Title: Quantum Liquids and Turbulent Black Holes

Date: Feb 24, 2014 02:00 PM

URL: http://pirsa.org/14020141

Abstract: The physics of black hole horizons is intimately connected to the physics of quantum liquids. In this talk I will review the connection and draw lessons about quantum turbulence from black hole dynamics and vice versa. For example, gravitational dynamics reveal that quantum turbulence can behave very differently from normal fluid turbulence in 2d, with long-wavelength excitations rapidly dissolving into small fluctuations and dissipating as in a 3d normal liquid. Meanwhile, familiar facts about turbulent flows in normal fluids imply that the horizons of certain accreting black holes evolve in time as if they have a non-trivial fractal dimension.

Quantum Liquids and Turbulent Black Holes

Perimeter Institute
Feb 2014

Allan Adams
MIT

Take Home Messages

Black Hole Horizons can have Fractal-like properties*

A Adams, P Chesler, H Liu
arxiv:1307.7267
PRL, In Review

Turbulence in 2d Superfluids Drives Energy to the UV*

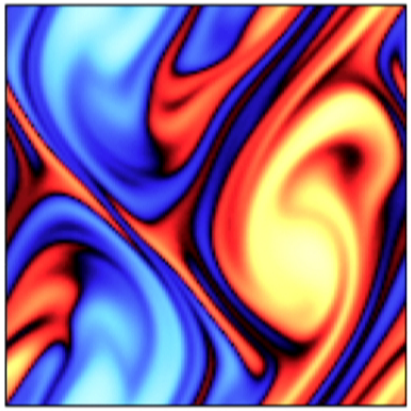
A Adams, P Chesler, H Liu
arxiv: 1212.0281
Science 341 (6144) 2013

* Important details will be added as we go

Basic Tool: Holography

Holography relates

Quantum Fluids and Classical Gravity



Quantum Liquids and Turbulent Black Holes

1. Introduction to Holography
2. Turbulent Black Holes
3. Turbulent Superfluids
4. Conclusions

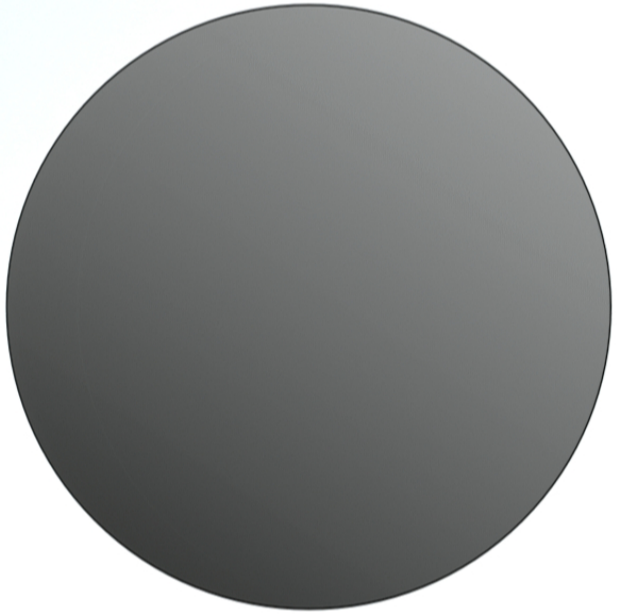
Intro to Holography

General Relativity		Hydrodynamics
$(t,\, x,\, y,\, z)$	$\leftrightarrow$	$(t,\, x,\, y)$
$G_{\mu\nu} = \Lambda g_{\mu\nu}$	$\leftrightarrow$	$D_t\, v + \nabla P = \eta\, \nabla^2 v$
Black Hole Horizons	$\leftrightarrow$	*Dissipation*
Classical Limit	$\leftrightarrow$	*Strong Interactions*

Some entries in the Holographic Dictionary

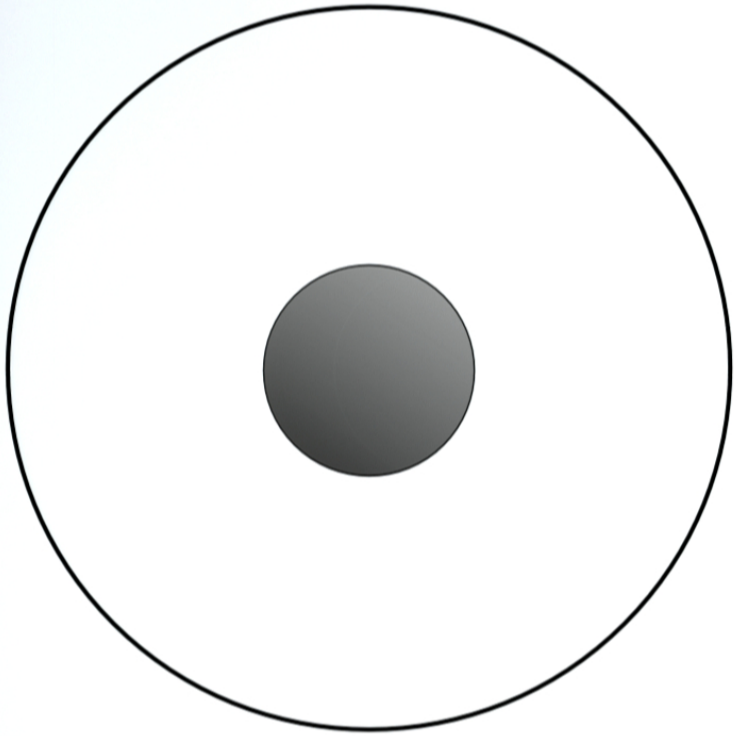
Intro to Holography

Intro to Holography



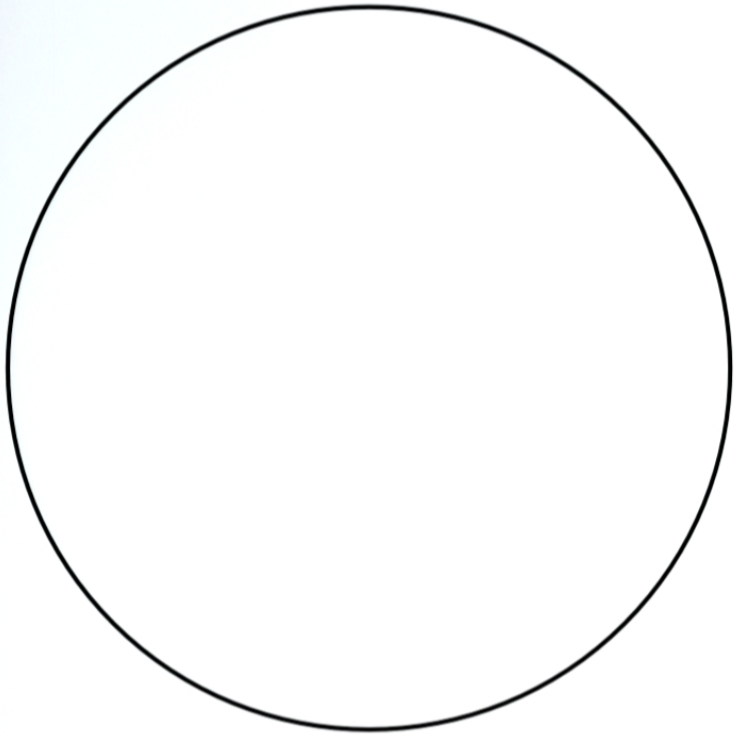
The Horizon of a 3d Black Hole behaves like a 2d Liquid!

