Title: Psi-epistemic models are exponentially bad at explaining the distinguishability of quantum states

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Abstract: The status of the quantum state is perhaps the most controversial issue in the foundations of quantum theory. Is it an epistemic state (representing knowledge, information, or belief) or an ontic state (a direct reflection of reality)? In the ontological models framework, quantum states correspond to probability measures over more fundamental states of reality. The quantum state is then ontic if every pair of pure states corresponds to a pair of measures that do not overlap, and is otherwise epistemic. Recently, several authors have derived theorems that aim to show that the quantum state must be ontic in this framework. Each of these theorems involve auxiliary assumptions of varying degrees of plausibility. Without such assumptions, it has been shown that models exist in which the quantum state is epistemic. However, the definition of an epistemic quantum state used in these works is extremely permissive. Only two quantum states need correspond to overlapping measures and furthermore the amount of overlap may be arbitrarily small. In order to provide an explanation of quantum phenomena such as no-cloning and the indistinguishability of pure states, the amount of overlap should be comparable to the inner product of the quantum states. In this talk, I show, without making auxiliary assumptions, that the ratio of overlap to inner product must go to zero exponentially in Hilbert space dimension for some families of states. This is done by connecting the overlap to Kochen-Specker noncontextuality, from which we infer that any contextuality inequality gives a bound on the ratio of overlap to inner product.

ψ-epistemic models are exponentially bad at explaining the distinguishability of quantum states

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Based on:
arXiv:1401.7996
PRL 110:120401 (2013) arXiv:1208.5132
Review article: to appear

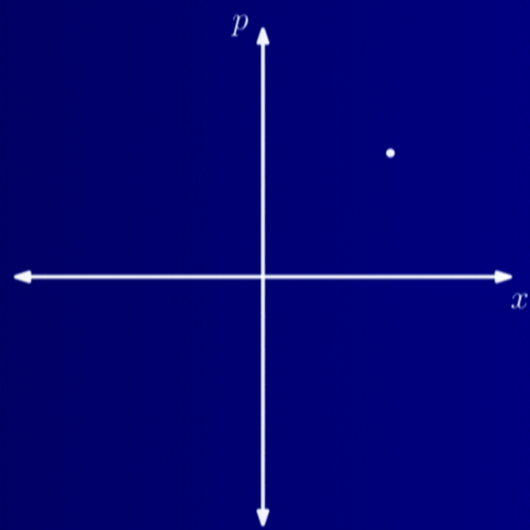
18th February 2014

ψ-epistemic vs. ψ-ontic

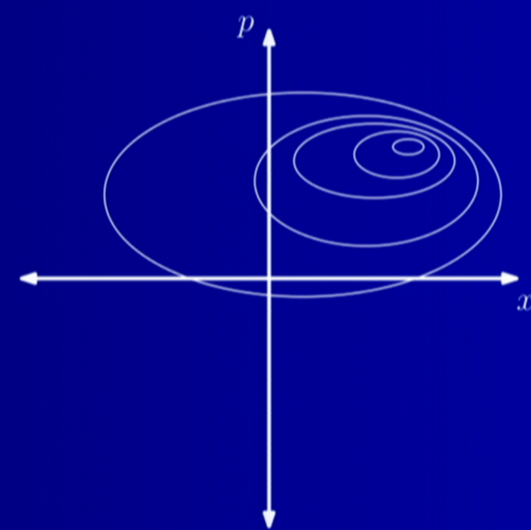
- *Ontic state*: a state of reality.
  - ψ-*ontic*: the quantum state is ontic.
- *Epistemic state*: a state of knowledge or information.
  - ψ-*epistemic*: the quantum state is epistemic.

Classical states

Ontic state



Epistemic state



ψ-epistemicists

Source: http://en.wikipedia.org/

There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature. — Niels Bohr[a]

[t]he ψ-function is to be understood as the description not of a single system but of an ensemble of systems. — Albert Einstein[b]

[a] Quoted in A. Petersen, "The philosophy of Niels Bohr", *Bulletin of the Atomic Scientists* Vol. 19, No. 7 (1963)

[b] P. A. Schilpp, ed., *Albert Einstein: Philosopher Scientist* (Open Court, 1949)

Interpretations of quantum theory

	ψ-epistemic	ψ-ontic
Anti-realist	Copenhagen neo-Copenhagen (e.g. QBism, Healey, Peres Mermin, Zeilinger)	
Realist	Einstein Ballentine? Spekkens Me ?	Dirac-von Neumann Many worlds Bohmian mechanics Spontaneous collapse Modal interpretations

