Title: Numerical detection of symmetry protected and symmetry enriched topological phases

Date: Feb 25, 2014 03:30 PM

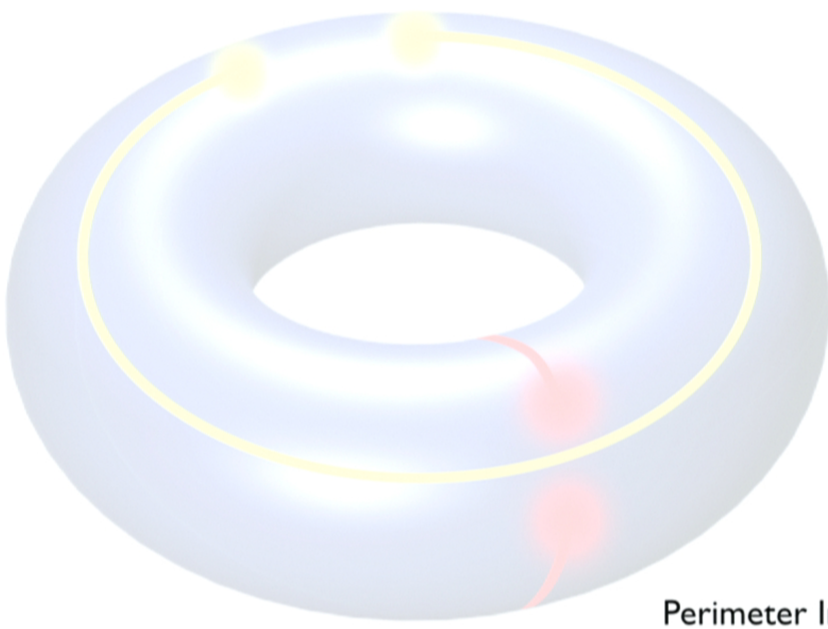
URL: http://pirsa.org/14020140

Abstract: A topological phase is a phase of matter which cannot be characterized by a local order parameter. We first introduce non-local order parameters that can detect symmetry protected topological (SPT) phases in 1D systems and then show how to generalize the idea to detect symmetry enriched topological (SET) phases in 2D. SET phases are new structures that occur in topologically ordered systems in the presence of symmetries. We introduce simple methods to detect the SET order directly from a complete set of topologically degenerate ground state wave functions. We first show how to directly determine the characteristic symmetry fractionalization of the quasiparticles from the reduced density matrix of the minimally entangled states. Second, we show how a simple generalization of a non-local order parameter can be measured to detect SETs. The usefulness of the proposed approached is demonstrated by examining two concrete model states which exhibit SET: (i) a spin-1 model on the honeycomb lattice and (ii) the resonating valence bond state on a kagome lattice. We conclude that the spin-1 model and the RVB state are in the same SET phases.

Detection of symmetry protected and symmetry enriched topological phases

Frank Pollmann

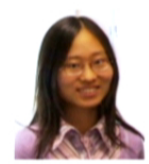
Max-Planck-Institut für komplexer Systeme, Dresden, Germany



Perimeter Institute, Feb. 25 2014

Detection of symmetry protected and symmetry enriched topological phases

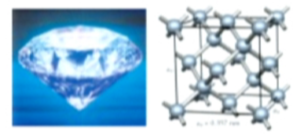
Overview

- Topological phases in condensed matter
- Detection of topological phases
  - **Symmetry protected topological phases in 1D**
    FP. and A. M. Turner, Phys. Rev. B 86, 125441 (2012)
    
  - **Symmetry enriched topological phases**
    C.-Y. Huang, X. Chen, and FP (arXiv:1312.3093)
    
    
- Summary

Perimeter Institute, Feb. 25 2014

Matter occurs in different phases

- Different phases of matter are usually understood using **spontaneous symmetry breaking**
  - **Crystals**: translation and rotation symmetry broken

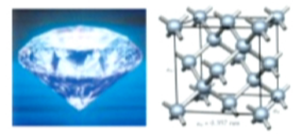


  - **Magnets**: spin rotation and TR symmetry broken

[Landau]

Matter occurs in different phases

- Different phases of matter are usually understood using **spontaneous symmetry breaking**
  - **Crystals**: translation and rotation symmetry broken



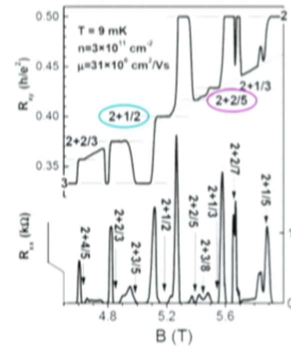
  - **Magnets**: spin rotation and TR symmetry broken

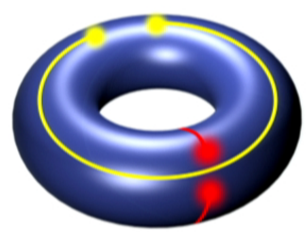
[Landau]

- **Local order parameters** distinguish different phases
- Ising model: **magnetization** as order parameter

Matter occurs in different phases

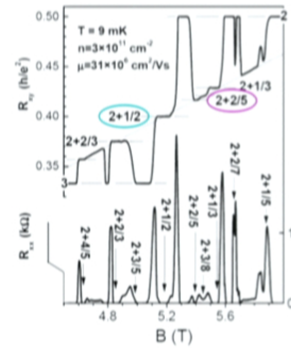
- **Fractional quantum Hall effect** represents new type of matter with **unusual properties**
  [Tsui, Stormer '82, Laughlin '83]
  - No symmetry breaking
  - Fractional charges and fractional statistics
  - Degeneracies that depend on the topology



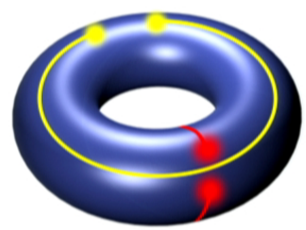


Matter occurs in different phases

- **Fractional quantum Hall effect** represents new type of matter with **unusual properties** [Tsui, Stormer '82, Laughlin '83]
  - No symmetry breaking
  - Fractional charges and fractional statistics
  - Degeneracies that depend on the topology



- **Quantum Hall states characterized by topological properties: "Topological order"** [Wen '91]



Matter occurs in different phases

- Distinguish two different kinds of topological order

Intrinsic: Cannot by transformed adiabatically to a "trivial" phase by **any path**

- Fractional quantum Hall
- Spin liquids
- …

Matter occurs in different phases

- Distinguish two different kinds of topological order

Intrinsic: Cannot by transformed adiabatically to a "trivial" phase by **any path**

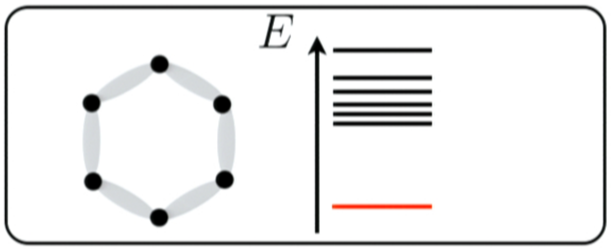
- Fractional quantum Hall
- Spin liquids
- …

Symmetry protected: Cannot by transformed adiabatically to a "trivial" phase by **symmetry preserving paths**

- Haldane phase
- Topological Insulators
- …

1D symmetry protected topological phases

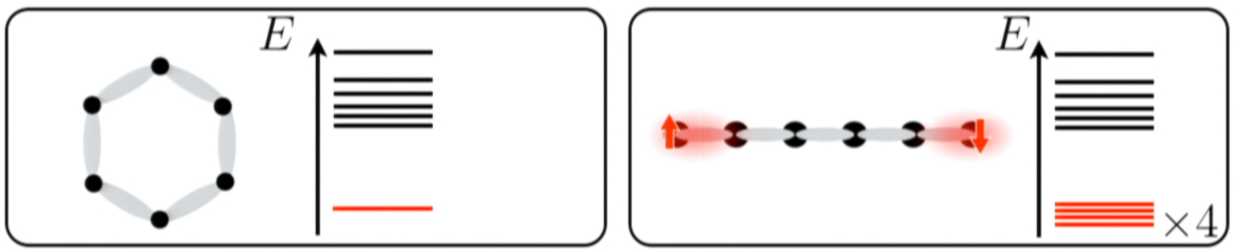
- Spin-1 Heisenberg chain $H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$
  - **Haldane phase:** Gapped in the bulk and no symmetry breaking (Spin rotation, TR, Inversion, TI) [Haldane '83]



AKLT: Each spin-1 splits up into two spin-1/2 [Affleck et al. '87]

1D symmetry protected topological phases

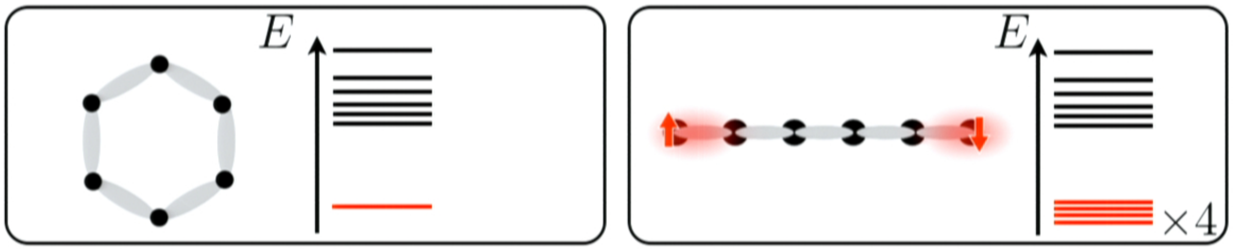
- Spin-1 Heisenberg chain $H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$
  - **Haldane phase:** Gapped in the bulk and no symmetry breaking (Spin rotation, TR, Inversion, TI) [Haldane '83]
  - Characterized by S=1/2 excitations at the edges



AKLT: Each spin-1 splits up into two spin-1/2 [Affleck et al. '87]

1D symmetry protected topological phases

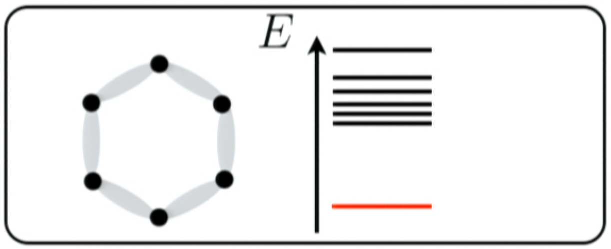
- Spin-1 Heisenberg chain $H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$
  - **Haldane phase:** Gapped in the bulk and no symmetry breaking (Spin rotation, TR, Inversion, TI) [Haldane '83]
  - Characterized by S=1/2 excitations at the edges

