

Title: Causality, holography, and entanglement entropy

Date: Feb 20, 2014 01:00 PM

URL: <http://pirsa.org/14020146>

Abstract: After giving a brief overview of holographic entanglement entropy formulas, I will explore a curious feature they imply: when the bulk spacetime includes a black hole, the entanglement entropies often appear to depend on the spacetime geometry inside the horizon. I will ask whether this implies any loss of causality in the field theory. To answer this question, I will present a new general-relativity theorem concerning the causal structure of asymptotically AdS spacetimes, which implies an interesting relationship between bulk and boundary causal domains.

Causality, holography, and entanglement entropy

w/ Hubeny, Lawrence, Rangamani

I. Causality + EE in QFT

II. Holographic EE formulas

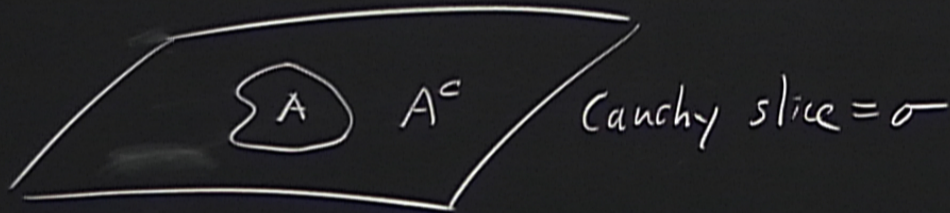
III. HRT + causality

IV. A theorem

V. The dual of P_A

CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARDER
OR THE BOARDER'S BOARD
OR THE BOARDER'S BOARD

I.



State: ρ_σ

Hilbert space: $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{A^c}$

Reduce state: $\rho_A = \text{Tr}_{\mathcal{H}_{A^c}} \rho_\sigma$

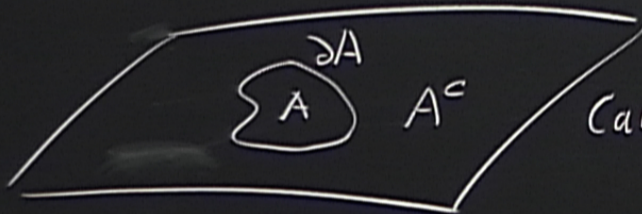
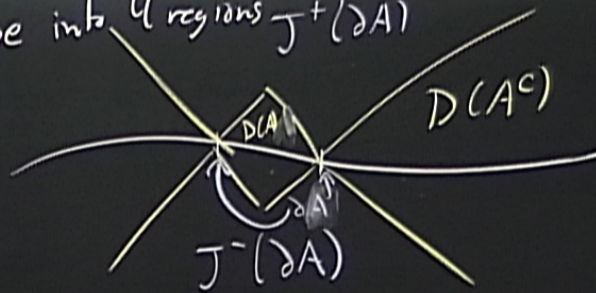
EE: $S(A) = -\text{tr} \rho_A \ln \rho_A$

CAUTION

BE CAREFUL AND ALWAYS USE PROTECTIVE EQUIPMENT
WHEN OPERATING OR MAINTAINING THIS EQUIPMENT.
DO NOT OPERATE OR MAINTAIN THIS EQUIPMENT
UNLESS YOU ARE TRAINED AND QUALIFIED TO DO SO.
ALWAYS WEAR YOUR SAFETY GEAR.

I.

Decompose into 4 regions $J^+(\partial A)$



Cauchy slice = σ



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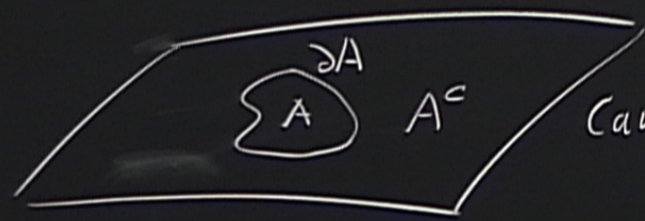
EE: $S(A) = -\text{tr} \rho_A \ln \rho_A$

$$\rho_{A'} = U \rho_A U^{-1} \quad S(A') = S(A)$$

$S(A), \rho_A$ unchanged in σ''