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Overview

Introduction

Arguments for Epistemic Quantum States

Arguments for Ontic Quantum States

Ontological Models

ψ-ontology theorems

ψ-epistemic models

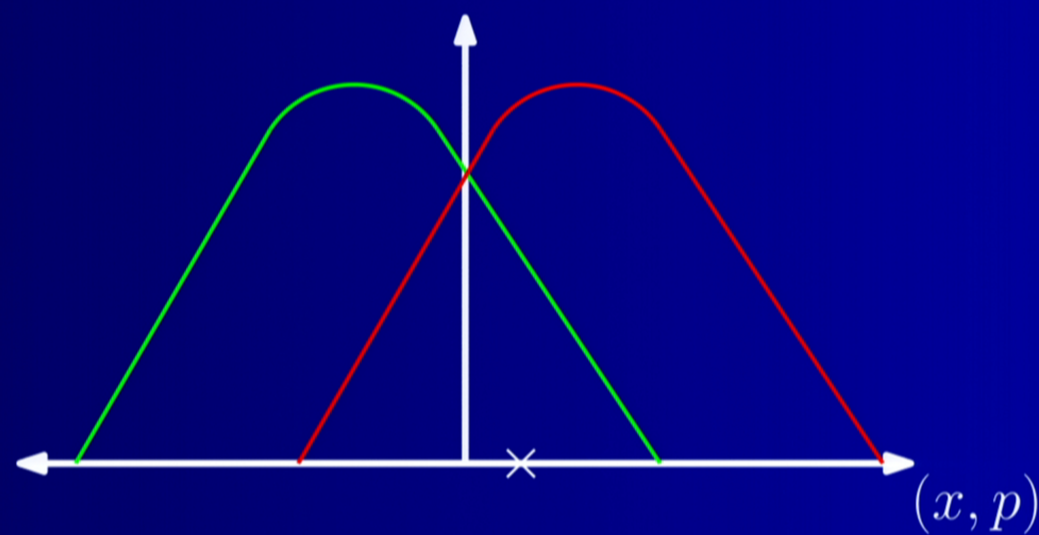
Overlap measures

Overlap bounds

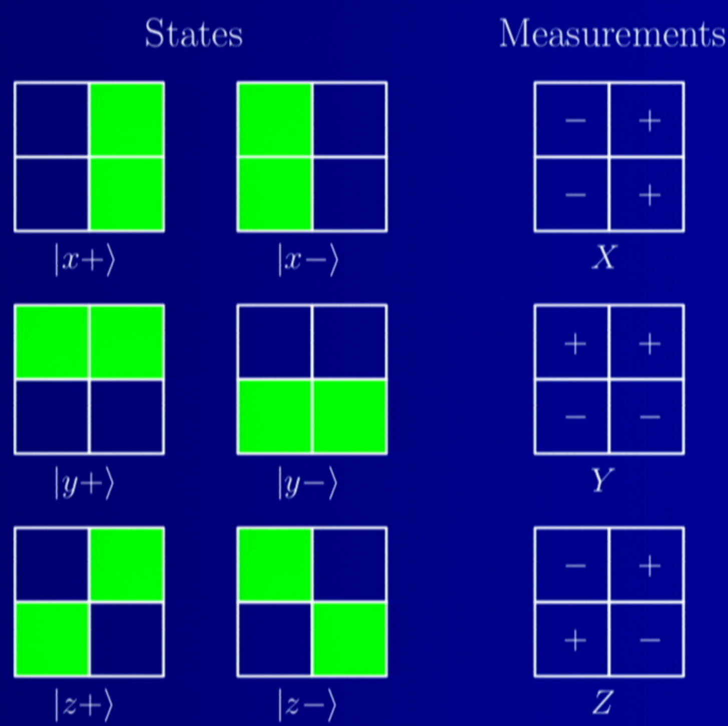
Conclusions

Arguments for Epistemic Quantum States

Epistemic states overlap



Spekkens' toy theory



R. W. Spekkens, *Phys. Rev. A* 75(3):032110 (2007) arXiv:quant-ph/0401052

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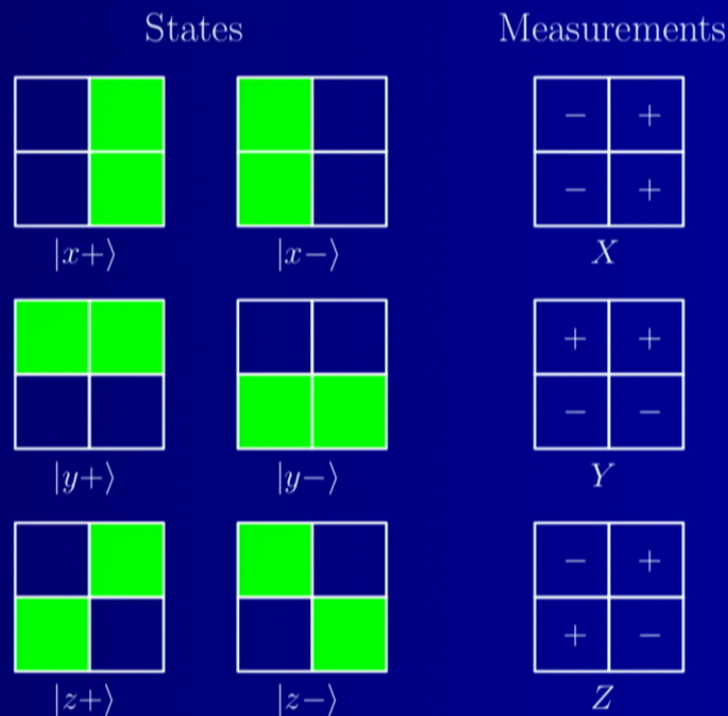

