Title: Evolution of cosmological black holes: exact solutions, accretion and scalar fields

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Abstract: Systems which contemplate the gravitational interaction between compact objects and the matter content in a cosmological environment constitute an important problem which has been studied since the early days of General Relativity. The generalized McVittie black hole is a simple exact solution to this problem, which provides us with insight on some of its known physical aspects, as well as hints to new mechanisms which arise from a formal treatment. We review some properties of this solution and its matter source, which can be interpreted as a classical fluid but is also an exact solution to a nontrivial scalar field theory.

Evolution of cosmological black holes: Exact solutions, accretion and scalar fields

Daniel C. Guariento

based on

A. M. da Silva, M. Fontanini, DCG, E. Abdalla, [1207.1086], [1212.0155]

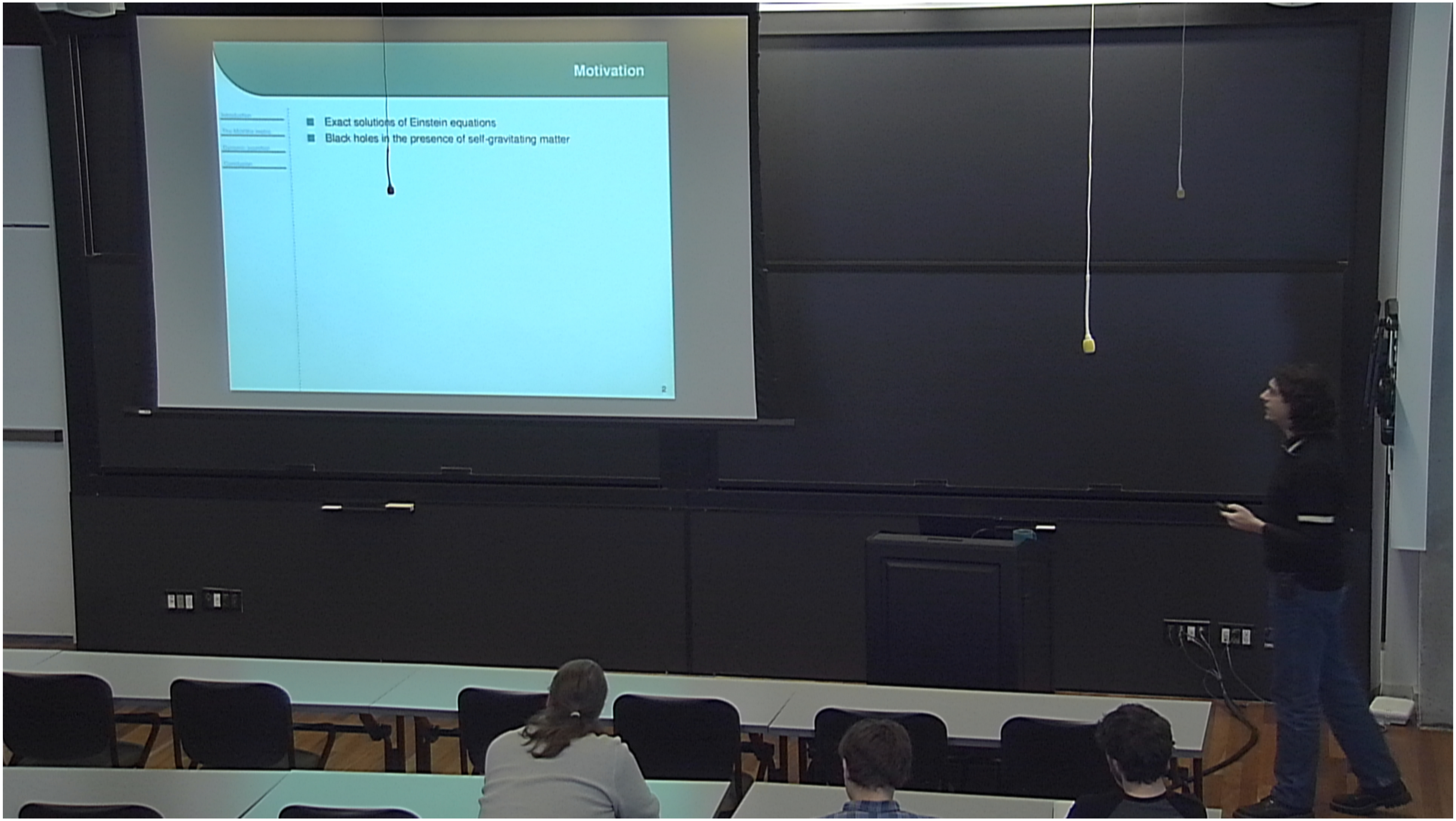
E. Abdalla, N. Afshordi, M. Fontanini, DCG. E. Papantonopoulos, [1312.3682]

N. Afshordi, M. Fontanini, DCG, [1402.xxxx]

PI Cosmo Seminar

Motivation

- Exact solutions of Einstein equations
- Black holes in the presence of self-gravitating matter


Motivation
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  - Causal structure
  - Accretion through Einstein equations
  - Generalizations with other types of matter

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  - Generalizations with other types of matter
- Field theory:
  - Consistency analysis
  - Evolution and interaction from equations of motion

Overview

Classical (Bondi) accretion

- Spherically symmetric models

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Classical (Bondi) accretion

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- Homogeneous distribution of particles at spatial infinity
- Capture cross-section depends on the last circular orbit ($r_{\mathsf{eff}} = 2r_{\mathsf{G}}$)
  - ☐ Non-relativistic particles

$$\sigma_{\mathsf{M}} = 4\pi \left(\frac{r_{\mathsf{G}}}{v_\infty}\right)^2 \tag{1}$$

  - ☐ Ultra-relativistic particles (radiation)

$$\sigma_{\mathsf{R}} = \frac{27}{4}\pi r_{\mathsf{G}}^2 = 27\pi m^2 \tag{2}$$

Mass variation from the energy-momentum tensor

- 4-momentum in a box of volume V

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- Energy variation through the horizon of a Schwarzschild black hole

[DCG, J. E. Horvath, 1111.0585]

$$\frac{\mathrm{d}E_{\text{inside}}}{\mathrm{d}\tau} = \frac{\mathrm{d}m}{\mathrm{d}\tau} = \frac{\mathrm{d}E_{\text{outside}}}{\mathrm{d}\tau} = \mathcal{A} T^{\mu\nu} u_\mu \sigma_\nu$$

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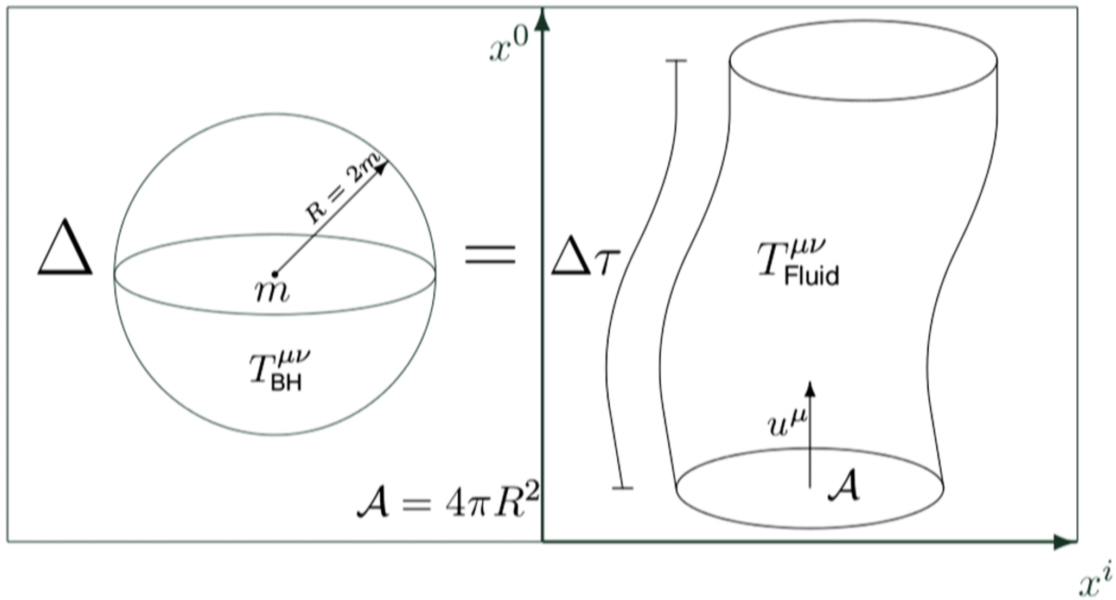
$$\boxed{\frac{\mathrm{d}m}{\mathrm{d}t} = \mathcal{A}T_0{}^1}$$

Energy-momentum flow across a 3-surface



Accretion of a test fluid

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- Accretion of dark energy and radiation with Hawking evaporation

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$$\frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{A(m)}{m^2} + m^2\left[27\pi\rho_{\mathrm{rad}}(T) + 16\pi(1+w)\rho_{\mathrm{DE}}\right]$$

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- Black-hole mass depends on densities at any given time
  - Example: accretion in a ΛCDM background

    [J. A. S. Lima, S. H. Pereira, J. E. Horvath, DCG, 0808.0860]

$$m = \frac{m_\mathrm{i}}{1 + m_\mathrm{i}\sqrt{\frac{8\pi}{3G}A^2}\left\{\left[\rho_\Lambda + \rho_\mathrm{c}^\mathrm{i}\right]^{1/2} - \left[\rho_\Lambda + \rho_\mathrm{c}^\mathrm{i}\frac{\Omega_\Lambda}{\Omega_\mathrm{c}}\frac{1}{\sinh^2\left(\frac{3}{2}H_0\sqrt{\Omega\Lambda}t\right)}\right]^{1/2}\right\}}$$

