Title: The emergent of the chiral spin liquid in kagome Heisenberg model

Date: Feb 20, 2014 11:00 AM

URL: http://pirsa.org/14020149

Abstract: The quantum spin liquid is an emergent new state of matter, which has attracted a lot of recent attention. In particular, the time reversal symmetry broken spin liquid (Kalmeyer et. al. and Wen et. al.), characterized by the chiral ordering and fractionalized quasi-particle as a realization of the fractional quantum Hall state had been proposed for more than 20 years, but never identified as the true ground state in any more generic (e.g. Heisenberg spin exchange) models with time reversal symmetry. Here I will report two concrete examples where chiral spin liquid (CSL) emerge for a range of parameter space for kagome J1-J2-J3 (three nearest neighbors) model based on accurate density matrix renormalization group (DMRG) simulations. We identify long-range chiral ordering, ground state degeneracy, characteristic entanglement spectra, and the fractionalized topological Chern number to establish the topological state in such a system as the long-sought CSL. We further explicitly extract the modular matrix, which captures all the information of the fractional statistics of the quasi-particles in the system. I will also discuss the close relation of our model to some frustrated kagome antiferromagnets, and make a conjecture that J1 kagome model is near an unconventional critical point. I will conclude the talk with some discussions of the open directions.

Collaborators

Chiral spin liquid and quantum phase transition

ShouShu Gong and Wei Zhu(CSUN, postdocs)

Yinchen He (PhD student, Fudan Univ.)

Yan Chen (Fudan)

Research supported by NSF and DOE

We also thank: Olexei Motrunich (Caltech) &
Matthew Fisher (UCSB) for square/honeycomb
F. D. M. Haldane for modular matrix of FCI

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Donna N. Sheng
California State Univ. Northridge

Talk prepared for PI, Feb. 20, 2014

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Outline

I. Introduction:

Time reversal invariant states in Heisenberg models: Z2, U1 spin liquids vs. valence bond solid

Time reversal symmetry (TRS) broken Chiral spin liquid proposed 25 years ago---building block for exotic theory of strongly correlated systems

II. Chiral Spin Liquid in Kagome J1-J2-J3 Model

Motivation---Connecting between two Z2 phases

Identifying Chiral Spin Liquid: Topological degeneracy and Entanglement spectra;

Chiral order and spontaneously TRS broken; Half-integer quantized Chern number; Modular matrix

III. Summary

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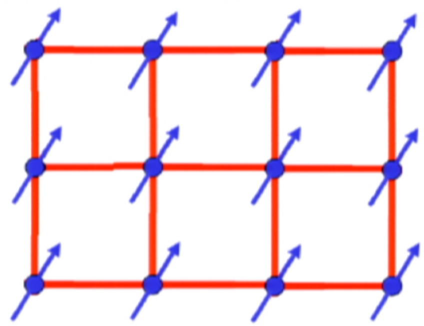
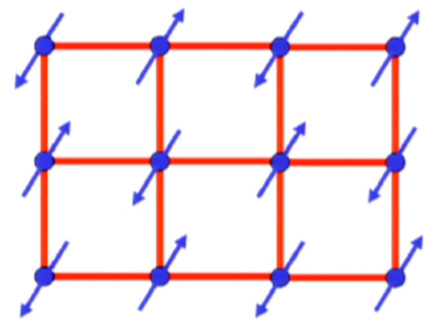
III. Summary

Magnetic systems intend to develop orders by breaking symmetry

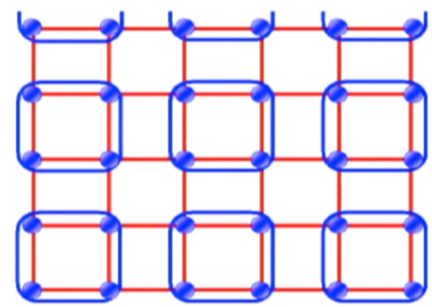


Ferromagnetic order



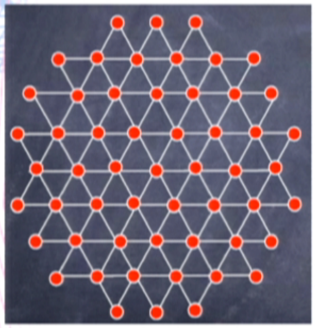
Antiferromagnetic order with NN J1

Valence Bond Solid (VBS) order

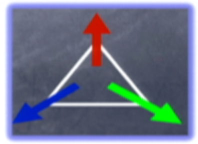


Plaquette VBS (possibly identified in J1-J2 square model)

Spin liquids in Heisenberg models with increasing frustration



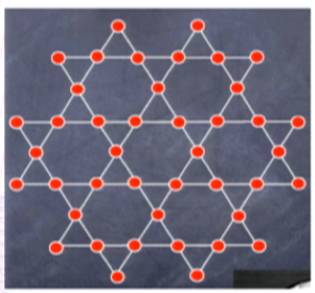
Triangular lattice



Three-sublattice AF order

Resonating valence bond (P. W. Anderson), RVB is a quantum spin liquid (SL): deconfined quasiparticles

Kagome lattice



- Larger geometrical frustration
- Smaller coordination number

Jiang, Wang, Balents (2012)
Depenbrock et al. (2012).

A Z2 SL?

Spin Liquid
H.C. Jiang, Z.Y. Weng, DNS (2008), not converged DMRG for wider systems

Yan, Huse, White, Science (2010)---new milestone: A possible gapped spin liquid

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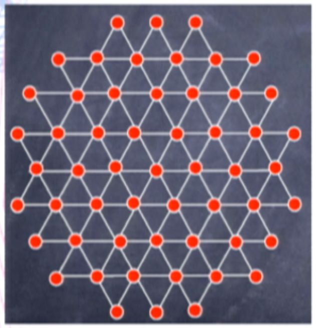

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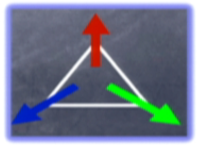

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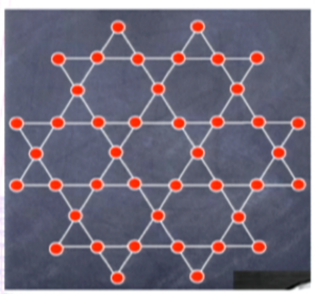

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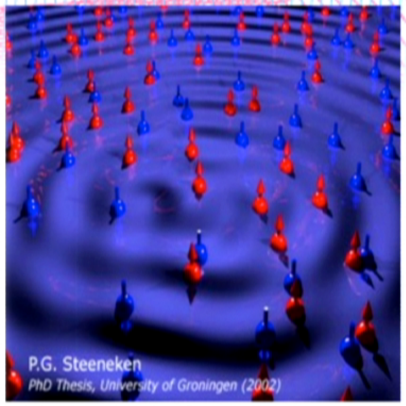
Spin Liquid (SL) State

A new state of matter with no symmetry broken, with topological order and fractionalization

X. G. Wen (1990, 1991)

Z2 SL in contrived theoretical models
Wen (1990,1991), Kivelson, Rokhsar, Sethna (1987), Senthil, Fisher (2000), Balents, Fisher, Girvin (2002) Moessner and Sondhi, (2001), Senthil, Motrunich (2002) Senthil, Vishwanath, Balents, Sachdev, Fisher (2004) Balents (2010) ………



Gapped SL State

Gaps to all spin excitations

exponential decay correlations

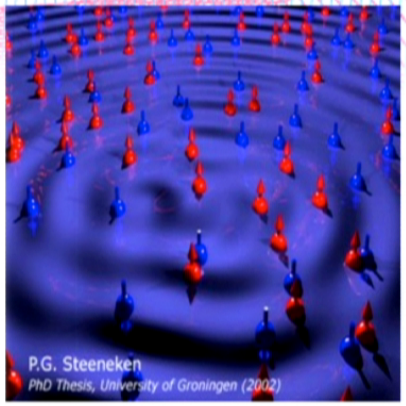
J1-J2-J3 (Ising) kagome, Balents et al (2002), DNS&Balents 2005: A concrete example of Z2 SL

Algebraic Spin Liquid

Gapless excitations

Power-law decay of spin correlations

Spin Liquid (SL) State



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Topological Entanglement Entropy (TEE) for topological order (positive identification of Z2 SL for kagome J1)

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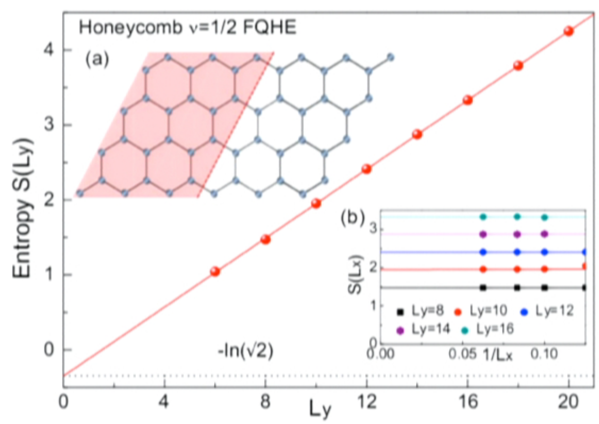
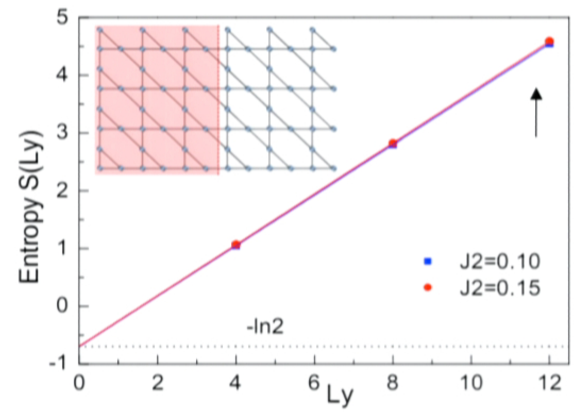


Jiang et al, flat band model for ½ FQHE: fitting is robust



The S at Ly=12 (6 unit cells) for kagome systems requires a scaling vs. m (kept states) or truncation error

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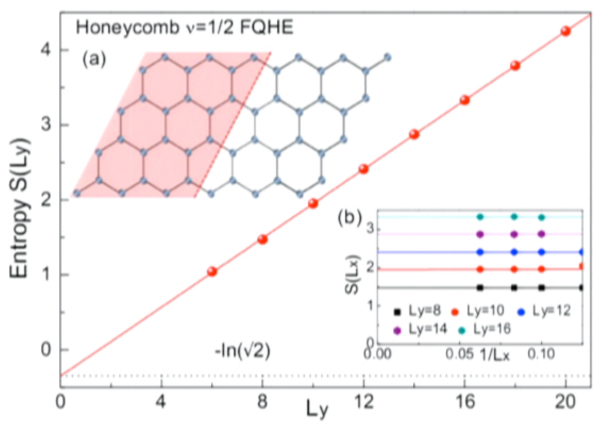

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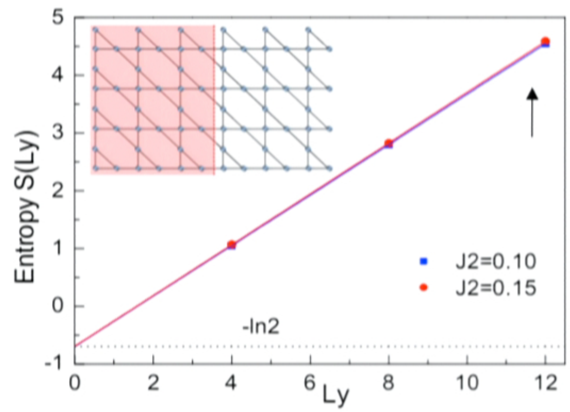