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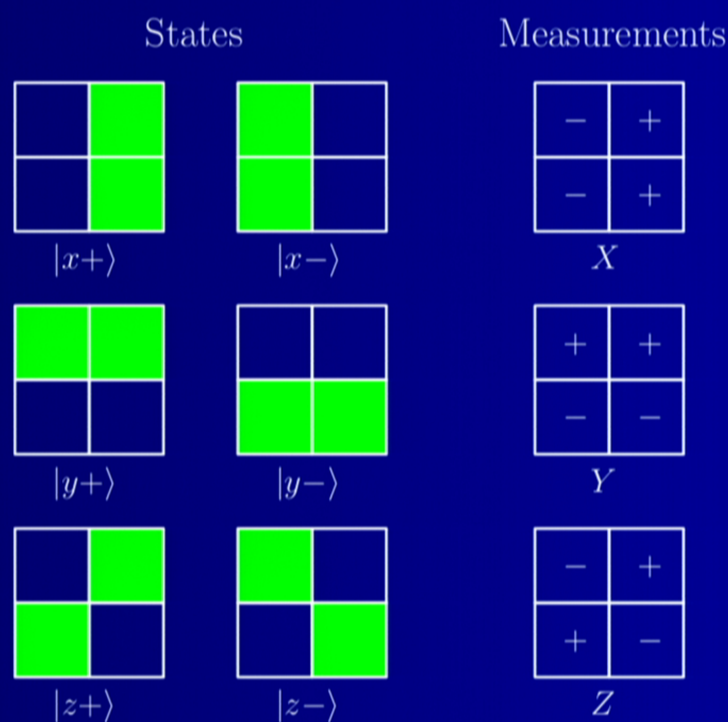

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Other arguments

- Collapse of the wavefunction
- Generalized probability theory
- Excess baggage

Arguments for ontic quantum states

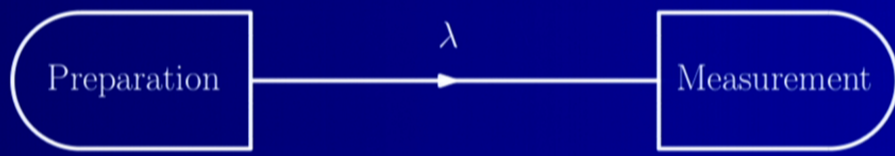
- Interference
- Eigenvalue-eigenstate link
- Lack of imagination
- Quantum computing

Prepare-and-measure experiments: Quantum description

$|\psi\rangle \in \mathbb{C}^d$ $\qquad$ $M = \{|a\rangle, |b\rangle, \ldots\}$

$$\text{Prob}(a|\psi, M) = |\langle a|\psi\rangle|^2$$

Prepare-and-measure experiments: Ontological description



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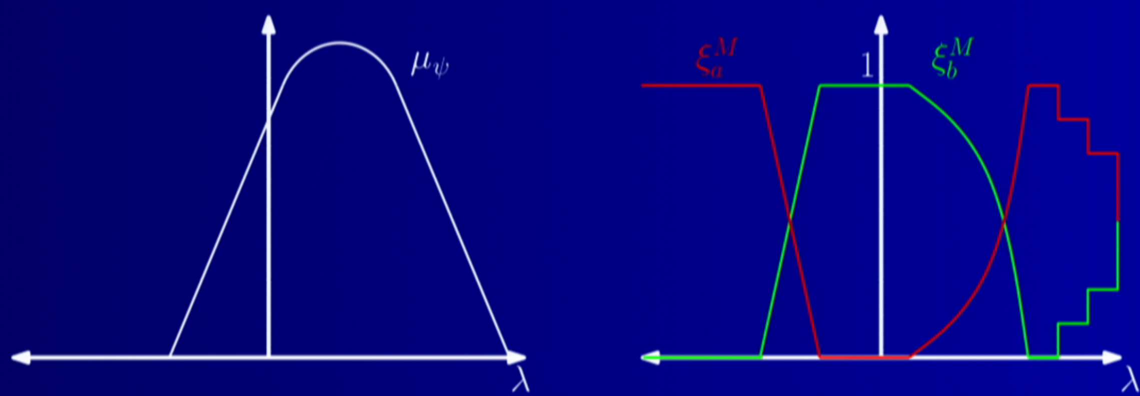

$$\mathrm{Prob}(a|\psi, M) = \int \xi_a^M(\lambda) d\mu_\psi$$

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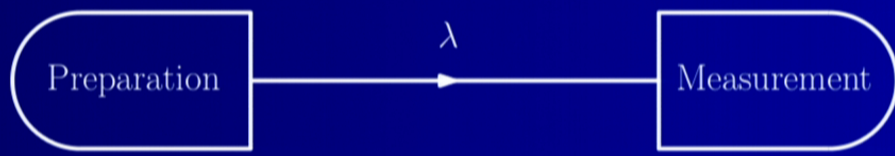

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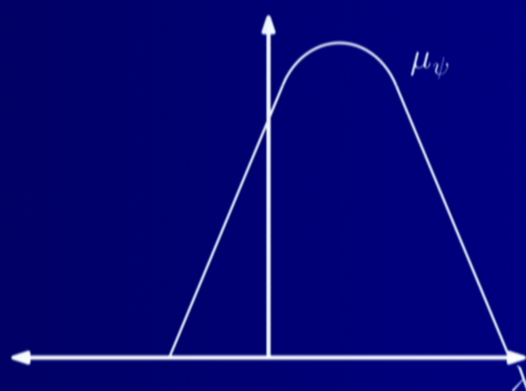