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ΛCDM accretion

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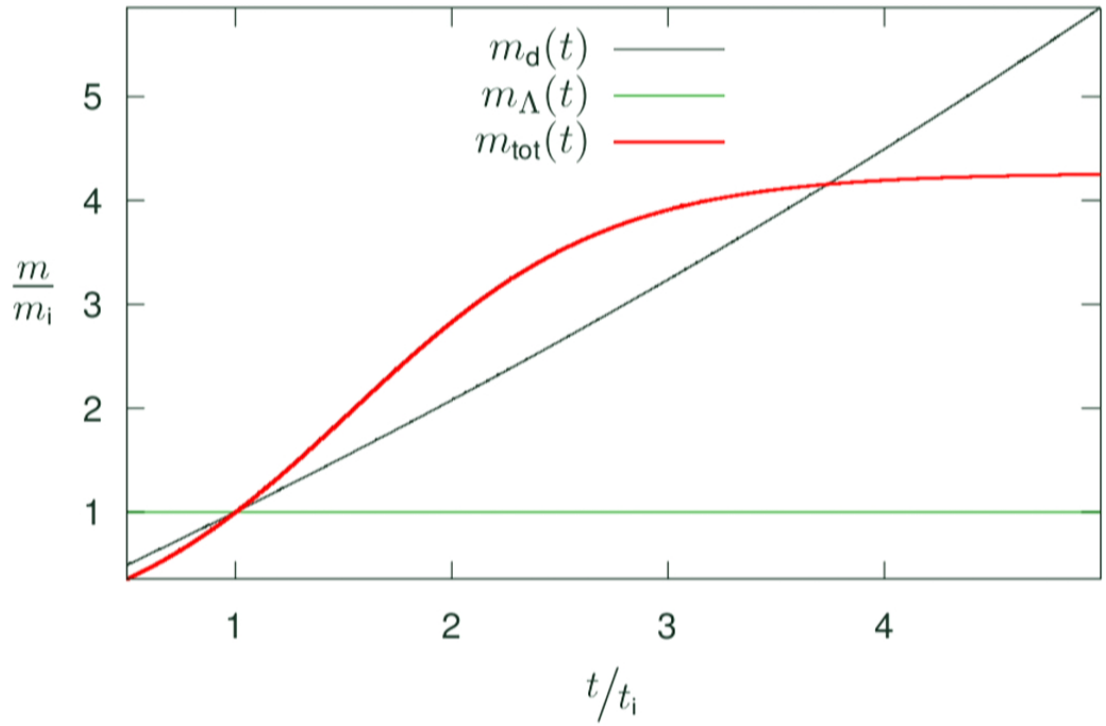
Mass variation from the energy-momentum tensor
Accretion of a test fluid
The McVittie metric
Dynamic accretion
Conclusion



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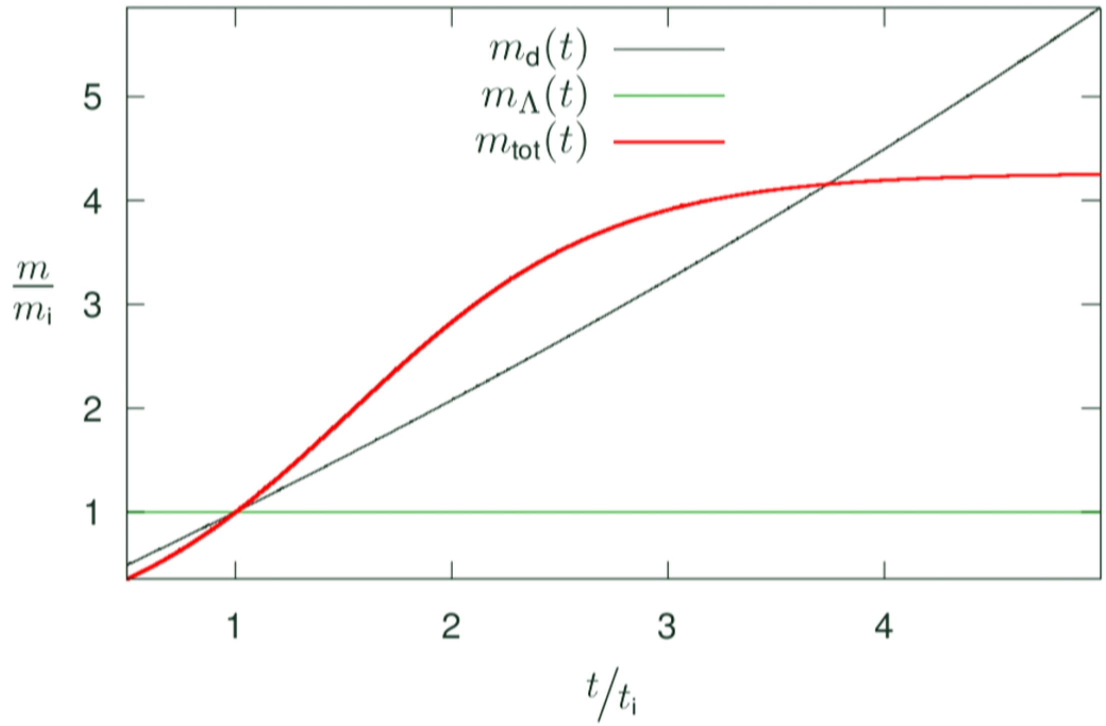

Problems and limitations

- Bondi-like test fluid accretion is unrealistic

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- Bondi-like test fluid accretion is unrealistic
  - m can diverge if ρ_{DM} is too high
  - Asymptotically Bondi-Hoyle accretion is inefficient
  - Does not reproduce observed black hole masses with realistic initial conditions
- Baryon physics at small scales is more important

  [M. A. M. Armijo, J. A. de Freitas Pacheco, 1008.4150]

  - Energy loss due to dispersion makes accretion more efficient

The McVittie metric

The McVittie metric

- Cosmological black holes: McVittie solution [McVittie, MNRAS **93**,325 (1933)]

$$ds^2 = -\frac{\left(1-\frac{m}{2a(t)\hat{r}}\right)^2}{\left(1+\frac{m}{2a(t)\hat{r}}\right)^2}dt^2 + a^2(t)\left(1+\frac{m}{2a(t)\hat{r}}\right)^4\left(d\hat{r}^2+\hat{r}^2 d\Omega^2\right)$$

  - $a(t)$ constant: Schwarzschild metric
  - $m = 0$: FLRW metric

Metric features

- Past spacelike singularity at $a = \frac{m}{2\hat{r}}$
- Event horizons only defined if $H \equiv \frac{\dot{a}}{a}$ constant as $t \to \infty$

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- Past spacelike singularity at $a = \frac{m}{2\hat{r}}$
- Event horizons only defined if $H \equiv \frac{\dot{a}}{a}$ constant as $t \to \infty$
- Fluid has homogeneous density

$$\rho(t) = \frac{3}{8\pi} H^2$$

  - Expansion is homogeneous (Hubble flow) and shear-free
  - Mean extrinsic curvature is constant on comoving foliation

$$K^{\alpha}{}_{\alpha} = 3H$$

- Pressure is inhomogeneous

$$p(\hat{r}, t) = \frac{1}{8\pi}\left[-3H^2 + 2\dot{H}\left(\frac{m + 2a\hat{r}}{m - 2a\hat{r}}\right)\right]$$

Areal radius coordinates

- Causal structure is more easily seen on non-comoving coordinates
- Areal radius

$$r = a\left(1 + \frac{m}{2\hat{r}}\right)^2 \hat{r}$$

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- Two branches: $\begin{cases} 0 < \hat{r} < \frac{a}{2m} & \text{(not used)} \\ \frac{a}{2m} < \hat{r} < \infty \implies 2m < r < \infty \end{cases}$

- McVittie in new (canonical) coordinates

[N. Kaloper, M. Kleban, D. Martin, 1003.4777]

$$\mathrm{d}s^2 = -R^2\mathrm{d}t^2 + \left[\frac{\mathrm{d}r}{R} - Hr\mathrm{d}t\right]^2 + r^2\mathrm{d}\Omega^2$$

where $\left(R = \sqrt{1 - \frac{2m}{r}}\right)$

Apparent horizons

- Apparent horizons: zero expansion of null radial geodesics

$$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)_{\pm} = R\,(rH \pm R) = 0$$

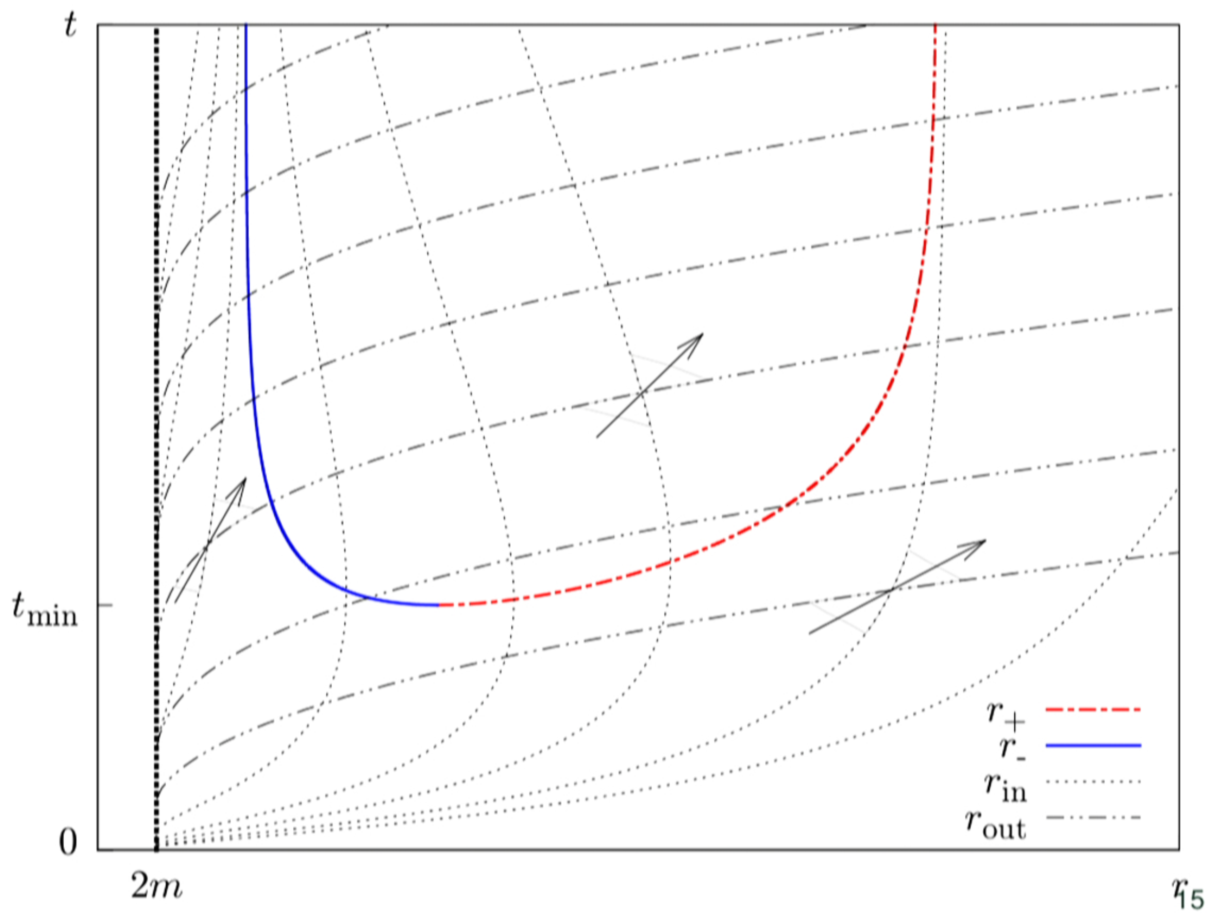
- Only ingoing geodesics have a solution