Intro to Holography



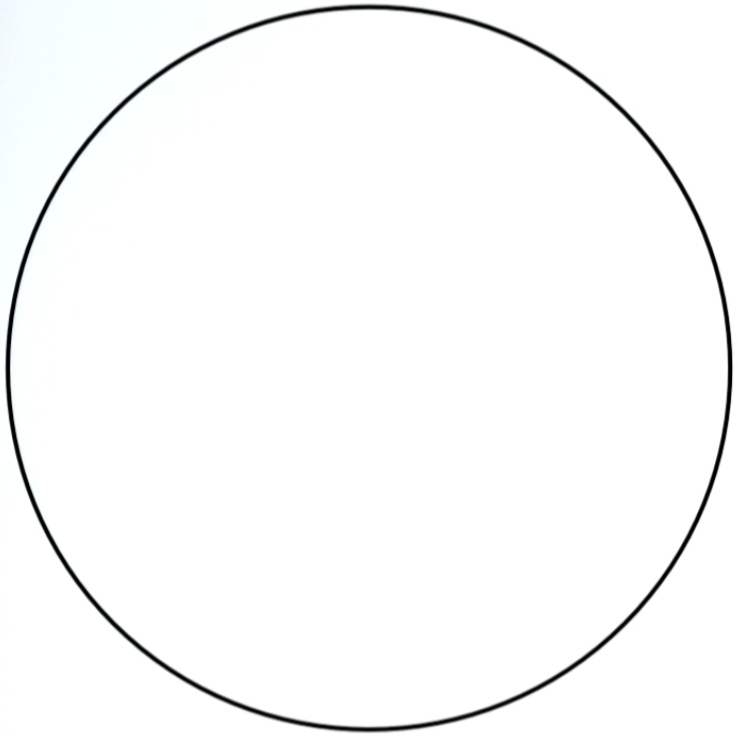
What is a harmonic trap for gravity?

Intro to Holography



Trap: Spacetime with Constant Negative Curvature

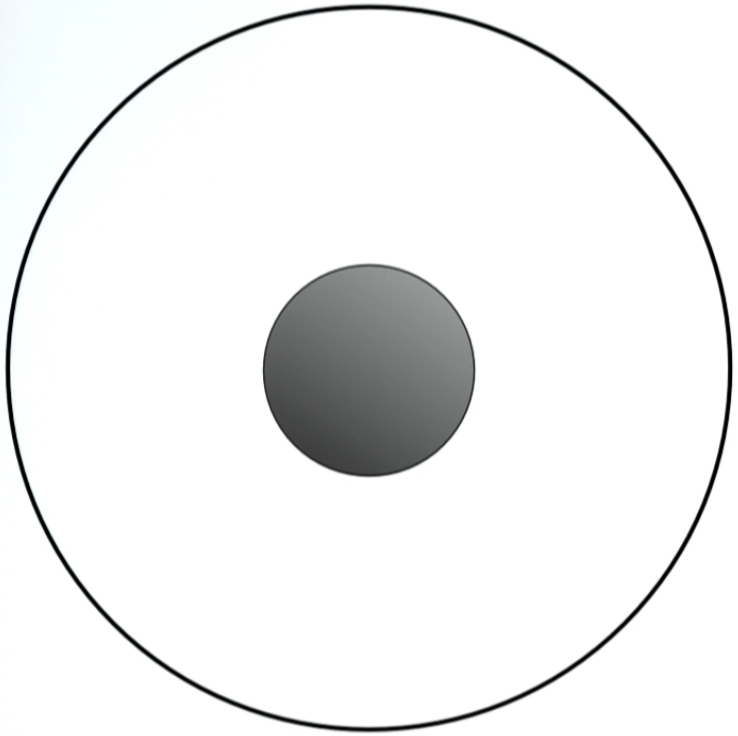
Intro to Holography



Trap: Spacetime with Constant Negative Curvature

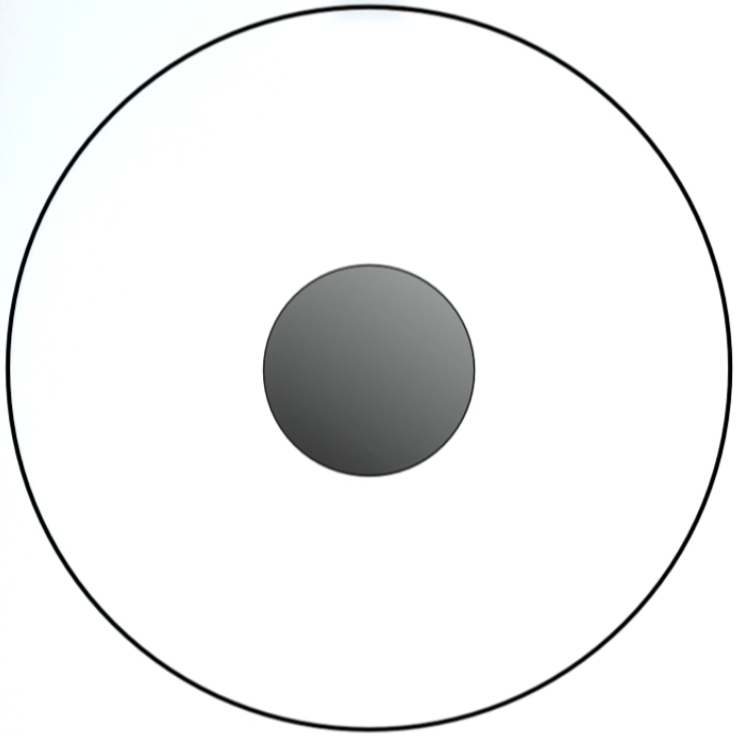
"Anti-de Sitter space" or AdS

Intro to Holography



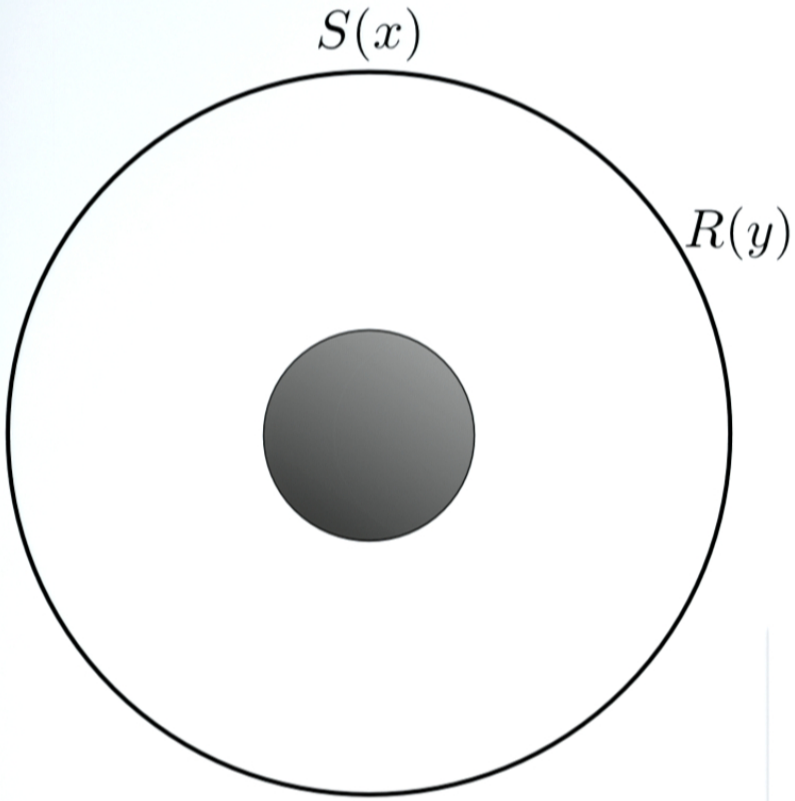
Black holes in AdS
can come to
equilibrium!

