Title: Probabilistic protocols in quantum information

Date: Feb 13, 2014 04:00 PM

URL: http://pirsa.org/14020138

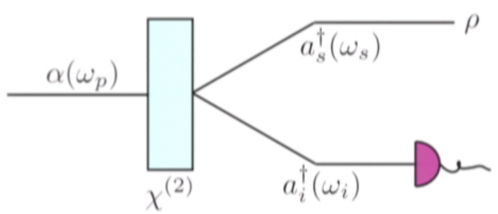
Abstract: Probabilistic protocols in quantum information are an attempt to improve performance by occasionally reporting a better result than could be expected from a deterministic protocol. Here we show that probabilistic protocols can never improve performance beyond the quantum limits on the corresponding deterministic protocol. To illustrate this result we examine three common probabilistic protocols: probabilistic amplification, weak value amplification, and probabilistic metrology. In each of these protocols we show explicitly that the optimal deterministic protocol is better than the corresponding probabilistic protocol when the probabilistic nature of the protocol is correctly accounted for.

Probabilistic protocols in quantum information

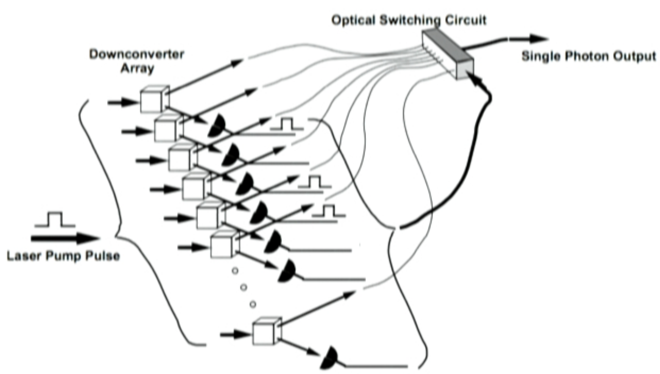
Joshua Combes
University of New Mexico
2014/02/13 @ Perimeter Institute

Some probabilistic protocols in QI

Probabilistic sources



Branczyk et al., NJP **12**, 063001 (2010)



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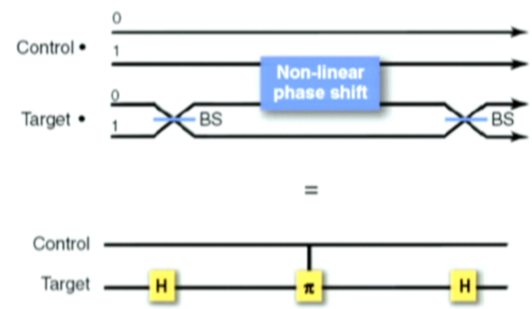
Probabilistic gates



O'Brien, Science, 318, 1467 (2007)

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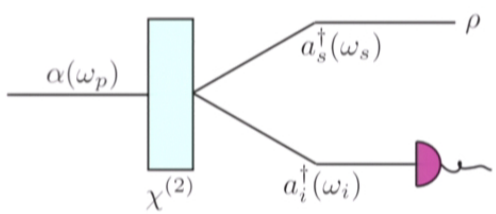

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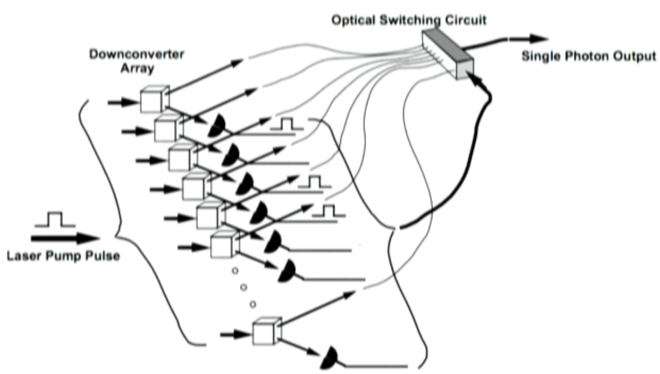

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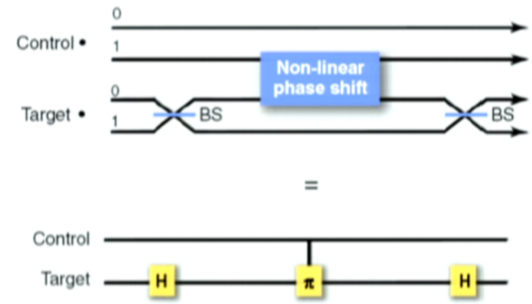

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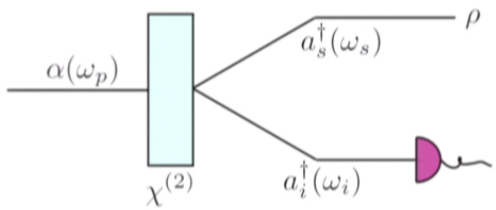

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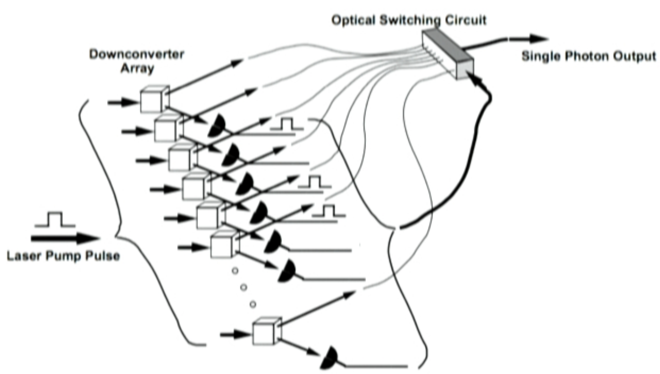

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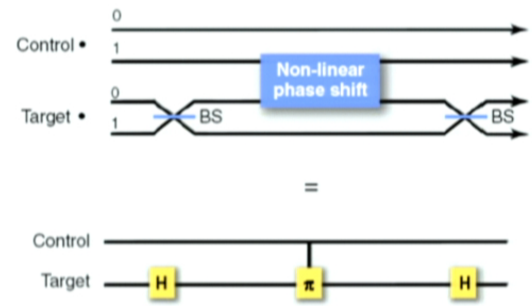

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Some probabilistic protocols in QI

1. Probabilistic noiseless amplification

2a. Probabilistic noiseless detection

2b. Probabilistic noiseless estimation

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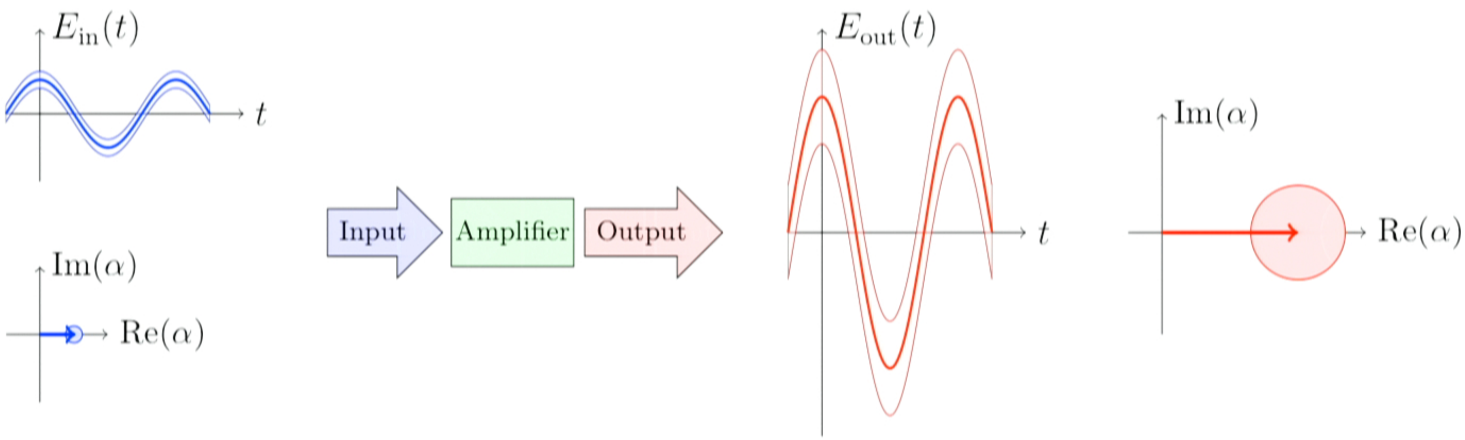
Weak value amplification