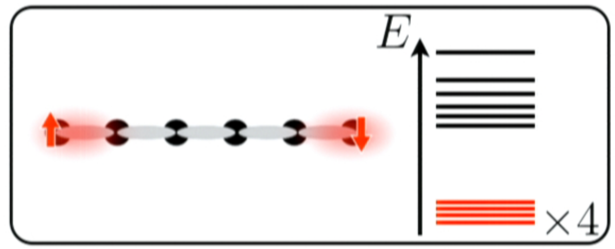


AKLT: Each spin-1 splits up into two spin-1/2 [Affleck et al. '87]

1D symmetry protected topological phases

- Spin-1 Heisenberg chain $H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$
  - **Haldane phase:** Gapped in the bulk and no symmetry breaking (Spin rotation, TR, Inversion, TI) [Haldane '83]
  - Characterized by S=1/2 excitations at the edges





AKLT: Each spin-1 splits up into two spin-1/2 [Affleck et al. '87]

- **Edge spins have been observed** in the NMR profile close to the chain ends of Mg-doped Y_2BaNiO [S.H. Glarum, et al.,]

1D symmetry protected topological phases

- **SPTP**: Hamiltonian and ground state have the same symmetry ($G_H = G\psi$) [Gu et al. '09]

FP, E. Berg, A.M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).
FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B **81**, 064439 (2010).

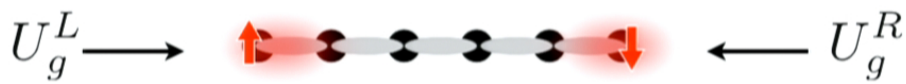
1D symmetry protected topological phases

- **SPTP**: Hamiltonian and ground state have the same symmetry $(G_H = G\psi)$ [Gu et al.'09]

FP, E. Berg, A.M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).
FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B **81**, 064439 (2010).

1D symmetry protected topological phases

- **SPTP**: Hamiltonian and ground state have the same symmetry $(G_H = G\psi)$ [Gu et al.'09]
- **Fractionalization** of symmetry operators $g \in G_\psi$



➡ **Projective representations:**

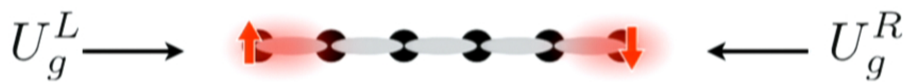
$$gh = k : U_g U_h = e^{i\phi_{gh}} U_k$$

FP, E. Berg, A.M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).

FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B **81**, 064439 (2010).

1D symmetry protected topological phases

- **SPTP**: Hamiltonian and ground state have the same symmetry ($G_H = G\psi$) [Gu et al.'09]
- **Fractionalization** of symmetry operators $g \in G_\psi$



➡ **Projective representations:**

$$gh = k : U_g U_h = e^{i\phi_{gh}} U_k$$

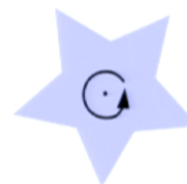
- Co-homology $H^2[G, U(1)]$: **SPTP are characterized by cohomology classes** (complete classification [Chen et al.'11])

FP, E. Berg, A.M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).
FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B **81**, 064439 (2010).

1D symmetry protected topological phases

Which symmetries stabilize topological phases?

- **Example** $\mathbb{Z}_n$: Rotation about single axis



$$R^n = \mathbb{1} \Rightarrow U_R^n = e^{i\phi}\mathbb{1}$$

  - Redefining $\tilde{U}_R = e^{-i\phi/n}U_R$ removes the phase

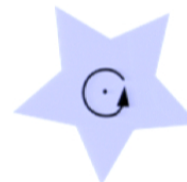
FP, E. Berg, A.M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).

FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B **81**, 064439 (2010).

1D symmetry protected topological phases

Which symmetries stabilize topological phases?

- **Example** $\mathbb{Z}_n$: Rotation about single axis



$$R^n = \mathbb{1} \Rightarrow U_R^n = e^{i\phi}\mathbb{1}$$

  - Redefining $\tilde{U}_R = e^{-i\phi/n} U_R$ removes the phase

FP, E. Berg, A.M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).
FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B **81**, 064439 (2010).

1D symmetry protected topological phases

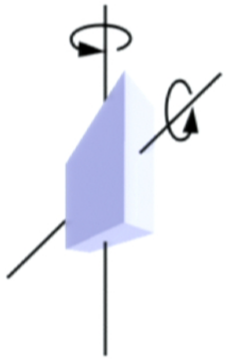
Which symmetries stabilize topological phases?

- **Example** $\mathbb{Z}_n$: Rotation about single axis

$$R^n = \mathbb{1} \Rightarrow U_R^n = e^{i\phi}\mathbb{1}$$

  - Redefining $\tilde{U}_R = e^{-i\phi/n} U_R$ removes the phase

- **Example** $\mathbb{Z}_2 \times \mathbb{Z}_2$: Phase for pairs



$$R_x R_z = R_z R_x \Rightarrow U_x U_z = e^{i\phi_{xz}} U_z U_x$$

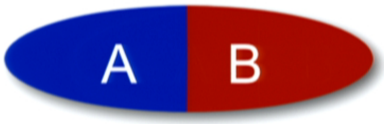
  - Phases $\phi = 0, \pi$ **cannot** be gauged away: **Distinct topological phases**

FP, E. Berg, A.M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).
FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B **81**, 064439 (2010).

1D symmetry protected topological phases

Schmidt decomposition (SVD)

- Decompose a state $|\psi\rangle$ into a superposition of product states:

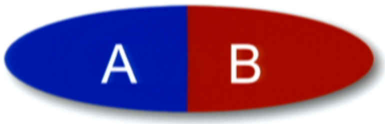


$$|\psi\rangle \quad = \quad \sum_{i=1}^{N_A}\sum_{j=1}^{N_B} C_{ij}|i\rangle_A|j\rangle_B = \sum_{\gamma}\lambda_\gamma|\phi_\gamma\rangle_A|\phi_\gamma\rangle_B$$

1D symmetry protected topological phases

Schmidt decomposition (SVD)

- Decompose a state $|\psi\rangle$ into a superposition of product states:



$$|\psi\rangle = \sum_{i=1}^{N_A}\sum_{j=1}^{N_B} C_{ij}|i\rangle_A|j\rangle_B = \sum_{\gamma}\lambda_\gamma|\phi_\gamma\rangle_A|\phi_\gamma\rangle_B$$

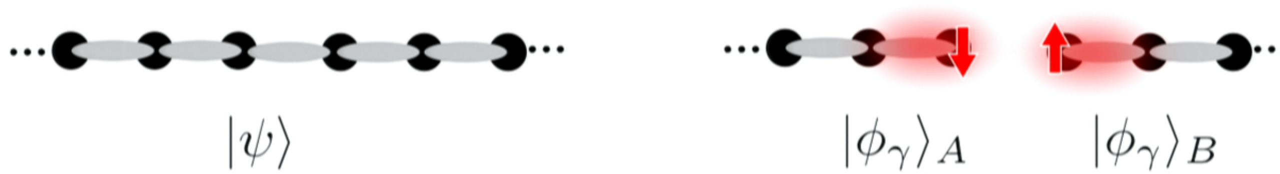
1D symmetry protected topological phases

Schmidt decomposition (SVD)

- Decompose a state $|\psi\rangle$ into a superposition of product states:

$$|\psi\rangle \quad = \quad \sum_{i=1}^{N_A} \sum_{j=1}^{N_B} C_{ij} |i\rangle_A |j\rangle_B = \sum_{\gamma} \lambda_\gamma |\phi_\gamma\rangle_A |\phi_\gamma\rangle_B$$

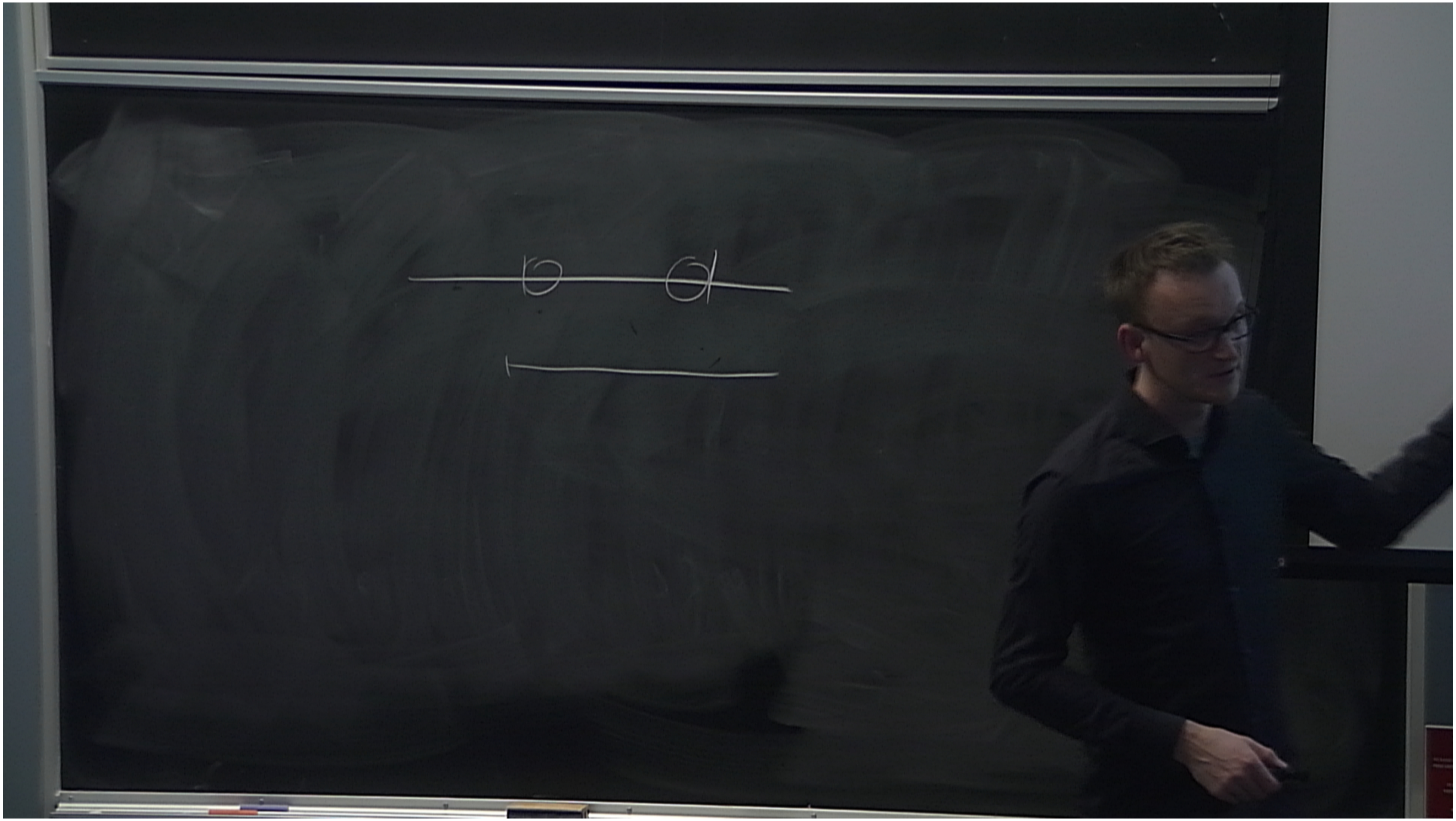
- "Artificial" edges give access to the edge modes