Intro to Holography



Study Black Holes
by perturbing them

Intro to Holography

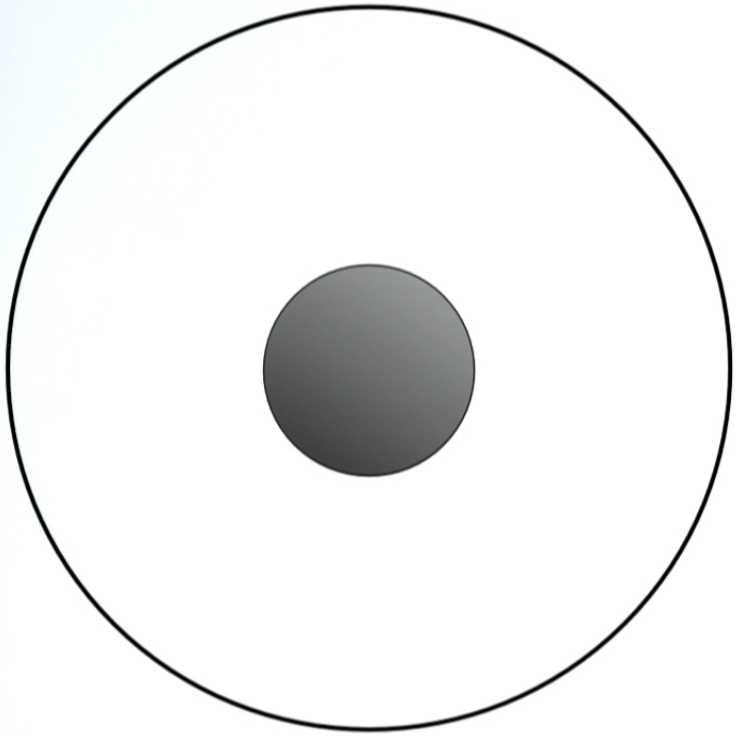


Study Black Holes
by perturbing them

$$R(y) = \int G(y|x)S(x)$$

Holography:
These correlators define
a QFT on boundary

Intro to Holography



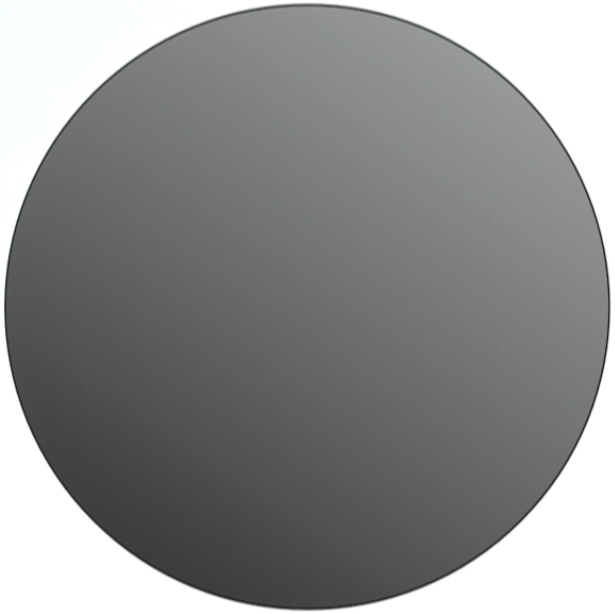
At finite density, low energy:

QFT reduces to Hydro

Holography:

Low energy physics of
Black Holes in 3+1 AdS
=
Hydro in 2+1

Turbulent Black Holes



If Horizons behave like liquids, we should be able to stir them and watch them develop turbulence via the Einstein Equations

$$G_{\mu\nu} = \Lambda g_{\mu\nu}$$

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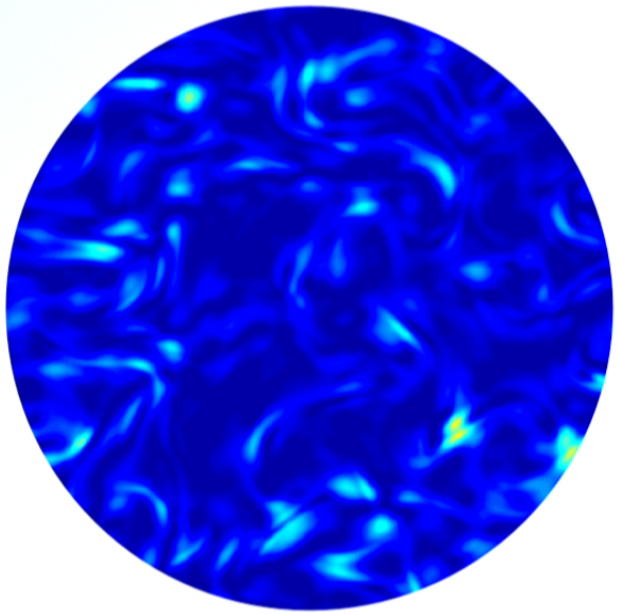
PRL, In Review

Turbulent Black Holes



Turbulence

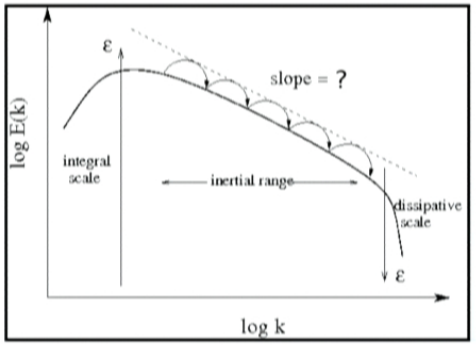
Working Def: On-average Homogeneous, Isotropic, Quasi-steady-state

Kolmogorov scaling

$$E(k) = C\,\epsilon^{2/3}\,k^{-5/3}$$

From dimensional analysis

$$\langle [\delta v(r)]^n \rangle = C_n\,\epsilon^{n/3}\,r^{n/3}$$

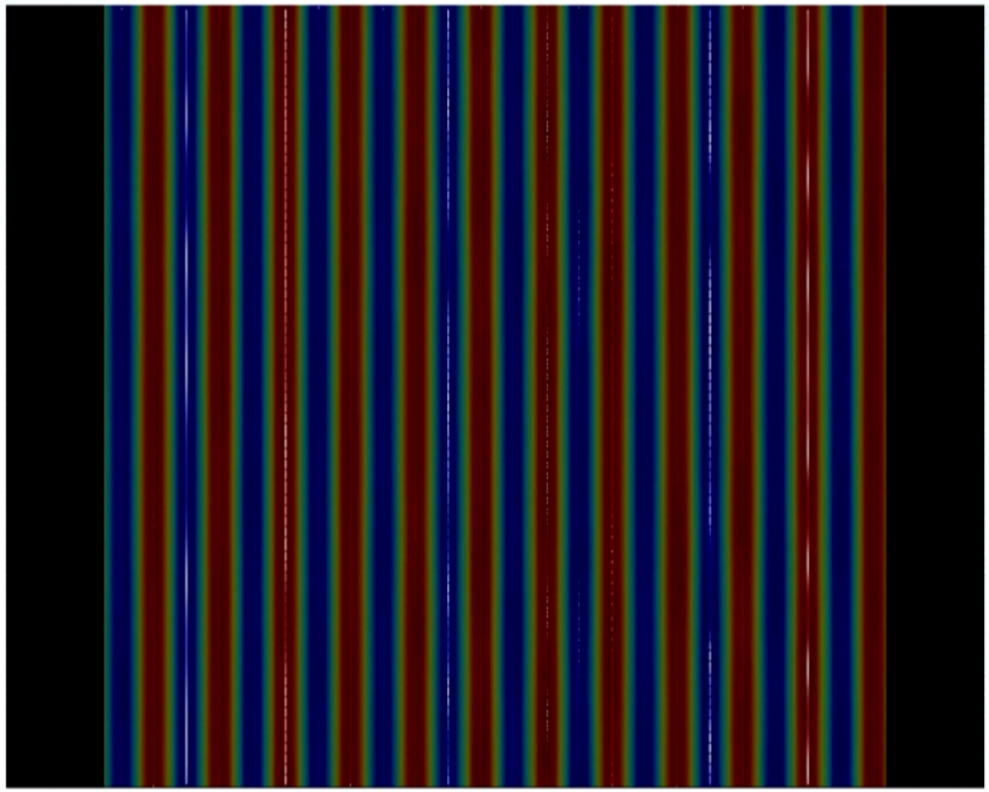


Cascade: Direct or Inverse?