Probabilistic metrology

A linear amplifier has an output proportional to its input

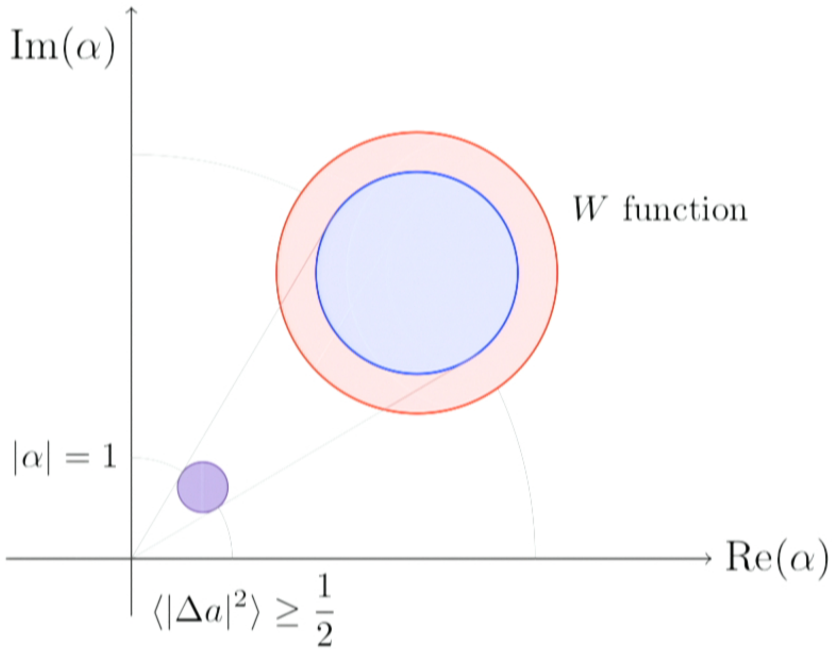
$$E(t) = \frac{1}{2}(ae^{-i\omega t} + a^\dagger e^{i\omega t}) = \frac{1}{\sqrt{2}}(x_1 \cos\omega t + x_2 \sin\omega t) \qquad a = \frac{1}{\sqrt{2}}(x_1 + ix_2)$$



$$\langle E_{\text{in}}(t)\rangle = \text{Re}(\langle a\rangle e^{-i\omega t}) = \cos(\omega t)$$

$$\langle E_{\text{out}}(t)\rangle = g\cos(\omega t)$$

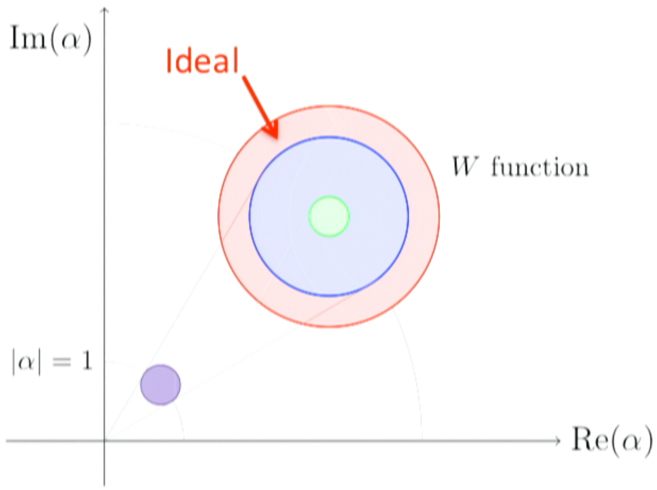
Ideal linear amplification



$$a_{\mathrm{out}} = g a_{\mathrm{in}}$$

$$a_{\mathrm{out}} = g a_{\mathrm{in}} + L^{\dagger}$$

Probabilistic amplification



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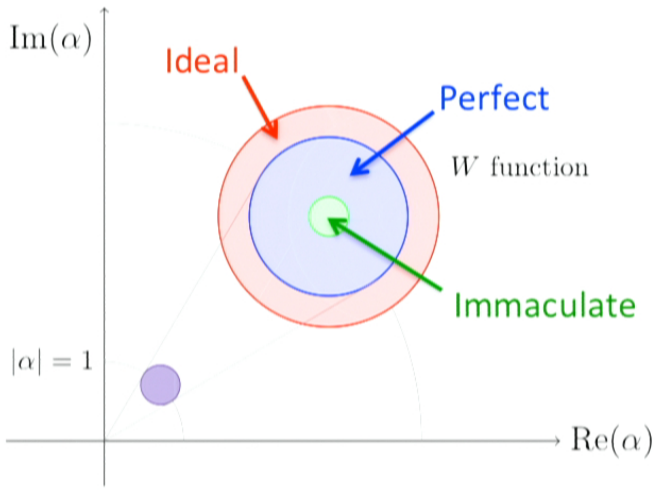
Immaculate amplification



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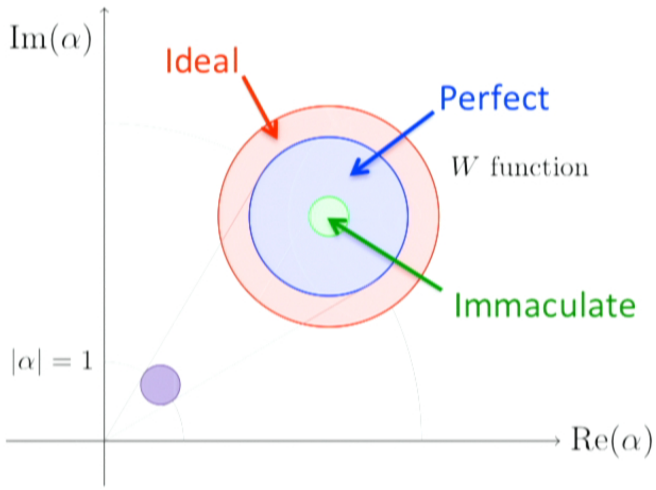

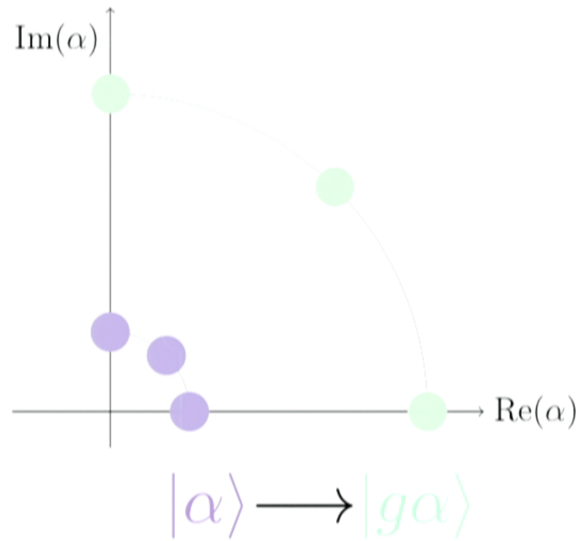

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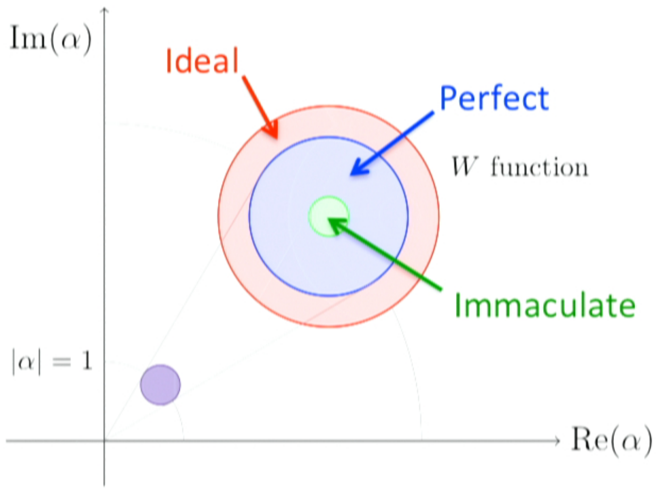
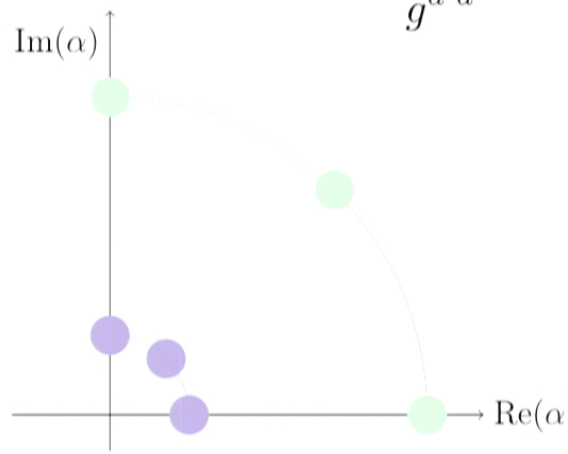


