Title: Cluster Polylogarithms for Scattering Amplitudes

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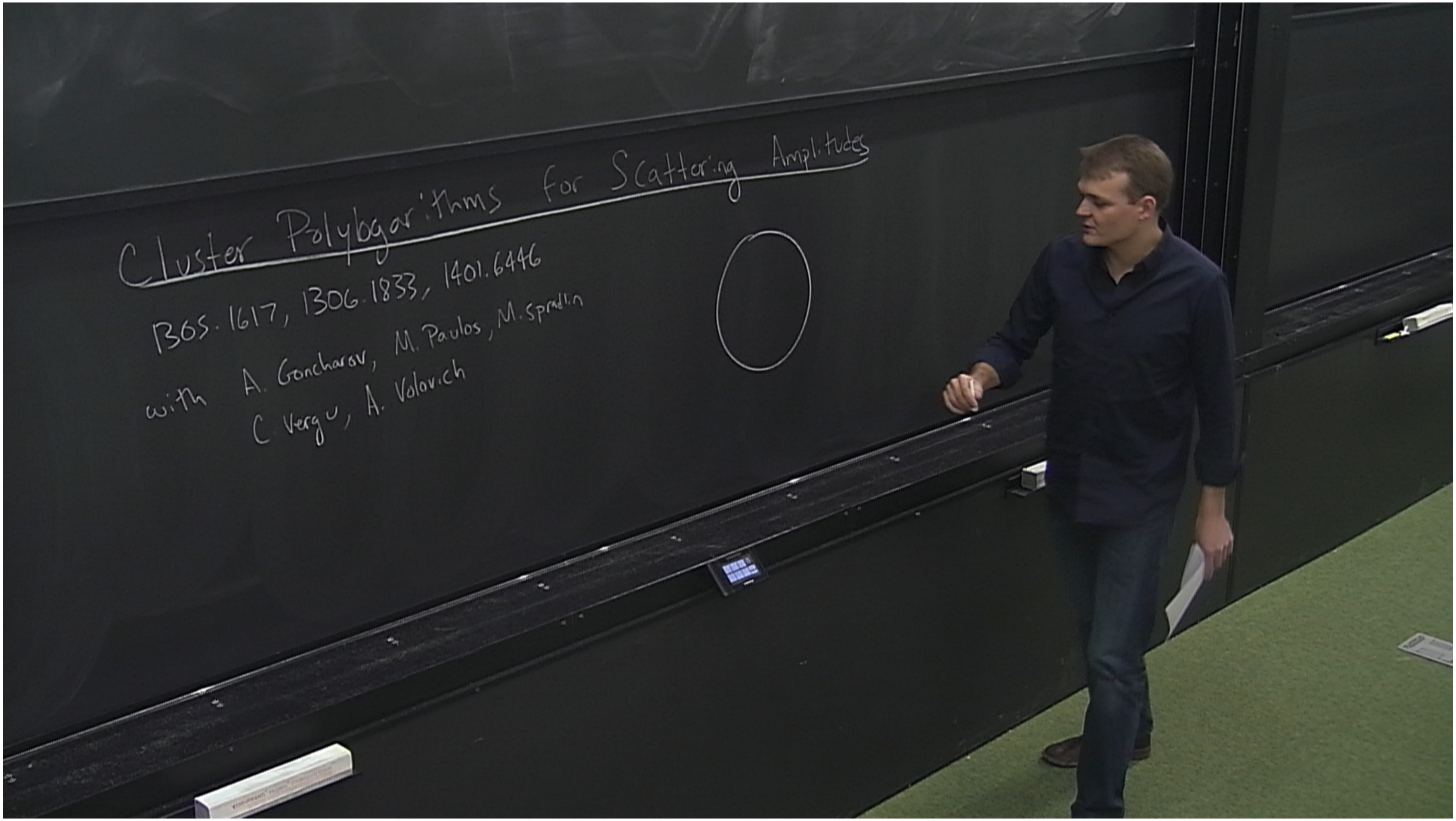
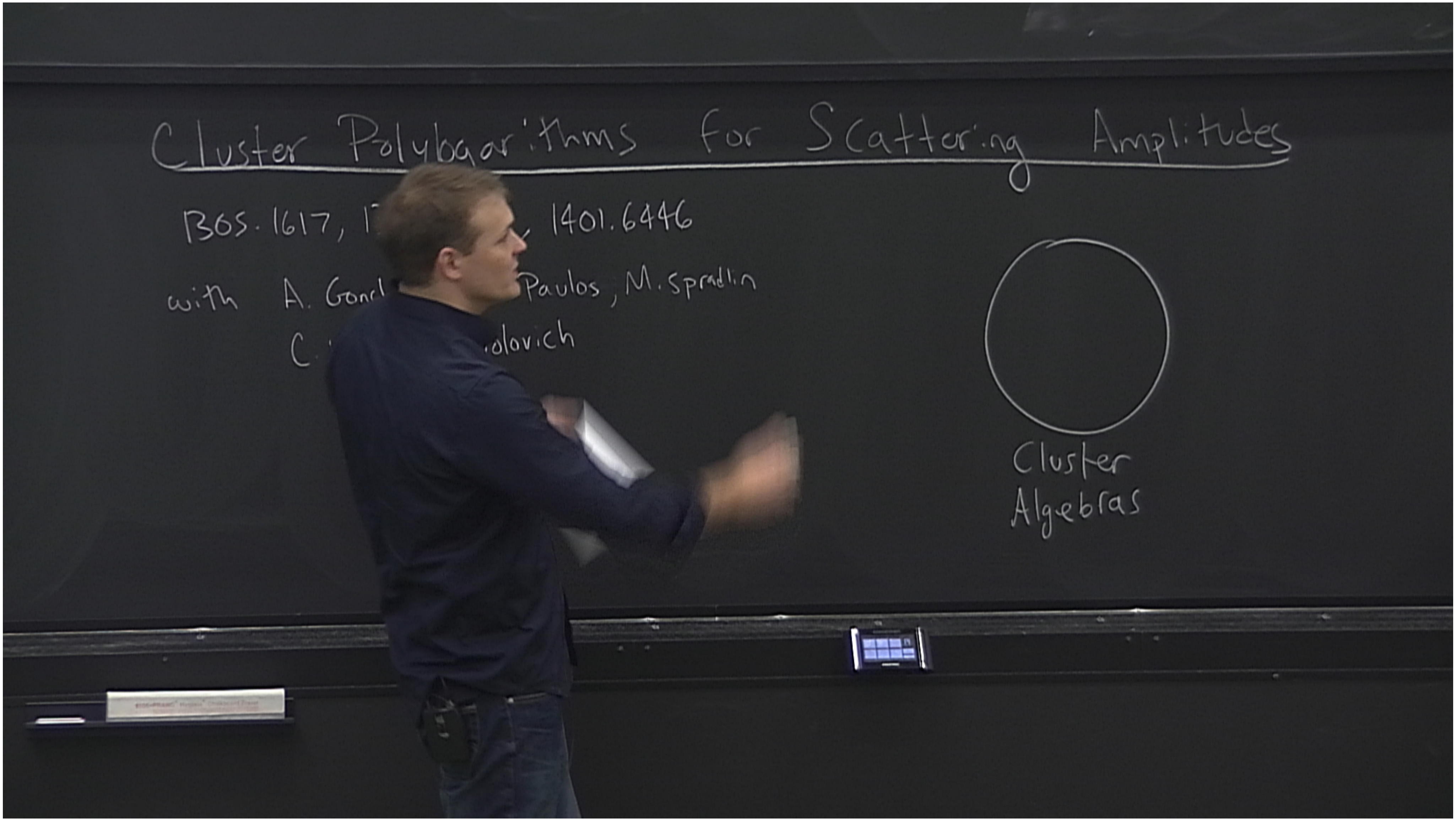
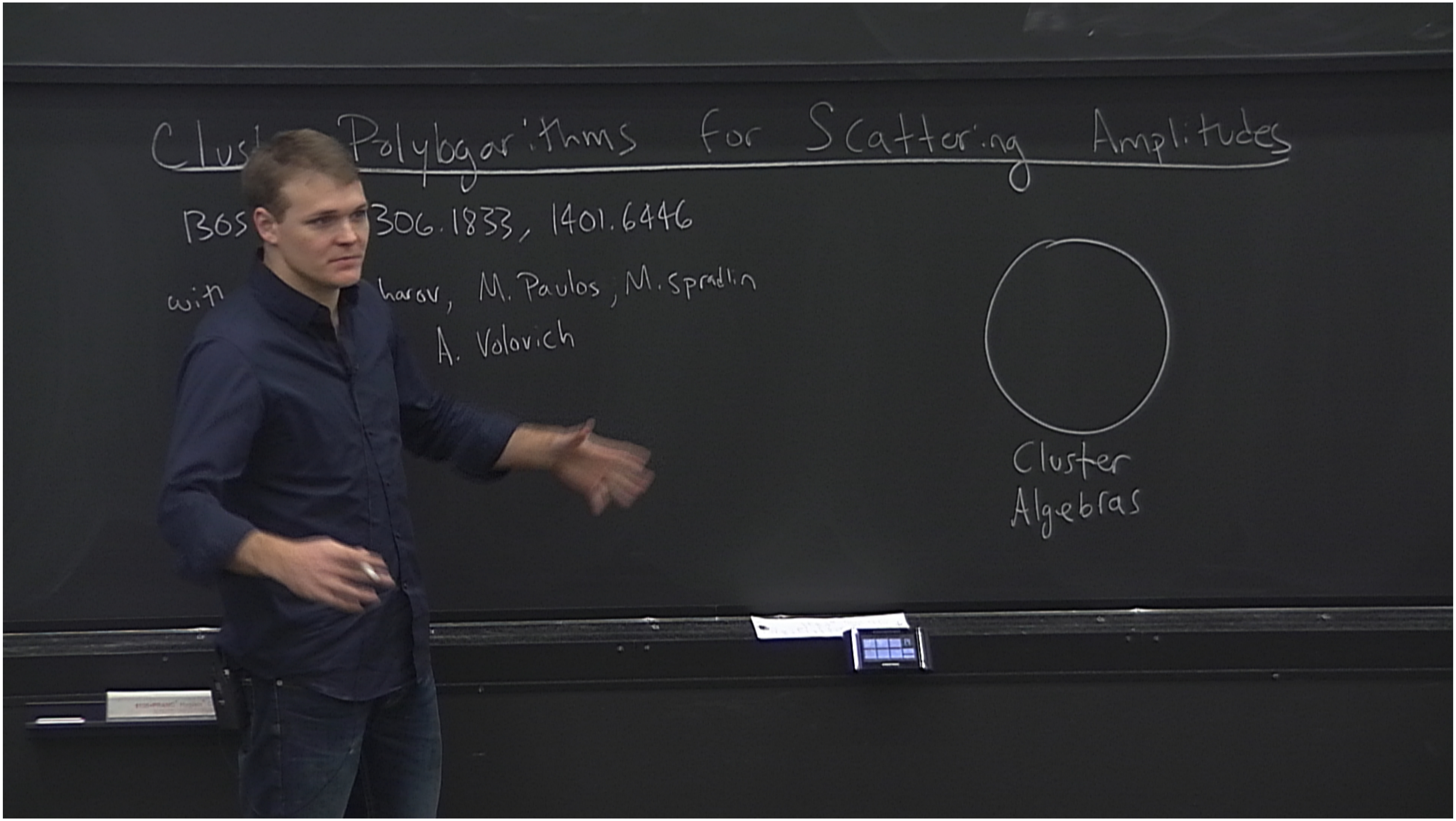
Abstract: Motivated by the cluster structure of two-loop scattering amplitudes in N = 4 Yang-Mills theory we define cluster polylogarithm functions. We find that all such functions of weight 4 are made up of a single simple building block associated to the A2 cluster algebra. Adding the requirement of locality on generalized Stasheff polytopes, we find that these A2 building blocks arrange themselves to form a unique function associated to the A3 cluster algebra. ThisA3 function manifests all of the cluster algebraic structure of the two-loop n-particle MHV amplitudes for all n, and we use it to provide an explicit representation for the most complicated part of the n = 7 amplitude as an example.


Cluster Polylogarithms for Scattering Amplitudes
1305.1617, 1306.1833, 1401.6446


Cluster Polylogarithms for Scattering Amplitudes
1305.1617, 1306.1833, 1401.6446
with A. Goncharov, M. Paulos, M. Spradlin
C. Vergu, A. Volovich


Cluster Polylogarithms for Scattering Amplitudes
1305.1617,
1401.6446
with
Paulos, M. Spradlin
Cluster
Algebras


Polylogarithms for Scattering Amplitudes
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harov, M. Paulos, M. Spradlin
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Cluster
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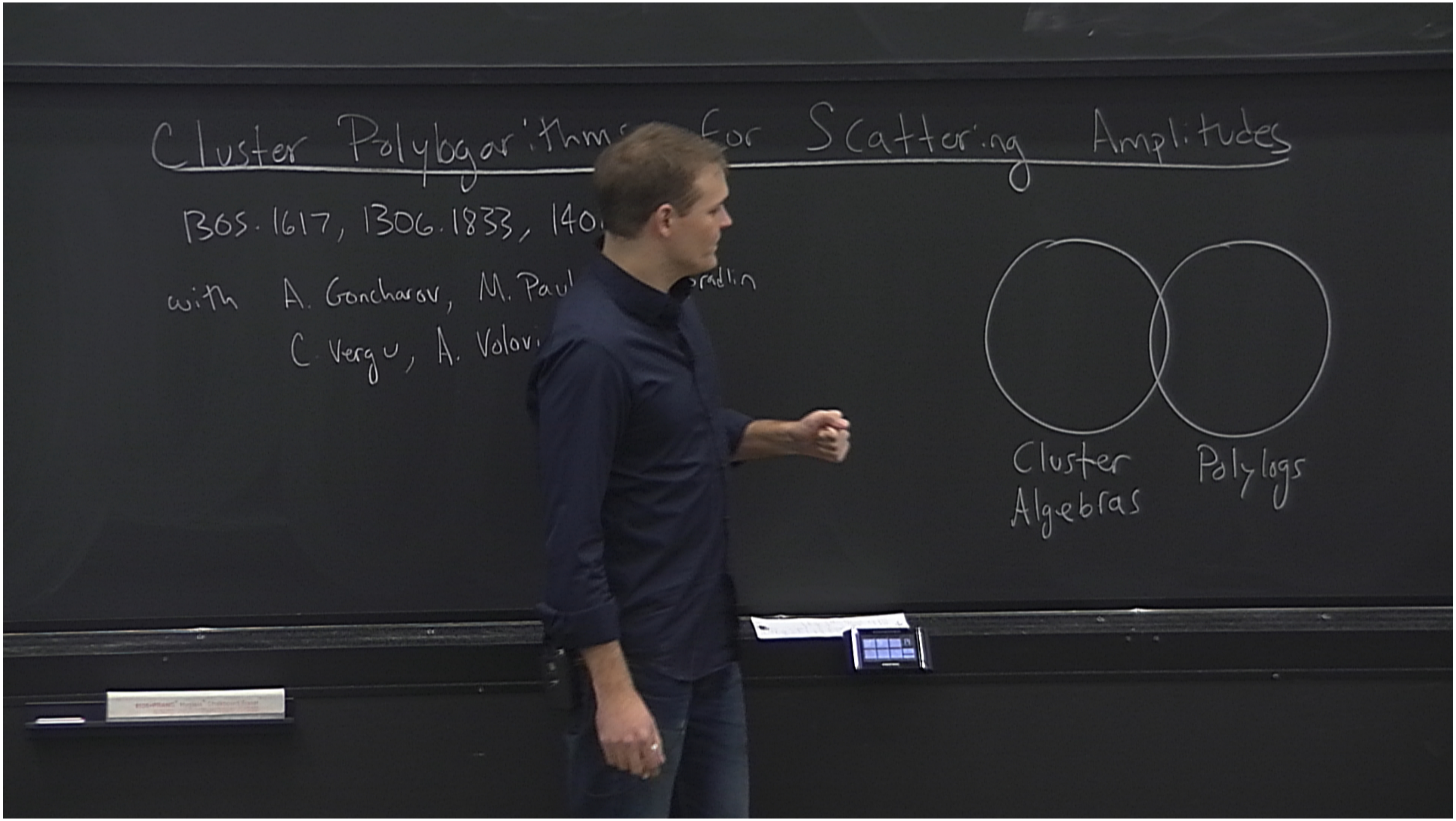
Cluster Polylogarithm
for Scattering Amplitudes
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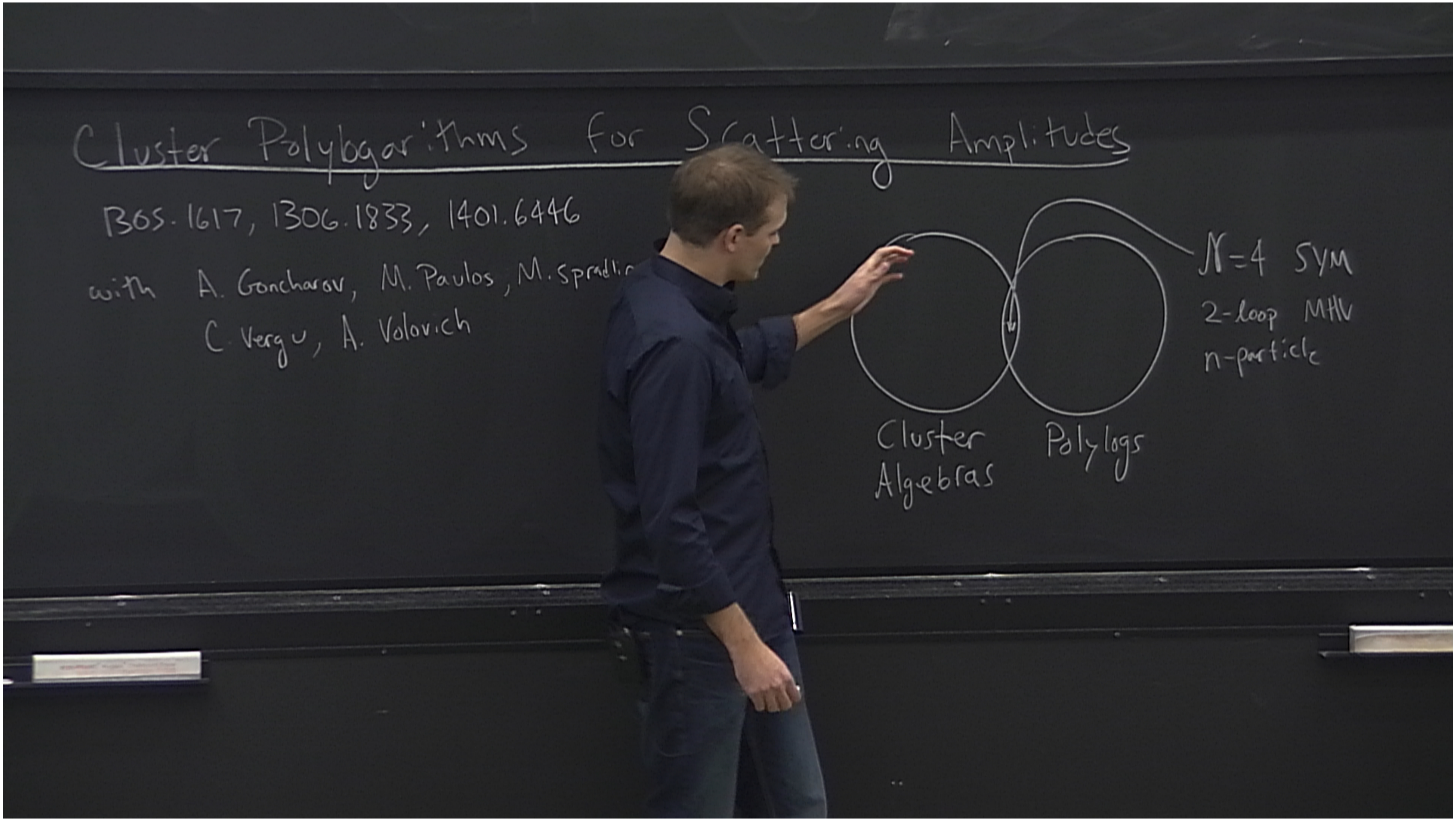
with A. Goncharov, M. Paul
radlin
C. Vergu, A. Volovi
Cluster Algebras
Polylogs


Cluster Polylogarithms for Scattering Amplitudes
1305.1617, 1306.1833, 1401.6446
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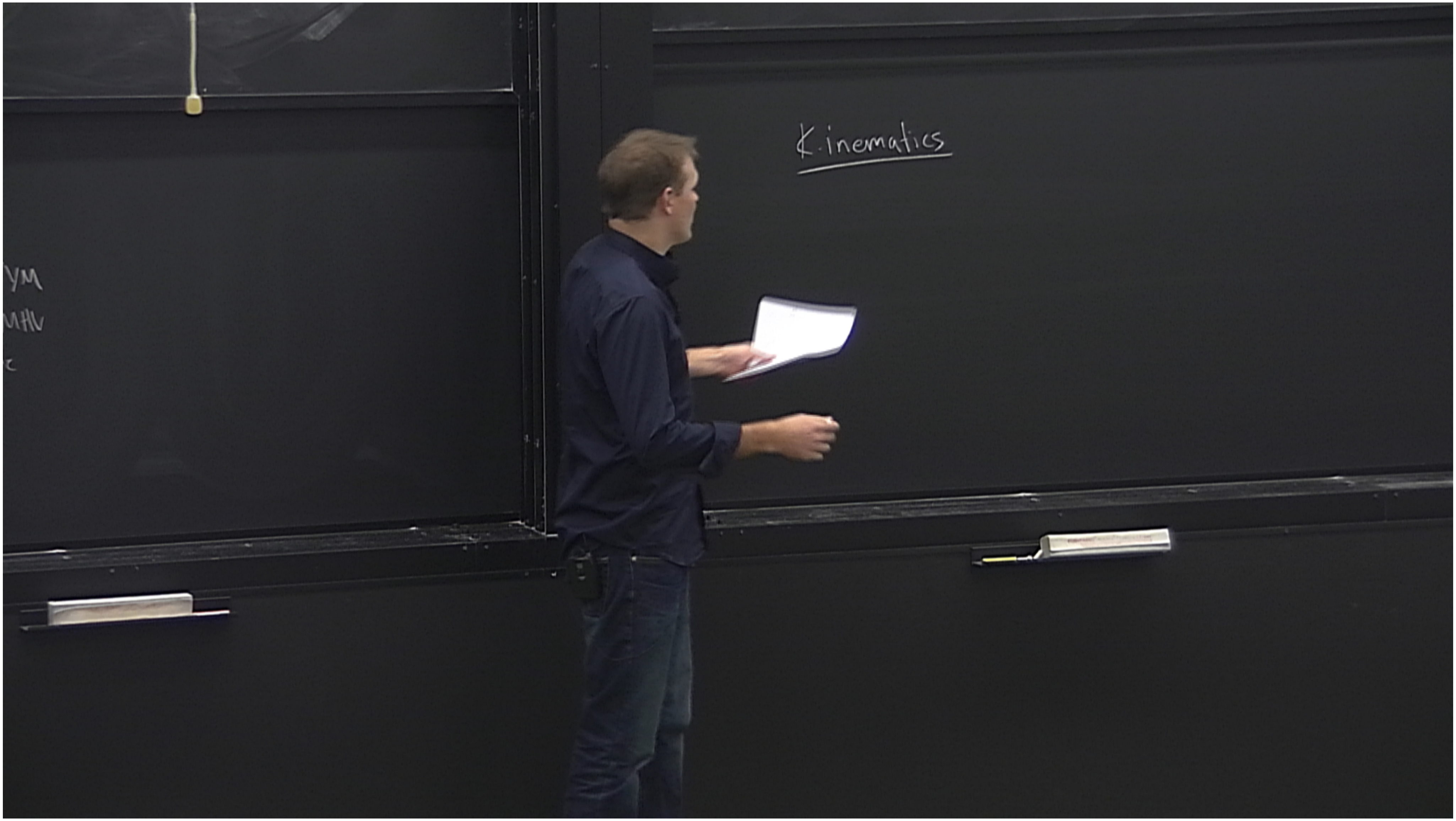
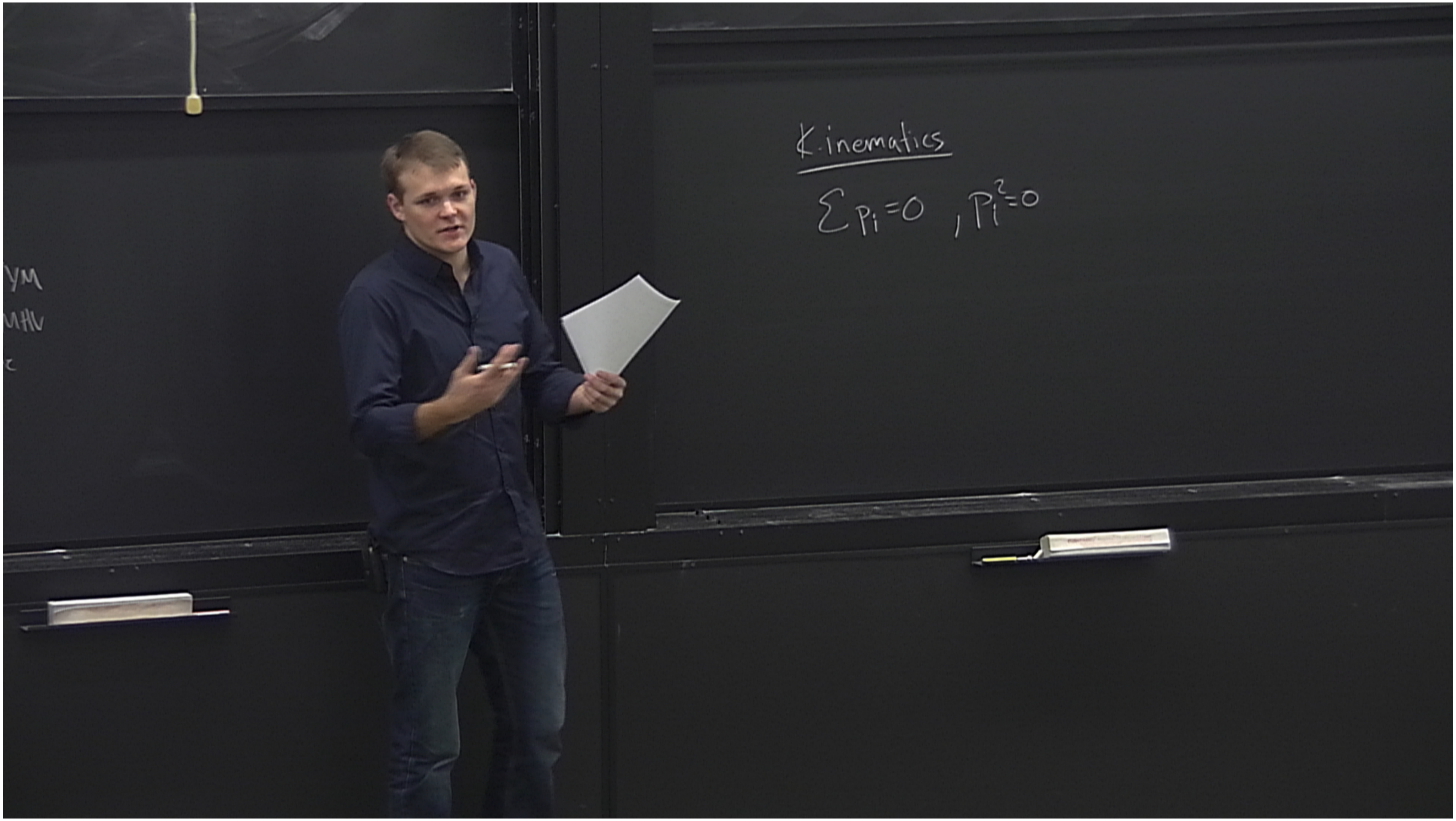
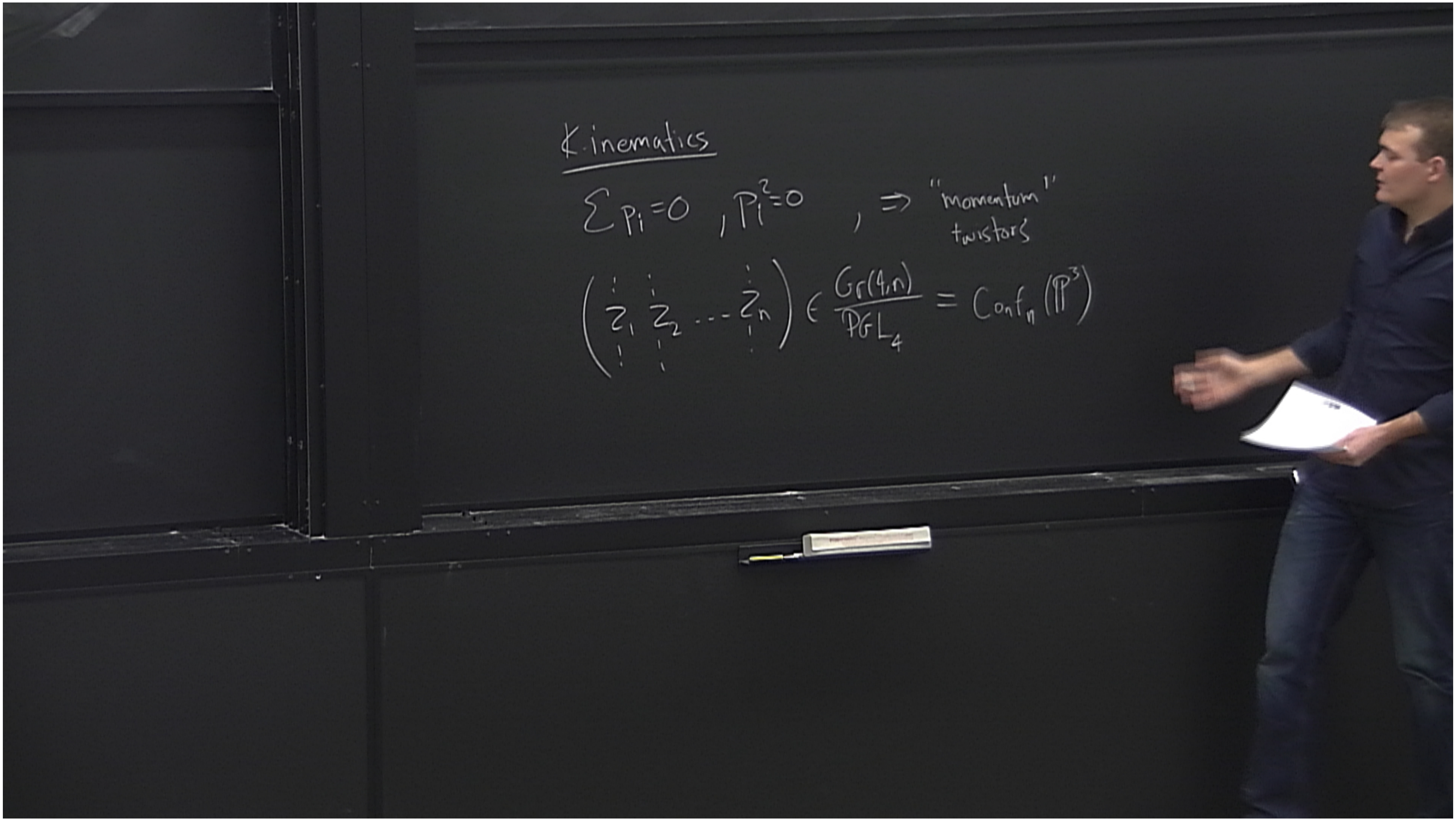
C. Vergu, A. Volovich
N=4 SYM
2-loop MHV
n-particle
Cluster Algebras
Polylogs