1D symmetry protected topological phases

- Representation of a one-dimensional many-body quantum state $|\psi\rangle = \sum_{i_1,\ldots,i_N} \psi_{i_1,\ldots,i_N} |i_1,\ldots,i_N\rangle$ as **matrix product state (MPS)** in canonical form [Vidal '02]

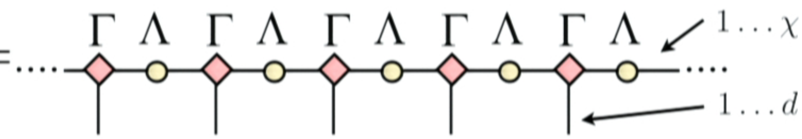
$$\psi_{\ldots,j_1,j_2,\ldots} = \ldots$$



1D symmetry protected topological phases

- Representation of a one-dimensional many-body quantum state $|\psi\rangle = \sum_{i_1,\ldots,i_N} \psi_{i_1,\ldots,i_N} |i_1,\ldots,i_N\rangle$ as **matrix product state (MPS)** in canonical form [Vidal '02]

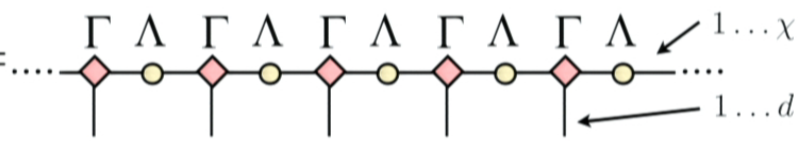
$\psi_{\ldots,j_1,j_2,\ldots} =$



1D symmetry protected topological phases

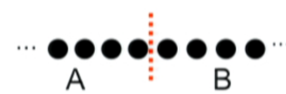
- Representation of a one-dimensional many-body quantum state $|\psi\rangle = \sum_{i_1,\ldots,i_N} \psi_{i_1,\ldots,i_N} |i_1,\ldots,i_N\rangle$ as **matrix product state (MPS)** in canonical form [Vidal '02]

$\psi_{\ldots,j_1,j_2,\ldots} =$



- Matrices are directly related to the Schmidt decomposition

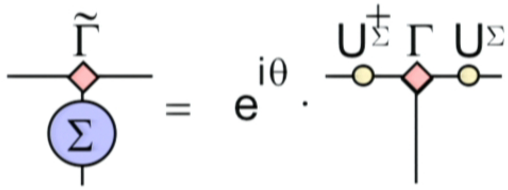
$[\phi_\alpha]_{j_1,j_2\ldots} = \alpha$



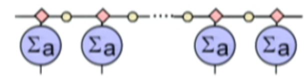
1D symmetry protected topological phases

- Transformation of an MPS under **symmetry operations**

[Perez-Garcia '07]



$$, \quad [U_\Sigma, \Lambda] = 0$$



...wave function  only changes by a phase

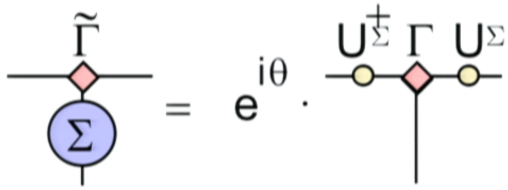
FP, E. Berg, A.M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).

FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B **81**, 064439 (2010).

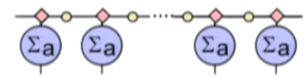
1D symmetry protected topological phases

- Transformation of an MPS under **symmetry operations**

[Perez-Garcia '07]



, $[U_\Sigma, \Lambda] = 0$



...wave function 

only changes by a phase

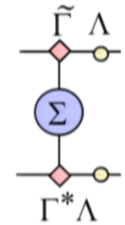
- Time reversal ($\Gamma_j \to \Gamma_j^*$) and inversion ($\Gamma_j \to \Gamma_j^T$) have a similar form

FP, E. Berg, A.M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).
FP, A.M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B **81**, 064439 (2010).

1D Symmetry protected topological phases

- Get U_Σ from the **dominant eigenvector** X of the **generalized transfermatrix** ($U_\Sigma = X^\dagger$)

$$T^{\Sigma}_{(\alpha\alpha');(\beta\beta')} = \sum_{j,j'} \Sigma_{jj'} \tilde{\Gamma}_{j',\alpha\beta} \Gamma^*_{j,\alpha'\beta'} \Lambda_\beta \Lambda_{\beta'}$$

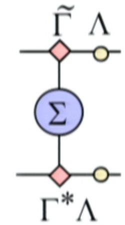


FP and A.M. Turner, Phys. Rev. B **86**, 125441 (2012).

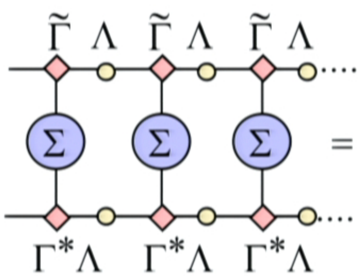
1D Symmetry protected topological phases

- Get U_Σ from the **dominant eigenvector** X of the **generalized transfermatrix** ($U_\Sigma = X^\dagger$)

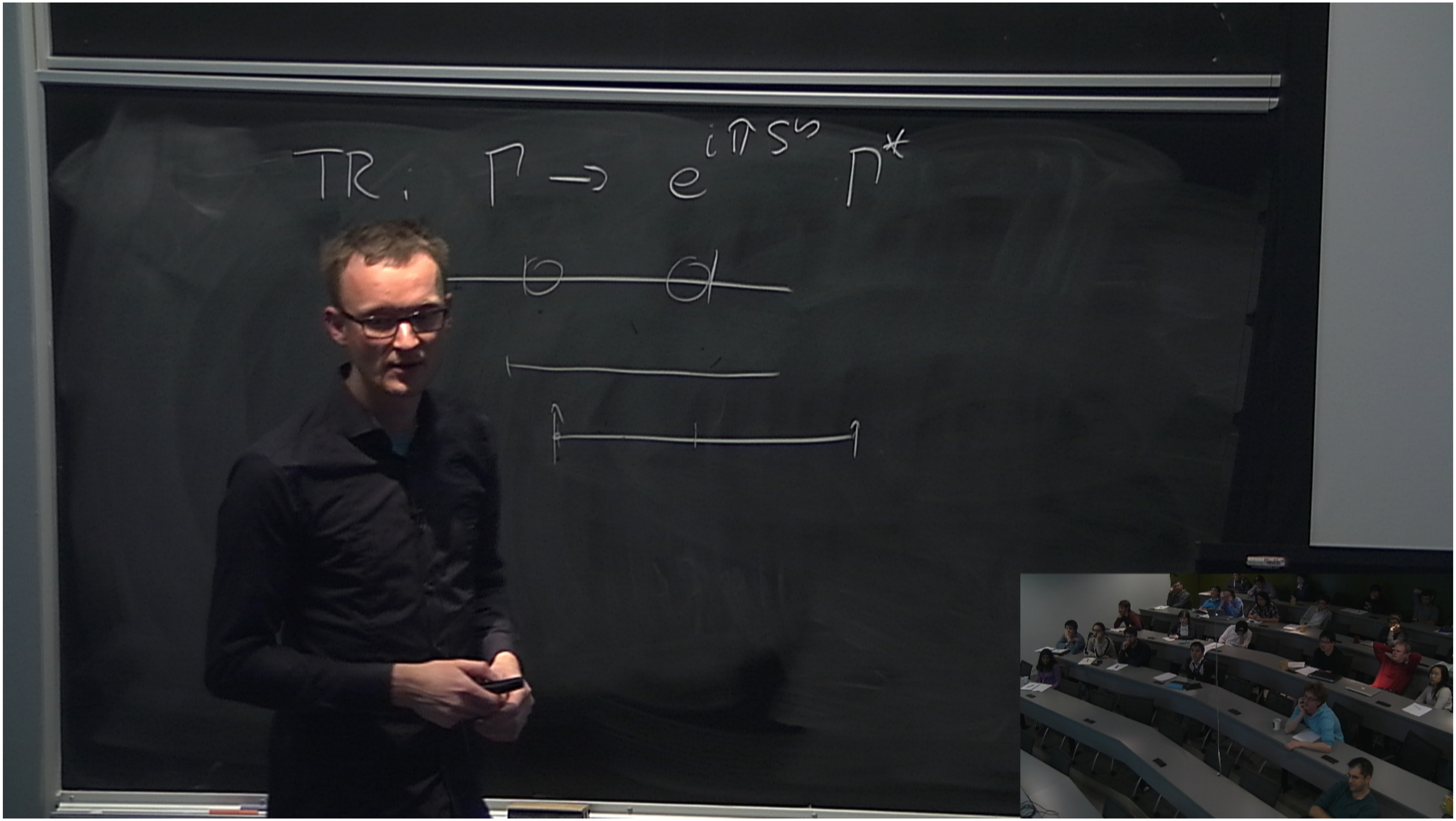
$$T^{\Sigma}_{(\alpha\alpha');(\beta\beta')} = \sum_{j,j'} \Sigma_{jj'} \tilde{\Gamma}_{j',\alpha\beta} \Gamma^*_{j,\alpha'\beta'} \Lambda_\beta \Lambda_{\beta'}$$



- Overlap of transformed Schmidt states



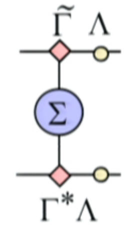
FP and A.M. Turner, Phys. Rev. B **86**, 125441 (2012).