$$\mathcal{E} = \omega^2\,, \quad \omega = \nabla \times v$$

Carrasco, Lehner, Myers, Reula, Singh

Turbulent Black Holes



3+1 Numerical Relativity

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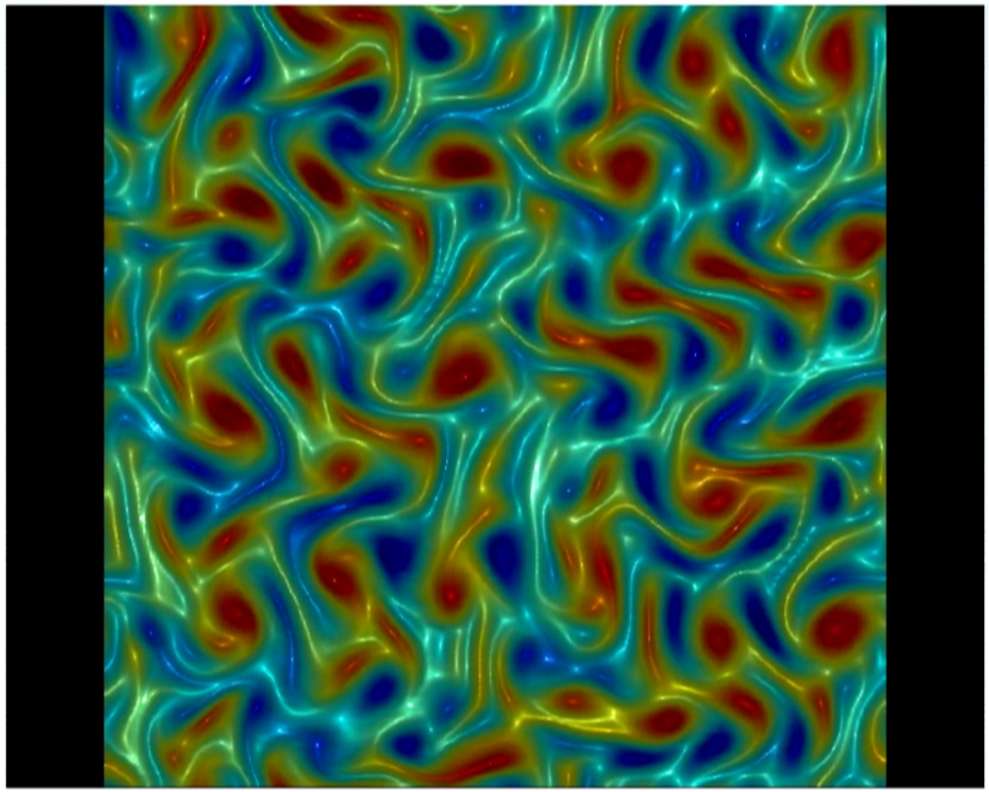

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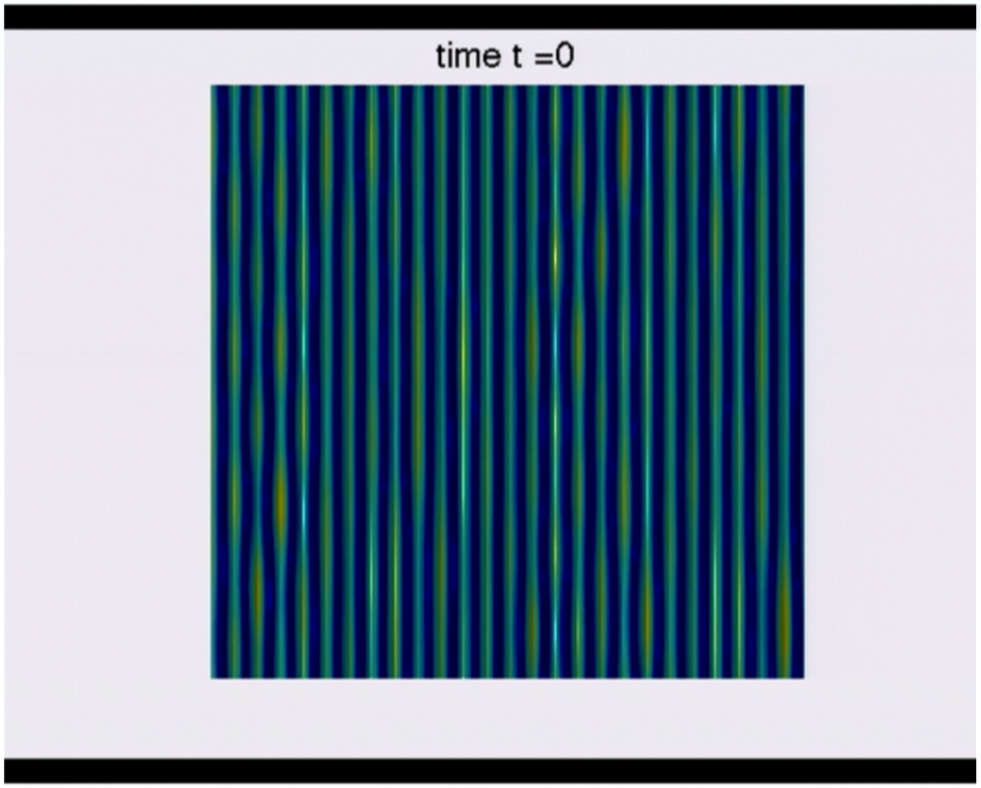
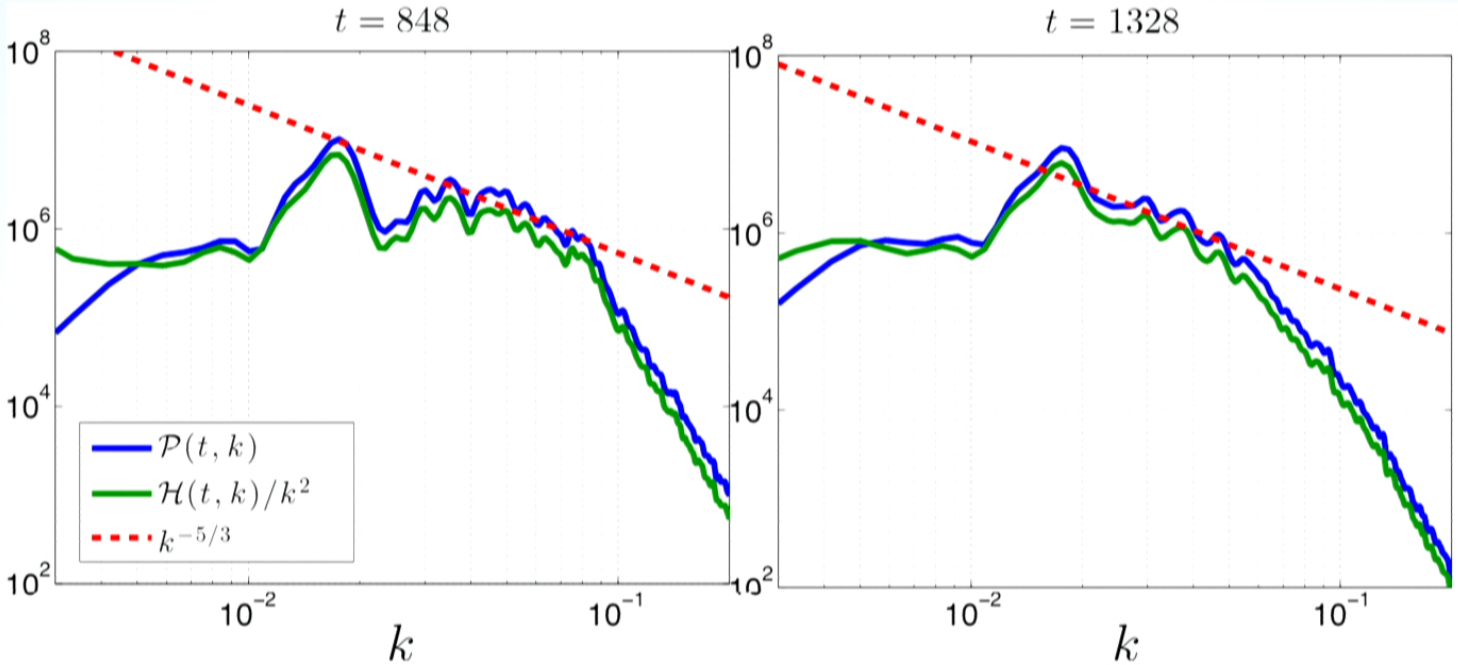


Horizon Area Density

A Adams, P Chesler, H Liu
arxiv:1307.7267
PRL, In Review

Turbulent Black Holes



Blue: Power spectrum of Kinetic Energy P(t,k)

Green: Power spectrum of Horizon Curvature A(t,k)

Note: $\mathcal{A}(t,k) \sim k^2 \mathcal{P}(t,k)$.