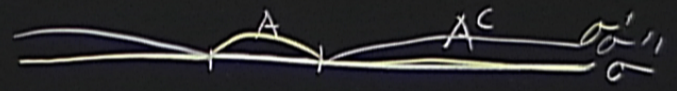
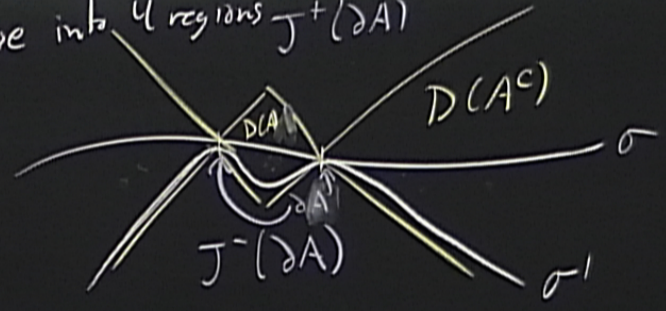
$\Rightarrow S(A)$ actually depends on $D(A)$

I.



Cauchy slice = σ

Decompose into 4 regions $J^+(\partial A)$



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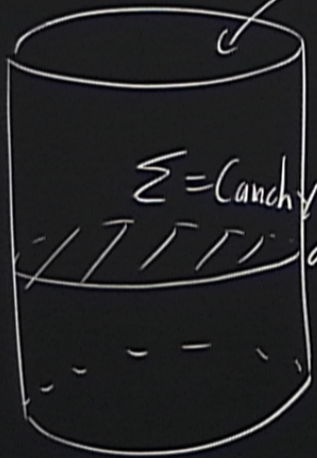
$S(A), \rho_A$ unchanged in σ
 $\Rightarrow S(A)$ actually depends on $D(A)$
 Evolving to the future, local perturbation in $D(A), D(A^c), J^+(\partial A)$
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can't affect $S(A)$.
 $D(A), D(A^c), J^-(\partial A)$

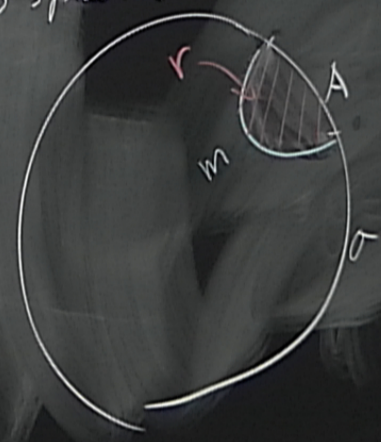
II.

Agin, Jafferis, Kasko, MH. EE for a disk in 2+1 dim'd free Maxwell theory.
Holographic QFT: strongly coupled, large N \Rightarrow bulk spacetime described by classical Einstein gravity

Ryu-Takayanagi '06: Static, $\sigma = \text{constant}$ - the slice
asymptotically AdS spacetime
 $m = \text{minimal surface in bulk}$
s.t. \exists region r s.t. $\partial r = m \cup A$



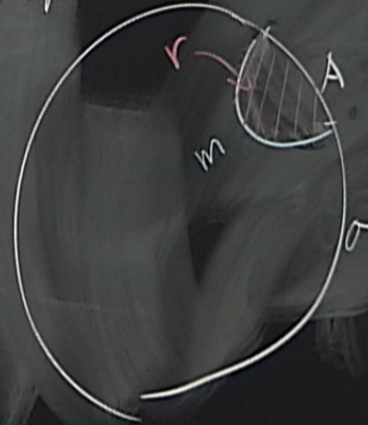
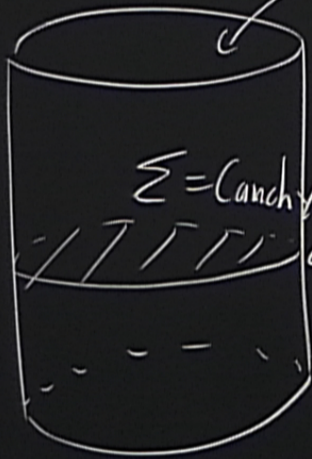
$\Sigma = \text{Cauchy slice for bulk}$