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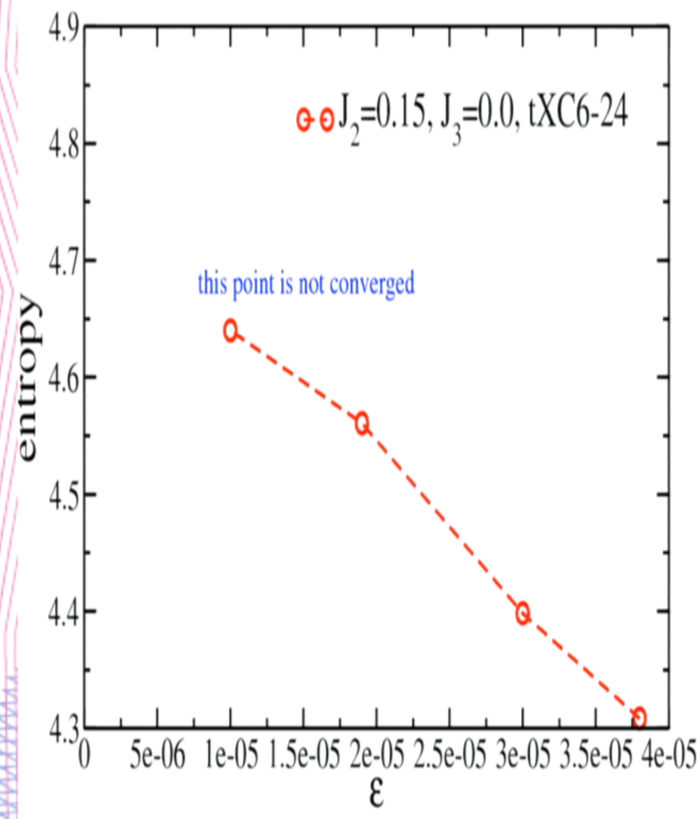
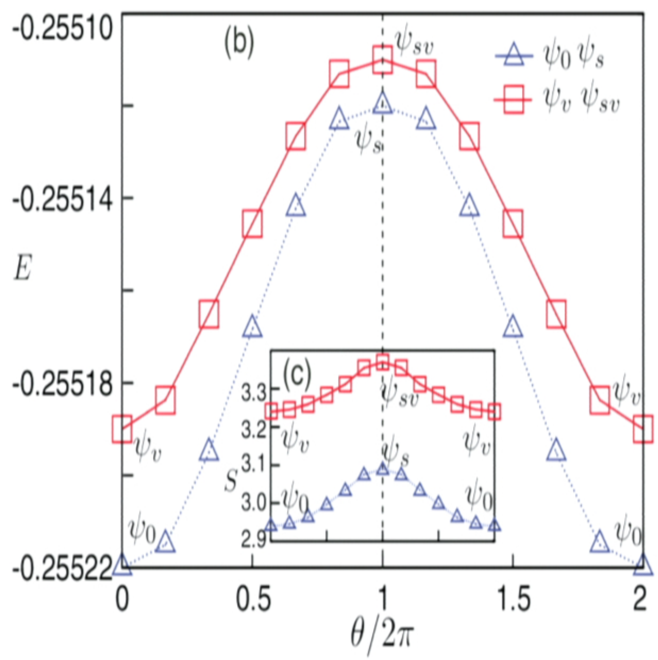
Four top. Sectors in J1-J2-J3 model, desired for J1 model



for J1-J2-J3 model
with large Jz,
we find the right 4 top. states



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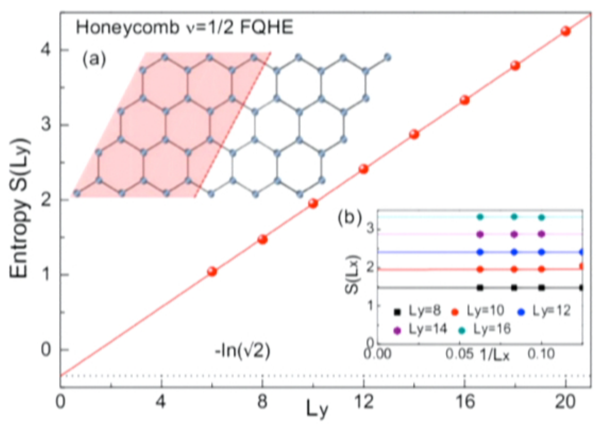

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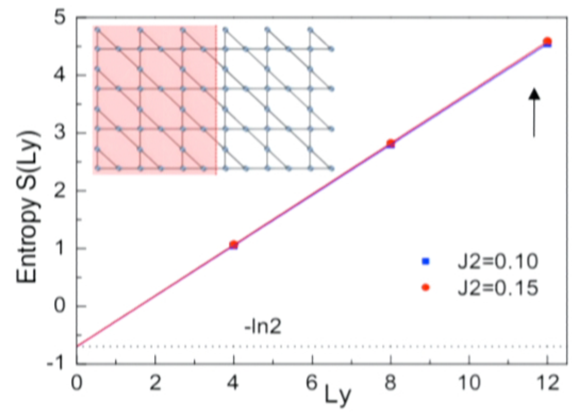

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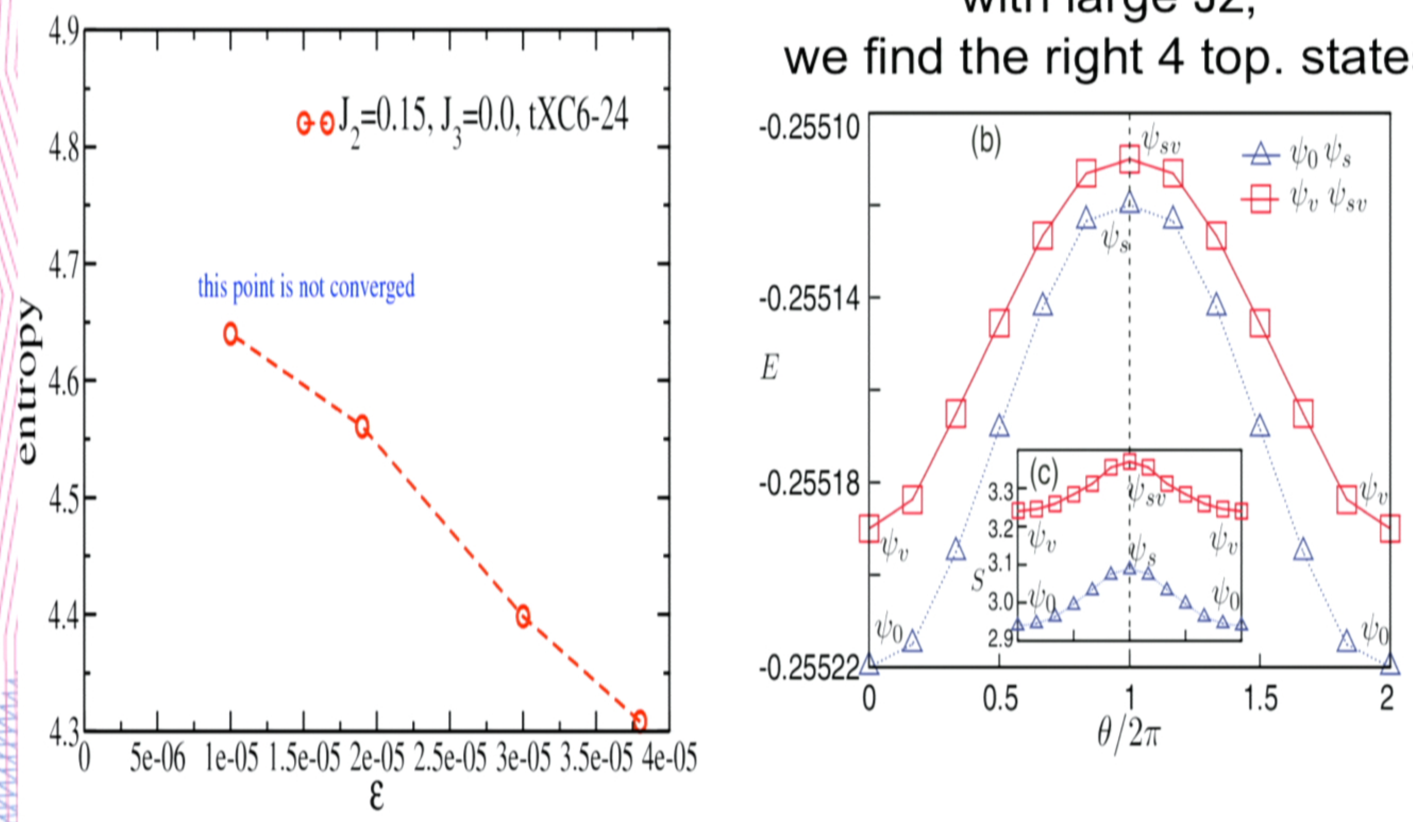

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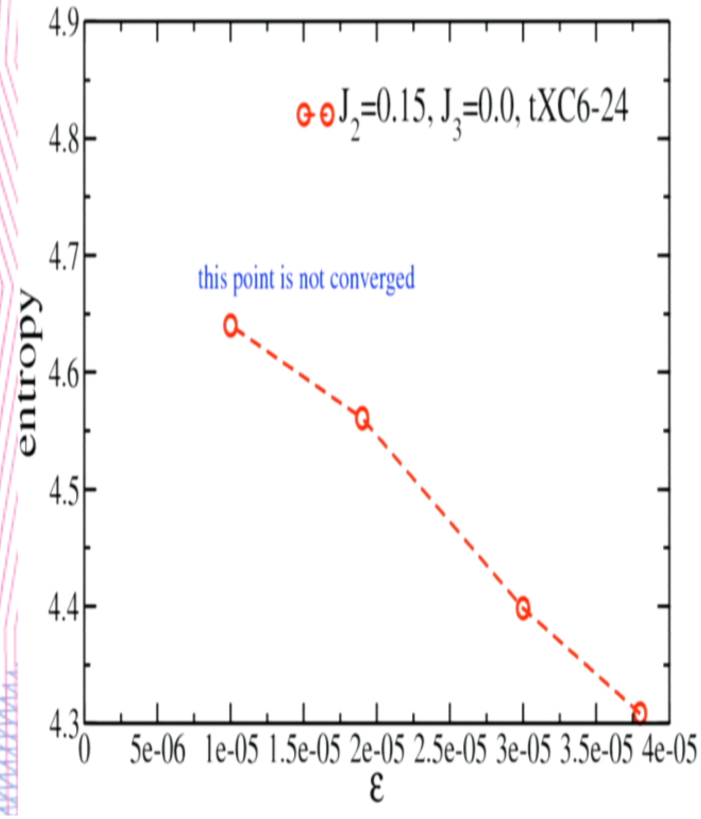

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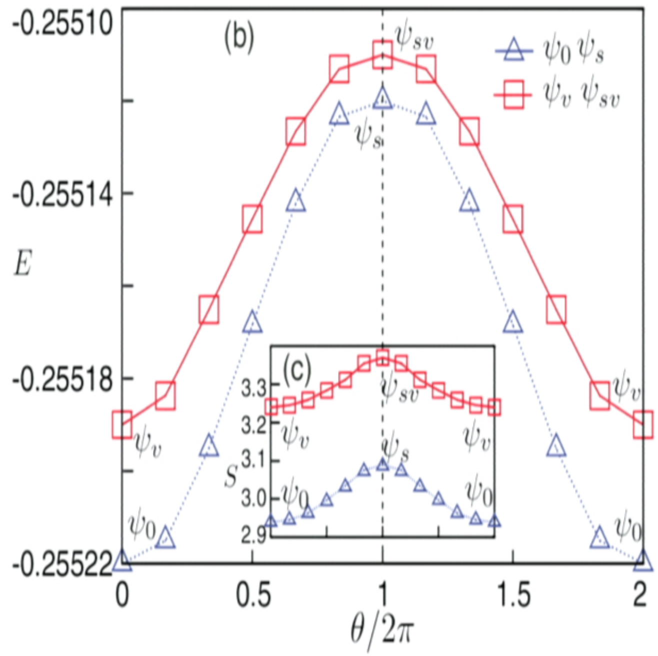

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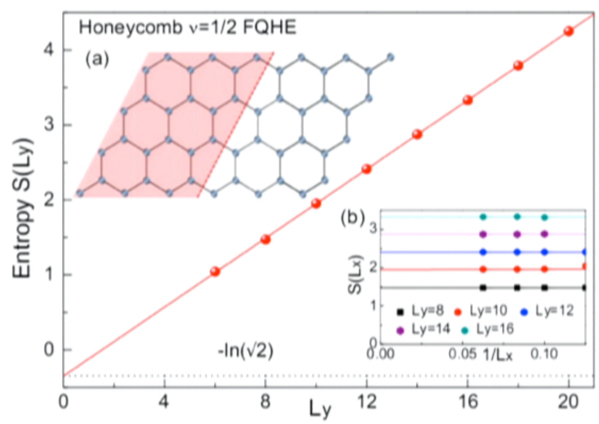

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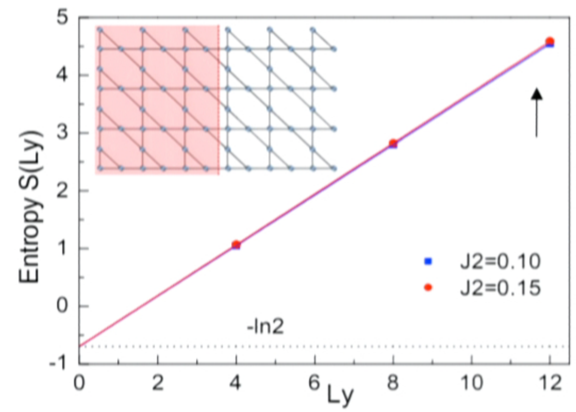

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Spin-1/2 KAF: Experimental findings

$ZnCu_3(OH)_6Cl_2$

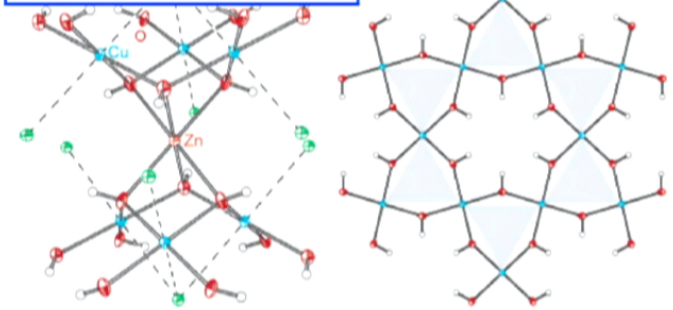
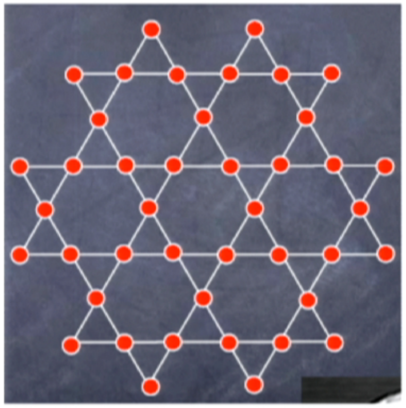


Figure 1. Crystal structure of Zn−paratacamite (1), $Zn_{0.33}Cu_{3.67}(OH)_6Cl_2$.



