Title: Nonlocal cosmology

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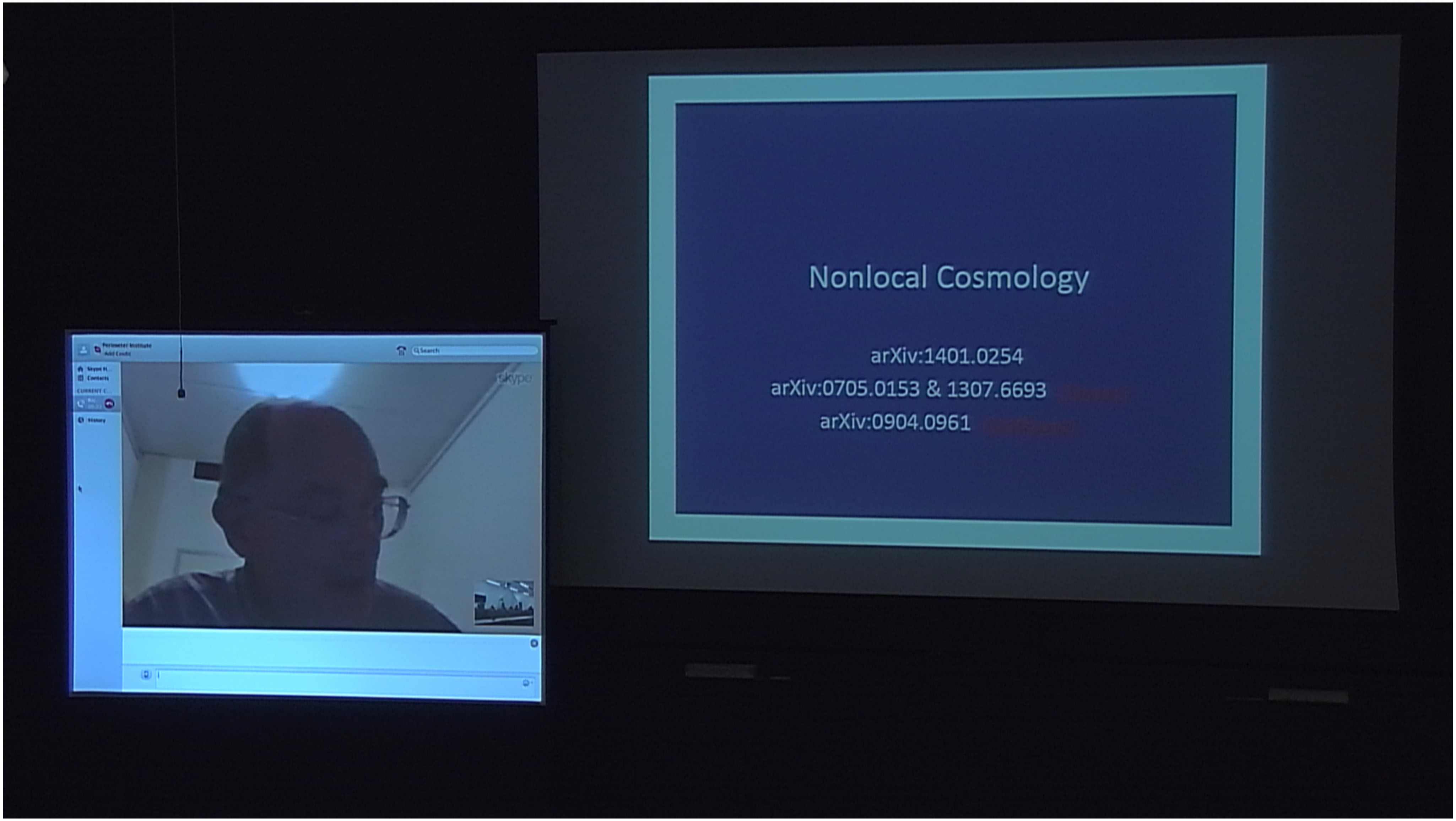
Abstract: I review a class of nonlocally modified gravity models which were proposed to explain the current phase of cosmic acceleration without dark energy. Among the topics considered are deriving causal and conserved field equations, adjusting the model to make it support a given expansion history, why these models do not require an elaborate screening mechanism to evade solar system tests, degrees of freedom and kinetic stability, and the negative verdict of structure formation. Although these simple models are not consistent with data on the growth of cosmic structures many of their features are likely to carry over to more complicated models which are in better agreement with the data.

