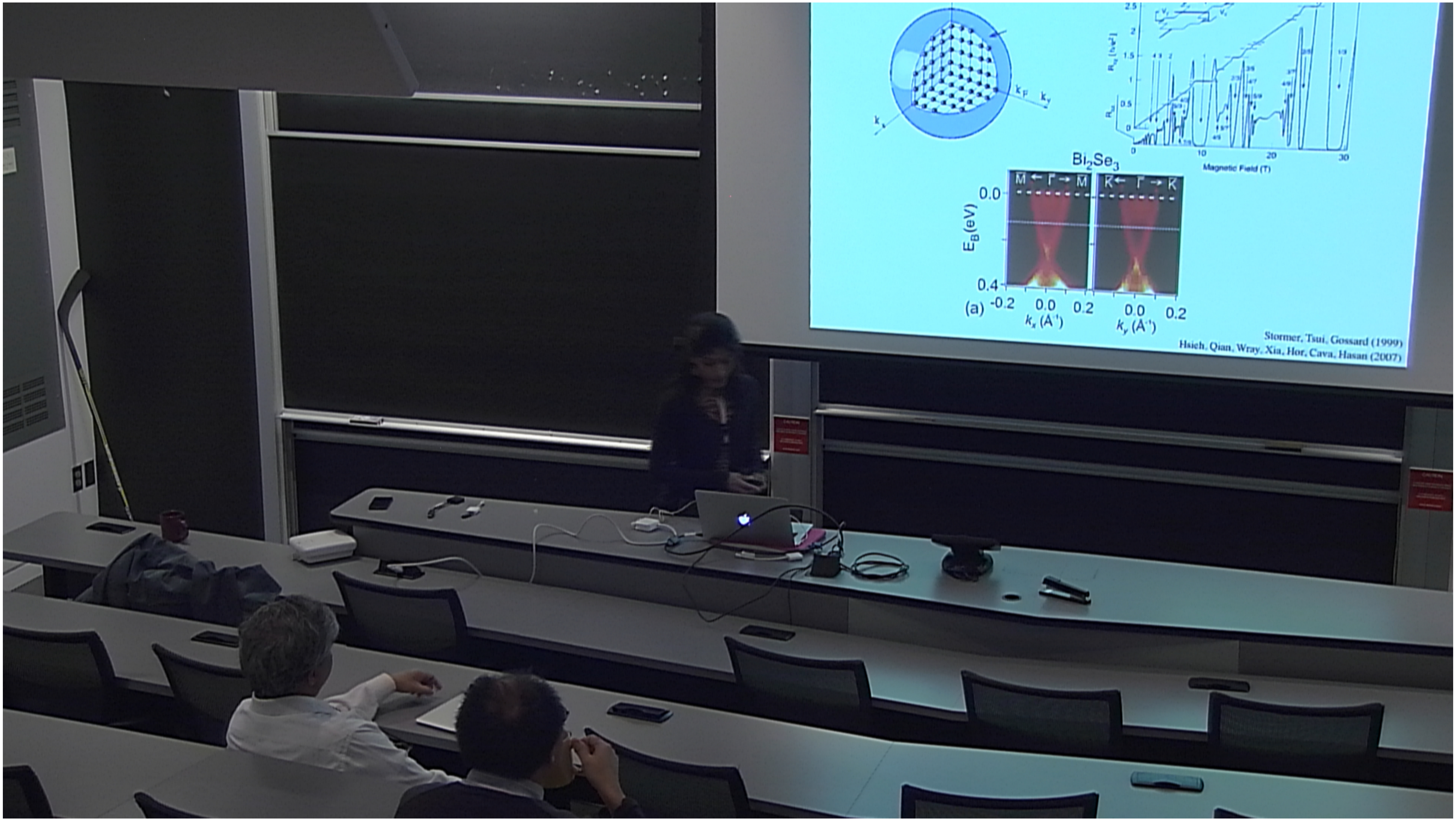
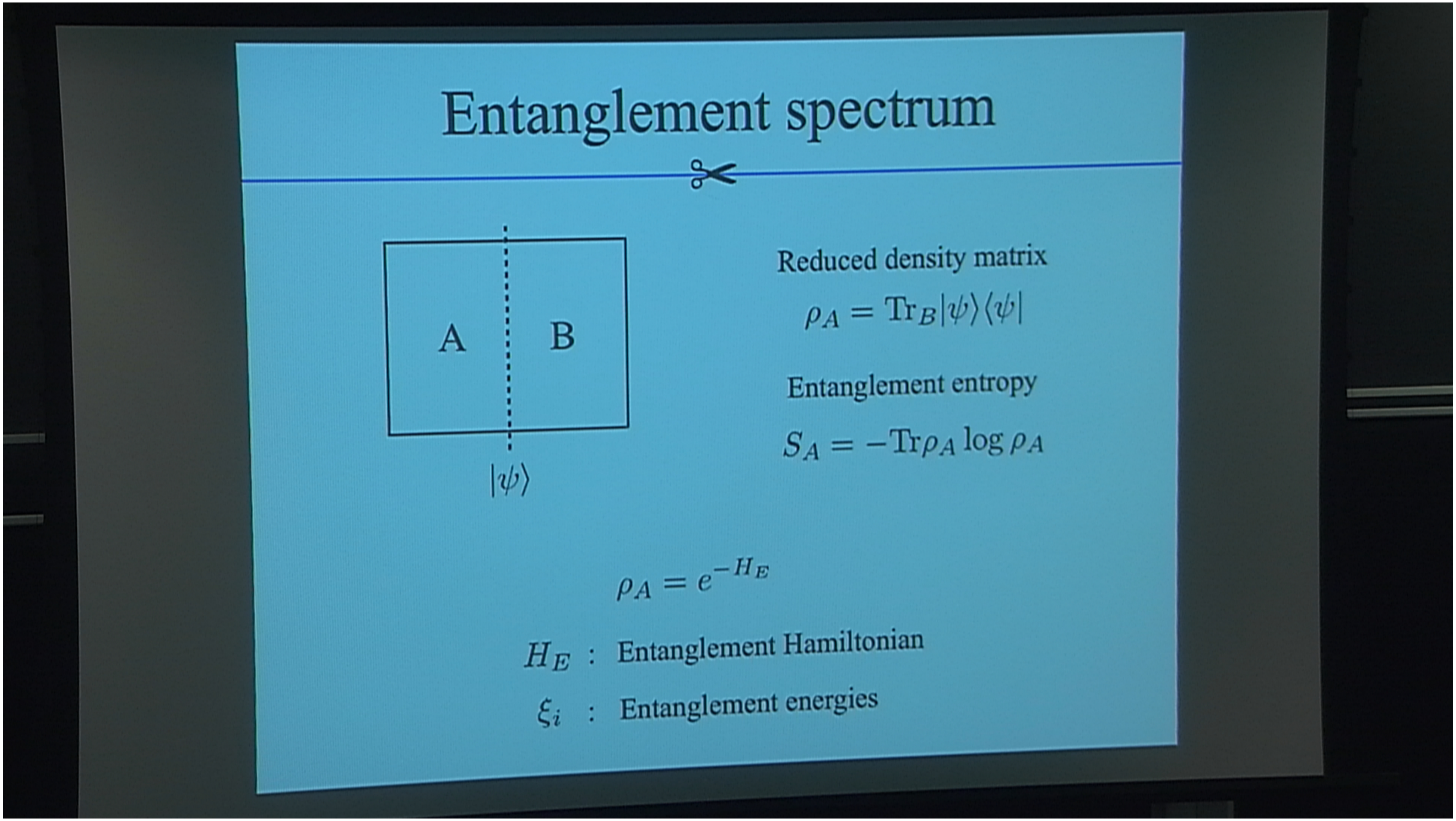
Title: How universal is the entanglement spectrum?