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PRL, In Review

Turbulent Black Holes

Fluid Power Spectrum sets
Horizon Curvature Spectrum

$$\mathcal{A}(t,k) \sim k^2 \mathcal{P}(t,k)$$

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Turbulent Black Holes

Fluid Power Spectrum sets Horizon Curvature Spectrum

$$\mathcal{A}(t,k) \sim k^2 \mathcal{P}(t,k)$$

Kolmogorov Scaling Gives

$$\mathcal{A}(t,k) \sim k^{1/3}$$

Curvature determines Growth of Horizon Area

$$\frac{dA}{dt} = \int dk\, \mathcal{A}(t,k)$$

So the Area grows as

$$\frac{dA}{dt} \sim k_{max}^{4/3}$$

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arxiv:1307.7267
PRL, In Review

Take Home Messages

Black Hole Horizons can have Fractal-like properties

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Turbulent Black Holes: An Aside

The metric ansatz:

$$(ds^2)^{(n)} = \frac{k^{(n)}}{r^2}\, u_\mu\, u_\nu\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu - 2\, h^{(n)}\, u_\mu\, \mathrm{d}x^\mu\, \mathrm{d}r - r^2\, h^{(n)}\, P_{\mu\nu}\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu$$
$$-\frac{1}{r}\left(j_\nu^{(n)}\, u_\mu + j_\mu^{(n)}\, u_\nu\right)\mathrm{d}x^\mu\, \mathrm{d}x^\nu + r^2\, \alpha_{\mu\nu}^{(n)}\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu$$

Einstein's equations

$$\frac{1}{r^4}\frac{d}{dr}\left[r^4 \frac{d}{dr} h^{(n)}(r)\right] = S_h^{(n)}(r)$$
$$\frac{d}{dr}\left[-\frac{2}{r}\, k^{(n)}(r) + \left(1 - 4r^3\right) h^{(n)}(r)\right] = S_k^{(n)}(r)$$
$$\frac{r}{2}\frac{d}{dr}\left[\frac{1}{r^2}\frac{d}{dr} j_i^{(n)}(r)\right] = S_j^{(n)}(r)_i$$
$$\frac{d}{dr}\left[-\frac{1}{2}\, r^4\, f(r)\, \frac{d}{dr}\alpha_{ij}^{(n)}(r)\right] = S_\alpha^{(n)}(r)_{ij}$$

$$\Pi^{\mu\nu}_{\alpha\beta} = \frac{1}{2}P^\mu_\alpha P^\nu_\beta + \frac{1}{2}P^\mu_\beta P^\nu_\alpha - \frac{1}{2}P^{\mu\nu}P_{\alpha\beta} \qquad \Sigma_2^{\mu\nu} = \Pi^{\mu\nu}_{\alpha\beta}(\sigma^\alpha_\lambda \omega^{\beta\lambda}) \qquad \sigma^{\mu\nu} = \Pi^{\mu\nu}_{\alpha\beta}(\partial^\alpha u^\beta)$$
$$\omega^{\mu\nu} = \frac{1}{2}P^\mu_\alpha P^\nu_\beta(\partial^\alpha u^\beta - \partial^\beta u^\alpha) \qquad \Sigma_1^{\mu\nu} = \Pi^{\mu\nu}_{\alpha\beta}(\mathcal{D}\sigma^{\alpha\beta} + \frac{1}{2}\sigma^{\alpha\beta}\partial_\lambda u^\lambda)$$

The Boundary Stress Tensor:

$$^{\mu\nu} = {}^{3}{}_{1}(\ ^{\mu\nu} + 3\ ^{\mu}\ ^{\nu}) + {}^{2}{}_{2}\ ^{\mu\nu} + {}_{3}\Sigma_1^{\mu\nu} + {}_{4}\Sigma_2^{\mu\nu}$$

This solves the Einstein equations if the fluid velocity solves Hydro,

$$\partial_\mu T^{\mu\nu} = 0$$

Turbulent Black Holes: An Aside

So instead of solving the 3+1 Einstein equation

$$G_{\mu\nu} = \Lambda g_{\mu\nu}$$

We can solve the 2+1 Hydro equation

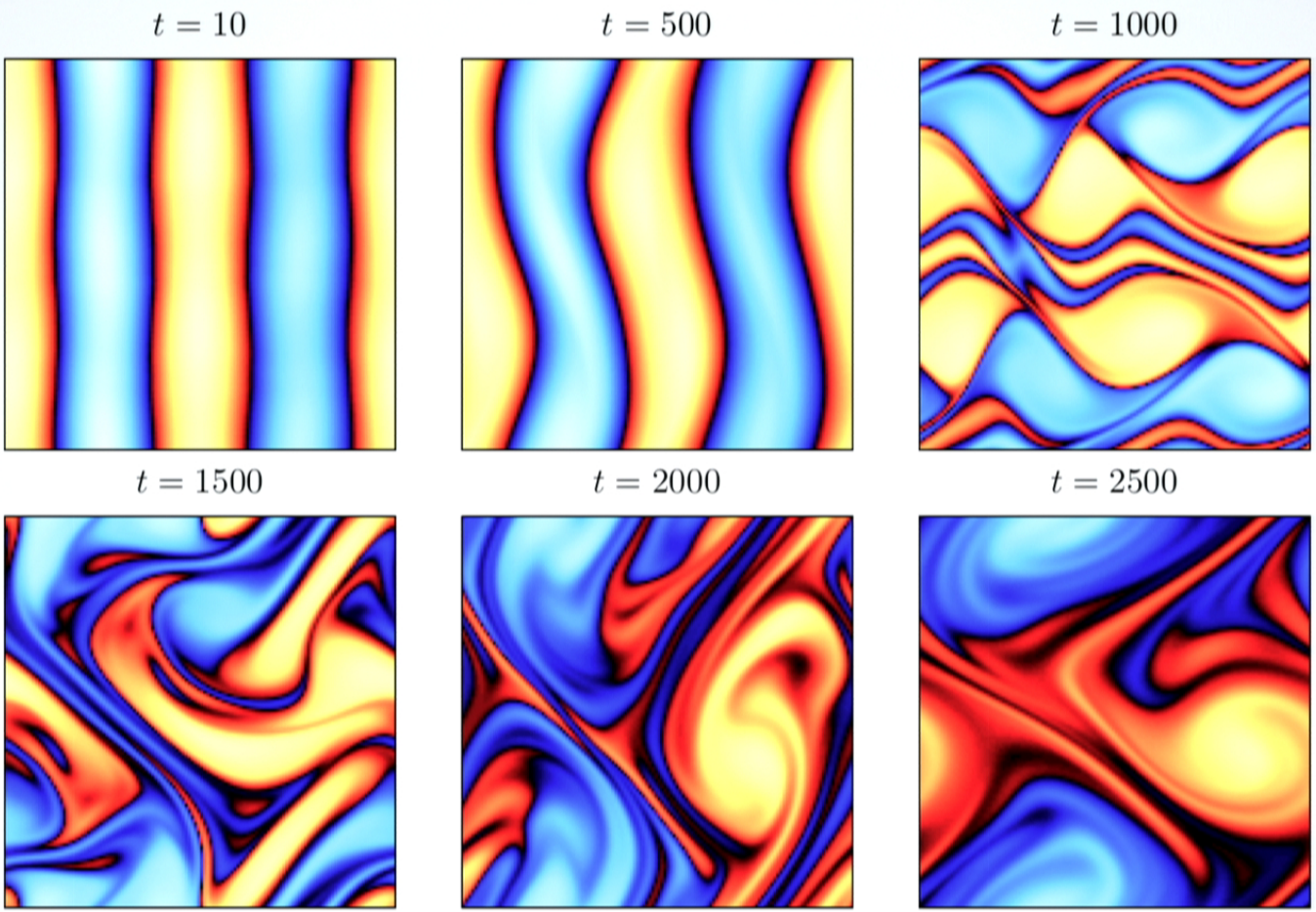
$$\partial_\mu T^{\mu\nu} = 0$$

And plug into the Fluid/Gravity metric.

Our job is thus to solve the resulting conservation equations numerically.

See also: Carrasco, Lehner, Myers, Reula, Singh

I gave this problem to a couple of undergrads...


$t = 10$
$t = 500$
$t = 1000$
$t = 1500$
$t = 2000$
$t = 2500$

After hunting and killing many, many bugs, my students and I re-derived the analytic results and found a few small but non-trivial errors in Sk, since confirmed by the author. Good Undergrads!