Nonlocal Cosmology

arXiv:1401.0254

arXiv:0705.0153 & 1307.6693 (Deser)

arXiv:0904.0961 (Deffayet)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Problem: What is making the universe accelerate?

- FLRW: $ds^2 = -dt^2 + a^2(t)\, d\boldsymbol{x} \cdot d\boldsymbol{x}$
  - ❖ $H(t) = \frac{\dot{a}}{a}$ ➔ $H_0 \sim 67 \frac{\text{km}}{\text{s-Mpc}}$
  - ❖ $q(t) \equiv -1 - \frac{\dot{H}}{H^2}$ ➔ $q_0 \sim -.54$
- General Relativity with $\frac{a_0}{a(t)} \equiv 1 + z$
  - ❖ $3H^2 = 3H_0^2\, [\Omega_r\, (1+z)^4 + \Omega_m\, (1+z)^3 + \Omega_\Lambda]$
  - ❖ $-2\dot{H} - 3H^2 = 3H_0^2\, [\tfrac{1}{3}\Omega_r\, (1+z)^4 + 0 - \Omega_\Lambda]$
- ΛCDM works
  - ❖ $\Omega_r \sim 8.5 \times 10^{-5},\ \Omega_m \sim .306,\ \Omega_\Lambda \sim .692$
  - ❖ But why is $G\Lambda$ so small and why dominant NOW?

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Scalar Quintessence Works

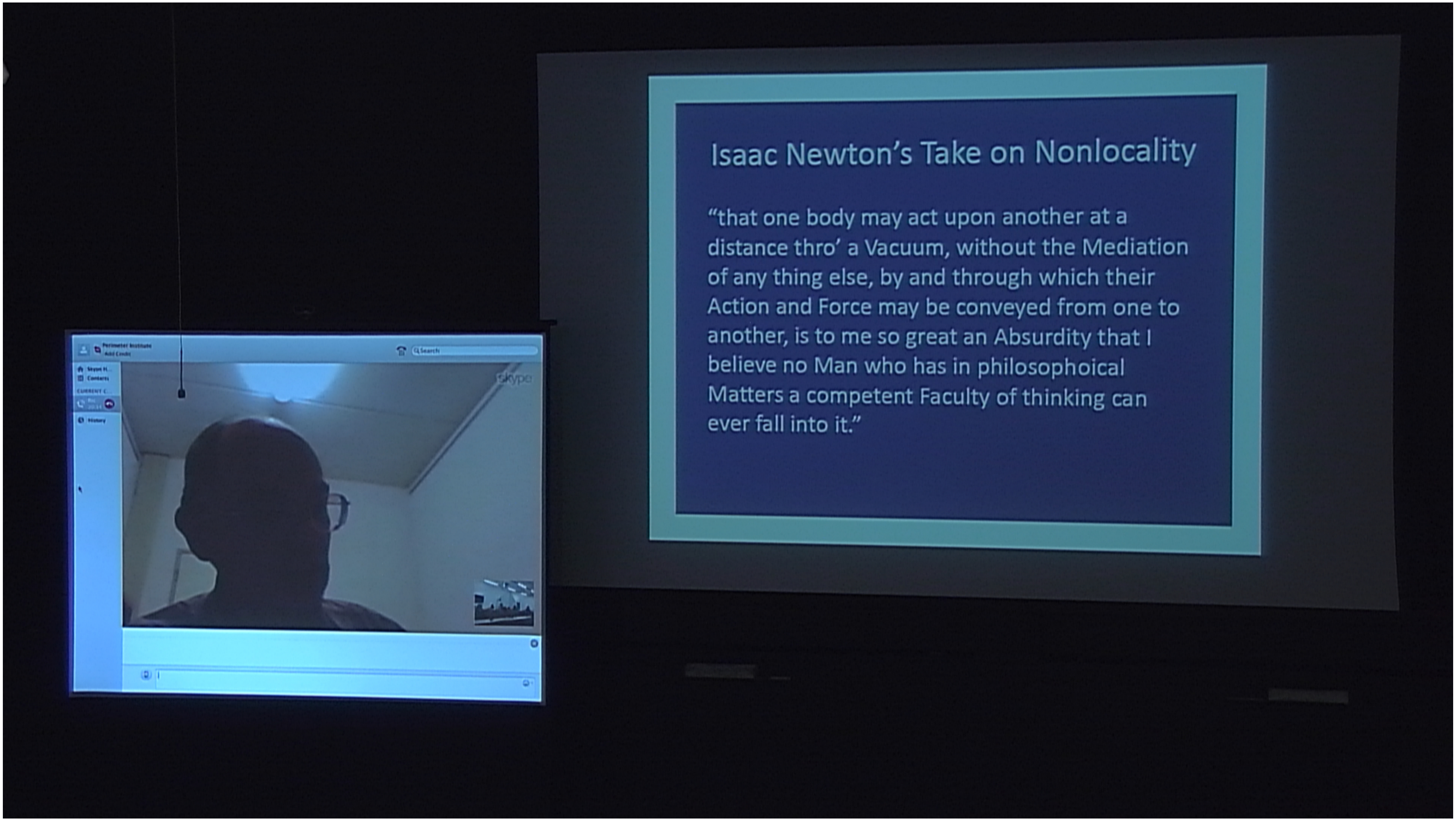
- $\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - V(\varphi)\sqrt{-g}$
  - $3H^2 = 8\pi G\,[\frac{1}{2}\dot{f}^2 + V(f)]$
  - $-2\dot{H}-3H^2 = 8\pi G\,[\frac{1}{2}\dot{f}^2 - V(f)]$
- Given $a(t)$ → Reconstruct $V(\varphi)$
  - $-2\dot{H}(t) = 8\pi G\,\dot{f}^2(t)$ → $f(t) = f_0 \pm \int_0^t dt' \sqrt{\frac{-\dot{H}(t')}{4\pi G}}$
  - Monotonic → t[f]
  - $\dot{H}(t) + 3H^2(t) = 8\pi G\,V$ → $V(f) = \frac{\dot{H}(t[f]) + 3H^2(t[f])}{8\pi G}$
- But who ordered that!
  - Why is $\varphi(t, \boldsymbol{x}) \sim f(t)$ so homogeneous?
  - Why is $G^2 V(f)$ so small?
  - Why is there no observed scalar force?