Date: Feb 14, 2014 09:00 AM

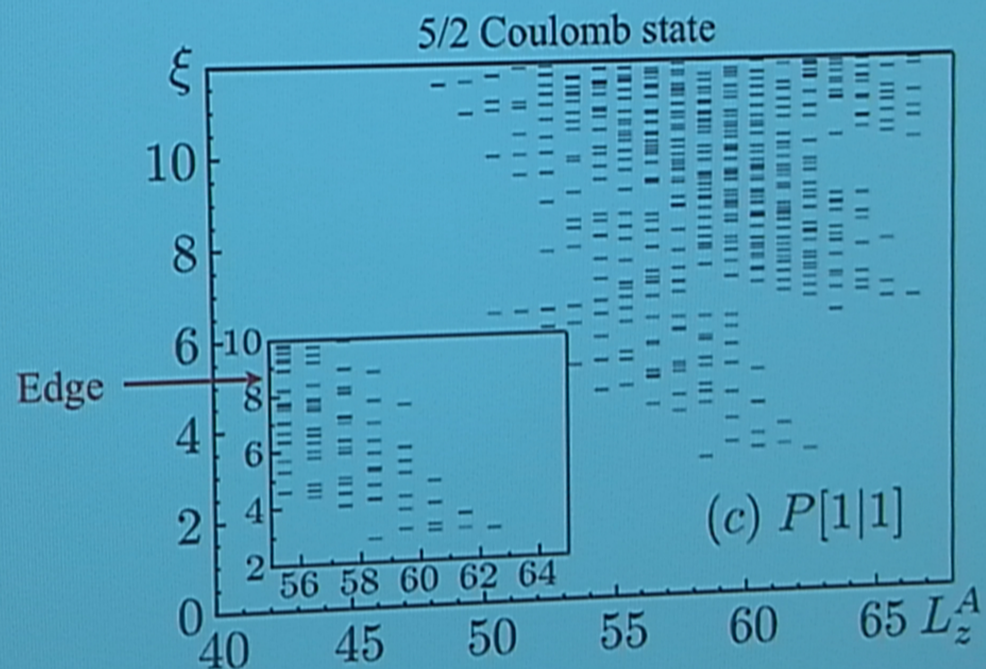
URL: http://pirsa.org/14020132

Abstract: It is now commonly believed that the ground state entanglement spectrum (ES) exhibits universal features characteristic of a given phase. In this talk, I will present evidence to the contrary. I will show that the entanglement Hamiltonian can undergo quantum phase transitions in which its ground state and low energy spectrum exhibit singular changes, even when the physical system remains in the same phase. For broken symmetry problems, this implies that the ES and the Renyi entropies can mislead entirely, while for quantum Hall systems the ES has much less universal content than assumed to date. I will also discuss the consequences of the eigenstate thermalization hypothesis for the entanglement Hamiltonian, showing that a pure state in a sub-system can capture the properties of the reduced density matrix.

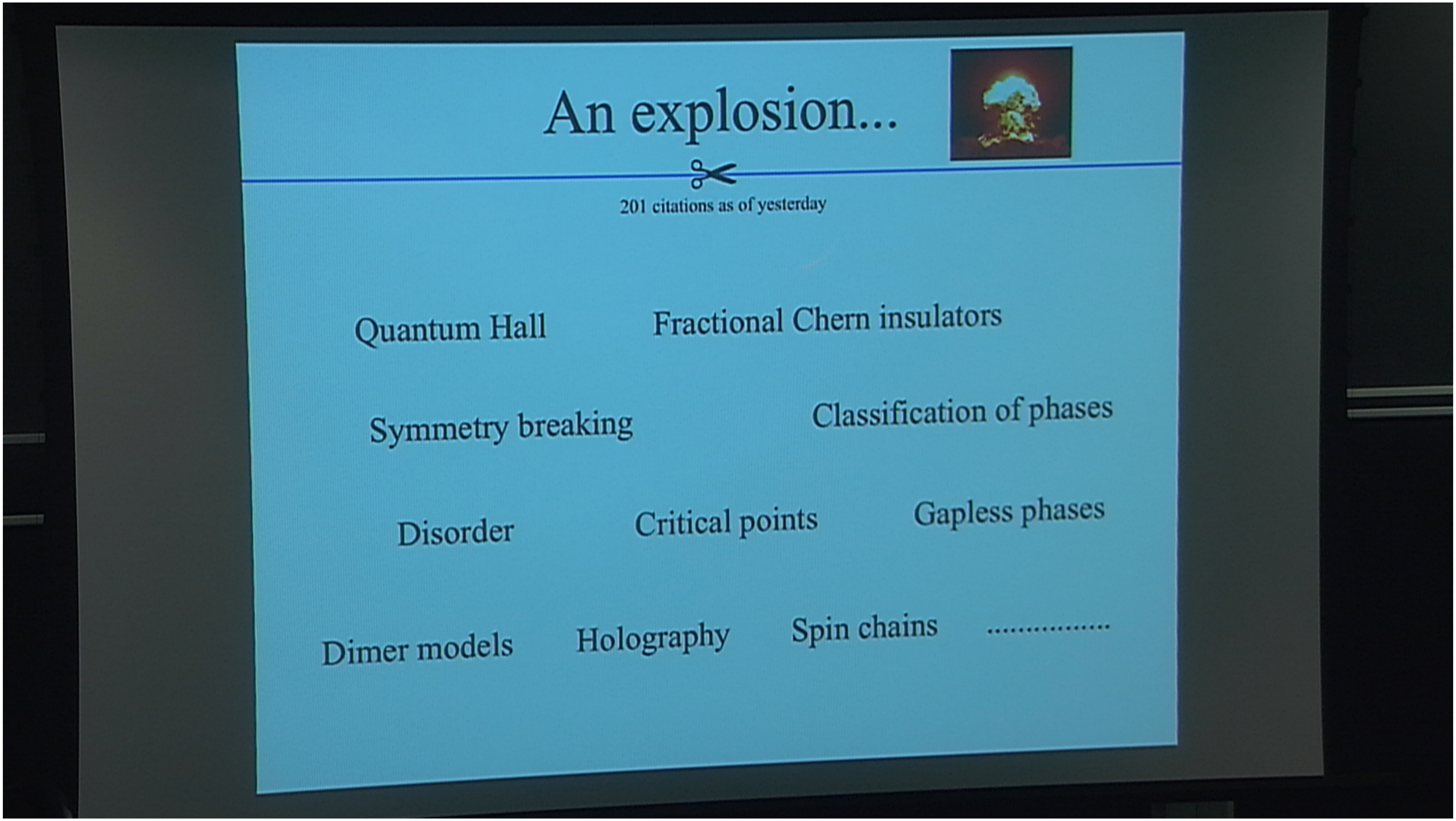

Bi2Se3
Magnetic Field (T)
E_B(eV)
k_x (Å⁻¹)
k_y (Å⁻¹)
(a)
Stormer, Tsui, Gossard (1999)
Hsieh, Qian, Wray, Xia, Hor, Cava, Hasan (2007)

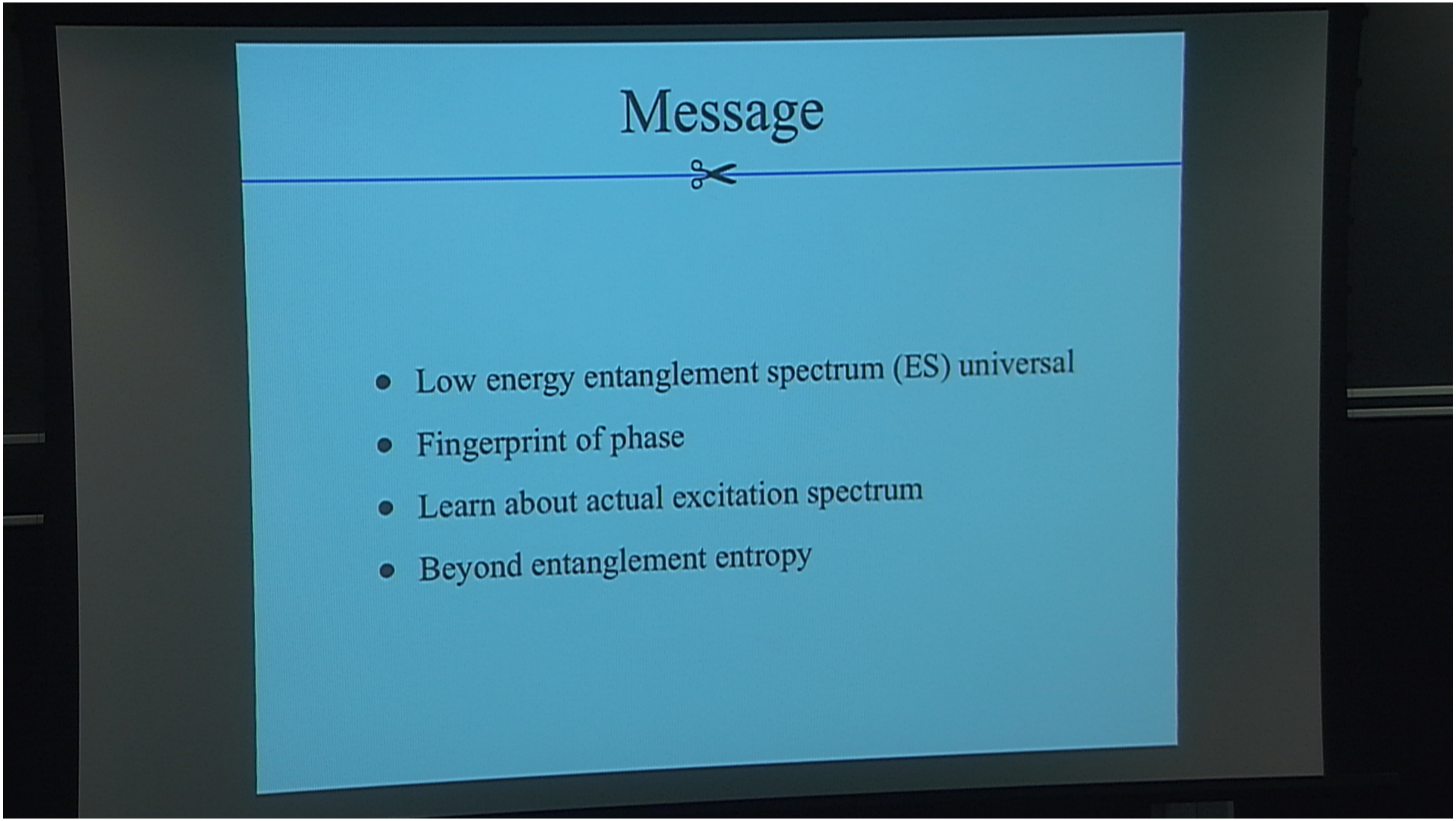

Entanglement spectrum
A
B
$|\psi\rangle$
Reduced density matrix
$\rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi|$
Entanglement entropy
$S_A = -\mathrm{Tr}\rho_A \log \rho_A$
$\rho_A = e^{-H_E}$
H_E : Entanglement Hamiltonian
ξ_i : Entanglement energies