$g^{a^\dagger a}$



$$|\alpha\rangle \longrightarrow |g\alpha\rangle$$

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Figures of merit

$p(\checkmark)$

F

Immaculate amplification proposals and experiments

- Approximately 20 theory papers
- Application: improved QKD ? + 6 papers

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nature physics | LETTERS
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Noise-powered probabilistic concentration of phase information

Mario A. Usuga[1,2†], Christian R. Müller[1,3†], Christoffer Wittmann[1,3], Petr Marek[4], Radim Filip[4], Christoph Marquardt[1,3], Gerd Leuchs[1,3] and Ulrik L. Andersen[2]*

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Single Kraus operator: $K = g^{a^\dagger a}$

$P_N K \qquad P_N = \sum_{n=0}^{N} |n\rangle\langle n|$

$$|\alpha\rangle \to \frac{P_N K\,|\alpha\rangle}{\sqrt{p(\checkmark\,|\alpha)}}$$

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Phase insensitivity

$$\mathcal{R}_\theta \rho = e^{i\theta a^\dagger a} \rho e^{-i\theta a^\dagger a}$$

$$\mathcal{R}_\theta \circ \mathcal{A} = \mathcal{A} \circ \mathcal{R}_\theta$$

Physical (trace non-increasing)

$$K^\dagger P_N K \leq I$$

Immaculate amplification

High fidelity operating region: $\bar{\alpha} < \sqrt{N}/g$

$$F(\bar{\alpha}) \approx 1 \qquad p(\checkmark|\bar{\alpha}) = \frac{e^{-\bar{\alpha}^2}}{g^{2N}}$$

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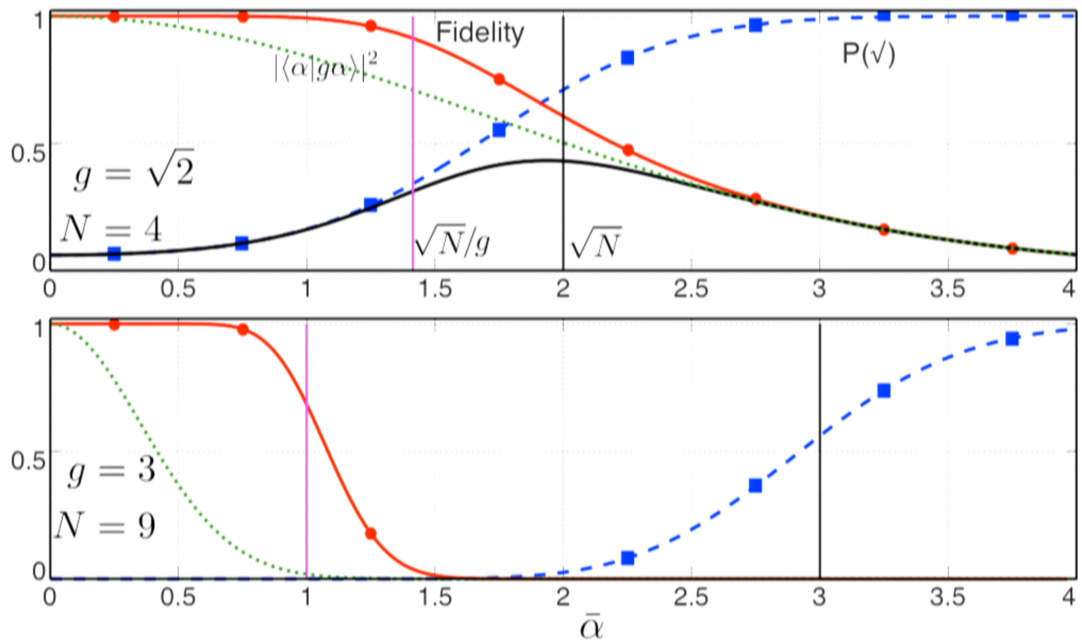

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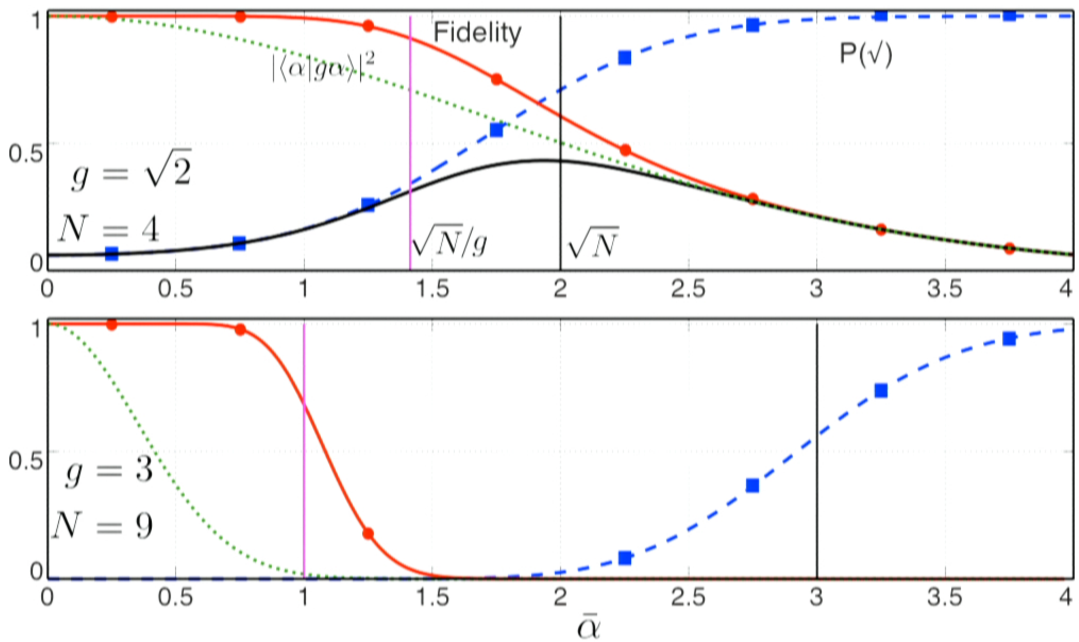