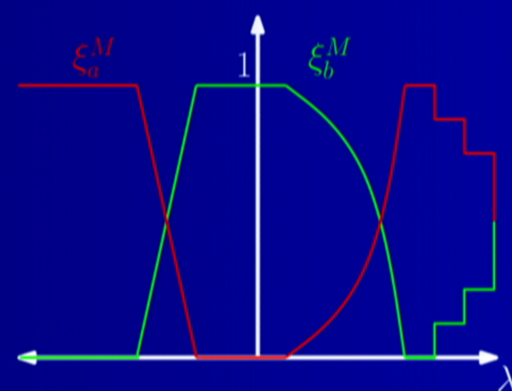

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Formal definition

An ontological model for $\mathbb{C}^d$ consists of:

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An ontological model for $\mathbb{C}^d$ consists of:

- A measurable space (Λ, Σ).
- For each state $|\psi\rangle \in \mathbb{C}^d$, a probability measure $\mu_\psi : \Sigma \to [0,1]$.
- For each orthonormal basis $M = \{|a\rangle, |b\rangle, \ldots\}$, a set of response functions $\xi_a^M : \Lambda \to [0,1]$ satisfying

$$\forall \lambda, \sum_{|a\rangle \in M} \xi_a^M(\lambda) = 1.$$

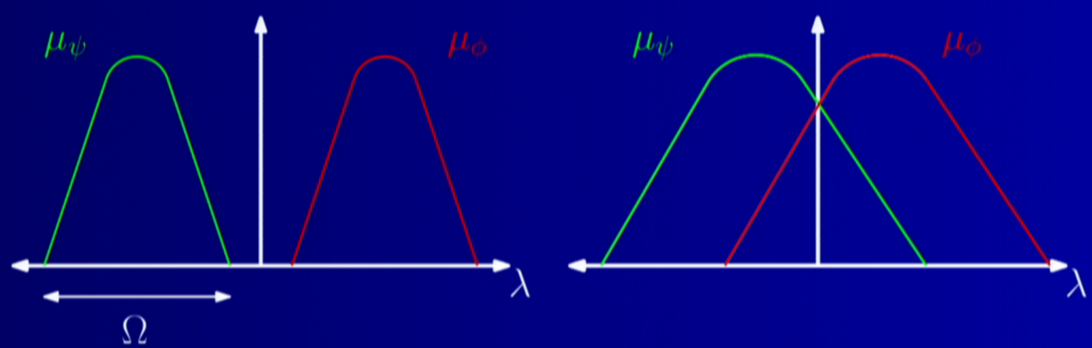
The model is required to reproduce the quantum predictions, i.e.

$$\int_\Lambda \xi_a^M(\lambda) d\mu_\psi = |\langle a|\psi\rangle|^2 .$$

ψ-ontic and ψ-epsitemic models

- $|\psi\rangle$ and $|\phi\rangle$ are *ontologically distinct* in an ontological model if there exists $\Omega \in \Sigma$ s.t.

$$\mu_\psi(\Omega) = 1 \qquad \mu_\phi(\Omega) = 0.$$



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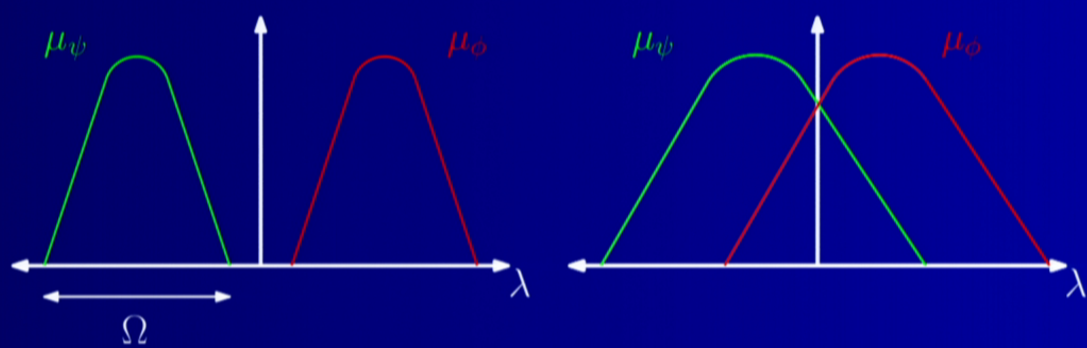

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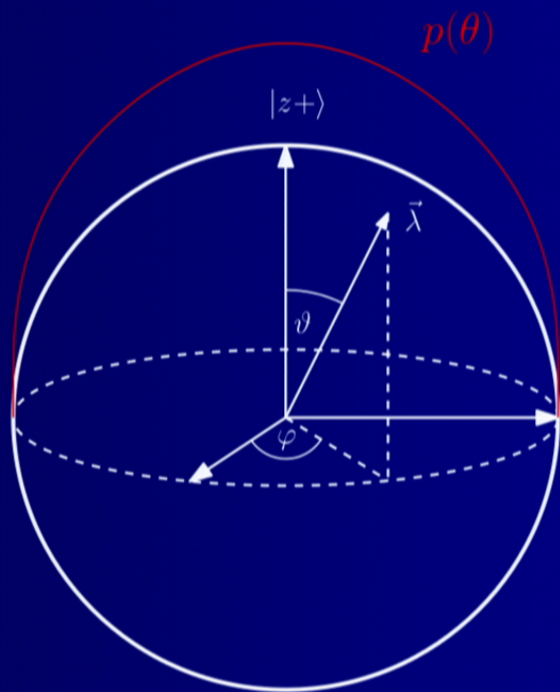
ψ-ontology theorems

