

Title: Many-body localization with dipoles

Date: Feb 14, 2014 11:45 AM

URL: <http://pirsa.org/14020129>

Abstract: Statistical mechanics is the framework that connects thermodynamics to the microscopic world. It hinges on the assumption of equilibration; when equilibration fails, so does much of our understanding. In isolated quantum systems, this breakdown is captured by the phenomenon known as many-body localization. This breakdown manifests in a variety of ways, as elucidated by much recent theoretical and numerical work. Many-body localized phases violate Ohm's law and Fourier's law as they conduct neither charge nor heat; they can exhibit symmetry breaking and/or topological orders in dimensions normally forbidden by Mermin-Wagner arguments; they hold potential as strongly interacting quantum computers due to the slow decay of local coherence. In this talk, I will briefly introduce the basic phenomena of many-body localization and review its theoretical status. To date, none of these phenomena has been observed in an experimental system, in part because of the isolation required to avoid thermalization. I will consider several dipolar systems which we believe to be ideal platforms for the realization of MBL phases and for investigating the associated delocalization phase transition. The presence of strong interactions in these systems underlies their potential for exploring physics beyond that of single particle Anderson localization. However, the power law of the dipolar interaction immediately raises the question: can localization in real space persist in the presence of such long-range interactions? I will review and extend several arguments producing criteria for localization in the presence of power laws and present small-scale numerics regarding the MBL transition in several of the proposed dipolar systems. Associated preprint: N. Yao, CRL, S. Gopalakrishnan, M. Knap, M. Mueller, E. Demler., M. Lukin arXiv:1311.7151

Many-body localization with dipoles

Chris Laumann (UW)

Norman Yao, Sarang Gopalakrishnan,
Michael Knap, Markus Müller,
Eugene Demler, Mikhail Lukin

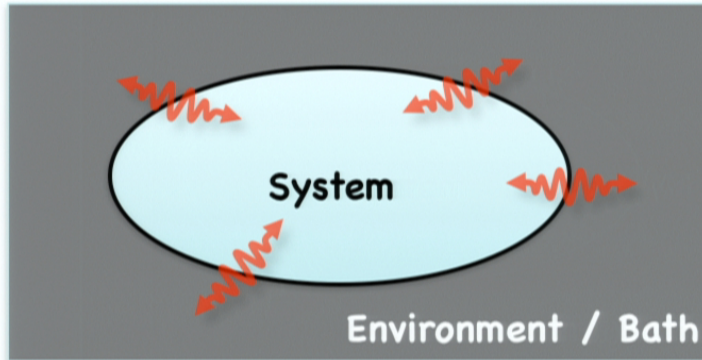


arXiv:1311.7151

Perimeter Institute, "Emergence in Complex Systems" February 14, 2014

Motivation

Statistical mechanics relates thermodynamics to the microscopic world



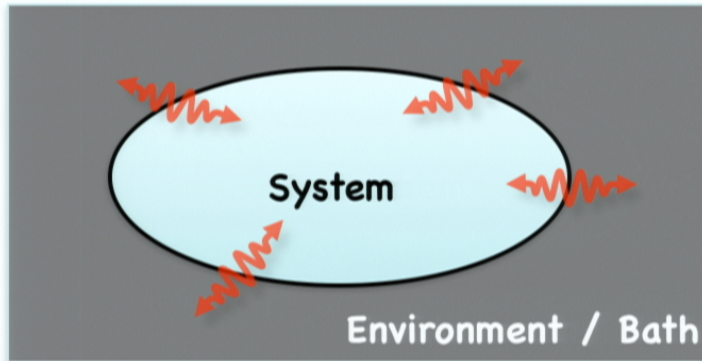
Equilibration requires exchange:

Energy, particles ...

Motivation

Input: DVI - 1920x1080p@60Hz
Output: SDI - 1920x1080i@60Hz

Statistical mechanics relates thermodynamics to the microscopic world

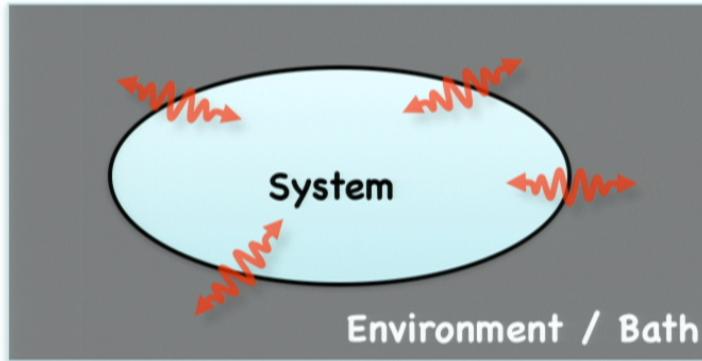


Equilibration requires exchange:

Energy, particles ...

Motivation

Statistical mechanics relates thermodynamics to the microscopic world



Equilibration requires exchange:

Energy, particles ...

What if there is no environment?

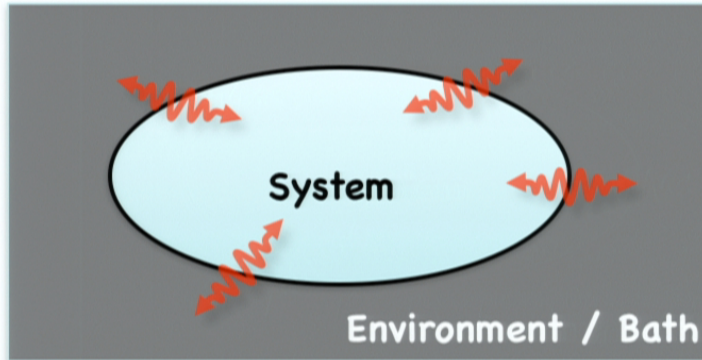


"Ergodic" system behaves as its own heat bath.

Many-body localization breaks ergodicity.