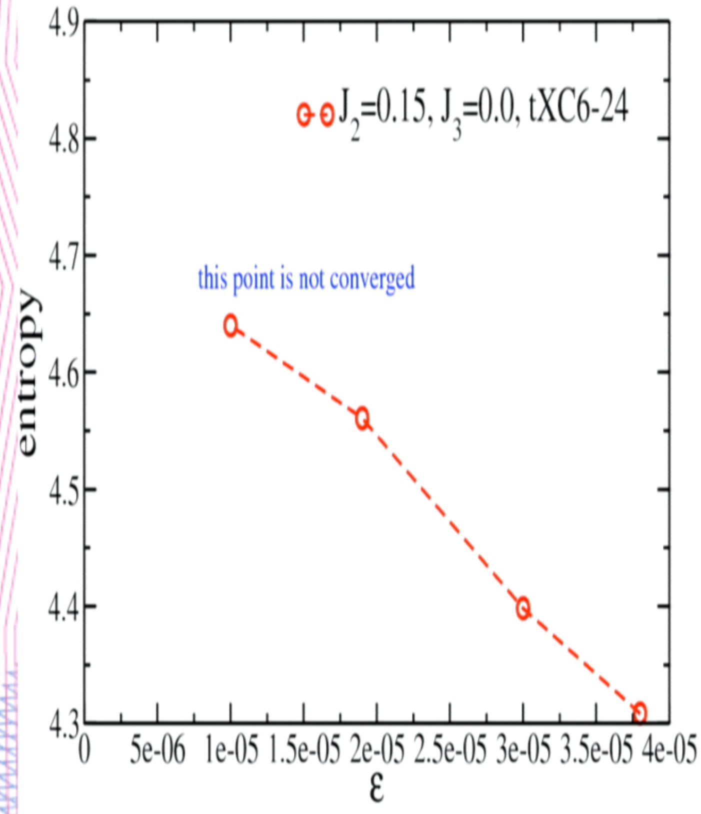
- ✓ No magnetic order down to 50 mK
- ✓ No observable spin gap down to 0.1 meV

⇒ SL Phase, but different from identified by DMRG

Gapless SL for kagome,
Y. Iqbal et al,arXiv:1209.1858

J. S. Helton et al., Phys. Rev. Lett. 98, 107204 (2007); O. Ofer et al., arXiv:condmat/0610540.
P. Mendels et al., Phys. Rev. Lett. 98, 077204 (2007). A. Olariu et al., Phys. Rev. Lett. 100, 087202 (2008).

Four top. Sectors in J1-J2-J3 model, desired for J1 model



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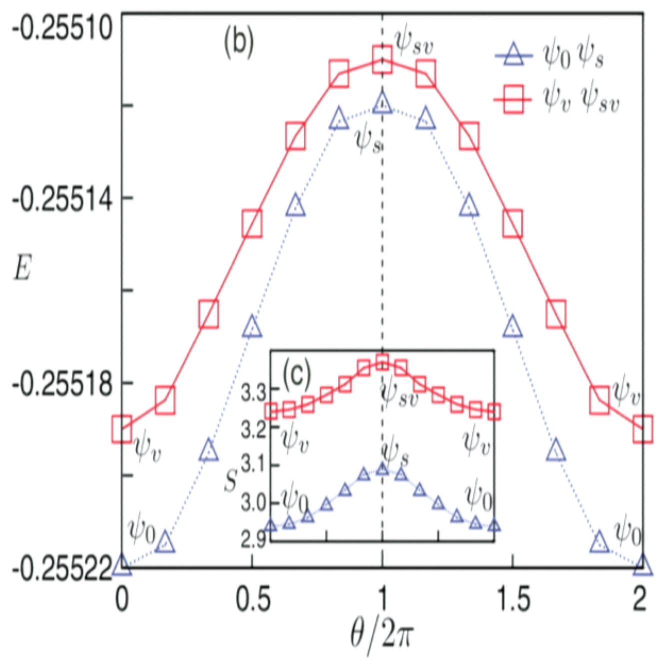


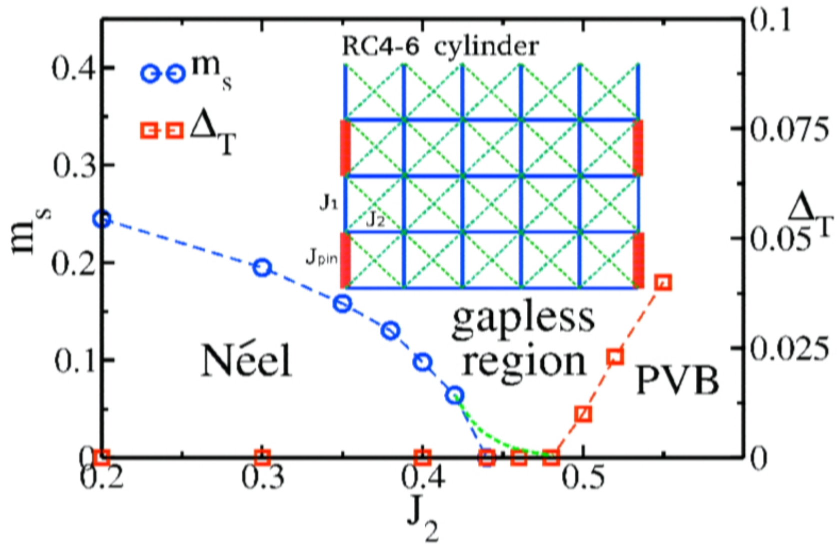
Possible gapped spin liquid vs. PVBS on square J1-J2 models

Slave-particle construction for gapless SL on square J1-J2
W. J. Hu et al. (2013)

Tensor network for square J1-J2, gapless SL
Wang, Gu, Wen, Verstrate (2011)
L. Wang et al, 2013

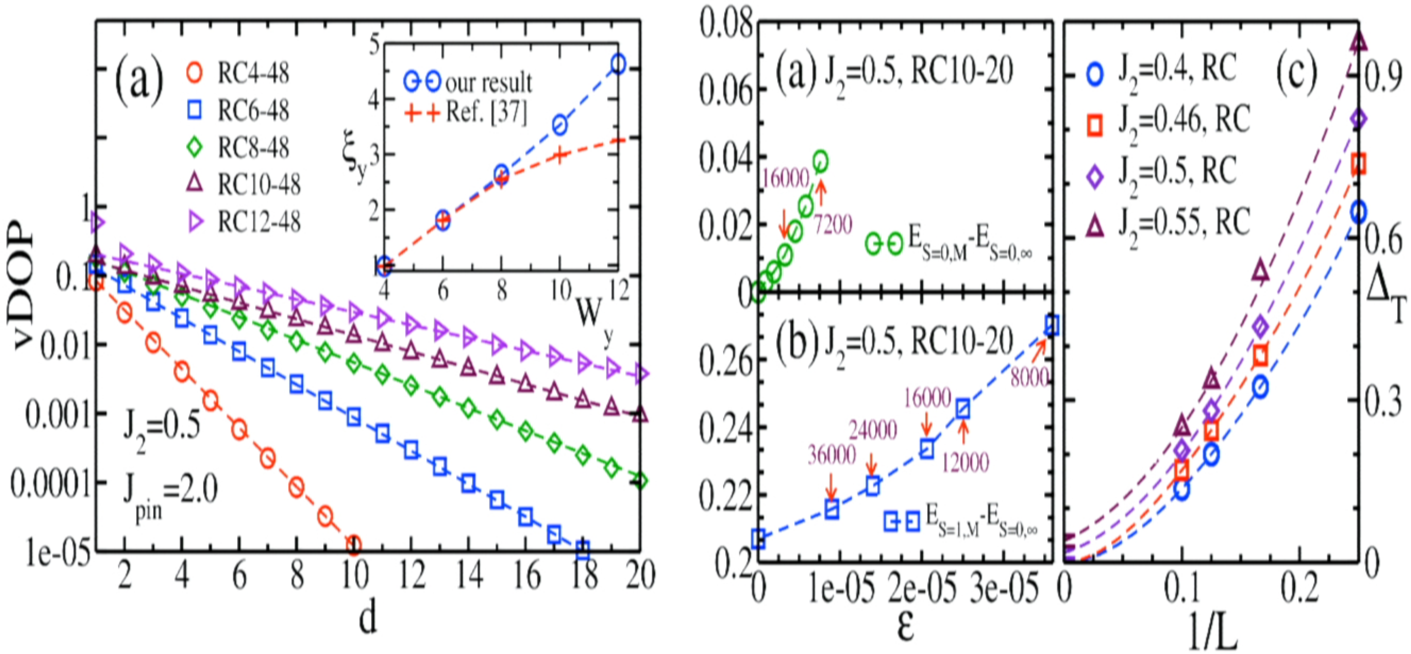
DMRG for square J1-J2 (Jiang, Yao, Balents) 2011: gapped Z2 SL

Sandvik PRB 2012



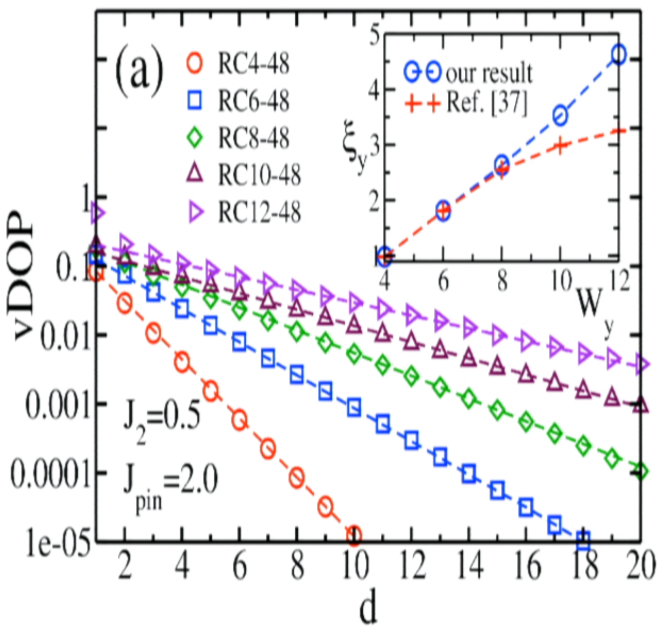
Higher accuracy DMRG found PVBS state
S. S. Gong, W.Zhu, DNS, L.Motrunch, Fisher (2013)

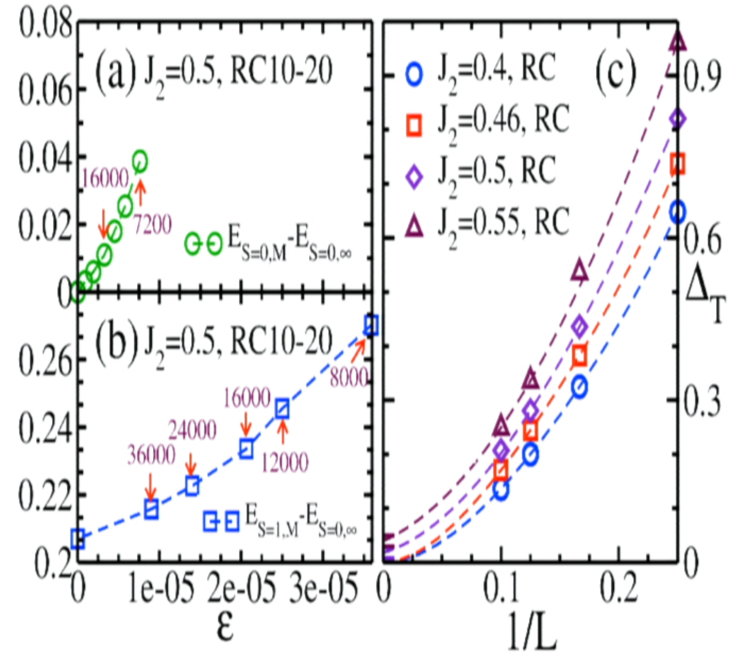
Improved accuracy on square J1-J2 model for DMRG leads to PVB