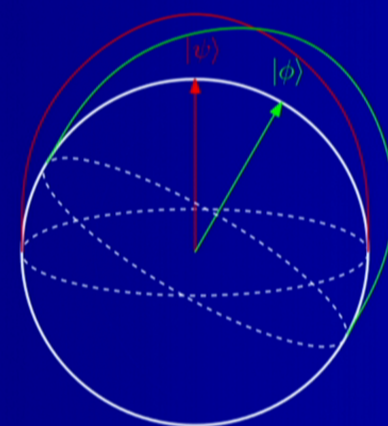
- The Pusey-Barrett-Rudolph theorem: M. Pusey et. al., *Nature Physics*, 8:475–478 (2012) arXiv:1111.3328
- Hardy's theorem: L. Hardy, *Int. J. Mod. Phys. B*, 27:1345012 (2013) arXiv:1205.1439
- The Colbeck-Renner theorem: R. Colbeck and R. Renner, arXiv:1312.7353 (2013).

The Kochen-Specker model for a qubit



$$\mu_{z+}(\Omega) = \int_\Omega p(\vartheta) \sin\vartheta d\vartheta d\varphi$$

$$p(\vartheta) = \begin{cases} \frac{1}{\pi}\cos\vartheta, & 0 \leq \vartheta \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < \vartheta \leq \pi \end{cases}$$



S. Kochen and E. Specker, *J. Math. Mech.*, 17:59–87 (1967)

Models for arbitrary finite dimension

- Lewis et. al. provided a ψ-epsitemic model for all finite d.
  - P. G. Lewis et. al., *Phys. Rev. Lett.* 109:150404 (2012) arXiv:1201.6554
- Aaronson et. al. provided a similar model in which every pair of nonorthogonal states is ontologically indistinct.
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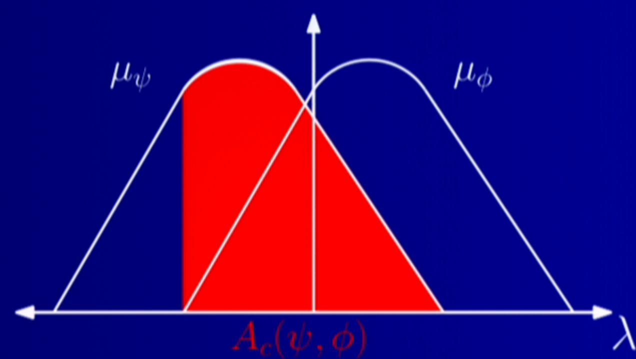
Asymmetric overlap

- *Classical asymmetric overlap*:

$$A_c(\psi,\phi) := \inf_{\{\Omega\in\Sigma \mid \mu_\phi(\Omega)=1\}} \mu_\psi(\Omega)$$



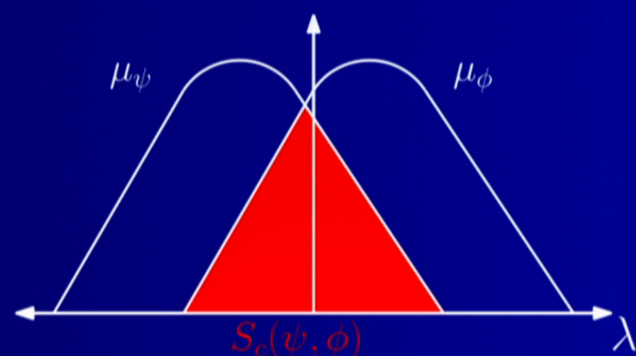
- An ontological model is *maximally ψ-epistemic* if

$$A_c(\psi,\phi) = |\langle\phi|\psi\rangle|^2$$

Classical Symmetric overlap

- *Classical symmetric overlap*:

$$S_c(\psi, \phi) := \inf_{\Omega \in \Sigma} [\mu_\psi(\Omega) + \mu_\phi(\Lambda \backslash \Omega)]$$



- Optimal success probability of distinguishing $|\psi\rangle$ and $|\phi\rangle$ if you know λ:

$$p_c(\psi, \phi) = \frac{1}{2}(2 - S_c(\psi, \phi))$$

Relationships between overlap measures

- Classical overlap measures:

$$S_c(\psi,\phi) \leq A_c(\psi,\phi)$$

- Quantum overlap measures:
  - $S_q(\psi,\phi) = 1 - \sqrt{1 - |\langle\phi|\psi\rangle|^2}$
  - $S_q(\psi,\phi) \geq \frac{1}{2}\,|\langle\phi|\psi\rangle|^2$

- Hence:

$$\frac{S_c(\psi,\phi)}{S_q(\psi,\phi)} \leq 2\frac{A_c(\psi,\phi)}{|\langle\phi|\psi\rangle|^2}.$$