$f(R)$ models don't really work

- $\mathcal{L} = \frac{f(R)\sqrt{-g}}{16\pi G}$
- Unique solution which gives ΛCDM is . . .
  - ❖ $f(R) = R - 2\Lambda$
  - ❖ Dunsby et al., arXiv:1005.2205
- Hence deviations occur even at 0^{th} order!
- And there are other problems
  - ❖ Why now? ➔ new scales
  - ❖ New scalar DoF ➔ needs screening

Modifications of Gravity

- $f(R)$ only local, invariant, stable & $g_{\mu\nu}$-based
- Retain locality and sacrifice invariance
  - ❖Horava gravity
  - ❖Massive gravitons
- Retain invariance and sacrifice locality for:
  - ❖Summing QIR effects from primordial inflation
  - ❖Explaining late time acceleration w/o Dark Energy
  - ❖Explaining galactic structure w/o Dark Matter


Isaac Newton's Take on Nonlocality
"that one body may act upon another at a distance thro' a Vacuum, without the Mediation of any thing else, by and through which their Action and Force may be conveyed from one to another, is to me so great an Absurdity that I believe no Man who has in philosophoical Matters a competent Faculty of thinking can ever fall into it."

Was Newton too Harsh?

- I don't think so
  - ❖Fundamental theory is local
  - ❖But quantum effective field equations are not
  - ❖ $M = 0$ loops could give big IR corrections
- Primordial Inflation ➔ IR gravitons
  - ❖$N(t,k) = \left[\frac{Ha(t)}{2ck}\right]^2$ for EVERY wave vector
  - ❖Perhaps their attraction stops inflation
  - ❖Late time modifications from vacuum polarization
  - ❖Would affect large scales most
- But for now, just model-building

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Late-Time Acceleration
(arXiv:0705.0153 with Deser)

