Title: Topological phases in graphene

Date: Feb 10, 2014 09:45 AM

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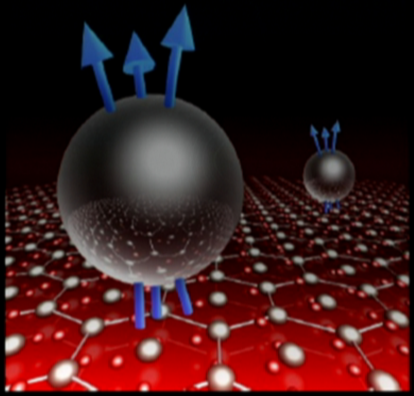
Abstract: As realized for the first time in 1980s, quantum many-body systems in reduced spatial dimensions can sometimes undergo a special type of ordering which does not break any symmetry but introduces long-range entanglement and emergent excitations that have radically different properties from their original constituents. Most of our experimental knowledge of such ``topological" phases of matter comes from studies of two-dimensional electron gases in GaAs semiconductors in high magnetic fields and at low temperatures. In the first part of this talk, I will give an introduction to these systems and review some latest theoretical developments related to their entanglement properties. In the second part, I will discuss new possibilities
for experimental realizations of topological phases in bilayer graphene. I will present evidence that this material supports an ``even-denominator" fractional state, related to the Moore-Read state, whose observation has recently been reported. Finally, I will outline several proposals based on the tunability of the electron-electron interactions in bilayer graphene which might enable further experimental progress beyond GaAs.

Topological phases in graphene

Z. Papić

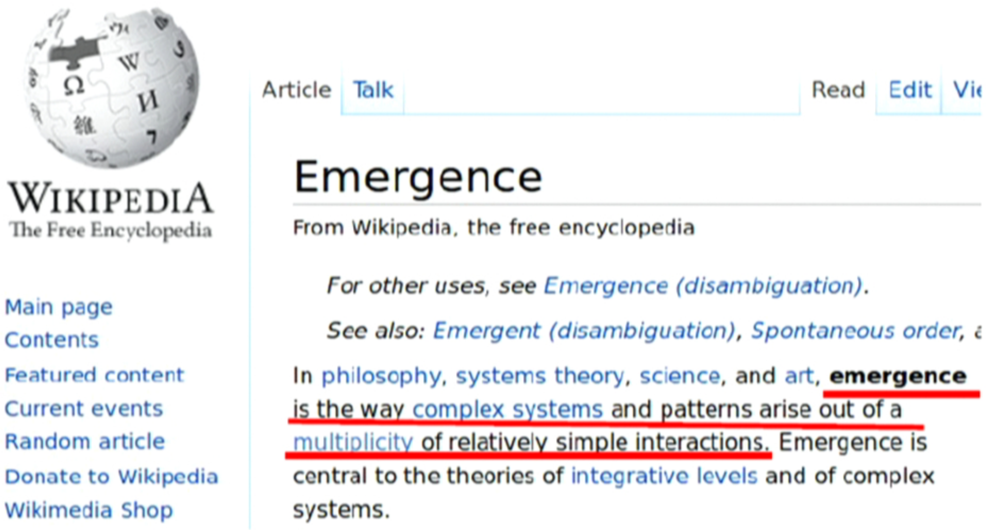


Perimeter, 10/02/2014

IQC Institute for Quantum Computing

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

PRINCETON UNIVERSITY



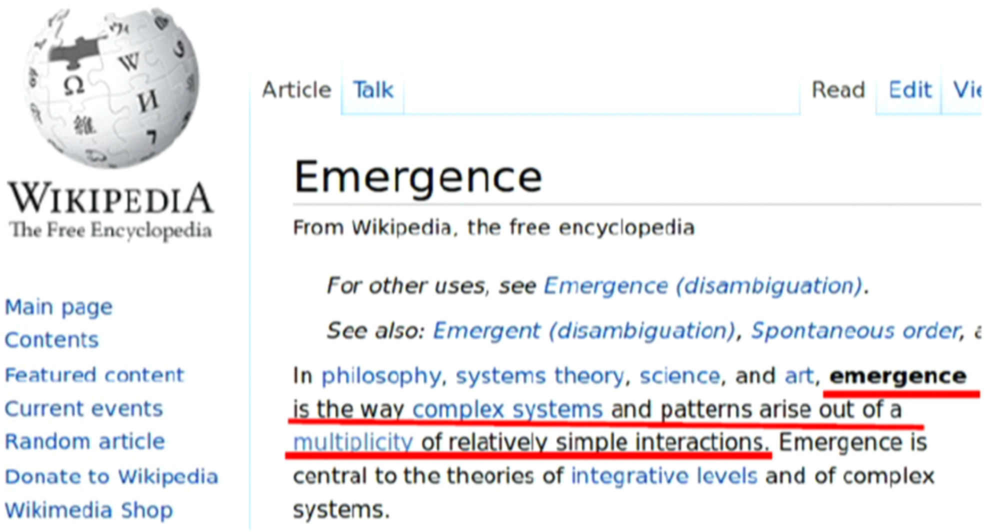
Emergence

From Wikipedia, the free encyclopedia

For other uses, see Emergence (disambiguation).

See also: Emergent (disambiguation), Spontaneous order,

In philosophy, systems theory, science, and art, **emergence** is the way complex systems and patterns arise out of a multiplicity of relatively simple interactions. Emergence is central to the theories of integrative levels and of complex systems.



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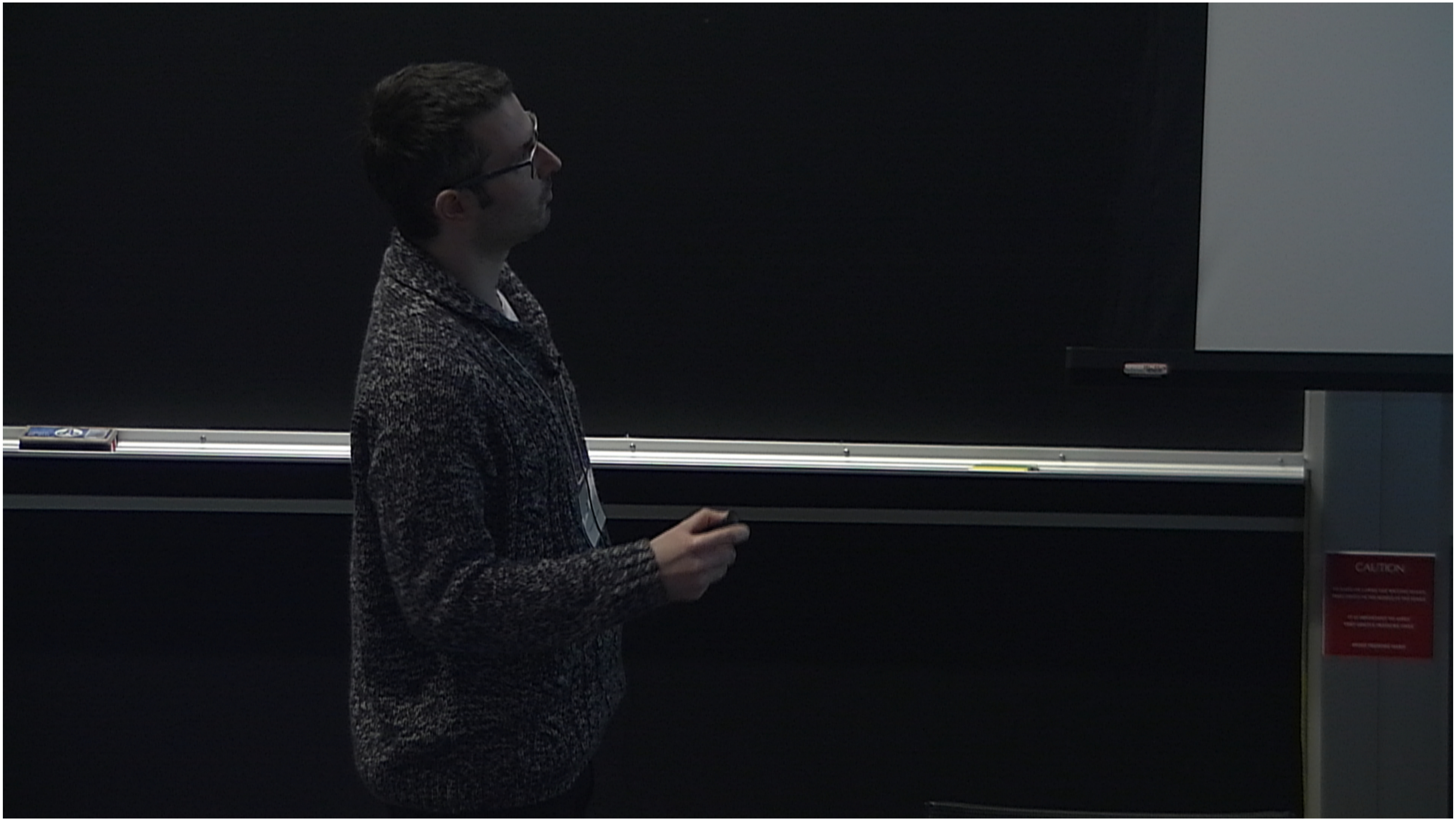
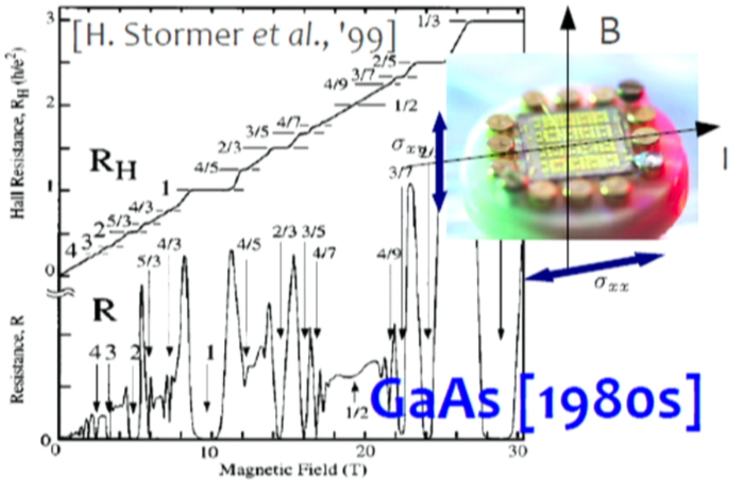
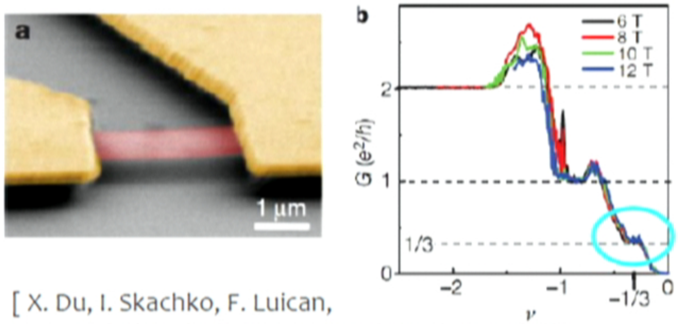

CAUTION

Quantum Hall effect: material realizations



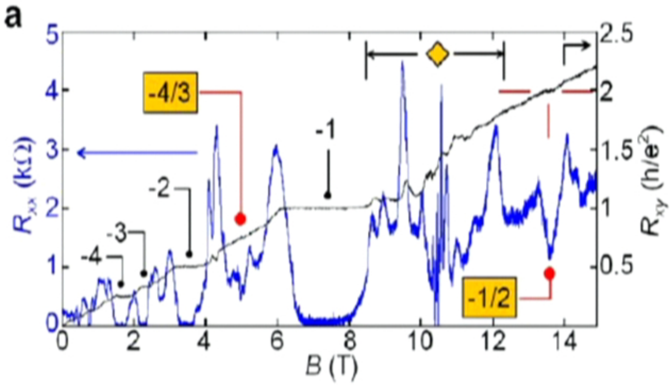
GaAs [1980s]

Graphene [2009]

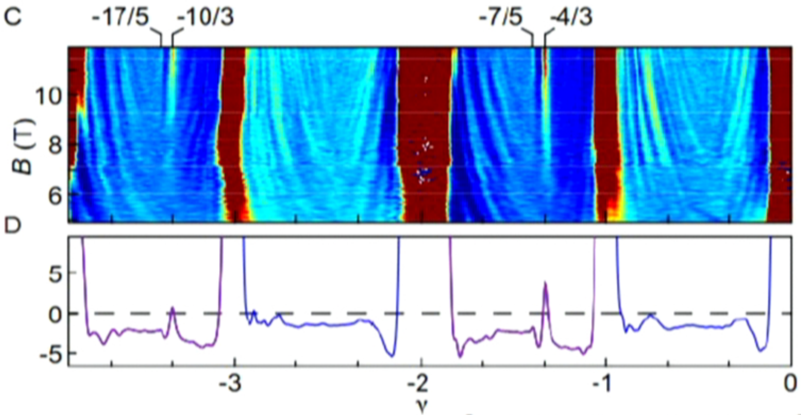


[X. Du, I. Skachko, F. Luican, E. Y. Andrei, Nature **462**, 192195 (2009); K. Bolotin, F. Ghahari, M. Shulman, H. Stormer, P. Kim, Nature 462, **196** (2009)]