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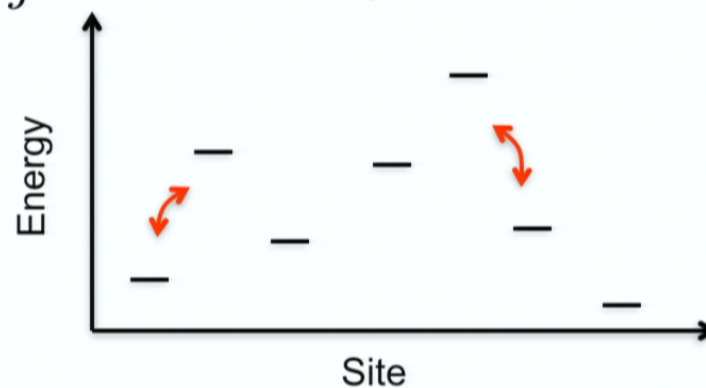


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Single-particle Localization

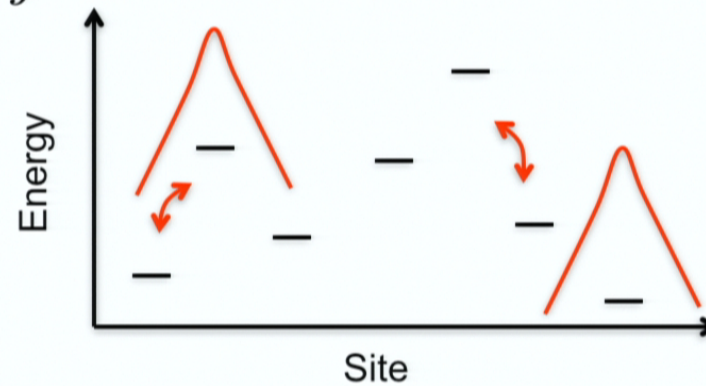
$$H = t \sum_{ij} a_i^\dagger a_j + \sum_i \mu_i n_i \quad \mu_i \in [-W/2, W/2]$$



P. W. Anderson, Phys. Rev. (1958)

Single-particle Localization

$$H = t \sum_{ij} a_i^\dagger a_j + \sum_i \mu_i n_i \quad \mu_i \in [-W/2, W/2]$$



Off-resonant hopping fails to hybridize sites at long-distances:

$$H = \sum_{\alpha} e_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}$$

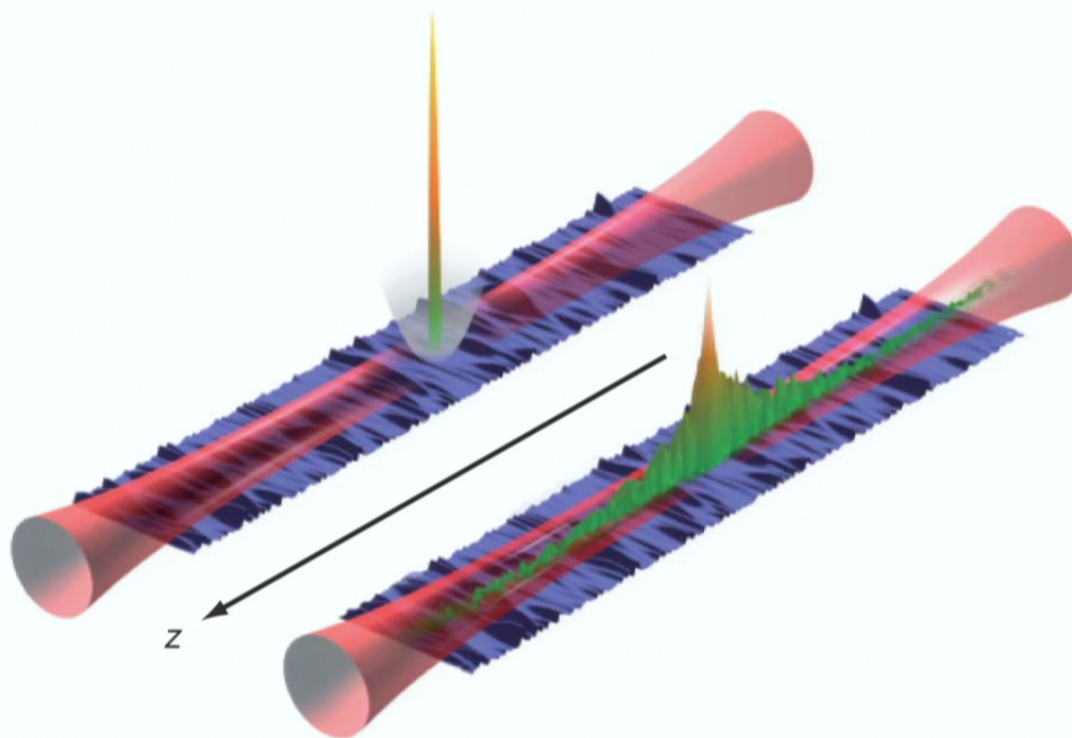
N

Localized

 $|\phi(r)|^2 \sim e^{-r/\xi}$

P. W. Anderson, Phys. Rev. (1958)

Observation of Localization

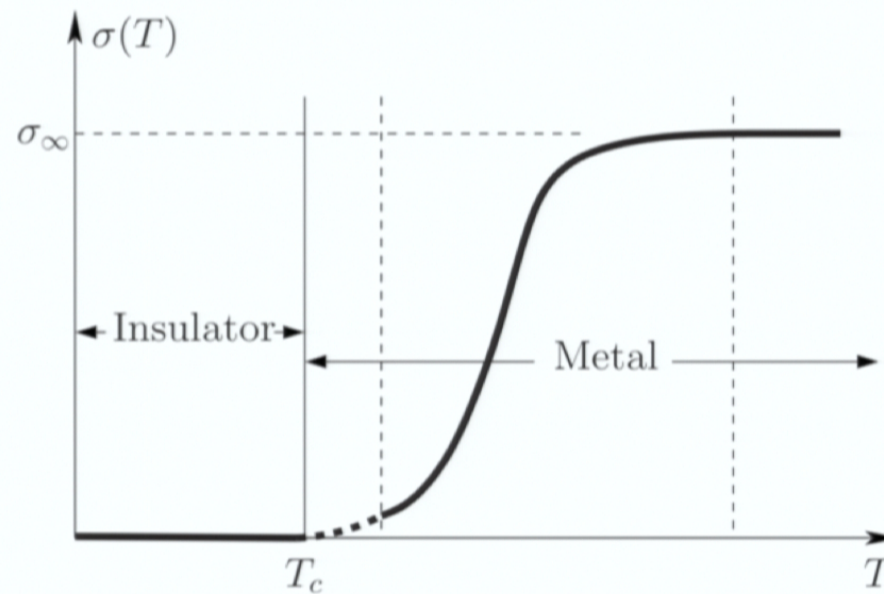


Anderson Localization observed in: Light waves, microwaves, sound waves, electron gases, matter waves in 1D (shown), ultracold fermions in 3D

Aspect Group -- Nature 453, 891-894 (2008)

Localization with interactions?