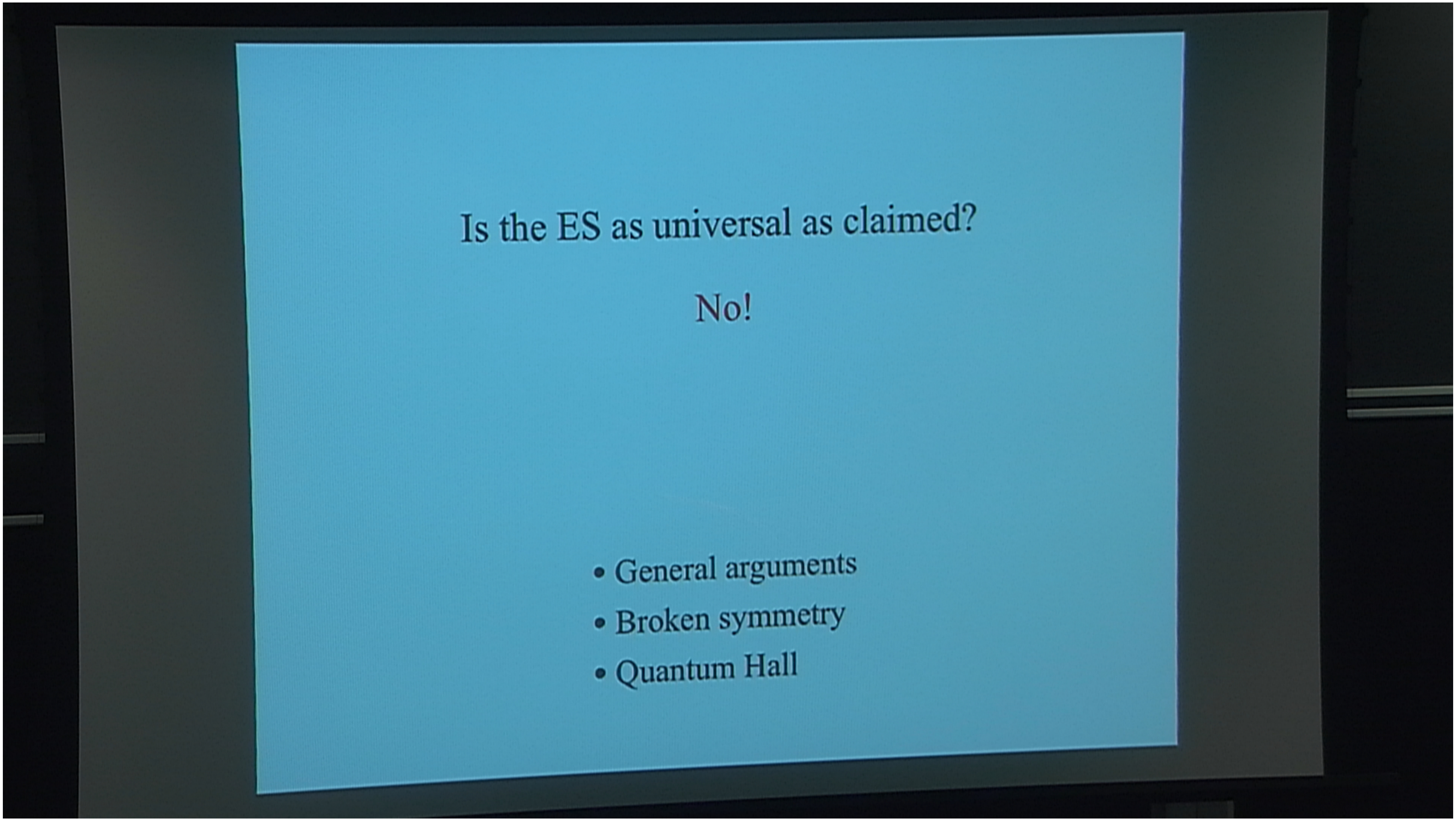
Fingerprint of topological order

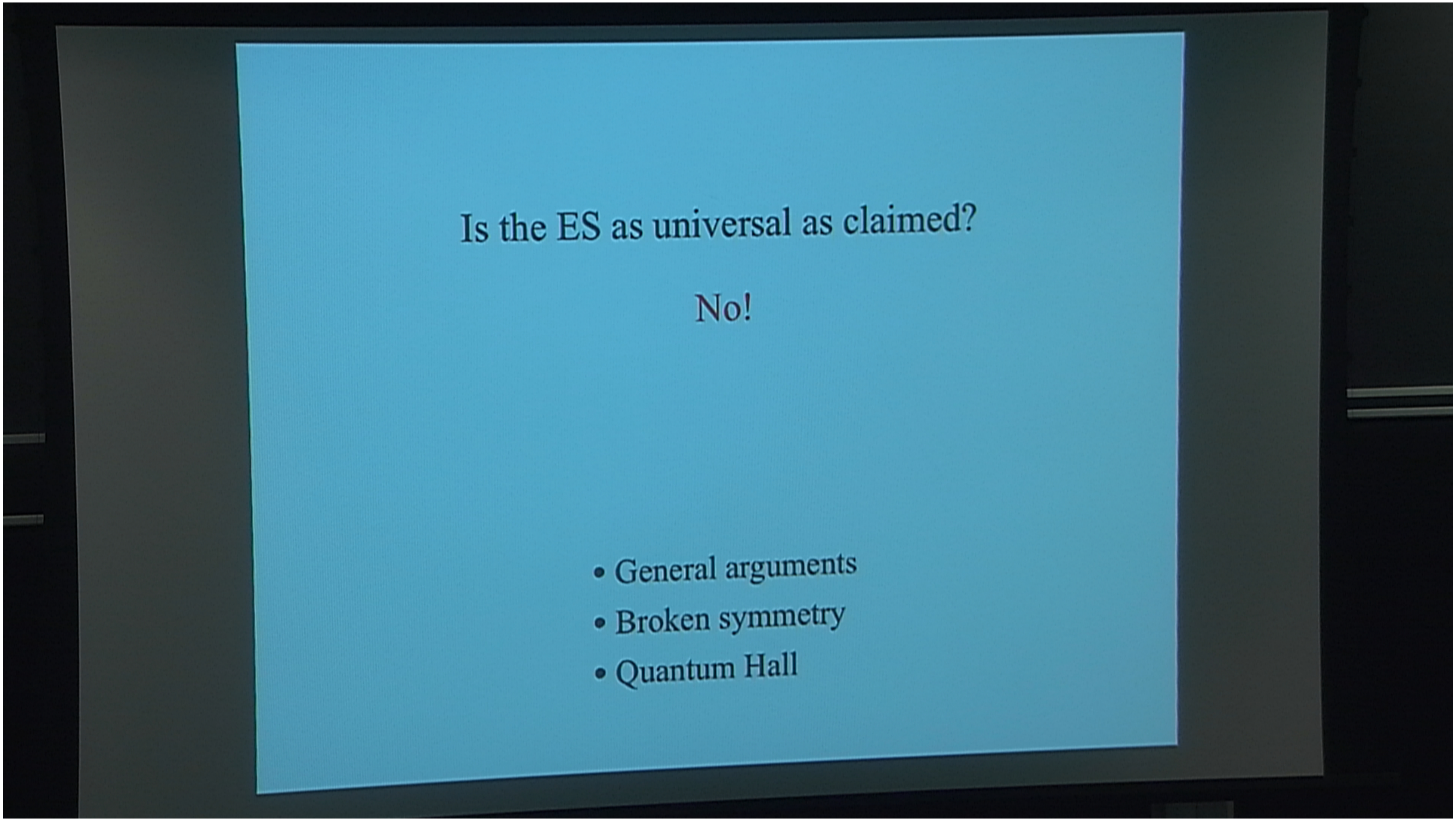


Li & Haldane (2008)


An explosion...
201 citations as of yesterday
Quantum Hall
Fractional Chern insulators
Symmetry breaking
Classification of phases
Disorder
Critical points
Gapless phases
Dimer models
Holography
Spin chains
....