$$1 - \frac{2m}{r} - Hr^2 = 0$$

- Real positive solutions only exist if $\frac{1}{3\sqrt{3}m} > H > 0$
  - r_+ Outer (cosmological) horizon
  - r_- Inner horizon

- If $H(t) \to H_0$ for $t \to \infty$ apparent horizons become Schwarszchild-de Sitter event horizons ($r_- \to r_\infty$)

Light cones and apparent horizons



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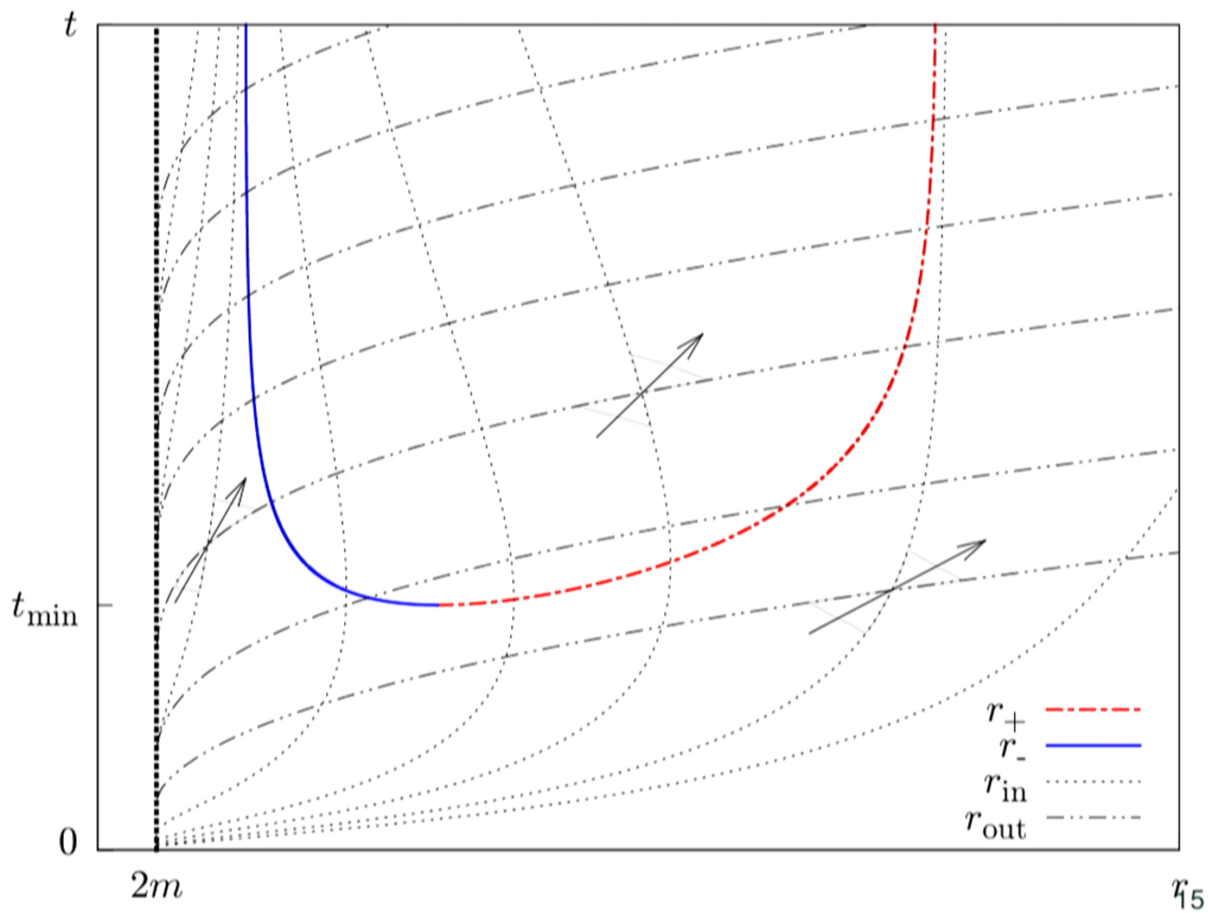
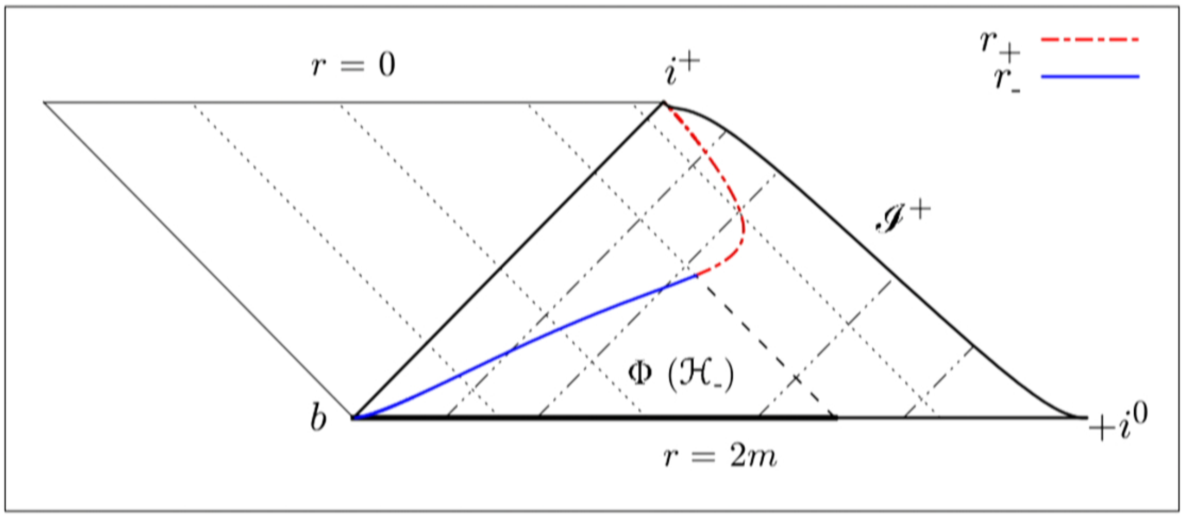


Asymptotic behavior

- Inner horizon is an *anti-trapping* surface for finite coordinate times
- Singular surface $r_* = 2m$ lies in the past of all events (McVittie big bang)

$$\frac{\mathrm{d}}{\mathrm{d}t}(r - r_*) = RrH + \mathcal{O}\left(R^2\right) > 0$$

- Geodesic completion with Schwarzschild-de Sitter



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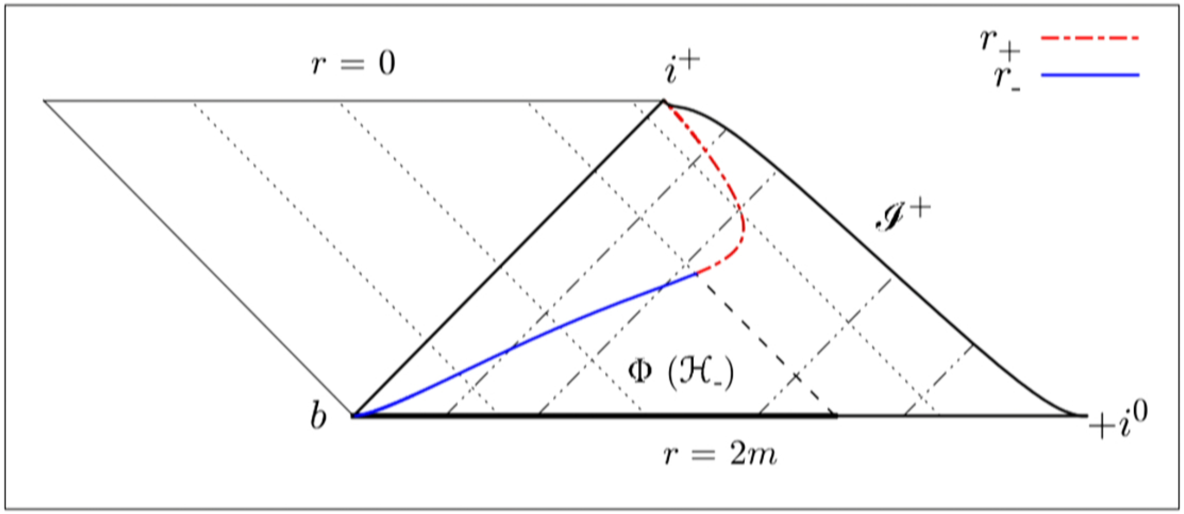