TR: Γ → e^{iπS^y} Γ*

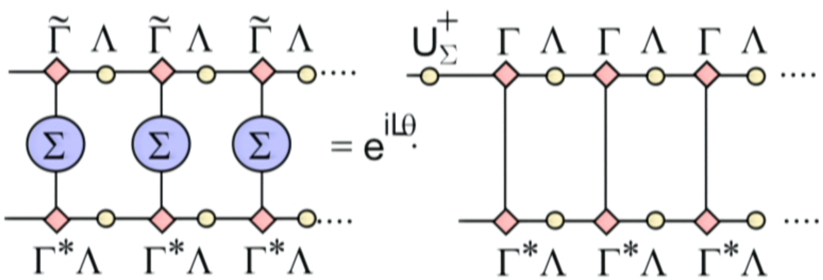
1D Symmetry protected topological phases

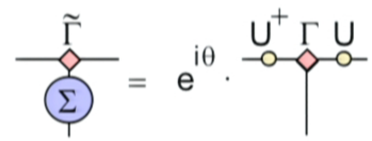
- Get U_Σ from the **dominant eigenvector** X of the **generalized transfermatrix** ($U_\Sigma = X^\dagger$)

$$T^{\Sigma}_{(\alpha\alpha');(\beta\beta')} = \sum_{j,j'} \Sigma_{jj'} \tilde{\Gamma}_{j',\alpha\beta} \Gamma^*_{j,\alpha'\beta'} \Lambda_\beta \Lambda_{\beta'}$$



- Overlap of transformed Schmidt states



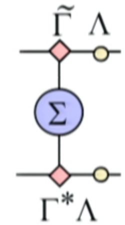


FP and A.M. Turner, Phys. Rev. B **86**, 125441 (2012).

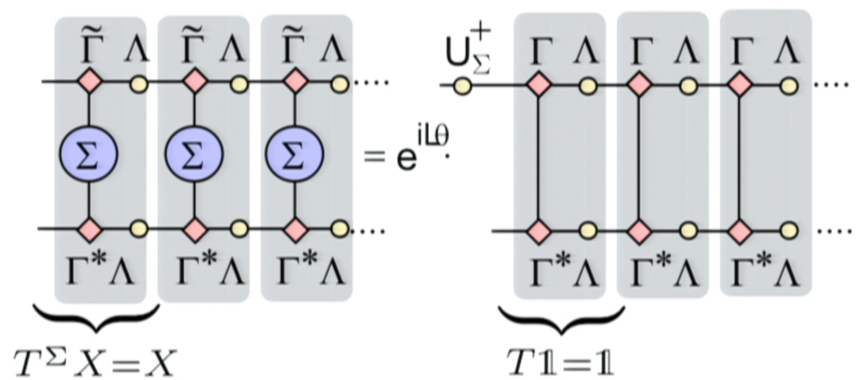
1D Symmetry protected topological phases

- Get U_Σ from the **dominant eigenvector** X of the **generalized transfermatrix** ($U_\Sigma = X^\dagger$)

$$T^{\Sigma}_{(\alpha\alpha');(\beta\beta')} = \sum_{j,j'} \Sigma_{jj'} \tilde{\Gamma}_{j',\alpha\beta} \Gamma^*_{j,\alpha'\beta'} \Lambda_\beta \Lambda_{\beta'}$$



- Overlap of transformed Schmidt states



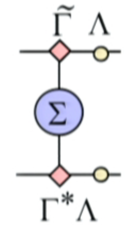
$\Leftrightarrow \quad U_\Sigma = X^\dagger$

FP and A.M. Turner, Phys. Rev. B **86**, 125441 (2012).

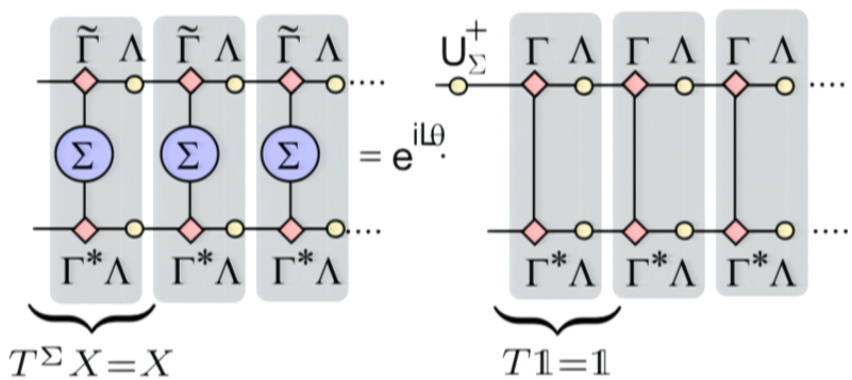
1D Symmetry protected topological phases

- Get U_Σ from the **dominant eigenvector** X of the **generalized transfermatrix** ($U_\Sigma = X^\dagger$)

$$T^{\Sigma}_{(\alpha\alpha');(\beta\beta')} = \sum_{j,j'} \Sigma_{jj'} \tilde{\Gamma}_{j',\alpha\beta} \Gamma^*_{j,\alpha'\beta'} \Lambda_\beta \Lambda_{\beta'}$$



- Overlap of transformed Schmidt states



$\Leftrightarrow$ $U_\Sigma = X^\dagger$

FP and A.M. Turner, Phys. Rev. B **86**, 125441 (2012).

1D Symmetry protected topological phases

- S=1 chain $H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + D \sum_j (S_j^z)^2$

- $\mathbb{Z}_2 \times \mathbb{Z}_2$ stabilizes Haldane phase

$$\mathcal{O}_{Z_2 \times Z_2} = \begin{cases} 0 & \text{if symmetry broken} \\ \frac{1}{\chi} \mathrm{tr}\left(U_x U_z U_x^\dagger U_z^\dagger\right) & \text{if symmetry not broken} \end{cases} .$$

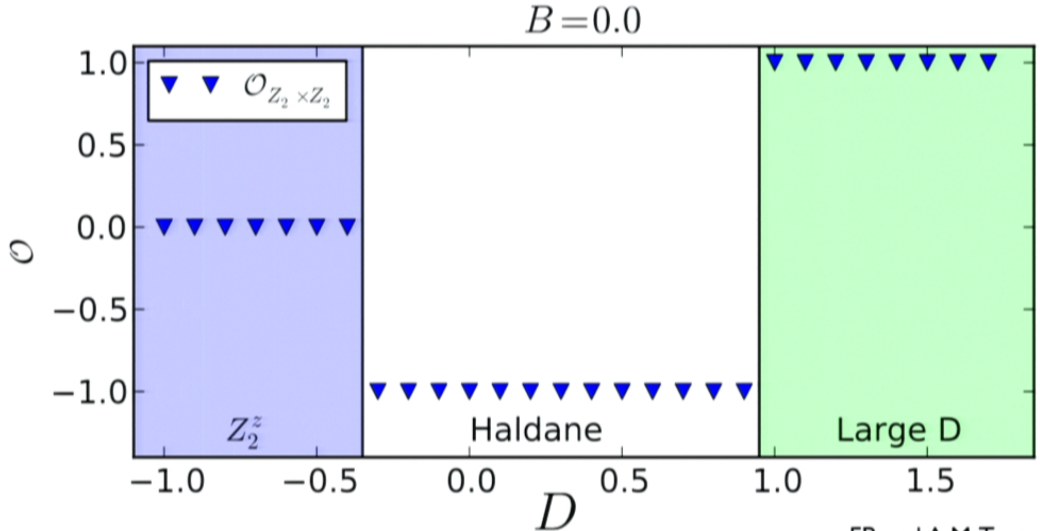
FP and A.M. Turner, Phys. Rev. B **86**, 125441 (2012).

1D Symmetry protected topological phases

- S=1 chain $H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + D \sum_j (S_j^z)^2$

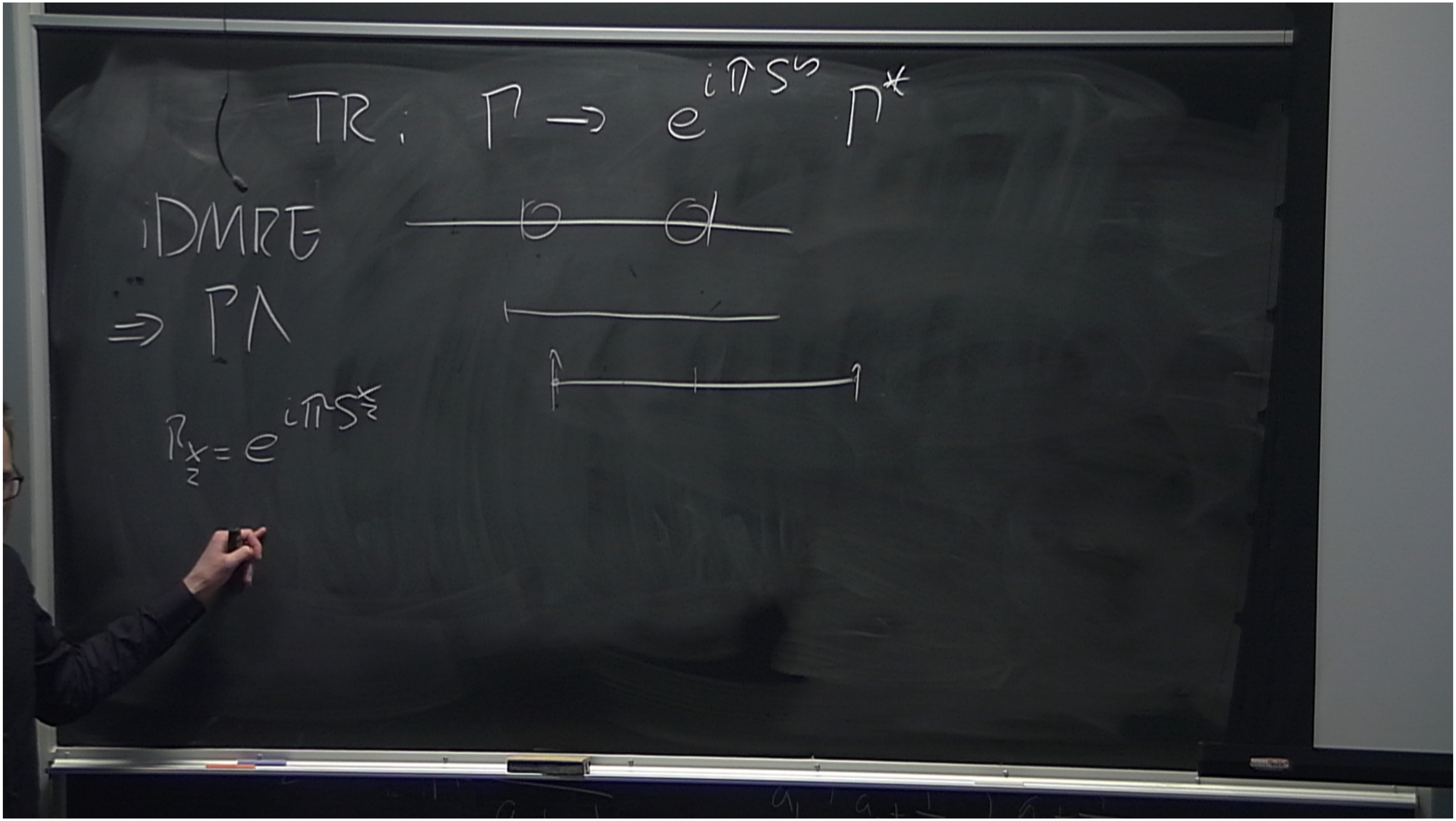
- $\mathbb{Z}_2 \times \mathbb{Z}_2$ stabilizes Haldane phase