Nathan Benjamin

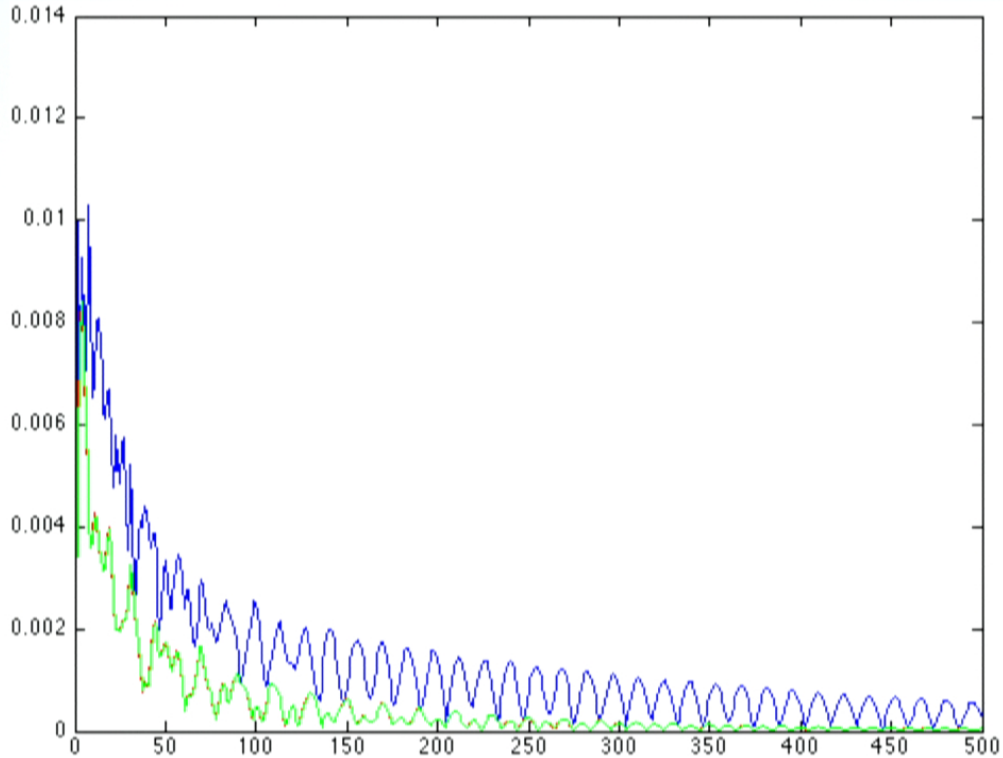
Wojciech Musial

$$S_h^{(2)}(r) = -\frac{1}{2\,r^4}\,\mathcal{S}_6 + F_1(r)\,\mathcal{S}_7$$

$$S_k^{(2)}(r) = 2\,\mathcal{S}_3 + \frac{1}{2}\,\mathcal{S}_5 - \frac{1+4\,r^3}{2\,r^3}\,\mathcal{S}_6 + F_2(r)\,\mathcal{S}_7$$

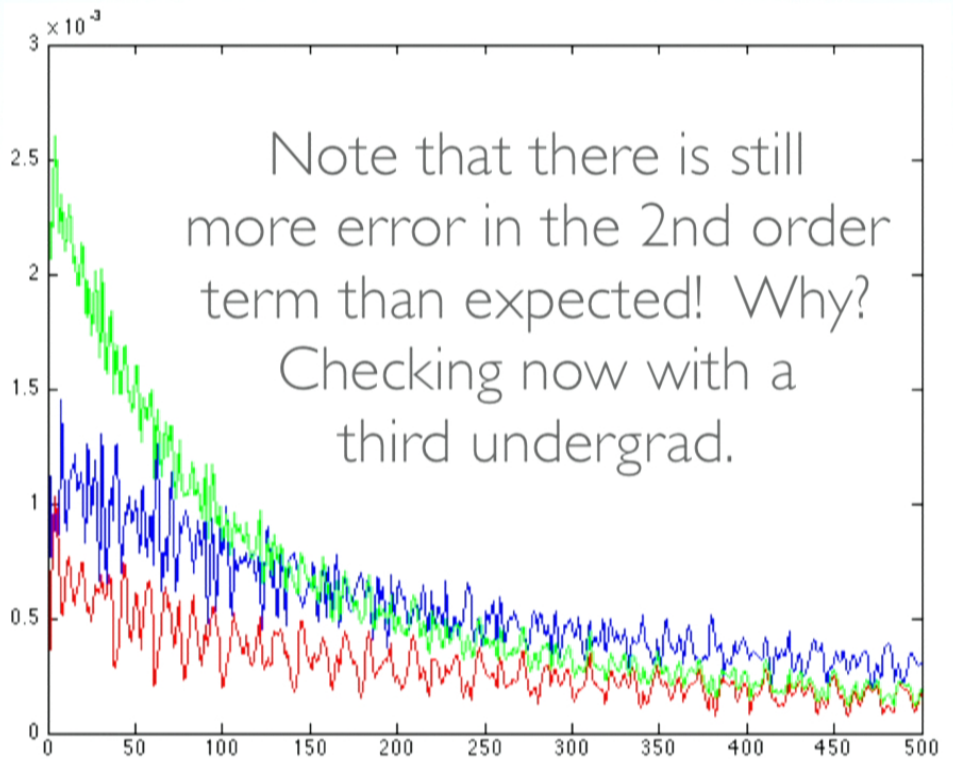
$$S_j^{(2)}(r) = \frac{1}{2\,r^2}\,\mathcal{V}_3 - \frac{1}{2\,r^2(1+r+r^2)}\,\mathcal{V}_4 + \frac{-1+r+r^2}{2\,r^2(1+r+r^2)}\,(\mathcal{V}_5 - \mathcal{V}_6)$$

$$S_\alpha^{(2)}(r) = F_3(r)\,(\mathcal{T}_2 + \mathcal{T}_3) + F_4(r)\,\mathcal{T}_4 + F_5(r)\,\mathcal{T}_5$$



Norm of the Einstein Tensor over time

Blue = 0th Order, Red = 1st Order, Green = 2nd Order



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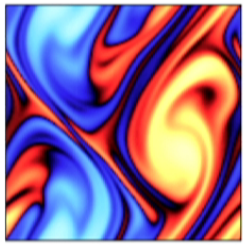
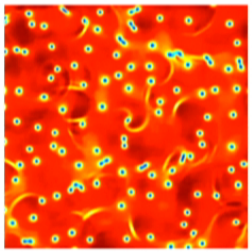
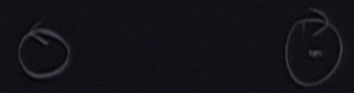
Turbulent Superfluids

Just Used  to study 

Now Use  to study 

$$\Omega = \int \omega^2$$

$\omega = 1$ $\omega = -1$



$\omega = +1$ $\omega = -1$

Turbulent Superfluids

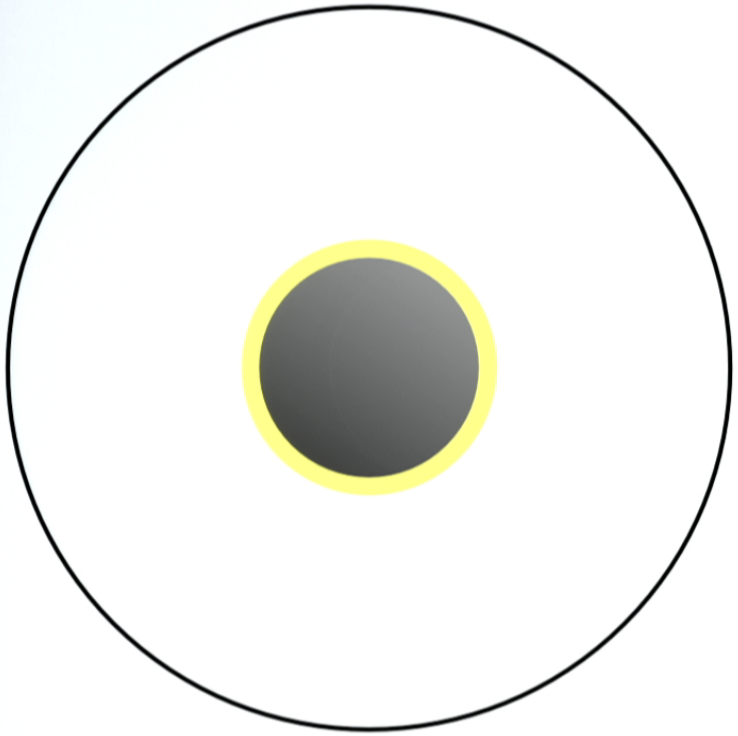
Long-standing Question:

Do 2d Superfluids Exhibit *Direct* or *Inverse* Cascades?