Message
Low energy entanglement spectrum (ES) universal
Fingerprint of phase
Learn about actual excitation spectrum
Beyond entanglement entropy


Is the ES as universal as claimed?
No!
• General arguments
• Broken symmetry
• Quantum Hall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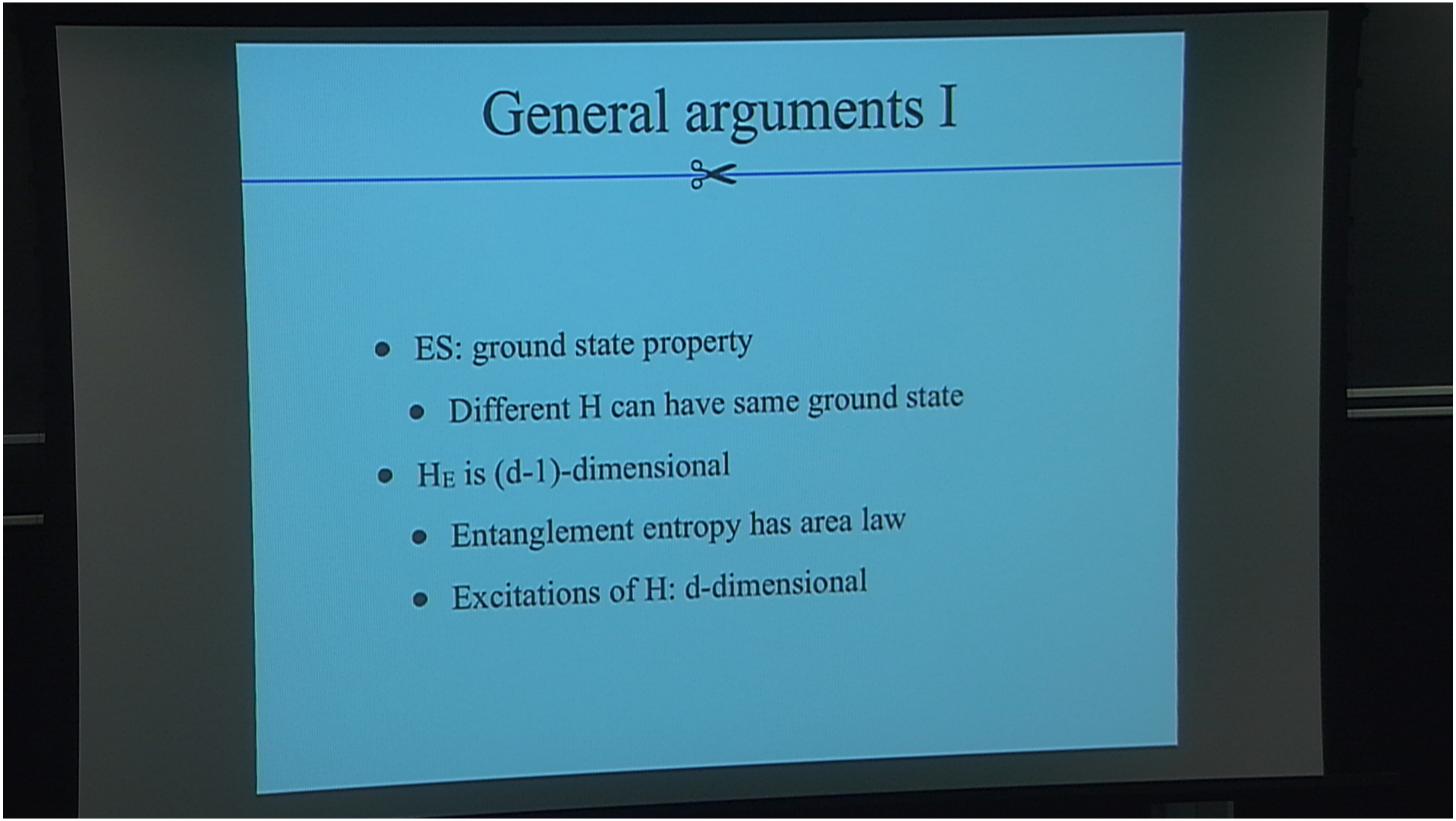
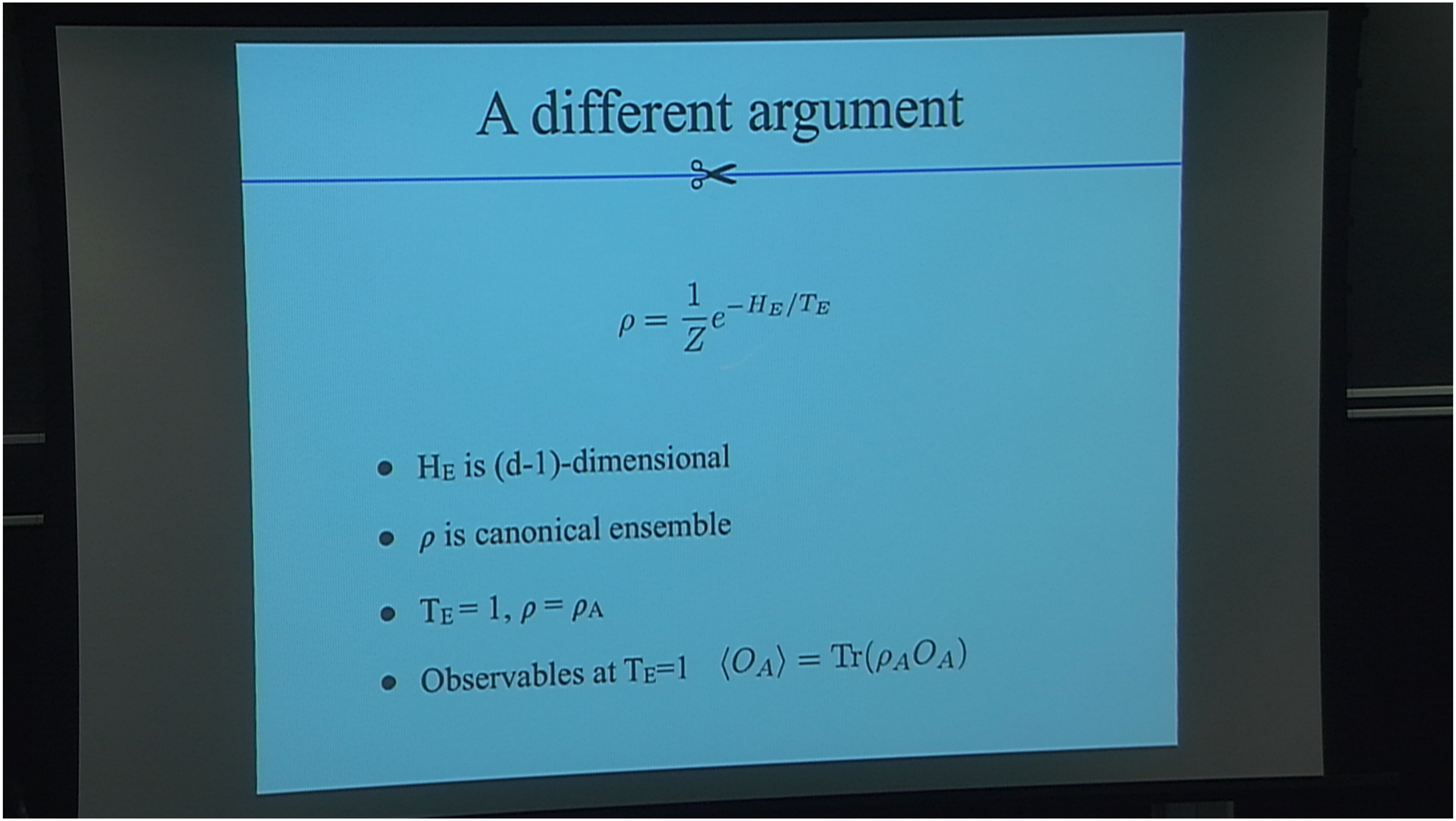

General arguments I
• ES: ground state property
• Different H can have same ground state
• H_E is (d-1)-dimensional
• Entanglement entropy has area law
• Excitations of H: d-dimensional

General arguments I

- ES: ground state property
  - Different H can have same ground state
- H_E is (d-1)-dimensional
  - Entanglement entropy has area law
  - Excitations of H: d-dimensional