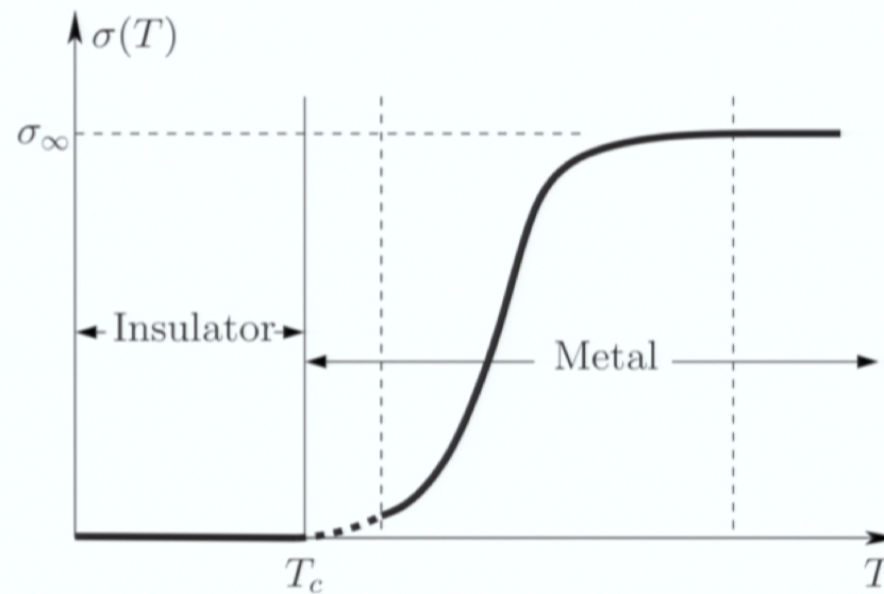
Can system remain insulating with extensive energy?



Basko, Aleiner, Altshuler, *Annals of Physics* (2006), Huse and Oganessian (2007), Pal and Huse (2009)

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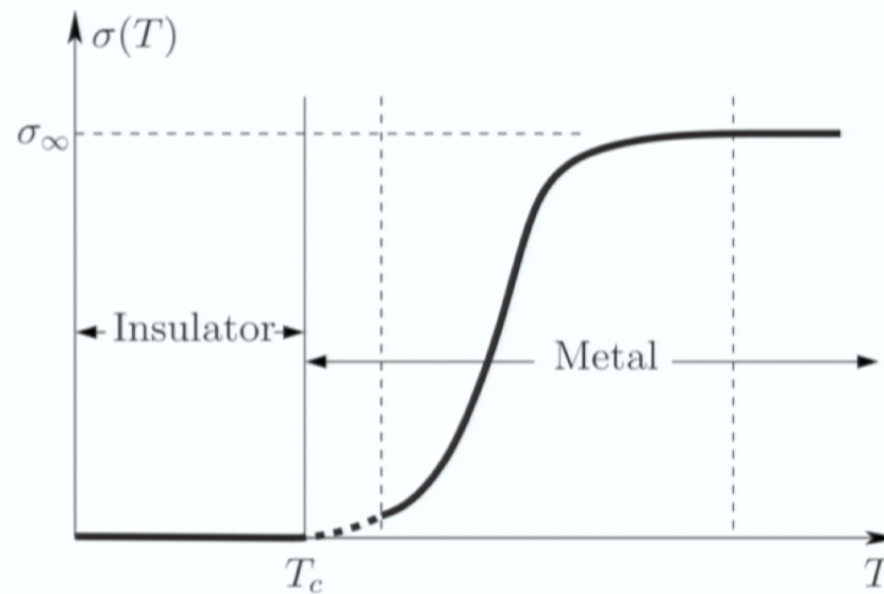
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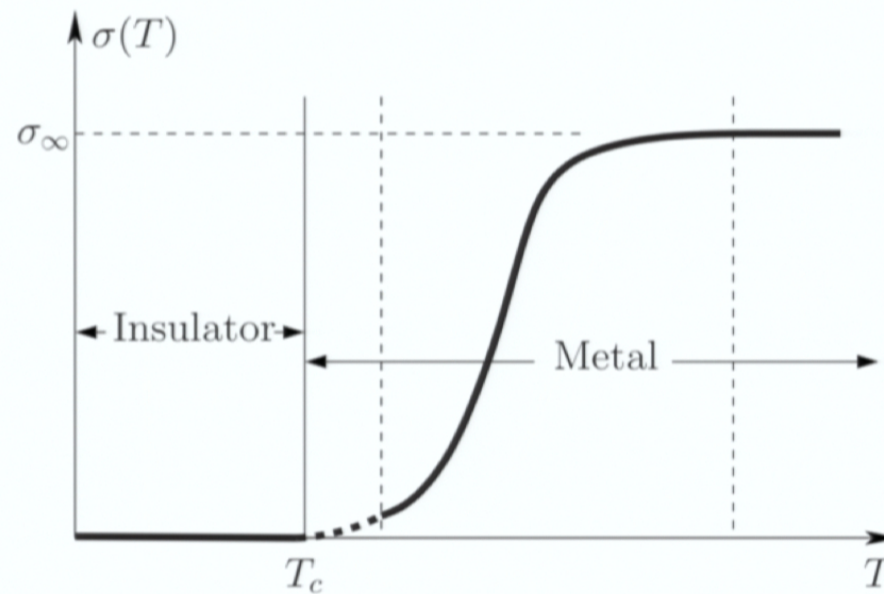
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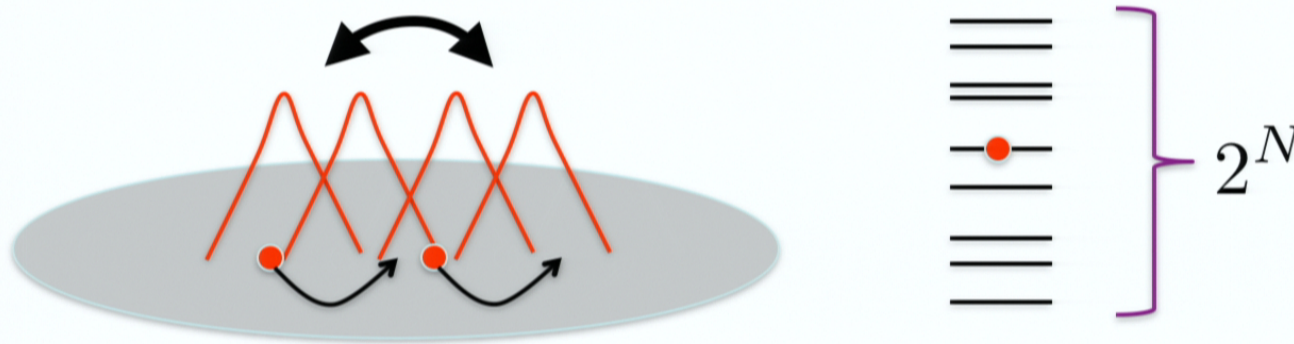
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Localization in Fock space

Start with single particle localized states and add in interactions:

$$H = \sum_{\alpha} e_{\alpha} a_{\alpha}^{\dagger} a_{\alpha} + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\gamma} a_{\delta}$$

Can weak V cause hybridization of localized many-particle states?