More immediate Question:

What is the Holographic description of a Superfluid?

Turbulent Superfluids



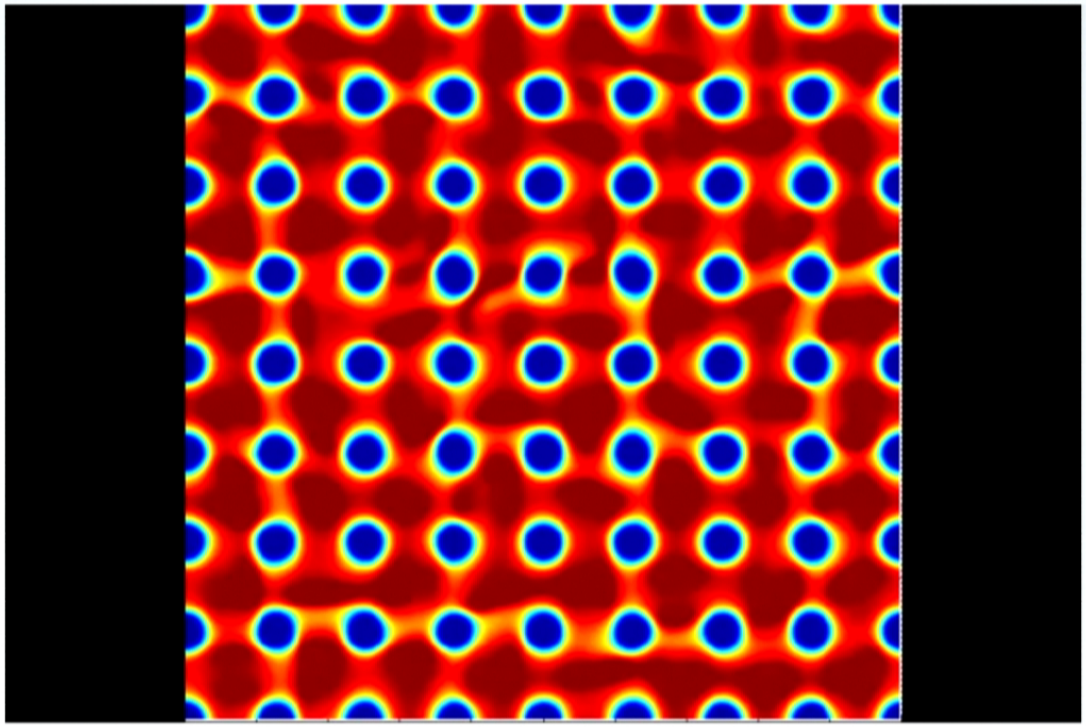
Dual of a Superfluid
is a
Charged Black Hole

$$G_{\mu\nu} = \Lambda g_{\mu\nu}$$

$$\nabla_\mu F^{\mu\nu} = J^\nu$$

$$\left(\mathcal{D}^2 - m^2\right)\Psi = 0$$

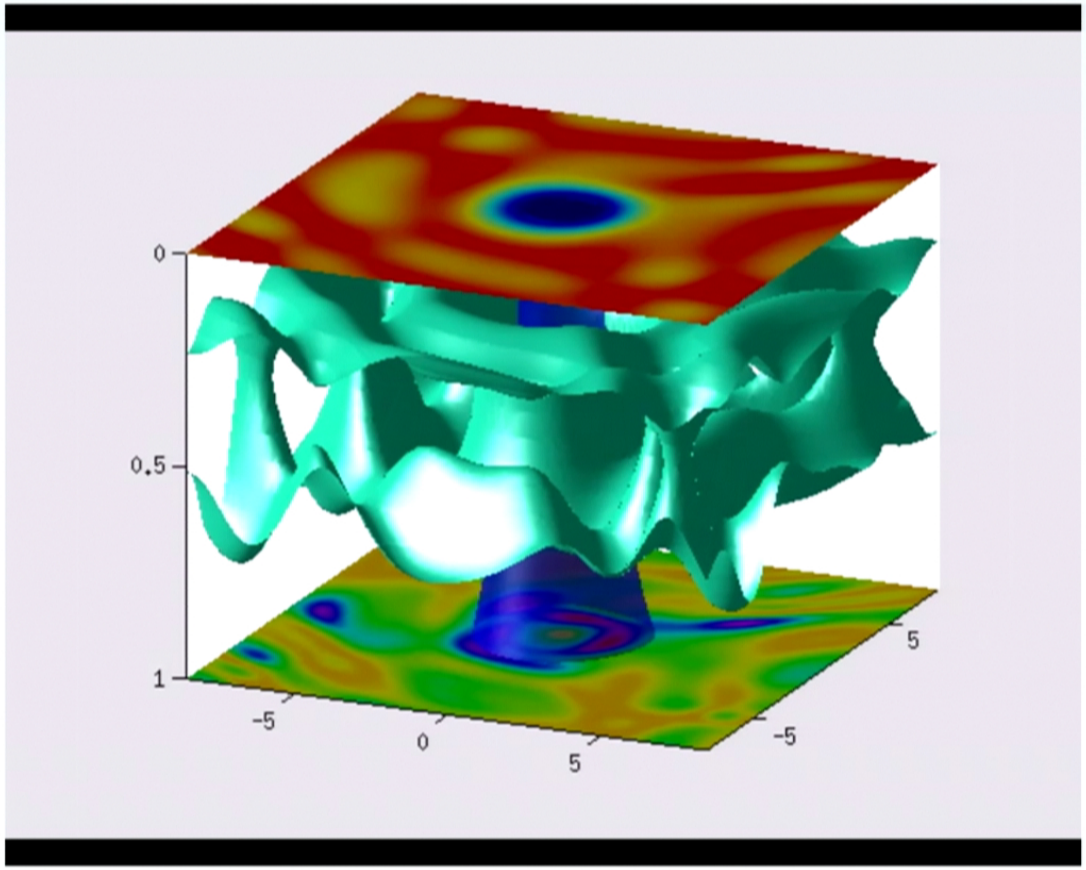
Turbulent Superfluids



Turbulent flow in a holographic superfluid

A Adams, P Chesler, H Liu
arxiv: 1212.0281
Science **341** (6144) 2013

Turbulent Superfluids



Inside the holographic spacetime.

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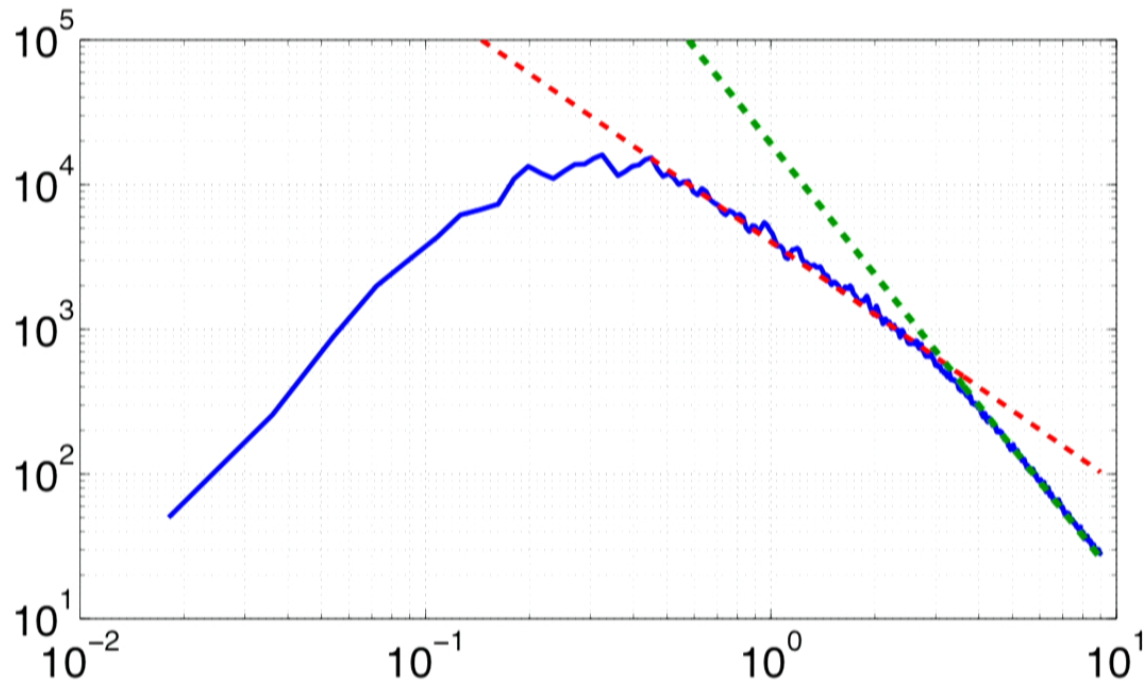
Turbulent Superfluids