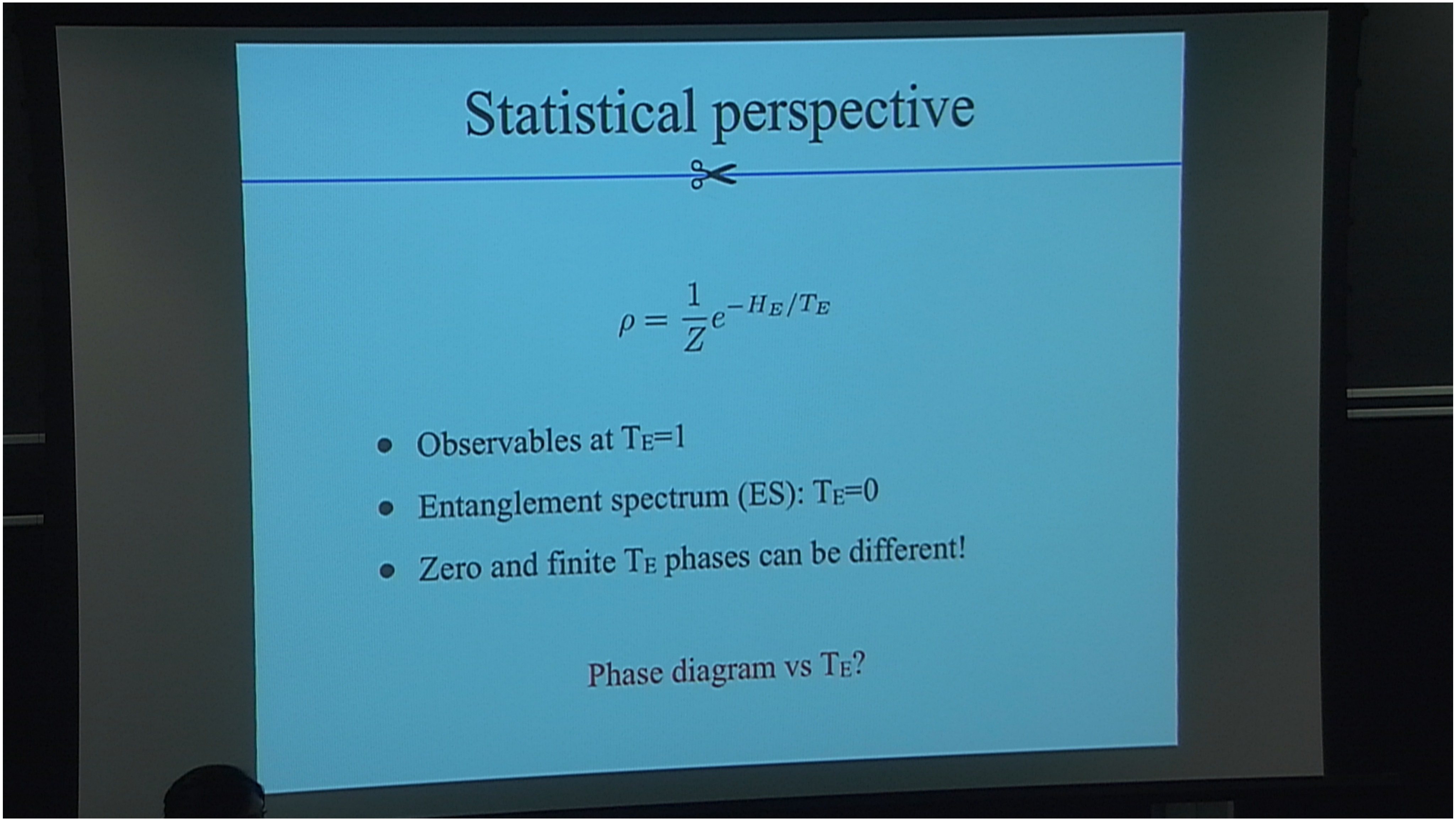

A different argument
$\rho = \frac{1}{Z} e^{-H_E/T_E}$
H_E is (d-1)-dimensional
ρ is canonical ensemble
$T_E = 1$, $\rho = \rho_A$
Observables at T_E=1 $\langle O_A \rangle = \mathrm{Tr}(\rho_A O_A)$

Statistical perspective

$$\rho = \frac{1}{Z} e^{-H_E/T_E}$$

- Observables at $T_E=1$
- Entanglement spectrum (ES): $T_E=0$
- Zero and finite T_E phases can be different!

Phase diagram vs T_E?