New: bilayer graphene [2013]



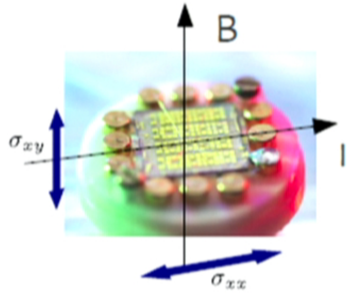
[D.-K. Ki *et al.* arXiv:1305.4761]



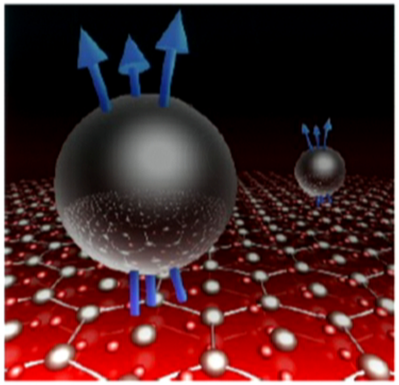
[A. Kou *et al.*, arXiv:1312.7033]

Outline

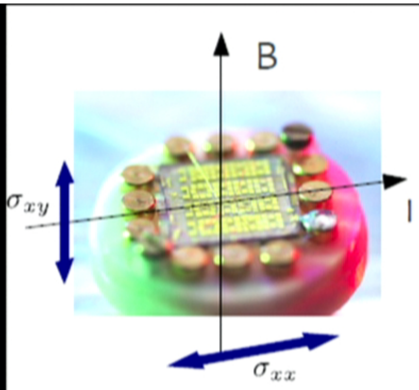
- Topological phases and the fractional quantum Hall effect
  - Introduction
  - Experimental realizations
  - Entanglement properties and description using Matrix-Product States (MPS)



- FQHE in bilayer graphene

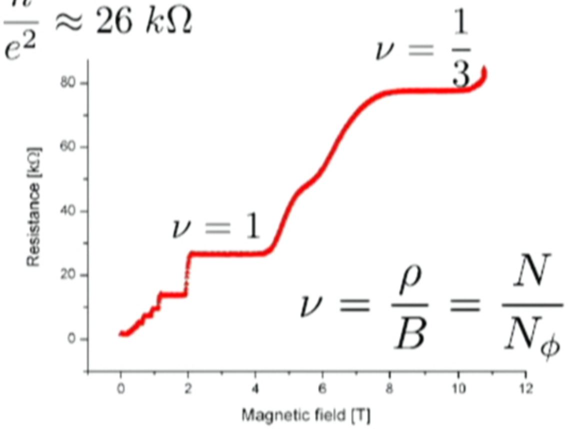


Quantum Hall effect



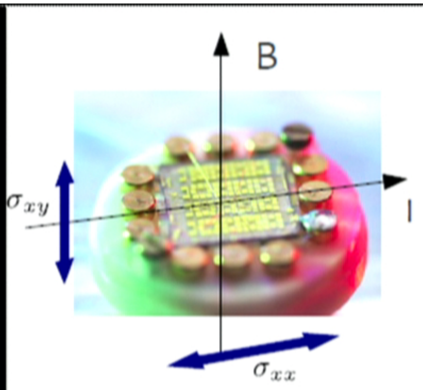
von Klitzing, Tsui, Stormer, Gossard

$$\frac{h}{e^2} \approx 26\ k\Omega$$



$$\nu = \frac{\rho}{B} = \frac{N}{N_\phi}$$

Quantum Hall effect



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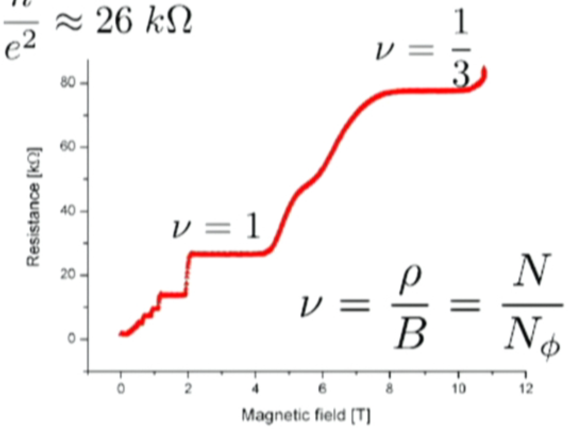

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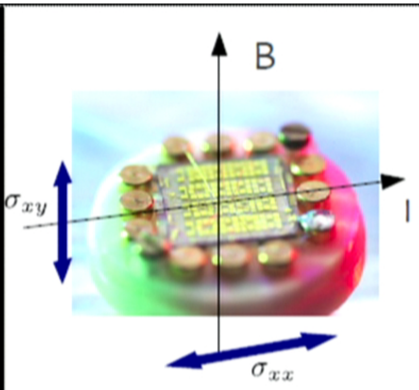

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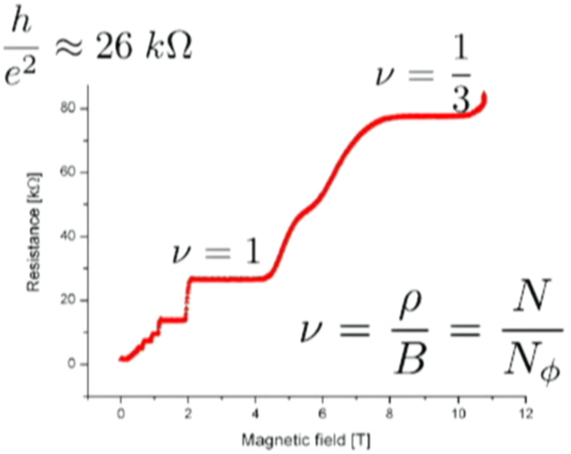

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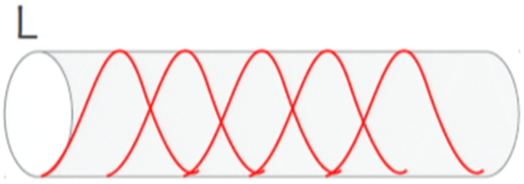

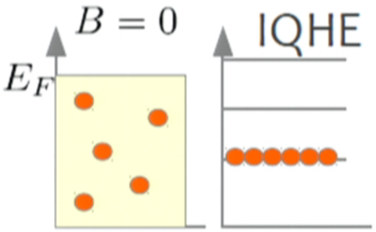

$$z = e^{\frac{2\pi}{L}(x+iy)}, \ell_B = \sqrt{h/eB}$$

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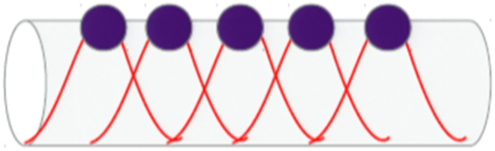

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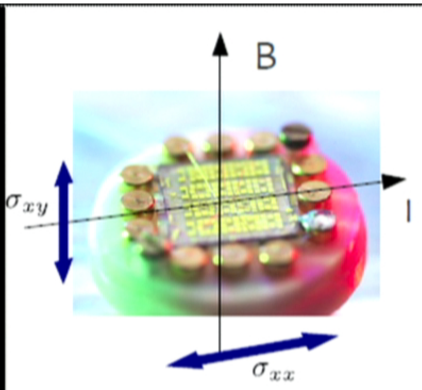

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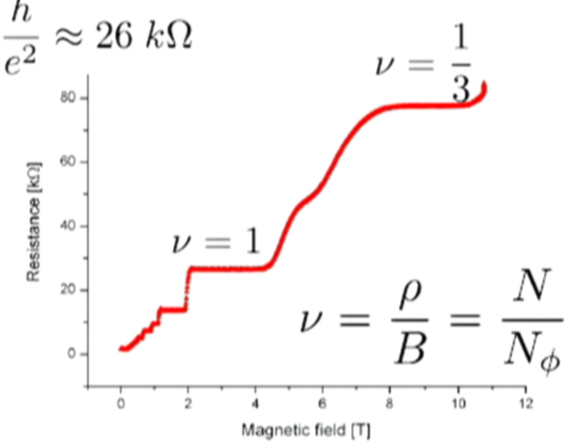

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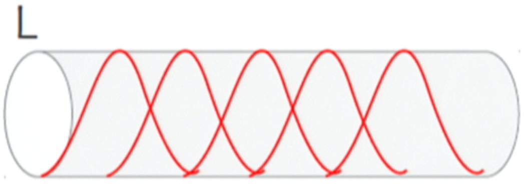

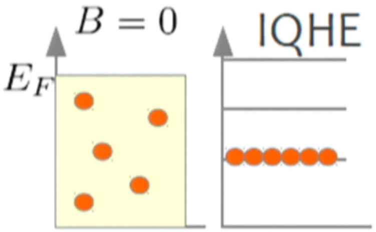

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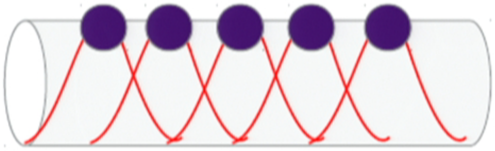
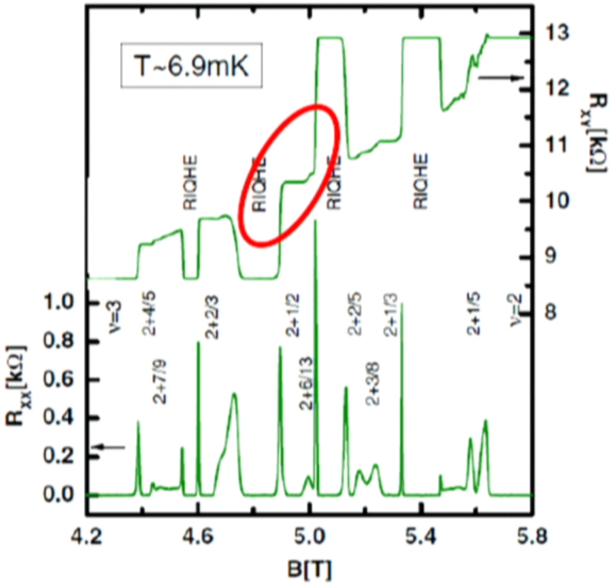


Non-Abelian physics in higher Landau levels

- $\Psi = |\text{Filled n} = 0, \uparrow, \downarrow\rangle \times \prod_{i<j}(z_i - z_j)^2$?



[Willett *et al.*, '89; Kumar *et al*, '10]

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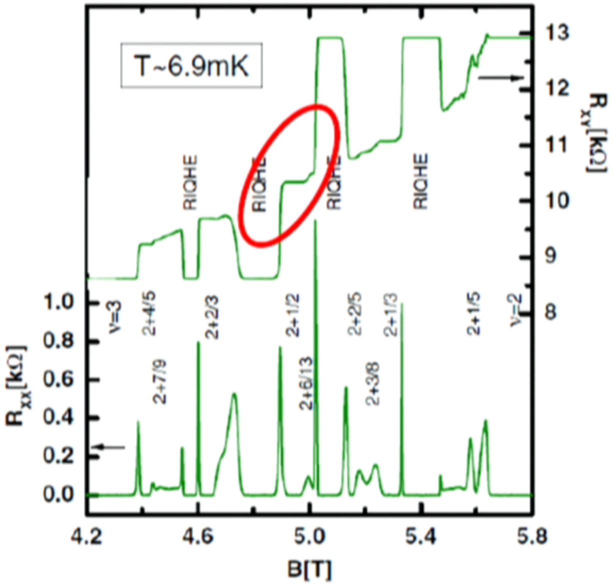

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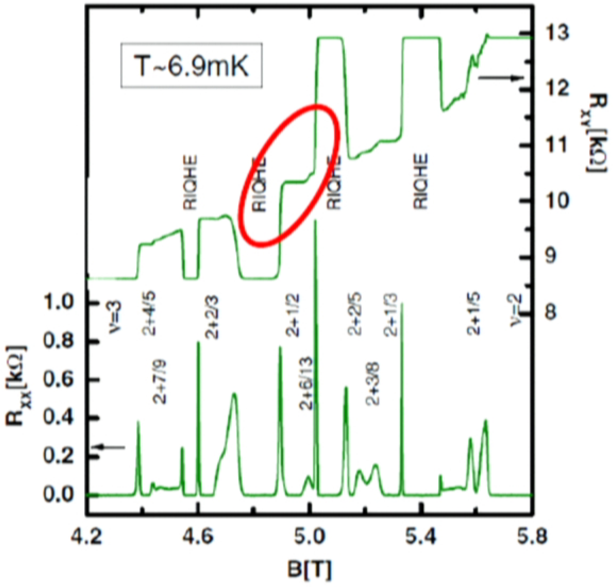