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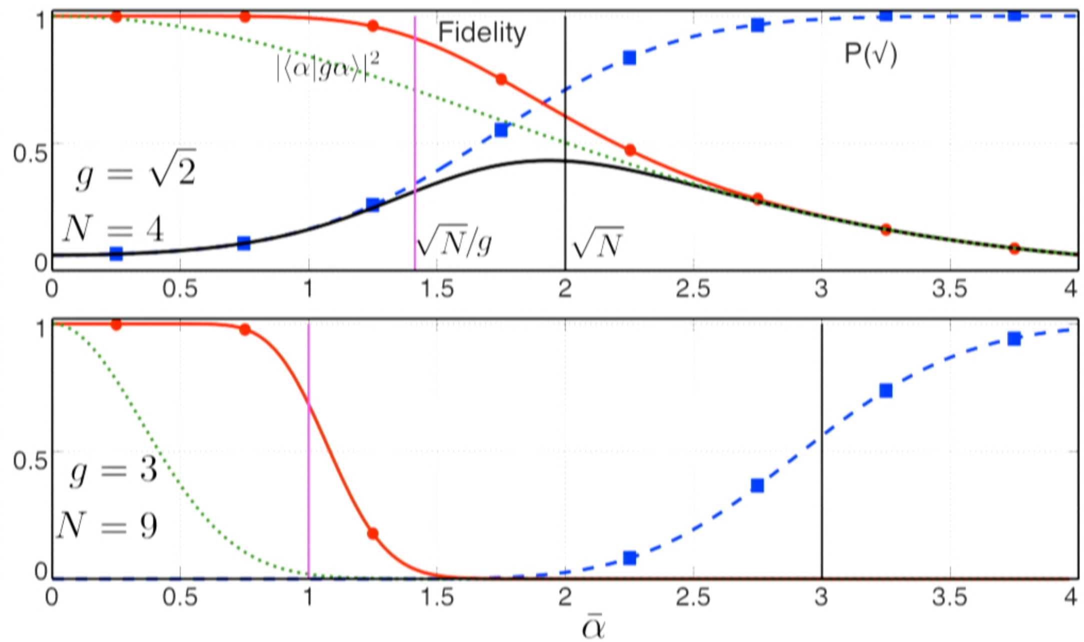

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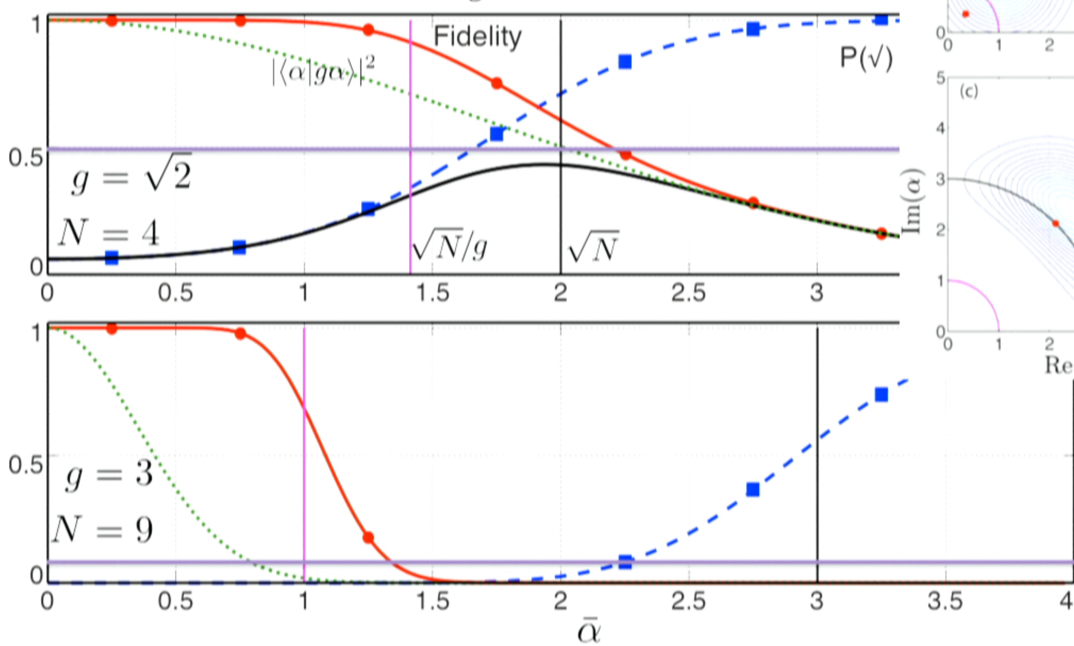

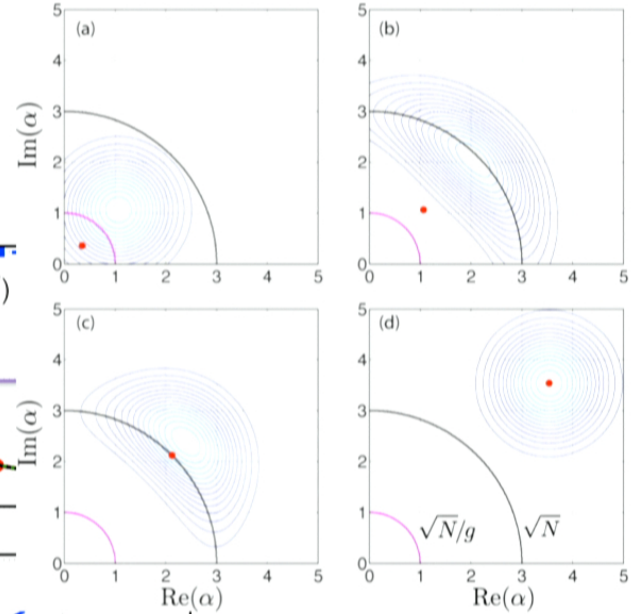

Immaculate amplification vs Ideal amplification

1. What are the quantum limits?

Immaculate amplification $$p(\checkmark|\bar{\alpha})F(\bar{\alpha}) = \frac{e^{-\bar{\alpha}^2}}{g^{2N}}$$

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Quantum limits on probabilistic amplifiers
Phys. Rev. A **88**, 033852 (2013)
Shashank Pandey, Zhang Jiang, Joshua Combes, Carlton M. Caves

Part II

a) weak value amplification

Weak values are suboptimal for estimation and detection
Phys. Rev. Lett. **112**, 040406 (2014).
Christopher Ferrie, Joshua Combes

b) probabilistic metrology

Probabilistic quantum metrology? Probably not.
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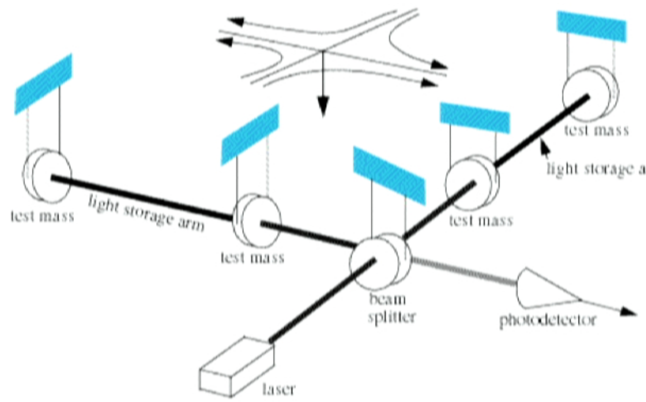
Detection vs Estimation

Did we detect a gravity wave?

What was the waveform ?



ligo.caltech.edu

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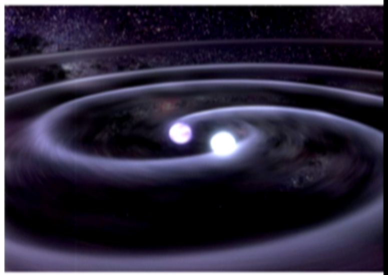

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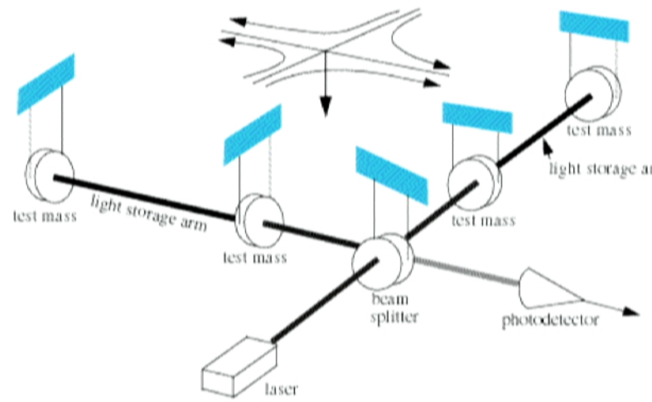

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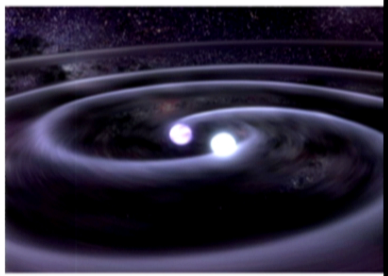