Statistical perspective
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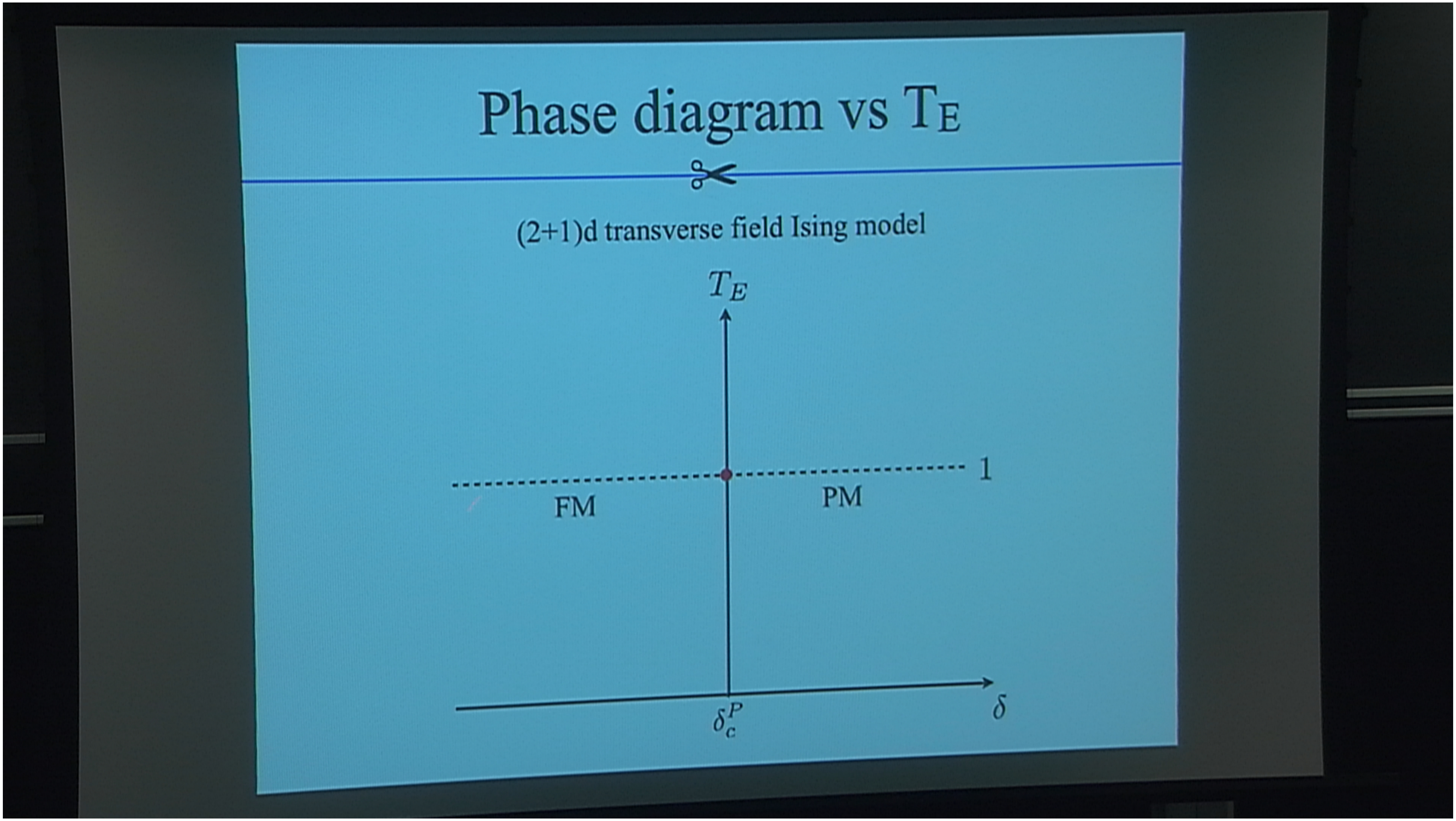
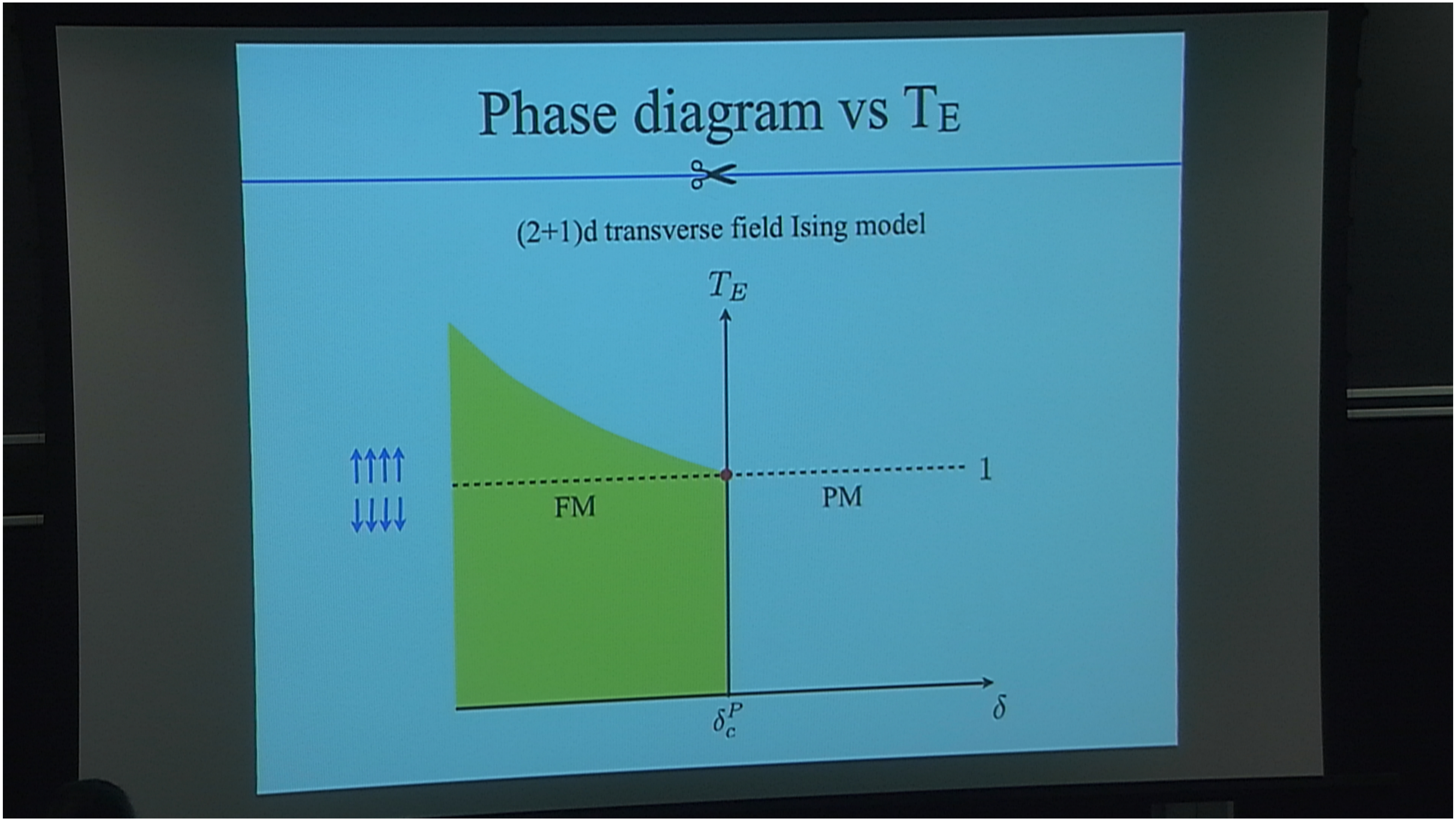
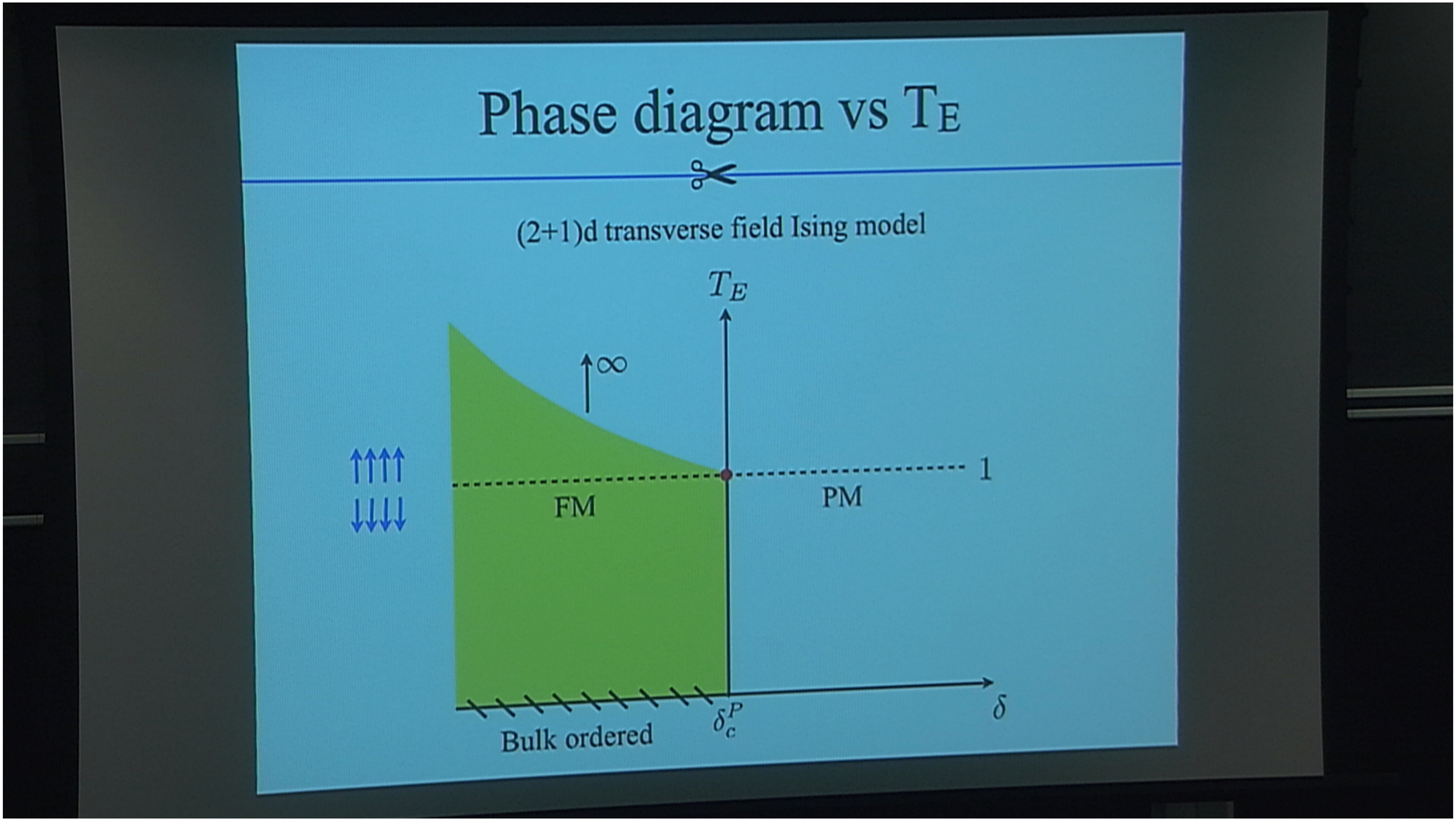
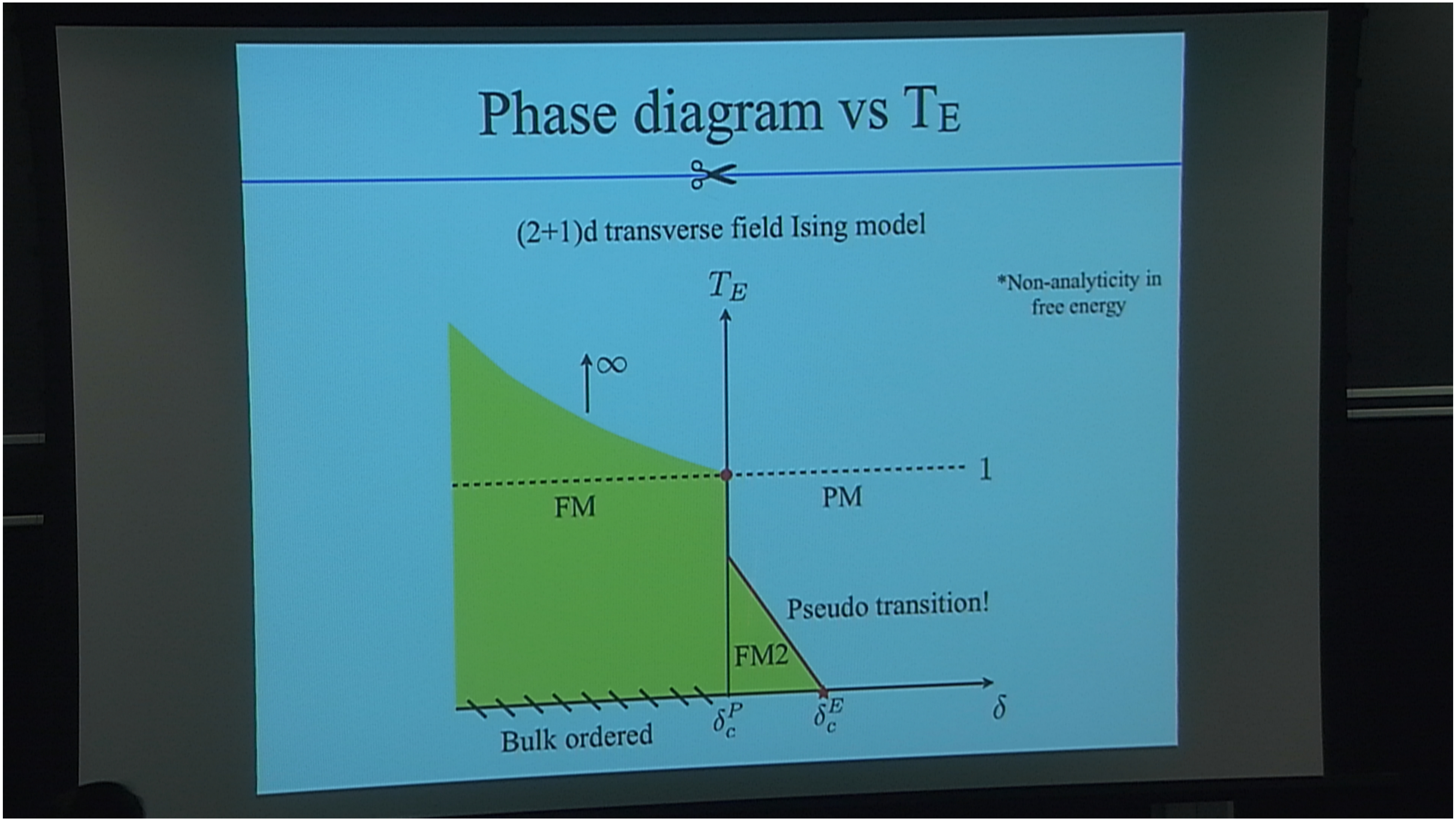
Observables at T_E=1
Entanglement spectrum (ES): T_E=0
Zero and finite T_E phases can be different!
Phase diagram vs T_E?