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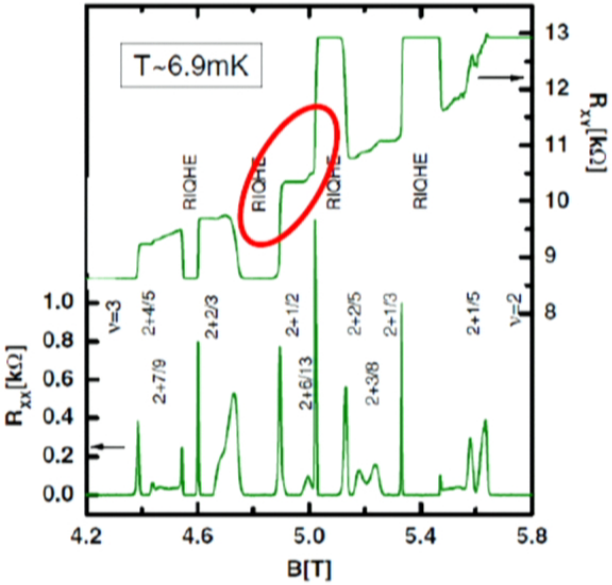

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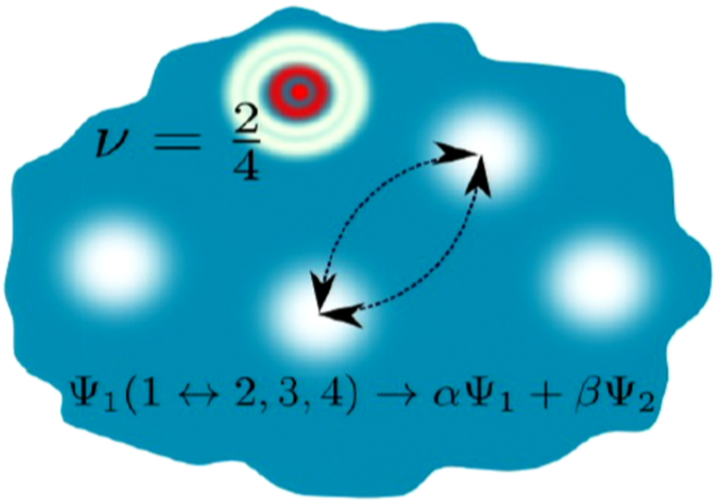
$$\Psi_{\text{Pf}} = \boxed{\text{Pf}(\frac{1}{z_i - z_j})} \prod_{i<j}(z_i - z_j)^2 \times |\text{Filled}\rangle$$

- example of a non-Abelian Ising phase



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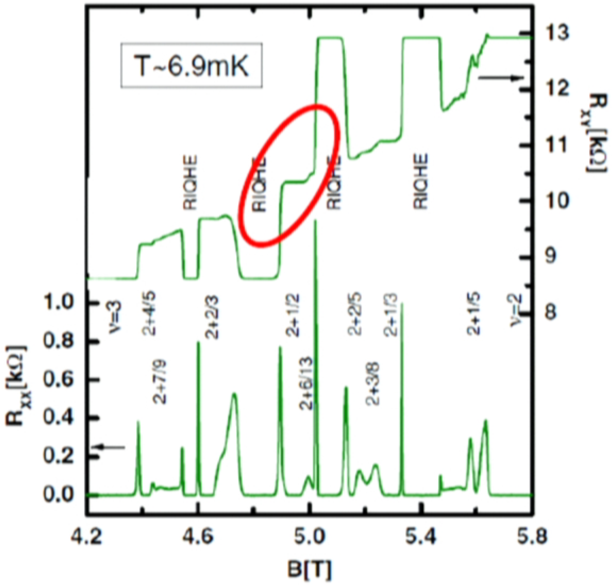

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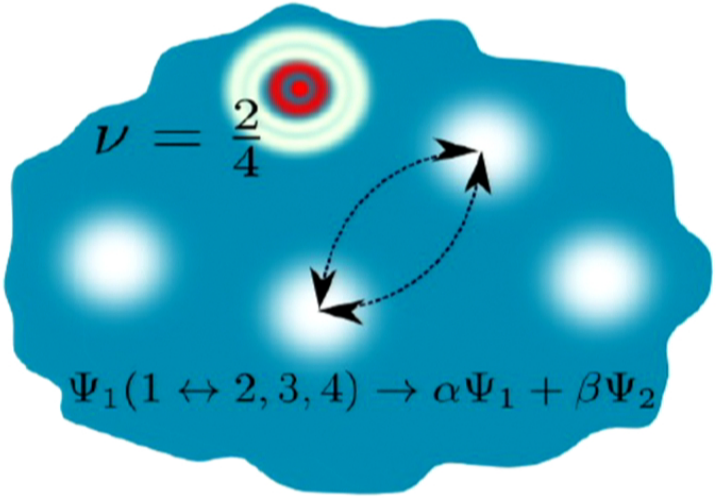


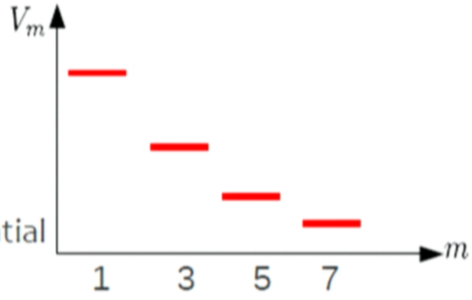
Numerical approaches: Exact diagonalization

$$H = \mathcal{P} \sum_{i<j} \frac{1}{|z_i - z_j|} \mathcal{P}$$

$\mathcal{P}$ - projection to one or several Landau levels

$$H = \sum_{m=1,3,\ldots} V_m \sum_{i<j} P_{ij}^m$$

Haldane pseudopotential



(interaction energy of 2 particles
In a state of relative momentum m)

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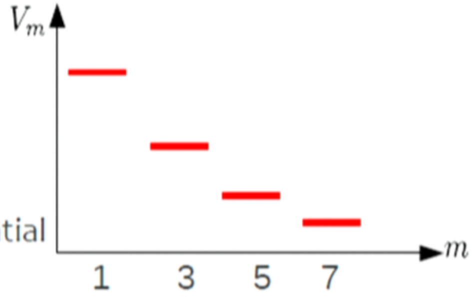
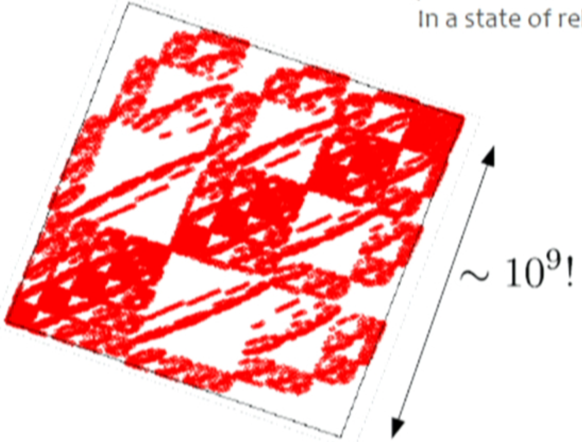
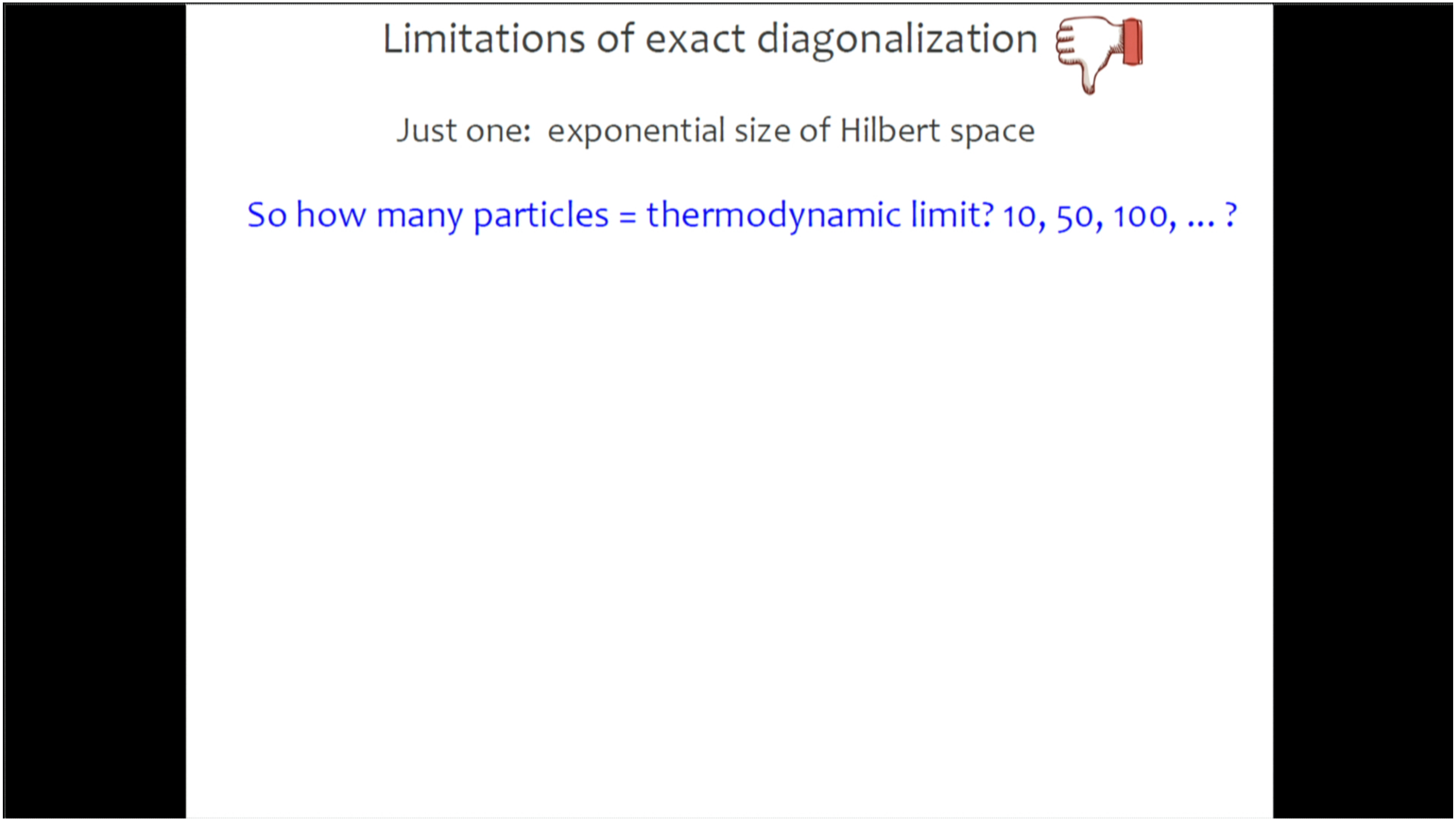


→ finite matrix



= overlaps, gaps, correlation functions, entanglement spectrum [Li,Haldane '08]

$$\Psi = \sum_{\{\sigma_i\}} c_{\{\sigma_i\}} |\sigma_1, \sigma_2, \dots, \sigma_N\rangle$$



Limitations of exact diagonalization

Just one: exponential size of Hilbert space

So how many particles = thermodynamic limit? 10, 50, 100, ... ?