Our decay length of VBS
Is growing with system width
about linearly

Improved accuracy on square J1-J2 model for DMRG leads to PVB





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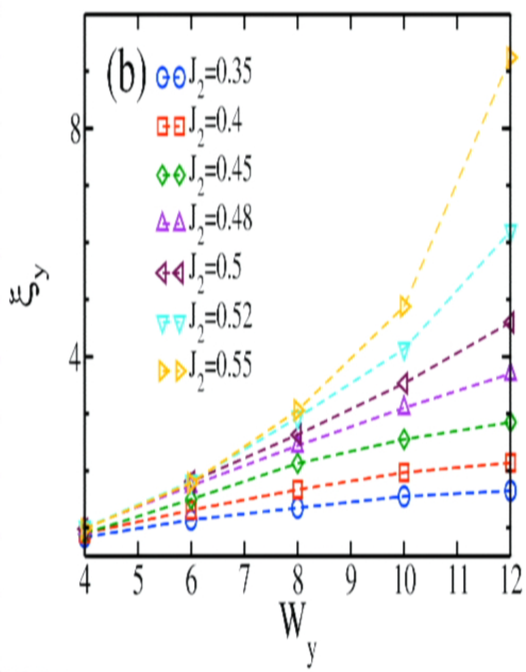
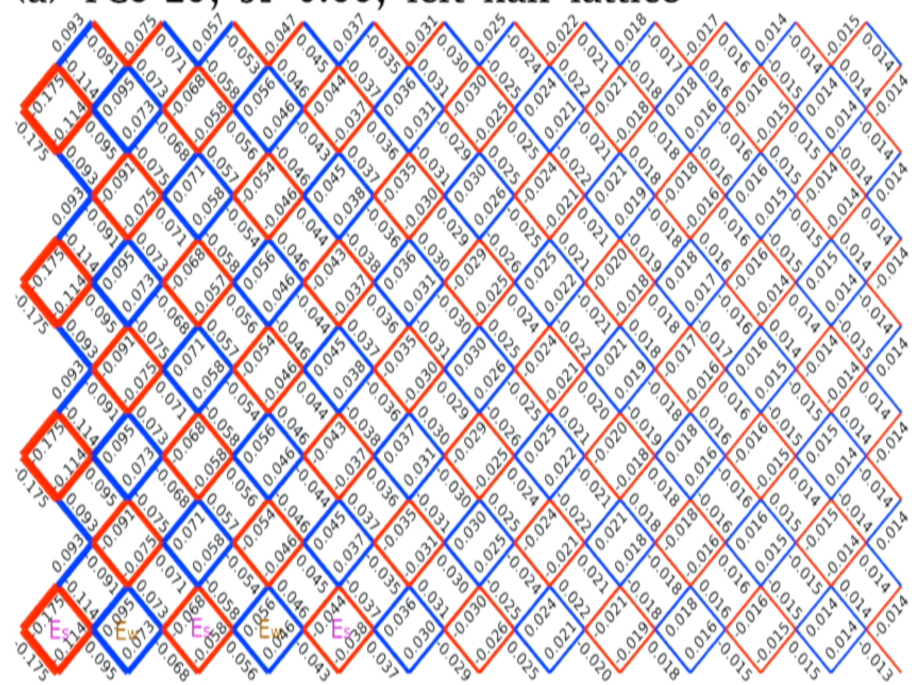
The intermediate phase is developing PVBS ordering for square J1-J2 model

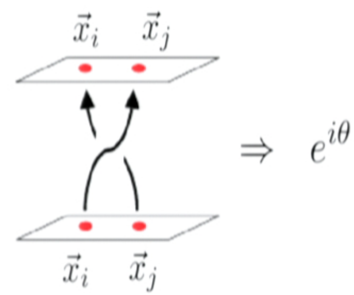


(a) TC8-25, J_2=0.55, left half lattice



Time reversal symmetry (TRS) broken chiral spin liquid (CSL): a bosonic FQHE state

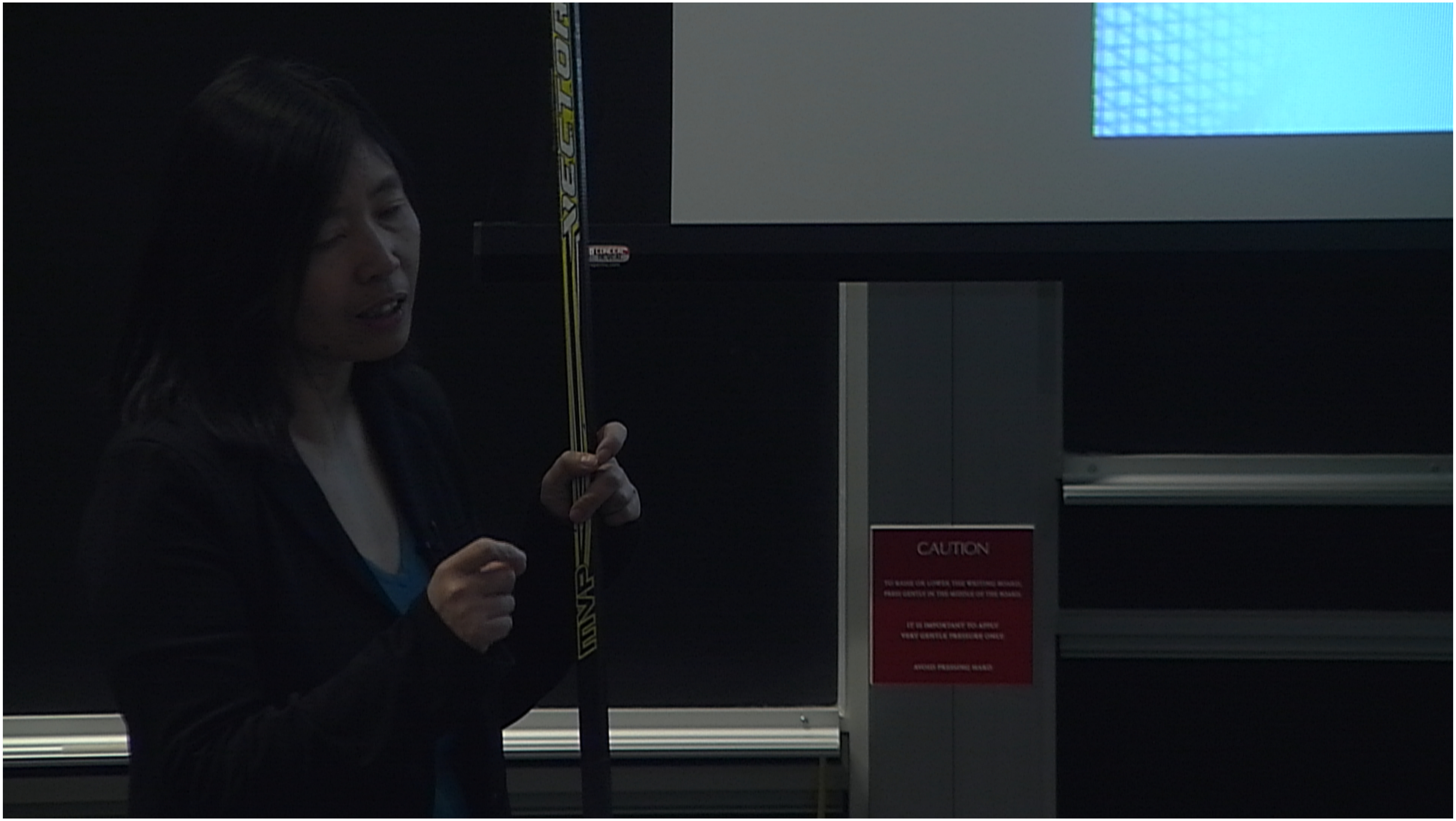
Kalmeyer and Laughlin 1987
Wen, Wilczek, Zee 1989



anyon quasiparticles obey fractional statistics, chiral ordering, TRS and parity broken spontaneously
Different from FQHE with B field as it emerges in TRS system

Haldane and Arovas 1995
Chern number to distinguish it from chiral spin state
Yang, Warman and Girvin 1993.
Yao and Kivelson: a contrived CSL state (for Kitaev model) 2007

Induce CSL with TRS broken terms
Schroeter et al 2007 Thomale et a. 2009
Nielsen et al. 2012 Bauer et al. 2013-2014 (Mott materials)
Approx. Methods find CSL
Hermele et al (2009), Messio et al.
For kagome J1-J2-J3 (2012)


CAUTION

Density matrix renormalization group (DMRG)
m=6000--10000 states

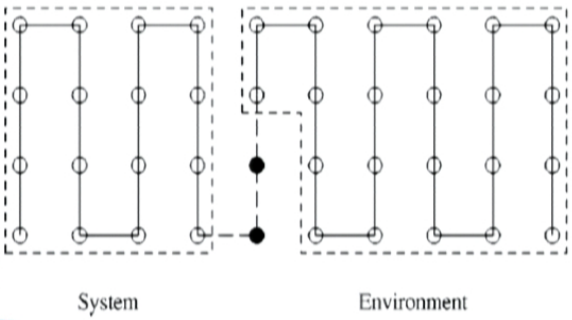
S. R. White 1992-1993

1D DMRG algorithm

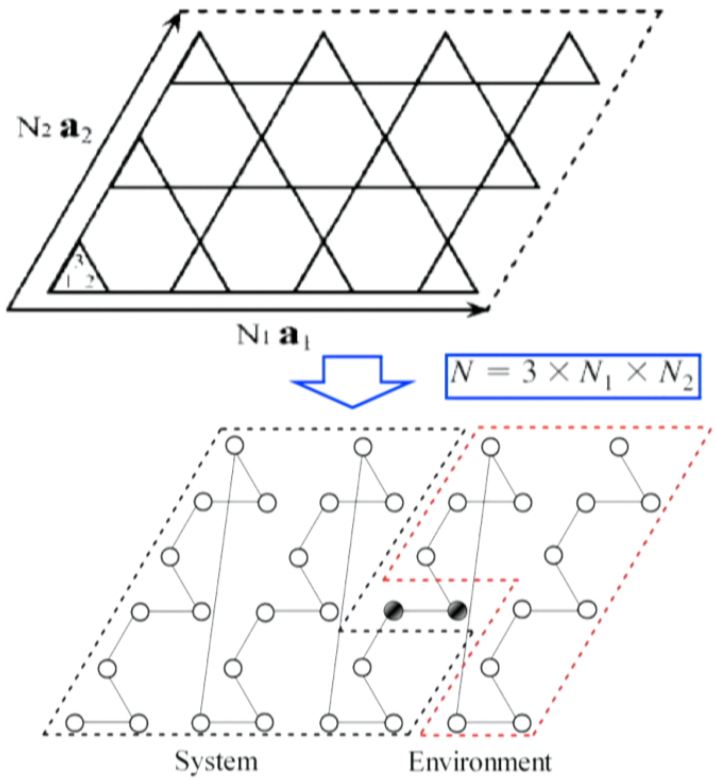
Mapping

Map a 2D lattice onto a 1D

Square lattice



Kagome lattice



DMRG with SU(2) Symmetry, matches to m=16,000---40,000 U(1) states

Reduce the dimension of the diagonalized Hilbert space; Preserve the SU(2) symmetry of the target states; Obtain more accurate results for the same kept optimal states compared to the common case with only U(1) symmetry.