Penrose diagrams

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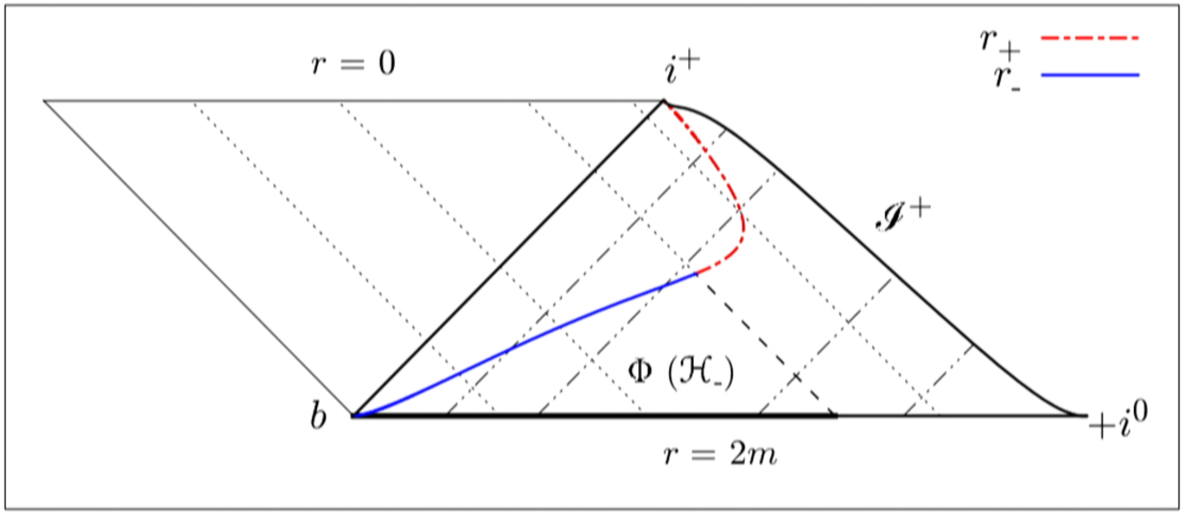

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Penrose diagrams
Geodesic completion with Schwarzschild-de Sitter
r = 0
r = 2m
r = 0
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Convergence to H_0

- Causal structure depends on cosmological history
- Horizon behavior at $t \to \infty$ depends on the set $\Phi(\mathcal{H}_-)$

Penrose diagrams

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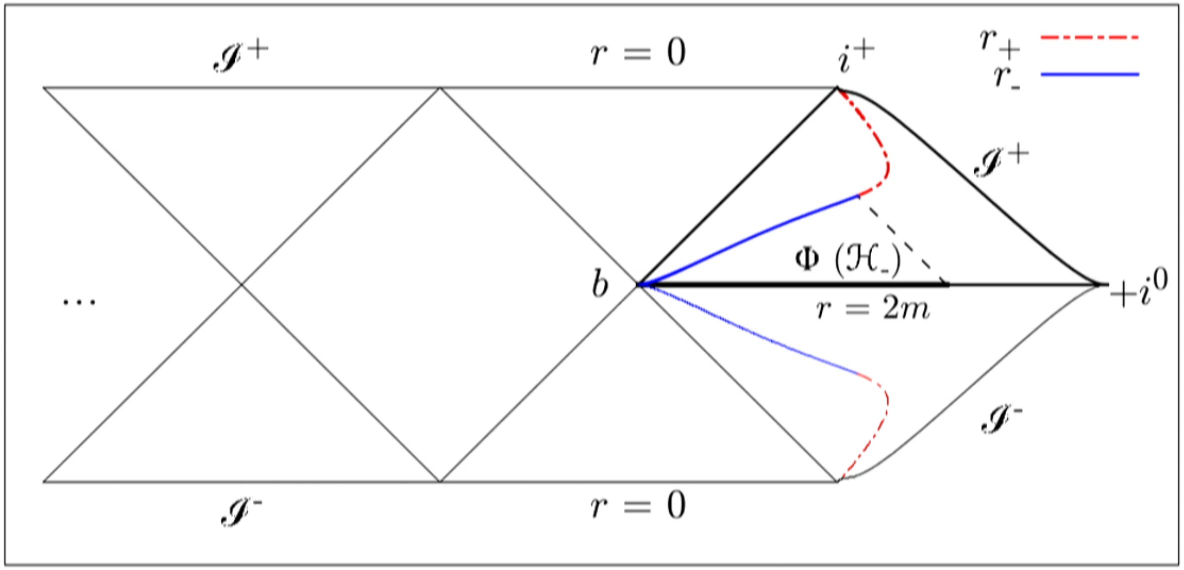
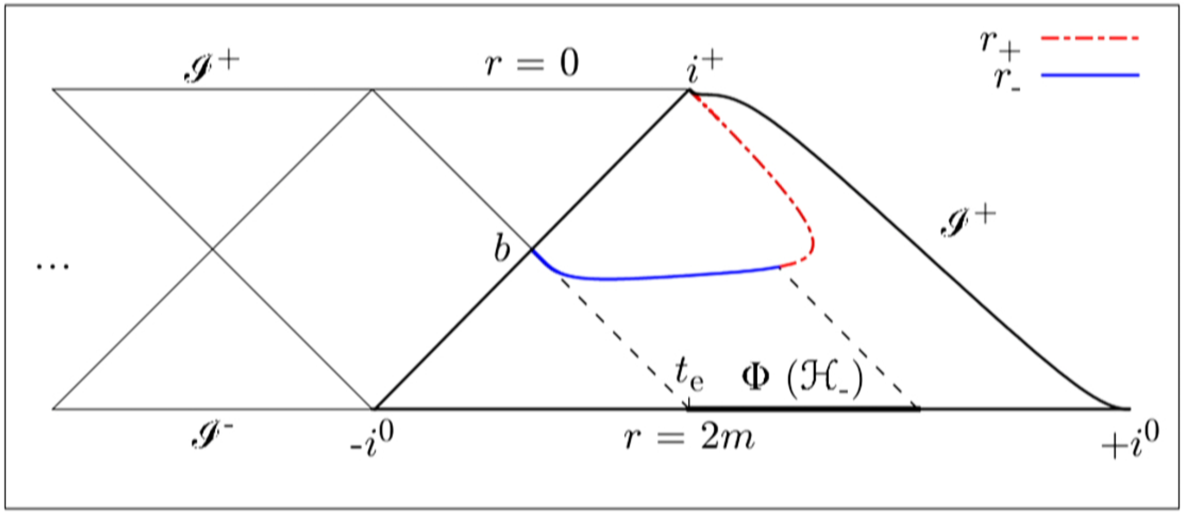


Convergence to H_0

- Causal structure depends on cosmological history
- Horizon behavior at $t \to \infty$ depends on the set $\Phi(\mathcal{H}_-)$
  - Φ non-compact
    - All causal curves departing $r_\star$ cross r_- before $t \to \infty$
    - Spacetime connects to the inner region of Schwarzschild-de Sitter


$\mathscr{I}^+$
$r = 0$
i^+
r_+
r_-
$\mathscr{I}^+$
b
...
t_e
Φ ($\mathcal{H}_-$)
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McVittie with compact Φ
$\mathscr{I}^+$
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Cosmological history

- We can determine the fate of null geodesics via the intermediate value theorem

  [A. M. da Silva, M. Fontanini, DCG, 1212.0155]

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- Bounding functions

$$F_+(t_i, t) = \int_{t_i}^{t} e^{(B-\delta)u} e^{-A\int_{t_i}^{u} \Delta H(s)\mathrm{d}s} \Delta H(u)\mathrm{d}u$$

$$F_-(t_i, t) = \int_{t_i}^{t} e^{(B+\bar{\delta})u} e^{-A\int_{t_i}^{u} \Delta H(s)\mathrm{d}s} \Delta H(u)\mathrm{d}u$$

$(A = R(r_\infty) + r_\infty R'(r_\infty), B = R(r_\infty)\,(R'(r_\infty) - H_0), \Delta H = H - H_0, t_i > 0)$

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Causal structure of McVittie depends on how fast $H \to H_0$.

Example: ΛCDM

- Convergence depends on the sign of η

$$\eta \equiv \frac{B}{3H_0} - 1 = \frac{R(r_\infty)}{3}\left[\frac{R'(r_\infty)}{H_0} - 1\right] - 1$$

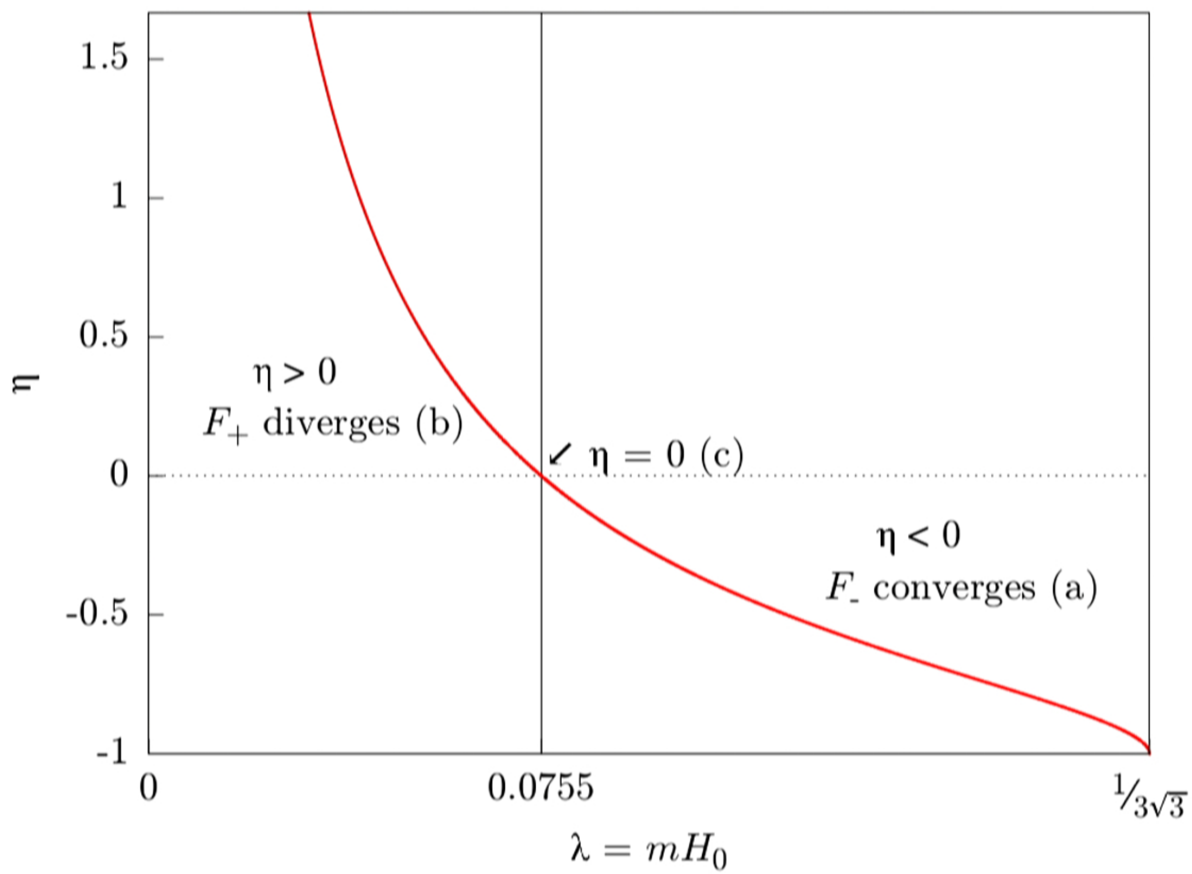
- Inner horizon on the limit $H \to H_0$

$$r_\infty = \frac{2}{H_0\sqrt{3}} \cos\left[\frac{\pi}{3} + \frac{1}{3}\arccos\left(3\sqrt{3}mH_0\right)\right]$$