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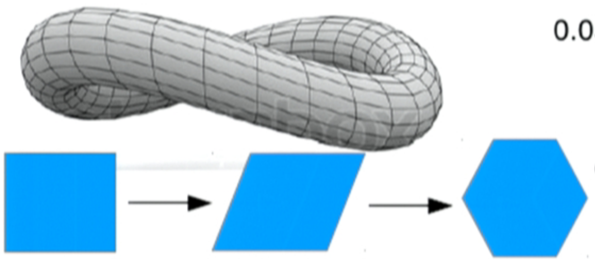
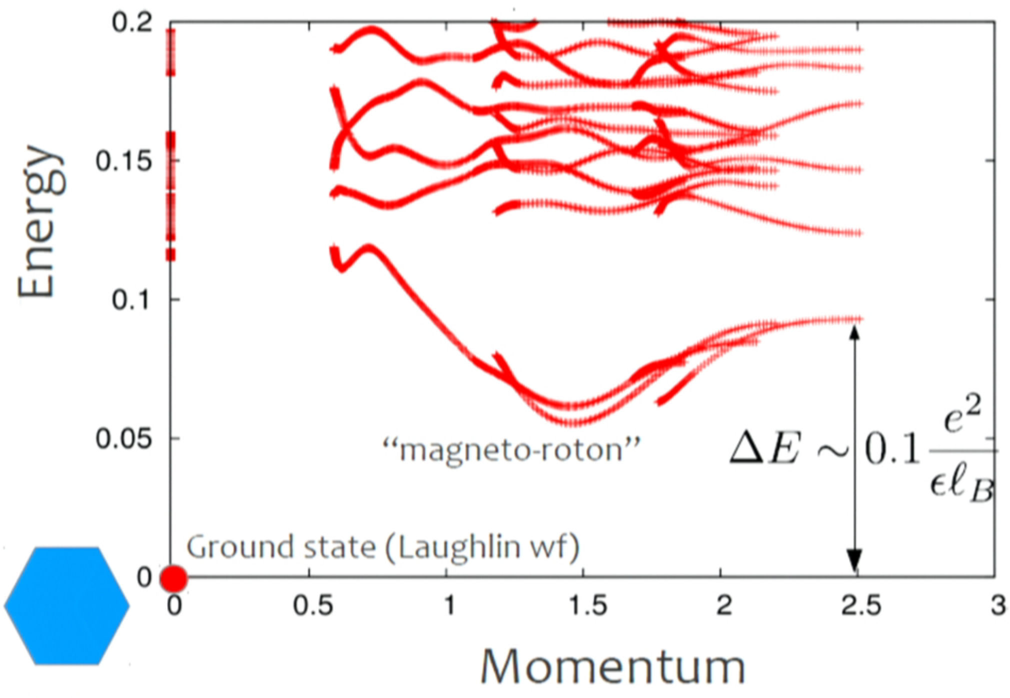
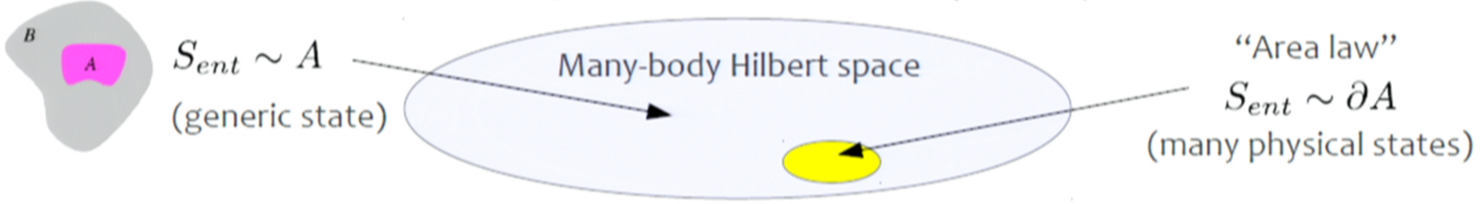
*at least for 1/3 state



(area-preserving deformations of the surface)



Matrix-product states (MPS)



$S_{ent} \sim A$

(generic state)

Many-body Hilbert space

"Area law"

$S_{ent} \sim \partial A$

(many physical states)

Physically relevant states typically live in a special corner of the many-body Hilbert space

[Verstraete, Cirac, Vidal...]

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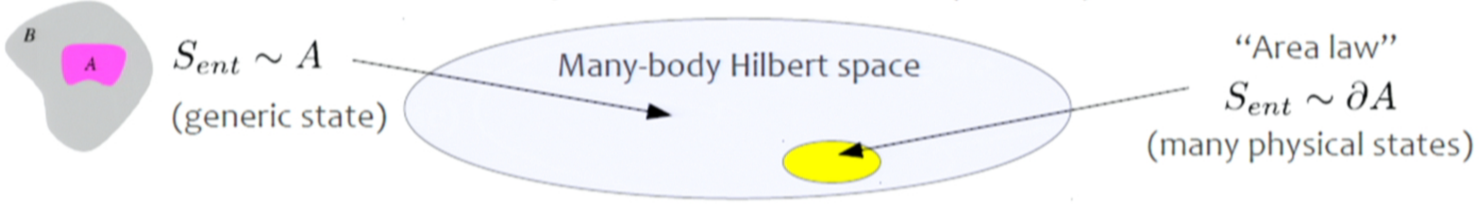

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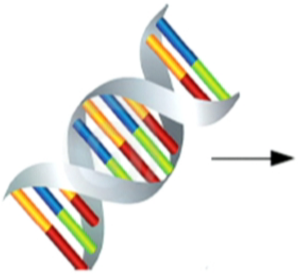
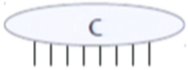


Matrix

1DMatrix-product state

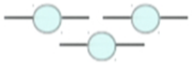
(in higher dimensions: tensors → "tensor networks")

$$\Psi = \sum_{\{\sigma_i\}} c_{\{\sigma_i\}} |\sigma_1, \sigma_2, \ldots, \sigma_N\rangle$$



Cost: 2^N

$$\Psi = \sum_{\{\sigma_i\}} \mathrm{Tr}(A^{\sigma_1} \ldots A^{\sigma_N}) |\sigma_1, \sigma_2, \ldots, \sigma_N\rangle$$

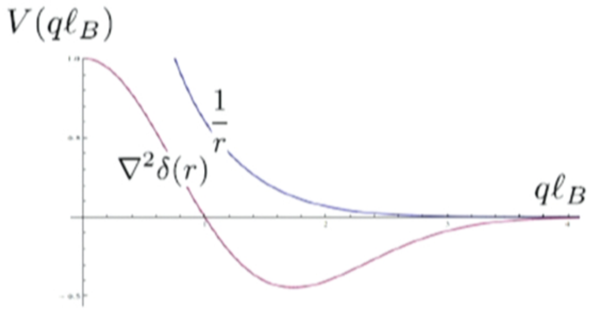


$O(N)$

Efficient to simulate!

- Physical properties can be computed directly from matrices *A*

Quantum Hall states and parent Hamiltonians



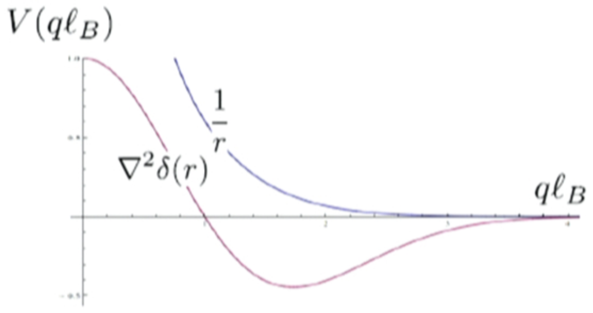
$$\mathcal{P}\frac{1}{|\mathbf{r}_i-\mathbf{r}_j|}\mathcal{P} = \underbrace{\mathcal{P}\nabla^2\delta(\mathbf{r}_i-\mathbf{r}_j)\mathcal{P}}_{V_1 \text{ Haldane pseudopotential}} + \dots$$

[Haldane '83, Trugman and Kivelson '83]

$$\nu = \frac{1}{3}: \; V_1\Psi_{\text{Laughlin}} = 0$$

Coulomb ground state is adiabatically connected to Laughlin once higher pseudopotentials are turned on

Quantum Hall states and parent Hamiltonians



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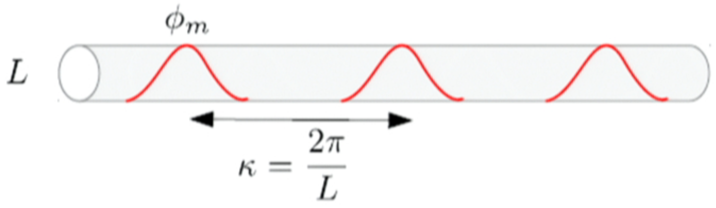
We want to study V_1 , but that is still too complicated – how to simplify it further?

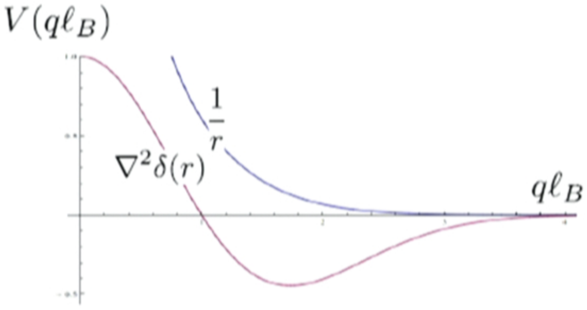
$$\langle \phi^*_{m_1}\phi^*_{m_2}|V_1|\phi_{m_3}\phi_{m_4}\rangle$$



$$V_1 = \sum_m \overset{\substack{\text{interaction}\\ \downarrow}}{e^{-\kappa^2 m^2}\mathrm{Poly}(m)} \overset{\substack{\text{geometry}\\ \downarrow}}{\sum_{i<j} P^m_{ij}}$$

[Seidel and Lee, Bergholtz and Karlhede '05]

Quantum Hall states and parent Hamiltonians



$$\color{red}{\mathcal{P}}\color{black}\frac{1}{|\mathbf{r}_i-\mathbf{r}_j|}\color{red}{\mathcal{P}}\color{black} = \underbrace{\color{red}{\mathcal{P}}\color{black}\nabla^2\delta(\mathbf{r}_i-\mathbf{r}_j)\color{red}{\mathcal{P}}}_{V_1 \text{ Haldane pseudopotential}} + \dots$$

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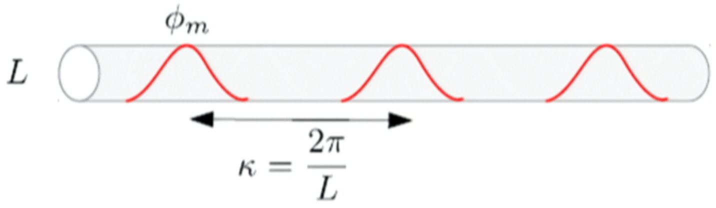

interaction geometry

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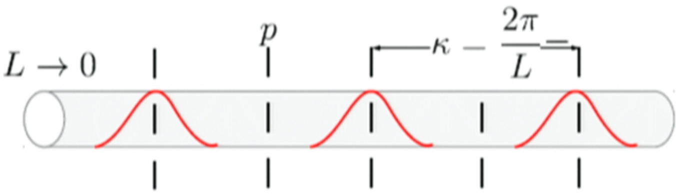
[Seidel and Lee, Bergholtz and Karlhede '05]

If perimeter becomes small, the Hamiltonian can be exactly solved.

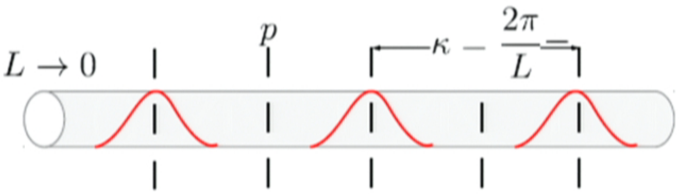
Solvable models and MPS for FQHE



$$V_1^0 = \sum_p e^{-\kappa^2/2} \hat{n}_{p+\frac{1}{2}} \hat{n}_{p-\frac{1}{2}} + (4e^{-2\kappa^2}) \hat{n}_{p+1} \hat{n}_{p-1}$$

~~...11...~~ ~~...101...~~

Solvable models and MPS for FQHE



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~~...11...~~ ~~...101...~~

$$\Psi_L^0 = |...100100100...\rangle$$

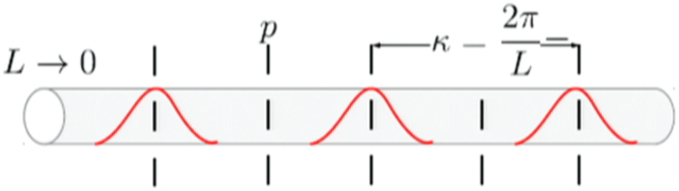
$$S_{ent} = 0 \qquad V_1 \Psi_L^0 = 0$$

$$V_1^1 = V_1^0 + \sum_p (3e^{-5\kappa^2}) c^\dagger_{p+\frac{3}{2}} c^\dagger_{p-\frac{3}{2}} c_{p-\frac{1}{2}} c_{p+\frac{1}{2}} + h.c.$$

$$+ \sum_p (9e^{-9\kappa^2/2}) \hat{n}_{p+\frac{3}{2}} \hat{n}_{p-\frac{3}{2}}$$

~~...1001...~~

Solvable models and MPS for FQHE



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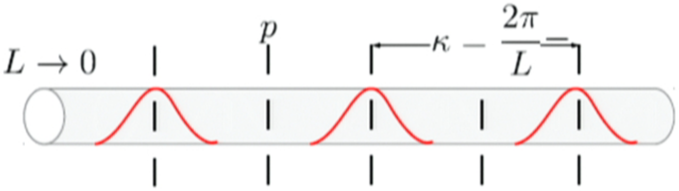
~~...1001...~~

$$\Psi_L^1 = |...100100\underline{1001}001001...\rangle - 3e^{-2\kappa^2}|...100100\underline{0110}001001...\rangle + ...$$

[Nakamura *et. al*, '12]

$V_1\Psi_L^1 = 0$ because $V_1^1 = \sum_p A_p^\dagger A_p + B_p^\dagger B_p$

Solvable models and MPS for FQHE



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$\cancel{\ldots 11 \ldots}$ $\cancel{\ldots 101 \ldots}$

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$$+ \sum_p (9e^{-9\kappa^2/2}) \hat{n}_{p+\frac{3}{2}} \hat{n}_{p-\frac{3}{2}}$$

$\cancel{\ldots 1001 \ldots}$

$$\Psi_L^1 = |\ldots 100100\underline{1001}001001 \ldots\rangle - 3e^{-2\kappa^2} |\ldots 100100\underline{0110}001001 \ldots\rangle + \ldots$$