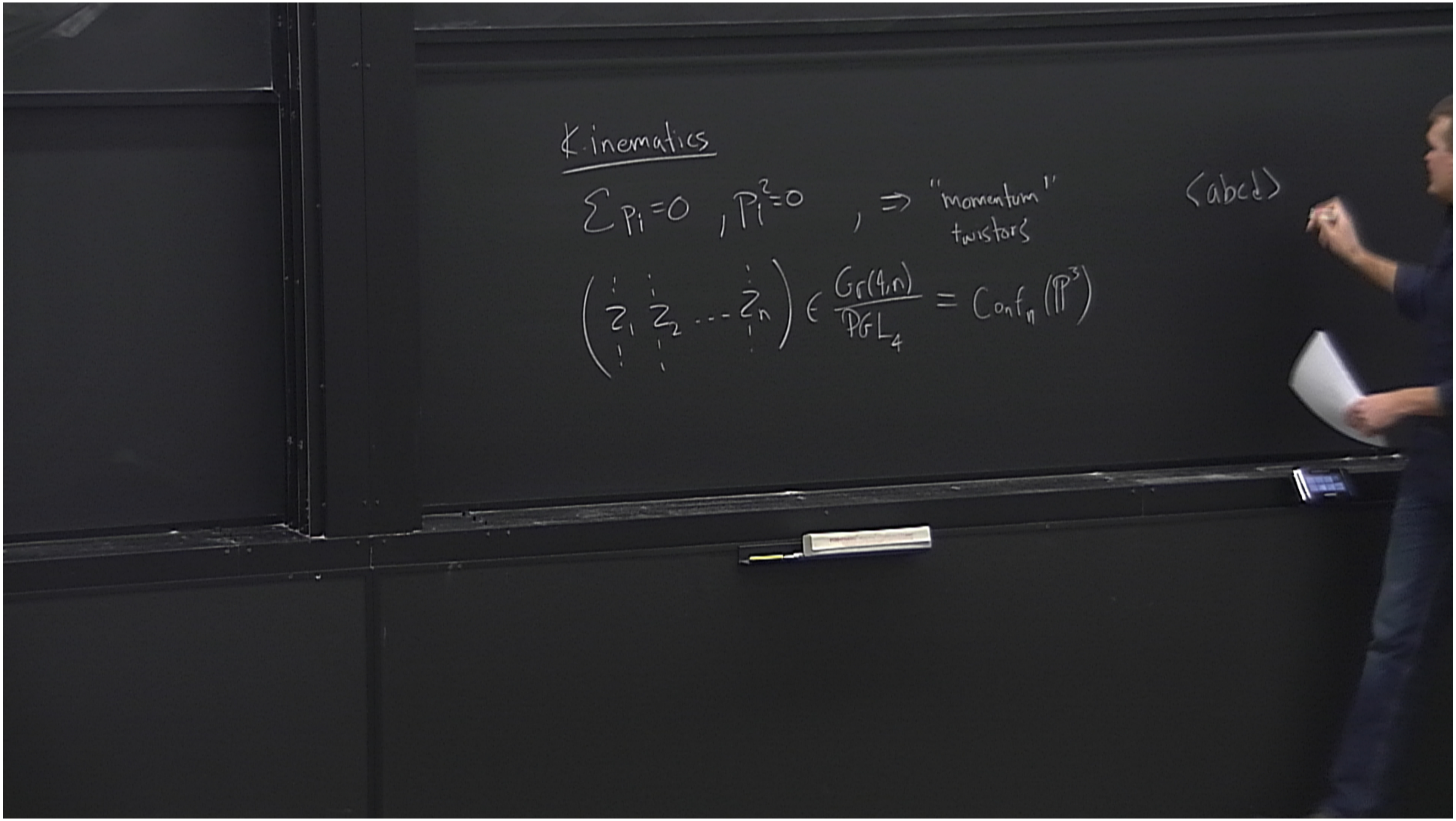

Kinematics
YM
MHV

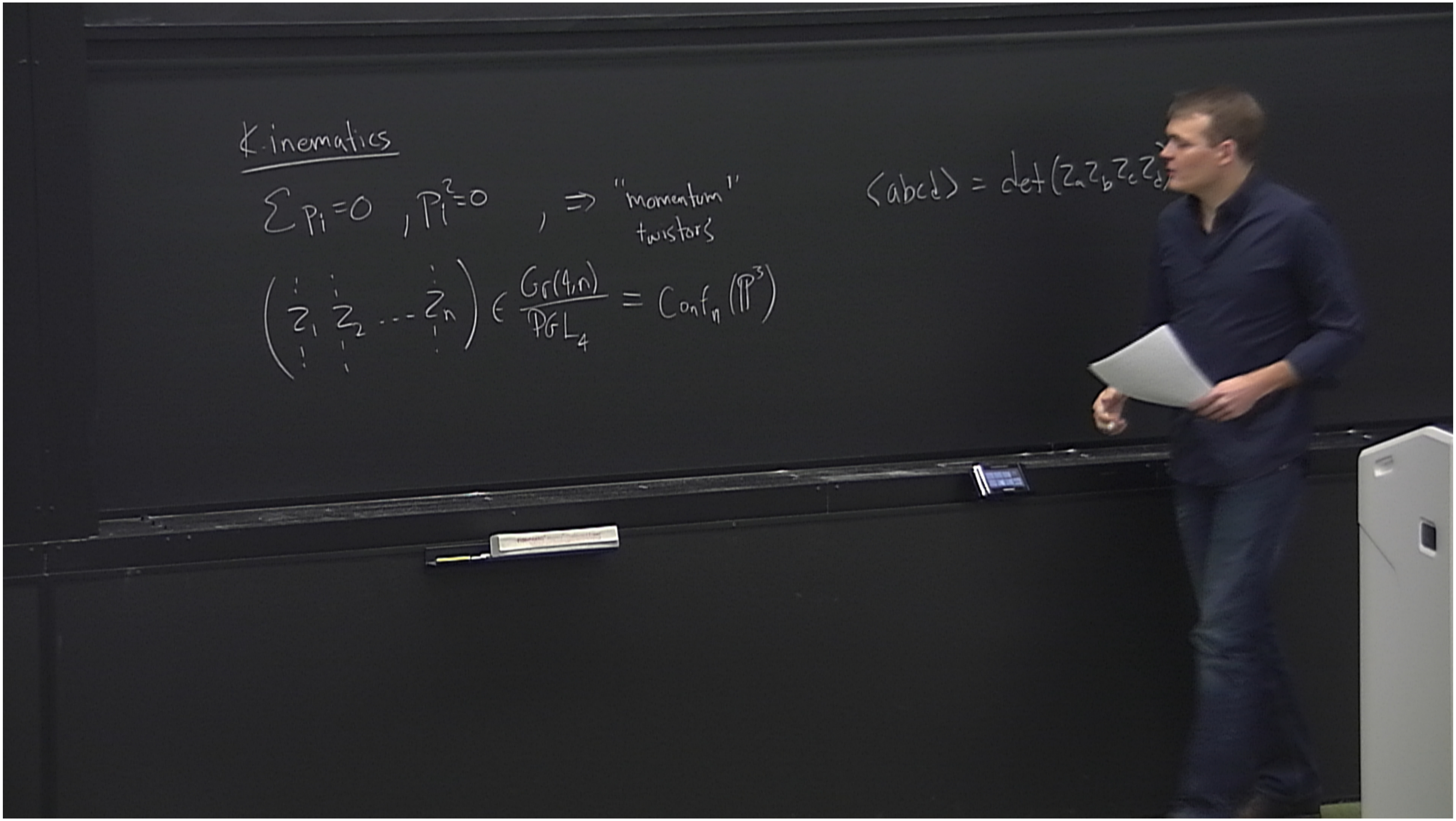

Kinematics
$\sum p_i = 0$, $p_i^2 = 0$
YM
MHV

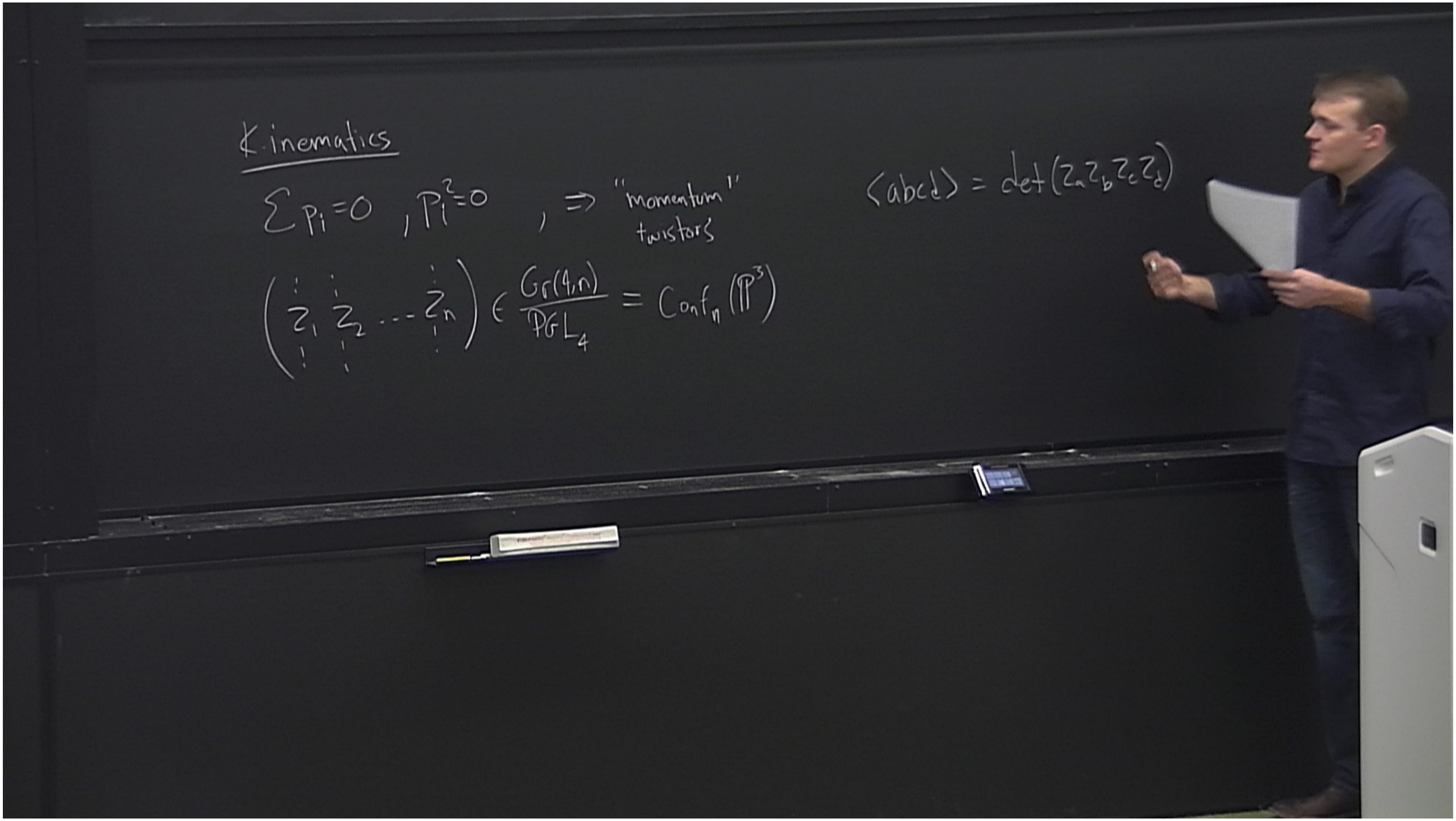

Kinematics
Σ p_i = 0 , p_i^2 = 0 , ⇒ "momentum" twistors
YM
MHV

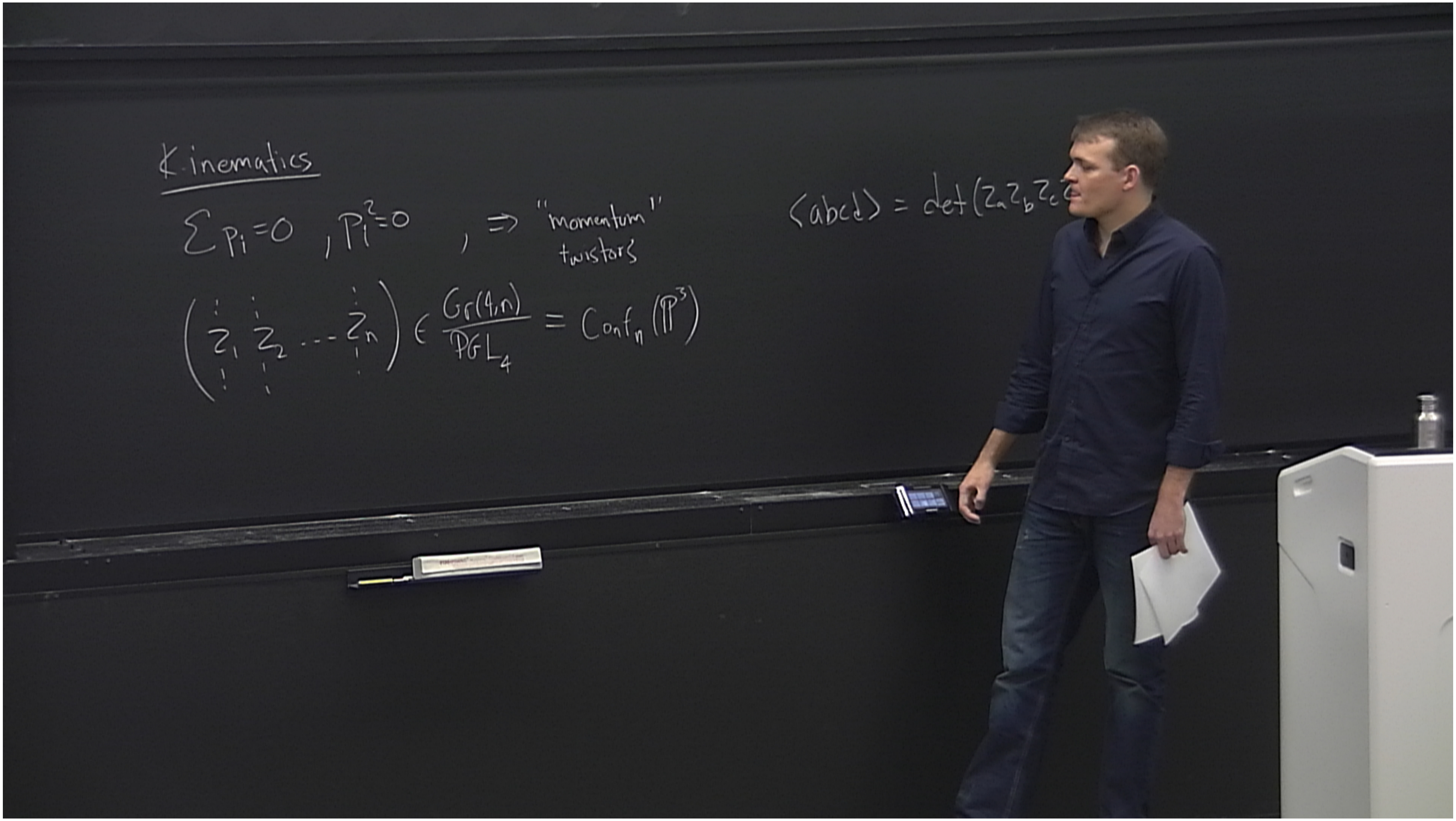

Kinematics
Σ pi = 0 , pi² = 0 , ⇒ "momentum" twistors
(Z1 Z2 ... Zn) ∈ Gr(4,n)
YM
MHV

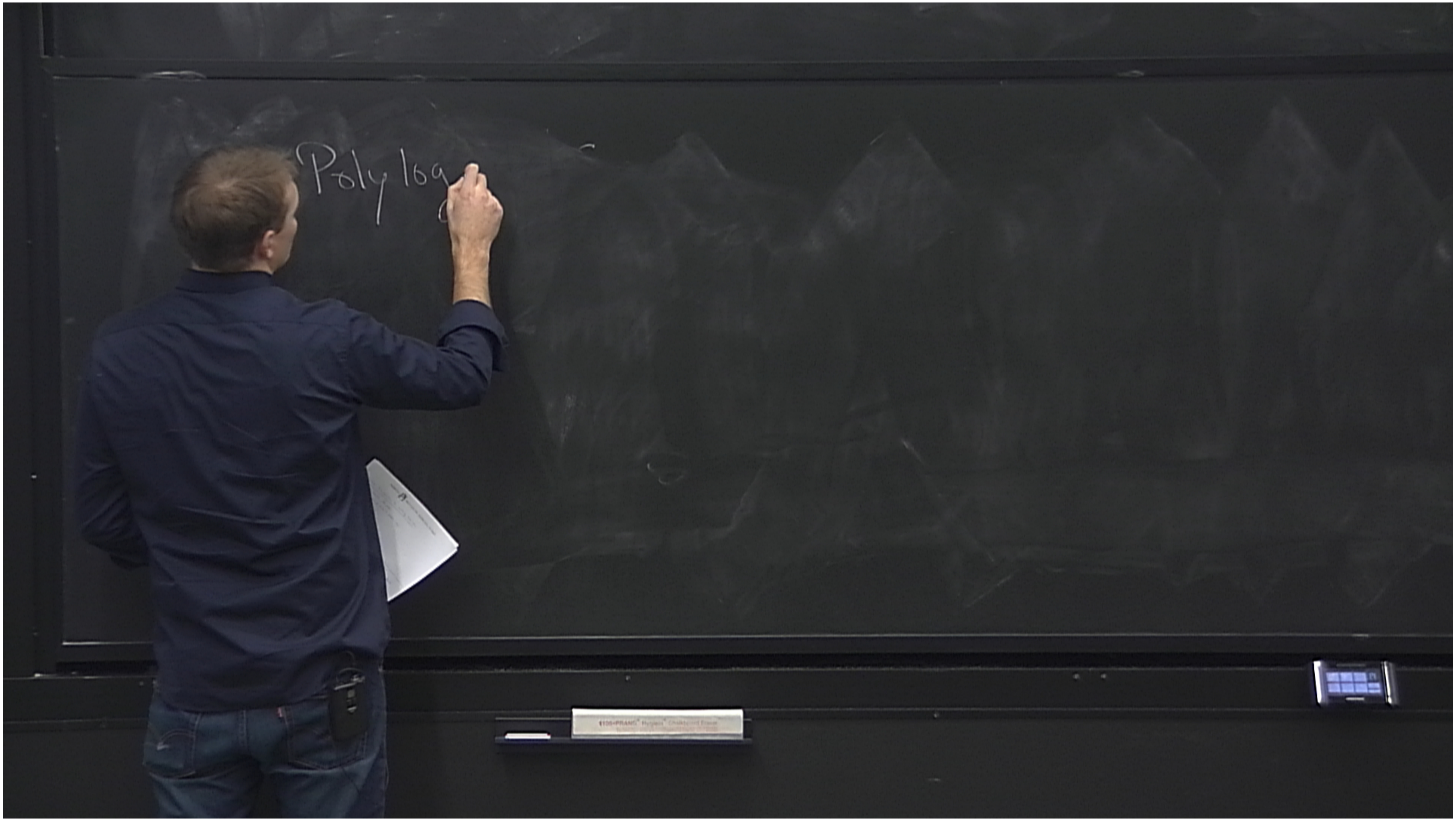

Kinematics
Σ p_i = 0 , p_i^2 = 0 , ⇒ "momentum" twistors
(Z_1 Z_2 ... Z_n) ∈ Gr(4,n)/PGL_4 = Conf_n(P^3)


Kinematics
$\sum p_i = 0$, $p_i^2 = 0$, ⇒ "momentum" twistors
$\begin{pmatrix} \vdots & \vdots & & \vdots \\ Z_1 & Z_2 & \cdots & Z_n \\ \vdots & \vdots & & \vdots \end{pmatrix} \in \frac{Gr(4,n)}{PGL_4} = Conf_n(\mathbb{P}^3)$
$\langle abcd \rangle$