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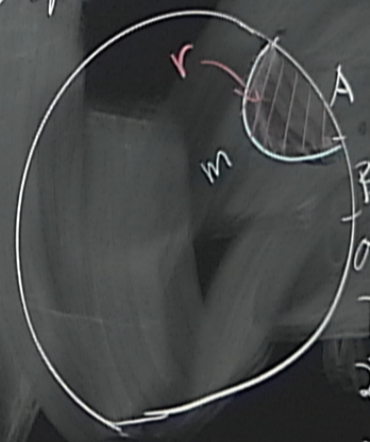
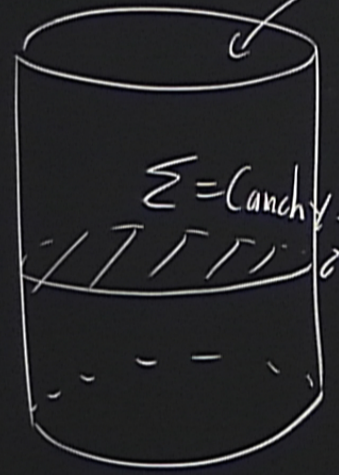
$$S(A) = \frac{1}{4G_N} \text{area}(m)$$

Evidence for RT:

- 1) Agree w/ first-principles calculations of EE's; general argument
- 2) Obeys strong subadditivity + all other known general properties of EE (MH'13)
- 3) Always gives reasonable answers.

Agon, Jafferis, Kasko, MH: EE for a disk in $z=0$ plane
 Holographic QFT: strongly coupled, large N \Rightarrow bulk spacetime described by classical Einstein gravity

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 asymptotically AdS spacetime



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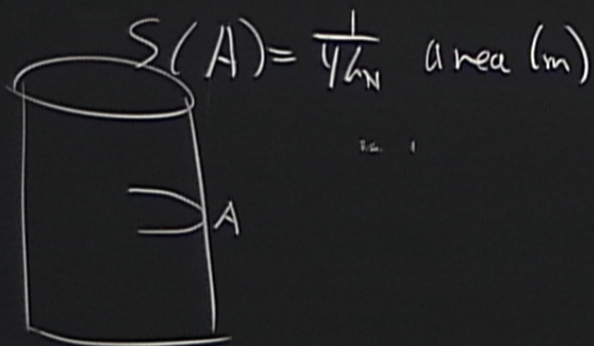
σ Properties of map r

- 1) $r(\sigma) = \Sigma$
- 2) $r(AB) > r(A)$ monotone
- 3) $r(A) \cap r(B) = \emptyset$

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Huhtani-Rangamani-Takayanagi '07: covariant




$$S(A) = \frac{1}{4G_N} \text{area}(A)$$

$m = \text{extremal codimension-2 surface s.t.}$
 $\exists (\text{spacelike}) r \text{ s.t. } \partial r = m \cup A$

Evidence for RT:

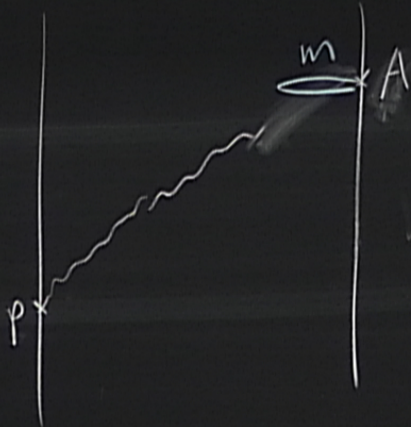
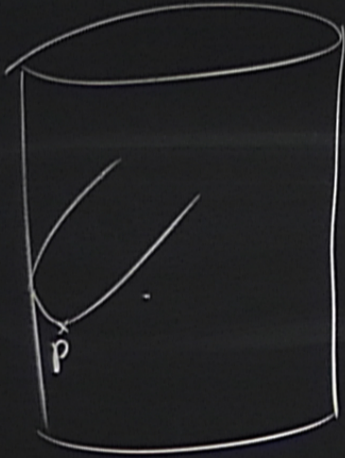
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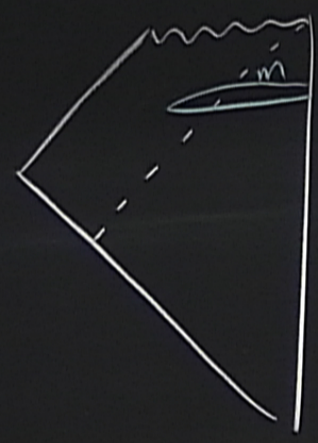
The diagram shows a cylinder representing a spacetime region. A shaded cross-section is labeled 'm', and the boundary of the cylinder is labeled 'A'. A red arrow points from the text 'm = extremal codimension-2 surface s.t. ∃ (spacelike) r s.t. ∂r = m ∪ A' to the shaded cross-section 'm'.