$$\mathcal{O}_{Z_2 \times Z_2} = \begin{cases} 0 & \text{if symmetry broken} \\ \frac{1}{\chi} \mathrm{tr}\left(U_x U_z U_x^\dagger U_z^\dagger\right) & \text{if symmetry not broken} \end{cases} .$$



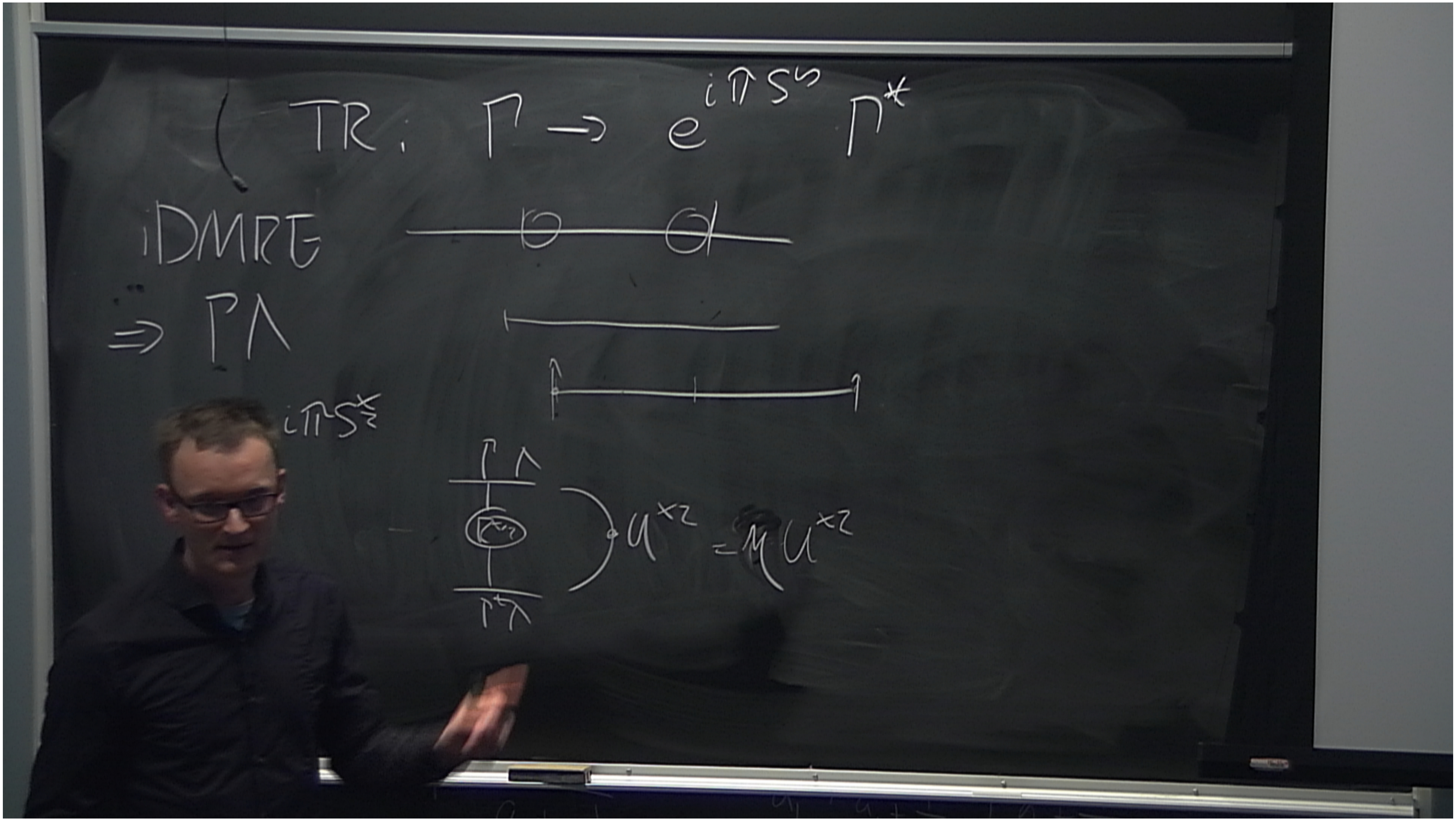
iMPS obtained using the iTEBD / iDRMG method

FP and A.M. Turner, Phys. Rev. B **86**, 125441 (2012).


TR: Γ → e^{iπS^y} Γ*
iDMRG
⇒ ΓΛ
Γ_{x/z} = e^{iπS^{x/z}}


TR:
iDMRG


TR:
iDMRG


TR:
iDMRG


TR:
iDMRG

1D Symmetry protected topological phases

- **Non-local order parameter** "detects" cohomology class from ground state wave function

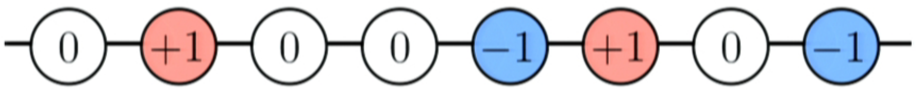
FP and A.M. Turner, Phys. Rev. B **86**, 125441 (2012).

1D Symmetry protected topological phases

- **Non-local order parameter** "detects" cohomology class from ground state wave function

- **String order** in the case of a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetric phase

[den Nijs '89]

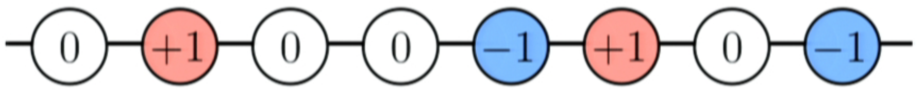


FP and A.M. Turner, Phys. Rev. B **86**, 125441 (2012).

1D Symmetry protected topological phases

- **Non-local order parameter** "detects" cohomology class from ground state wave function
- **String order** in the case of a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetric phase

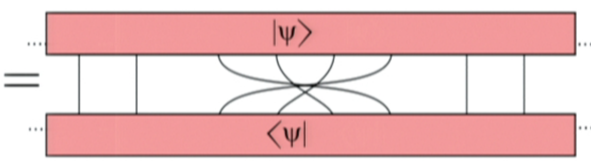
[den Nijs '89]

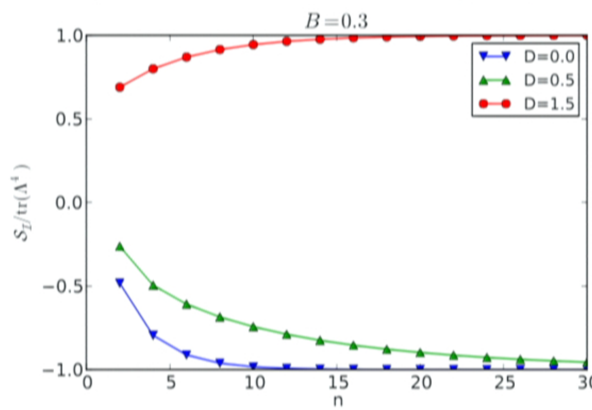


- **Inversion** symmetry based order parameter distinguishes Haldane and trivial Phase