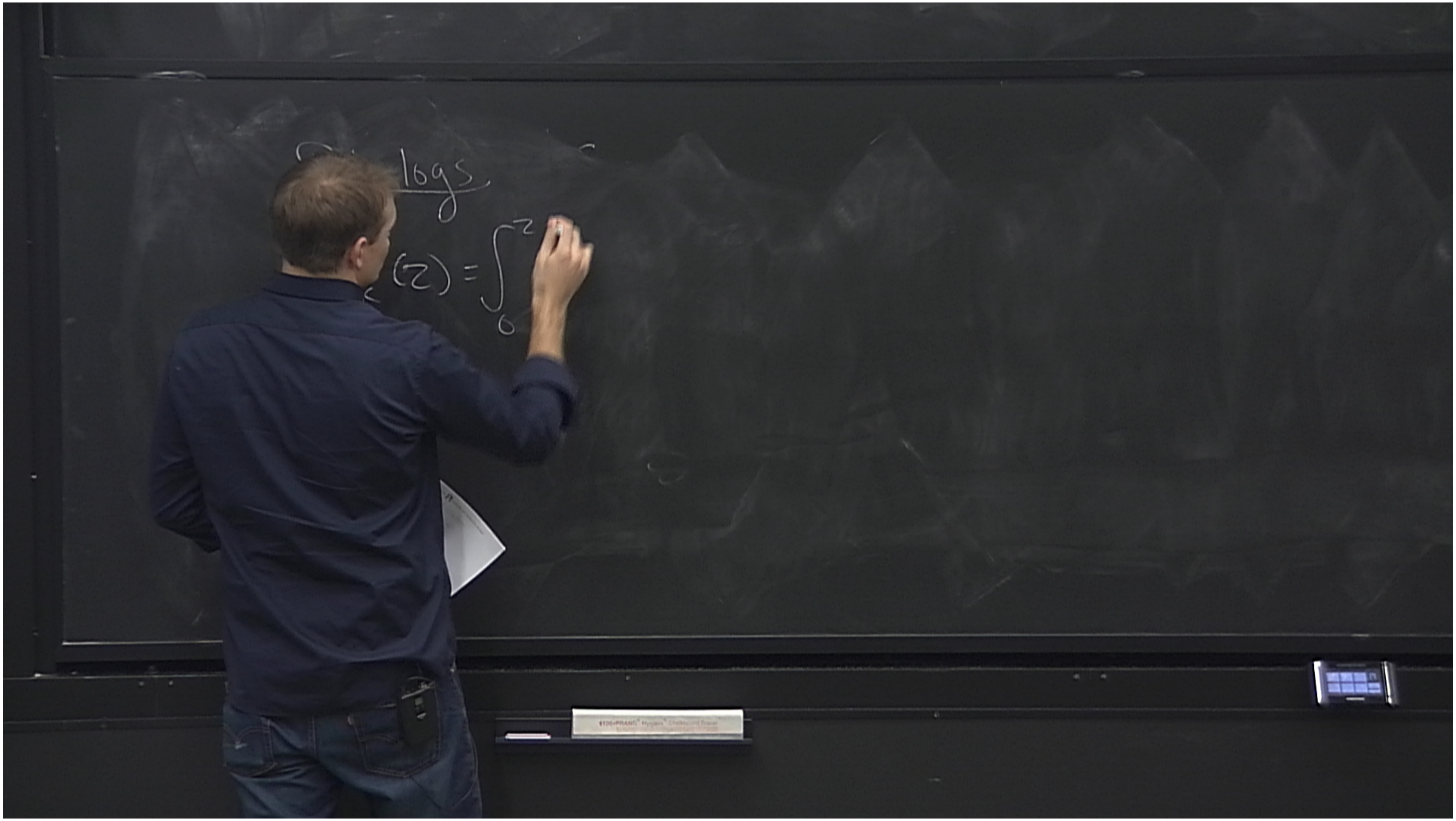

Kinematics
Σ p_i = 0 , p_i^2 = 0 , ⇒ "momentum" twistors
(Z_1 Z_2 ... Z_n) ∈ Gr(4,n) / PGL_4 = Conf_n(P^3)
⟨abcd⟩ = det(Z_a Z_b Z_c Z_d

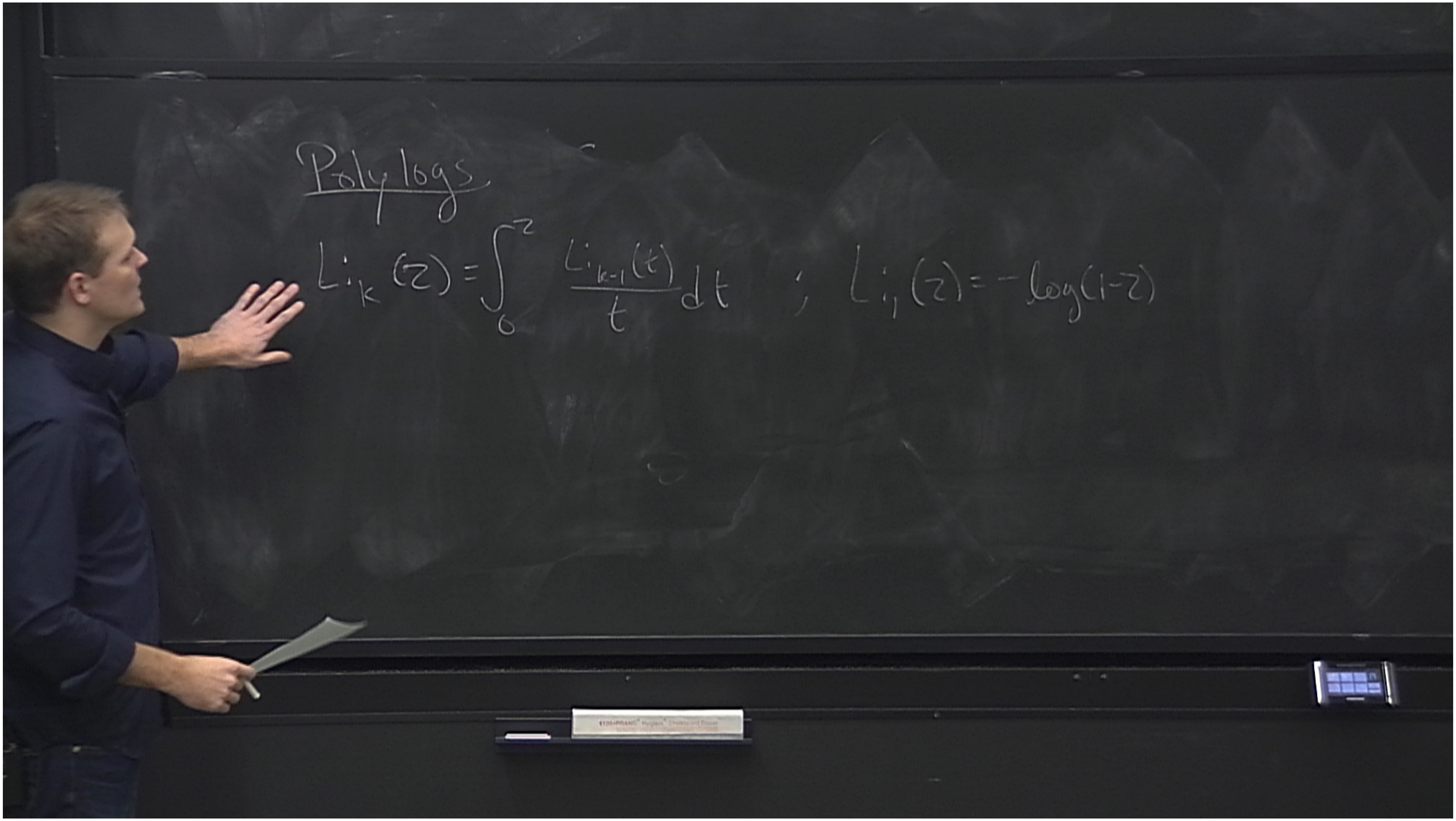

Kinematics
$\sum p_i = 0$, $p_i^2 = 0$, $\Rightarrow$ "momentum twistors"
$\langle abcd \rangle = \det(Z_a Z_b Z_c Z_d)$
$\begin{pmatrix} \vdots & \vdots & & \vdots \\ Z_1 & Z_2 & \cdots & Z_n \\ \vdots & \vdots & & \vdots \end{pmatrix} \in \frac{Gr(4,n)}{PGL_4} = Conf_n(\mathbb{P}^3)$

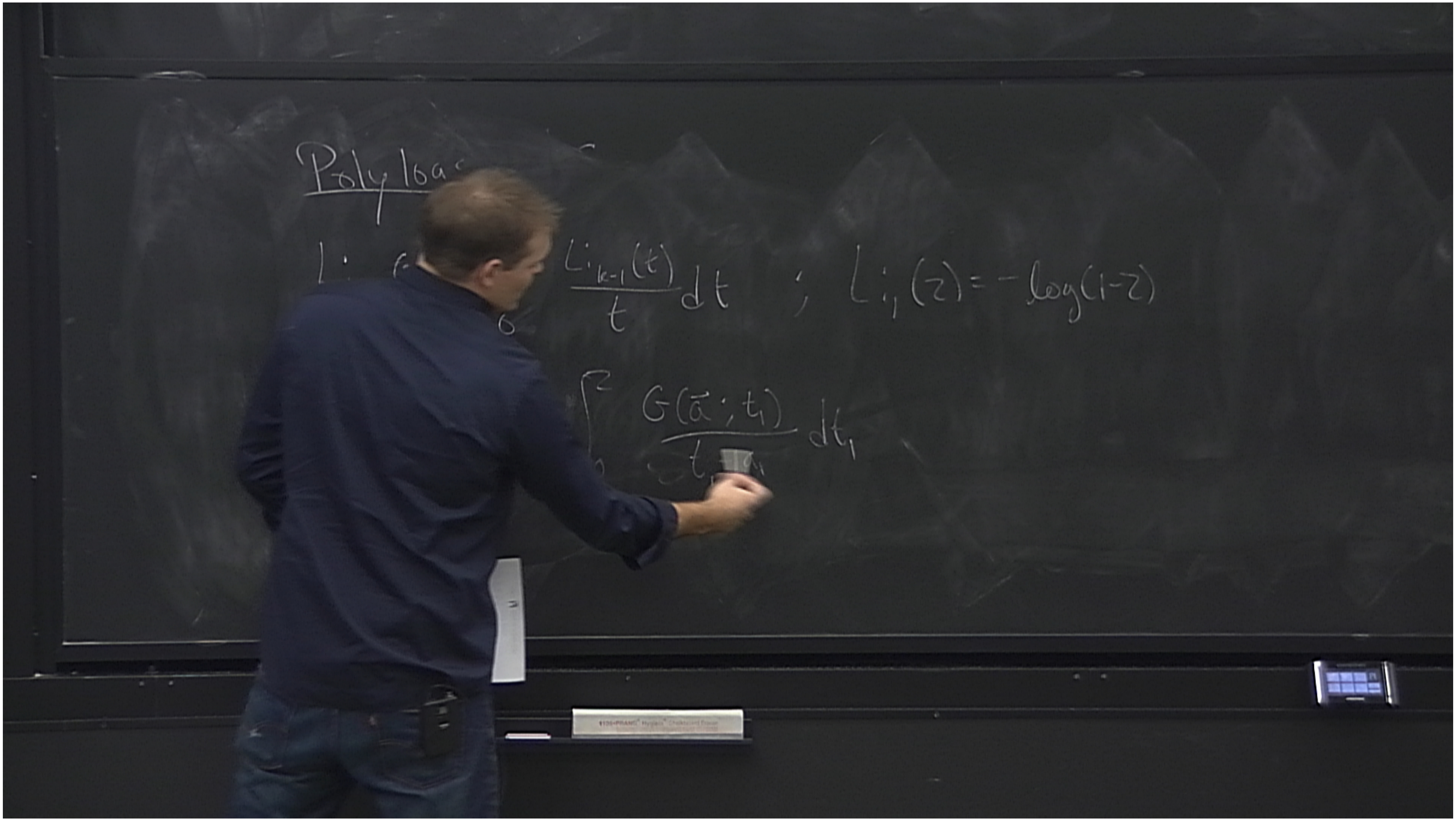

Kinematics
Σ p_i = 0 , p_i^2 = 0 , ⇒ "momentum" twistors
(Z_1 Z_2 ... Z_n) ∈ Gr(4,n) / PGL_4 = Conf_n (P^3)
<abcd> = det(Z_a Z_b Z_c