III Causality + holography

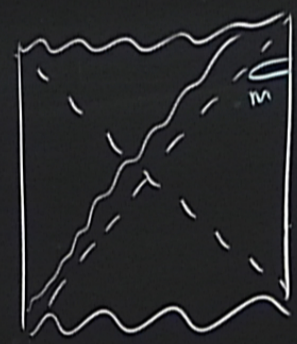


Are there shortcuts through the bulk?
Gao-Wald '00: No, if bulk matter fields obey null energy condition.

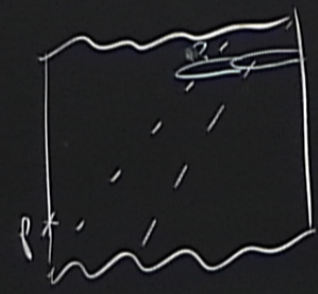
Abaño-Arreastia-Aparicio-Lopez '10. m goes behind event horizon
+ apparent



Bug or feature?



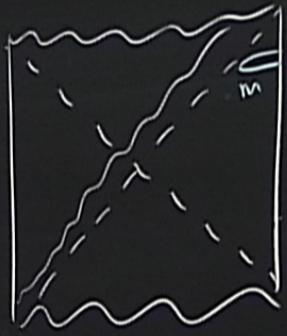
External BH
Entangled state of 2 non-interacting CFTs
(EPR pair)



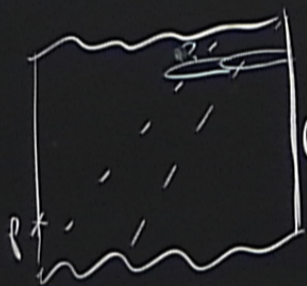
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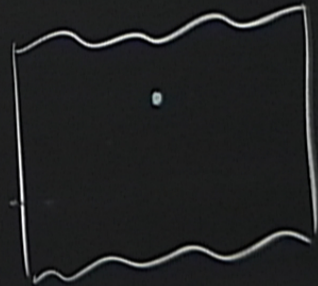
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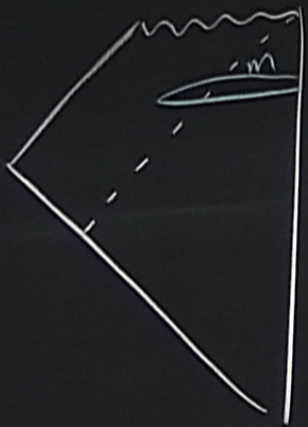
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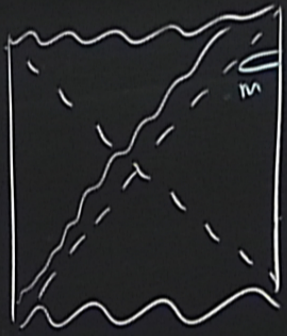
Ex: $A = CFT_2$
 $\partial A = \phi$



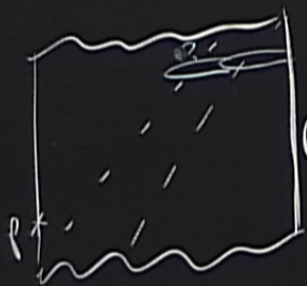
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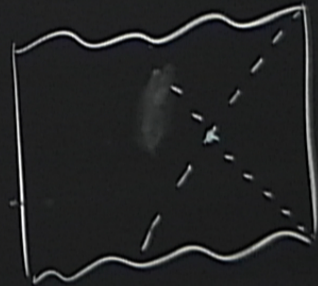
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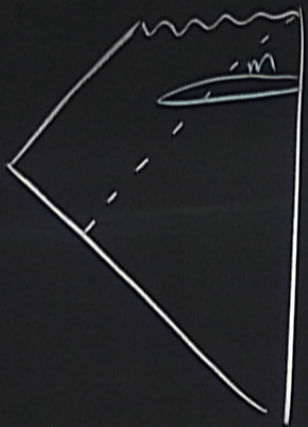
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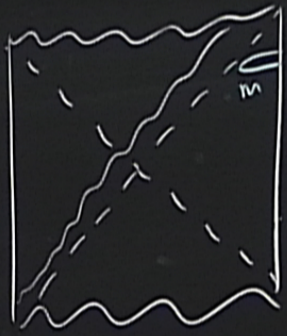
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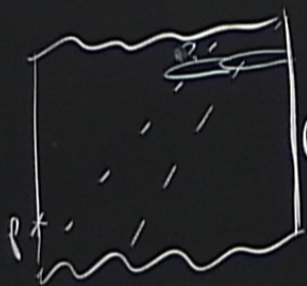
Abaño-Arrestita-Aparicio-Lopez '10: m goes behind event horizon + apparent