- η depends only on the product $mH_0 \equiv \lambda$
- Non-extreme Schwarzschild-de Sitter at infinity

$$0 < \lambda < \frac{1}{3\sqrt{3}}$$

Example: ΛCDM



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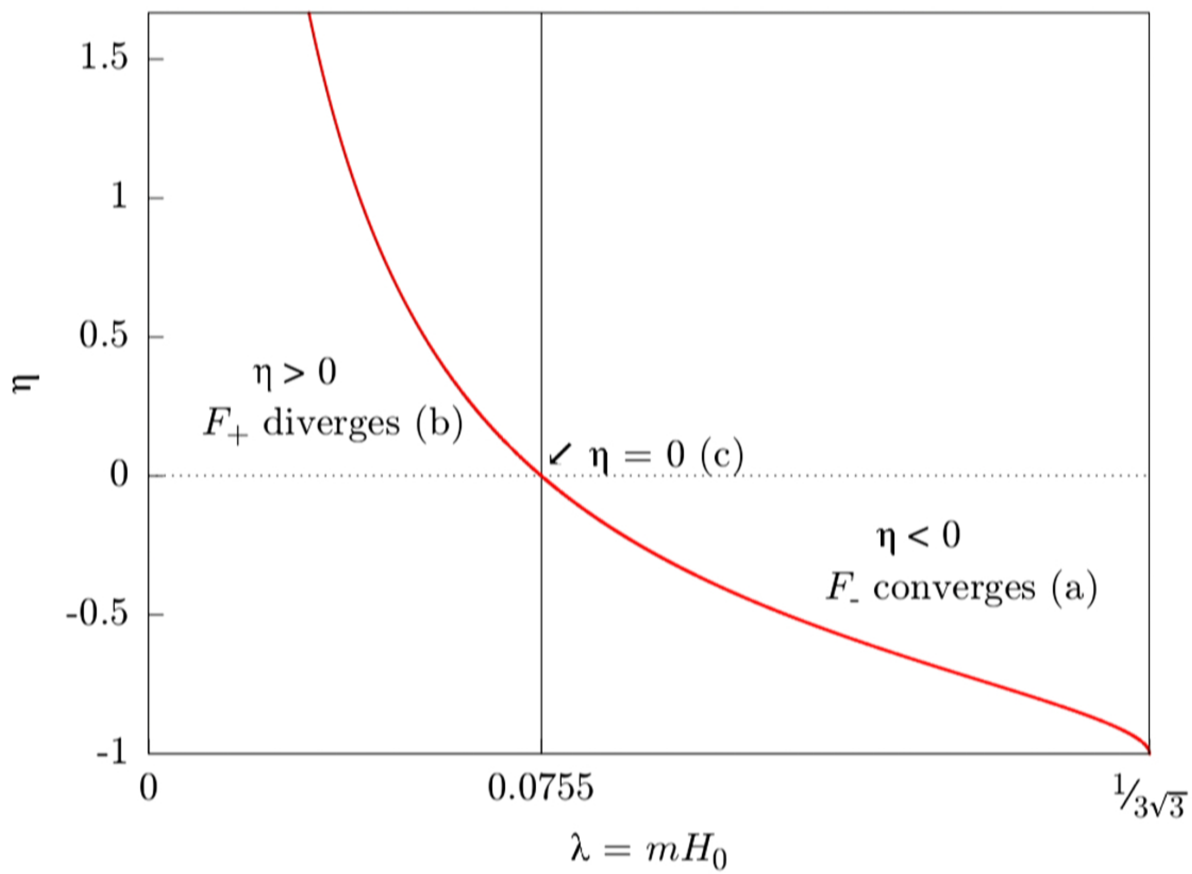
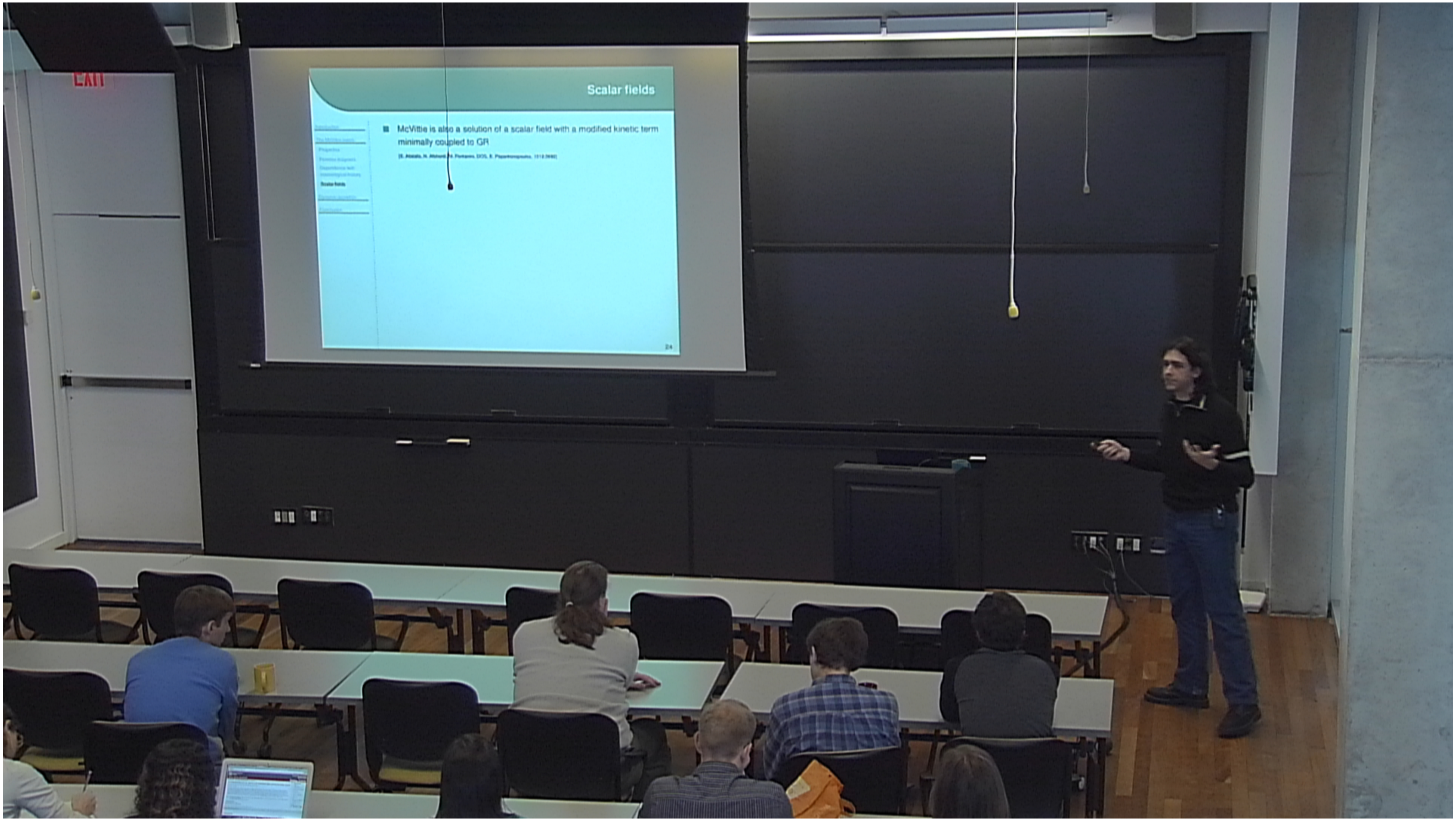


Scalar fields

- McVittie is also a solution of a scalar field with a modified kinetic term minimally coupled to GR

[E. Abdalla, N. Afshordi, M. Fontanini, DCG, E. Papantonopoulos, 1312.3682]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Scalar fields
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- *Cuscuton* field

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\left[\mu^2\sqrt{-g^{\alpha\beta}\phi_{;\alpha}\phi_{;\beta}} - V(\phi)\right]$$

- Field has constant $K^\alpha{}_\alpha$ on homogeneous surfaces

$$K^\alpha{}_\alpha = \frac{1}{\mu^2}\frac{\mathrm{d}V}{\mathrm{d}\phi} = 3H(t)$$

- Einstein equations and equations of motion give consistent results

$$V(\phi) = -6\pi\mu^4(\phi + C)^2 = \frac{3}{8\pi}H^2$$

- Uniform expansion, shear-free solutions are *unique* to this type of field

Accretion of multiple fluids

- Two non-interacting perfect fluids

$$T_\mu{}^\nu = (\rho_1 + p_1) u_\mu u^\nu + p_1 \delta_\mu^\nu + \rho_2 v_\mu v^\nu$$