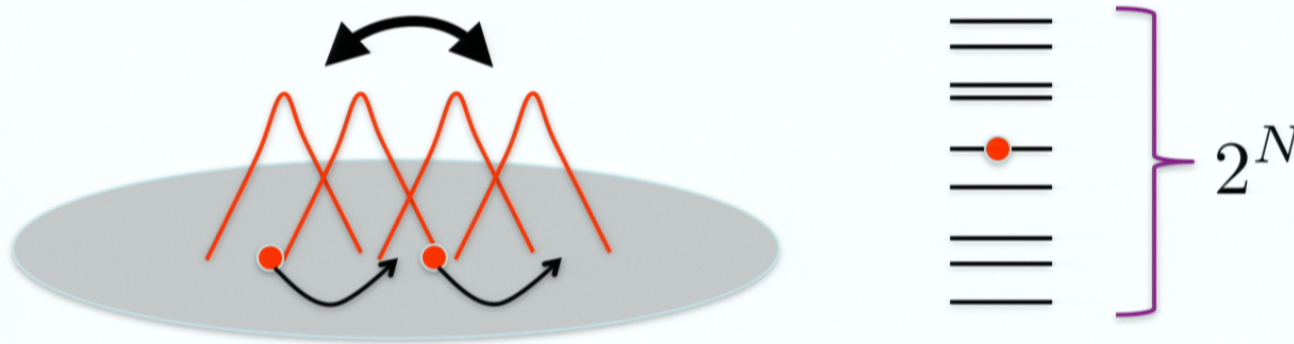
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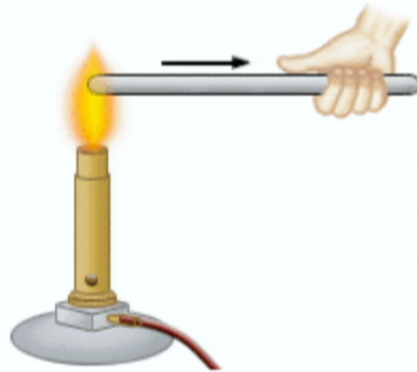
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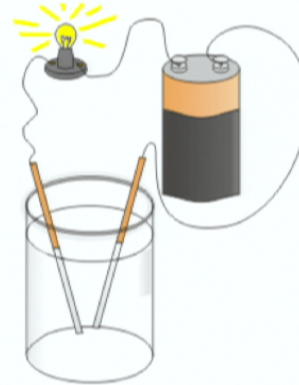
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A few consequences...

1) Breakdown of equilibrium transport phenomena

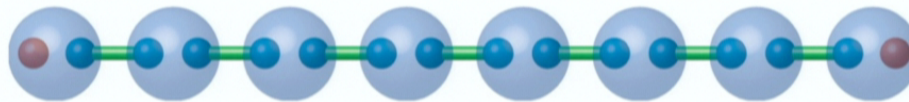


Fourier's Law



Ohm's Law

2) May protect order not allowed in equilibrium by freezing defects



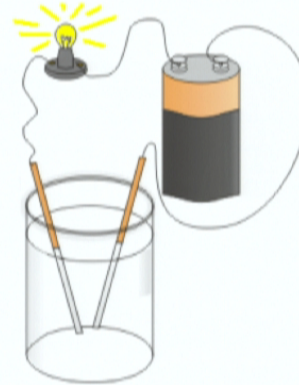
David A. Huse, Rahul Nandkishore, Vadim Oganesyan, Arijeet Pal, S.L.Sondhi, arXiv:1304.1158

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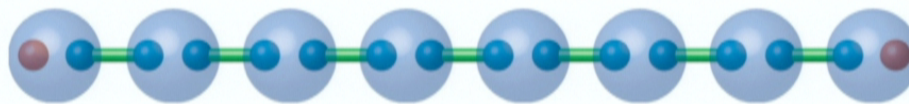


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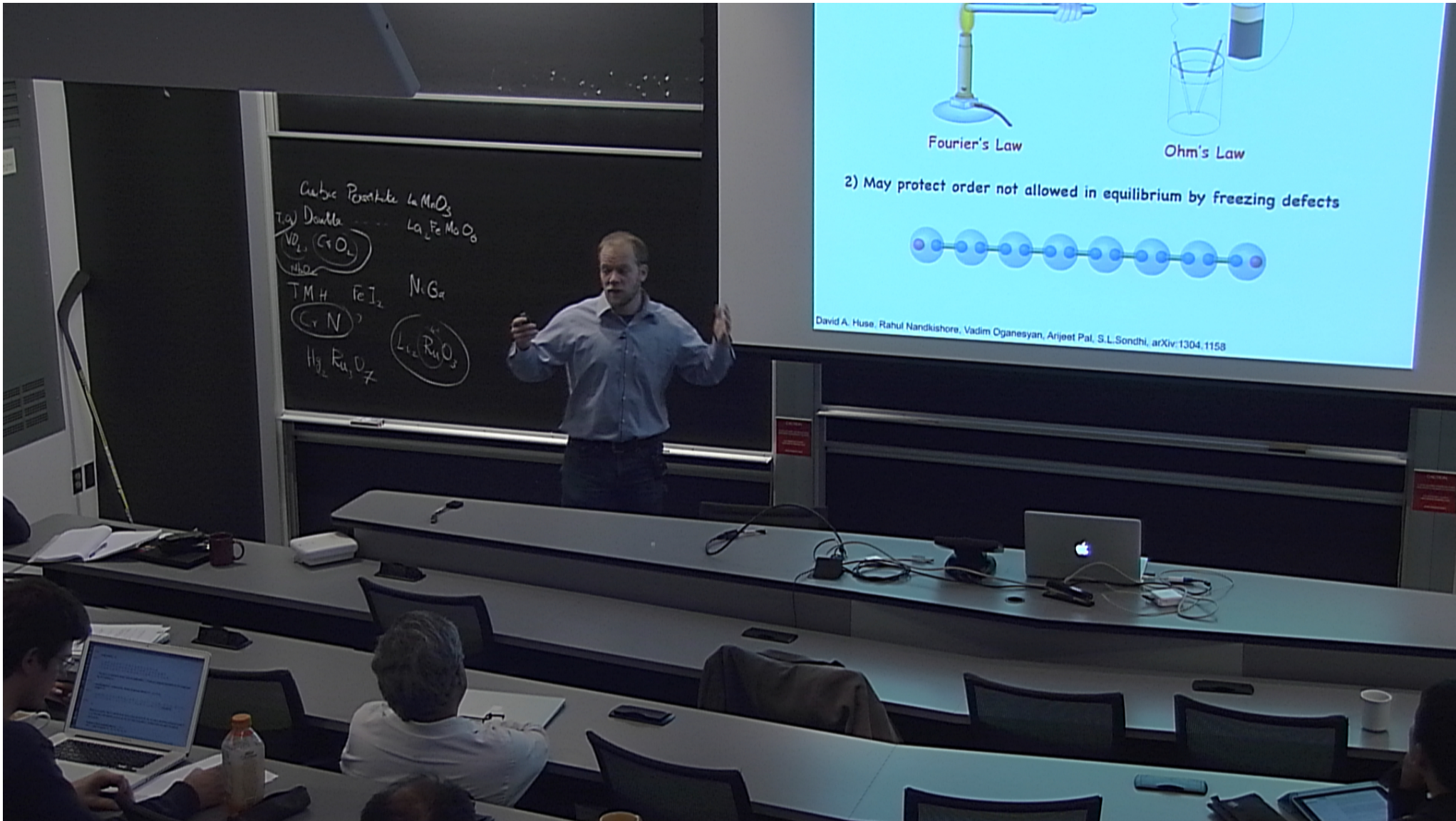