Previous results

- Define:

$$k(\psi,\phi) = \frac{A_c(\psi,\phi)}{|\langle\phi|\psi\rangle|^2}.$$

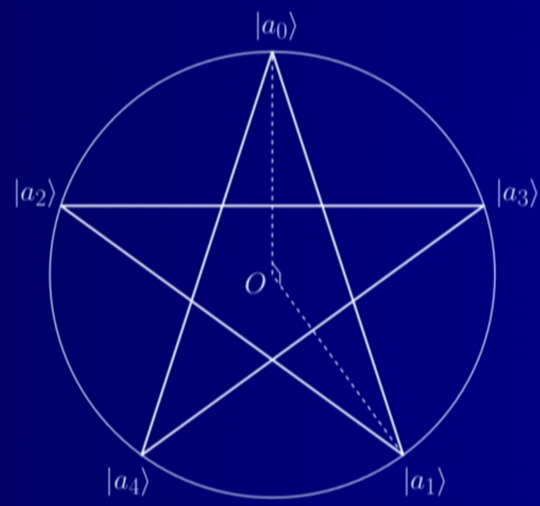
- Maroney showed $k(\psi,\phi) < 1$ for some states. ML and Maroney showed this follows from KS theorem.
- Barrett et. al. exhibited a family of states in $\mathbb{C}^d$ such that:

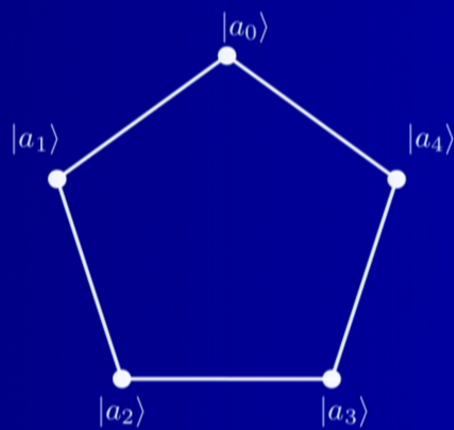
$$k(\psi,\phi) \leq \frac{1}{d}.$$

- Today: $k(\psi,\phi) \leq de^{-cd}$ for d divisible by 4.

Orthogonality graphs

- Example: Klyachko states
  - $|a_j\rangle = \sin\vartheta\cos\varphi_j\,|0\rangle + \sin\vartheta\sin\varphi_j\,|1\rangle + \cos\vartheta\,|2\rangle$
  - $\varphi_j = \frac{4\pi j}{5}$ and $\cos\vartheta = \frac{1}{\sqrt[4]{5}}$





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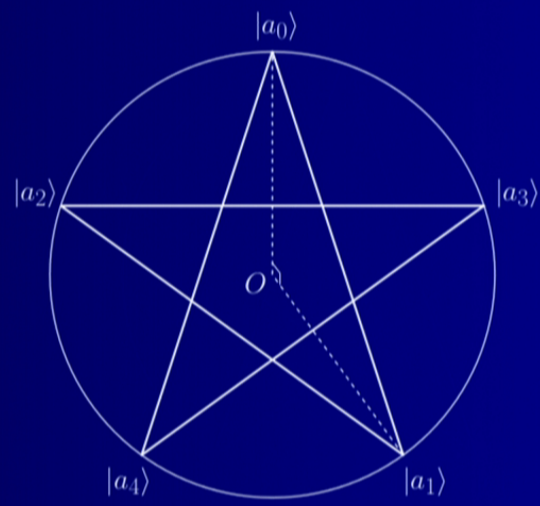
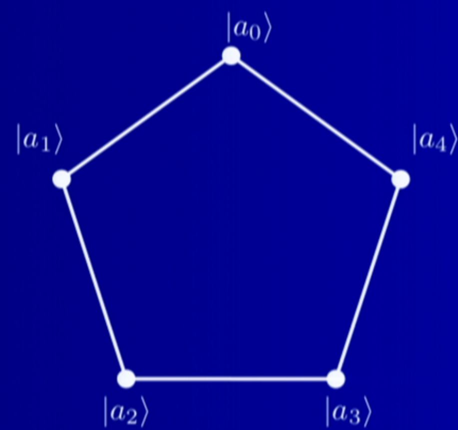
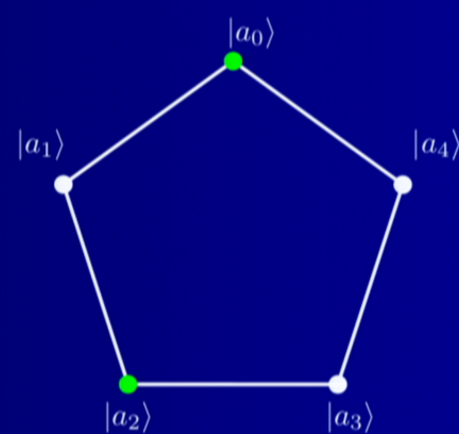




Independence number

- The *independence number* $\alpha(G)$ of a graph G is the cardinality of the largest subset of vertices such that no two vertices are connected by an edge.