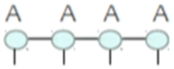
[Nakamura *et. al*, '12]

$V_1 \Psi_L^1 = 0$ because $V_1^1 = \sum_p A_p^\dagger A_p + B_p^\dagger B_p$

Mapping to spin-1 chain:

$010 \to |0\rangle$
$001 \to |+\rangle$
$100 \to |-\rangle$

$|\Psi_L^1\rangle$



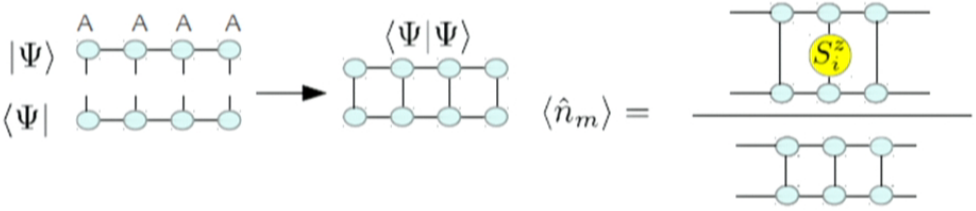
$$A = \begin{bmatrix} |0\rangle & |+\rangle \\ -3e^{-4\kappa^2}|-\rangle & 0 \end{bmatrix}$$

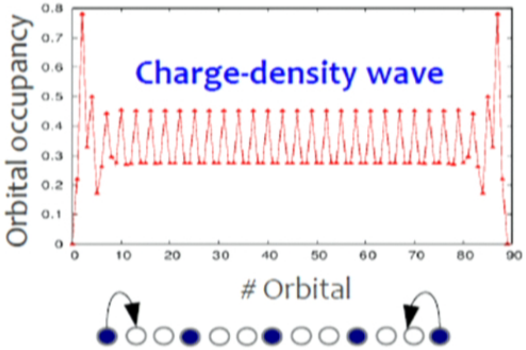
$S_{ent} \to \log 4$

Can be generalized to non-Abelian (Read-Rezayi), non-unitary states (Gaffnian, Haffnian etc.)

[ZP, Regnault, Bernevig,to appear]

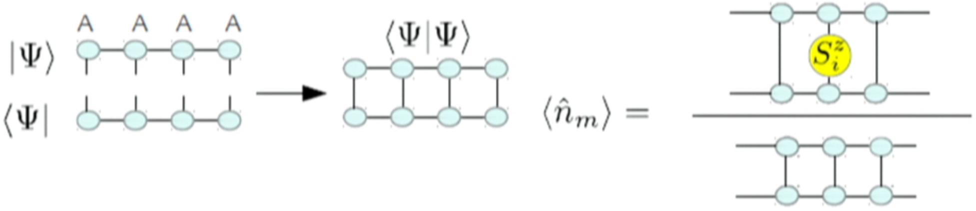
Physical properties via MPS

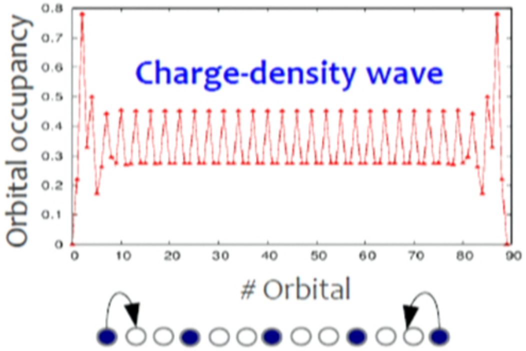




Particles must be allowed to hop even from very large distances to get a liquid!

Physical properties via MPS





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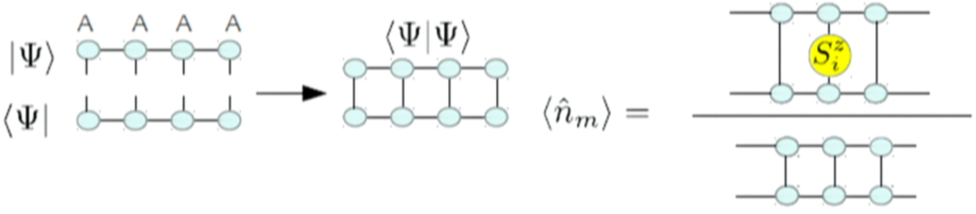
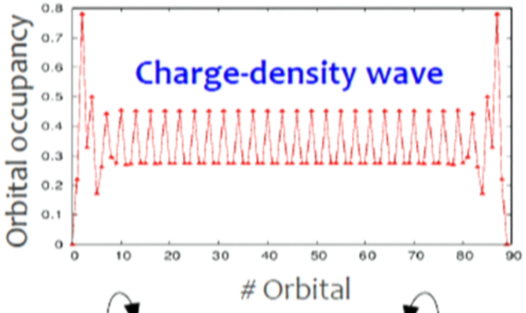
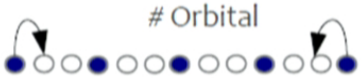


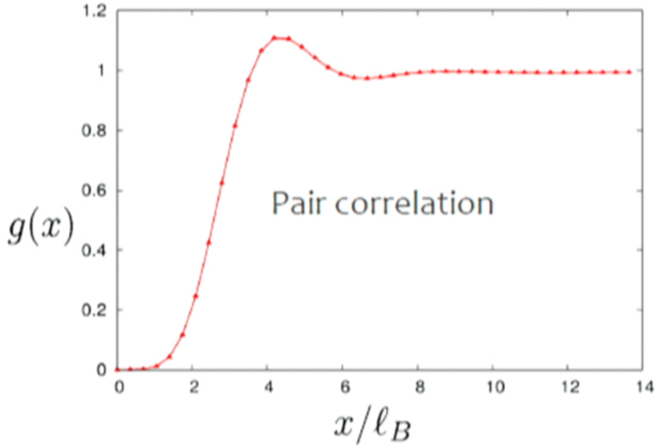


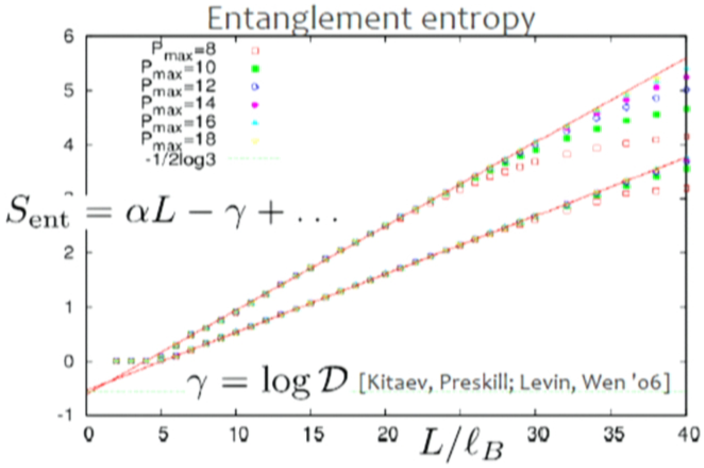


Particles must be allowed to hop
even from very large distances
to get a liquid!

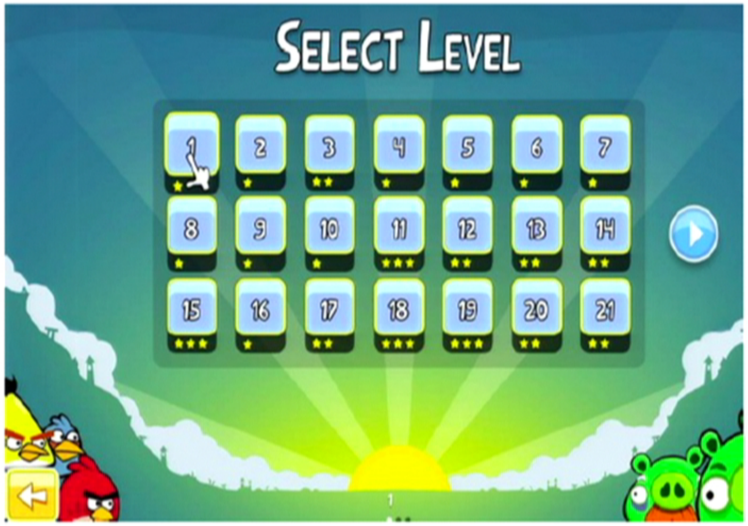
- If we continue the expansion or use Conformal Field Theory:

  [Zaletel and Mong, '12; Estienne, ZP, Regnault, Bernevig '12]

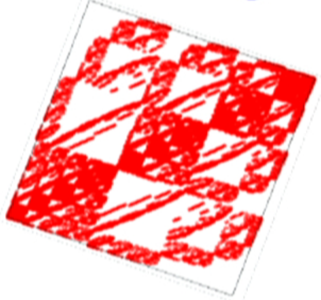




What's next?



- **Level 1: Exact diagonalization** ✓



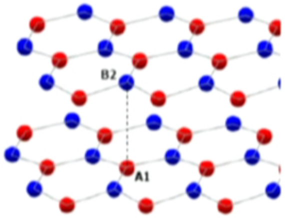
- **Level 2: Analytic MPS for model states** ✓
- **Level 3: Density-matrix renormalization group for generic interactions**

[Previous efforts: N. Shibata and D. Yoshioka,
A. Feiguin, J. Zhao and D. N. Sheng,
M. Zaletel, R. Mong and F. Pollmann]

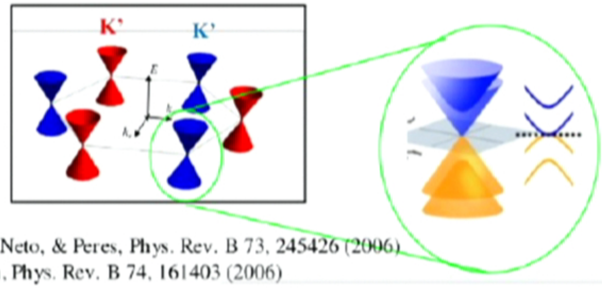
Why?

To understand the role of Landau-level mixing

Bilayer graphene in a magnetic field



Bernal stacked

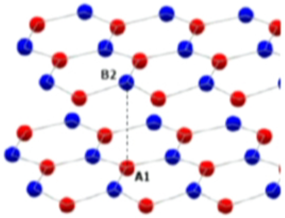


Guinea, Neto, & Peres, Phys. Rev. B 73, 245426 (2006)
McCann, Phys. Rev. B 74, 161403 (2006)

$$H = \frac{1}{2m}\begin{bmatrix} 0 & (k_x + ik_y)^2 \\ (k_x - ik_y)^2 & 0 \end{bmatrix}$$

$$k_x + ik_y \to a^\dagger/\sqrt{2}\ell_B$$

Bilayer graphene in a magnetic field



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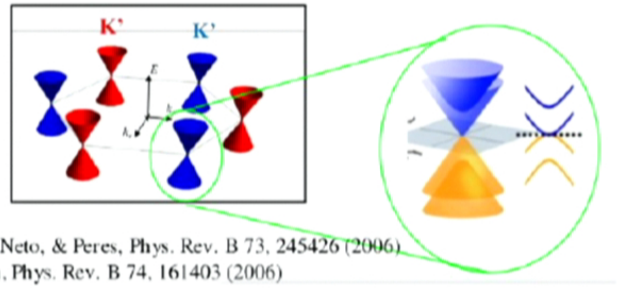

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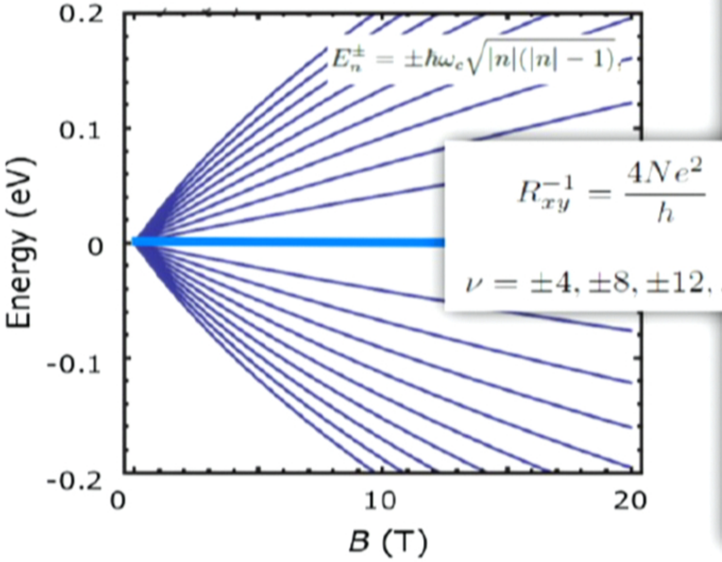
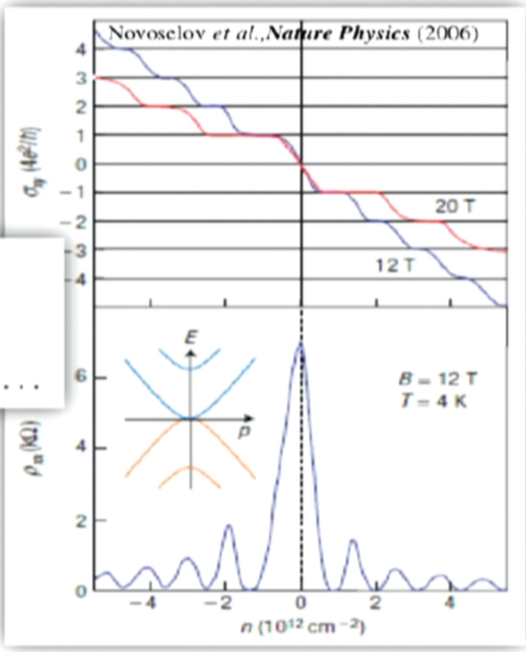


$$R_{xy}^{-1} = \frac{4Ne^2}{h}$$

$$\nu = \pm 4, \pm 8, \pm 12, \ldots$$



Novel aspects of FQHE in bilayer graphene

- New aspects of fractional quantum Hall physics:

LLs are **multiply degenerate**

Coulomb interaction is **screened**:

$$V(k) = \frac{V_0(k)}{1 + a \tanh(bk\ell_B^2)/k\ell_B}$$

[E. V. Gorbar, V. P. Gusynin, V. A. Miransky, JETP Lett. 91, 314 (2010);
K. Snizhko, V. Cheianov, S. H. Simon, PRB 85, 201415(R) (2012)]

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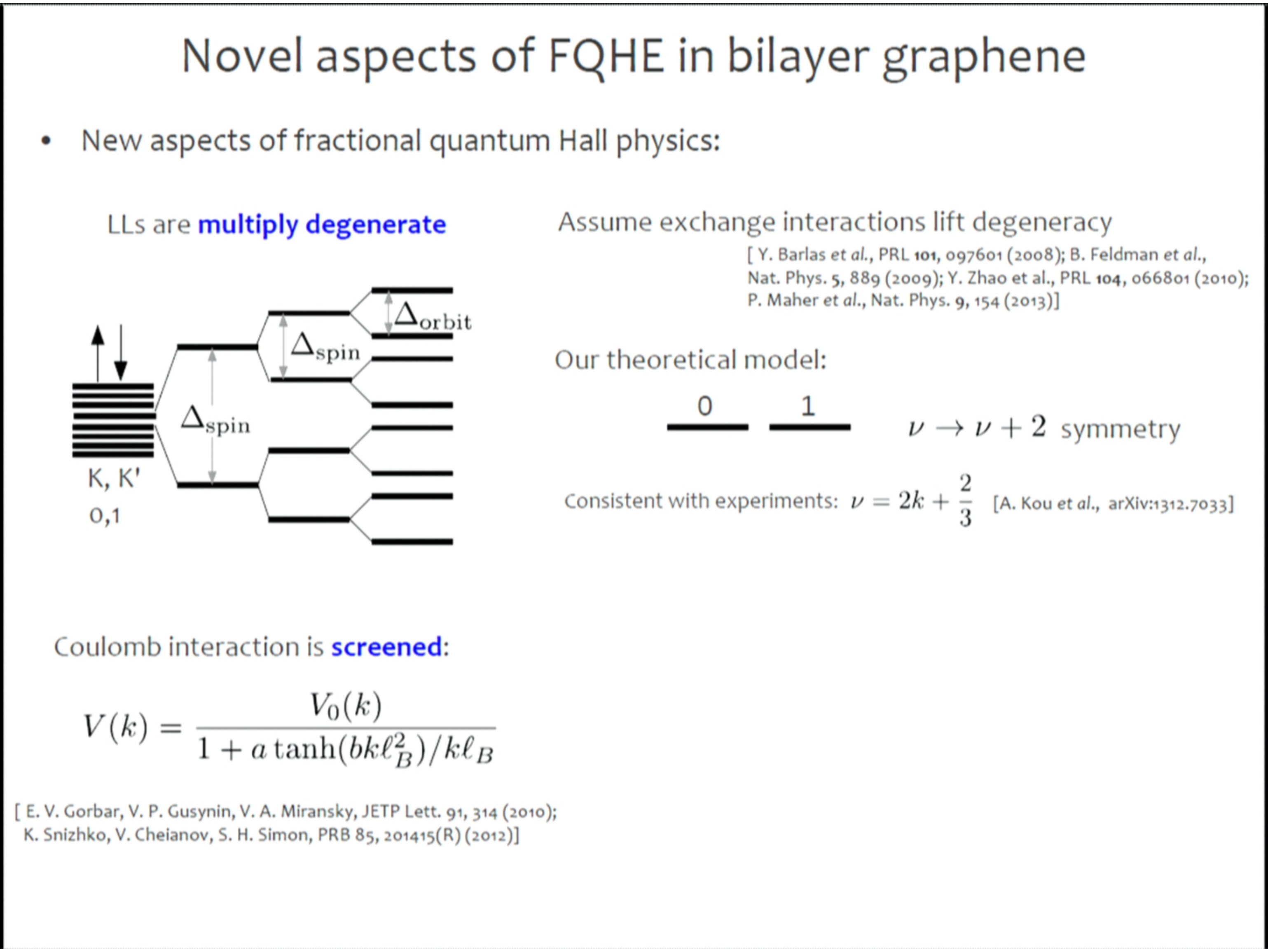

Novel aspects of FQHE in bilayer graphene

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LLs are **multiply degenerate**

Assume exchange interactions lift degeneracy

[Y. Barlas *et al.*, PRL **101**, 097601 (2008); B. Feldman *et al.*, Nat. Phys. **5**, 889 (2009); Y. Zhao et al., PRL **104**, 066801 (2010); P. Maher *et al.*, Nat. Phys. **9**, 154 (2013)]

Our theoretical model:

$\nu \to \nu + 2$ symmetry

Consistent with experiments: $\nu = 2k + \frac{2}{3}$ [A. Kou *et al.*, arXiv:1312.7033]

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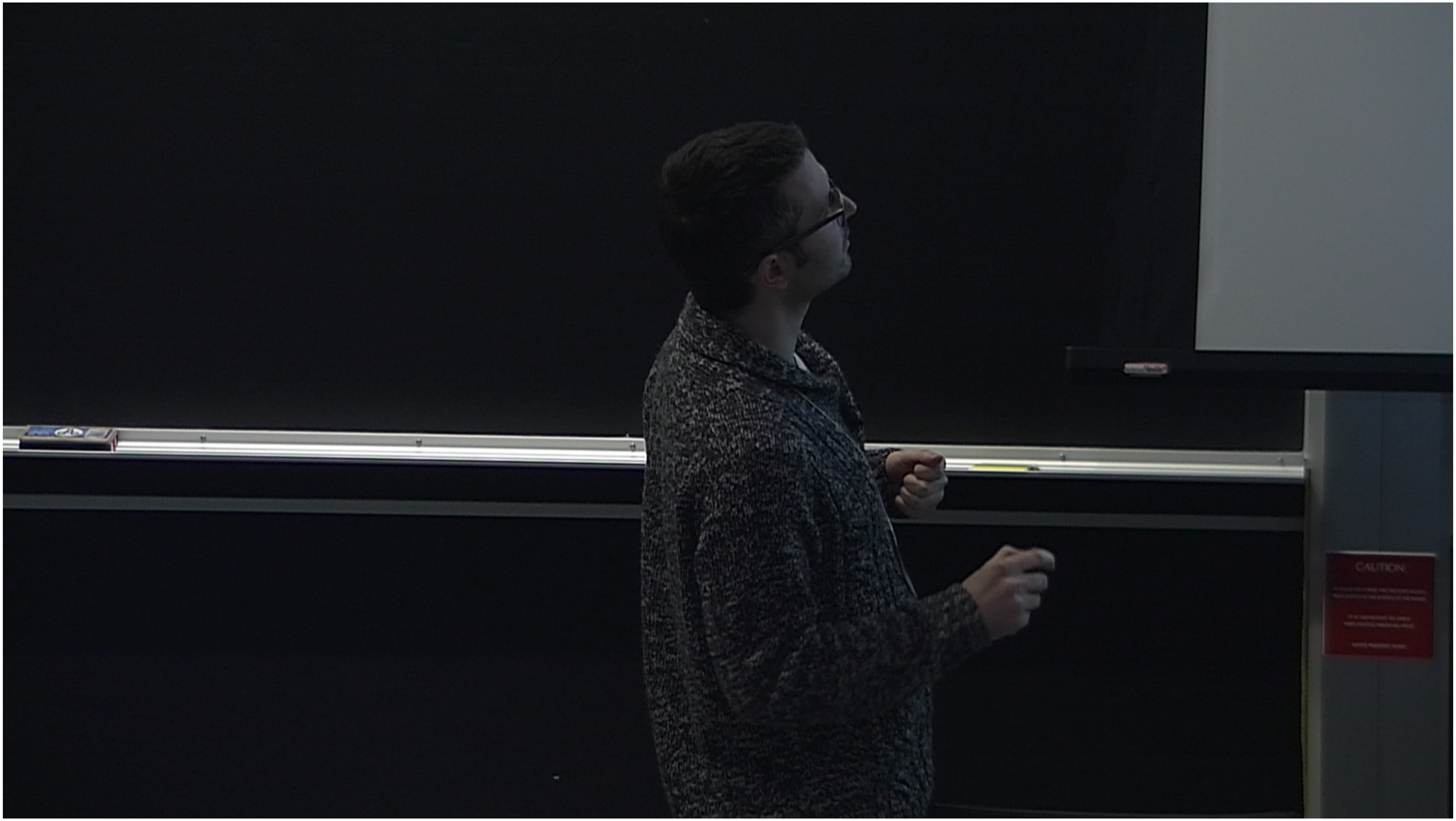
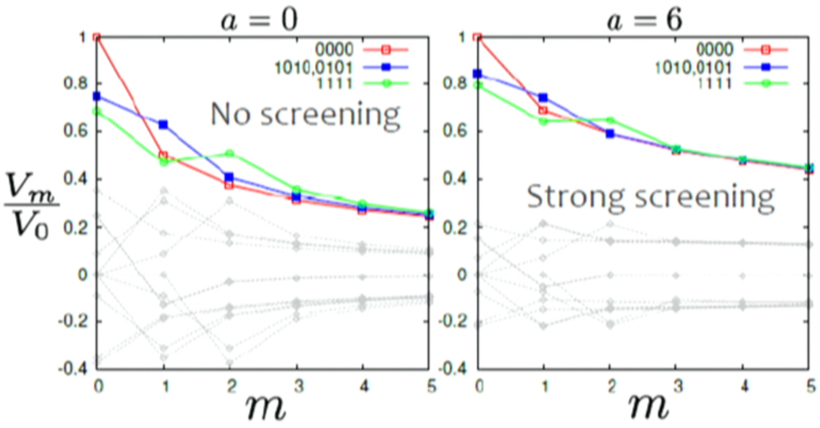

CAUTION

(1) What fractions are stable?

- Interactions are very complicated (16 types of Haldane pseudopotentials!)



$$V_m^{\{n_i\}} = \int d^2k e^{-k^2} V(k) F_{n_1}^{n_3}(k) F_{n_2}^{n_4}(-k) F_{m'}^{m}(k\sqrt{2})$$

$$m' = m + n_3 + n_4 - n_1 - n_2$$

$$V(r) \to \ln(a/r), \quad 1 \ll r/\ell_B \ll a$$

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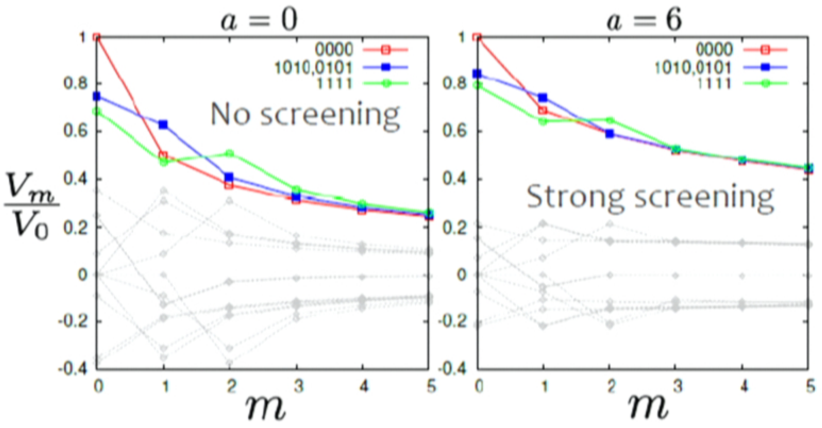