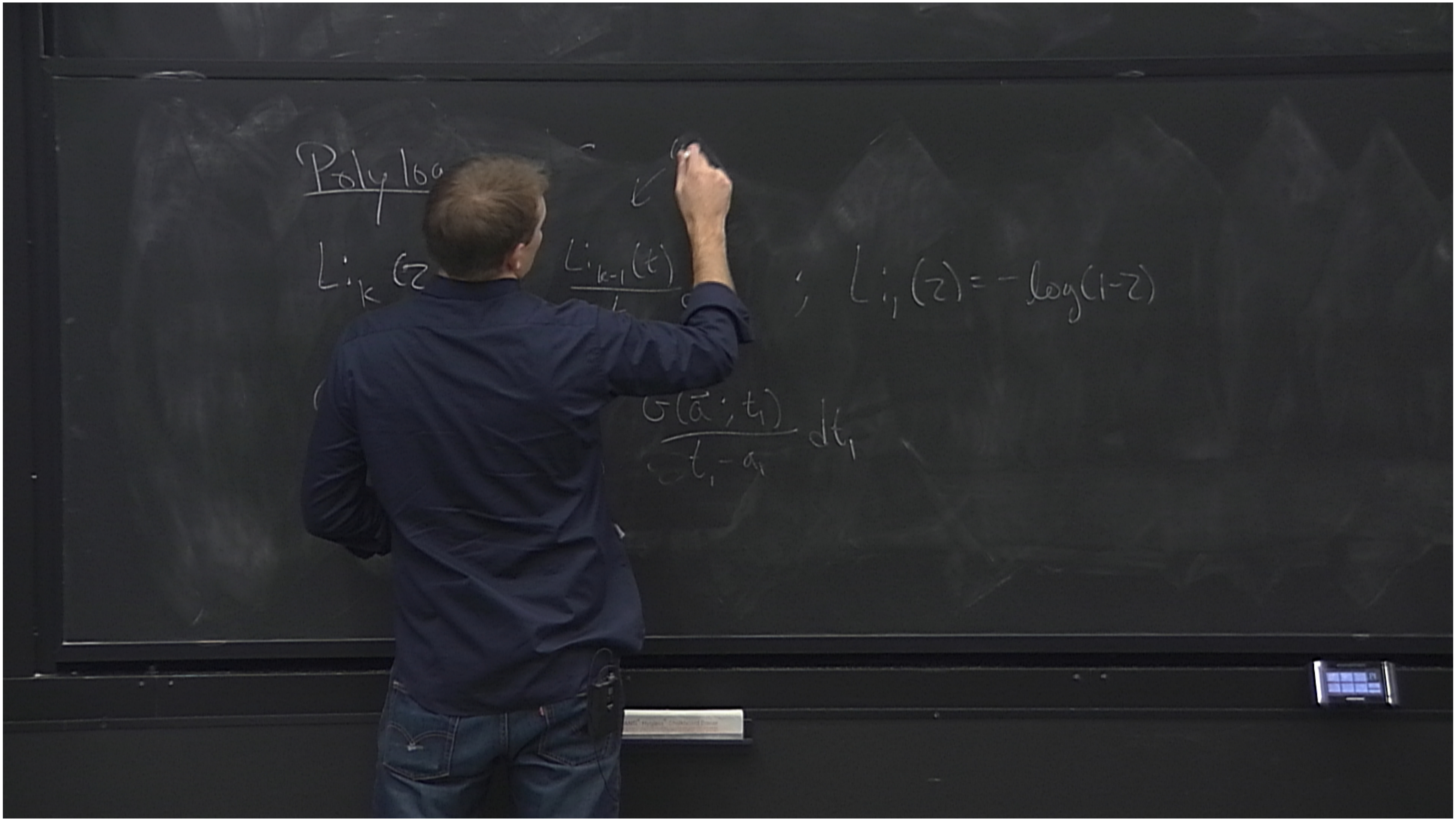

Poly log

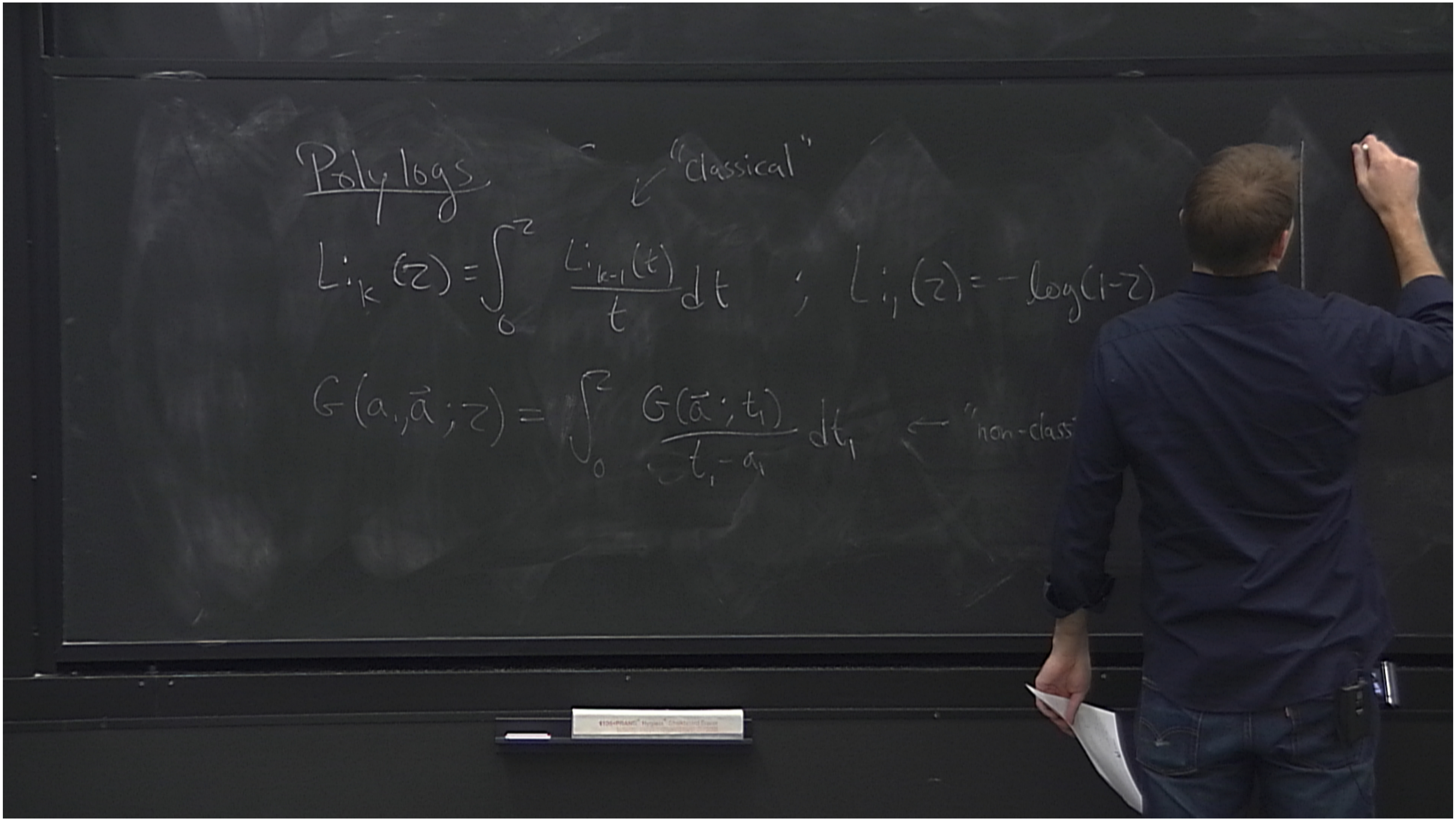

Polylogs

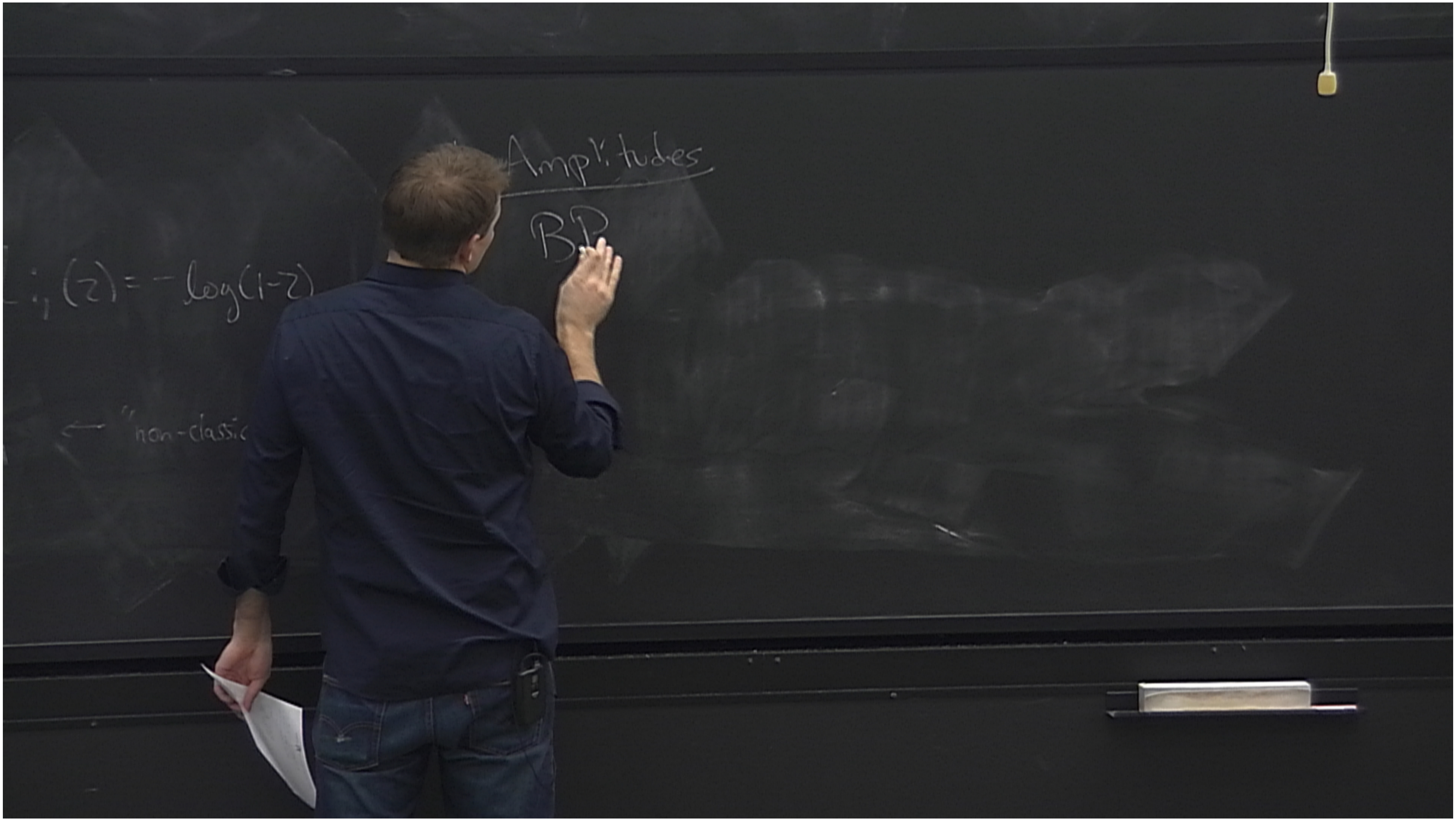

logs

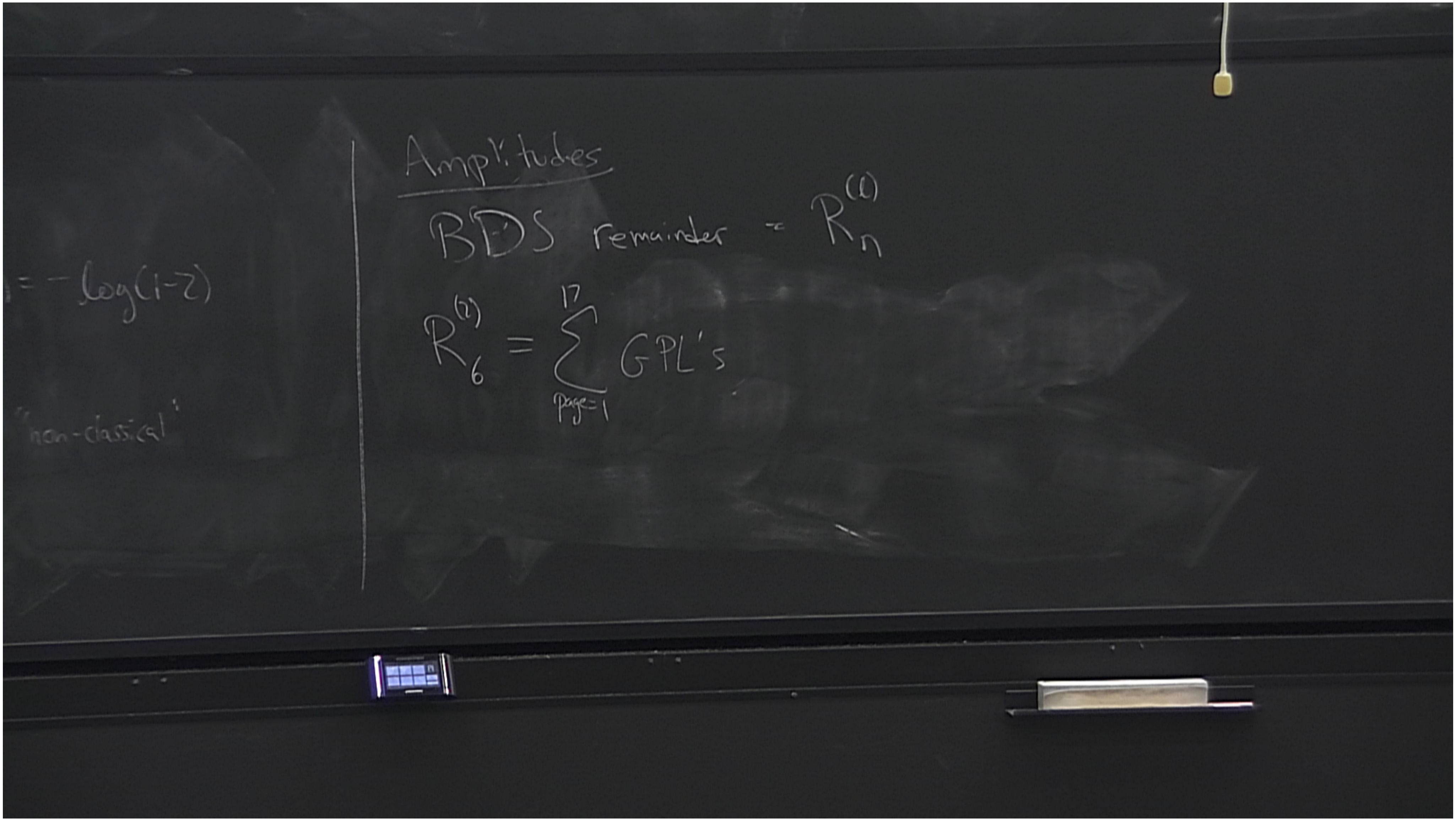

Polylogs
$Li_k(z) = \int_0^z \frac{Li_{k-1}(t)}{t} dt$; $Li_1(z) = -\log(1-z)$

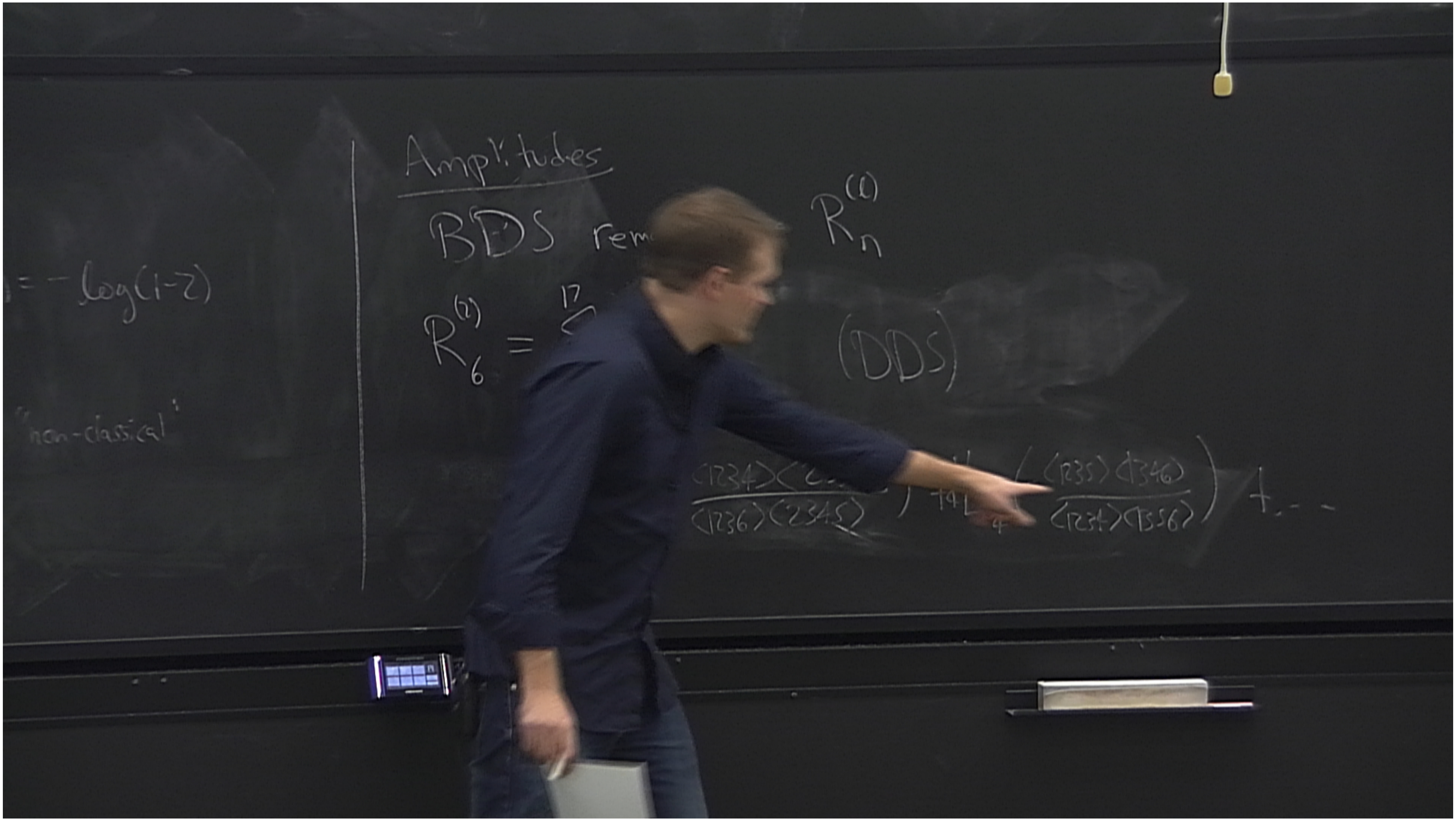

Li_1(z) = -log(1-z)

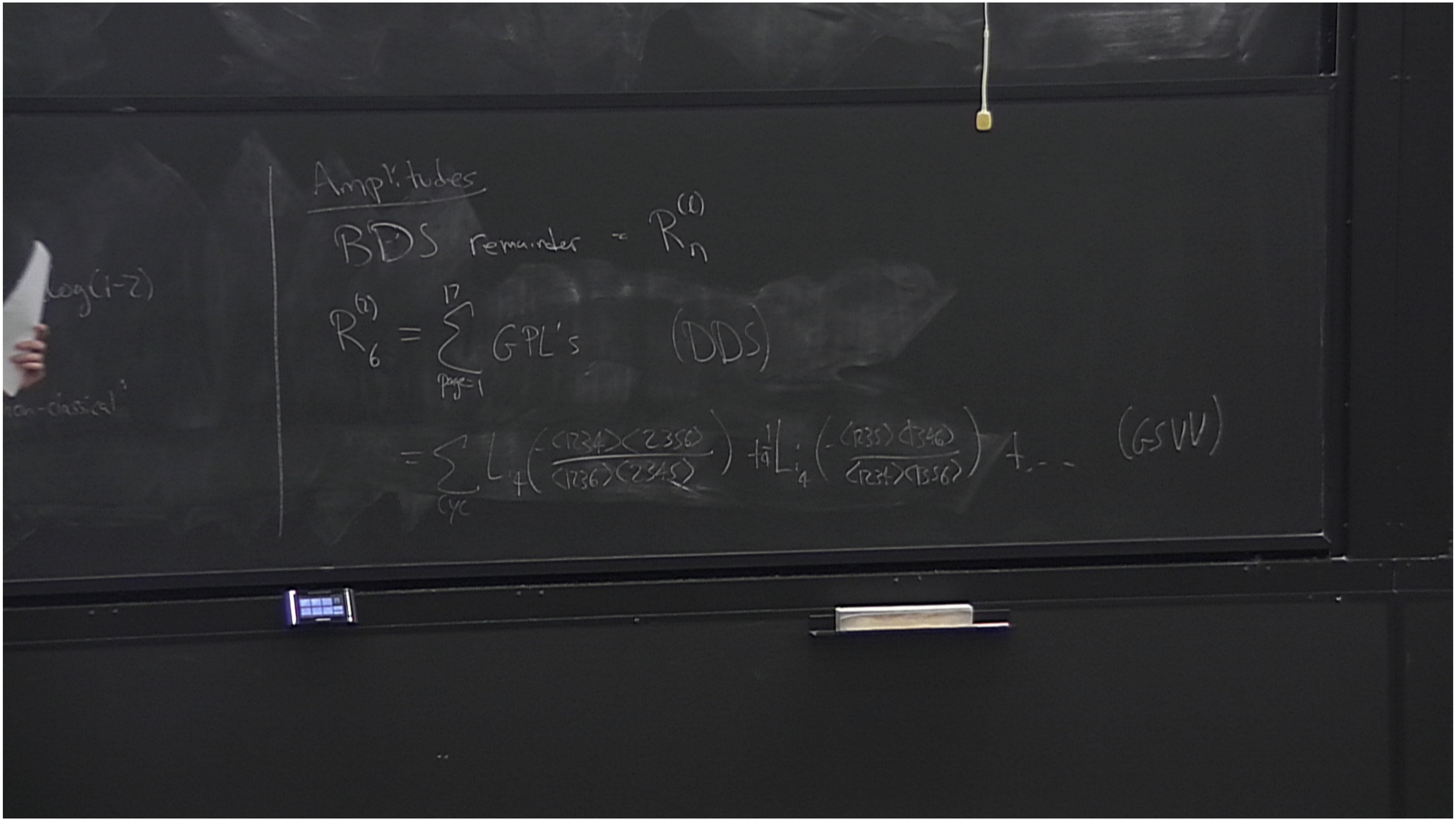

Polylog
; Li₁(z) = −log(1−z)
dt₁

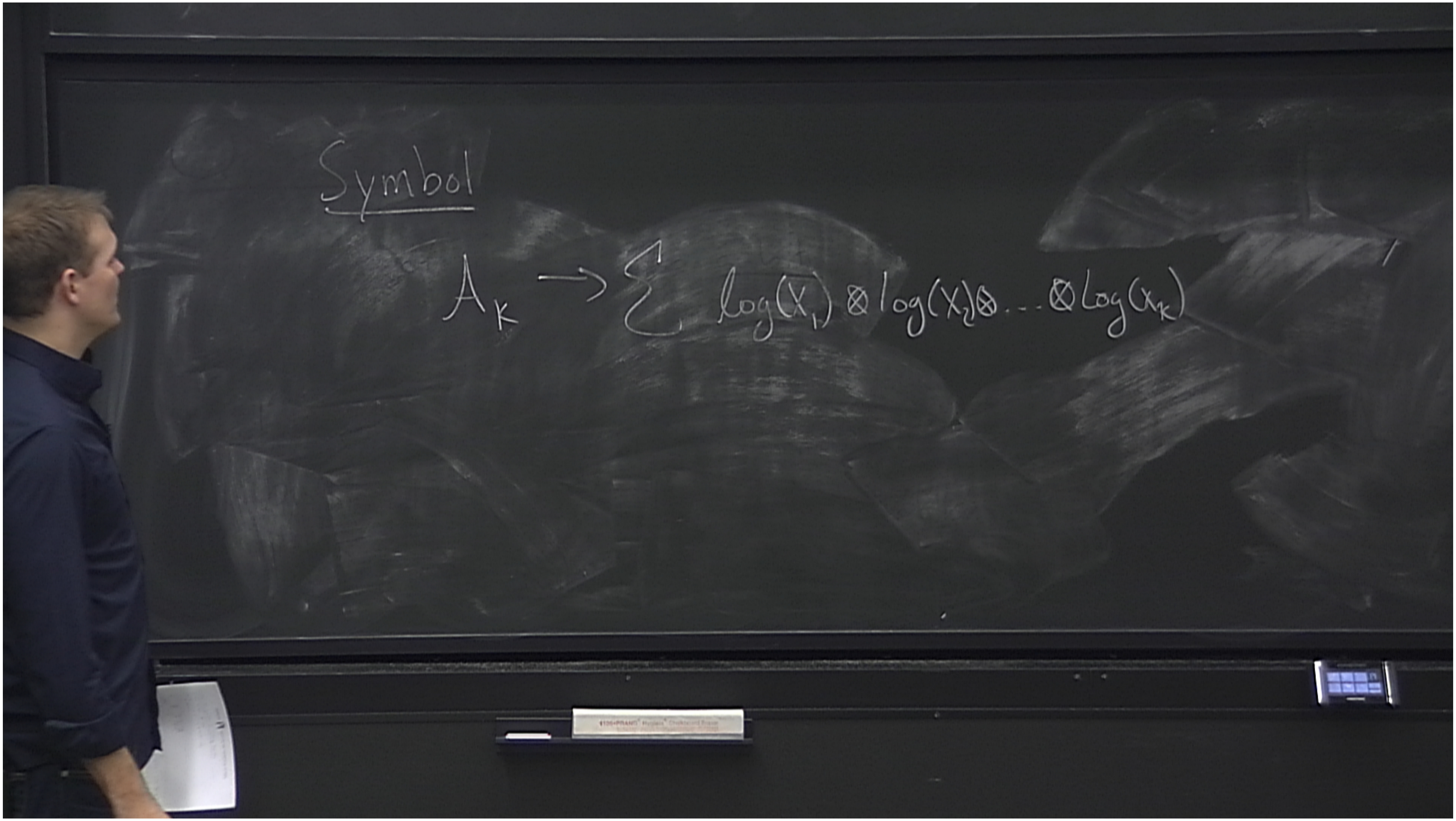

Polylogs
"classical"
$Li_k(z) = \int_0^z \frac{Li_{k-1}(t)}{t} dt \; ; \; Li_1(z) = -\log(1-z)$
$G(a_1, \vec{a}; z) = \int_0^z \frac{G(\vec{a}; t_1)}{t_1 - a_1} dt_1$
← "non-class

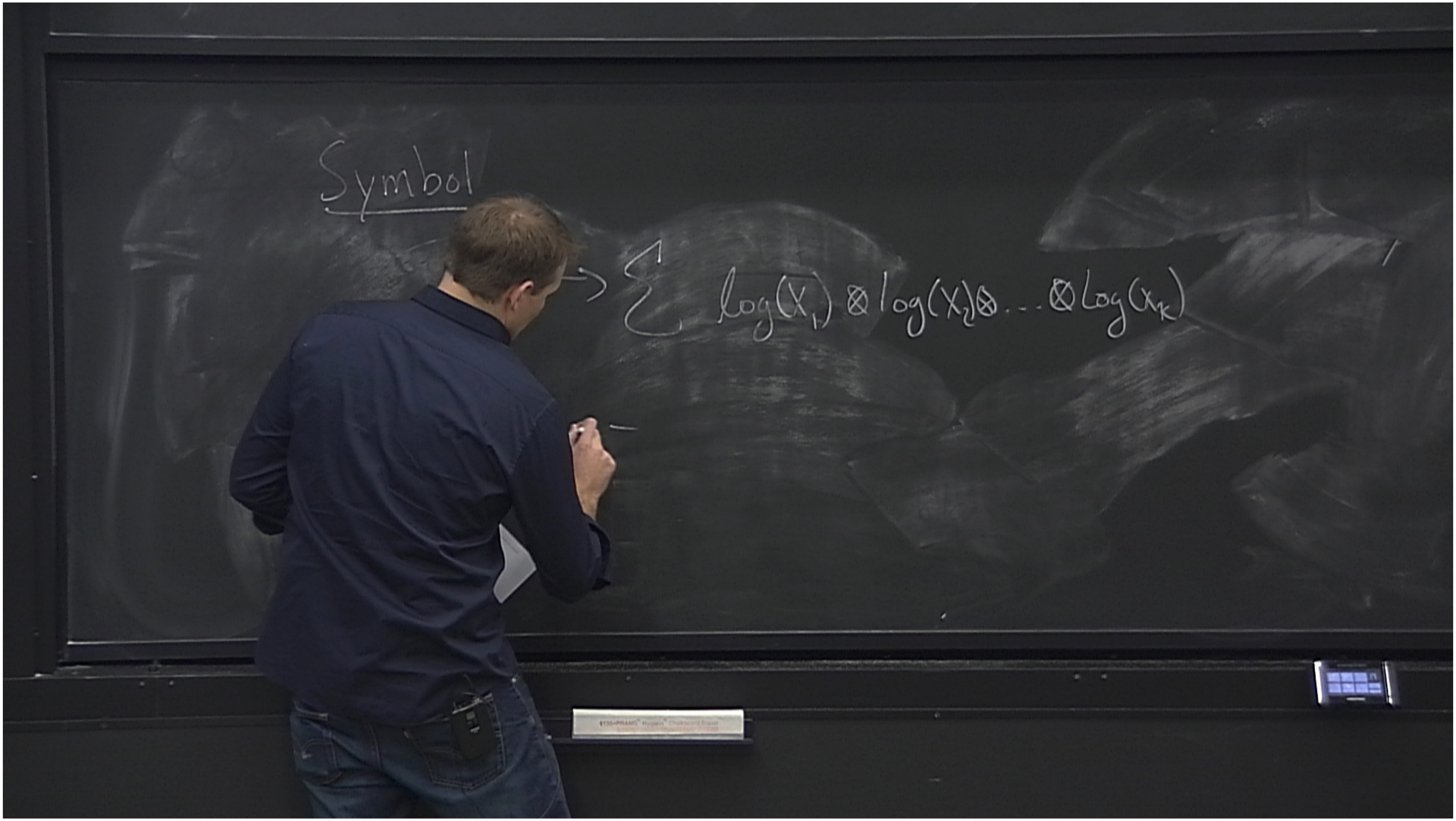

Amplitudes
BP
$Li_1(z) = -\log(1-z)$
← "non-classic

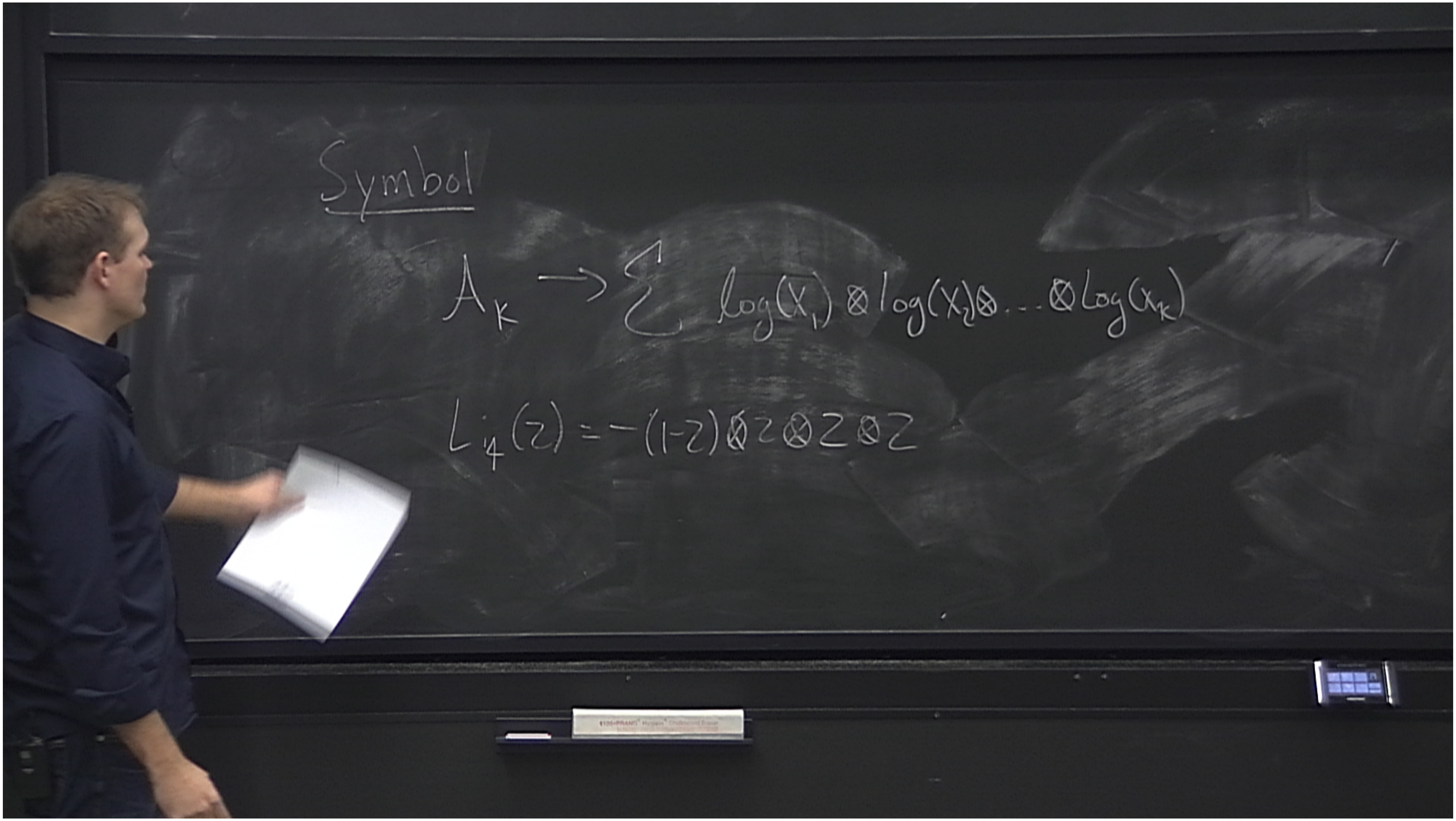

= − log(1−z)
"non-classical"
Amplitudes
BDS remainder = $R_n^{(l)}$
$R_6^{(2)} = \sum_{page=1}^{17}$ GPL's

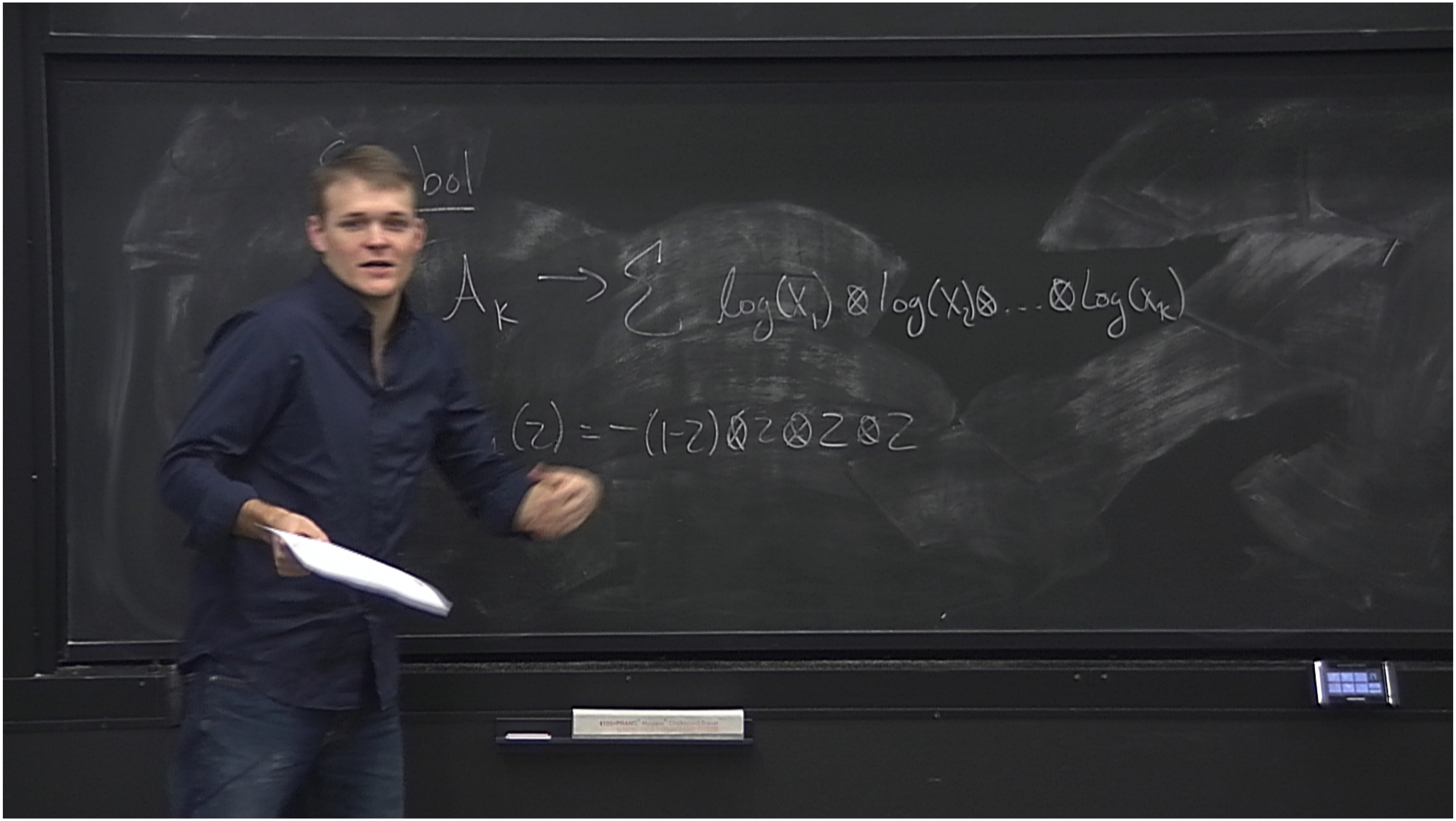

Amplitudes
BDS rem
(DDS)
-log(1-z)
"non-classical"
+ . . .