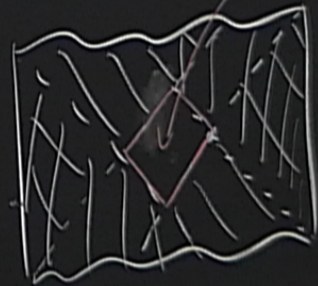
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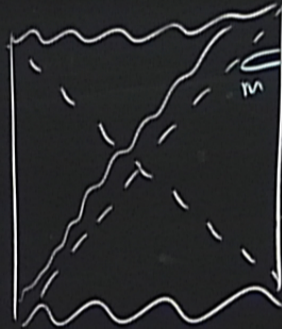
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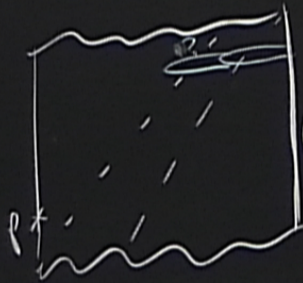


Bug or feature?



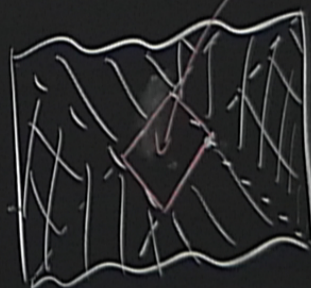
External BH

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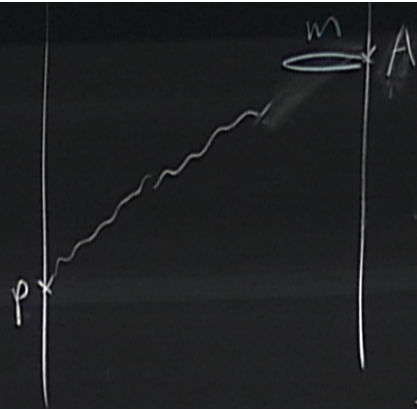
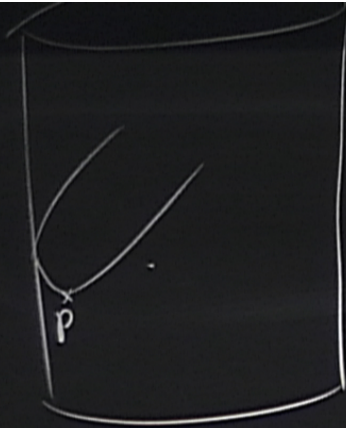


(CFT₂)

Ex: $A = CFT_2$
 $\partial A = \phi$

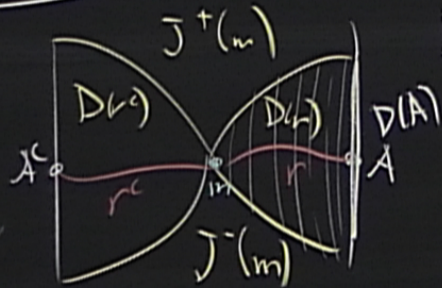


For consistency of HRT,
need m 's causal shadow.



Gao-Wald '00. No, if fields obey null energy condition.

Thm:



$$\text{NEC} \Rightarrow D(r) \cap \text{boundary} = D(A)$$

$$D(r) \cap \text{ " } = D(A^c)$$

$$J^{\pm}(m) \cap \text{ " } = J^{\pm}(\partial A)$$

\Rightarrow HRT is consistent w/ causality

CAUTION