I. P. McCulloch and M. Gulacsi, Europhys. Lett. **57**, 852 (2002);

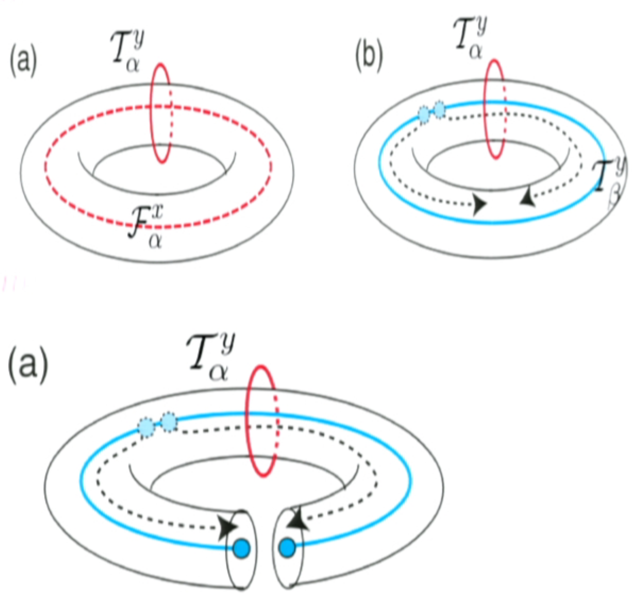
G. Alvarez, arXiv:1003.1919., not generally used for frustrated spin systems

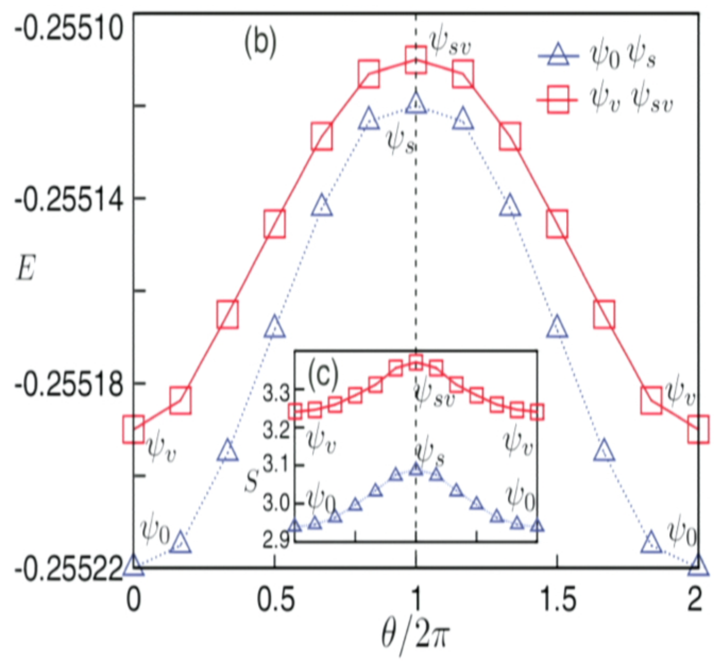
Depenbrock et al., PRL 109, 067201 (2012).

SSG, W. Zhu, DNS, Motrunich,Fisher (2013-2014)

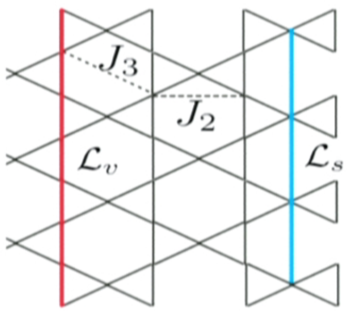
Adiabatic DMRG: by pinning and inserting flux on cylinder, we can access different top. sectors for Z2 SL and CSL

J1-J2-J3 model with large Jz (or small Jxy1)
He, DNS, Chen PRB(2014)

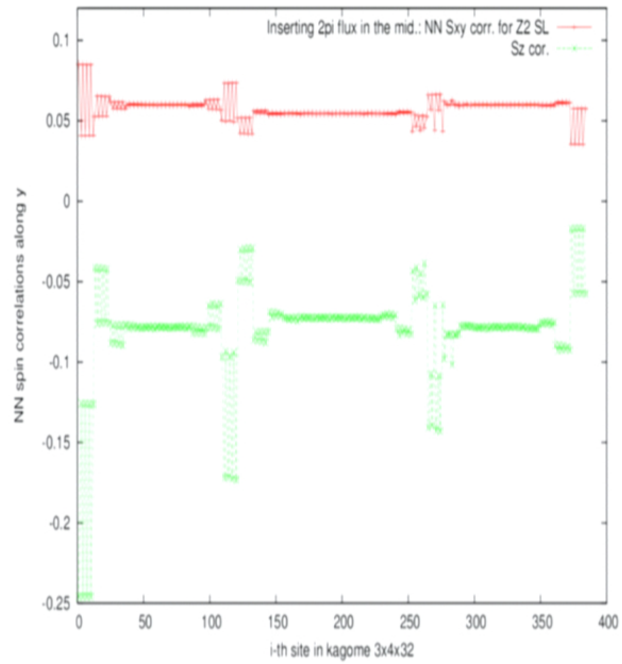




For Z2 SL at large Jz limit (J1-J2-J3 model), we can identify 4 topological sectors

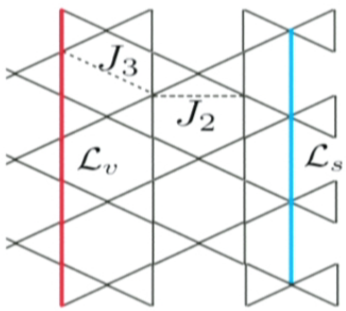


J^{xy}	State	$\mathcal{L}_v$	$\mathcal{L}_s$	E
−0.1	$\lvert\psi_0\rangle$	0.90	0.21	−0.25522
	$\lvert\psi_s\rangle$	0.91	−0.18	−0.25512
	$\lvert\psi_v\rangle$	−0.90	0.13	−0.25519
	$\lvert\psi_{sv}\rangle$	−0.90	−0.12	−0.25511
−0.05	$\lvert\psi_0\rangle$	0.98	0.20	−0.251199
	$\lvert\psi_s\rangle$	0.98	−0.20	−0.251197
	$\lvert\psi_v\rangle$	−0.98	0.12	−0.251194
	$\lvert\psi_{sv}\rangle$	−0.98	−0.11	−0.251192



For Z2 SL at large Jz limit (J1-J2-J3 model),

we can identify 4 topological sectors



J^{xy}	State	$\mathcal{L}_v$	$\mathcal{L}_s$	E
−0.1	$\lvert\psi_0\rangle$	0.90	0.21	−0.25522
	$\lvert\psi_s\rangle$	0.91	−0.18	−0.25512
	$\lvert\psi_v\rangle$	−0.90	0.13	−0.25519
	$\lvert\psi_{sv}\rangle$	−0.90	−0.12	−0.25511
−0.05	$\lvert\psi_0\rangle$	0.98	0.20	−0.251199
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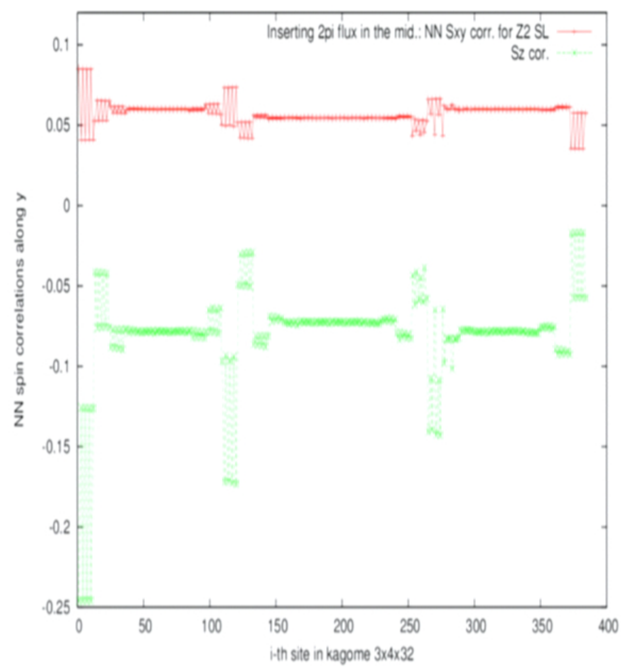
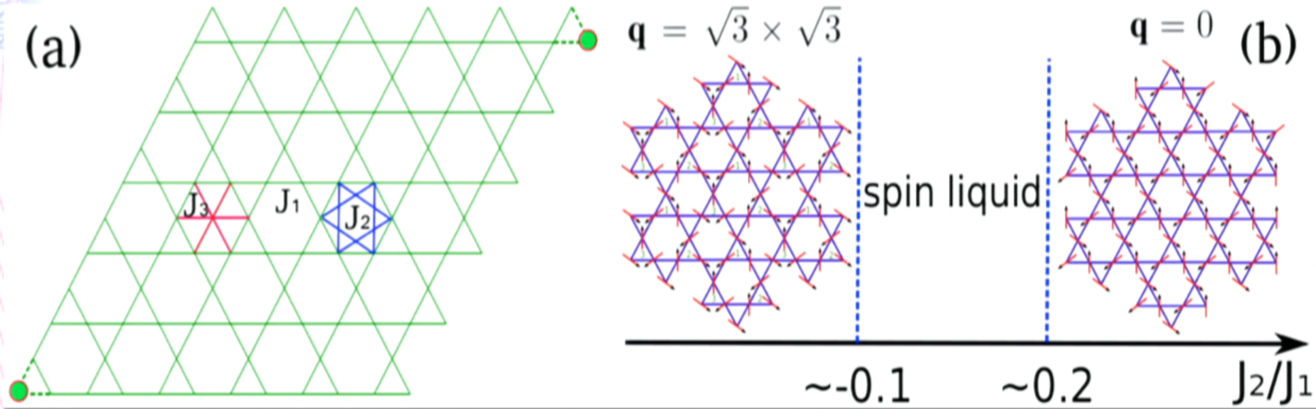
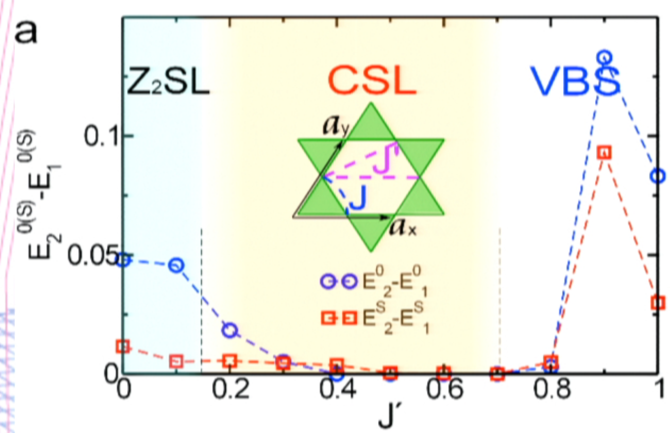


Chiral spin liquid on kagome with J2 ~ J3---avoiding conventional orders





slave-particle mean-field and classical consideration suggested CSL, Messio et al. for Kagome J1-J2-J3 PRB 2011 and PRL 2012

Our work: ArXiv1312.4519 1312.3461

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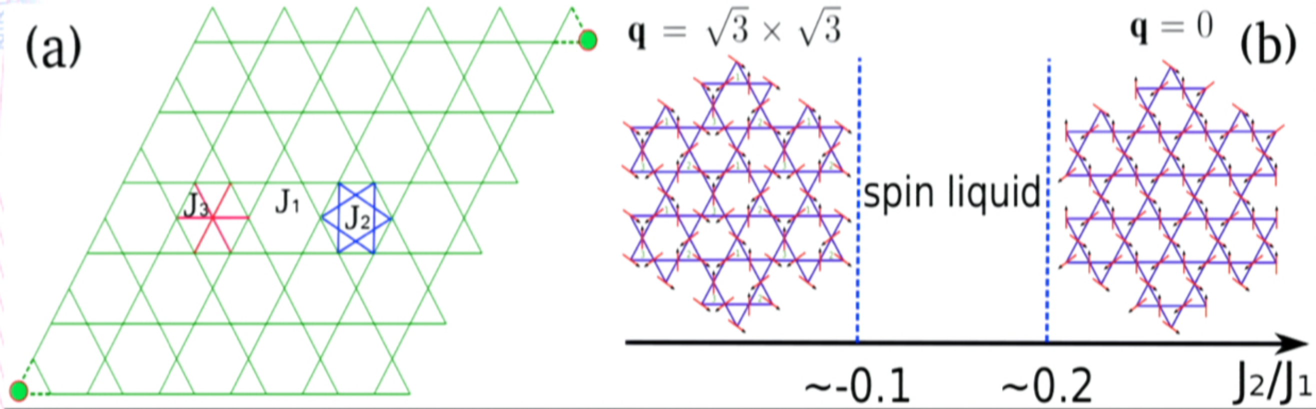