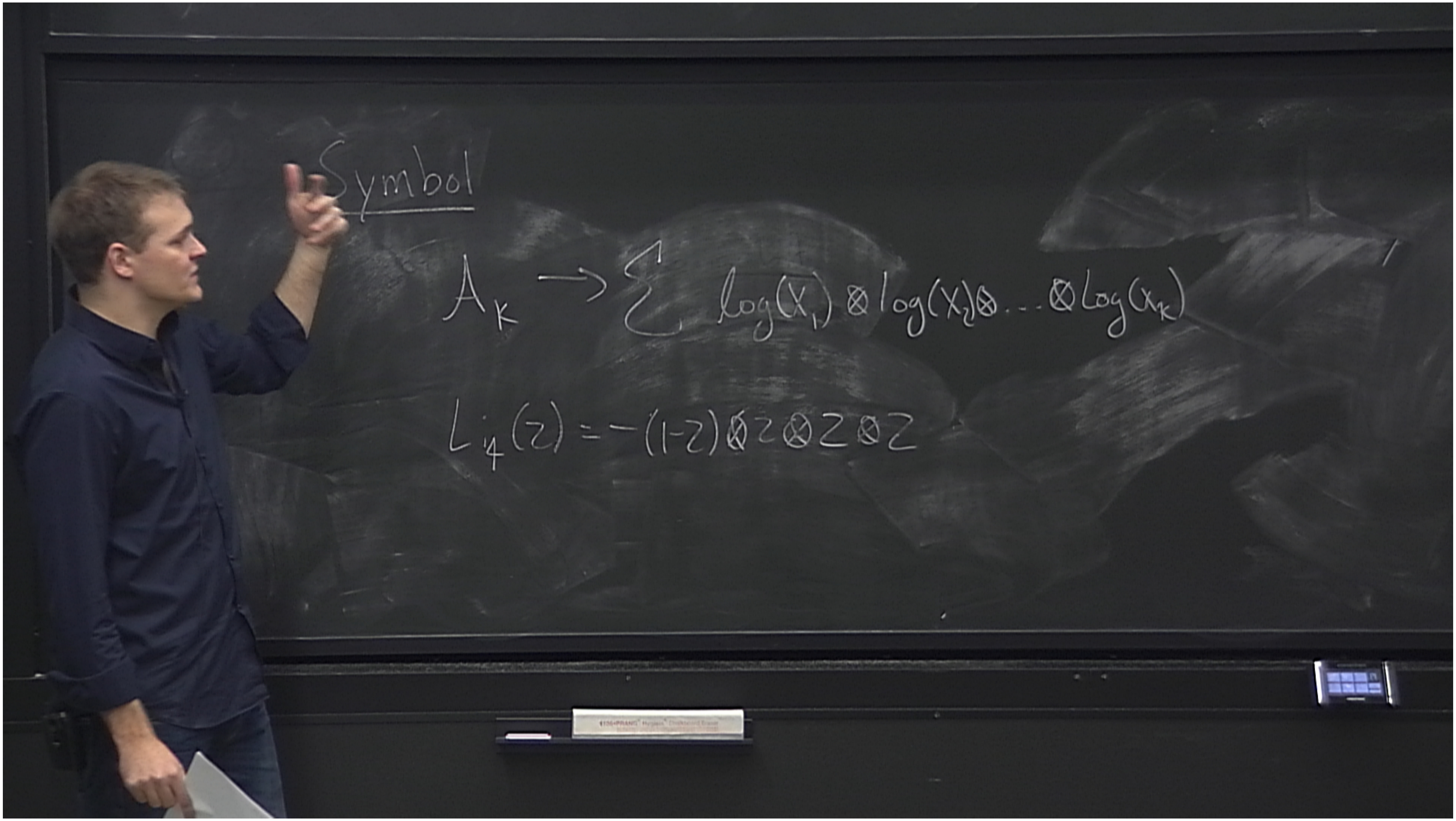

log(1-z)
non-classical
Amplitudes
BDS remainder = R_n^(l)
R_6^(2) = Σ_{page=1}^{17} GPL's (DDS)
= Σ_cyc L_4(- ⟨1234⟩⟨2356⟩ / ⟨1236⟩⟨2345⟩) + 1/4 L_4(- ⟨1235⟩⟨1346⟩ / ⟨1234⟩⟨1356⟩) + . . . (GSVV)

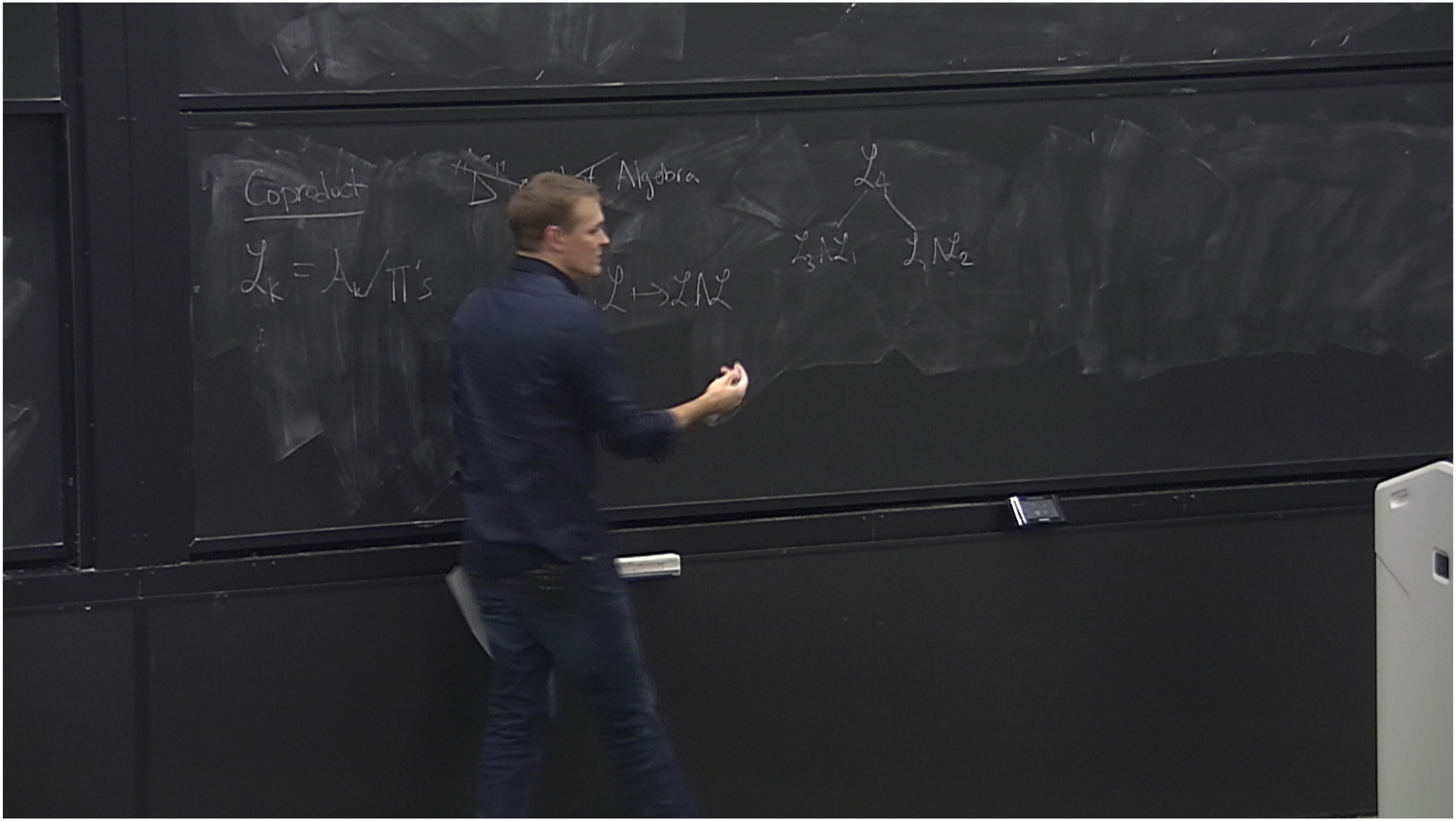

Symbol
$A_k \longrightarrow \sum \log(x_1) \otimes \log(x_2) \otimes \ldots \otimes \log(x_k)$

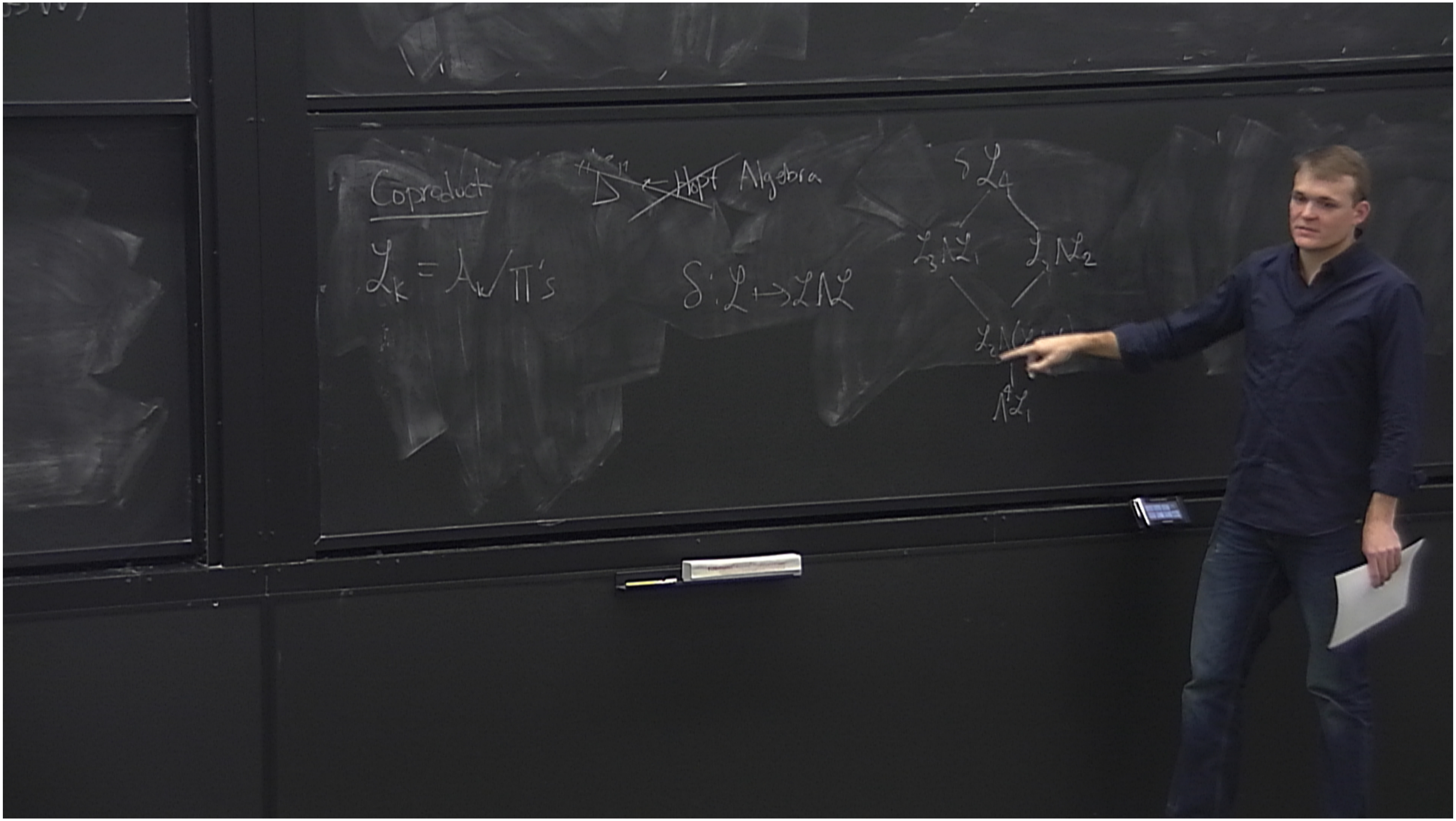

Symbol
→ Σ log(X1) ⊗ log(X2) ⊗ . . . ⊗ Log(Xk)

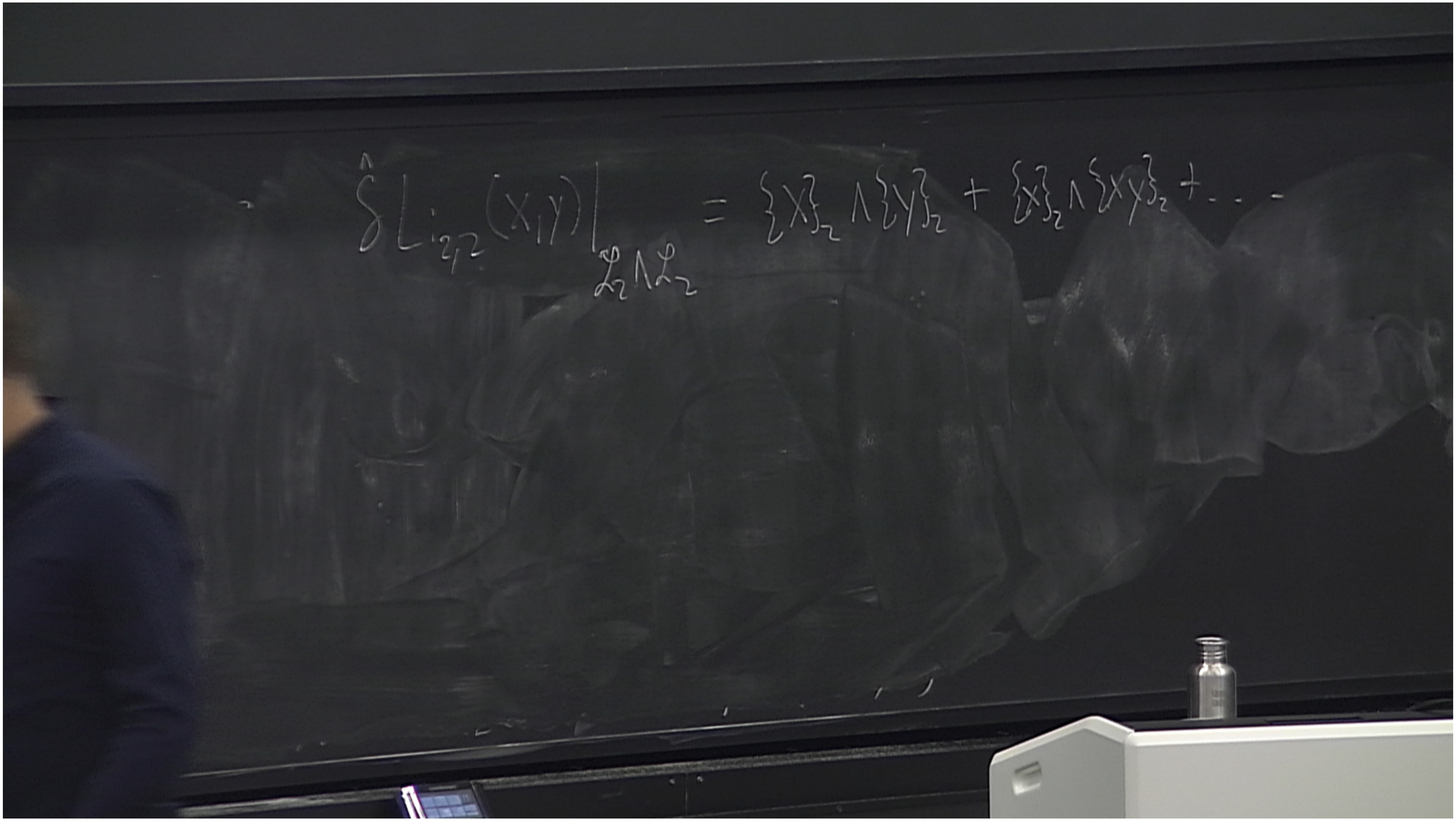

Symbol
$A_K \longrightarrow \sum \log(X_1) \otimes \log(X_2) \otimes \ldots \otimes \log(X_K)$
$Li_4(z) = -(1-z) \otimes z \otimes z \otimes z$

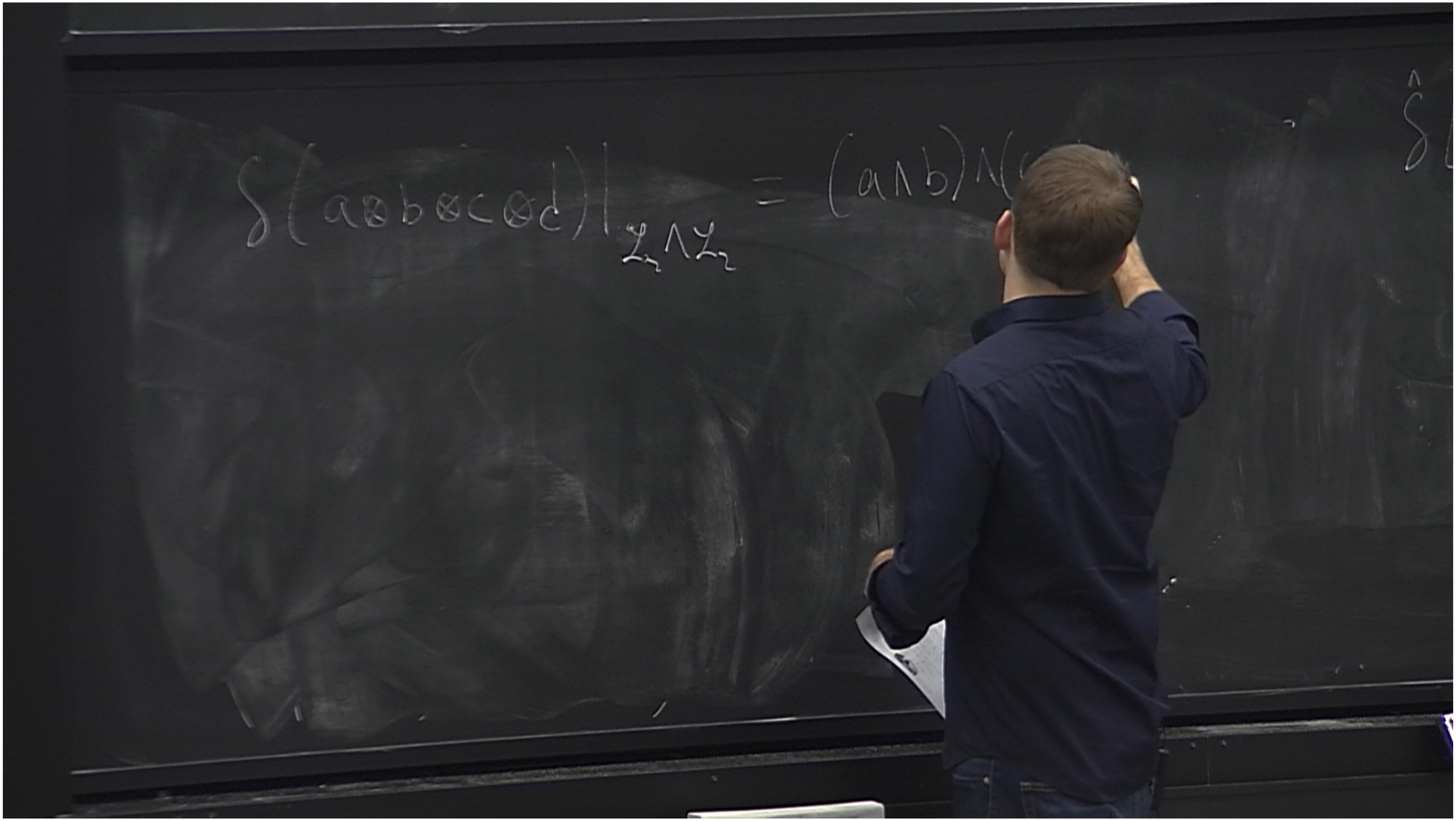

bol
A_K → Σ log(X_1) ⊗ log(X_2) ⊗ . . . ⊗ Log(x_K)
(z) = −(1−z) ⊗ z ⊗ Z ⊗ Z

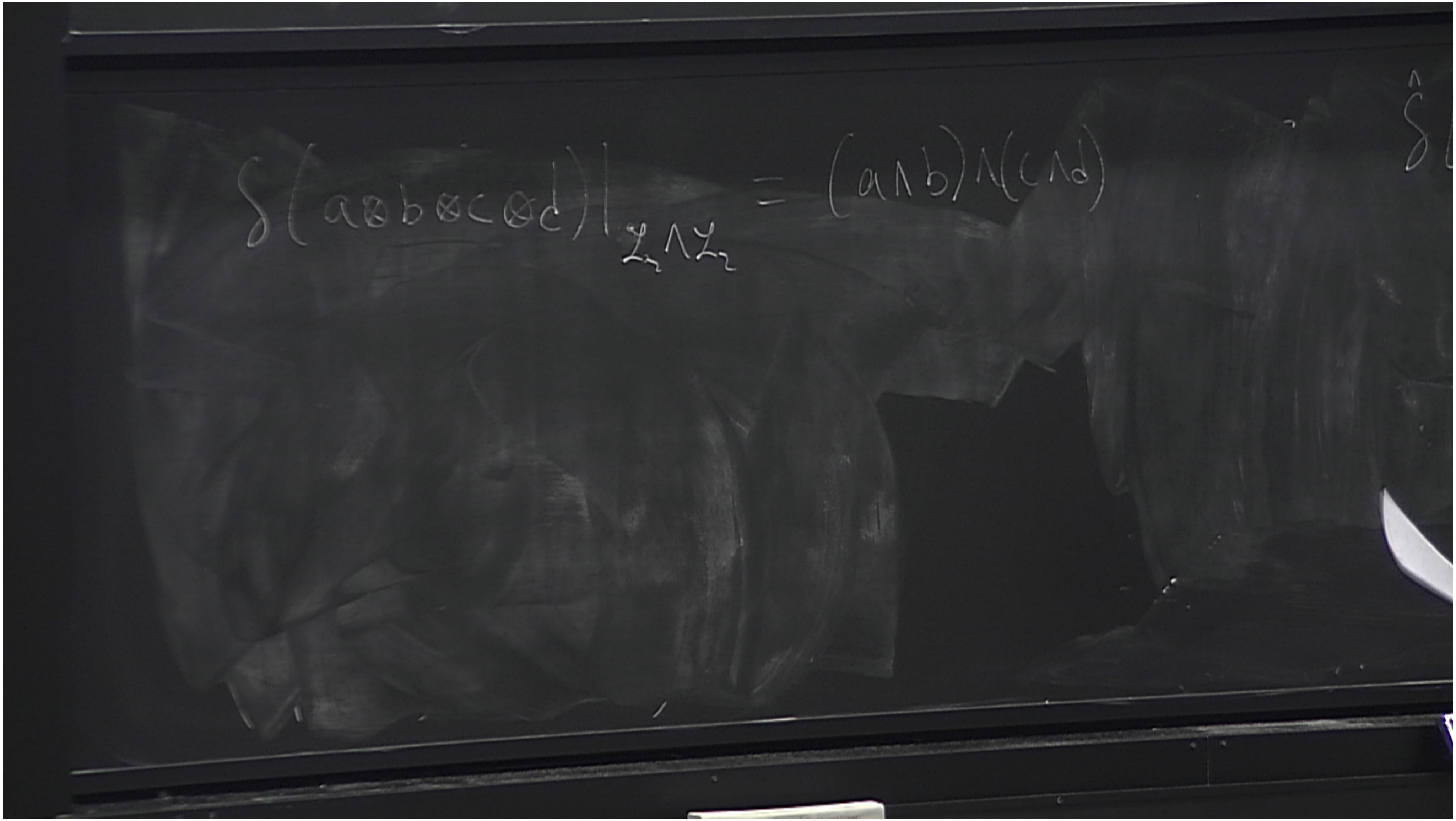

Symbol
$A_k \to \sum \log(x_1) \otimes \log(x_2) \otimes \ldots \otimes \log(x_k)$
$Li_4(z) = -(1-z) \otimes z \otimes z \otimes z$