- Example: $\alpha(G) = 2$



Main result

Theorem: Let V be a finite set of states in $\mathbb{C}^d$ an let $G = (V, E)$ be its orthogonality graph. For $|\psi\rangle \in \mathbb{C}^d$ define

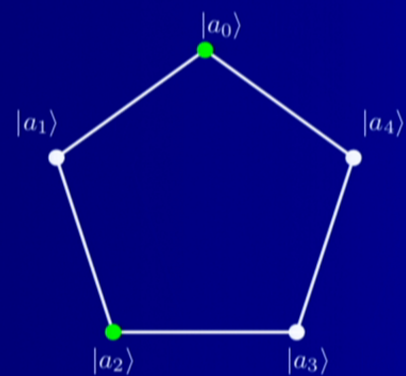
$$\bar{k}(\psi) = \frac{1}{|V|} \sum_{|a\rangle \in V} k(\psi, a).$$

Then, in any ontological model

$$\bar{k}(\psi) \leq \frac{\alpha(G)}{|V| \min_{|a\rangle \in V} |\langle a|\psi\rangle|^2}.$$

Bound from Klyatchko states

- $|a_j\rangle = \sin\vartheta\cos\varphi_j\,|0\rangle + \sin\vartheta\sin\varphi_j\,|1\rangle + \cos\vartheta\,|2\rangle$
- $\varphi_j = \frac{4\pi j}{5}$ and $\cos\vartheta = \frac{1}{\sqrt[4]{5}}$
- $|\psi\rangle = |2\rangle$



$$\bar{k}(\psi) \leq \frac{\alpha(G)}{5\min_j |\langle a_j|\psi\rangle|^2} = \frac{2}{5\times\frac{1}{\sqrt[4]{5}}} \sim 0.598$$

Orthogonality graphs

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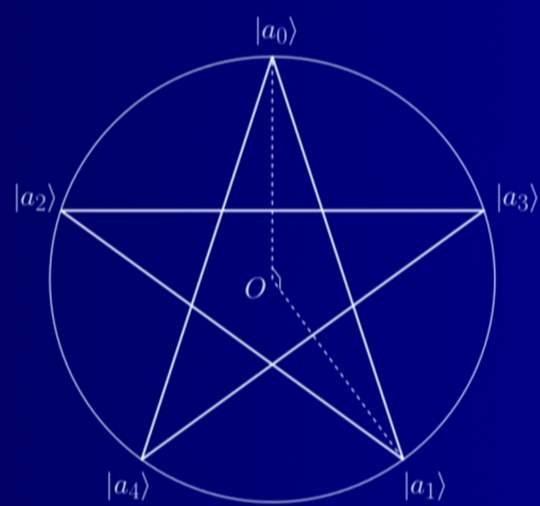

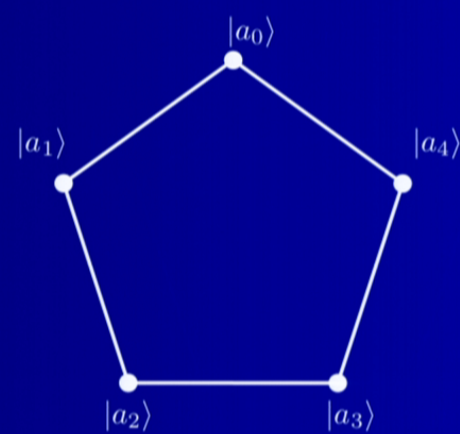

Exponential bound: Hadamard states

- For $\boldsymbol{x} = (x_0, x_1, \ldots, x_{d-1}) \in \{0,1\}^d$, let

$$|a_{\boldsymbol{x}}\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} (-1)^{x_j} |j\rangle .$$

- Let $|\psi\rangle = |0\rangle$.
- By Frankl-Rödl theorem[1], for d divisible by 4, there exists an $\epsilon > 0$ such that $\alpha(G) \leq (2-\epsilon)^d$.

$$\bar{k}(\psi) \leq \frac{\alpha(G)}{2^d \min_{\boldsymbol{x}\in\{0,1\}^d} |\langle a_{\boldsymbol{x}}|\psi\rangle|^2} = \frac{(2-\epsilon)^d}{2^d \times \frac{1}{d}} = de^{-cd}$$

$$c = \ln 2 - \ln(2-\epsilon)$$

[1] P. Frankl and V. Rödl, *Trans. Amer. Math. Soc.* 300:259 (1987)

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Proof of main result:1

- Let $\mathcal{M}$ be a covering set of bases for V.

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- For $M \in \mathcal{M}$, let

$$\Gamma_a^M = \{\lambda | \xi_a^M(\lambda) = 1\}$$

  – $\mu_a(\Gamma_a^M) = 1$ because $\int_\Lambda \xi_a^M(\lambda) d\mu_a = |\langle a|a\rangle|^2 = 1$.

- Let

$$\Gamma_a^{\mathcal{M}} = \cap_{\{M \in \mathcal{M} | |a\rangle \in M\}} \Gamma_a^M$$

  – $\mu_a(\Gamma_a^{\mathcal{M}}) = 1$ also.