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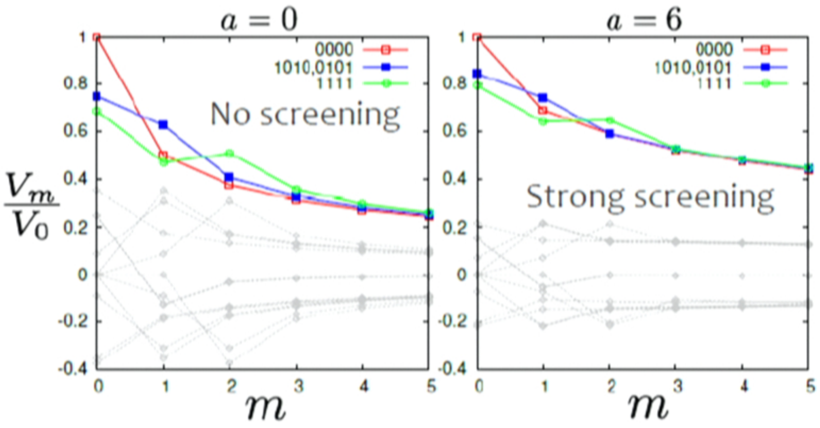

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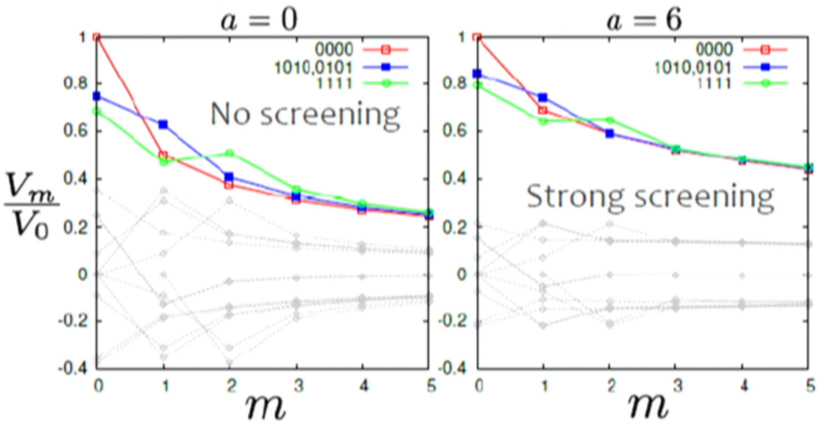

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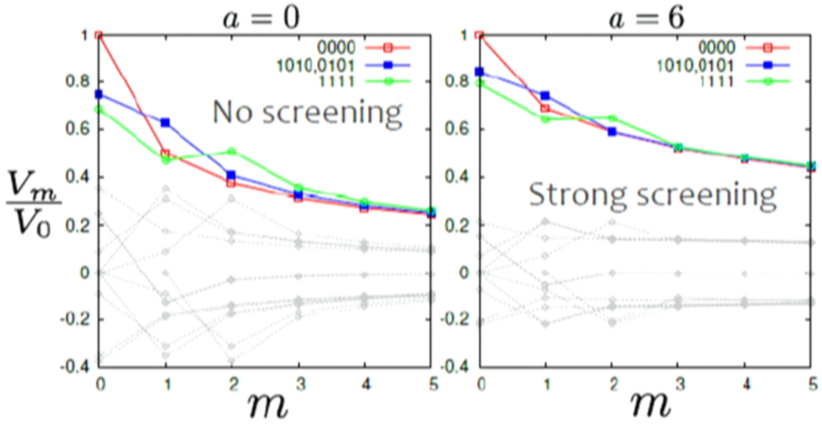

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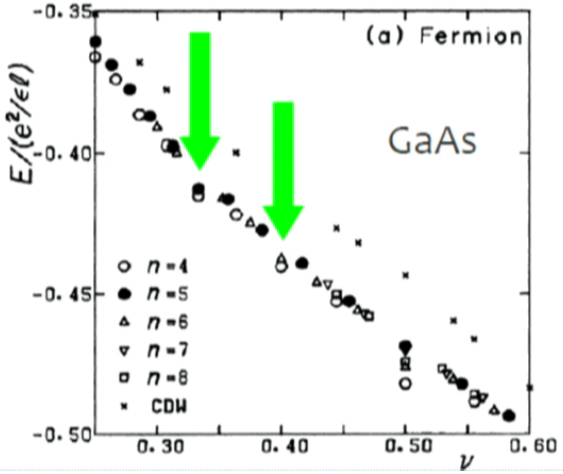
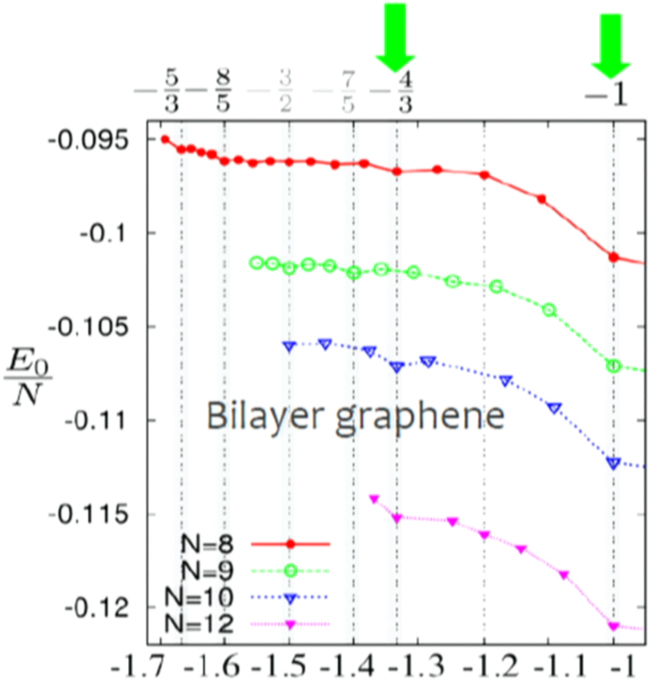
$$V(r) \to \ln(a/r), \;\; 1 \ll r/\ell_B \ll a$$

- What fractions are expected to be stable?

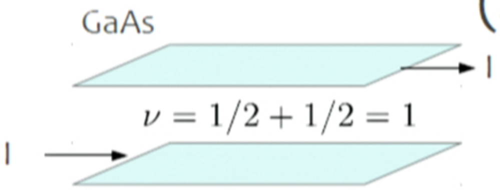
[Reprinted from D. Yoshioka, Phys. Rev. B 29, 6833 (1984)]





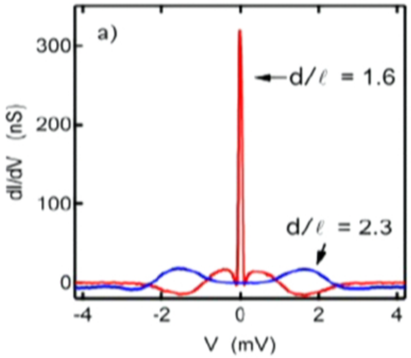
[ZP and D. Abanin, PRL **112**, 046602 (2014)]

(2) Collective modes

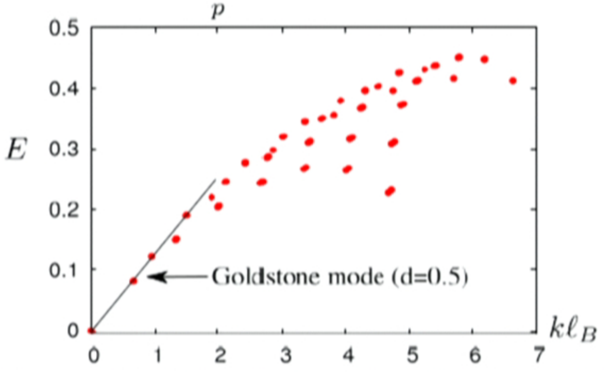


$$\nu = 1/2 + 1/2 = 1$$

[I. Spielman, J. Eisenstein, L.Pfeiffer, K. West, PRL 84, 5808 (2000)]



$$\Psi = \prod_p (|\uparrow_p\rangle + e^{i\phi}|\downarrow_p\rangle)$$



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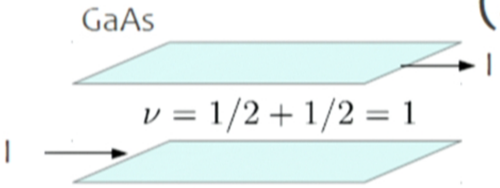

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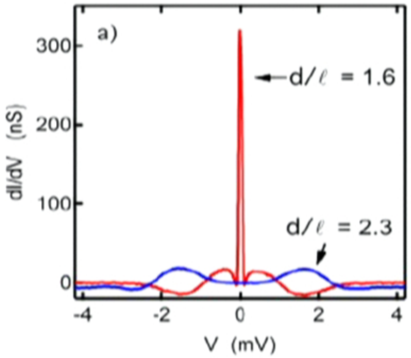

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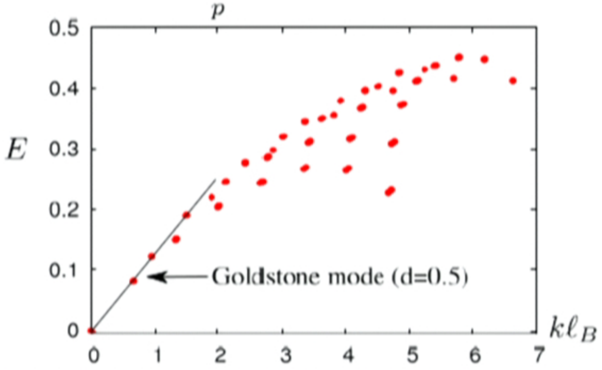
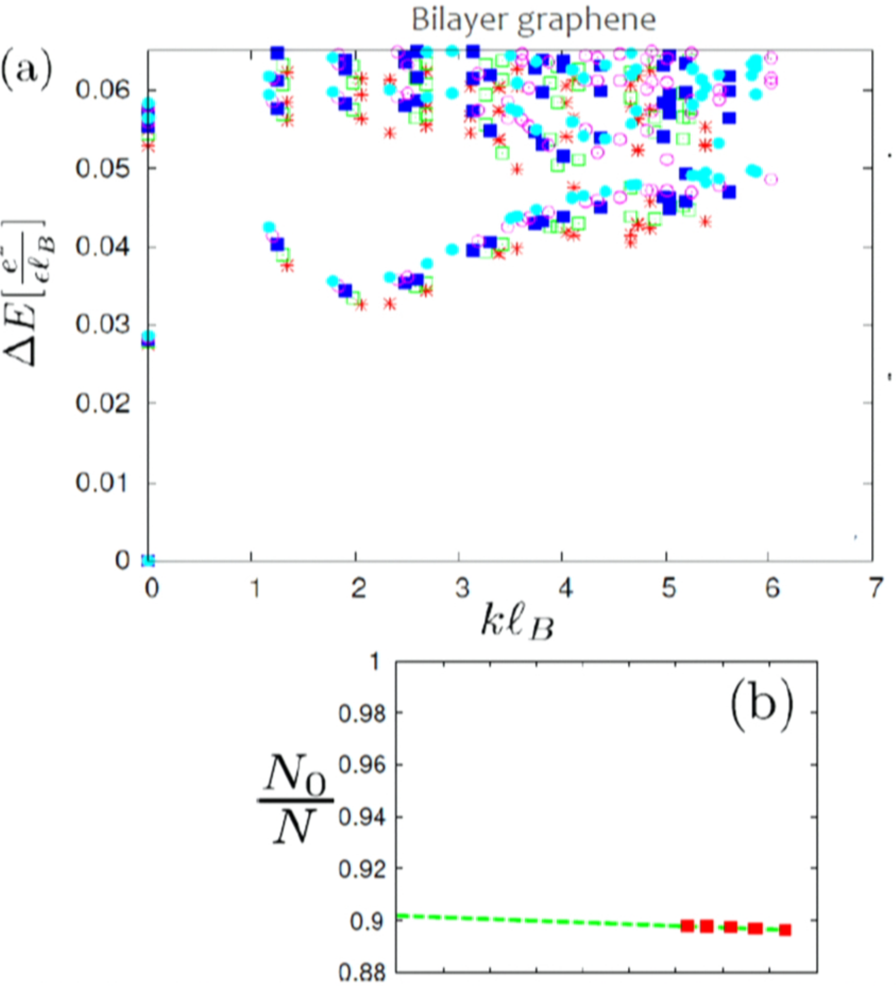
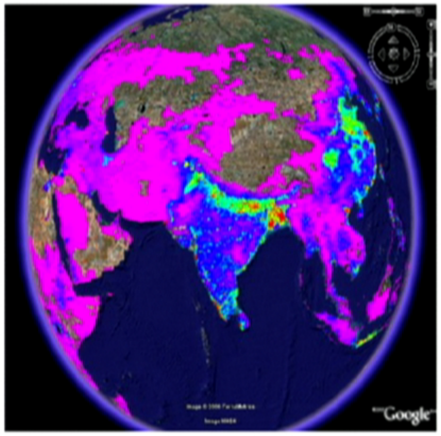
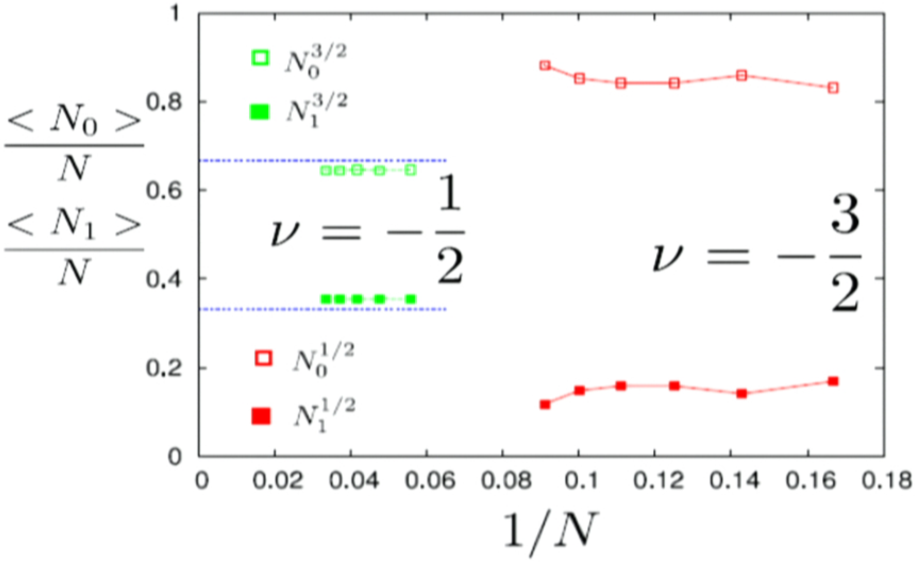