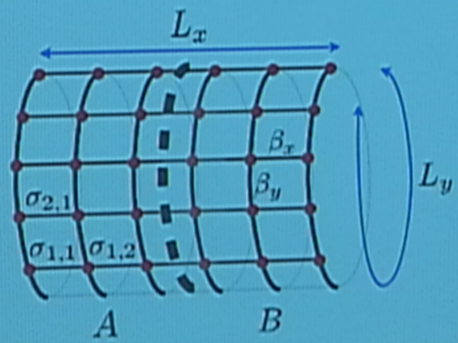
Phase diagram vs T_E
(2+1)d transverse field Ising model
T_E
1
FM
PM
δ_c^P
δ


Phase diagram vs TE
(2+1)d transverse field Ising model
T_E
1
FM
PM
δ_c^P
δ


Phase diagram vs TE
(2+1)d transverse field Ising model
T_E
$\uparrow\infty$
1
FM
PM
Bulk ordered
δ_c^P
δ


Phase diagram vs TE
(2+1)d transverse field Ising model
T_E
*Non-analyticity in free energy
∞
1
FM
PM
Pseudo transition!
FM2
Bulk ordered
δ_c^P
δ_c^E
δ

Rokshar-Kivelson Ising wavefunction

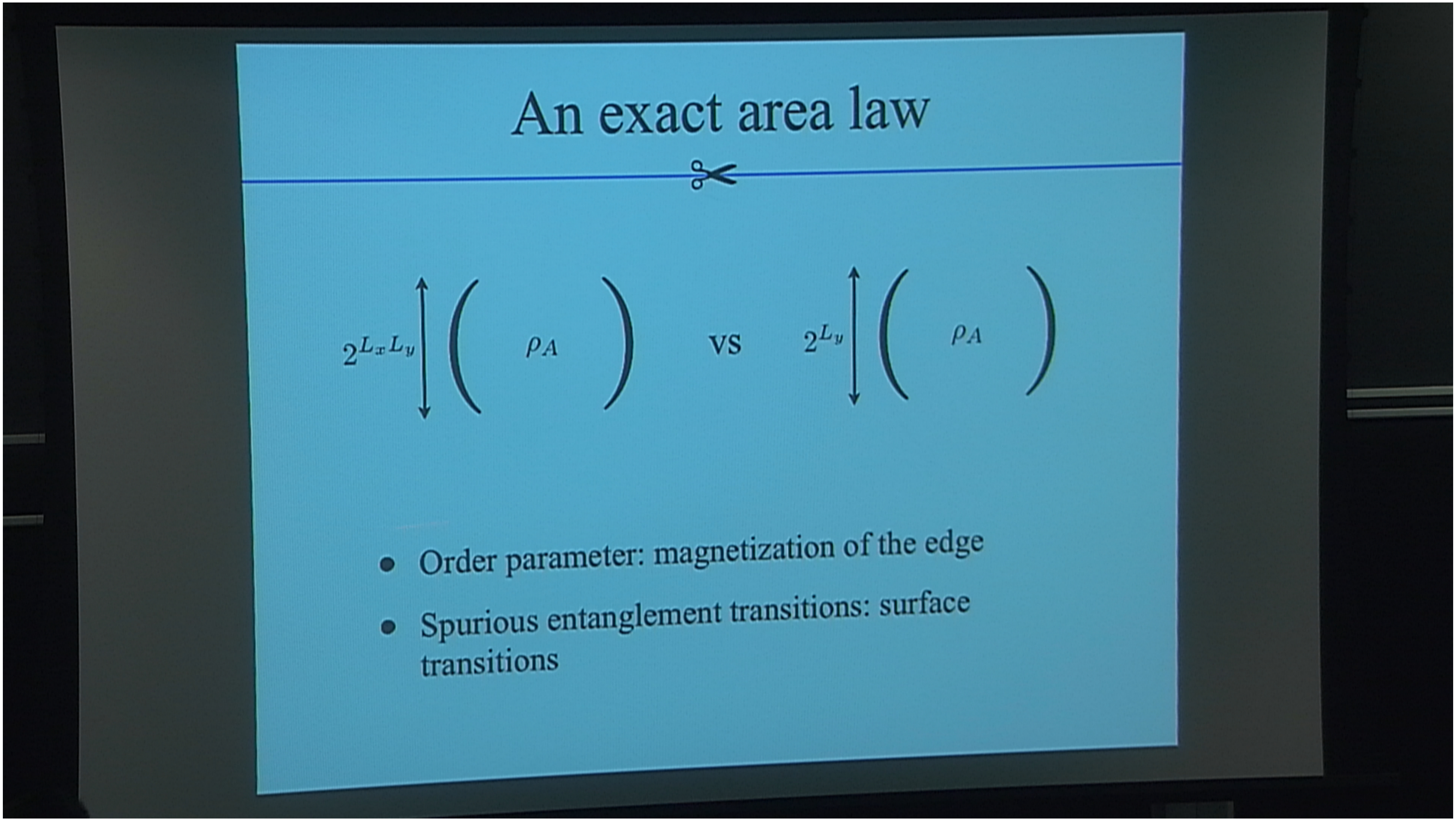


$$|\psi\rangle = \sum_{\sigma} e^{-E_{cl}/2} |\vec{\sigma}\rangle$$

E_{cl}: energy of the classical Ising model

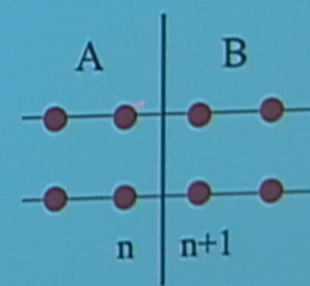
$$E_{cl}(\vec{\sigma}) = -\sum_{i,j} \beta_x \sigma^z_{i,j} \sigma^z_{i,j+1} + \beta_y \sigma^z_{i,j} \sigma^z_{i+1,j}$$

Rokshar, Kivelson (1988), Henley (2004), Castelnovo, Chamon, Mudry, Pujol (2005), Ardonne, Fendley, Fradkin (2004), Stephan, Misguich, Pasquier (2010), Verstraete, Wolf, Perez-Garcia, Cirac (2005, 2011)


An exact area law
$2^{L_x L_y}$ ρ_A vs 2^{L_y} ρ_A
Order parameter: magnetization of the edge
Spurious entanglement transitions: surface transitions

Result ($\beta_x \gg 1$)

- Decoupled chains (PM):