- Nonlocality via $\frac{1}{\Box}$ for $\Box \equiv \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\right)$
  - ❖ Retarded BC ➔ both $\frac{1}{\Box}$ and $\partial_t \frac{1}{\Box}$ vanish at $t = 0$
- Act it on R ➔ $X \equiv \frac{1}{\Box}R$ is dimensionless
- $\mathcal{L} = \frac{R[1+f(X)]\sqrt{-g}}{16\pi G}$
  - ❖ $f(X)$ the "nonlocal distortion function"
- Field equations: $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$\Delta G_{\mu\nu} = \left[G_{\mu\nu} + g_{\mu\nu}\Box - D_\mu D_\nu\right]\left(f(X) + \frac{1}{\Box}[Rf'(X)]\right)$$

$$+\left[\delta_\mu^{(\rho}\delta_\nu^{\sigma)} - \tfrac{1}{2}g_{\mu\nu}g^{\rho\sigma}\right]\partial_\rho X\,\partial_\sigma\left(\frac{1}{\Box}[I\quad(X)]\right)$$

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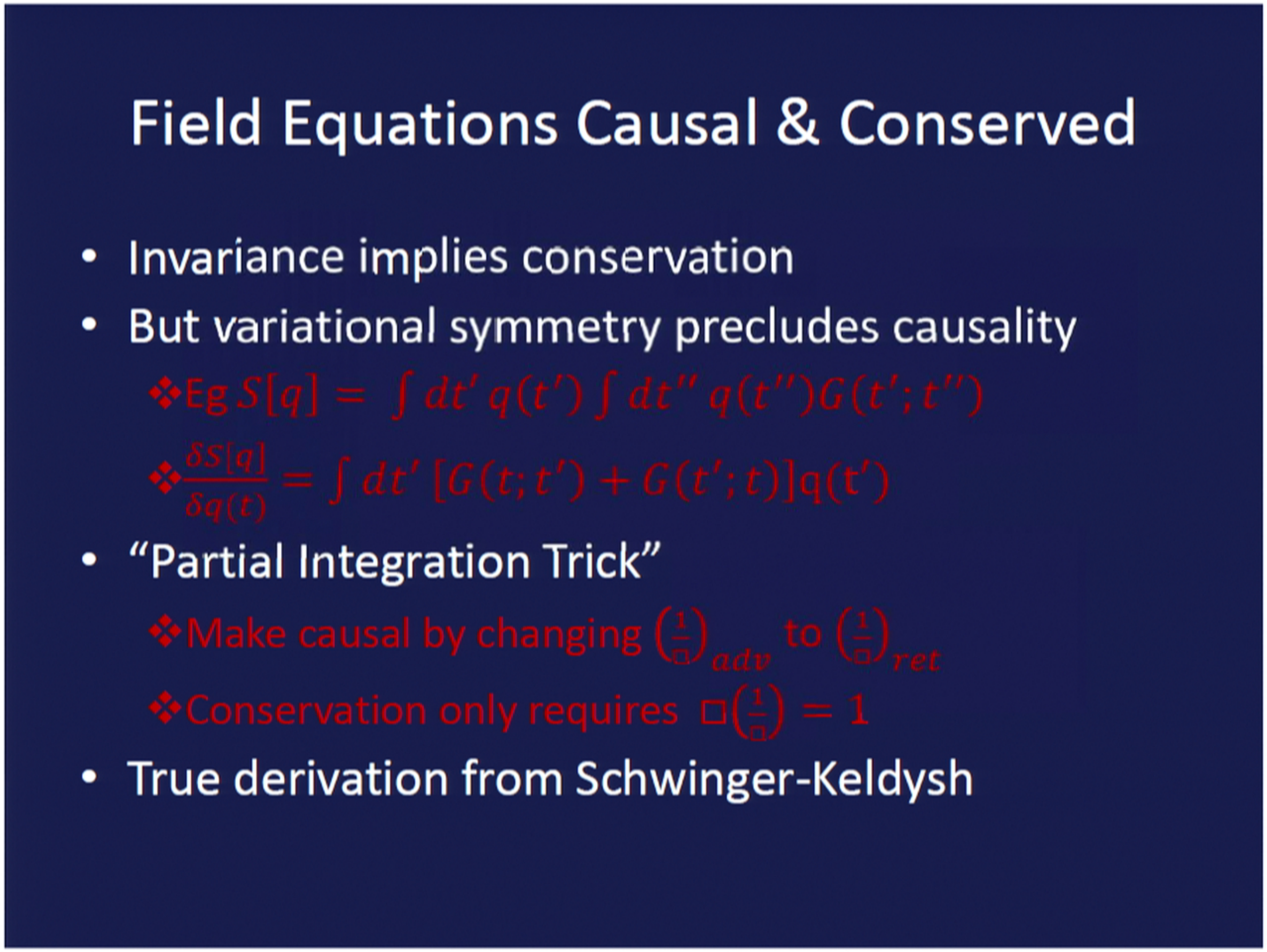
Field Equations Causal & Conserved
Invariance implies conservation
But variational symmetry precludes causality
Eg $S[q] = \int dt'\, q(t') \int dt''\, q(t'') G(t';t'')$
$\frac{\delta S[q]}{\delta q(t)} = \int dt' \, [G(t;t') + G(t';t)] q(t')$
"Partial Integration Trick"
Make causal by changing $\left(\frac{1}{\Box}\right)_{adv}$ to $\left(\frac{1}{\Box}\right)_{ret}$
Conservation only requires $\Box\left(\frac{1}{\Box}\right) = 1$
True derivation from Schwinger-Keldysh