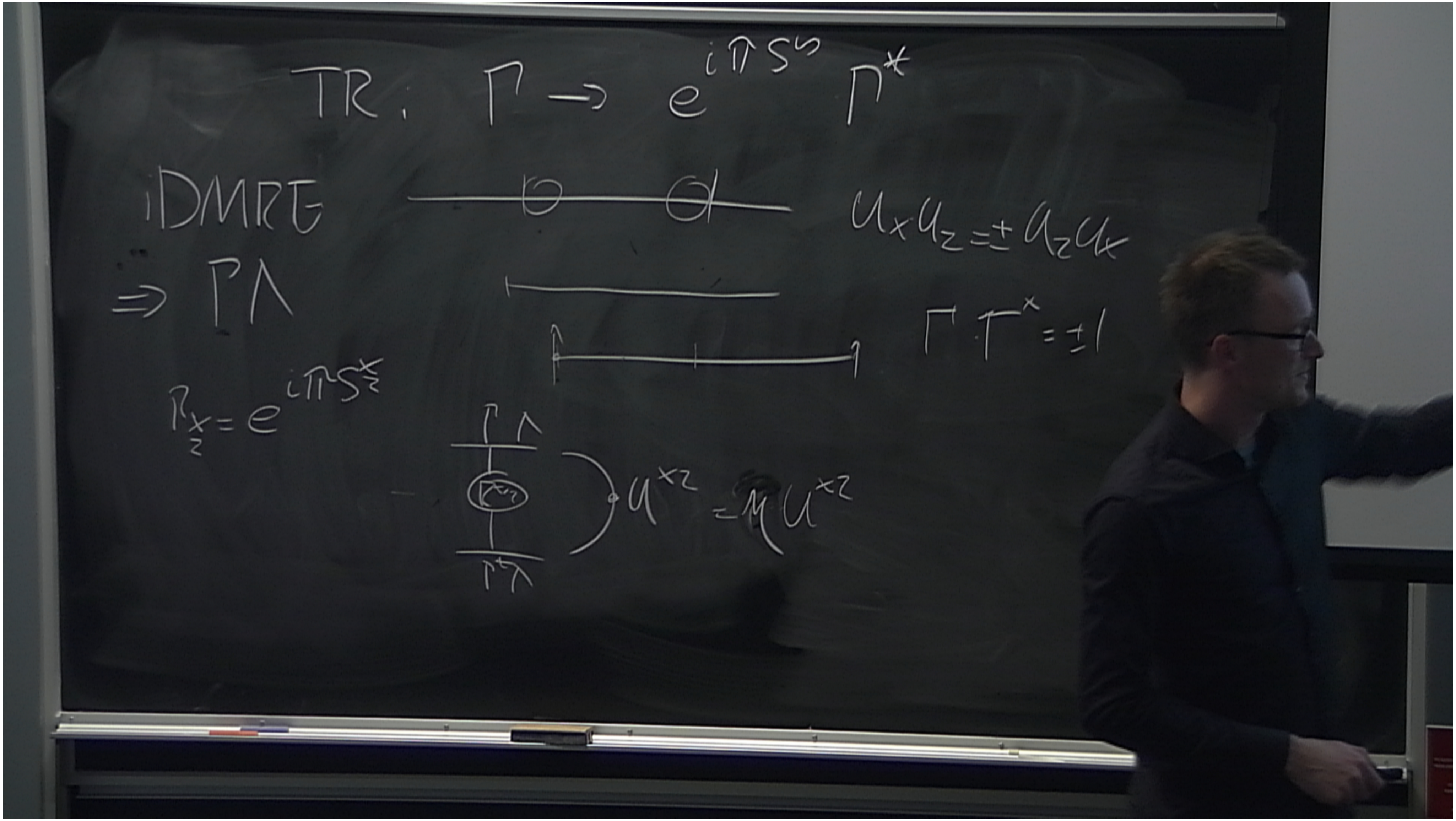
$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + D \sum_j (S_j^z)^2$$

$\mathcal{O}_I =$



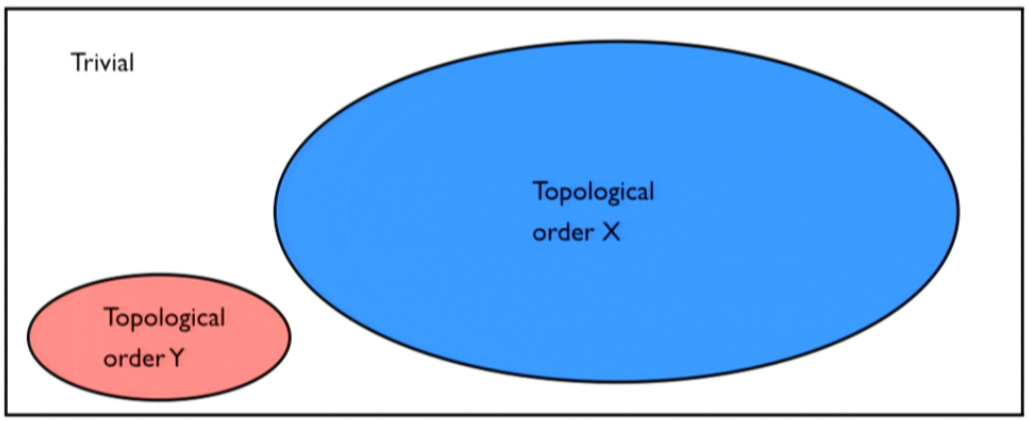


FP and A.M. Turner, Phys. Rev. B **86**, 125441 (2012).


TR: Γ → e^{iπS^y} Γ*
iDMRG
u_x u_z = ± u_z u_x
⇒ ΓΛ
Γ Γ* = ±1
Γ_{x/z} = e^{iπS^{x/z}}
ΓΛ
u^{xz}
u^{xz}

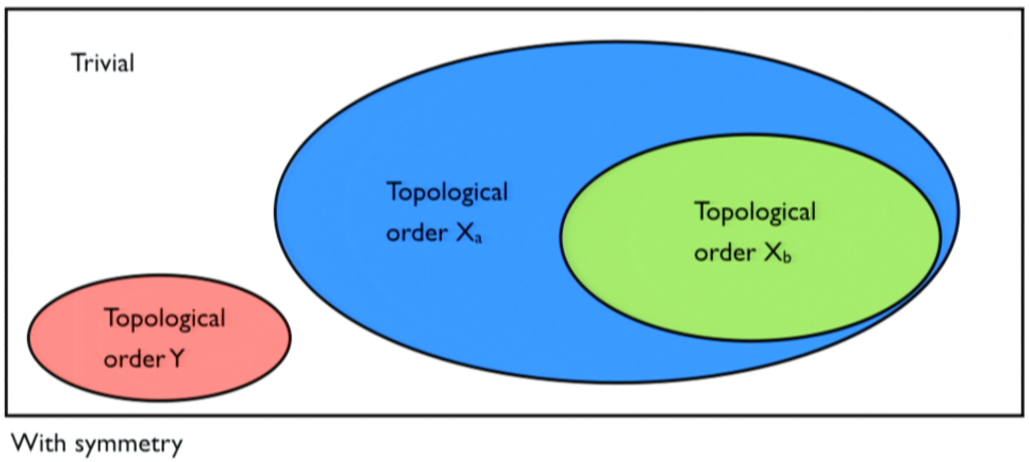
Symmetry enriched topological order

- **Intrinsic topological order** is characterized by its **anyonic quasi particles (QP)** (no symmetry required) [Wen '91]



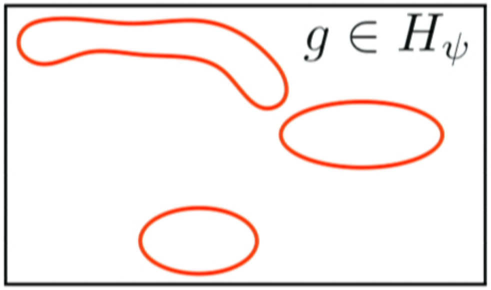
Symmetry enriched topological order

- **Intrinsic topological order** is characterized by its **anyonic quasi particles (QP)** (no symmetry required) [Wen '91]
- Topologically ordered systems have a **richer structure when symmetries are present** [Wang and Wen '12, Wen '13]



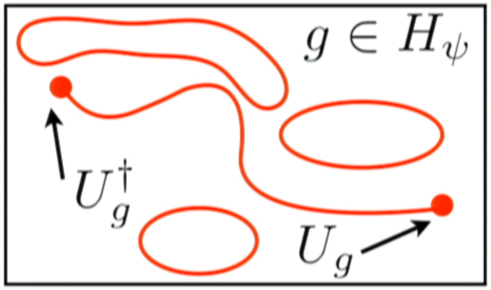
Symmetry enriched topological order

- Hamiltonian and ground state have the same symmetry ($G_H = G\psi$)



Symmetry enriched topological order

- Hamiltonian and ground state have the same symmetry ($G_H = G\psi$)



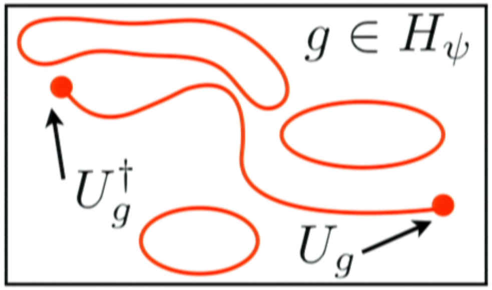
- **Fractionalization** of symmetry operations in terms of the anionic quasiparticles (QP)

➡ QP induce **projective representations:**

$$gh = k : U_g U_h = e^{i\phi_{gh}} U_k$$

Symmetry enriched topological order

- Hamiltonian and ground state have the same symmetry ($G_H = G\psi$)



- **Fractionalization** of symmetry operations in terms of the anionic quasiparticles (QP)

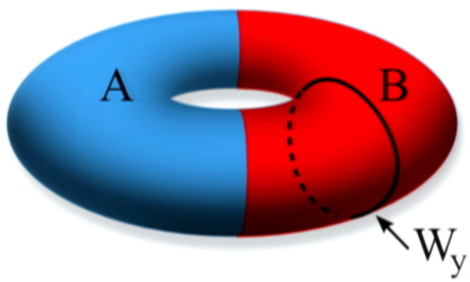
➡ QP induce **projective representations:**

$$gh = k : U_g U_h = e^{i\phi_{gh}} U_k$$

- Co-homology $H^2[G, U(1)]$: **Inequivalent projective representations classify different SET's**

Symmetry enriched topological order

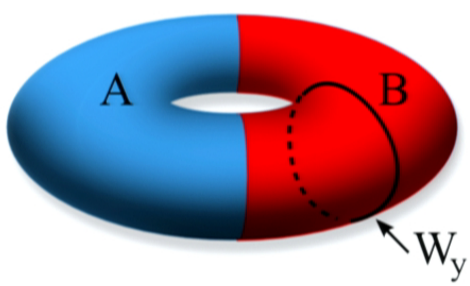
- **Topological degeneracy:** Ground state degeneracy and the torus is equal to the number of QP



Symmetry enriched topological order

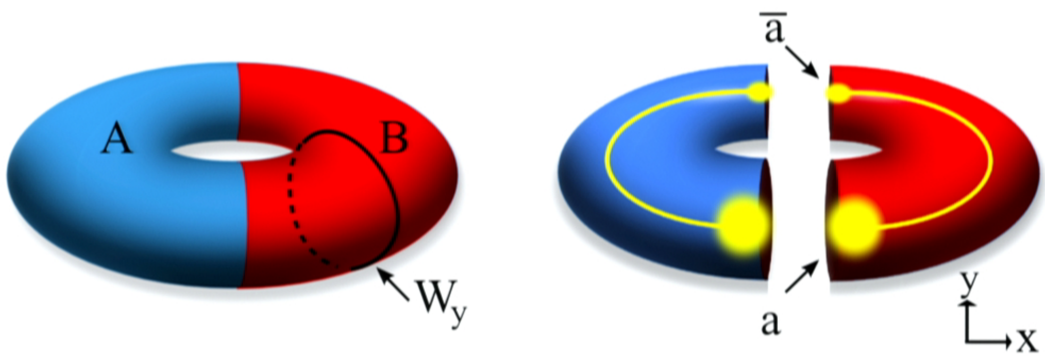
- **Topological degeneracy:** Ground state degeneracy and the torus is equal to the number of QP
- **Minimally entangled ground states (MES)** of a topological system on torus are eigenstates of Wilson loop operators W_y parallel to the bipartite cut