Proof of main result:2

- $$A_c(\psi, a) \leq \mu_\psi(\Gamma_a^{\mathcal{M}})$$
  $$\sum_{|a\rangle \in V} A_c(\psi, a) \leq \sum_{a \in V} \mu_\psi(\Gamma_a^{\mathcal{M}})$$

Proof of main result:2

- $$A_c(\psi, a) \leq \mu_\psi(\Gamma_a^{\mathcal{M}})$$
  $$\sum_{|a\rangle \in V} A_c(\psi, a) \leq \sum_{a \in V} \mu_\psi(\Gamma_a^{\mathcal{M}})$$

- Let
  $$\chi_a(\lambda) = \begin{cases} 1, & \lambda \in \Gamma_a^{\mathcal{M}} \\ 0, & \lambda \notin \Gamma_a^{\mathcal{M}} \end{cases}$$

- Then,
  $$\sum_{a \in V} \mu_\psi(\Gamma_a^{\mathcal{M}}) = \int_\Lambda \left[\sum_{a \in V} \chi_a(\lambda) \right] d\mu_\psi \leq \sup_{\lambda \in \Lambda} \left[\sum_{a \in V} \chi_a(\lambda) \right].$$

Proof of main result:3

- If $\langle a|b\rangle = 0$ then $\Gamma_a^M \cap \Gamma_b^M = \emptyset$ because $\xi_a^M(\lambda) + \xi_b^M(\lambda) \leq 1$.
- Hence, $\Gamma_a^{\mathcal{M}} \cap \Gamma_b^{\mathcal{M}} = \emptyset$.

Proof of main result:3

- If $\langle a|b\rangle = 0$ then $\Gamma_a^M \cap \Gamma_b^M = \emptyset$ because $\xi_a^M(\lambda) + \xi_b^M(\lambda) \leq 1$.
- Hence, $\Gamma_a^{\mathcal{M}} \cap \Gamma_b^{\mathcal{M}} = \emptyset$.
- Hence, if $\lambda \in \Gamma_a^{\mathcal{M}}$ then $\lambda \notin \Gamma_b^{\mathcal{M}}$ for any $|b\rangle \in V$ such that $(|a\rangle, |b\rangle) \in E$.

The connection to contextuality

- An ontological model for a set of bases $\mathcal{M}$ is *Kochen-Specker (KS) noncontextual* if it is:
  - *Outcome deterministic*: $\xi_a^M(\lambda) \in \{0, 1\}$.
  - *Measurement noncontextual*: $\xi_a^M = \xi_a^N$.

The connection to contextuality

- An ontological model for a set of bases $\mathcal{M}$ is *Kochen-Specker (KS) noncontextual* if it is:
  - *Outcome deterministic*: $\xi_a^M(\lambda) \in \{0, 1\}$.
  - *Measurement noncontextual*: $\xi_a^M = \xi_a^N$.
- If a model is KS noncontextual then it satisfies

$$\int_\Lambda \xi_a^M(\lambda) d\mu_\psi = \mu_\psi(\Gamma_a^{\mathcal{M}}).$$

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Summary and Open questions

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  - There exist pairs of states such that $k(\psi, \phi) \leq de^{-cd}$. The ψ-epsitemic explanations of indistinguishability, no-cloning, etc. get implausible for these states very radpidly for large d.
  - Any contextuality inequality can be used to derive an overlap bound.
- Open questions
  - Error analysis.
  - Best bounds in small dimensions.
  - Bounds with a fixed inner product.
  - Connection to communication complexity.

Exponential bound: Hadamard states

- For $\boldsymbol{x} = (x_0, x_1, \ldots, x_{d-1}) \in \{0,1\}^d$, let

$$|a_{\boldsymbol{x}}\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} (-1)^{x_j} |j\rangle .$$

- Let $|\psi\rangle = |0\rangle$.
- By Frankl-Rödl theorem[1], for d divisible by 4, there exists an $\epsilon > 0$ such that $\alpha(G) \leq (2-\epsilon)^d$.

$$\bar{k}(\psi) \leq \frac{\alpha(G)}{2^d \min_{\boldsymbol{x} \in \{0,1\}^d} |\langle a_{\boldsymbol{x}}|\psi\rangle|^2} = \frac{(2-\epsilon)^d}{2^d \times \frac{1}{d}} = de^{-cd}$$

$$c = \ln 2 - \ln(2-\epsilon)$$

[1] P. Frankl and V. Rödl, *Trans. Amer. Math. Soc.* 300:259 (1987)

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What now for ψ-epistemicists?

- Become neo-Copenhagen.
- Adopt a more exotic ontology:
  - Nonstandard logics and probability theories.
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- Become neo-Copenhagen.
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  - Nonstandard logics and probability theories.
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- Principle of minimal weirdness: QM is weird but an interpretation of QM should not be more weird than it has to be.
  - Suggests exploring exotic ontologies.

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Arguments for Ontic Quantum States
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ψ-epistemic models
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Independence number
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Klyatchko bound
Exponential bound
Main result
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Proof of main result:3
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Previous results

- Define:

$$k(\psi,\phi) = \frac{A_c(\psi,\phi)}{|\langle\phi|\psi\rangle|^2}.$$

- Maroney showed $k(\psi,\phi) < 1$ for some states. ML and Maroney showed this follows from KS theorem.

- Barrett et. al. exhibited a family of states in $\mathbb{C}^d$ such that:

$$k(\psi,\phi) \leq \frac{1}{d}.$$

- Today: $k(\psi,\phi) \leq de^{-cd}$ for d divisible by 4.