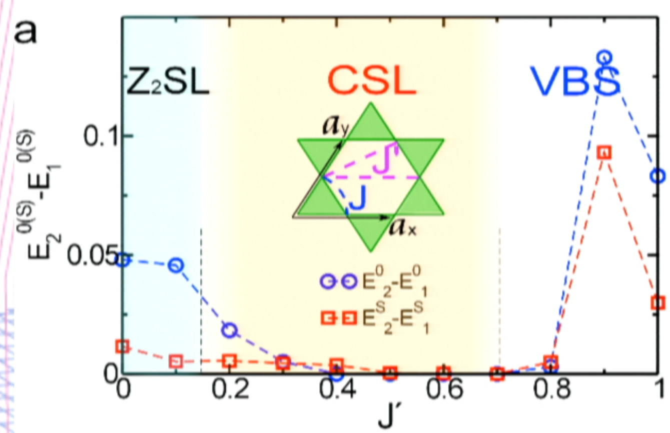

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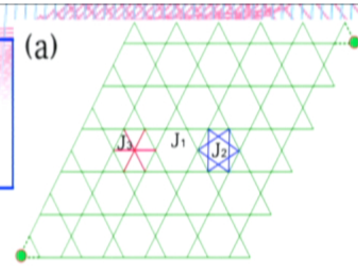
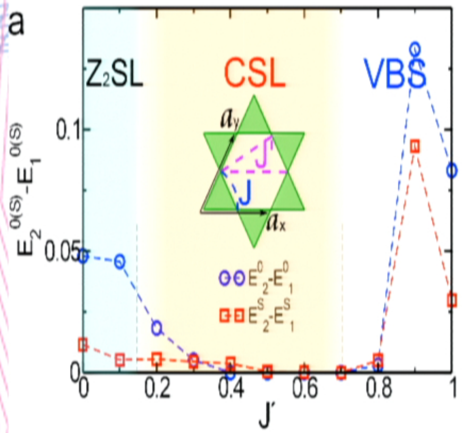
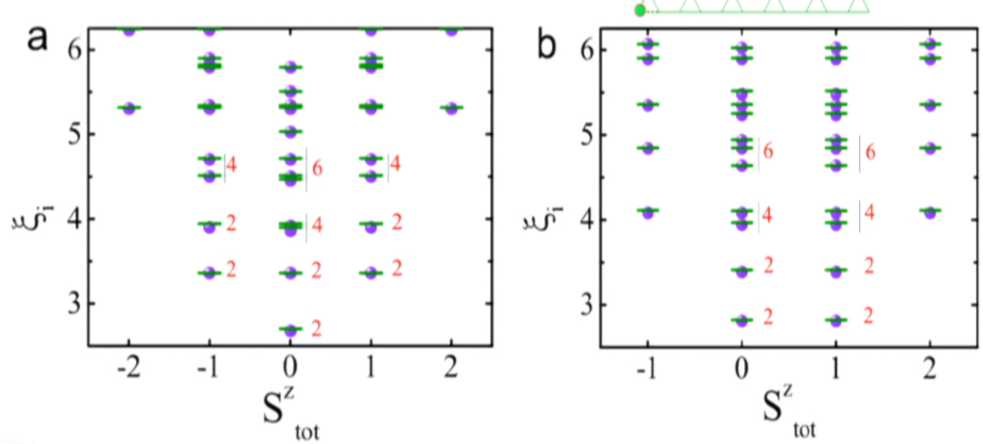
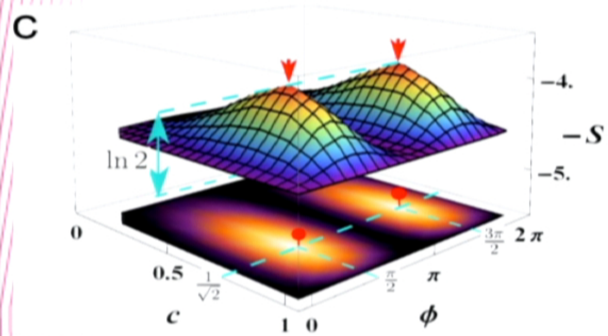
Four fold ground state degeneracy and entanglement spectra









Complex code selects TRS broken MES in DMRG

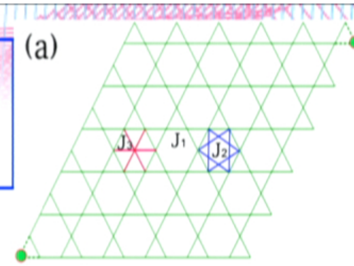
Spectrum of vacuum and spinon sectors---pinning or inserting flux

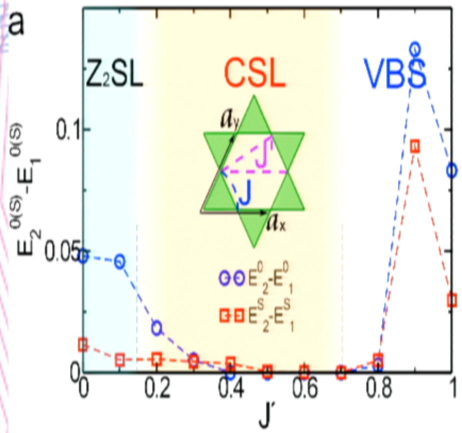
For real code, we get 2 degenerating ground states in each sector

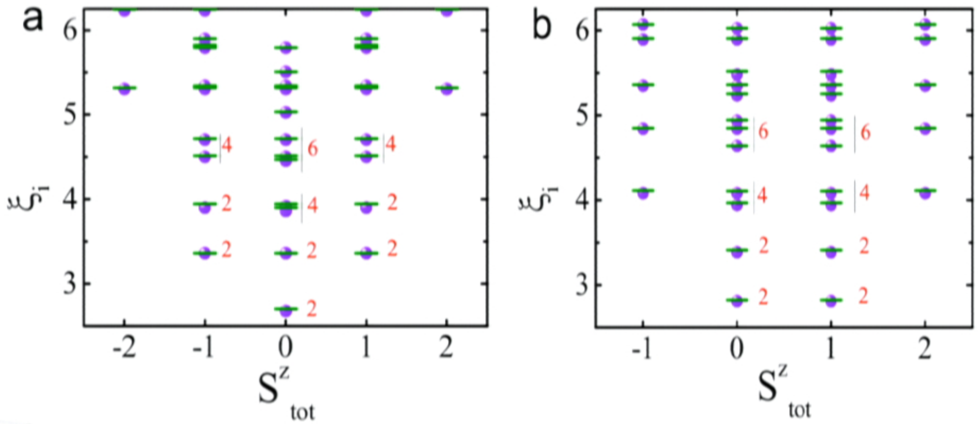
Minimum Entangled State (MES)

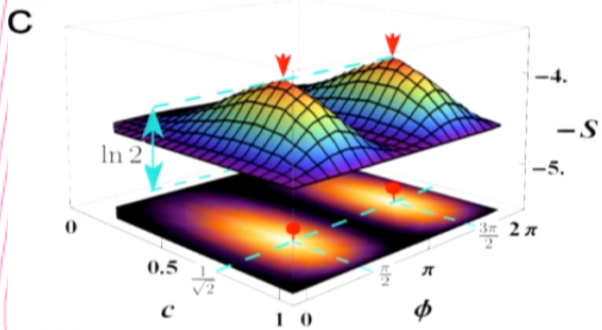
$$\frac{1}{\sqrt{2}}(|\psi_1^0\rangle \pm i|\psi_2^0\rangle).$$

Four fold ground state degeneracy and entanglement spectra









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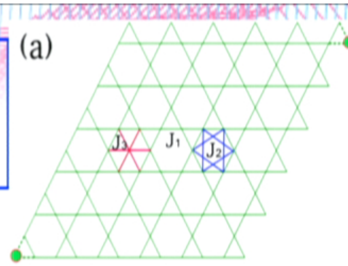

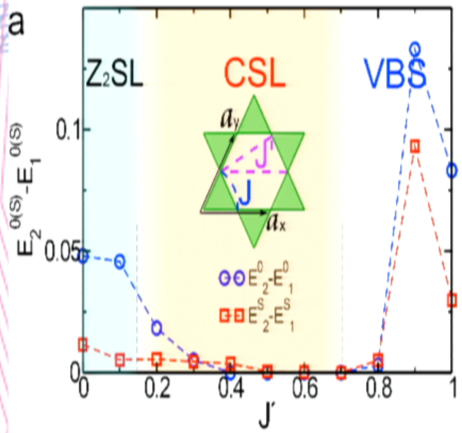

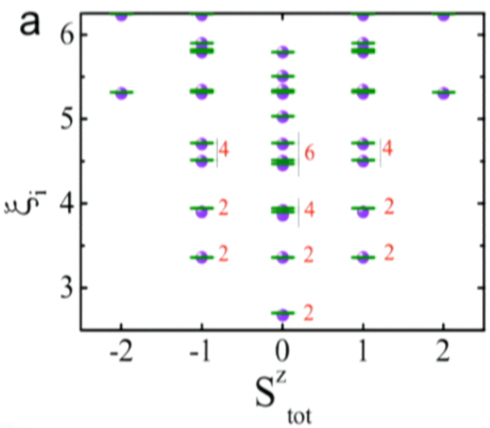

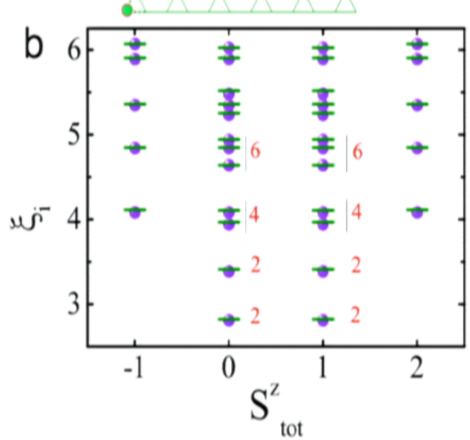

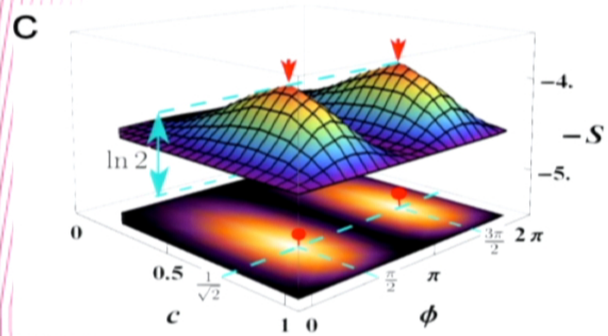

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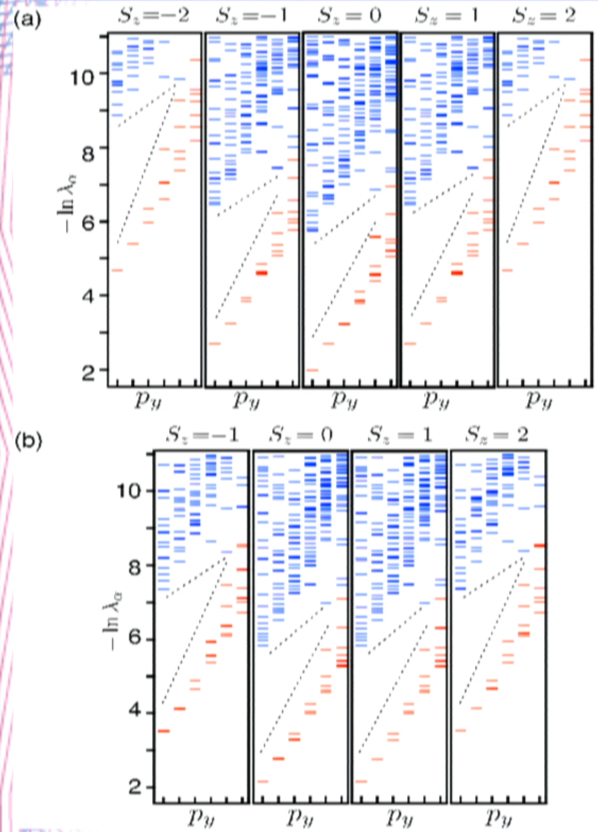
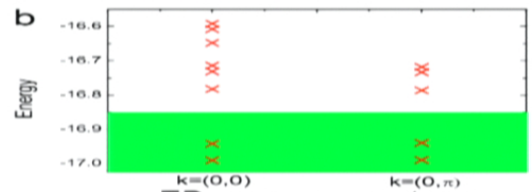
Entanglement spectrum and modular matrix: matching to 1/2 FQHE: J1+J2z=J3z=1



Edge 1-1-2-3-5 pattern for MES, just like 1/2 FQHE



ED energy spectrum

We then obtain the modular matrix

$$\mathcal{S} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} + \frac{10^{-2}}{\sqrt{2}} \begin{pmatrix} -0.42 & -2.2 \\ -1.26 & 0.76 - 0.15i \end{pmatrix} \approx \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},$$

and

$$\mathcal{U} = e^{-i(2\pi/24)} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \times \begin{pmatrix} e^{0.011i} & 0 \\ 0 & e^{-0.006i} \end{pmatrix} \approx e^{-i(2\pi/24)} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}.$$

X.G.Wen 1990, Y. Zhang et al 2012
Li, Haldane 2008
Cincio &Vidal2013, Zhu et al. 2013