- Spatial Ricci-isotropy only allows *phantom* equation of state

$$(\rho_1 + p_1)(u^r)^2 + \rho_2 (v^r)^2 = 0$$

- Violation of the weak energy condition

Accretion of general fluids

- McVittie class is too restrictive to the accretion of perfect fluids
- Imperfect fluid (heat conductivity χ, bulk viscosity ζ, ~~shear viscosity η~~)
  - McVittie class is shear-free

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- Imperfect fluid (heat conductivity χ, ~~bulk viscosity ζ~~, ~~shear viscosity η~~)
  - McVittie class is shear-free
  - Bulk viscosity reabsorbed into pressure

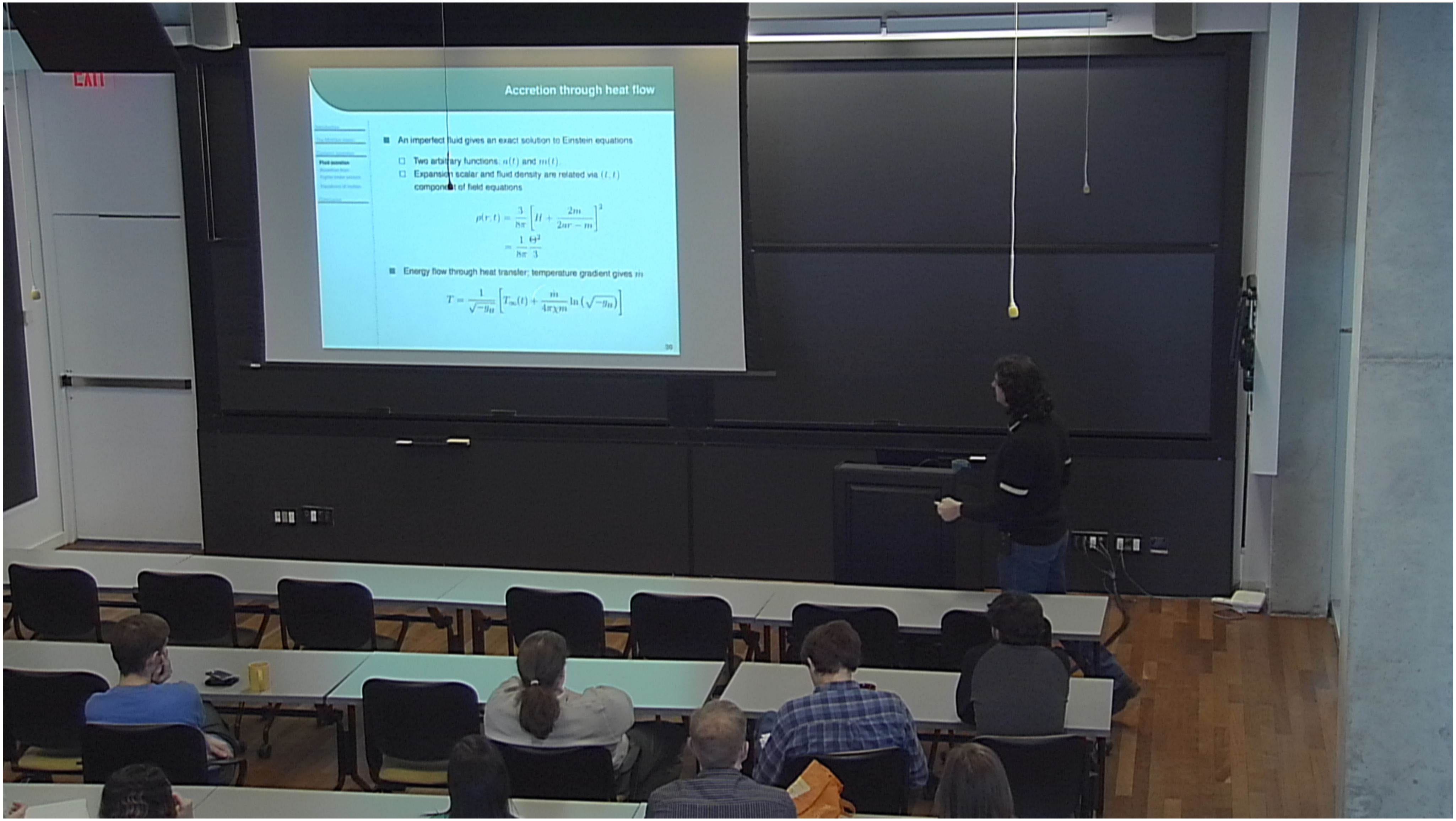
$$T^{r}{}_{r} = T^{\theta}{}_{\theta} = T^{\phi}{}_{\phi} = p - 3\zeta \left(H + \frac{2\dot{m}}{2ar - m} \right)$$

- Landau-Eckart hydrodynamical model
  - Fluid temperature has an extra term

$$T\sqrt{-g_{tt}} = T_{\infty}(t) + \frac{\dot{m}}{4\pi\chi m} \ln\left(\sqrt{-g_{tt}}\right)$$

Accretion through heat flow

- An imperfect fluid gives an exact solution to Einstein equations
  - Two arbitrary functions: $a(t)$ and $m(t)$.


Accretion through heat flow
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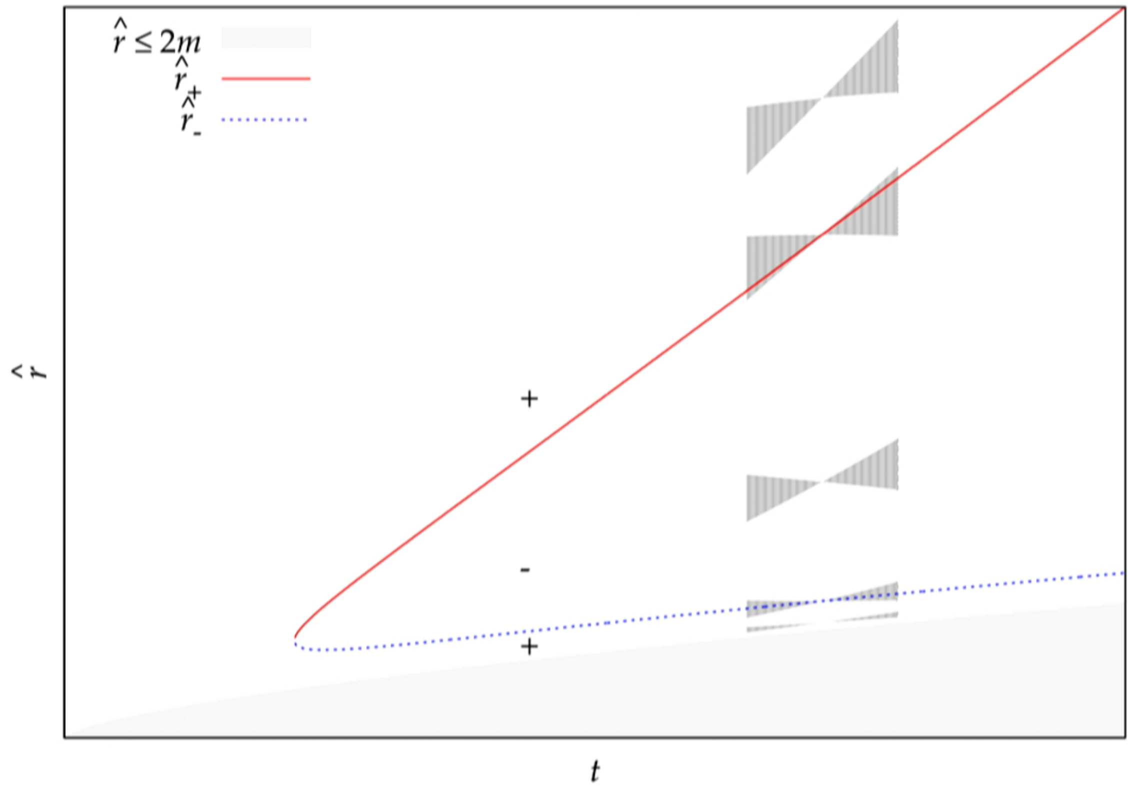
Light cones and apparent horizons