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Specialization to FLRW:

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x} \cdot d\boldsymbol{x}$$

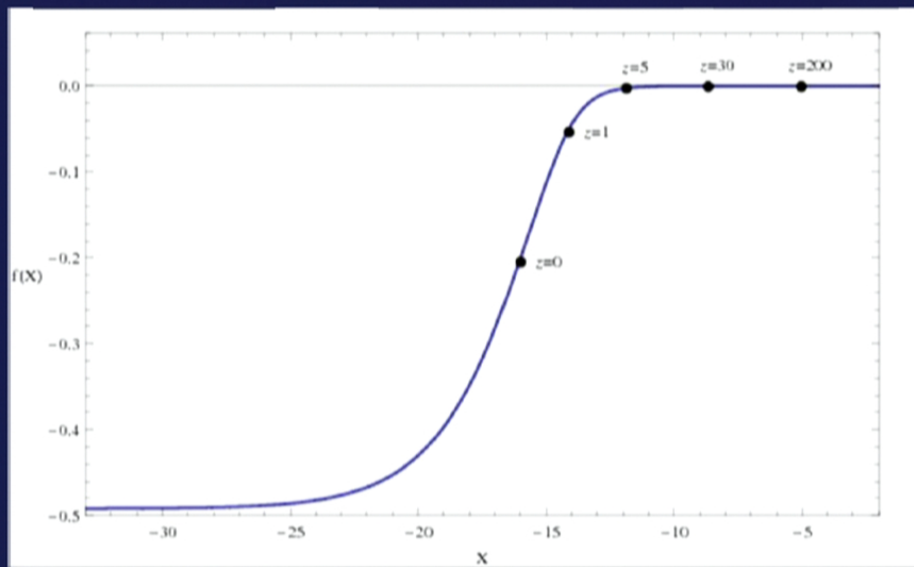
- $R = 6\dot{\mathrm{H}} + 12H^2$
- $\left[\frac{1}{\Box} f\right](t) = -\int_0^t \frac{dt'}{a^3(t')} \int_0^{t'} dt''\, a^3(t'') f(t'')$
- Two Built-In Delays:
  - ❖ $R = 0$ during Radiation domination $\left(H = \frac{1}{2t}\right)$