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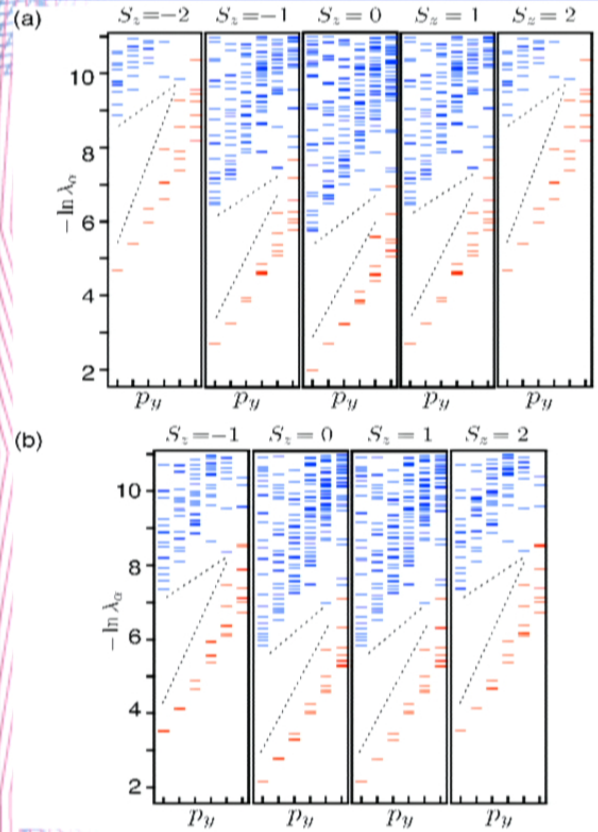

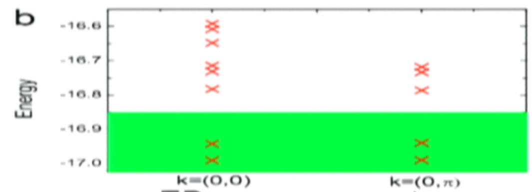

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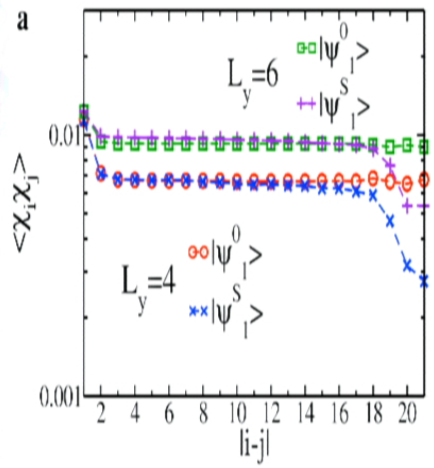
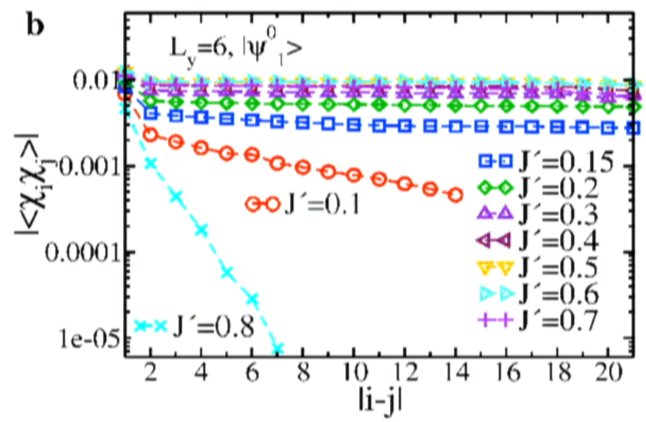
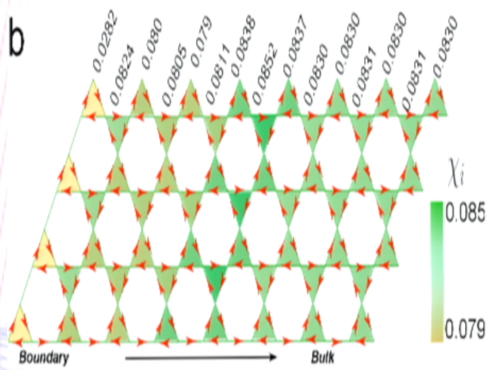
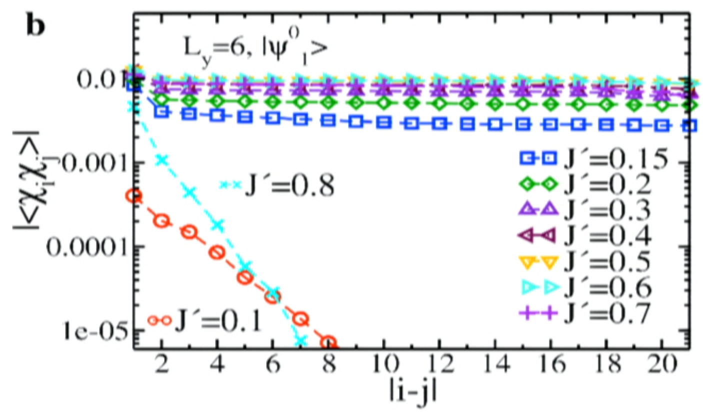
Long-range chiral order and correlation