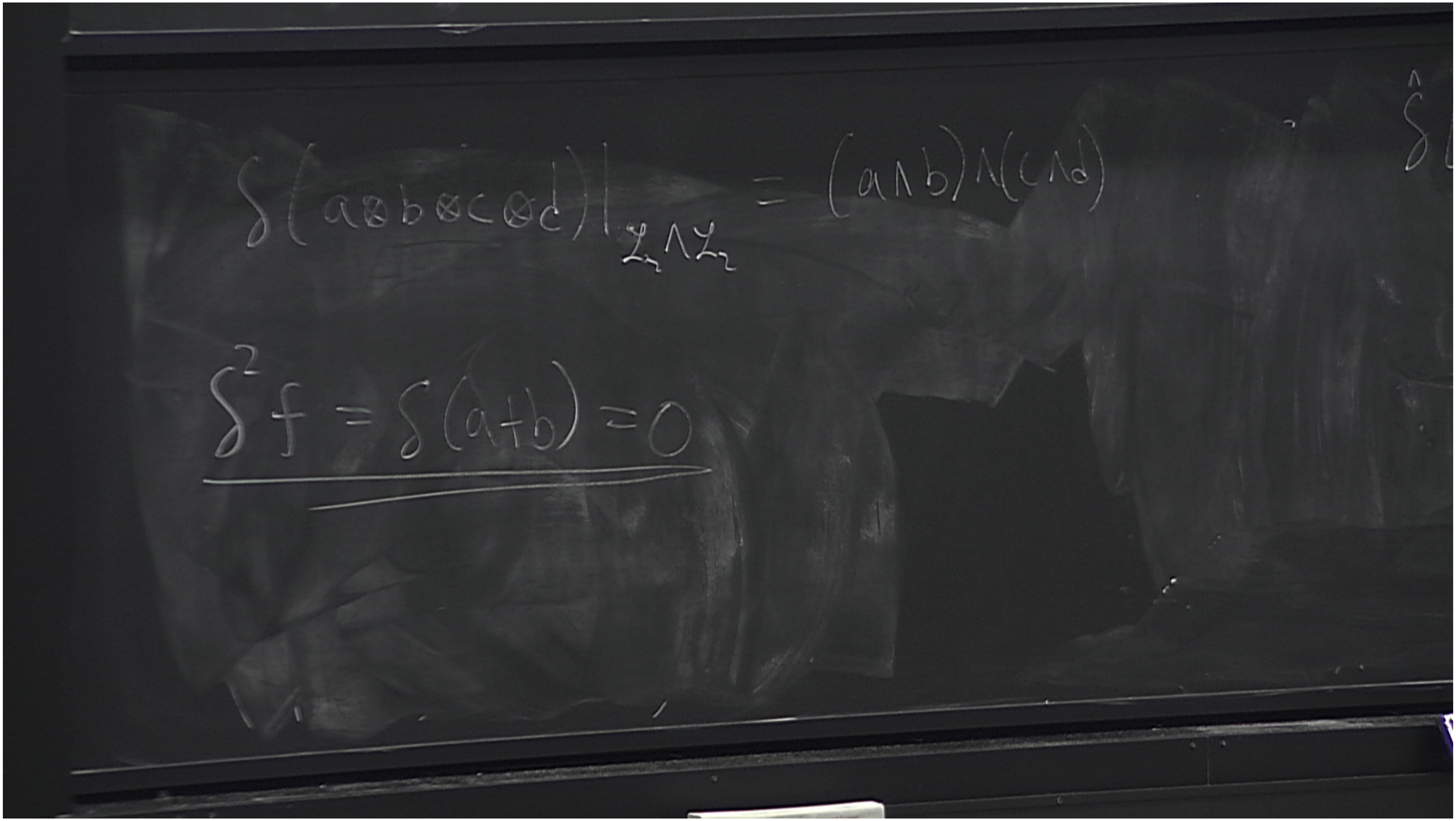

Coproduct
Algebra

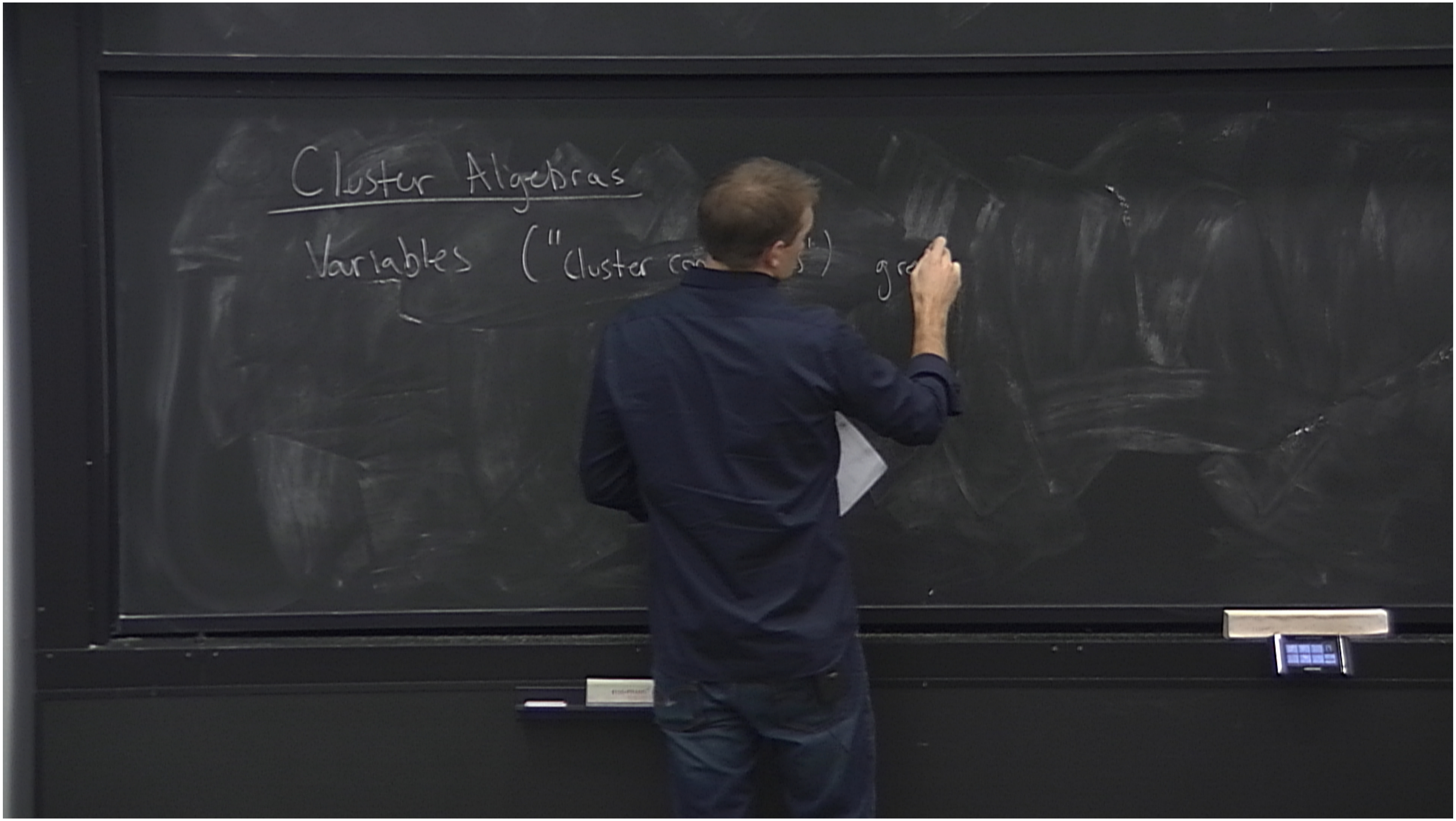

Coproduct
Hopf Algebra
δ: L ↦ L∧L
δL4
L3∧L1
L1∧L2
Λ4L1

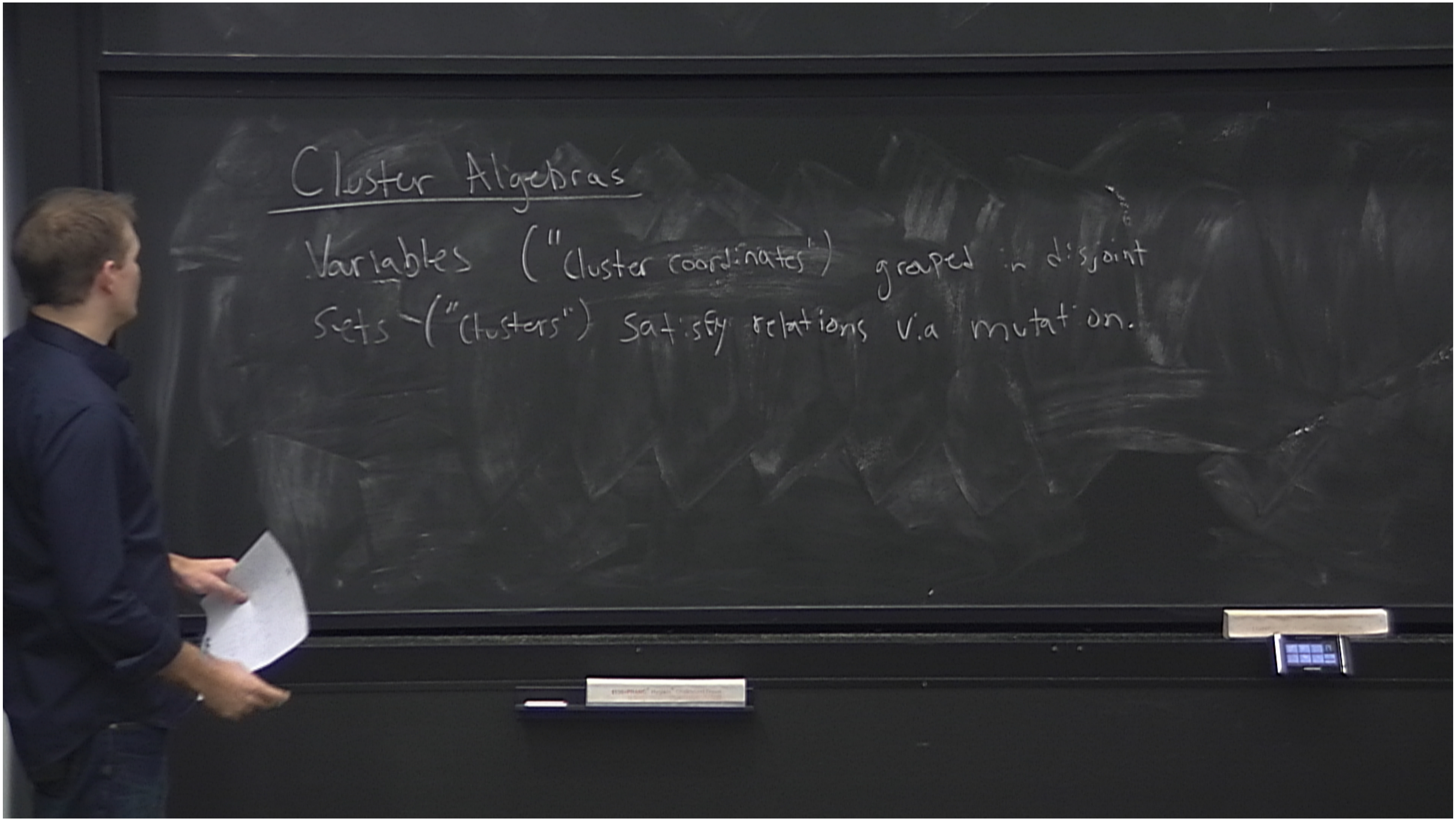

$\hat{\delta} Li_{2,2}(x,y)\big|_{\mathcal{L}_2 \wedge \mathcal{L}_2} = \{x\}_2 \wedge \{y\}_2 + \{x\}_2 \wedge \{xy\}_2 + \dots$

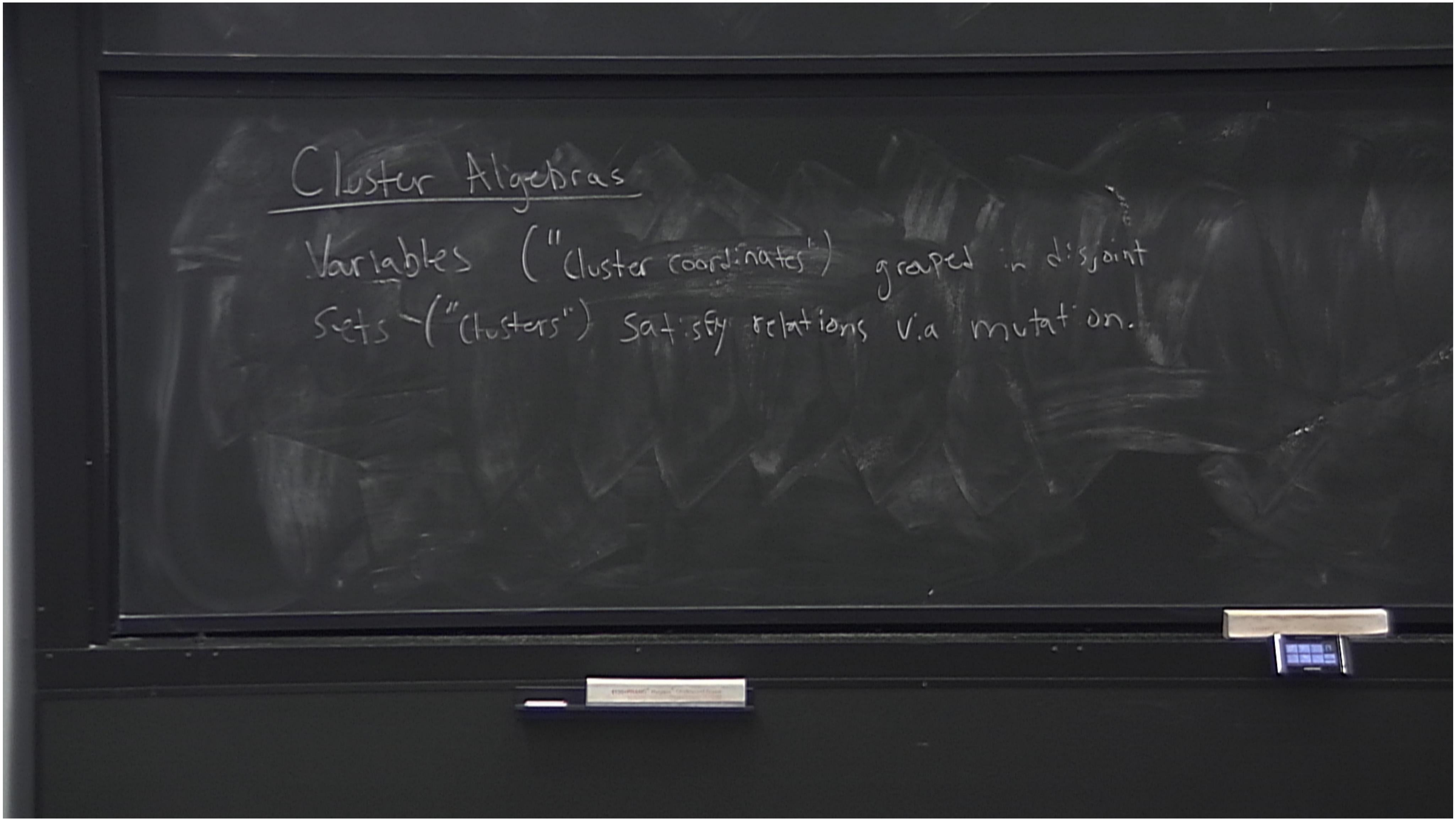

δ(a⊗b⊗c⊗d)|_{L_2∧L_2} = (a∧b)∧(


$\delta(a \otimes b \otimes c \otimes d)|_{\mathcal{L}_2 \wedge \mathcal{L}_2} = (a \wedge b) \wedge (c \wedge d)$


δ(a⊗b⊗c⊗d)|L2∧L2 = (a∧b)∧(c∧d)
δ²f = δ(a+b) = 0


Cluster Algebras
Variables ("cluster


Cluster Algebras
Variables ("cluster coordinates") grouped in disjoint
sets ("clusters") satisfy relations via mutation.


Cluster Algebras
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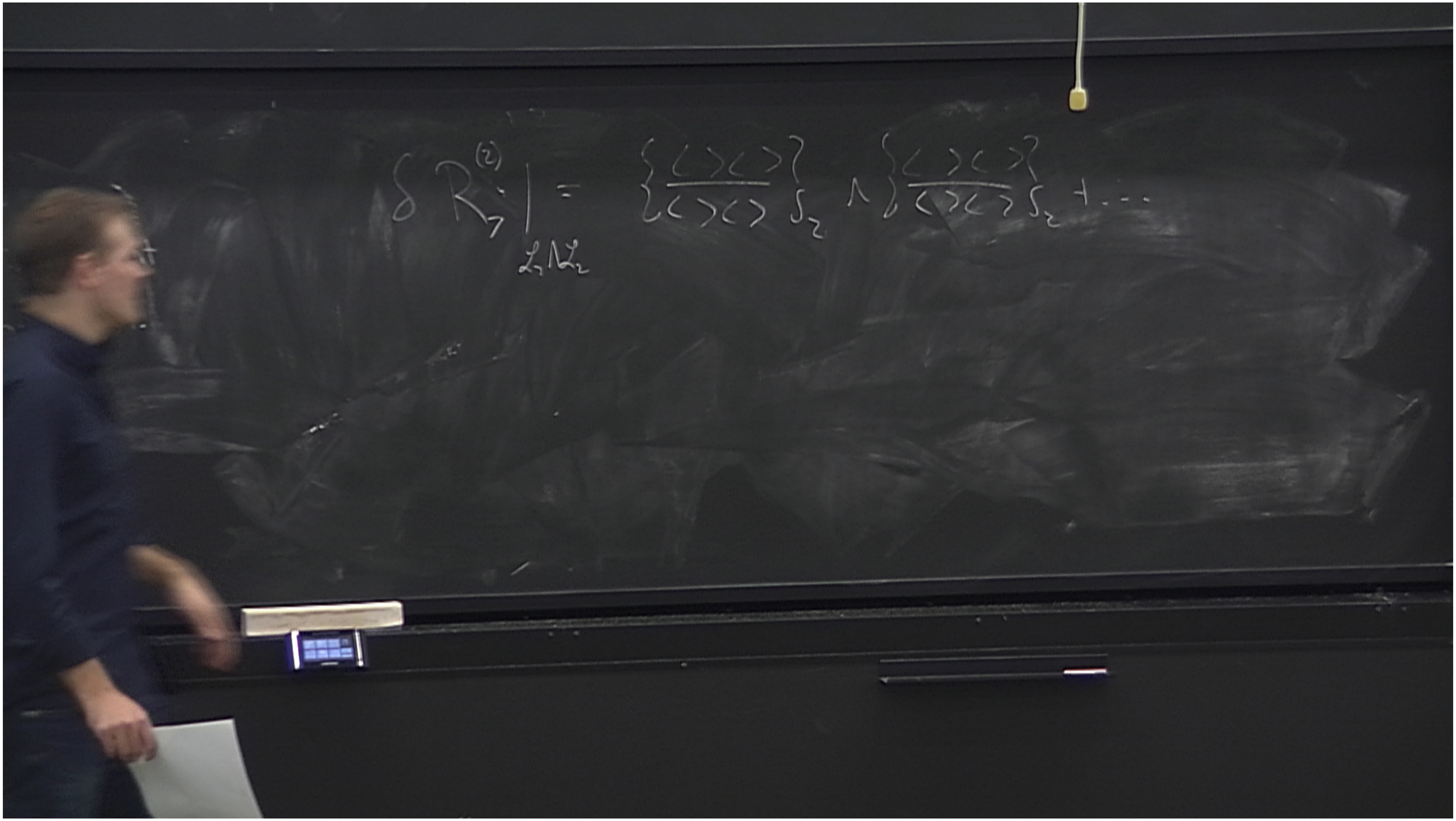
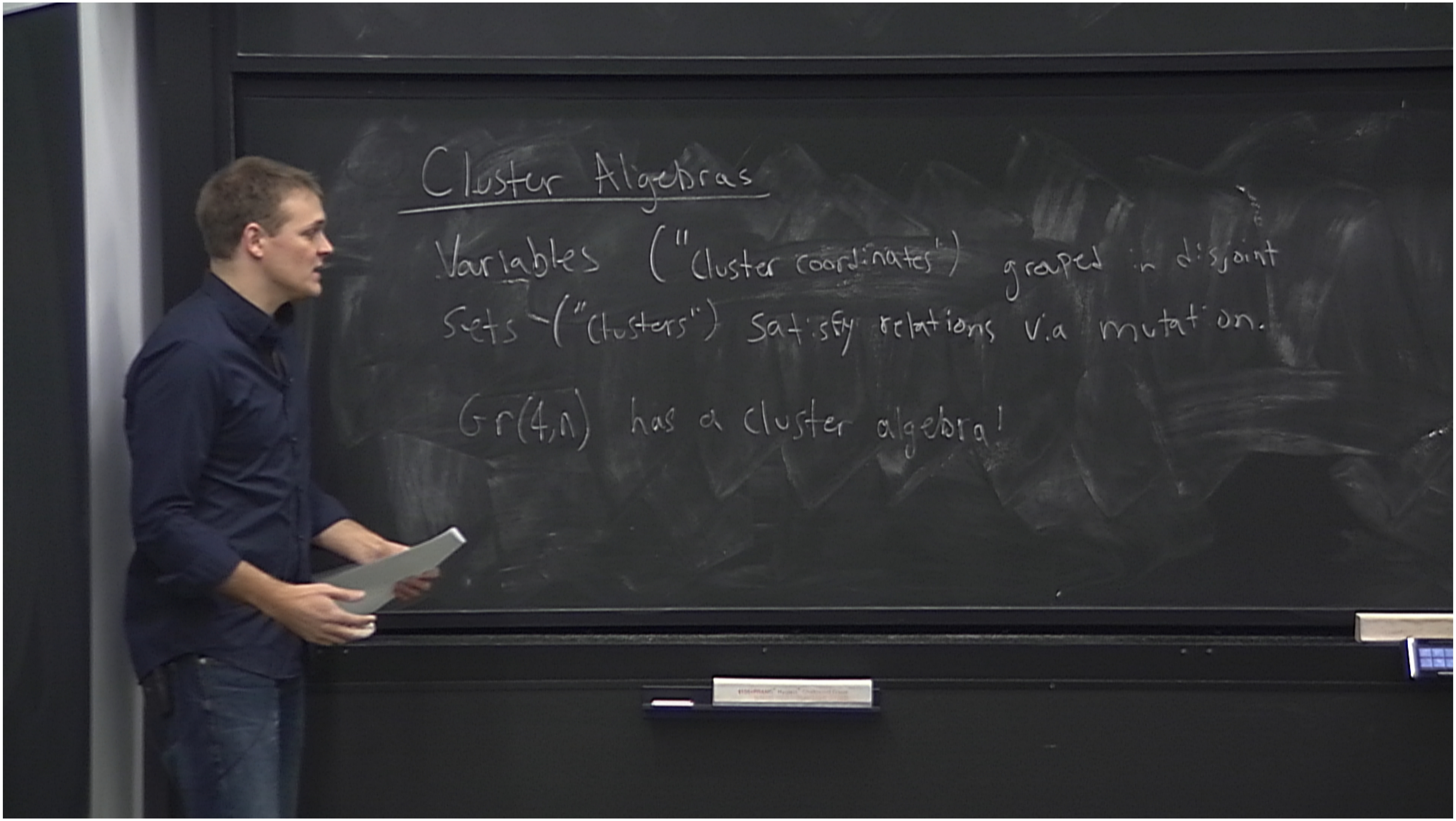
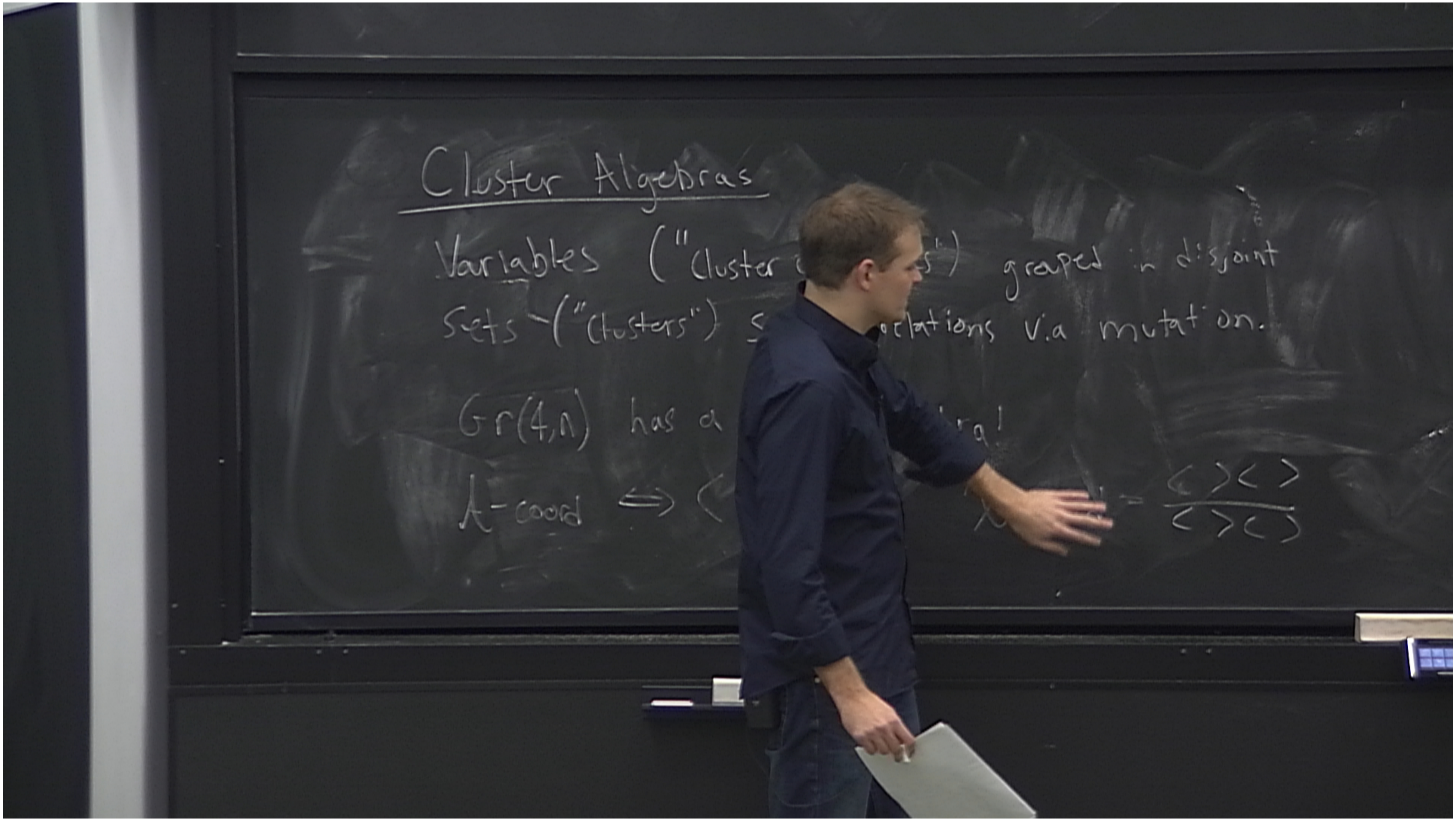

$\delta R_7^{(2)}\Big|_{L_1 \cap L_2} = \left\{\frac{\langle\,\rangle\langle\,\rangle}{\langle\,\rangle\langle\,\rangle}\right\}_{S_2} \wedge \left\{\frac{\langle\,\rangle\langle\,\rangle}{\langle\,\rangle\langle\,\rangle}\right\}_{S_2} + \dots$


Cluster Algebras
Variables ("cluster coordinates") grouped in disjoint
sets ("clusters") satisfy relations via mutation.
Gr(4,n) has a cluster algebra!