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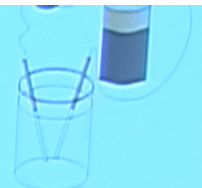
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Carbonate Perovskite LaMnO_3
TMH Double $\text{La}_2\text{FeMoO}_6$
 VO_2 CrO_2
 H_2O
TMH FeI_2 N.G.
 CrN
 H_2 Ru_3O_7
 Li_2RuO_3



2) May protect order not allowed in equilibrium by freezing defects



David A. Huse, Rahul Nandkishore, Vadim Oganesyan, Arijeet Pal, S.L.Sondhi, arXiv:1304.1158

SPT at $E > 0$

EG. Haldane phase in $d=1$



Ground state:

- (i) Long-range string order
- (ii) Two-state boundary modes
- (iii) Entanglement spectrum degeneracy

$$\sigma_{ij}^{\alpha} = -S_i^{\alpha} \left(\prod_{k=i+1}^{j-1} R_k^{\alpha} \right) S_j^{\alpha}$$

$\alpha \in x, y, z$

A. Chandran, V. Khemani, CRL, S.L.Sondhi arXiv:1310.1096.

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Disorder + small E

Localize 'triplon' defects

- (i) String 'glass' order
- (ii) Two-state boundary modes
- (iii) Entanglement spectrum degeneracy

A. Chandran, V. Khemani, CRL, S.L.Sondhi arXiv:1310.1096.

SPT at $E > 0$

EG. Haldane phase, protected by $D2 = Z2 \times Z2$



Frustration-free models

$$H_{AKLT} = \sum_{i,\alpha} J_i P_{i,i+1}^{(2)}$$

SO(3) symmetry

SPT at $E > 0$

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Frustration-free models

$$H_{AKLT} = \sum_{i,\alpha} J_i P_{i,i+1}^{(2)} \quad \text{SO(3) symmetry}$$

Defects have spin-1 degeneracy – BAA does not apply
unknown whether this can localize at finite energy density

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$$H_{BKLT} = \sum_{i,\alpha} P_{i,i+1}^{(2)} \left(J_i + c_i^\alpha (S_i^\alpha + S_{i+1}^\alpha)^2 + d_i^\alpha (S_i^\alpha + S_{i+1}^\alpha)^4 \right) P_{i,i+1}^{(2)}$$

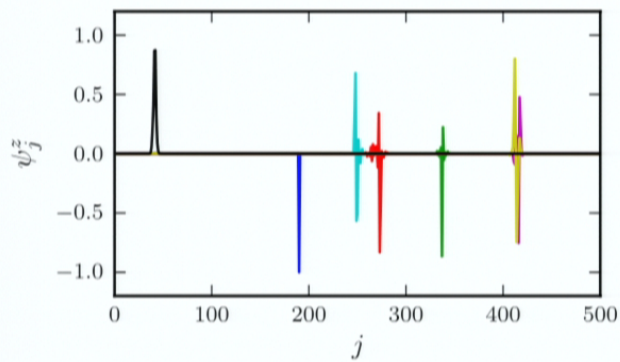
D = Z2xZ2 symmetry

SPT at $E > 0$

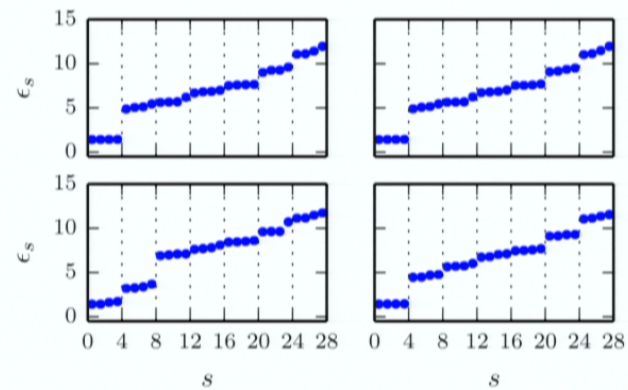
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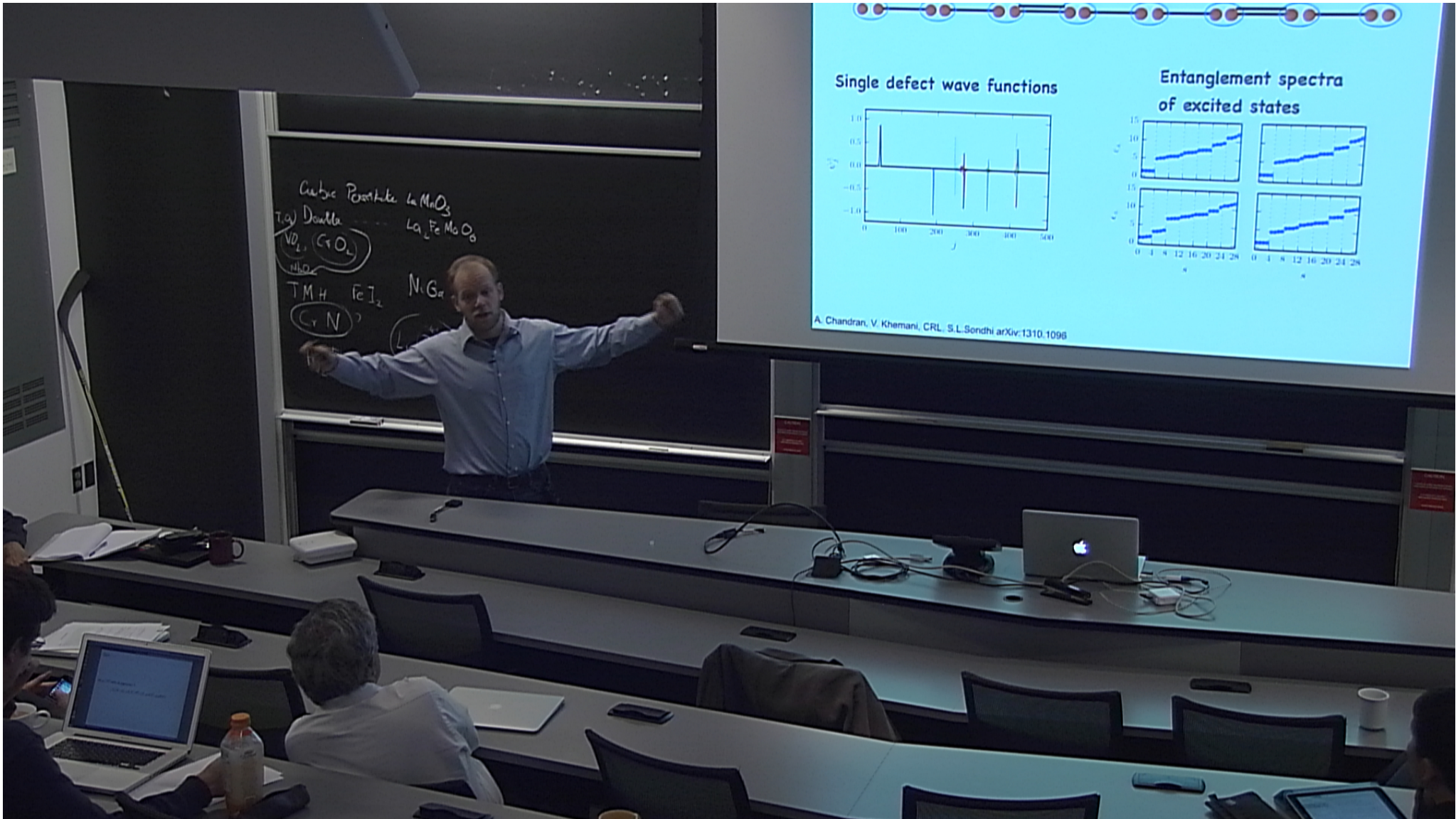
Single defect wave functions