(3) Even-denominator states and non-Abelian physics

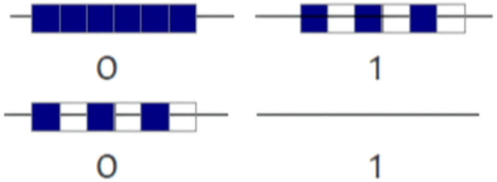
(1) Average populations of two sublevels





$\nu = -\frac{1}{2}$

$\nu = -\frac{3}{2}$



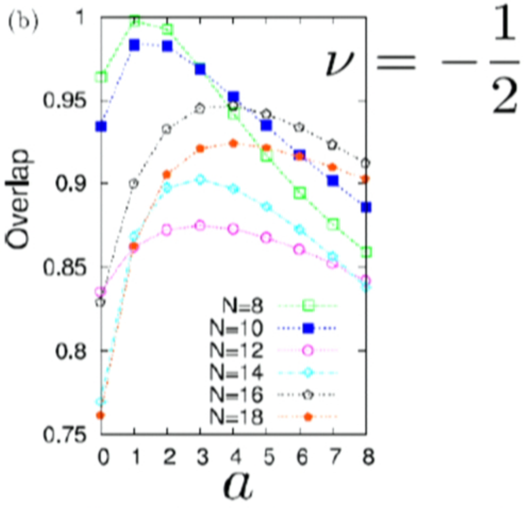
$$\frac{\langle N_0 \rangle}{N} \rightarrow \frac{2}{3} \quad \frac{\langle N_1 \rangle}{N} \rightarrow \frac{1}{3}$$

(3) Even-denominator states and non-Abelian physics

(2) Effect of screening: $V(k) = \frac{V_0(k)}{1 + a\tanh(k\ell_B^2/2)/k\ell_B}$

$$\langle \mathcal{P}_{n=1}\Psi_{\rm exact}|\Psi_{\rm Pfaffian}\rangle$$

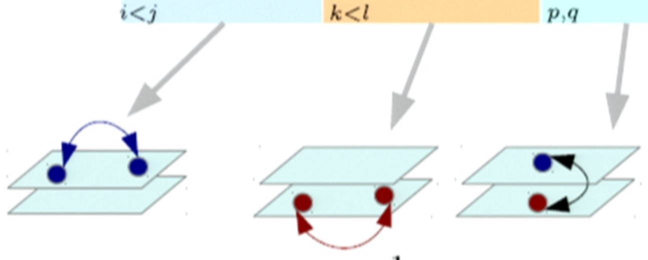
Screening improves overlap of -1/2 state with Pfaffian



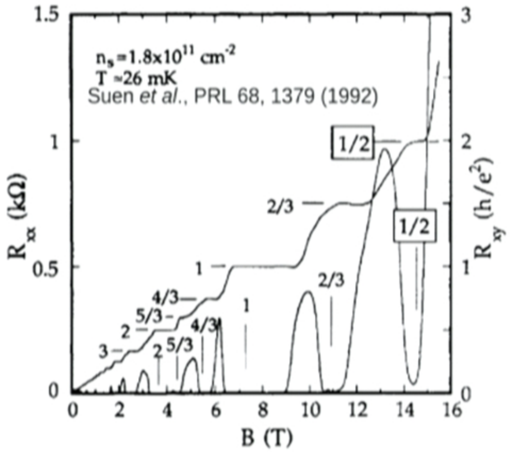
[ZP and D. Abanin, PRL **112**, 046602 (2014)]

(3) Can it be a two-component state?

$$\Psi_{331} = \prod_{i<j}(z_{i,0} - z_{j,0})^3 \prod_{k<l}(z_{k,1} - z_{l,1})^3 \prod_{p,q}(z_{p,0} - z_{q,1})$$



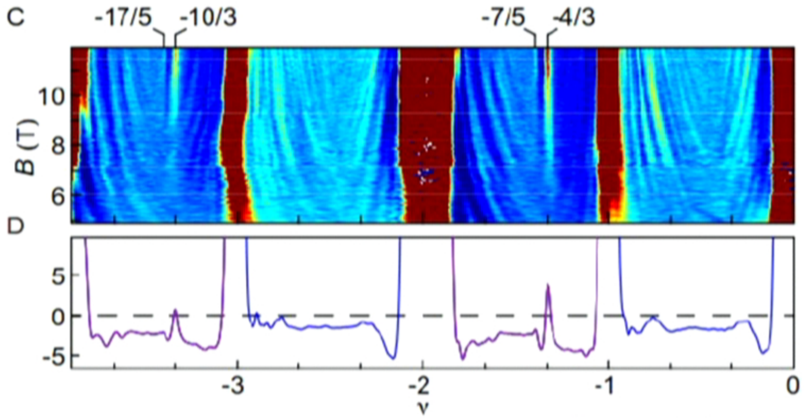
$$\nu_T = \frac{1}{2}$$



Experiments

- States obey $\nu \rightarrow \nu + 2$

$$\nu = 2k + \frac{2}{3}, \nu = 2k + \frac{3}{5},\ k = -2, -1, 0, 1$$



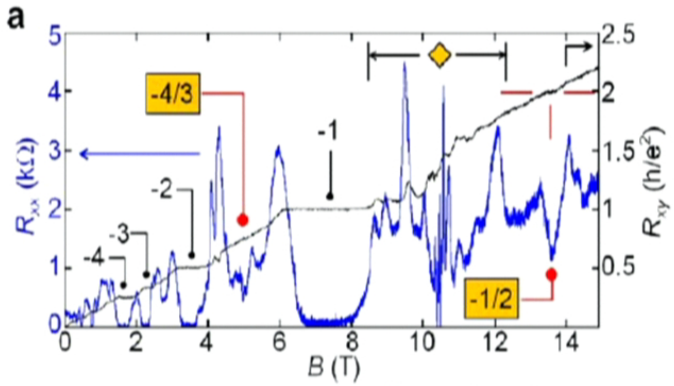
[A. Kou *et al.*, arXiv:1312.7033]

- Strongest fraction $\nu = -\frac{4}{3}$

(observed in both samples)

- Non-Abelian state $\nu = -\frac{1}{2}$

observed in one sample so far

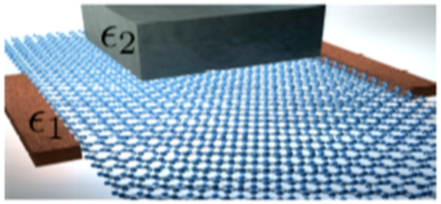


[D.-K. Ki *et al.* arXiv:1305.4761]

Prospects and outlook

- Because of their exposed surface, graphene materials allow for tuning of the effective interaction via **dielectric screening**

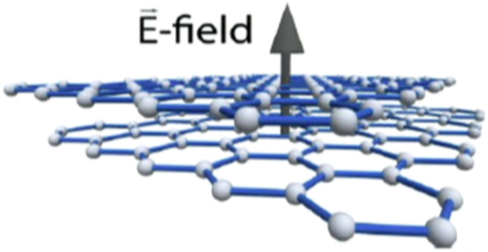
  This can be used to increase the excitation gaps



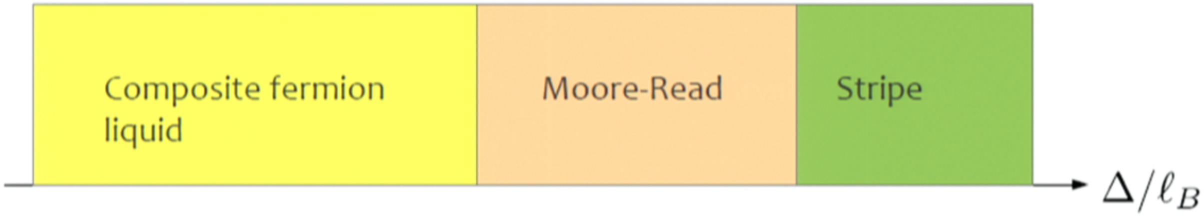
[ZP, R. Thomale, and D. Abanin, PRL **107**, 176602 (2011)]

- Bilayer graphene + perpendicular electric field

$$H = \begin{bmatrix} \Delta & \mathcal{M}_\lambda (k_x + i k_y)^2 \\ \mathcal{M}_\lambda (k_x - i k_y)^2 & -\Delta \end{bmatrix}$$



Can study **phase transitions** between different states as a function of electric field:



[ZP, D. Abanin, Y. Barlas, and R. Bhatt, PRB **84**, 241306(R) (2011); NJP **14**, 025009 (2012)]

- Can also apply parallel magnetic field [ZP, Phys. Rev. B **87**, 245315 (2013)]

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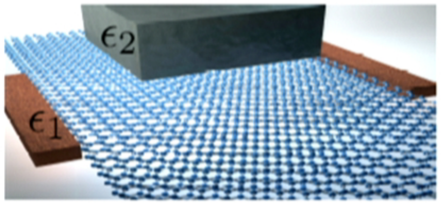

[ZP, R. Thomale, and D. Abanin, PRL **107**, 176602 (2011)]

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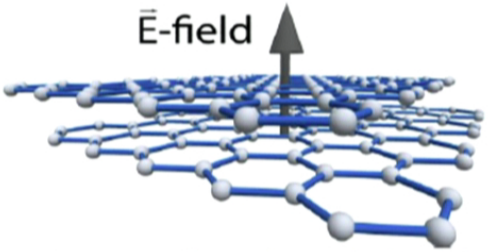

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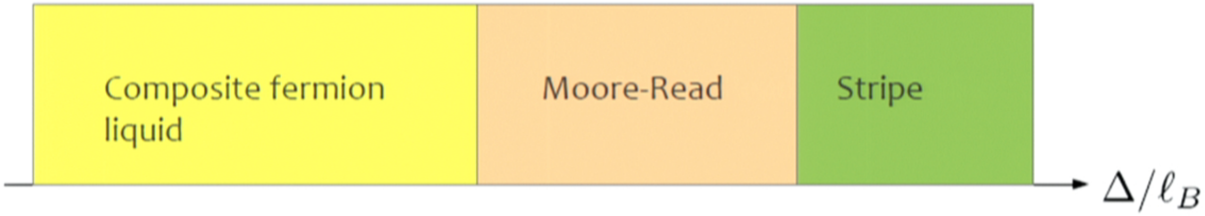

Composite fermion liquid

Moore-Read

Stripe

Δ/ℓ_B

[ZP, D. Abanin, Y. Barlas, and R. Bhatt, PRB **84**, 241306(R) (2011); NJP **14**, 025009 (2012)]

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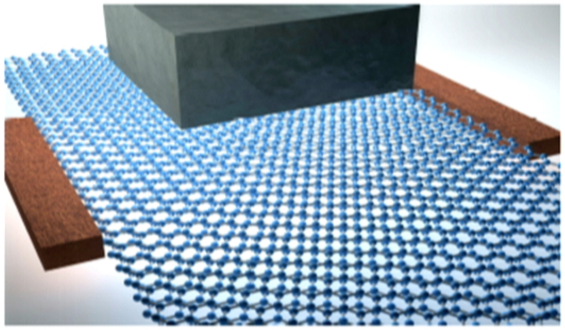
Acknowledgments

Graphene



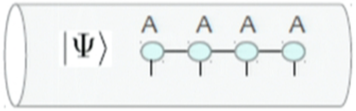
Dima Abanin (Perimeter)

Ravin Bhatt (Princeton)

Ronny Thomale (Wurzburg)

Yafis Barlas (UC Riverside)

Entanglement; MPS



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Nicolas Regnault (ENS Paris)

LL mixing and non-Abelian states

Duncan Haldane (Princeton)

Ed Rezayi (Calstate LA)

Thank you