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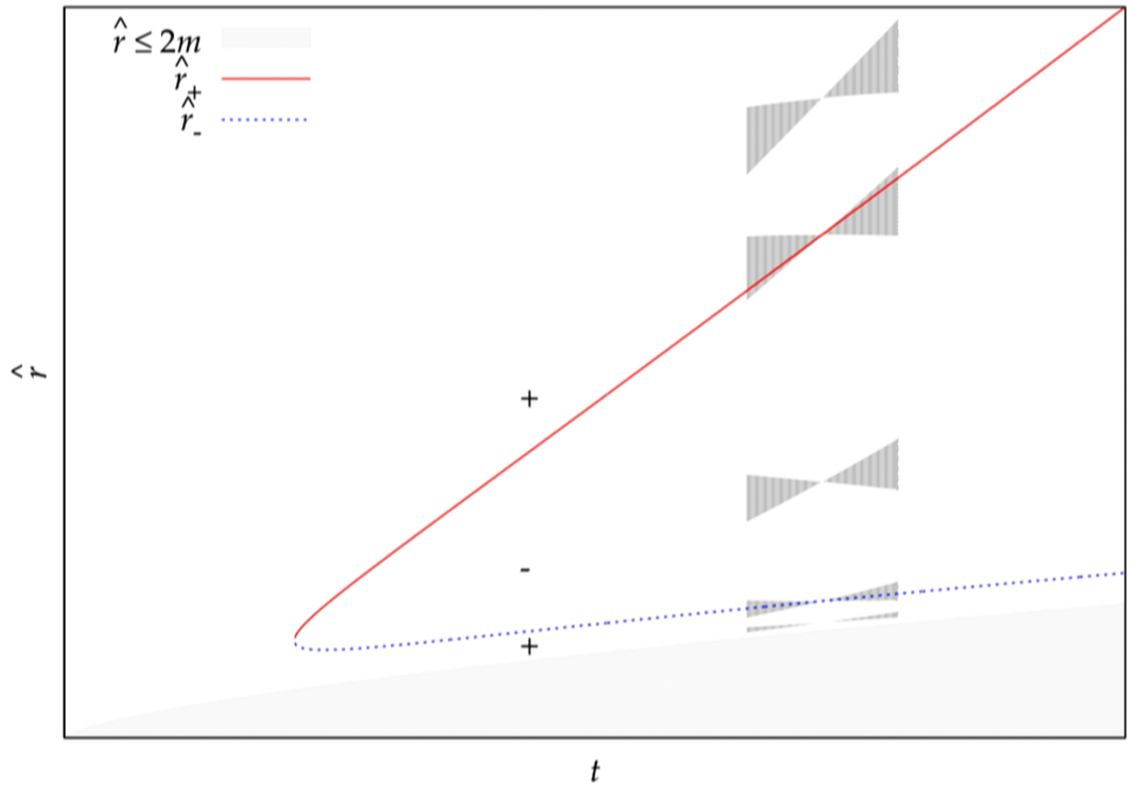
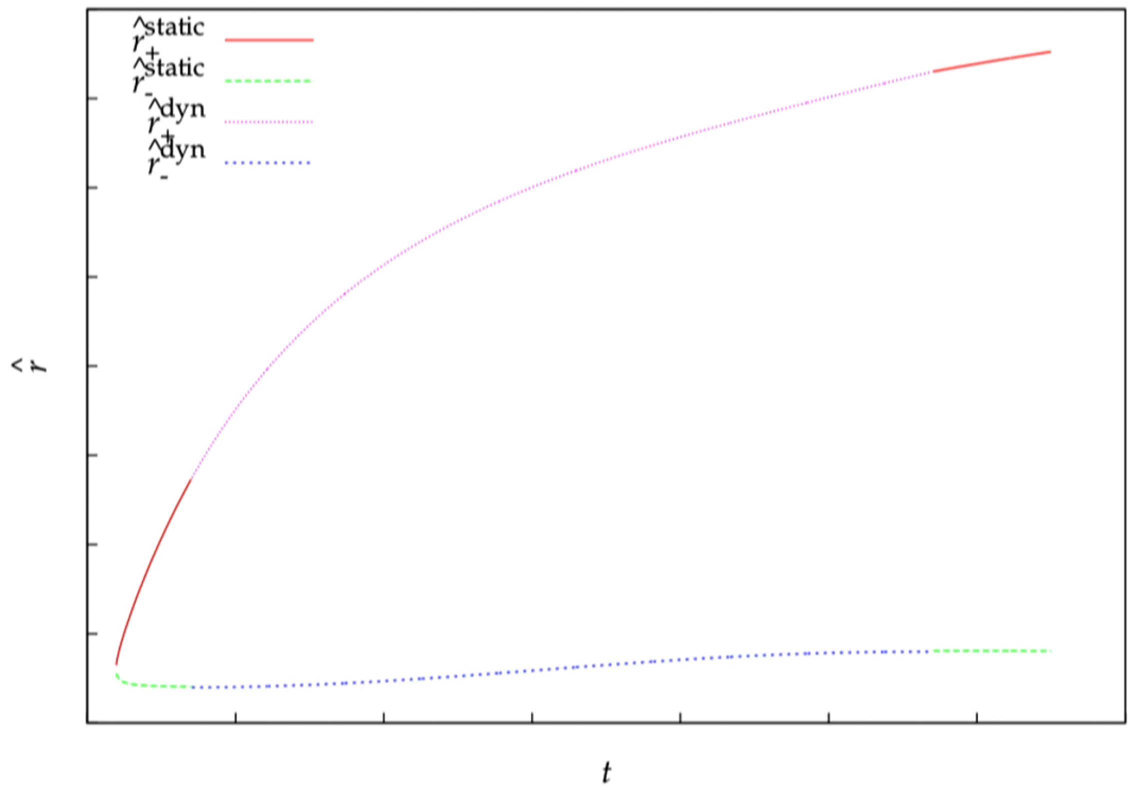


An example

- Toy model for the mass: accretion during a finite interval

$$m(t) = \begin{cases} 1 & t \leq t_0 \\ \frac{1}{2}\left[3 + \sin\left(\omega t + \phi\right)\right] & t_0 < t < t_1 \\ 2 & t \geq t_1 \end{cases}$$

Null geodesics



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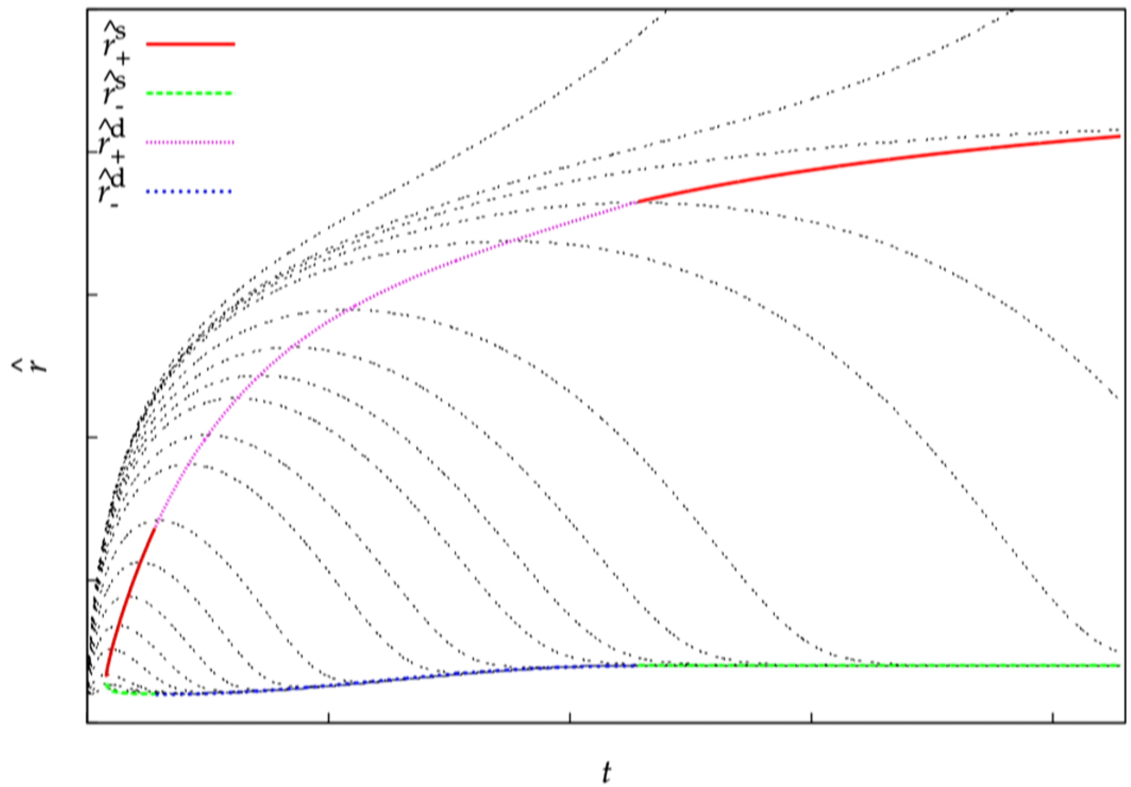

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Horndeski action

- Can we have a field as source for generalized McVittie?
  - Additional terms in the action must look like heat flow
  - Most general scalar action: Horndeski
- First term added to the k-essence action: *kinetic gravity braiding*

[C. Deffayet, O. Pujolàs, I. Sawicki, A. Vikman, 1008.0048]

$$S_\varphi = \int \mathrm{d}^4x \sqrt{-g}\,[K(X,\varphi) + G(X,\varphi)\Box\varphi]$$

with $\Box\varphi = g^{\alpha\beta}\varphi_{;\alpha\beta}$

- Up to total derivatives, the Lagrangian may be rewritten as

$$\begin{aligned}\mathcal{L} &= K + G\Box\varphi \\ &= K - G_{;\alpha}\varphi^{;\alpha} \\ &= K + 2XG_{,\varphi} - G_{,X}\varphi^{;\alpha}X_{;\alpha}\end{aligned}$$

Full Horndeski action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}R + \sum_{n=0}^{3}\mathcal{L}^{(n)}\right)$$

where

$$\begin{aligned}
\mathcal{L}^{(0)} &= K(X,\varphi) \\
\mathcal{L}^{(1)} &= G(X,\varphi)\,\Box\varphi \\
\mathcal{L}^{(2)} &= G^{(2)}(X,\varphi)_{,X}\left[(\Box\varphi)^2 - \varphi_{;\alpha\beta}\varphi^{;\alpha\beta}\right] + RG^{(2)}(X,\varphi) \\
\mathcal{L}^{(3)} &= G^{(3)}(X,\varphi)_{,X}\left[(\Box\varphi)^3 - 3\Box\varphi\,\varphi_{;\alpha\beta}\varphi^{;\alpha\beta} + 2\varphi_{;\alpha\beta}\varphi^{;\alpha\rho}\varphi_{;\rho}{}^{\beta}\right] \\
&\quad - 6G_{\mu\nu}\varphi^{;\mu\nu}G^{(3)}(X,\varphi)
\end{aligned}$$

- $\mathcal{L}^{(2)}$ and $\mathcal{L}^{(3)}$ components of $T_{\mu\nu}$ have non-vanishing anisotropic stress

- Energy-momentum tensor of the KGB term

$$T_{\mu\nu} = \left(K - G_{;\alpha}\varphi^{;\alpha}\right) g_{\mu\nu} + \left(K_{,X} + \Box\varphi G_{,X}\right) \varphi_{;\mu}\varphi_{;\nu} + 2G_{(;\mu}\varphi_{;\nu)}$$

- Equivalent fluid four-velocity u^μ

$$u^\mu = \frac{\varphi^{;\mu}}{\sqrt{2X}}$$

- Energy-momentum tensor of the equivalent fluid

$$\rho \equiv 2X\left(K_{,X} + \Box\varphi G_{,X}\right) - \sqrt{2X}\varphi_{;\alpha}G^{;\alpha}$$
$$p \equiv K - G_{;\alpha}\varphi^{;\alpha}$$
$$q^\mu \equiv \sqrt{2X} h^\mu{}_\alpha G^{;\alpha}$$

- tr and tt Einstein equations

$$-\frac{\dot{m}}{m\dot{\varphi}} = 8\pi X G_{,X}$$

$$-\frac{1}{3}\Theta^2 = 8\pi \left\{ K - 2X \left[G_{,\varphi} + K_{,X} + 3\sqrt{2X} G_{,X} \Theta \right] \right\}$$

