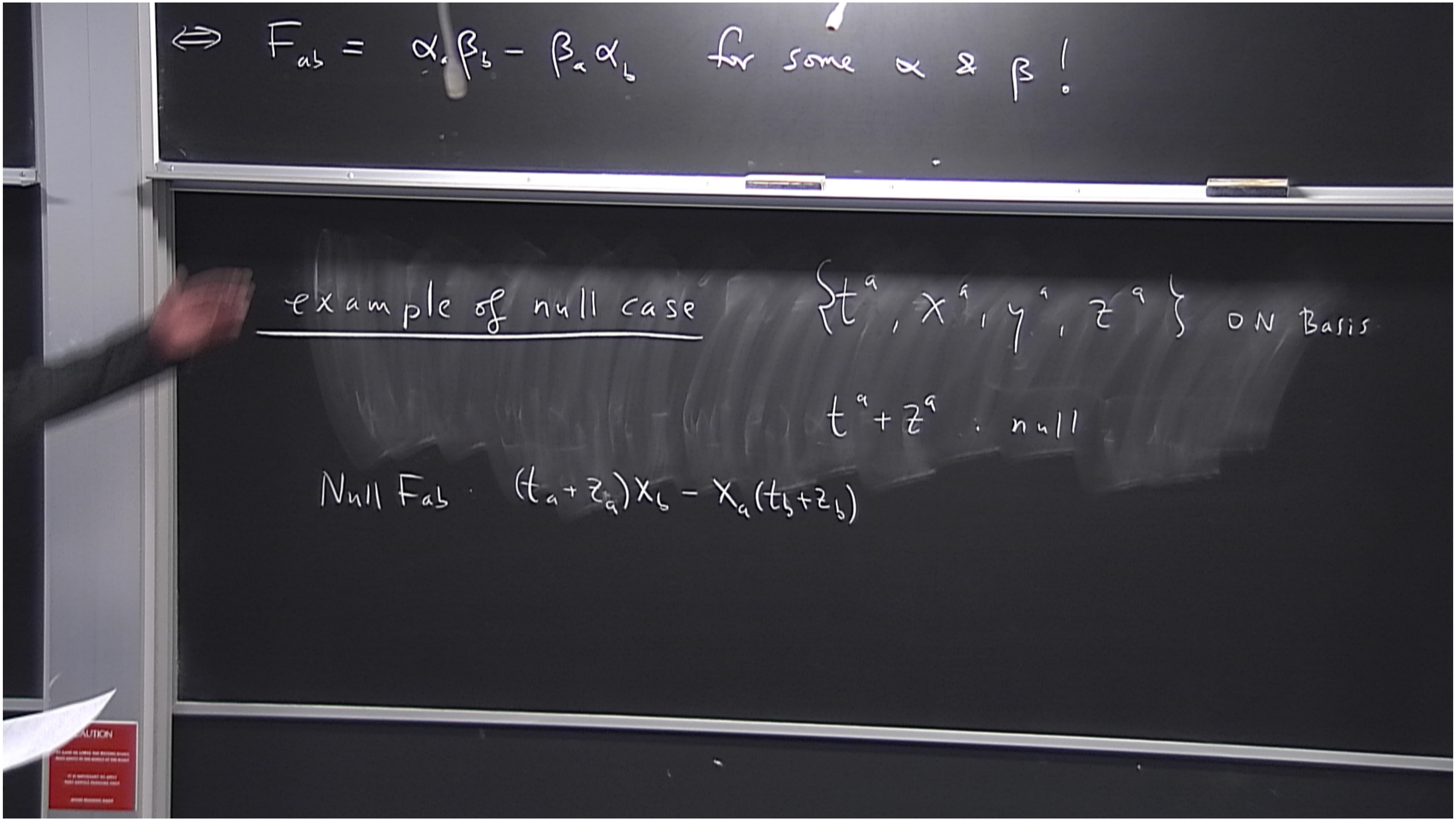
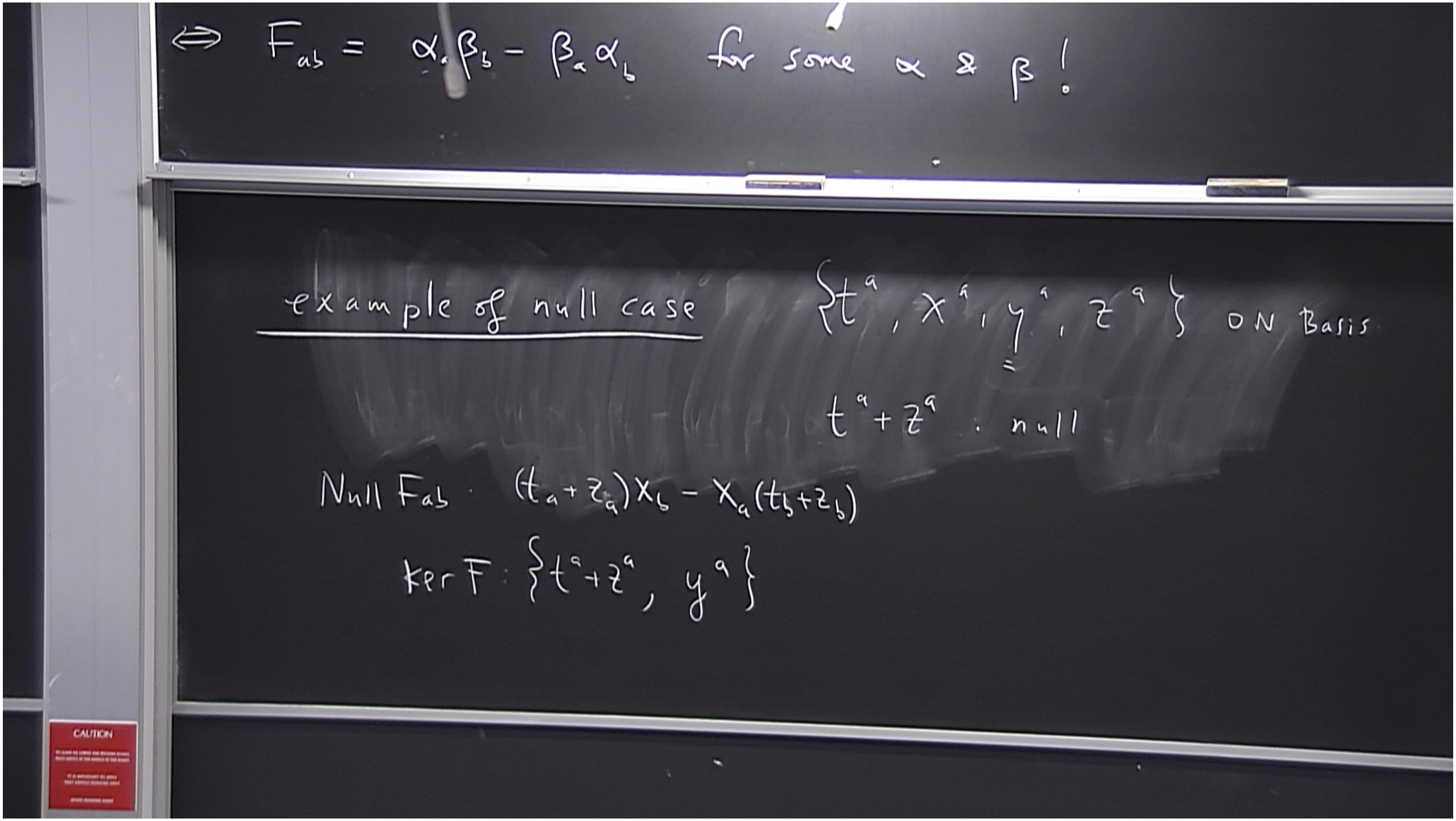

$\Leftrightarrow F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !
example of null case
$\{t^a, x^a, y^a, z^a\}$ ON Basis
$t^a + z^a$: null
Null F_{ab} : $(t_a + z_a) x_b - x_a (t_b + z_b)$


⇔ $F_{ab} = \alpha_a \beta_b - \beta_a \alpha_b$ for some α & β !
example of null case
$\{t^a, x^a, y^a, z^a\}$ ON Basis
$t^a + z^a$: null
Null F_{ab} : $(t_a + z_a) x_b - x_a (t_b + z_b)$
ker F : $\{t^a + z^a, y^a\}$