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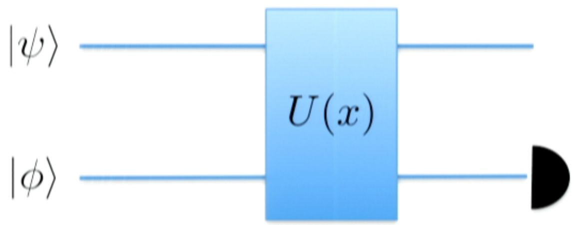
Weak value amplification & Probabilistic metrology



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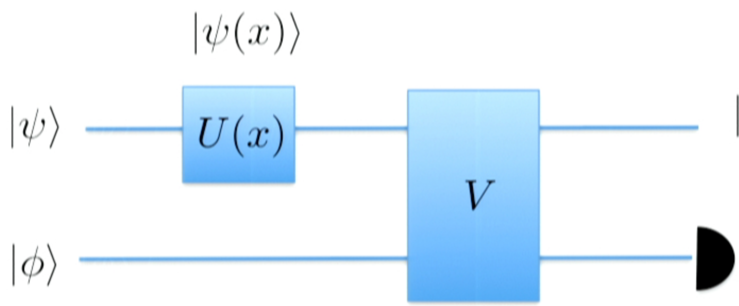

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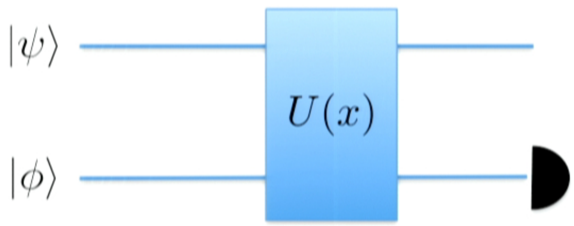

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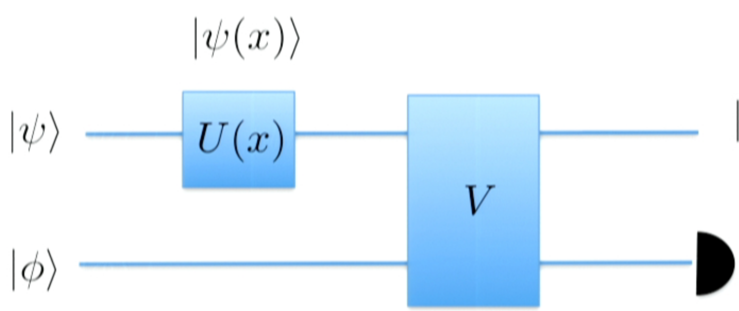

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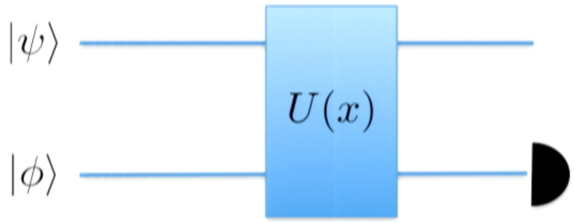

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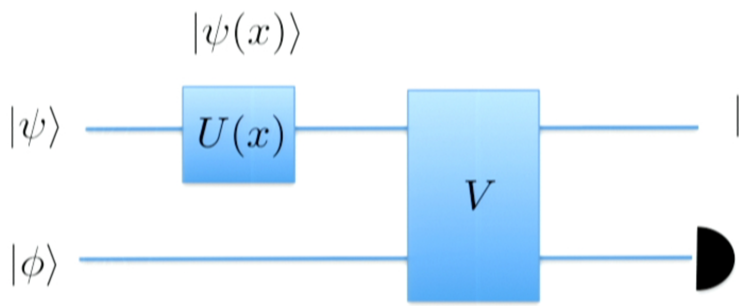

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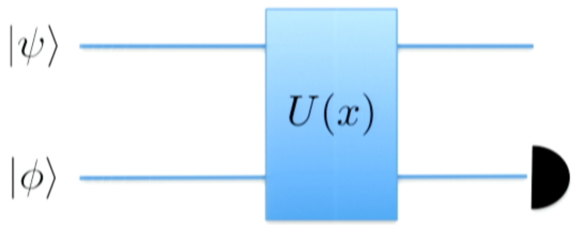

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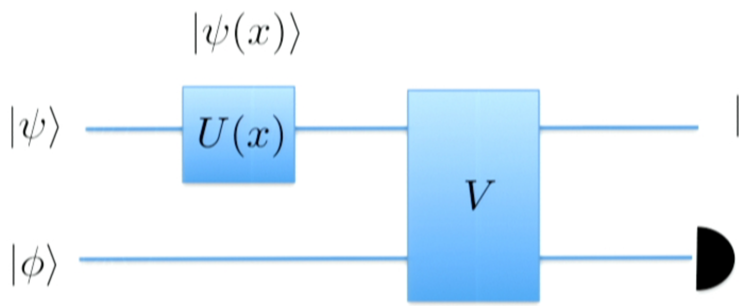

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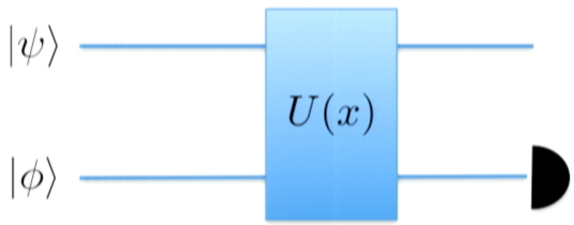

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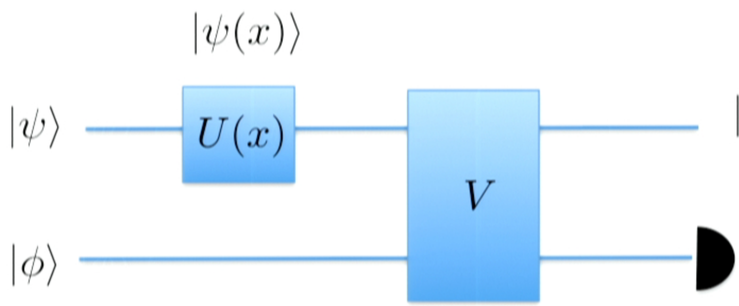

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Performance metric: likelihood-ratio test

H_0 is the null hypothesis

1. H_0 true; report H_0
2. H_0 true; report H_1
3. H_1 true; report H_1
4. H_1 true; report H_0

Performance metric: likelihood-ratio test

H_0 is the null hypothesis

1. H_0 true; report H_0
2. H_0 true; report H_1
3. H_1 true; report H_1
4. H_1 true; report H_0

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minimise

$$\Lambda(\text{data}) = \frac{p(\text{data}|H_0)}{p(\text{data}|H_1)} \qquad \begin{array}{l} \Lambda > c : \text{report } H_0 \\ \Lambda < c : \text{report } H_1 \end{array}$$

Performance metric: likelihood-ratio test

H_0 is the null hypothesis

1. H_0 true; report H_0
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minimise (items 2 and 4)

$$\Lambda(\text{data}) = \frac{p(\text{data}|H_0)}{p(\text{data}|H_1)}$$

$\Lambda > c$: report H_0

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Performance metric: likelihood-ratio test

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$$\Lambda(\text{data}) = \frac{p(\text{data}|H_0)}{p(\text{data}|H_1)}$$

$\Lambda > c$: report H_0

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$H_0 : x = 0$

$H_1 : x \neq 0$

$$\Lambda(\text{data}) = \frac{p(\text{data}|0)}{\max_x p(\text{data}|x)}$$

Performance metric: likelihood-ratio test

H_0 is the null hypothesis

Why likelihood-ratio test?