Kinetic Energy Spectrum Scaling

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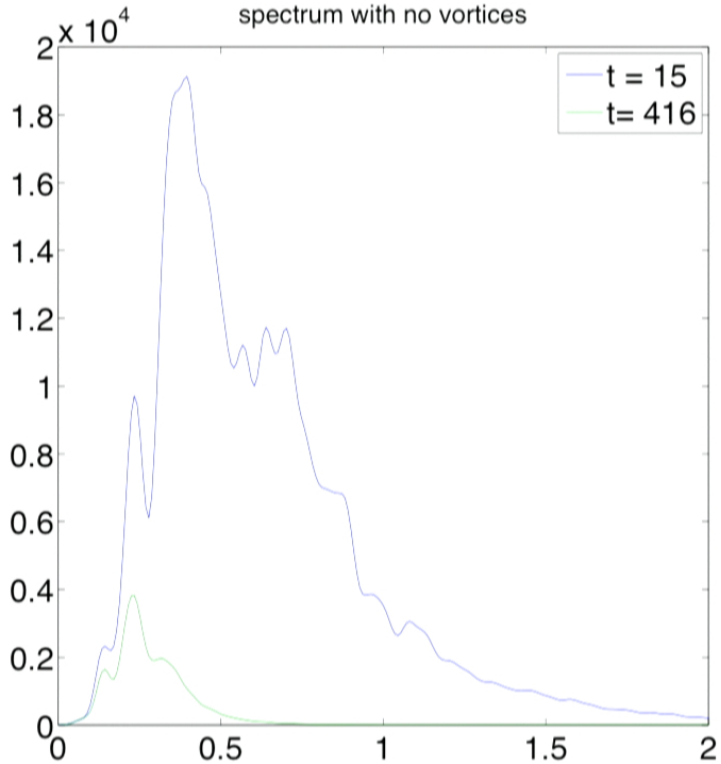
Turbulent Superfluids



No Scaling Without Vortices

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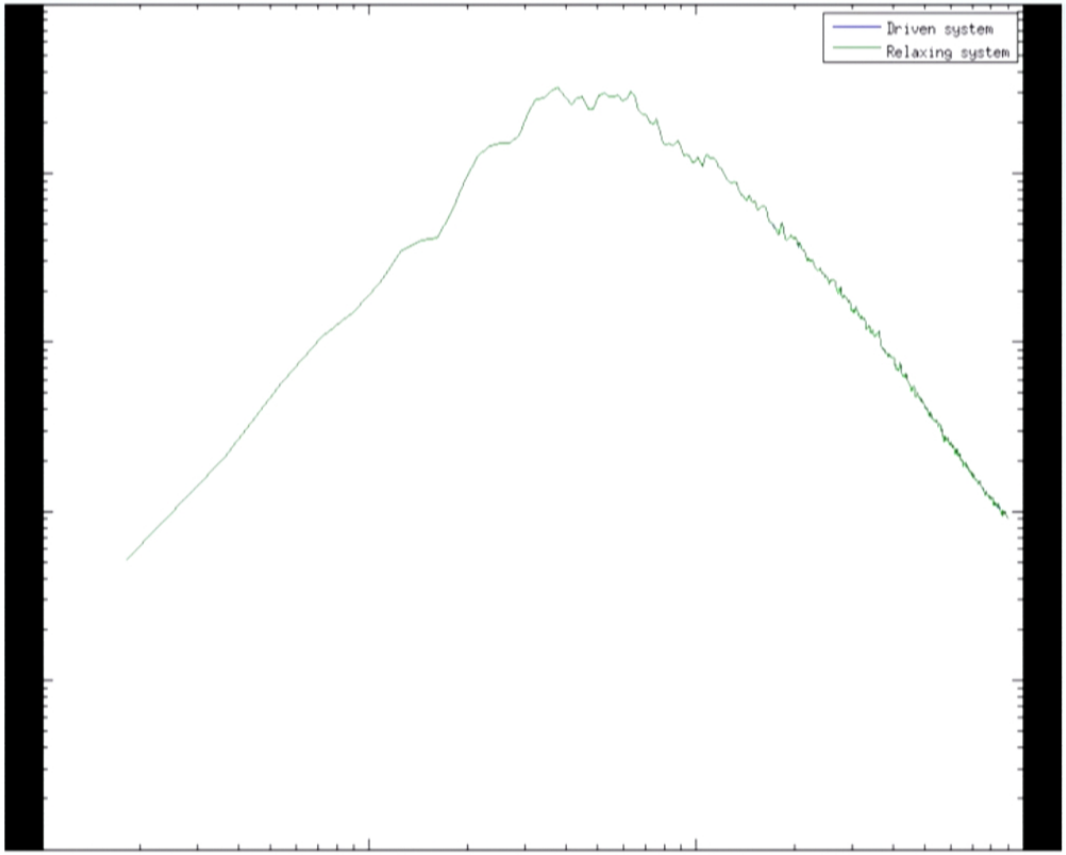
Turbulent Superfluids



Energy Injected in IR flows to UV

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Conclusions

* Fluid dynamics illuminates BH physics, and vice versa

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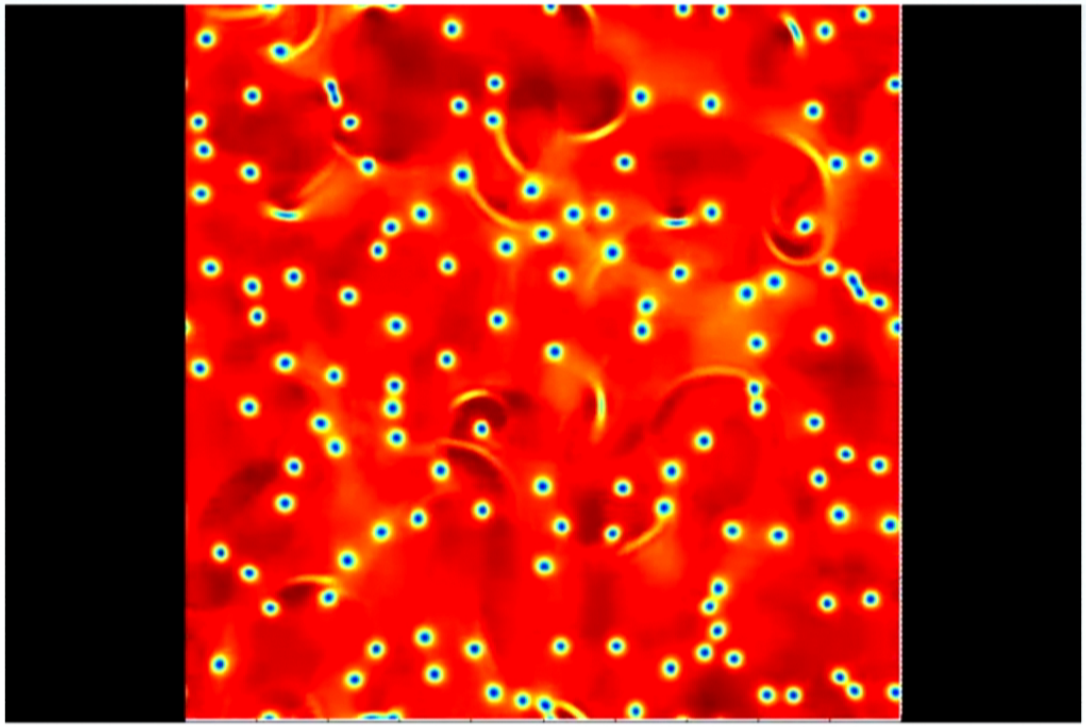
* Ongoing work / ideas
  - Turbulent (quasi-)Steady-State w/ Driving
  - Non-zero Net Vorticity
  - Soliton Physics
  - Disorder

Turbulent Superfluids



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