  [Zhang et al. '12; Grover et al. '12; Cincio & Vidal '12, Wen '90]



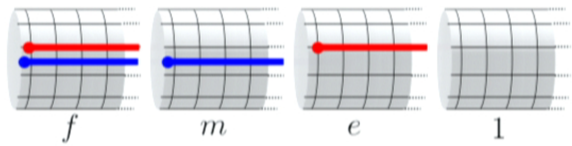
Symmetry enriched topological order

- **Topological degeneracy:** Ground state degeneracy and the torus is equal to the number of QP
- **Minimally entangled ground states (MES)** of a topological system on torus are eigenstates of Wilson loop operators W_y parallel to the bipartite cut
  [Zhang et al. '12; Grover et al. '12; Cincio & Vidal '12, Wen '90]
- QP of type a and are localized at the edges of the cut



Symmetry enriched topological order

- MES can also be obtained for an **infinite cylinder** (locally equivalent to a large torus) [Cincio & Vidal '12, Zaletel et al '13]
- A $\mathbb{Z}_2$ liquid (e.g., toric code) has four quasiparticle types [Kitaev '01]

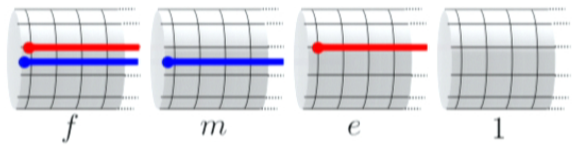


C.-Y. Huang, X. Chen, and FP, arXiv:1312.3093.

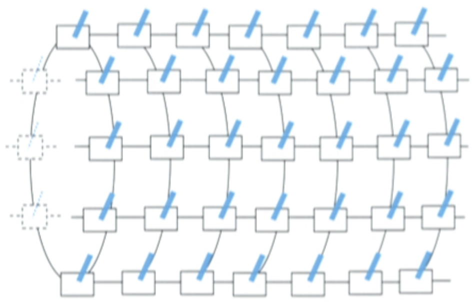
Symmetry enriched topological order

- MES can also be obtained for an **infinite cylinder** (locally equivalent to a large torus) [Cincio & Vidal '12, Zaletel et al '13]

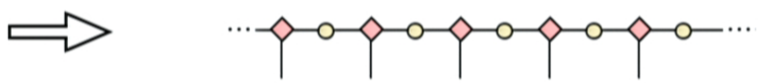
- A $\mathbb{Z}_2$ liquid (e.g., toric code) has four quasiparticle types [Kitaev '01]



- Map cylinder to an effective one-dimensional system: extract projective representations of QPs like in 1D SPTP



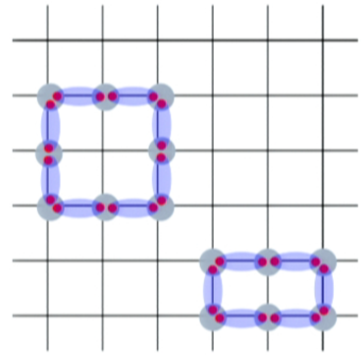
[Cirac et al '11]



C.-Y. Huang, X. Chen, and FP, arXiv:1312.3093.

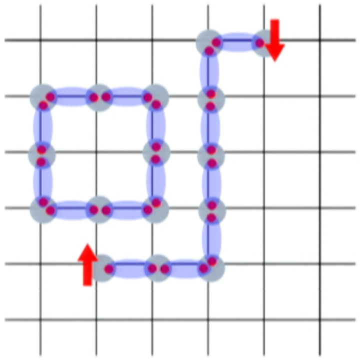
Symmetry enriched topological order

- $S = 1$ bosons model state: $\mathbb{Z}_2$ loop gas of AKLT chains



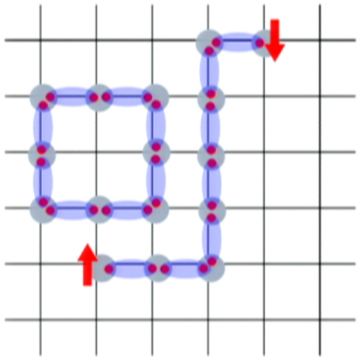
Symmetry enriched topological order

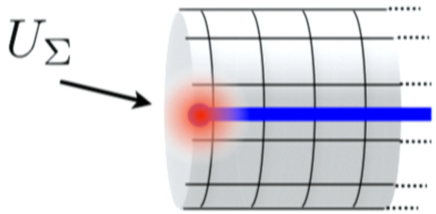
- $S = 1$ bosons model state: $\mathbb{Z}_2$ loop gas of AKLT chains
- e and f-particles in projective ($S = 1/2$) representation



Symmetry enriched topological order

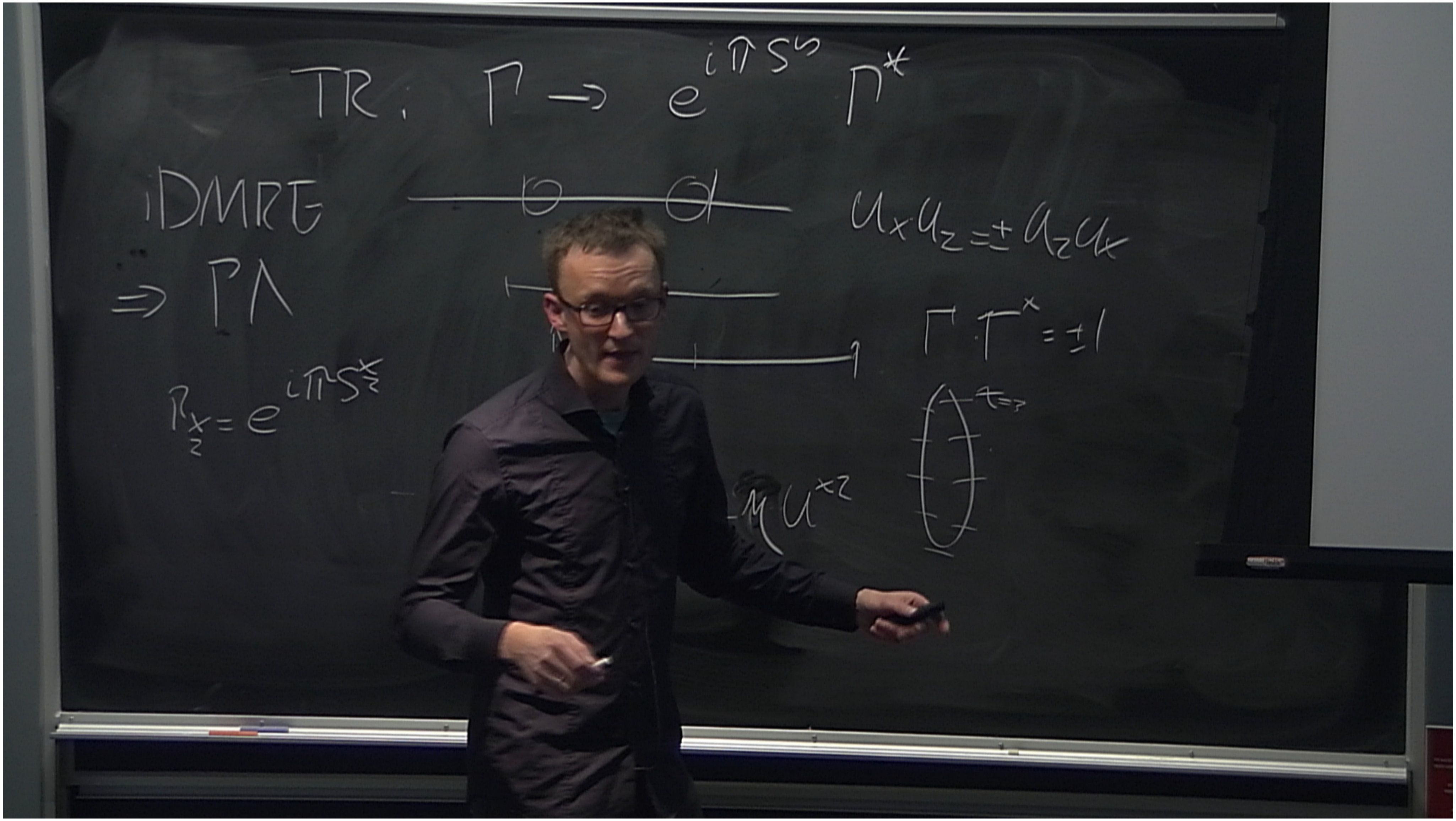
- $S = 1$ bosons model state: $\mathbb{Z}_2$ loop gas of AKLT chains
- e and f-particles in projective ($S = 1/2$) representation
- Represented by $\chi = 3$ **tensor product state**