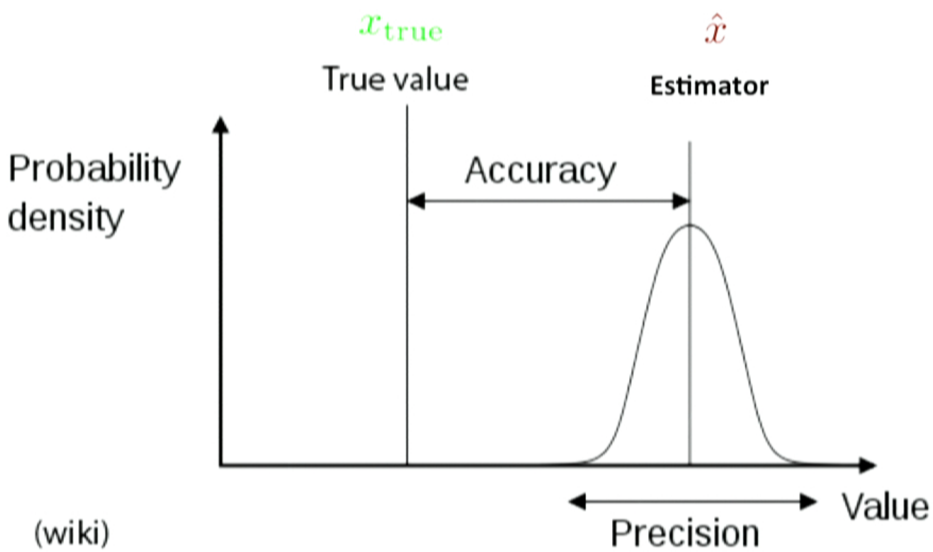
Minimise the occurrence of false positives: H_0 true; report H_1

$$\Lambda(\mathrm{data}) < c$$
$$p[\Lambda(\mathrm{data}) < c | H_0] < \alpha$$

Neyman-Person lemma:

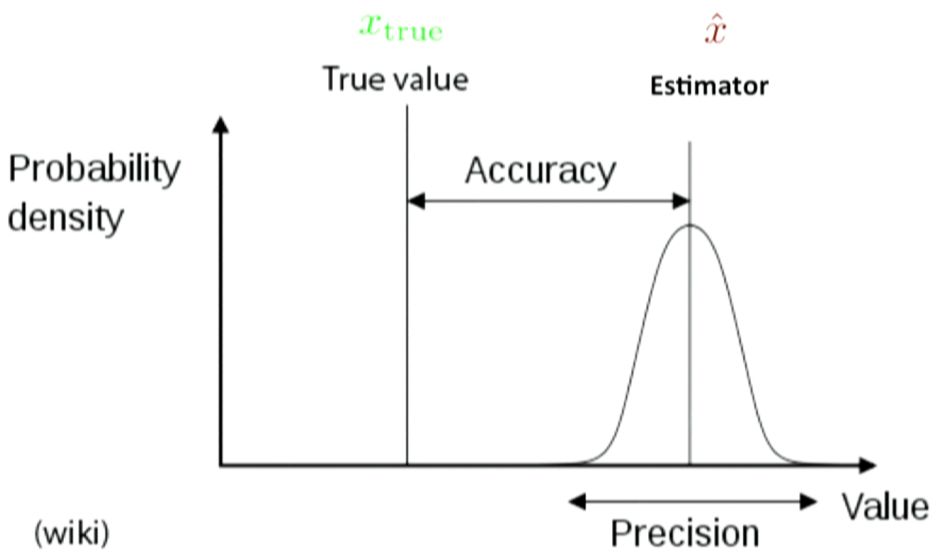
$\Lambda(\mathrm{data}) < c$ is the most "powerful" test of size α for threshold c .

Performance metric: Mean Square Error



(wiki)

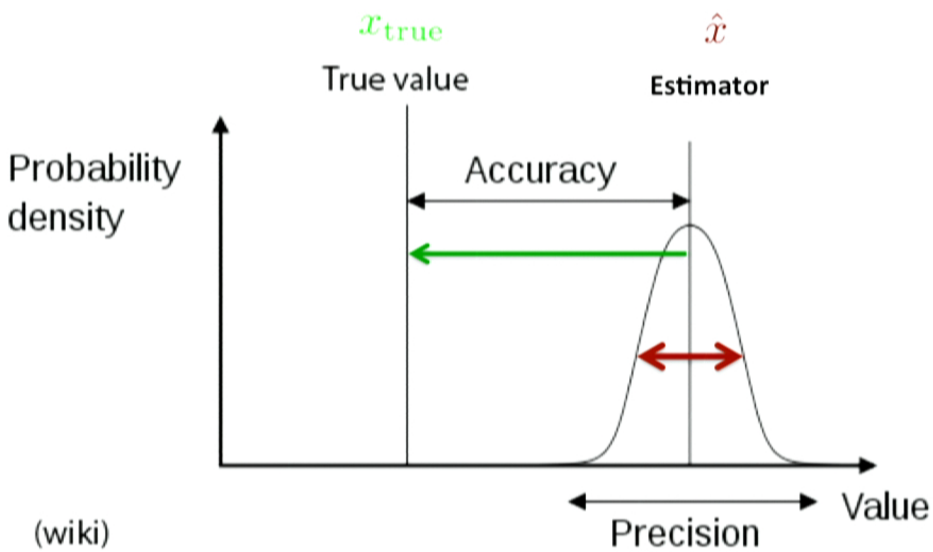
Performance metric: Mean Square Error



$$\mathbb{E}[x_{\text{true}} - \hat{x}] = 0$$

Unbiased estimator

Performance metric: Mean Square Error



$$\mathbb{E}[x_{\text{true}} - \hat{x}] = 0$$

Unbiased estimator

$$\begin{aligned} \text{MSE}(\hat{x}) &= \mathbb{E}[(x_{\text{true}} - \hat{x})^2] \\ &= \text{Var}(\hat{x}) \end{aligned}$$

Performance metric: Mean Square Error

Why Mean Square Error?

Any differentiable loss function $L(x, \hat{x}) = f(x - \hat{x})$

$$L(x, \hat{x}) \approx f(0) + f'(0)(x - \hat{x}) + \frac{1}{2}f''(0)(x - \hat{x})^2 = \frac{1}{2}f''(0)(x - \hat{x})^2$$

i.e. to first approximation MSE is locally the only relevant metric.

$$\begin{aligned} \mathrm{MSE}(\hat{x}) =& \mathbb{E}[(x_{\mathrm{true}} - \hat{x})^2] \\ =& \mathrm{Var}(\hat{x}) \end{aligned}$$

Performance metric: Mean Square Error

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Quantum Cramer-Rao Bound: $\mathrm{MSE}(\hat{x}) \geq \dfrac{1}{\sqrt{\nu} I_\rho(x)}$

Performance metric: Mean Square Error

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$$= \mathrm{Var}(\hat{x})$$

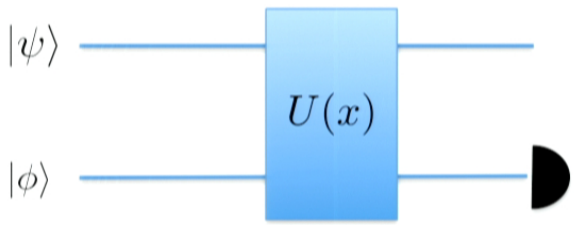
Quantum Cramer-Rao Bound: $\mathrm{MSE}(\hat{x}) \geq \frac{1}{\sqrt{\nu} I_\rho(x)}$

Helstrom (1967)

$$I_\rho(x) = \mathrm{Tr}\big(\rho(x) L(x)^2\big)$$

$$\frac{\partial}{\partial x}\rho(x) = \frac{1}{2}\Big(\rho(x)L(x) + L(x)\rho(x)\Big)$$

Weak value amplification



$$H = xO_A \otimes O_B$$

$$|\psi_{\text{better?}}(x)\rangle = \frac{M_k(x)\,|\psi\rangle}{\sqrt{\langle\psi|M_k(x)^\dagger M_k(x)|\psi\rangle}}$$

$$M_k(x) = \langle k|U(x)|\phi\rangle$$

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Knee et al., PRA **87**, 012115 (2013).
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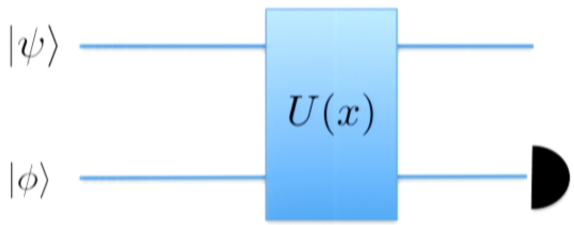