- solution of the tr equation

$$G(X, \varphi) = g_0(\varphi) \ln X + g_1(\varphi)$$

with

$$g_0(\varphi) = -\frac{1}{8\pi} \frac{\dot{m}}{m\dot{\varphi}}$$

Einstein equations

- Solution of tt equation

$$K = f_1(\varphi) + f_2(\varphi)\sqrt{X} + 2X\left[(2 - \ln X)g_0' - g_1' - 24\pi g_0^2\right]$$

- Plugging the solution into rr Einstein equation and solving for f_1 and f_2

$$f_1 = -\frac{3}{8\pi}(H - M)^2$$

$$f_2 = \frac{\sqrt{2}}{4\pi\dot{\varphi}}\left[H - M + 3M\left(\dot{H} - \dot{M}\right)\right]$$

$$\left(\frac{\dot{m}}{m} \equiv M \quad \text{and} \quad \frac{\dot{a}}{a} \equiv H\right)$$

Equation of motion

- Consistency check: equation of motion of the scalar

$$K_{,\varphi} + G_{,\varphi}\Box\varphi + \left[\left(K_{,X} + G_{,X}\Box\varphi\right)\varphi^{;\mu} + G^{;\mu}\right]_{;\mu} = 0$$

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- Inserting the solutions for G and K

$$G = g_0(\varphi)\ln X + g_1(\varphi)$$
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Action coefficients

- Functions g_0, f_1 and f_2 can be written in terms of φ

$$g_0 = -\frac{1}{8\pi}\frac{\mathrm{d}\,(\ln m)}{\mathrm{d}\varphi} \quad \Rightarrow \quad m(\varphi) = e^{-8\pi \int g_0 \mathrm{d}\varphi}$$

$$f_2 = -\frac{1}{\sqrt{3\pi}}\left(8\pi g_0 + \frac{f_1'}{\sqrt{-f_1}}\right)$$

$$H = \sqrt{\frac{-8\pi f_1}{3}} - 8\pi g_0 \dot{\varphi}$$

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  - Two functions necessary, plus $\varphi(t)$

- Compare with cuscuton case: $K(X,\varphi) = A(\varphi) + B(\varphi)\sqrt{X}$

$$A(\varphi) = \frac{3}{8\pi}H^2 \quad , \qquad B(\varphi) = \text{constant}$$

  - One function necessary, plus $\varphi(t)$

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  - Apparent horizons
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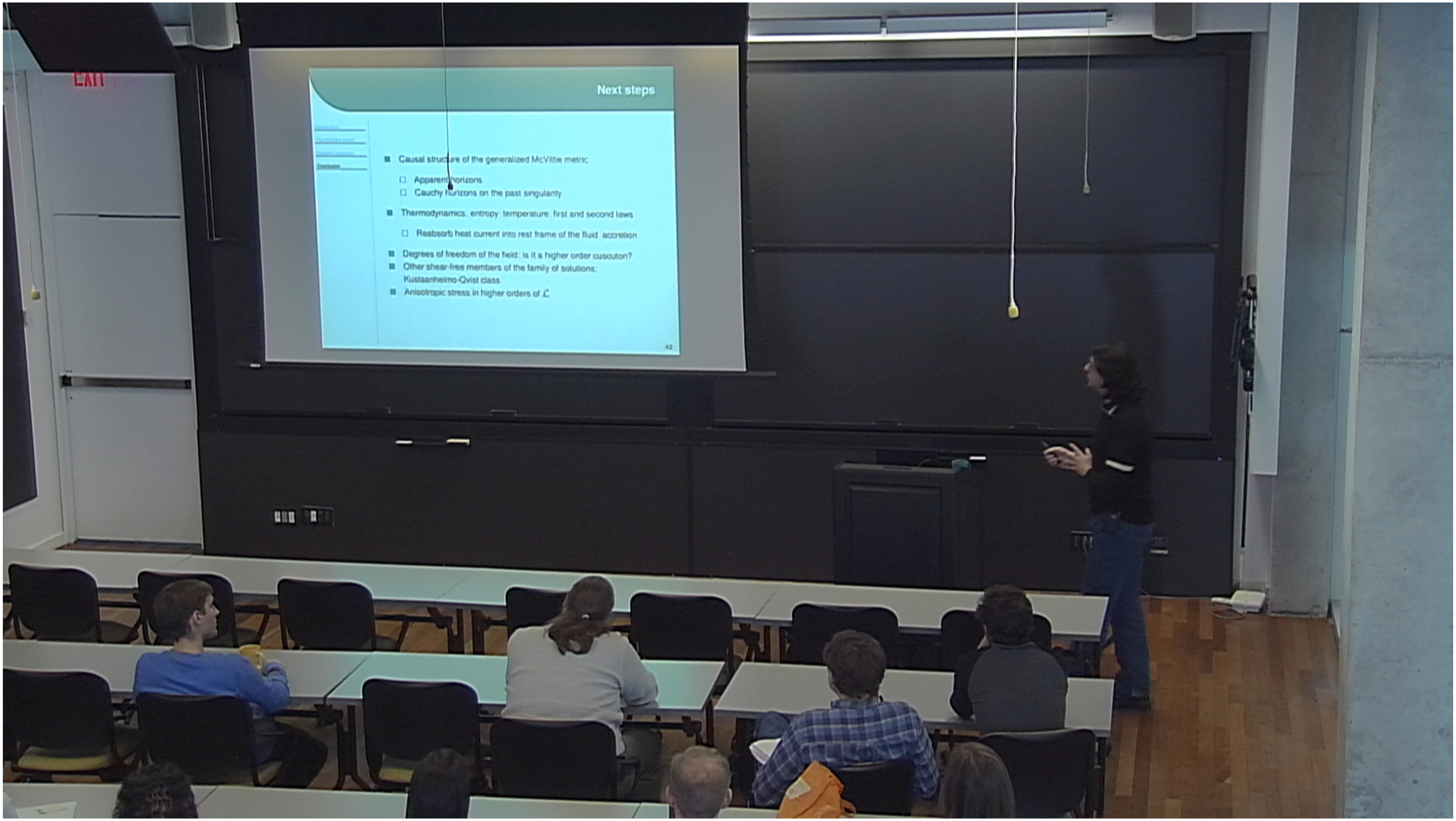
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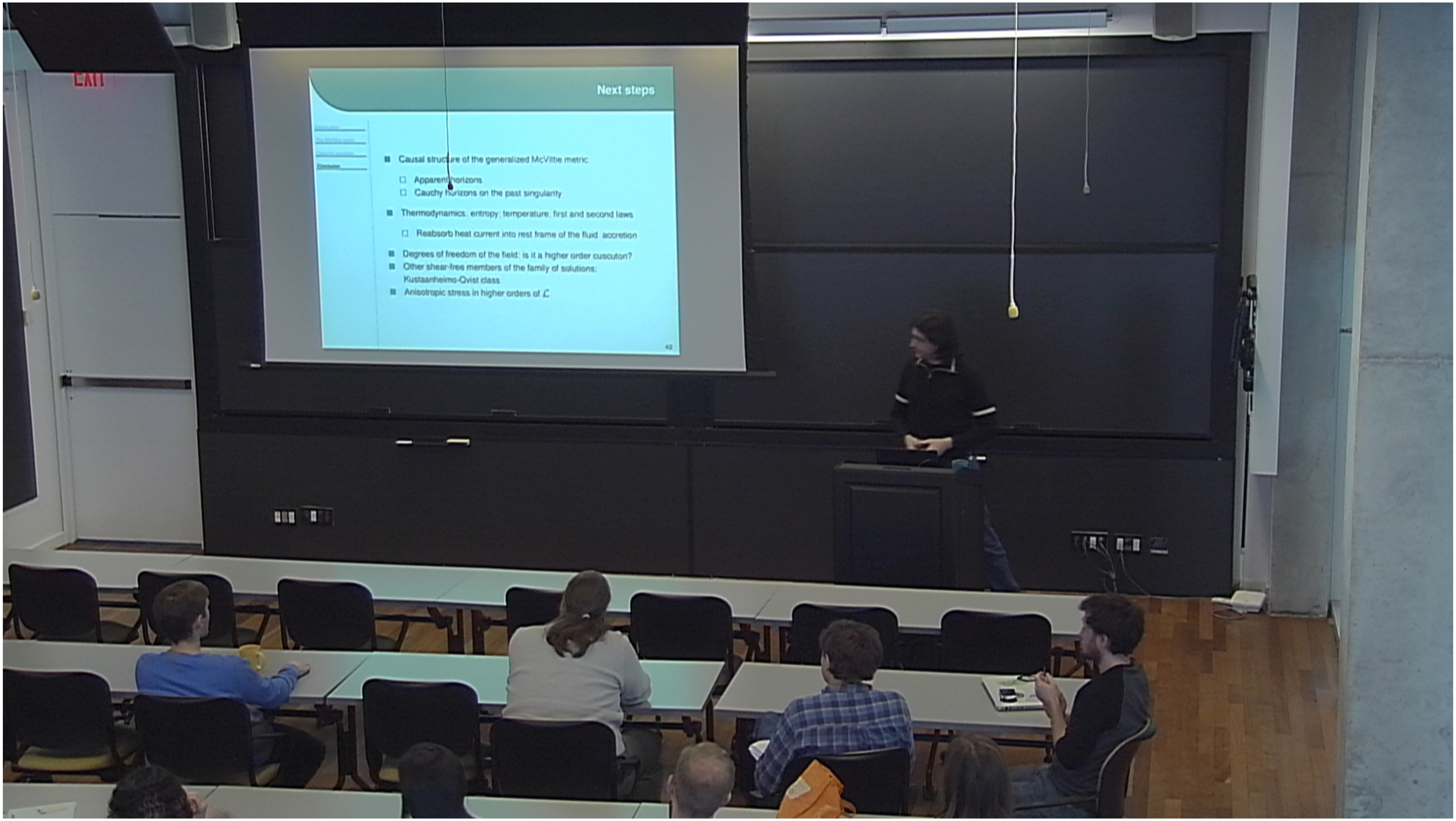
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