Entanglement spectra of excited states



A. Chandran, V. Khemani, CRL, S.L.Sondhi arXiv:1310.1096

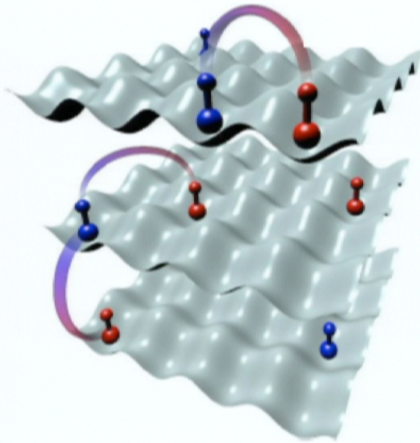


Quantum Optical Systems

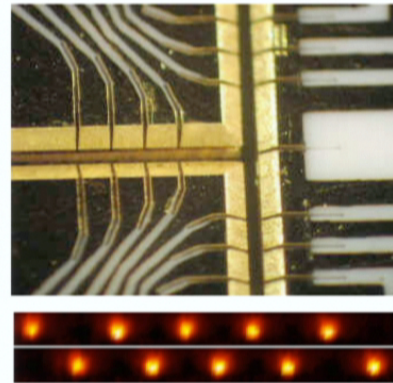
Polar Molecules

Trapped ions

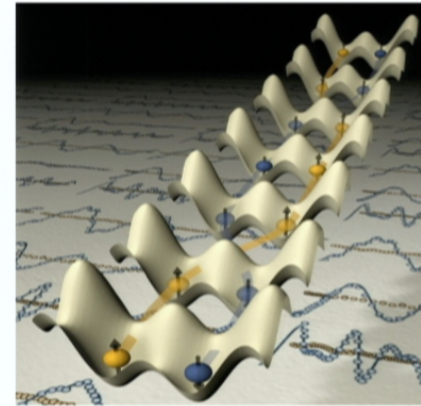
Cold Atoms



JILA group



JQI group



MPQ group

Isolated systems: No thermal bath!

Probes: Optical spectroscopy (observe localization in real space)!

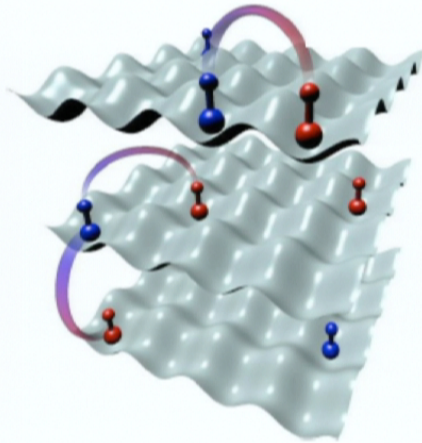
Challenge: Interactions typically long-range (dipolar, van der Waals)