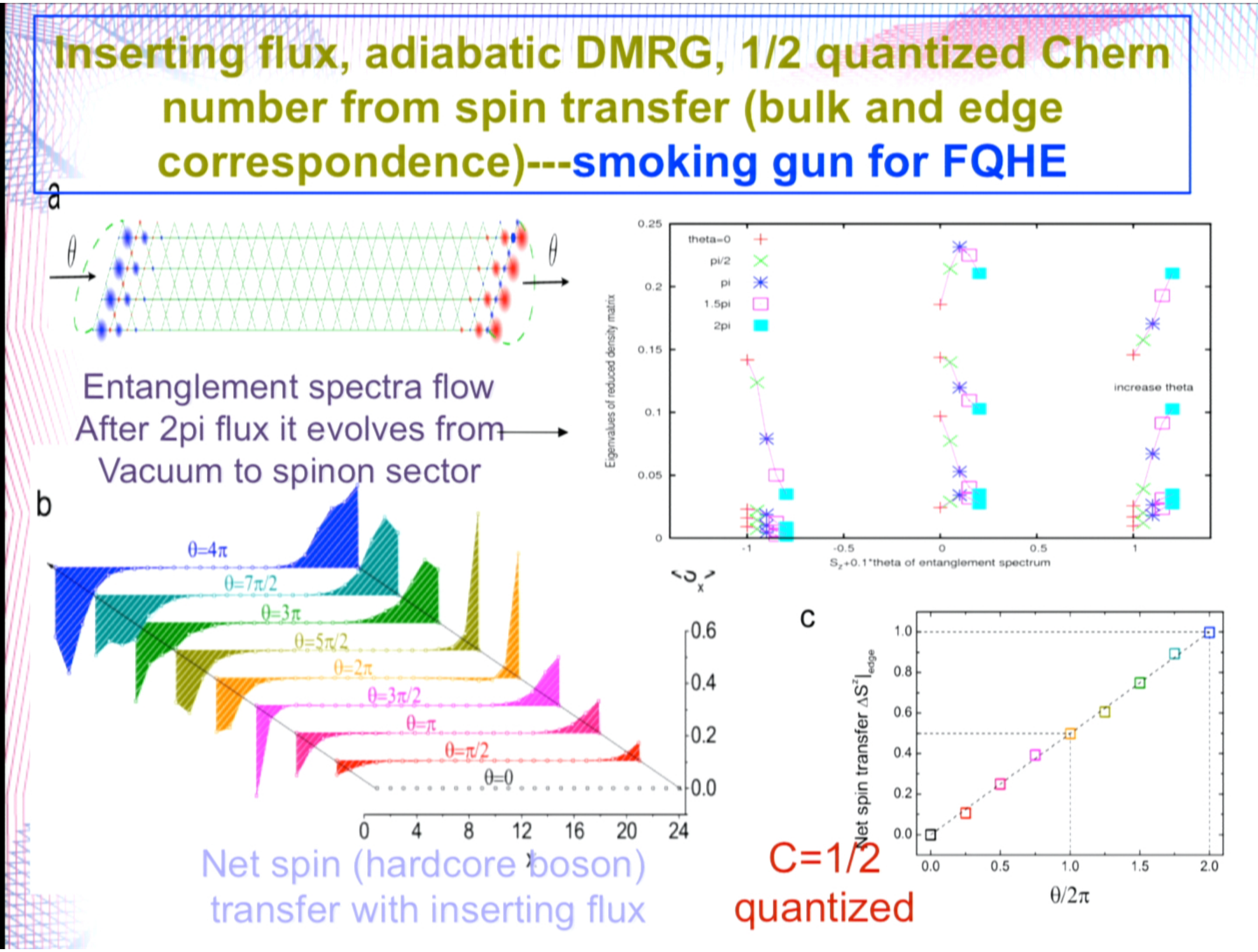


m=28000
U1 states

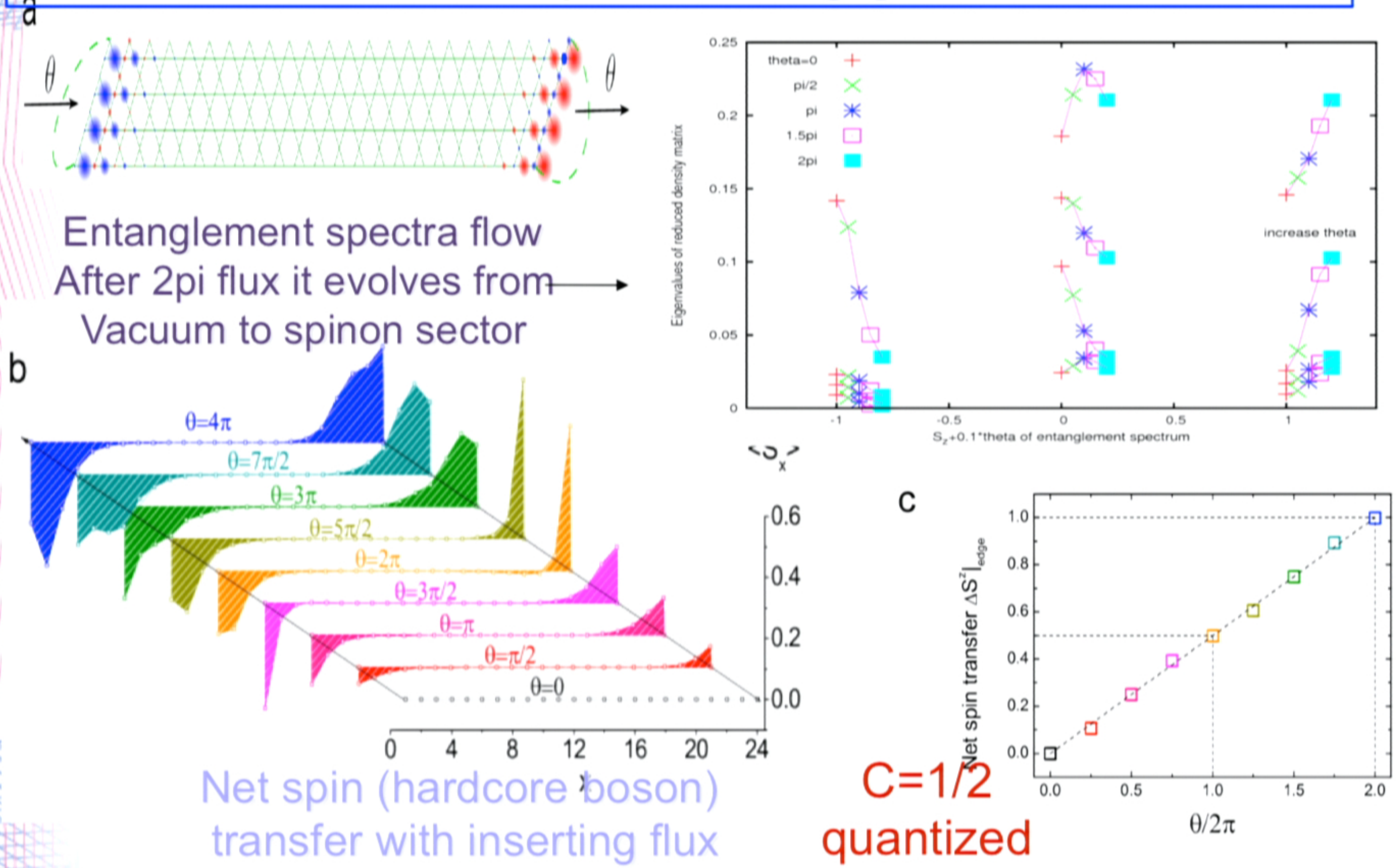




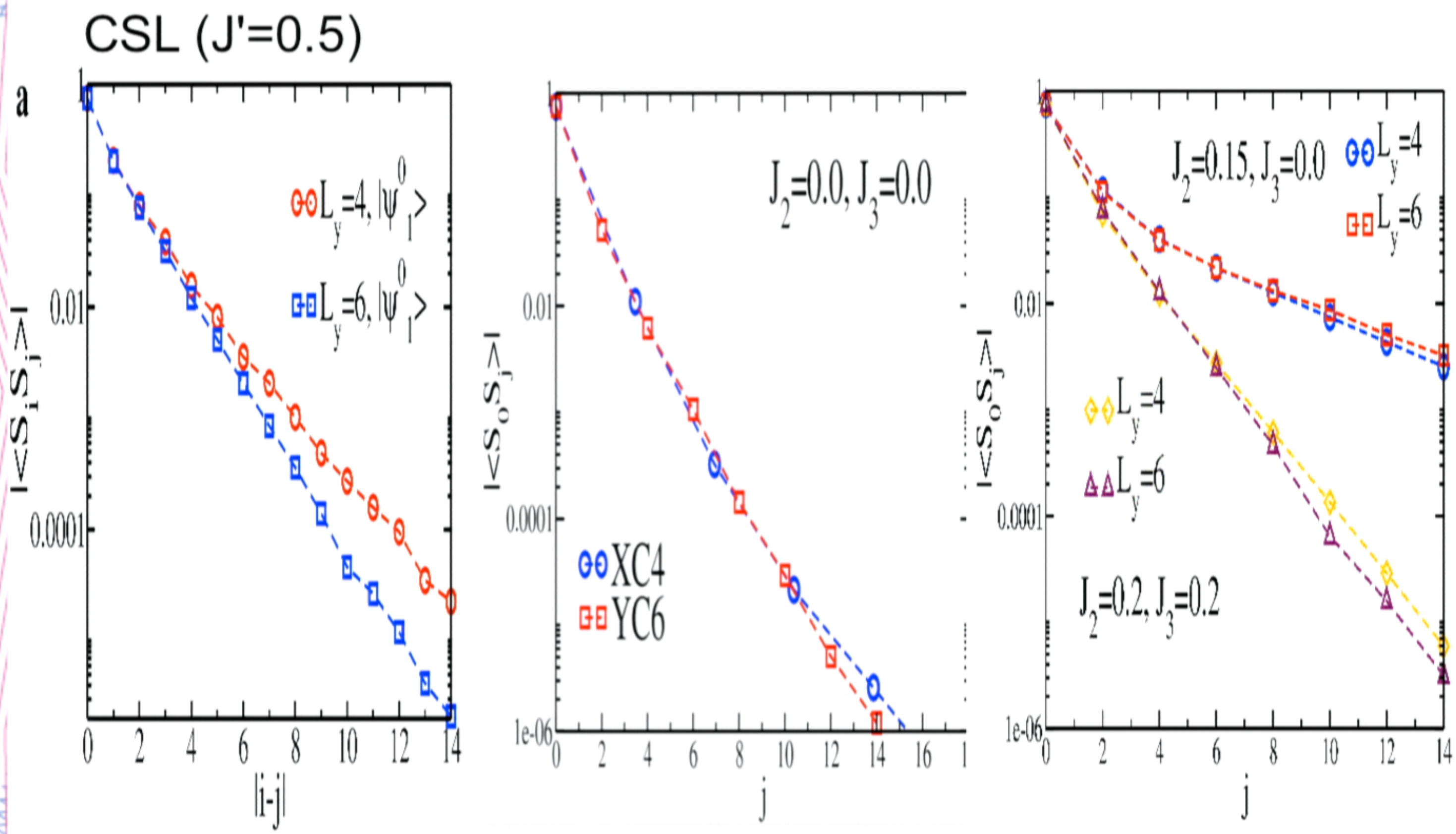
Convergence enhances chiral order near J'=0.1
(m=28000 vs 12000), X=(S1xS2)*S3


Inserting flux, adiabatic DMRG, 1/2 quantized Chern number from spin transfer (bulk and edge correspondence)---smoking gun for FQHE
a
θ
θ
Entanglement spectra flow
After 2pi flux it evolves from
Vacuum to spinon sector
theta=0
pi/2
pi
1.5pi
2pi
increase theta
Eigenvalues of reduced density matrix
S_z+0.1*theta of entanglement spectrum
b
θ=4π
θ=7π/2
θ=3π
θ=5π/2
θ=2π
θ=3π/2
θ=π
θ=π/2
θ=0
<S_x>
0.6
0.4
0.2
0.0
0
4
8
12
16
20
24
x
Net spin (hardcore boson)
transfer with inserting flux
c
Net spin transfer ΔS^z|_edge
θ/2π
C=1/2
quantized

Inserting flux, adiabatic DMRG, 1/2 quantized Chern number from spin transfer (bulk and edge correspondence)---smoking gun for FQHE



Comparison of the spin correlations with J1 and J1-J2 models



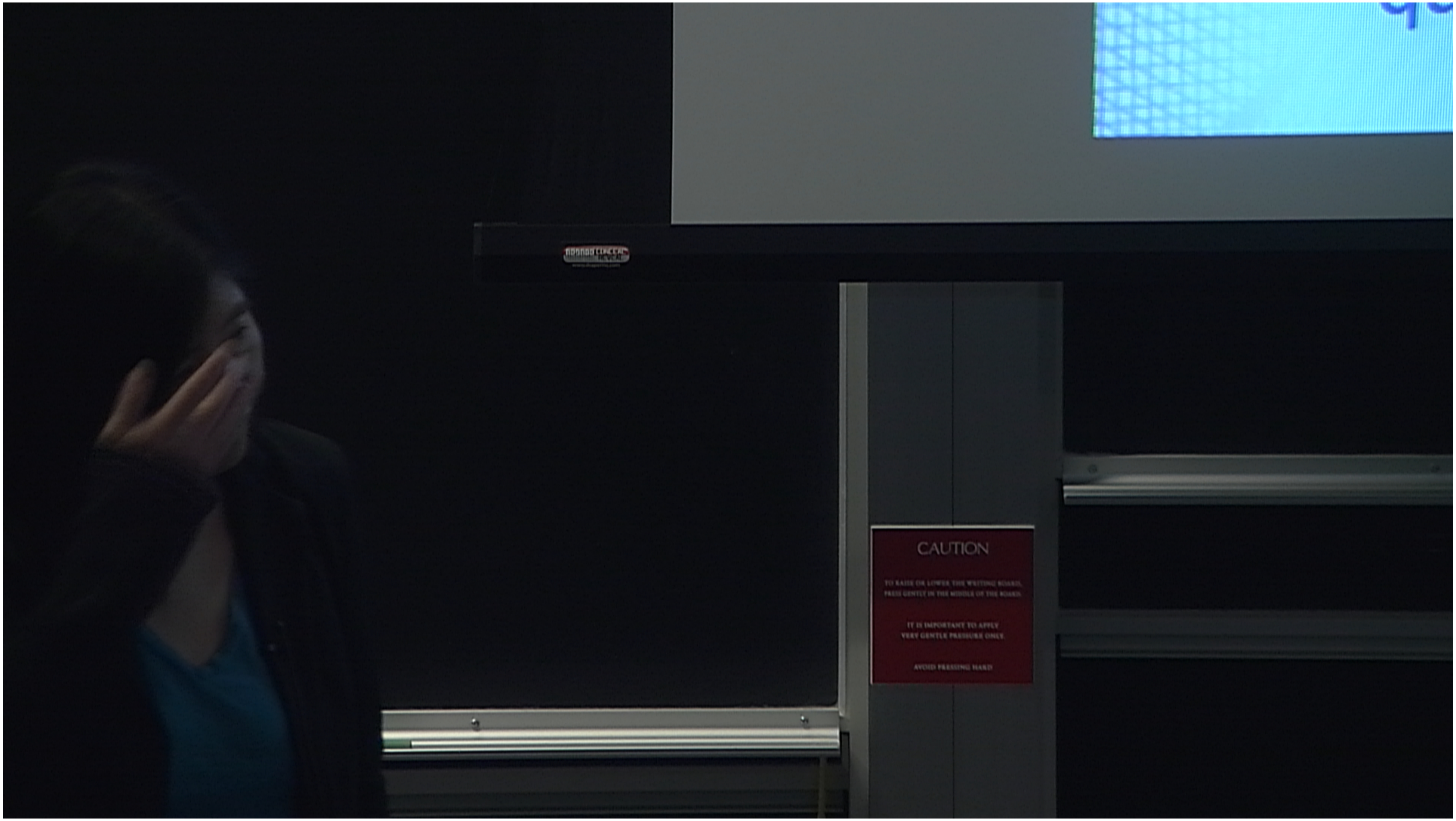
Summary: Chiral Spin Liquid in Kagome J1-J2-J3 Model

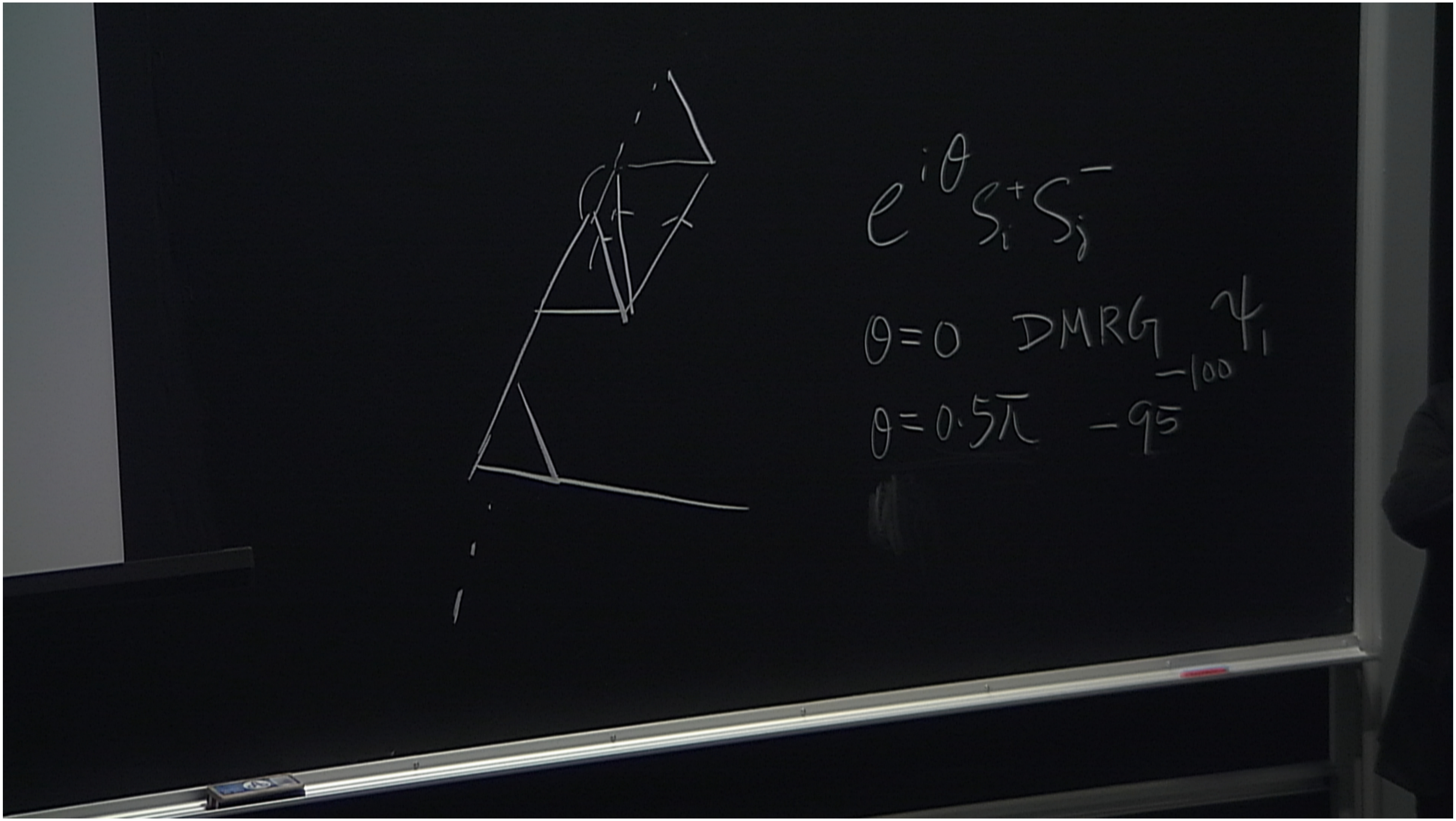
Two topological sectors (through pinning corner spins or flux) , their bulk energies are degenerating.

With in each sector, we find two near degenerating states--- their complex superposition give rise to two chiral states, exhibiting long-range chiral order and spontaneous TRS broken. Chiral states are MESs.

The entanglement spectrum follows the conform edge theory for 1/2 FQHE, with the same modular matrix ---- anyon emerges from such a CSL.

The topological Chern number is observed to be quantized at 1/2, protected by robust excitation gap.


CAUTION
IT IS IMPORTANT TO APPLY
VERY GENTLE PRESSURE ONLY
AVOID PRESSING HARD


$e^{i\theta} S_i^+ S_j^-$
$\theta = 0$ DMRG ψ_1
-100
$\theta = 0.5\pi$ -95