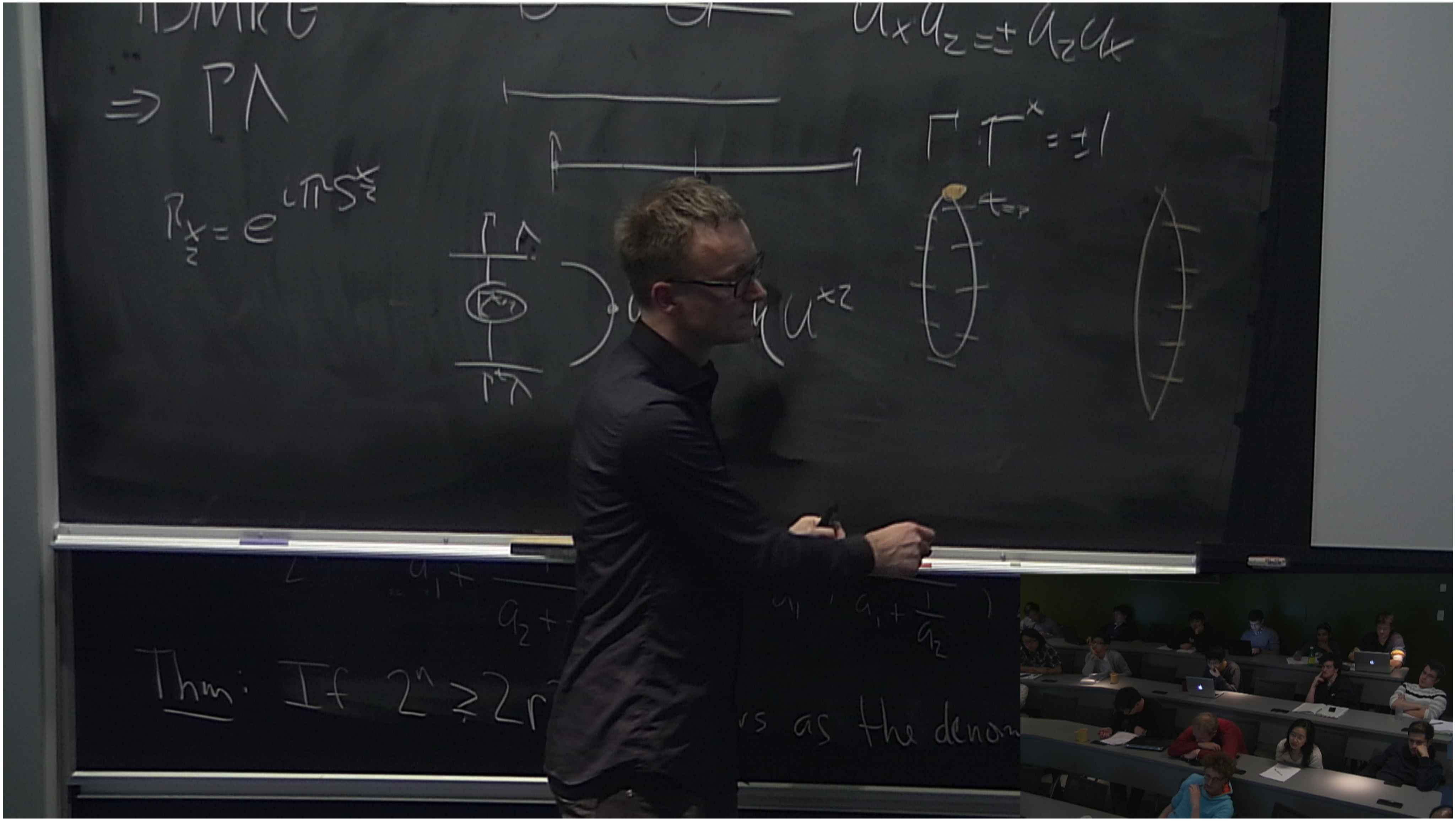




QP	1	e	m	f
$U_x U_z U_x^{-1} U_z^{-1}$	1	-1	1	-1

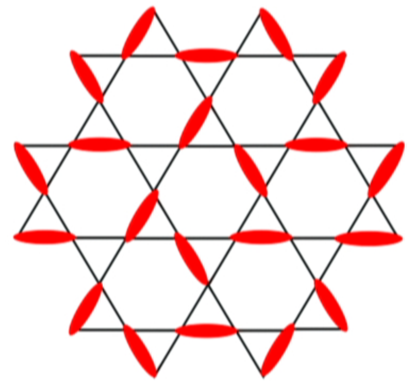
C.-Y. Huang, X. Chen, and FP, arXiv:1312.3093.


TR: Γ → e^{iπS^y} Γ*
iDMRG
⇒ PΛ
U_x U_z = ± U_z U_x
Γ Γ* = ±1
Γ_x = e^{iπS^x}



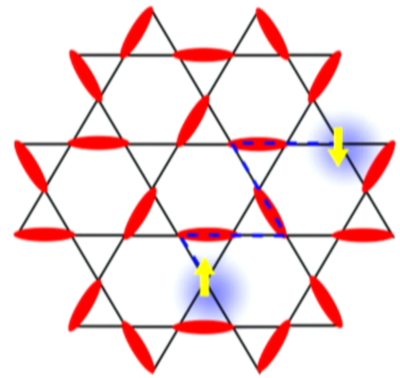
Symmetry enriched topological order

- RVB state on the kagome lattice:
  $\mathbb{Z}_2$ spin liquid states (spin 1/2 singlets)



Symmetry enriched topological order

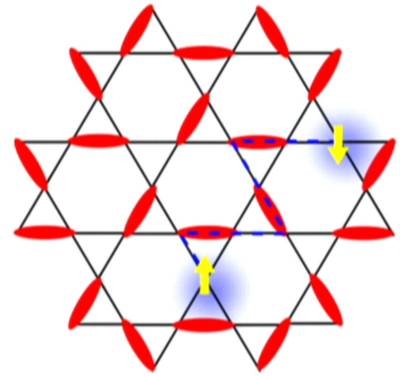
- RVB state on the kagome lattice: $\mathbb{Z}_2$ spin liquid states (spin 1/2 singlets)
- e and f-particles in projective ($S = 1/2$) representation



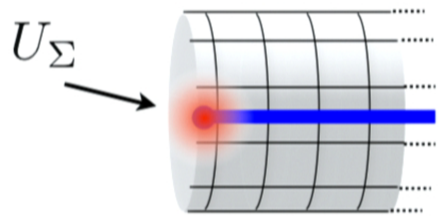
C.-Y. Huang, X. Chen, and FP, arXiv:1312.3093.

Symmetry enriched topological order

- RVB state on the kagome lattice: $\mathbb{Z}_2$ spin liquid states (spin 1/2 singlets)



- e and f-particles in projective ($S = 1/2$) representation
- Represented by $\chi = 3$ **tensor product state**

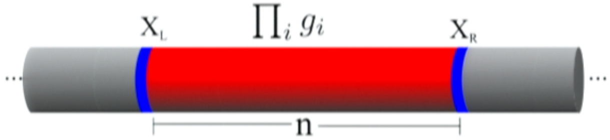


QP	1	e	m	f
$U_x U_z U_x^{-1} U_z^{-1}$	1	-1	1	-1

C.-Y. Huang, X. Chen, and FP, arXiv:1312.3093.

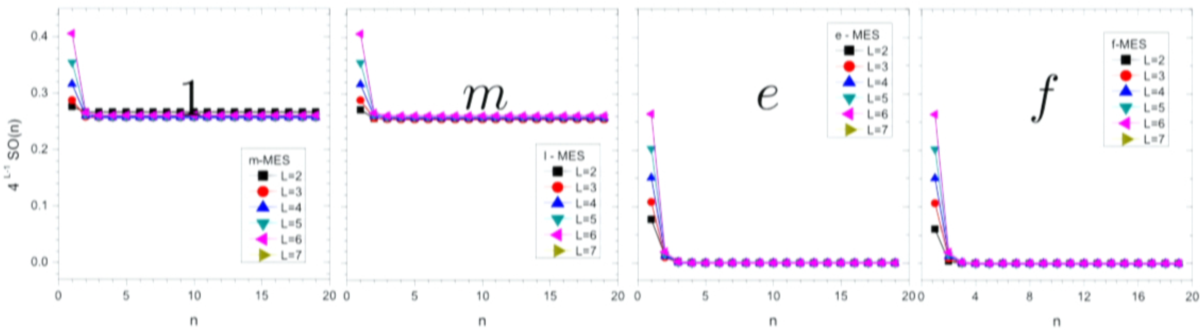
Symmetry enriched topological order

- "String order (SO) parameter": SO detects the projective representations of anyons



- Selection rule forces SO to vanish if edge spins are fractionalized

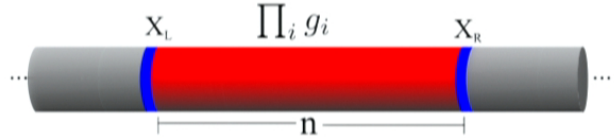
$$SO = \langle \psi_0 | \prod_{j=1}^{n} \exp(i S_j^z) | \psi_0 \rangle$$



C.-Y. Huang, X. Chen, and FP, arXiv:1312.3093

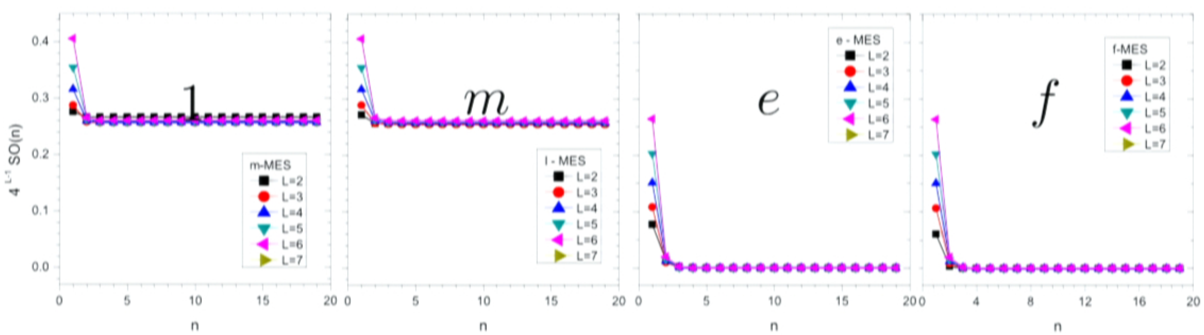
Symmetry enriched topological order

- "String order (SO) parameter": SO detects the projective representations of anyons



- Selection rule forces SO to vanish if edge spins are fractionalized

$$SO = \langle \psi_0 | \prod_{j=1}^{n} \exp(iS_j^z) | \psi_0 \rangle$$

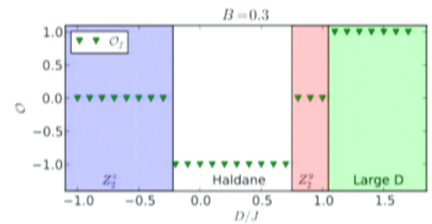


C.-Y. Huang, X. Chen, and FP, arXiv:1312.3093

Summary

- Non-local order parameters which detects/distinguishes symmetry protected topological phases in 1D
  - Can be obtained directly from a generalized transfermatrix
  - Expressions which can be evaluated using any numerical methods, e.g., Quantum Monte Carlo

FP. and A. M. Turner, Phys. Rev. B 86, 125441 (2012)



- Non-local order parameters which detects/distinguishes symmetry enriched topological phases in 2D

C.-Y. Huang, X. Chen, and FP, arXiv:1312.3093

Thank You!