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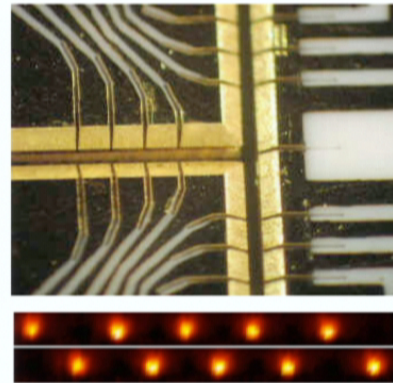
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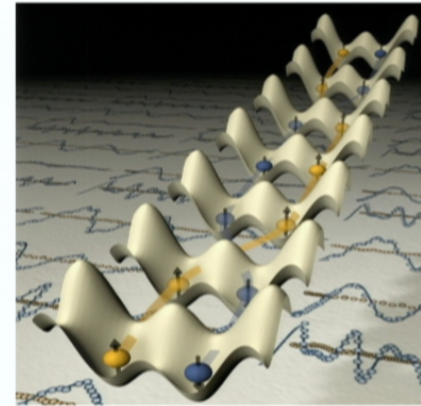
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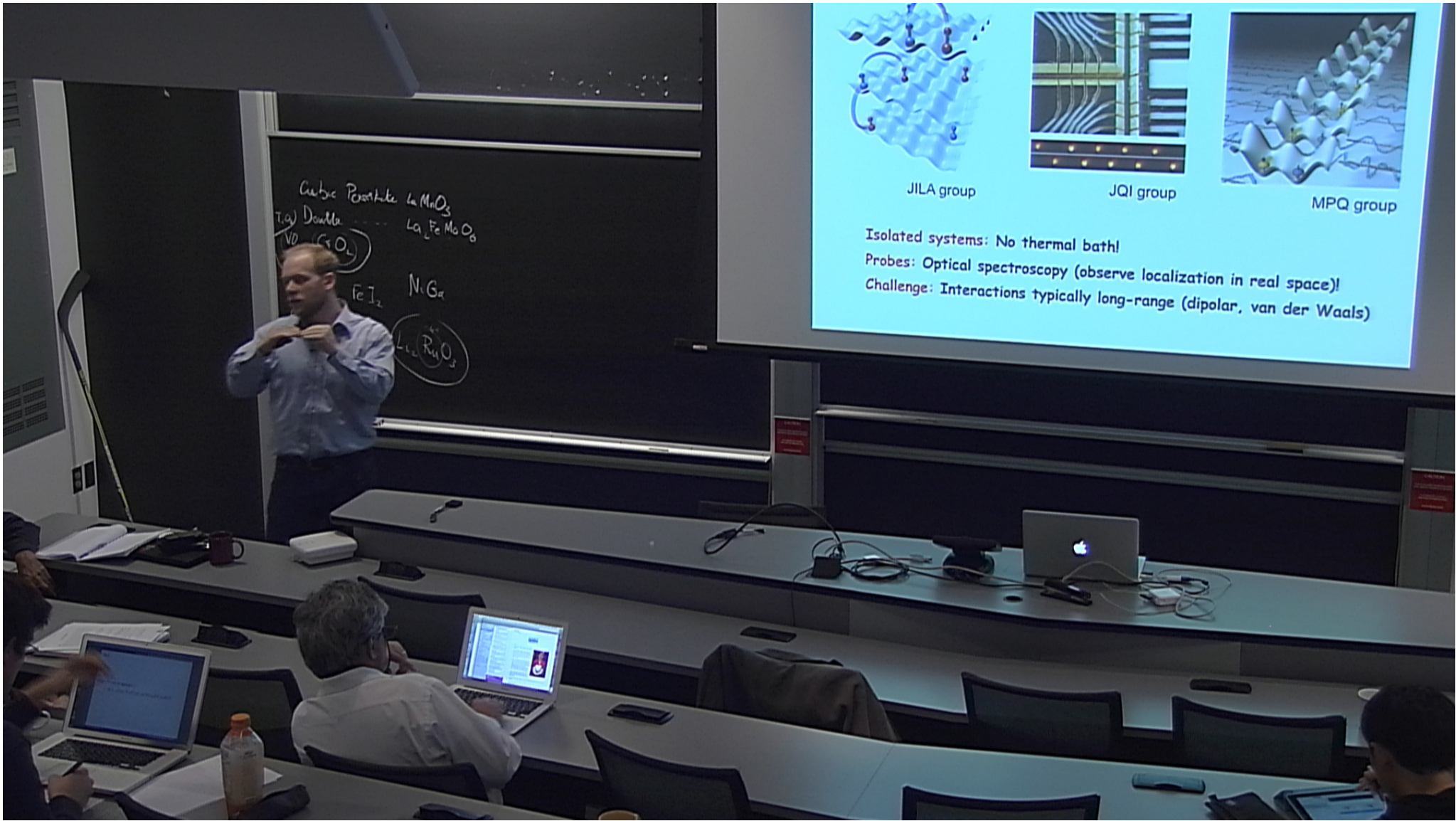


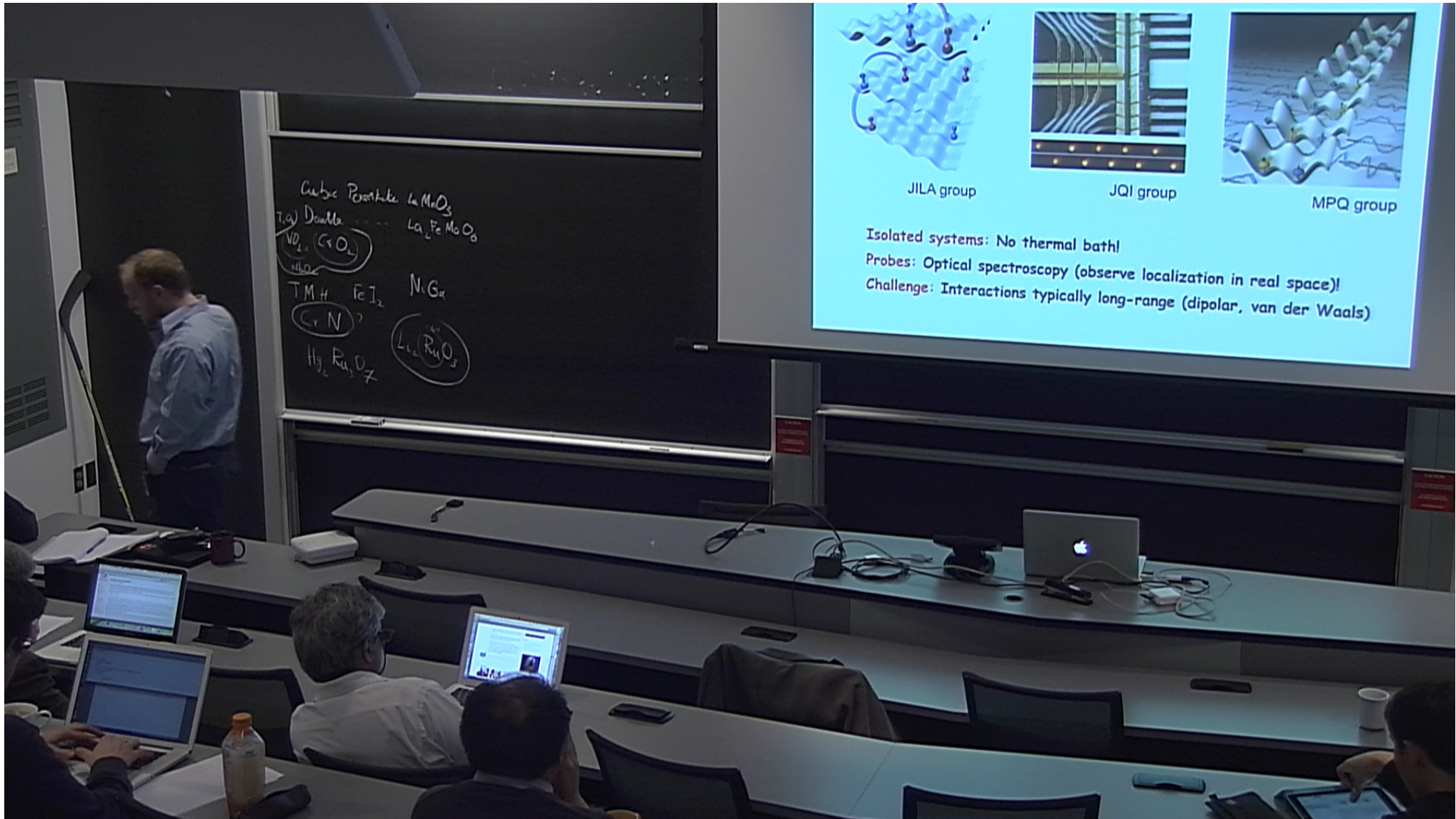
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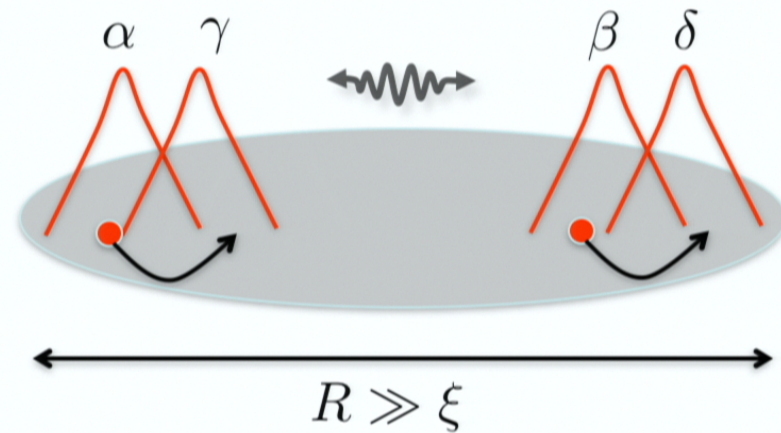
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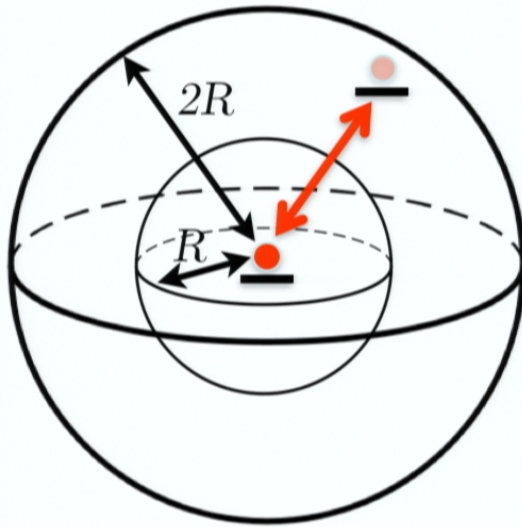
Can Localization Persist II ?