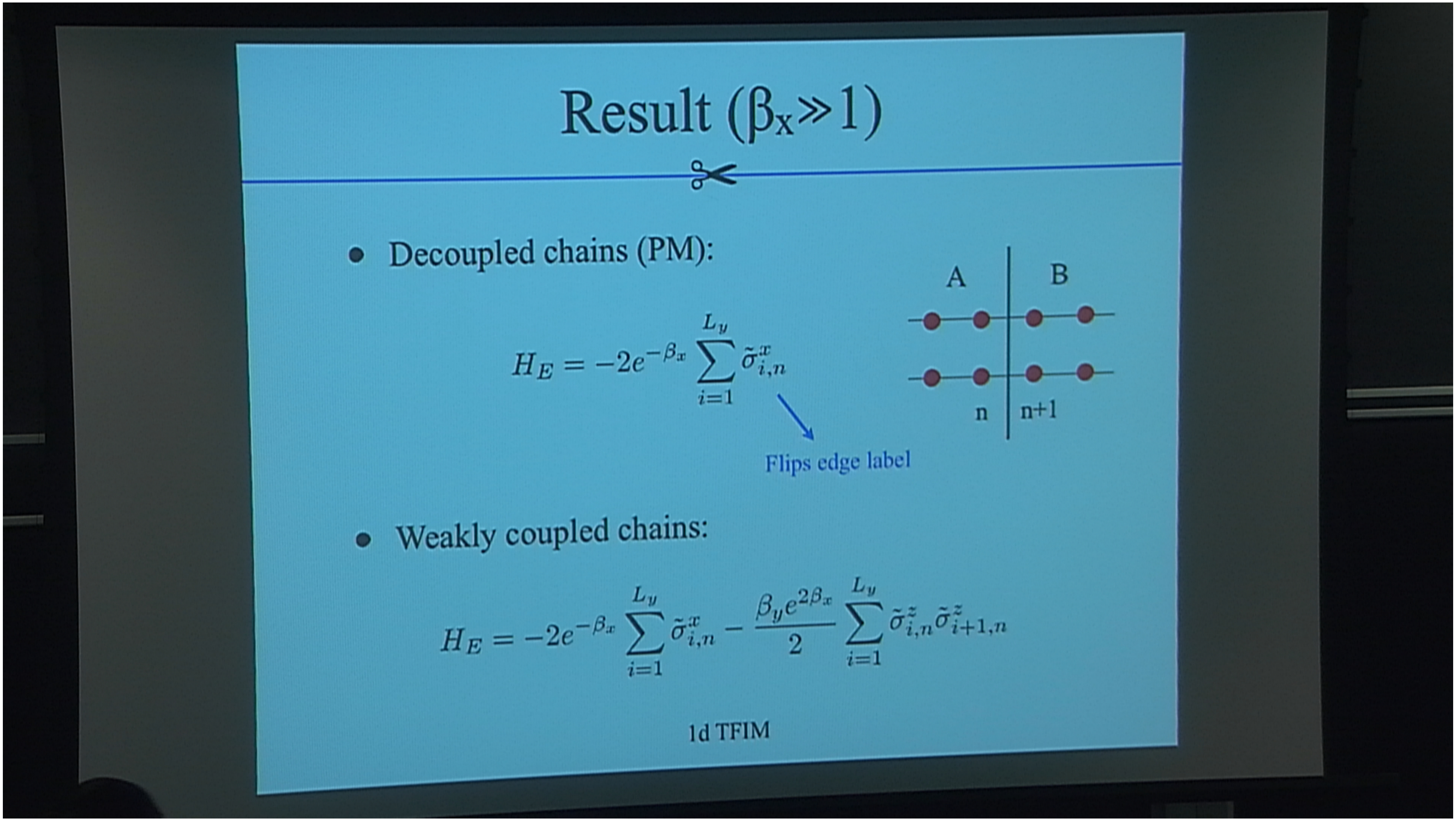
$$H_E = -2e^{-\beta_x} \sum_{i=1}^{L_y} \tilde{\sigma}^x_{i,n}$$

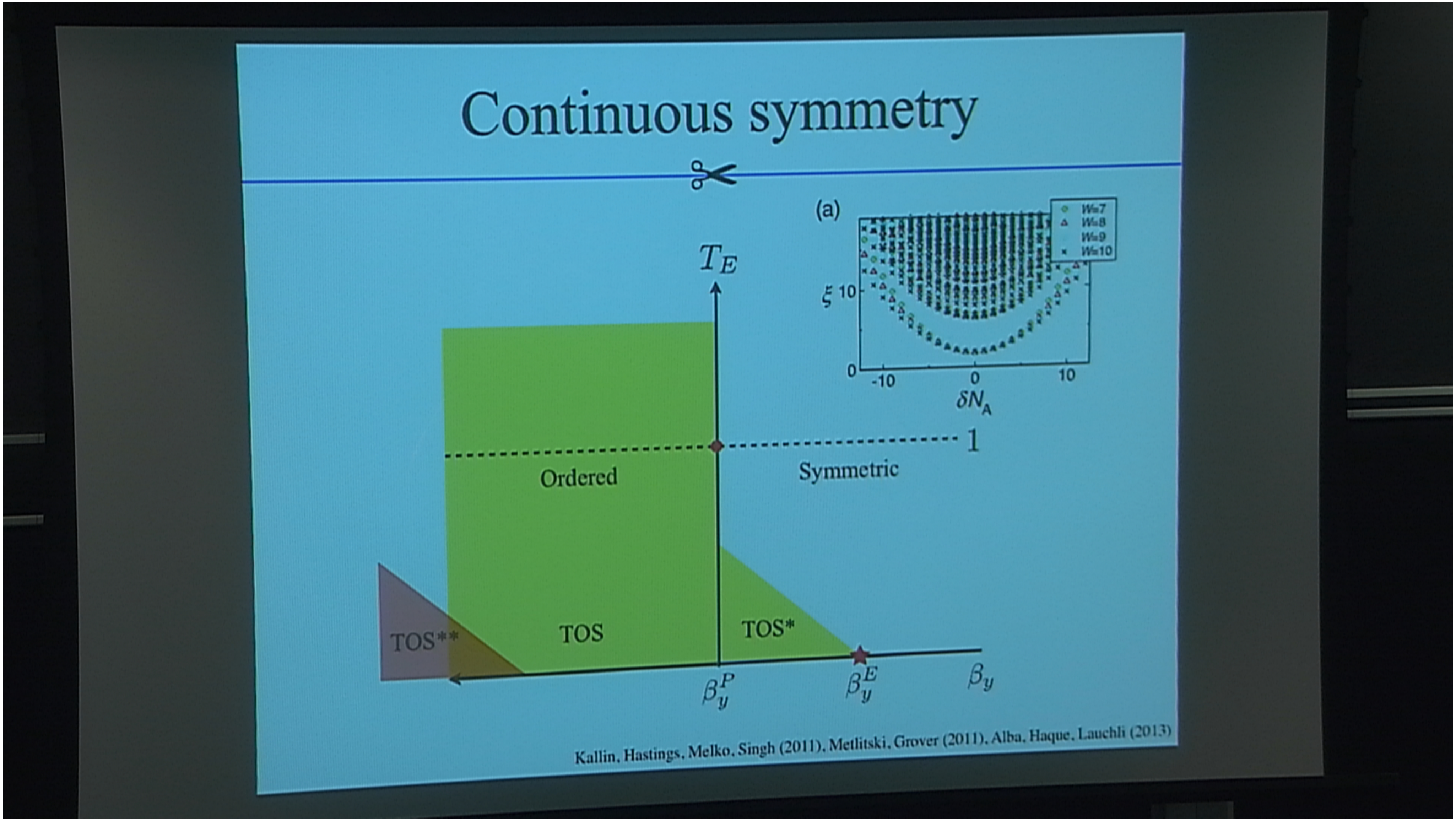
Flips edge label

- Weakly coupled chains:

$$H_E = -2e^{-\beta_x} \sum_{i=1}^{L_y} \tilde{\sigma}^x_{i,n} - \frac{\beta_y e^{2\beta_x}}{2} \sum_{i=1}^{L_y} \tilde{\sigma}^z_{i,n} \tilde{\sigma}^z_{i+1,n}$$

1d TFIM


Result ($\beta_x \gg 1$)
Decoupled chains (PM):
$$H_E = -2e^{-\beta_x} \sum_{i=1}^{L_y} \tilde{\sigma}^x_{i,n}$$
Flips edge label
A
B
n
n+1
Weakly coupled chains:
$$H_E = -2e^{-\beta_x} \sum_{i=1}^{L_y} \tilde{\sigma}^x_{i,n} - \frac{\beta_y e^{2\beta_x}}{2} \sum_{i=1}^{L_y} \tilde{\sigma}^z_{i,n} \tilde{\sigma}^z_{i+1,n}$$
1d TFIM


Continuous symmetry
(a)
W=7
W=8
W=9
W=10
ξ 10
0
-10
0
10
δN_A
T_E
1
Ordered
Symmetric
TOS**
TOS
TOS*
β_y^P
β_y^E
β_y
Kallin, Hastings, Melko, Singh (2011), Metlitski, Grover (2011), Alba, Haque, Lauchli (2013)