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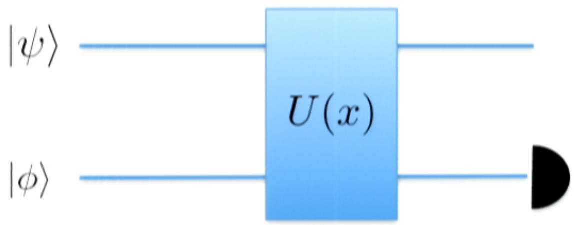

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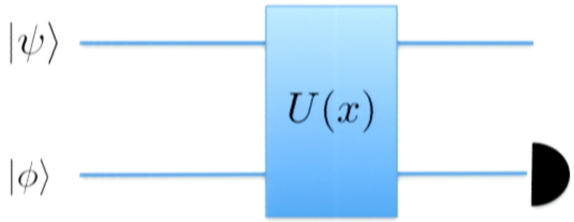

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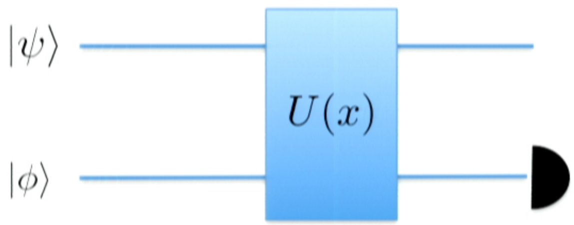

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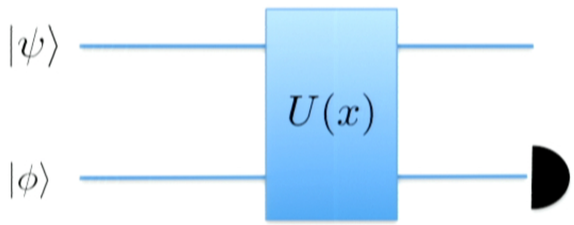

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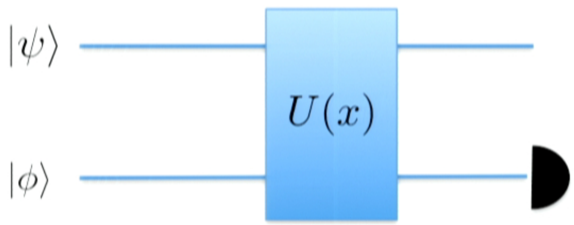

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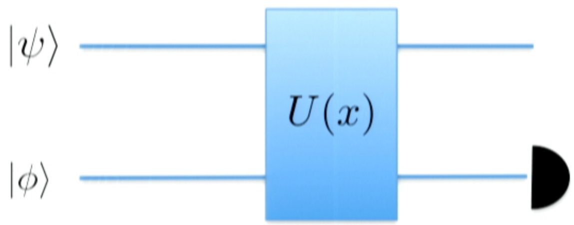
Weak value amplification

Results completely general do not require weak coupling limit



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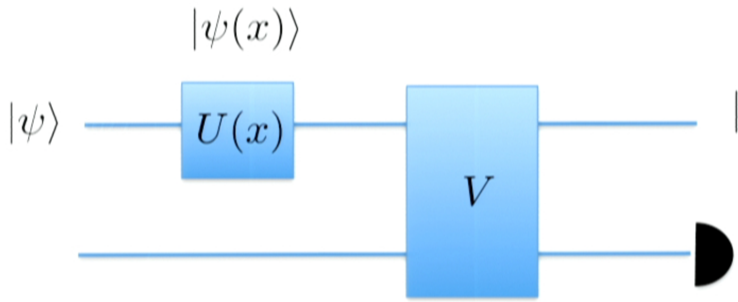
Probabilistic metrology



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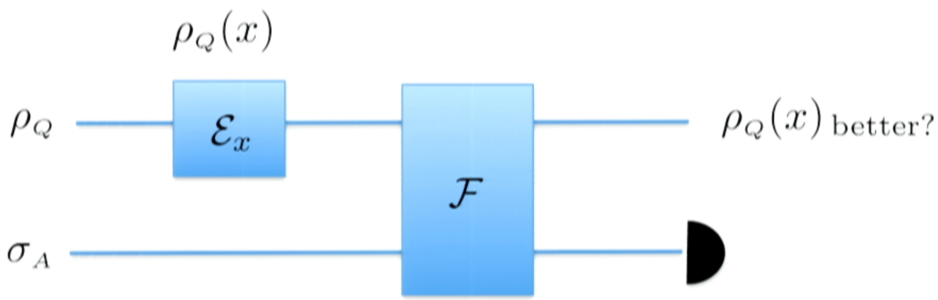

Probabilistic quantum metrology? Probably not.
arXiv:1309.6620
Joshua Combes, Christopher Ferrie, Zhang Jiang, Carlton M. Caves

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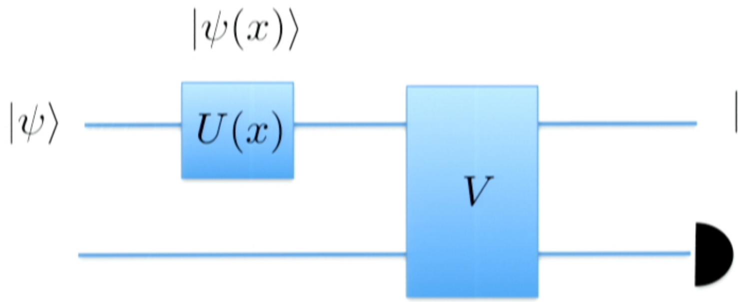

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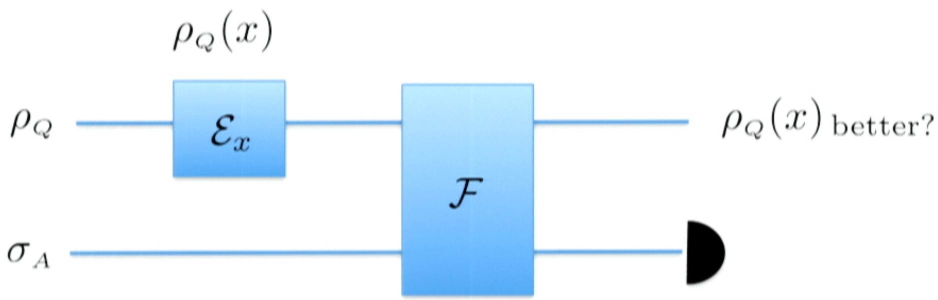

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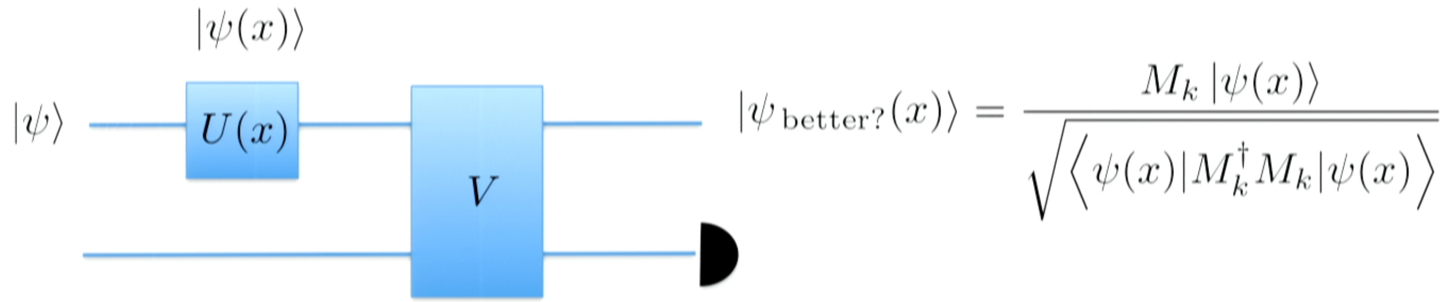