Cluster Algebras
Variables ("cluster
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Sets ("clusters")
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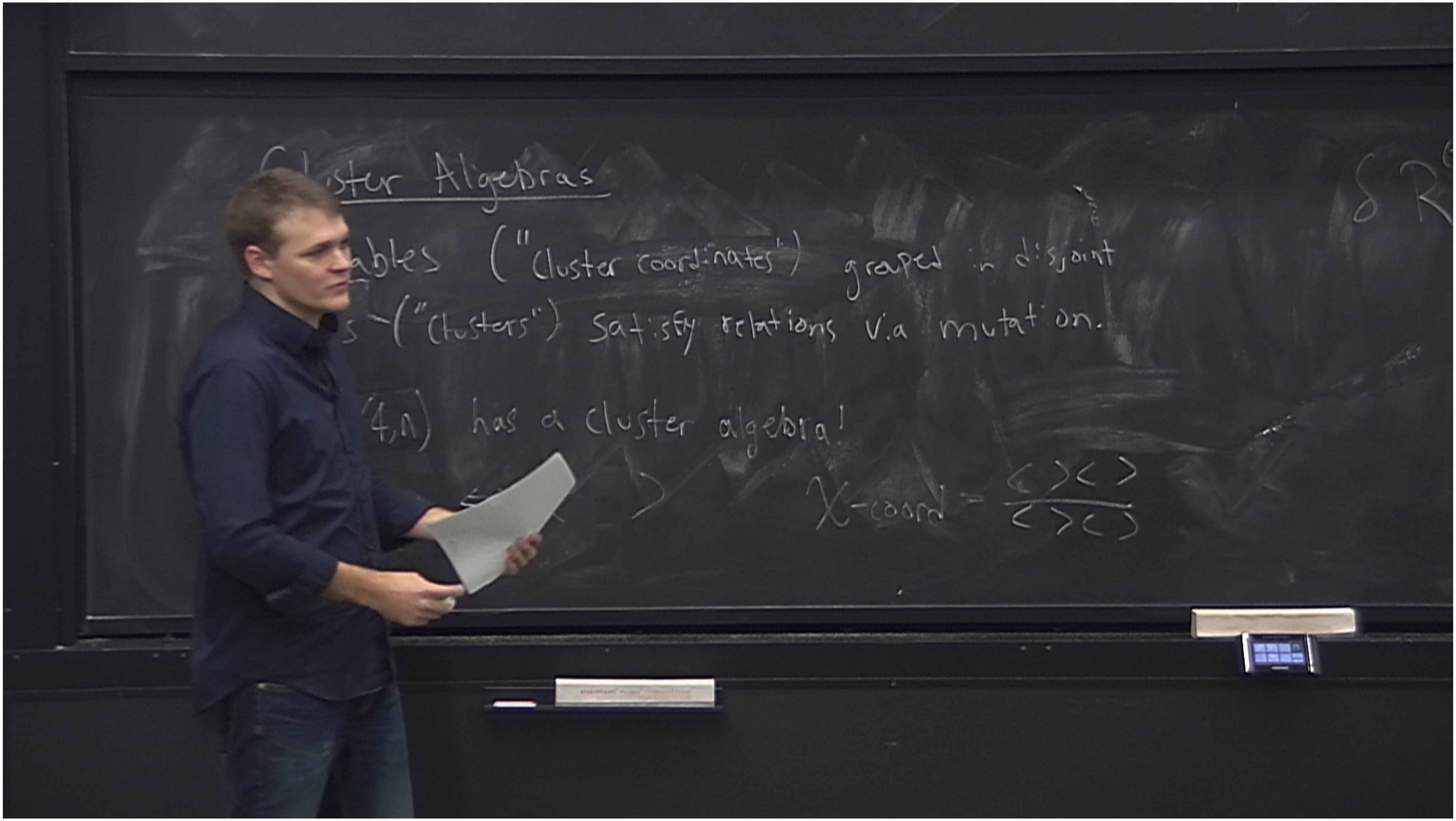
Gr(4,n) has a
λ-coord ⇔


ster Algebras
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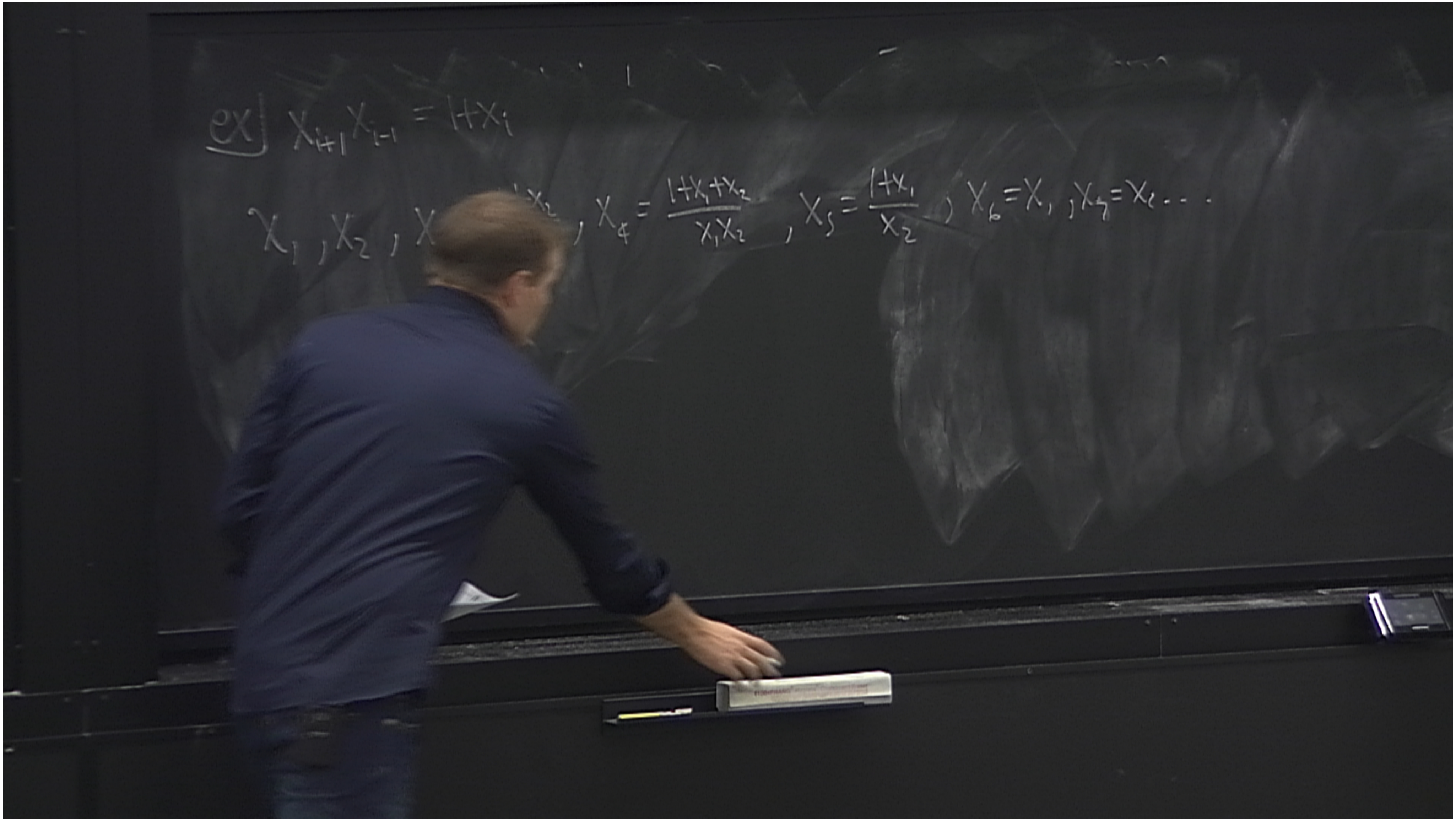
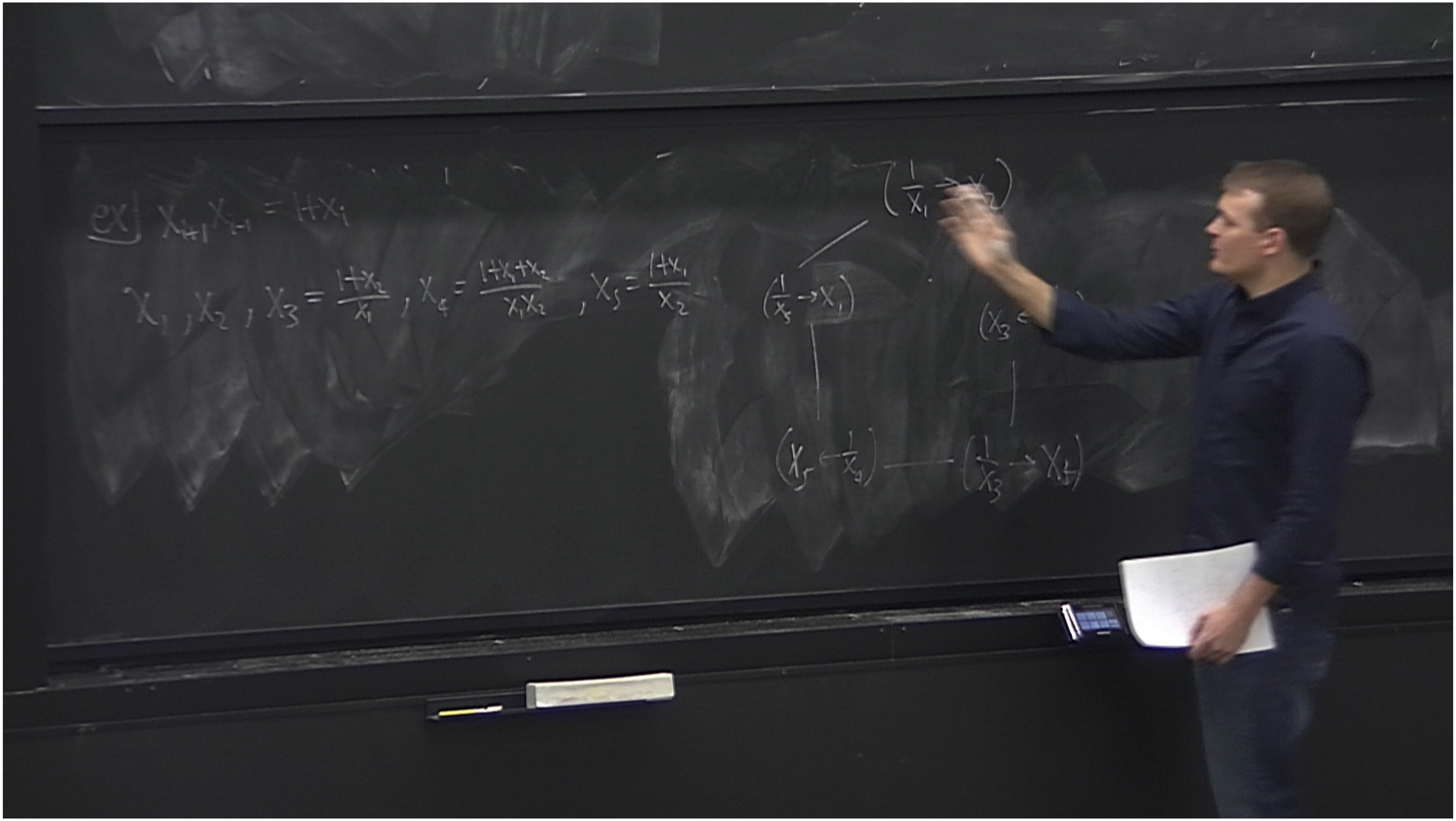
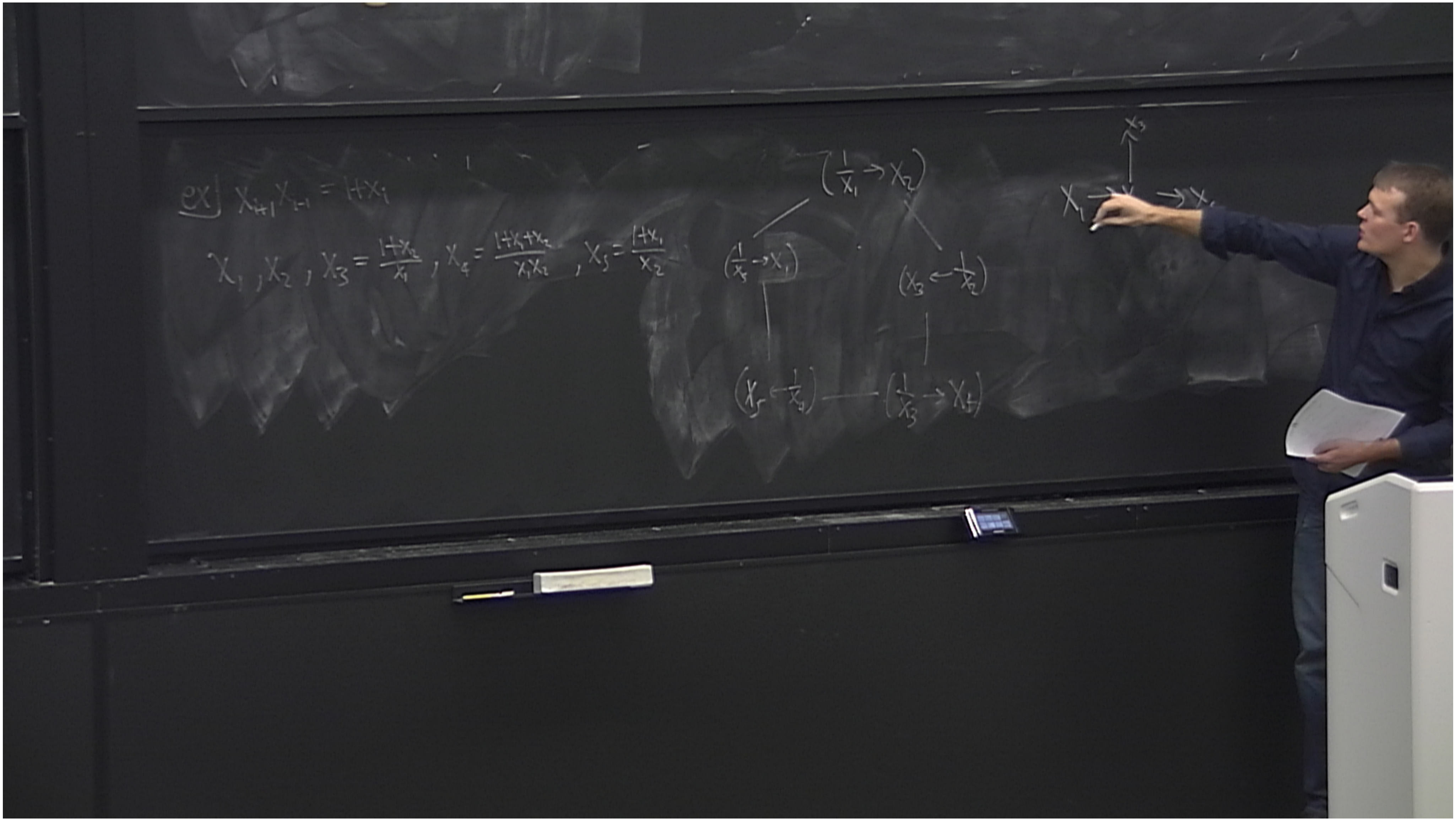
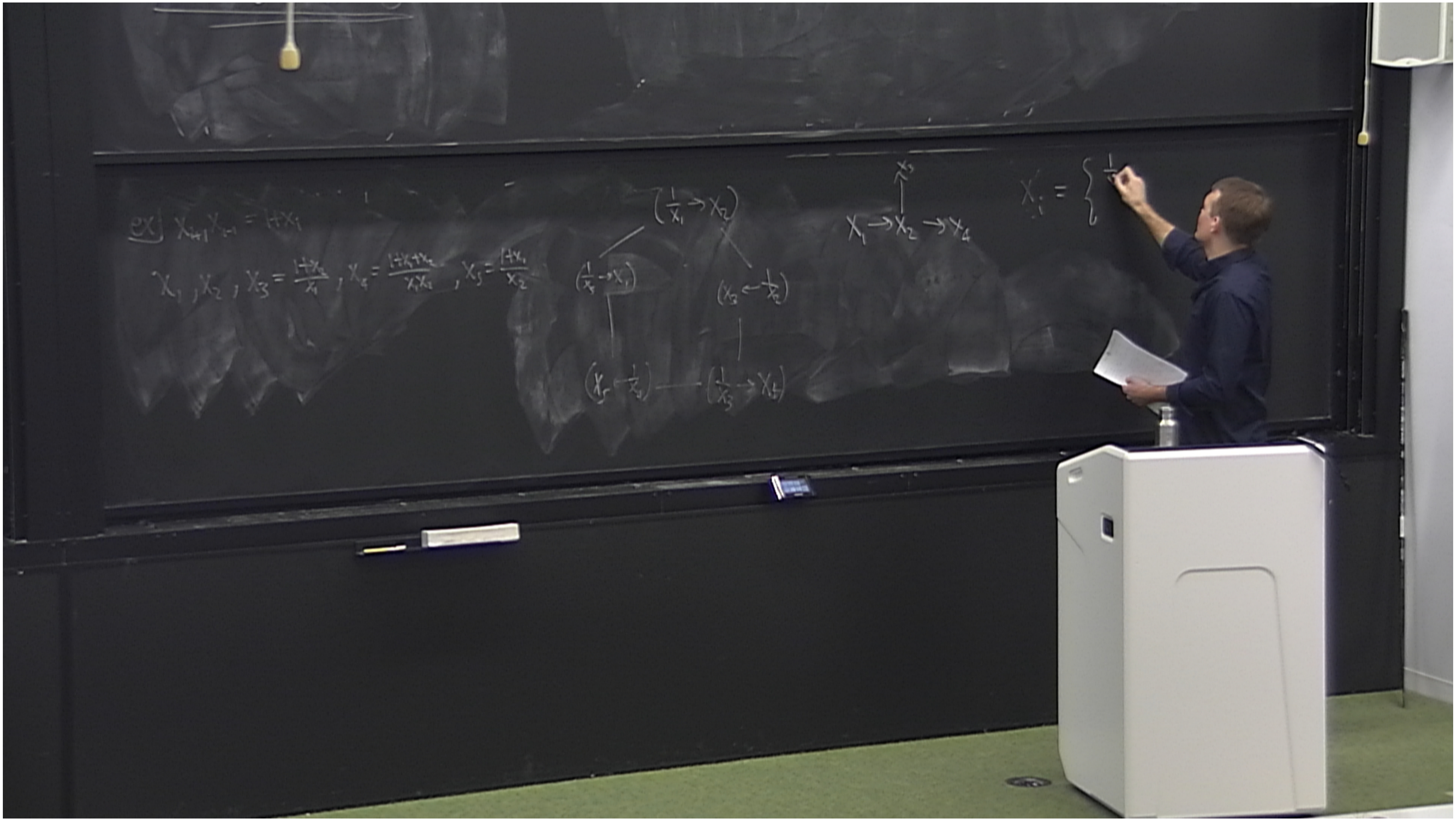
(4,n) has a cluster algebra!
χ-coord

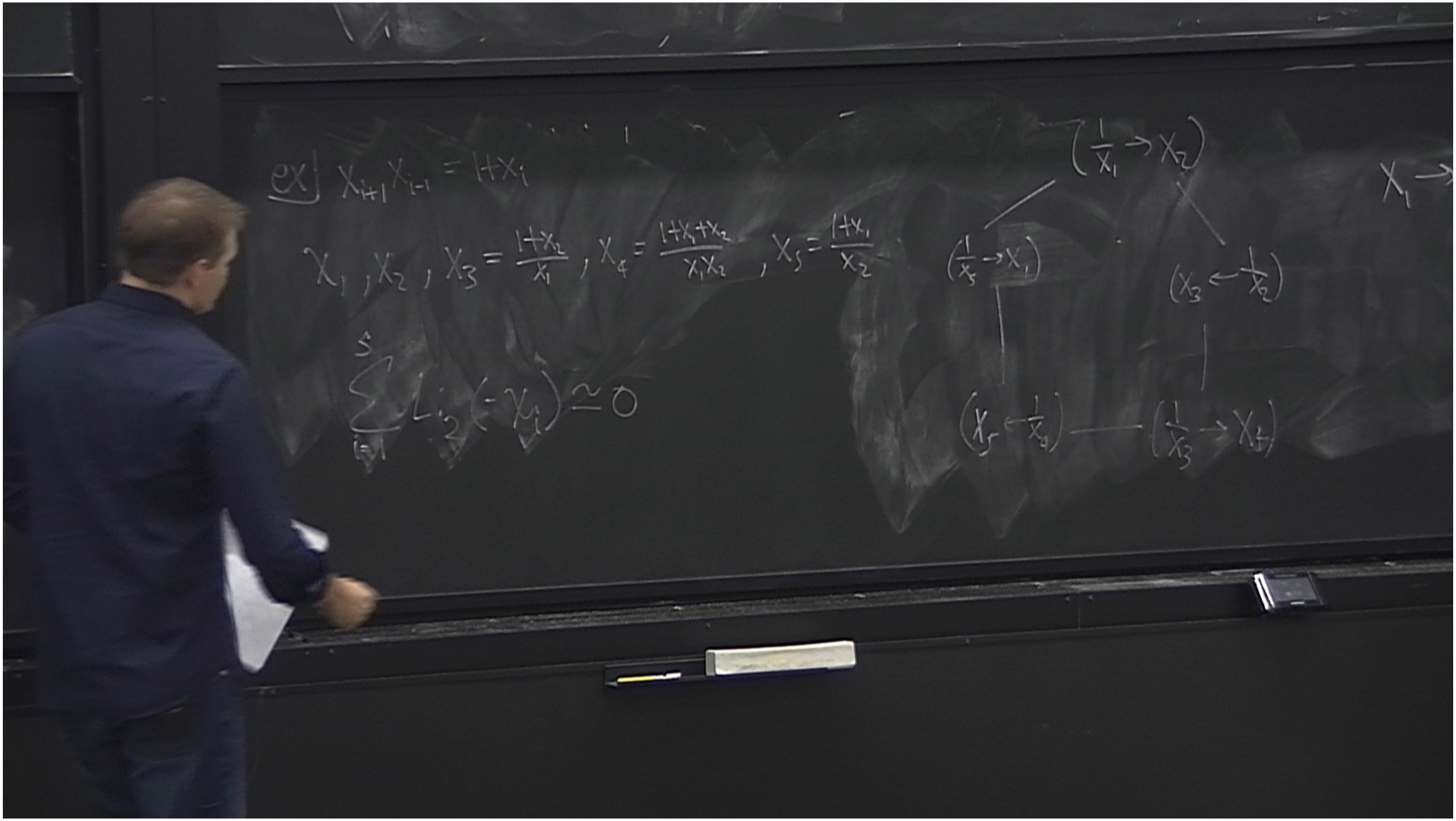

ex] X_{i+1} X_{i-1} = 1+X_i
X_1 , X_2 ,
X_4 = (1+X_1+X_2)/(X_1X_2) , X_5 = (1+X_1)/X_2 , X_6 = X_1 , X_7 = X_2 . . .

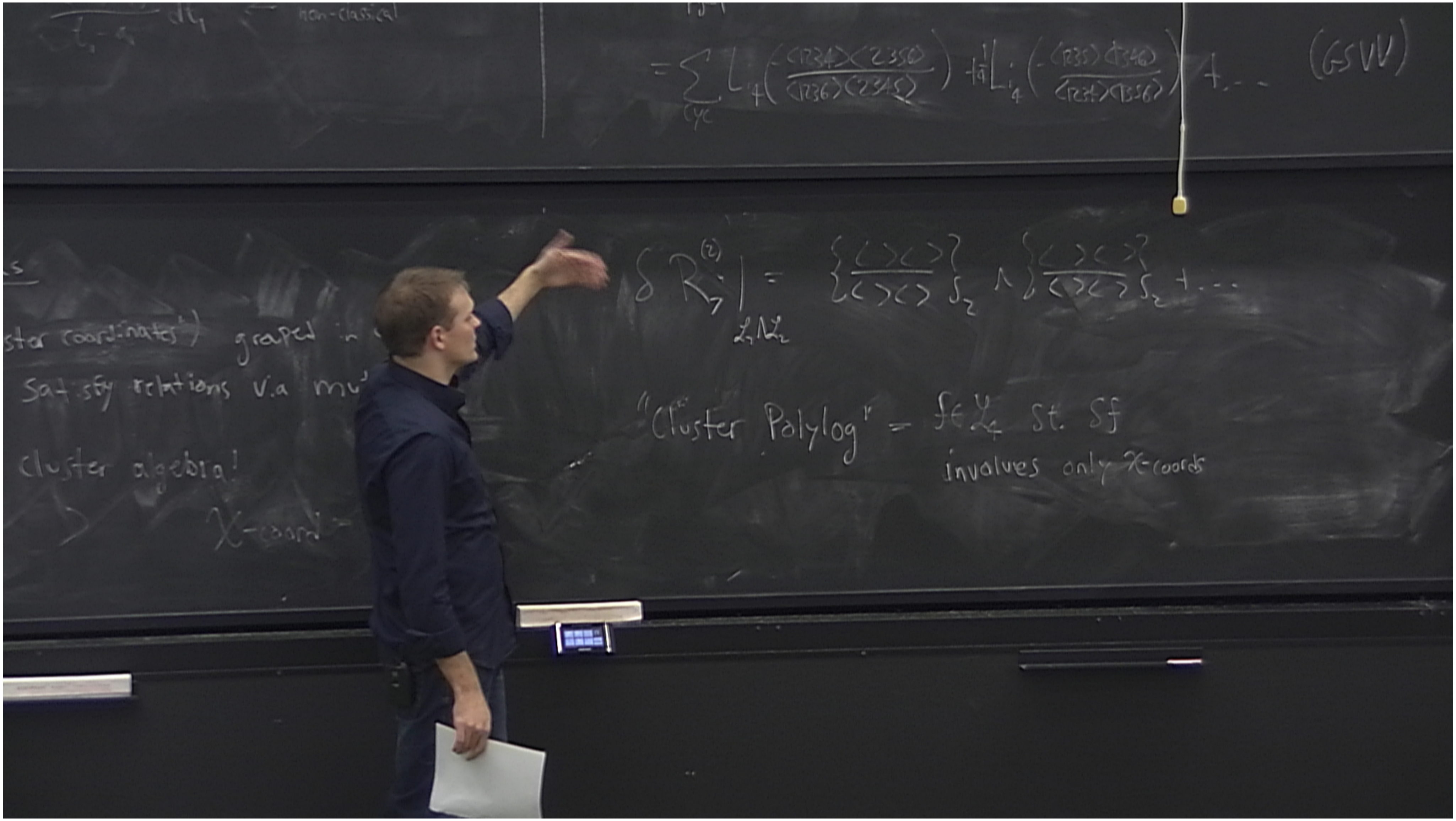


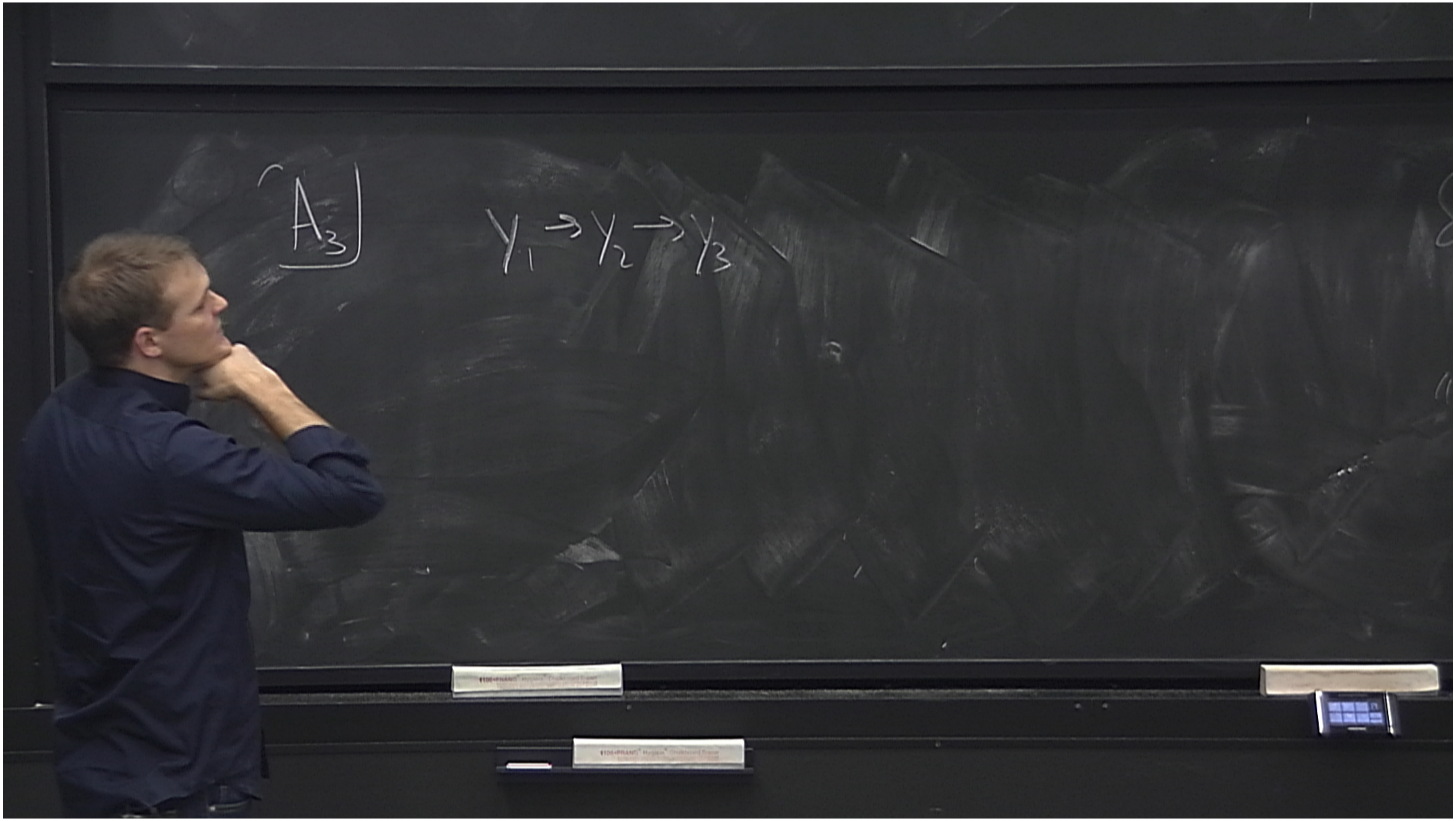
ex] $X_{i+1}X_{i-1} = 1+X_i$
$X_1, X_2, X_3 = \frac{1+X_2}{X_1}, X_4 = \frac{1+X_1+X_2}{X_1X_2}, X_5 = \frac{1+X_1}{X_2}$
$(\frac{1}{X_1} \to X_2)$
$(\frac{1}{X_5} \to X_1)$
$(X_3 \leftarrow$
$(X_5 \leftarrow \frac{1}{X_4})$
$(\frac{1}{X_3} \to X_4)$