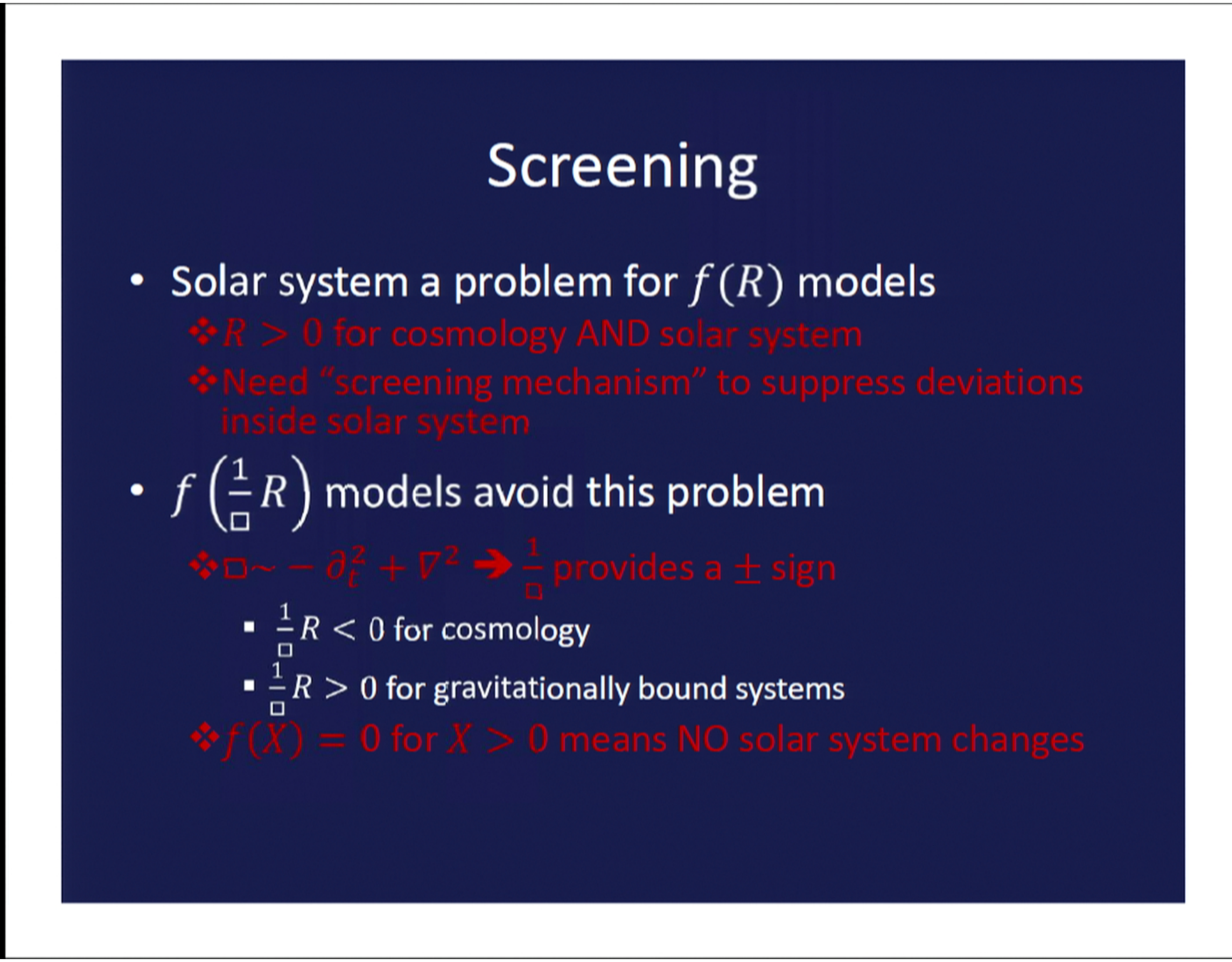
    ➔ No modification until $t_{\text{eq}} \sim 10^5$ years
  - ❖ $X = \frac{1}{\Box} R \sim -\frac{4}{3} \ln\left(\frac{t}{t_{\text{eq}}}\right)$ during Matter domination

    ➔ $X \sim -15$ at $t \sim 10^{10}$ years

Reconstructing ΛCDM (arXiv:0904.0961 with Deffayet)

$$f(X) \approx \tfrac{1}{4}\left[\tanh\left(\frac{X}{3} + \frac{11}{2}\right) - 1\right]$$




Screening
• Solar system a problem for $f(R)$ models
❖ $R > 0$ for cosmology AND solar system
❖ Need "screening mechanism" to suppress deviations inside solar system
• $f\left(\frac{1}{\Box}R\right)$ models avoid this problem
❖ $\Box \sim -\partial_t^2 + \nabla^2$ ➔ $\frac{1}{\Box}$ provides a $\pm$ sign
▪ $\frac{1}{\Box}R < 0$ for cosmology
▪ $\frac{1}{\Box}R > 0$ for gravitationally bound systems
❖ $f(X) = 0$ for $X > 0$ means NO solar system changes

Local Version Is Haunted
(Nojiri & Odintsov, arXiv:0708.0924)