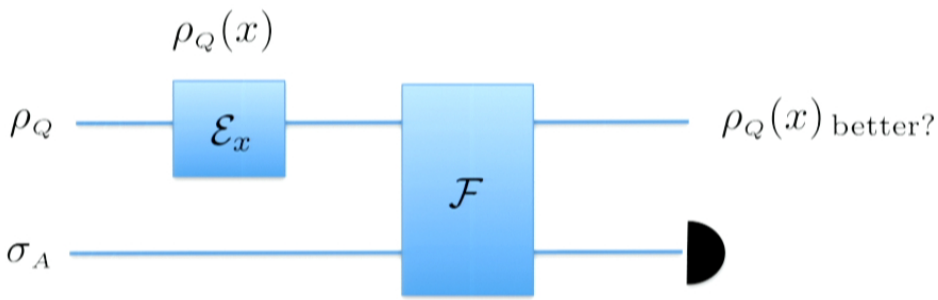

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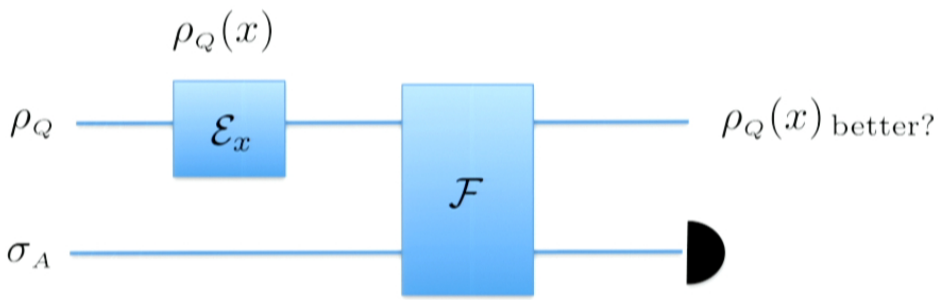

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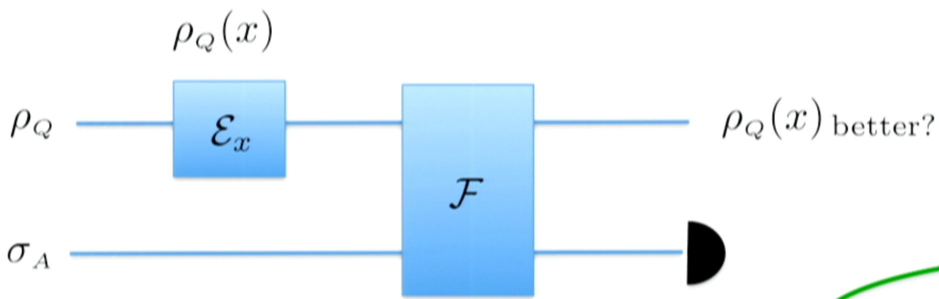

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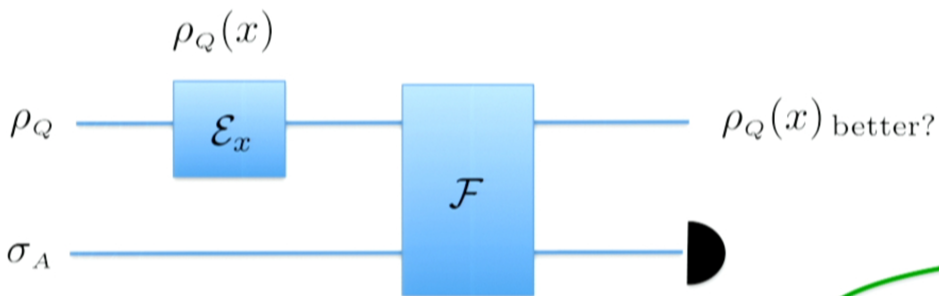

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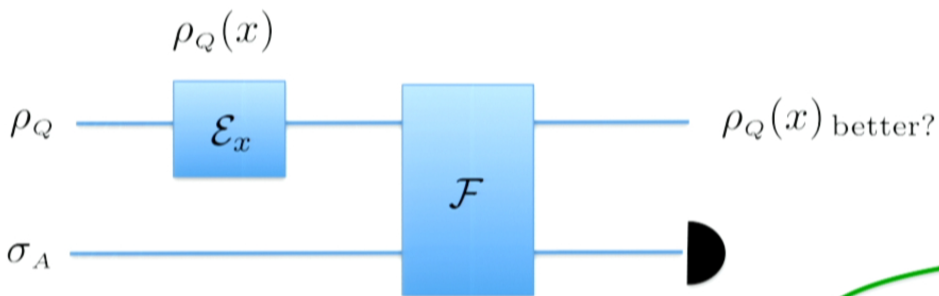

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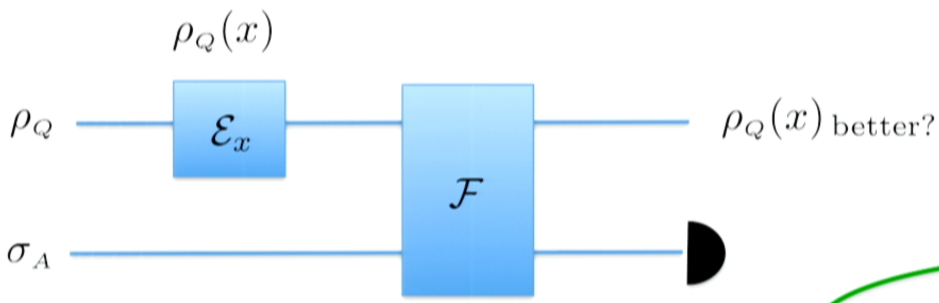

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Weak value amplification: objections

1. Point is detection not estimation
2. Technical noise
3. Optimal estimation is hard
4. Imaginary weak values
5. Wrong comparison
6. Can almost achieve equality

Probabilistic metrology

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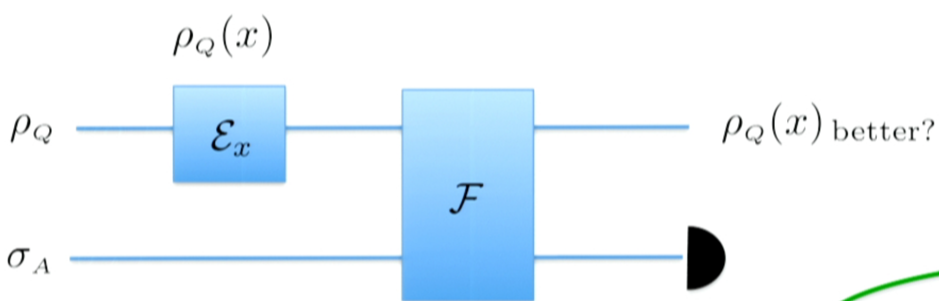

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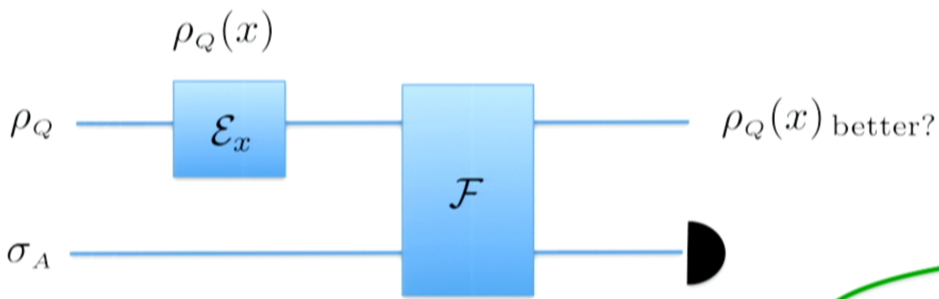

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Discussion

Benign Probabilistic Protocols

1. correspond to a physically allowed process
2. typically the success probability can approach 1

Exotic Probabilistic Protocols

1. correspond to a nonphysical process
2. increasing distinguishability of quantum states or channels
3. the success probability is typically small

Future directions

Conclusion

Probabilistic protocols are an attempt to evade the no-cloning theorem by praying that the quantum gods won't be paying attention all the time.

Unfortunately, our results indicate that the quantum ones are keenly alert and do not suffer hubris gladly.