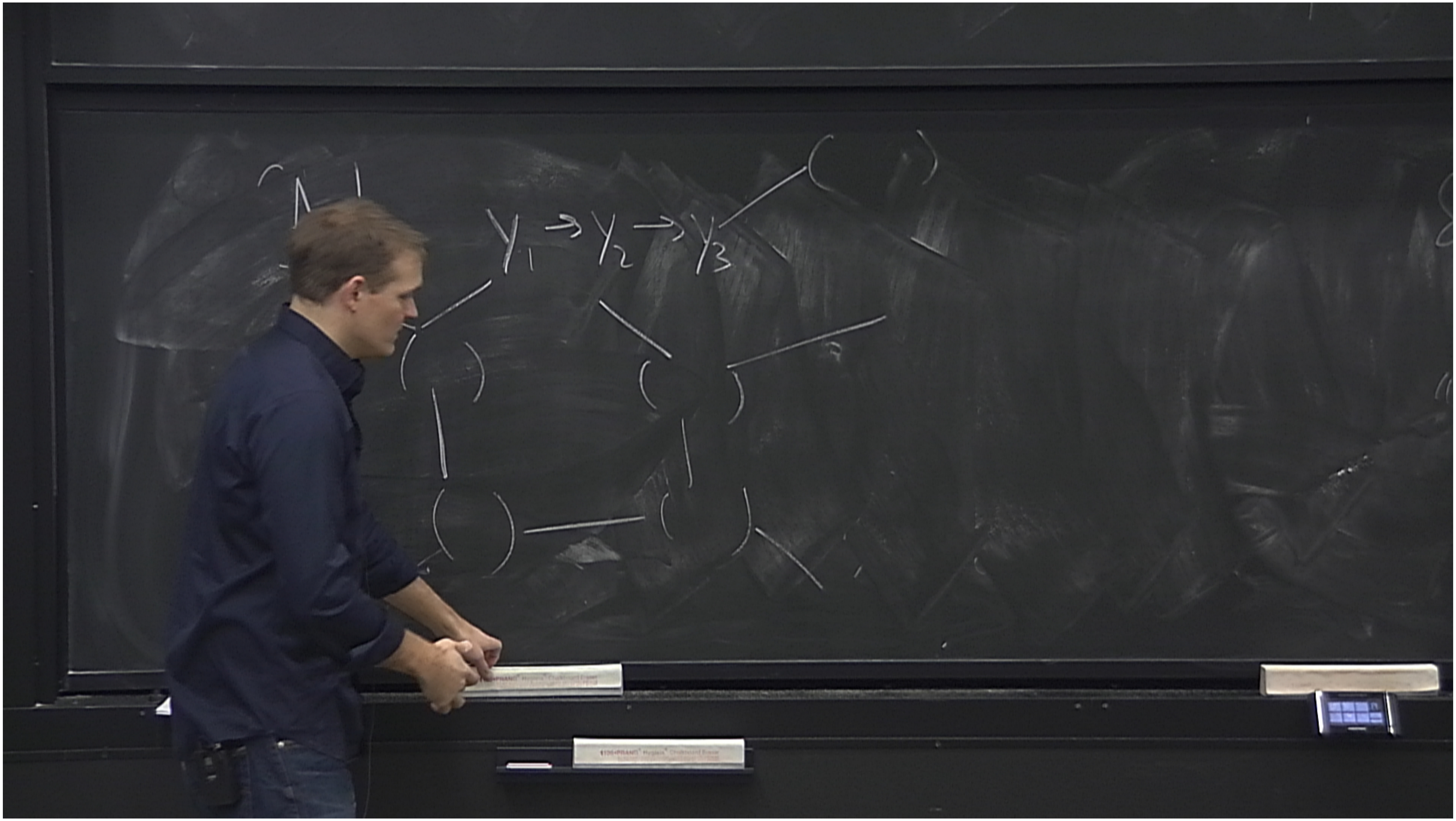

ex] $X_{i+1}X_{i-1} = 1+X_i$
$X_1, X_2, X_3 = \frac{1+X_2}{X_1}, X_4 = \frac{1+X_1+X_2}{X_1X_2}, X_5 = \frac{1+X_1}{X_2}$


ex)
X1 , X2 , X3 = 1+X2 / X1 , X4 = 1+X1+X2 / X1X2 , X5 = 1+X1 / X2
X1 → X2 → X4
X_i = {

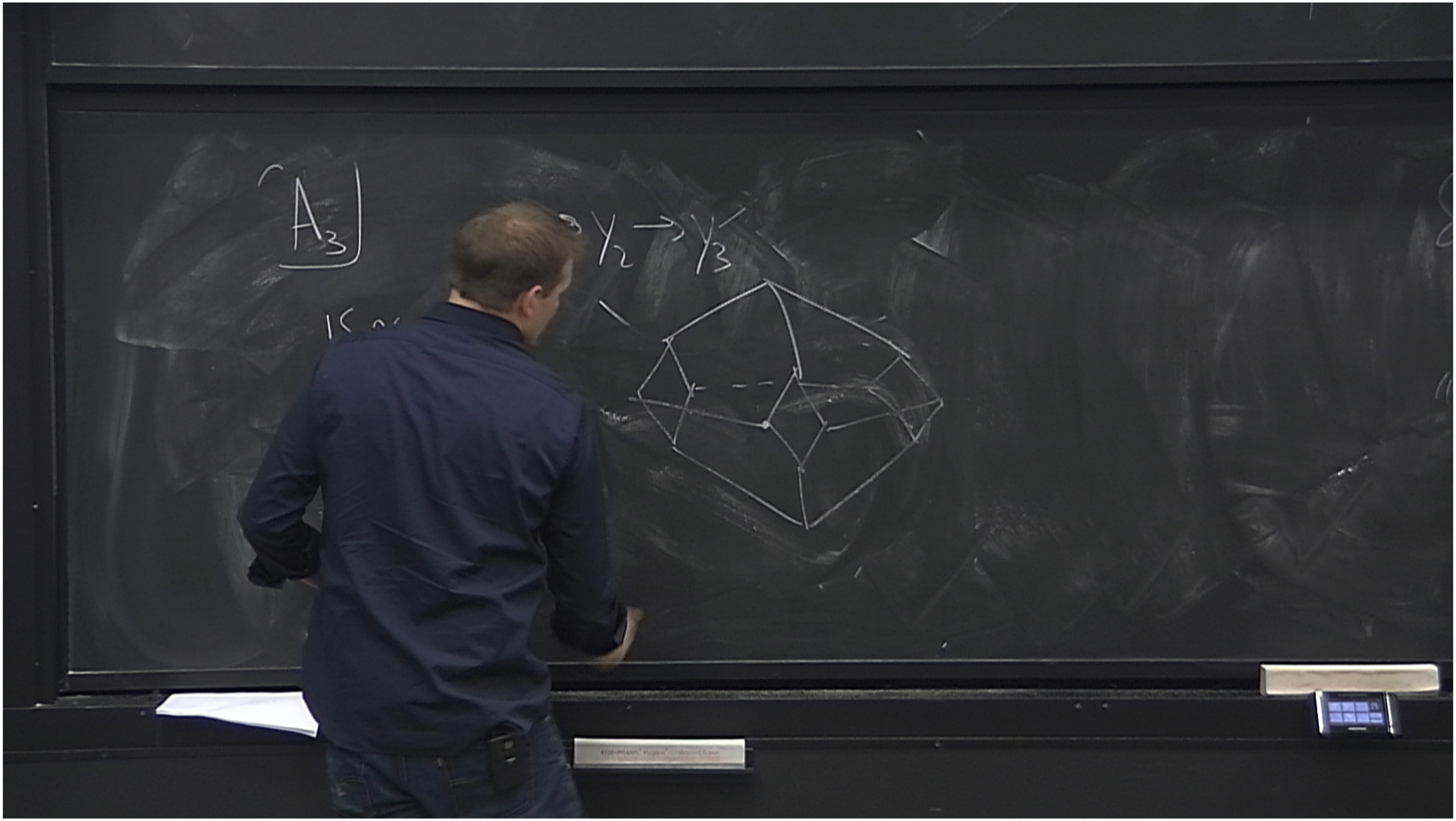

ex) X_{i+1} X_{i-1} = 1+X_i
X_1 , X_2 , X_3 = (1+X_2)/X_1 , X_4 = (1+X_1+X_2)/(X_1 X_2) , X_5 = (1+X_1)/X_2
∑_{i=1}^{5} Li_2(-X_i) ≃ 0
(1/X_1 → X_2)
(1/X_5 → X_1)
(X_3 ← 1/X_2)
(X_5 ← 1/X_4)
(1/X_3 → X_4)
X_1 →

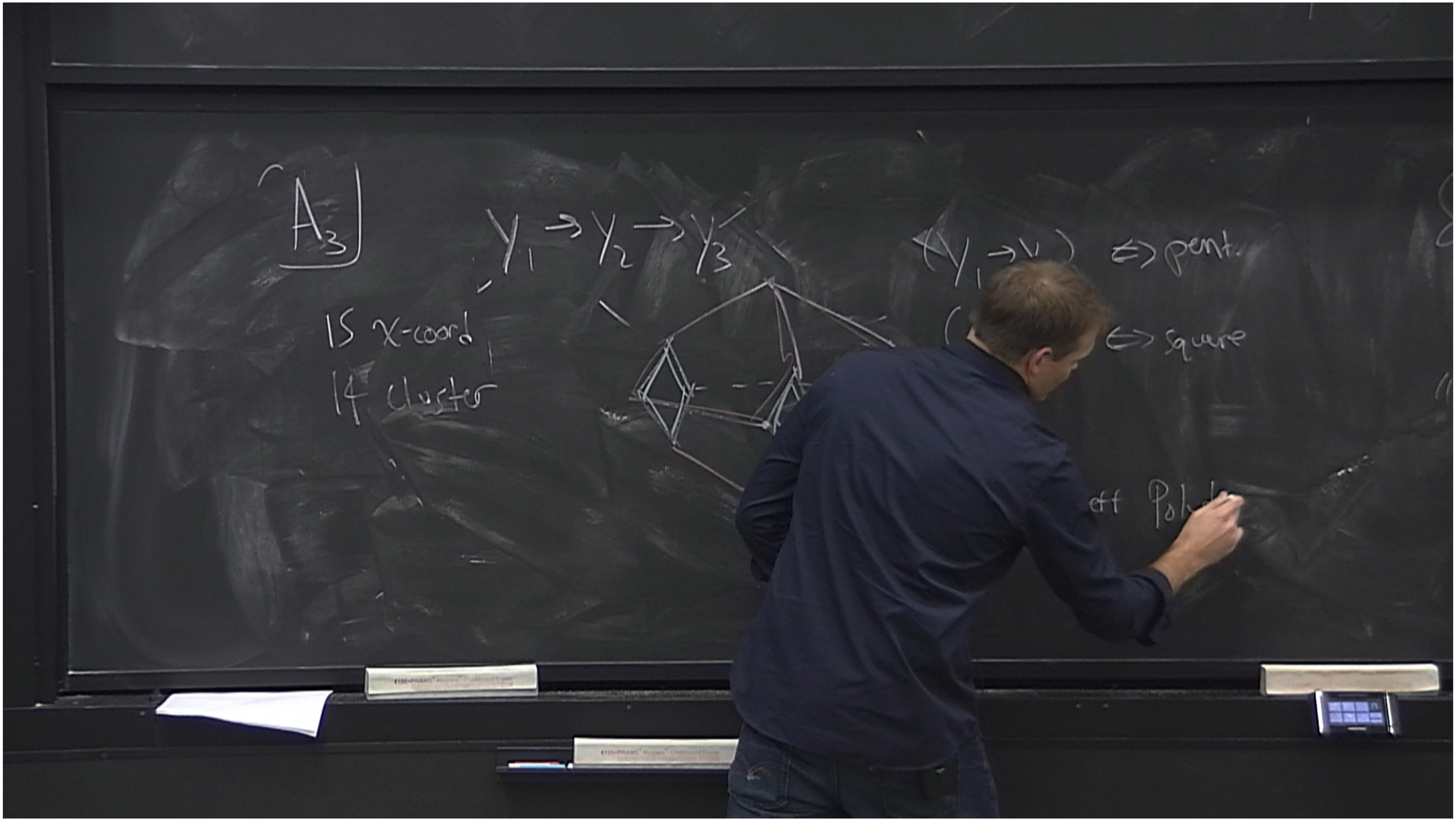

non-classical
(GSVV)
satisfy relations via
cluster algebra!
X-coord
"Cluster Polylog" = f ∈ L4 st. δf
involves only X-coords

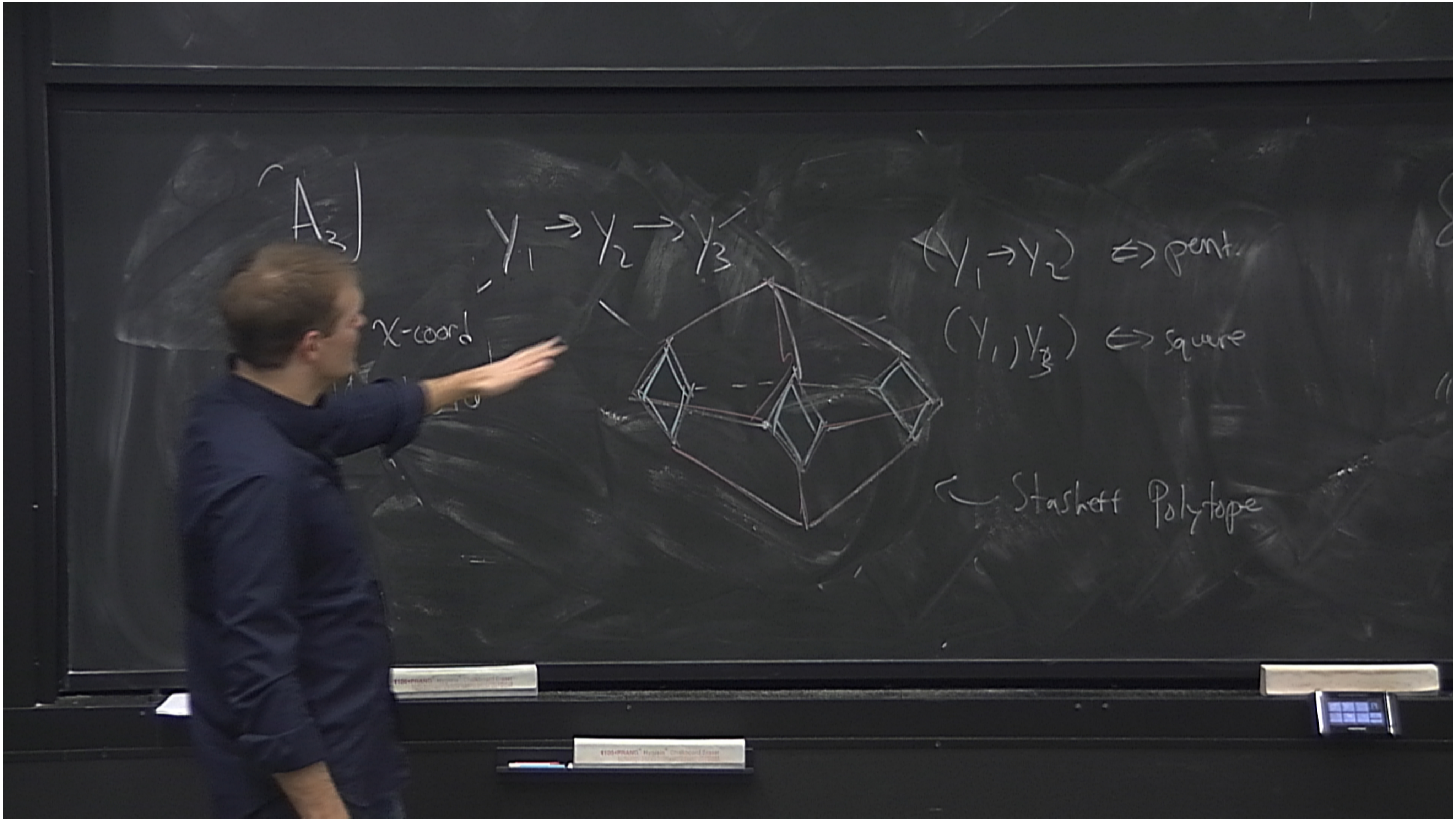

A_3
$Y_1 \rightarrow Y_2 \rightarrow Y_3$

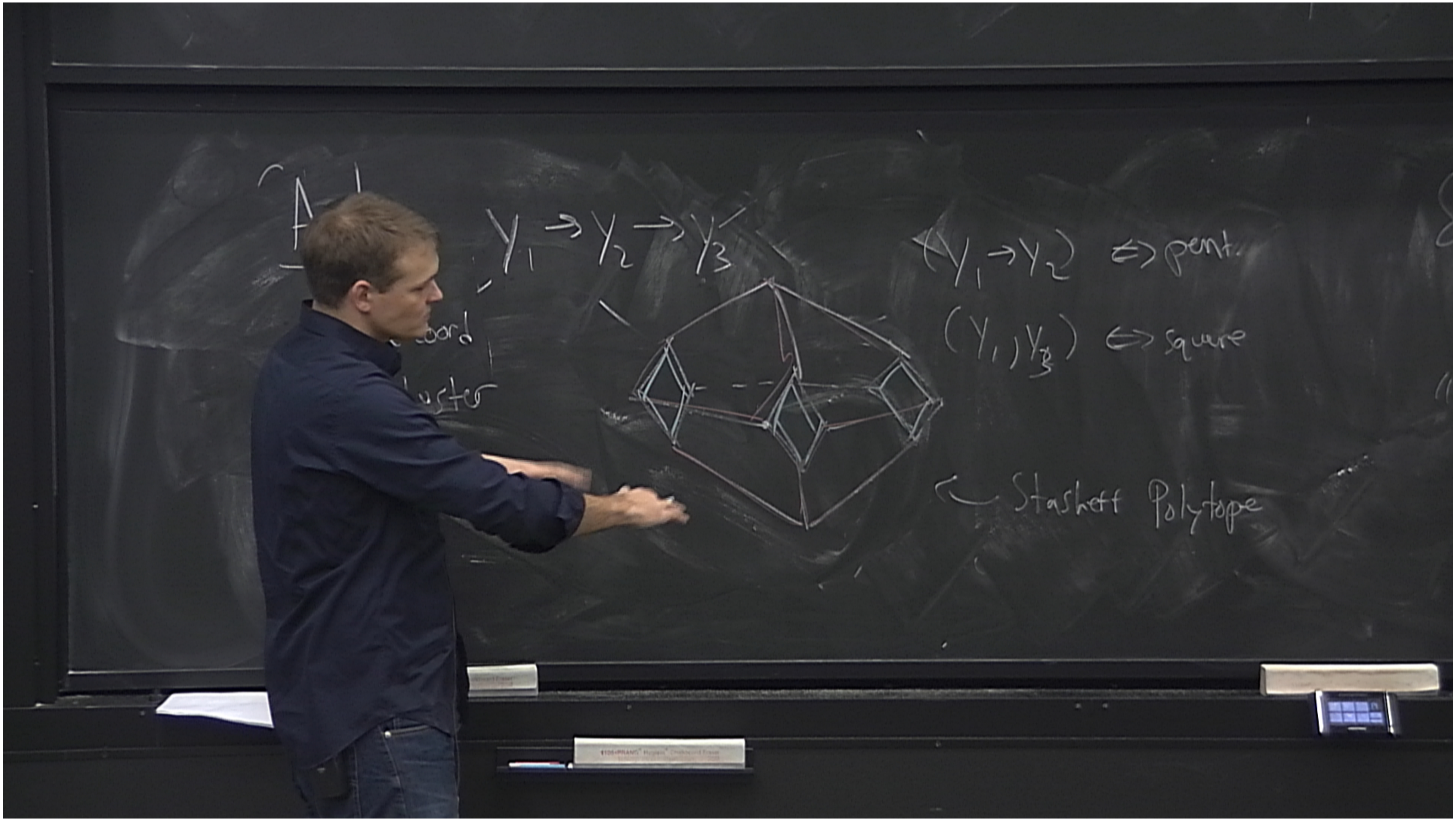


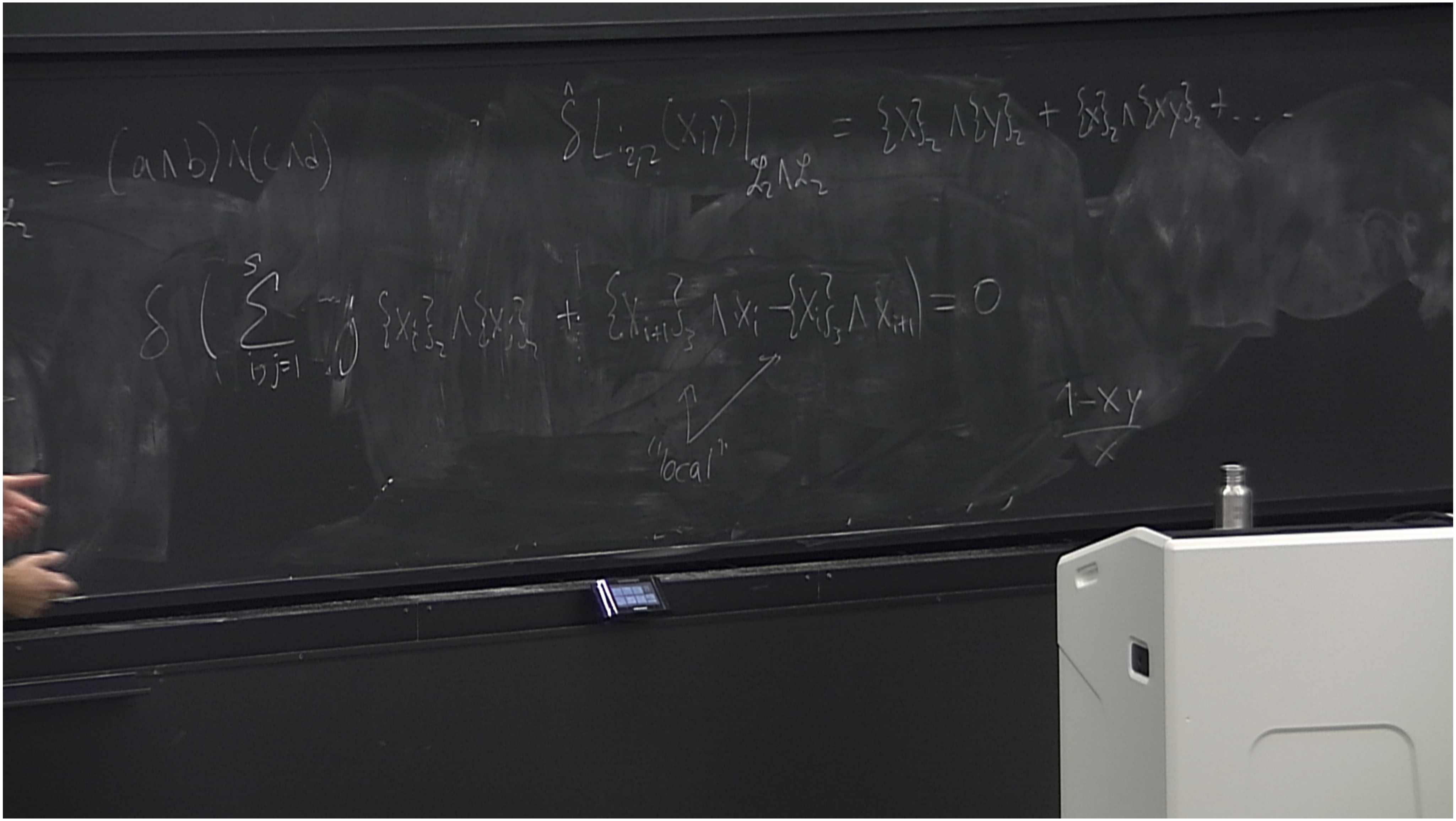
y1 → y2 → y3


A3
Y2 → Y3


A3
y1 → y2 → y3
15 x-coord
14 cluster
⇔ pent.
⇔ square


A3)
Y1 → Y2 → Y3
x-coord
(Y1 → Y2) ⟺ pent.
(Y1, Y3) ⟺ square
Stasheff Polytope


$Y_1 \to Y_2 \to Y_3$
$(Y_1 \to Y_2) \Leftrightarrow$ pent.
$(Y_1, Y_3) \Leftrightarrow$ square
Stasheff Polytope


= (a∧b)∧(c∧d)
"local"
1-xy
x