- $R\left[1 + f\left(\frac{1}{\Box}R\right)\right] \Rightarrow R[1 + f(\phi)] + \xi[\Box\phi - R]$
  - Varying with respect to ξ enforces $\phi = R$
  - NB both scalars have 2 pieces of initial value data
- $\xi\Box\phi \rightarrow -\partial_\mu\xi\partial_\nu\phi g^{\mu\nu}$
  $= -\frac{1}{4}\partial_\mu(\xi + \phi)\partial_\nu(\xi + \phi)g^{\mu\nu} + \frac{1}{4}\partial_\mu(\xi - \phi)\partial_\nu(\xi - \phi)g^{\mu\nu}$
- $\xi - \phi$ has negative kinetic energy
- Mixing with gravity doesn't help

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Initial Value Constraints Identical to General Relativity

- Recall $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$\Delta G_{\mu\nu} = \left[G_{\mu\nu} + g_{\mu\nu}\Box - D_\mu D_\nu\right]\left(f(X) + \tfrac{1}{\Box}[Rf'(X)]\right) + \left[\delta_\mu^{(\rho}\delta_\nu^{\sigma)} - \tfrac{1}{2} g_{\mu\nu} g^{\rho\sigma}\right]\partial_\rho X\, \partial_\sigma\left(\tfrac{1}{\Box}[Rf'(X)]\right)$$

- Retarded BC ➔ both $\frac{1}{\Box}$ & $\partial_t \frac{1}{\Box}$ vanish at $t = 0$
  - ❖ $f(X)$ also vanishes at $X = 0$
  - ❖ Only $\left[g_{\mu\nu}\Box - D_\mu D_\nu\right]\left\{f\left(\frac{1}{\Box}R\right) + \frac{1}{\Box}\left[Rf'\left(\frac{1}{\Box}R\right)\right]\right\} \neq 0$
- Synchronous constraints ➔ ΔG_{00} and ΔG_{0i}
  - ❖ $g_{00}\Box - D_0 D_0 = \tfrac{1}{2} h^{ij}\dot{h}_{ij}\partial_t - \Delta$ ➔ 0 at $t = 0$
  - ❖ $g_{0i}\Box - D_0 D_i = -\partial_0\partial_i + \tfrac{1}{2} h^{ik}\dot{h}_{ki}\partial_j$ ➔ 0 at $t = 0$

No Ghosts ➔ Check the ∂_t^2 Terms

- Recall $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$\Delta G_{\mu\nu} = \left[G_{\mu\nu} + g_{\mu\nu}\Box - D_\mu D_\nu\right]\left(f(X) + \tfrac{1}{\Box}[Rf'(X)]\right) + \left[\delta_\mu^{(\rho}\delta_\nu^{\sigma)} - \tfrac{1}{2}g_{\mu\nu}g^{\rho\sigma}\right]\partial_\rho X\,\partial_\sigma\left(\tfrac{1}{\Box}[Rf'(X)]\right)$$

- Dynamical equations ➔ $G_{ij} + \Delta G_{ij} = 8\pi G T_{ij}$
  - ❖ $g_{ij}\Box - D_i D_j = h_{ij}\Box + O(\partial_t)$
  - ❖ $\Delta G_{ij} = 2h_{ij} R f'(X) + O(\partial_t)$
  - ❖ $R_{ij} = \tfrac{1}{2}\ddot{h}_{ij} + O(\partial_t)$ and $R = h^{kl}\ddot{h}_{kl} + O(\partial_t)$
- $G_{ij} + \Delta G_{ij}$ ➔ $\tfrac{1}{2}\left\{1 + f(X) + \tfrac{1}{\Box}[Rf'(X)]\right\}h_{ij}$ +Irrelevant
  - ❖ No graviton ever becomes a ghost
  - ❖ Still might have a potential energy instability

A problem with how the model reproduces ΛCDM without Λ

- For FLRW with slowly varying $H(t)$
  - ❖ $G_{\mu\nu} + \Delta G_{\mu\nu} \approx \left\{1 + f(X) + \frac{1}{\Box}[Rf'(X)]\right\} G_{\mu\nu} = 8\pi G T_{\mu\nu}$
- This is effectively a time-varying Newton constant
  - ❖ $G_{eff}(t) = \frac{G}{1+f(X)+\frac{1}{\Box}[Rf'(X)]}$
  - ❖ Balances the Friedmann Eqn: $3H^2 \approx 8 \; G_{eff}(t) \times \frac{\rho_m}{a^3(t)}$
- But $G_{eff}(t)$ also strengthens the force of gravity
  - ❖ Not relevant for solar system
  - ❖ Should increase structure formation
  - ❖ Dodelson & Park have confirmed this, & it's bad

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