$$H = t \sum_{ij} a_i^\dagger a_j + \sum_i \mu_i n_i + \sum_{ij} \frac{V_{ij}}{R_{ij}^\beta} n_i n_j$$



Localization with Long-range Hopping

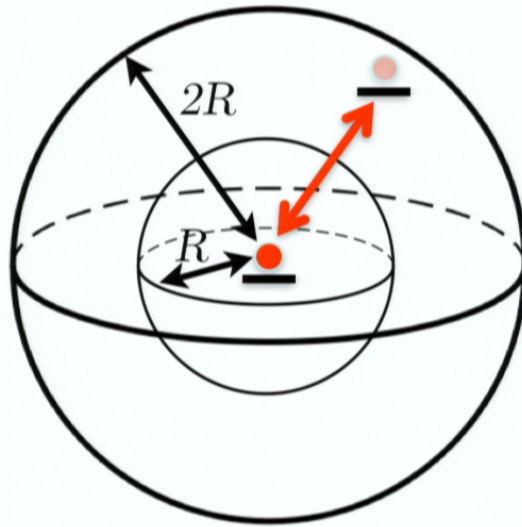
$$H = \sum_i \epsilon_i n_i - \sum_{ij} \frac{t_{ij}}{R_{ij}^\alpha} a_i^\dagger a_j$$



P. W. Anderson, Phys. Rev. (1958)

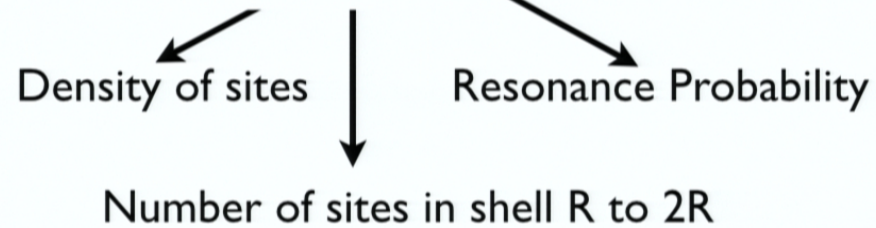
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Density of resonant pairs of sites at scale R :

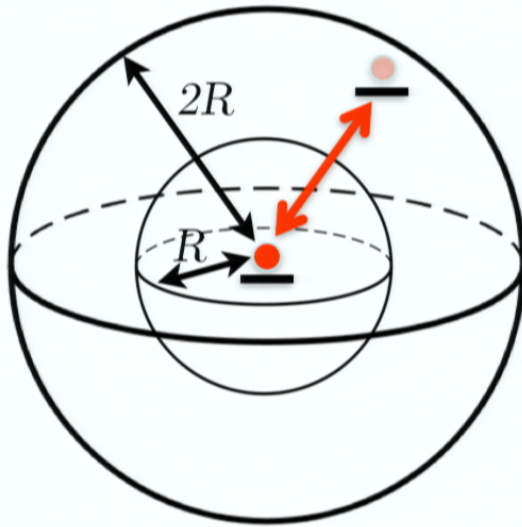
$$N(R) \sim (\rho R^d) \cdot \frac{t/R^\alpha}{W} \sim R^{d-\alpha}$$



P. W. Anderson, Phys. Rev. (1958)

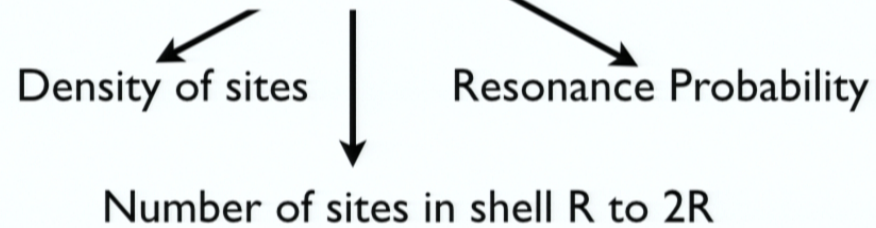
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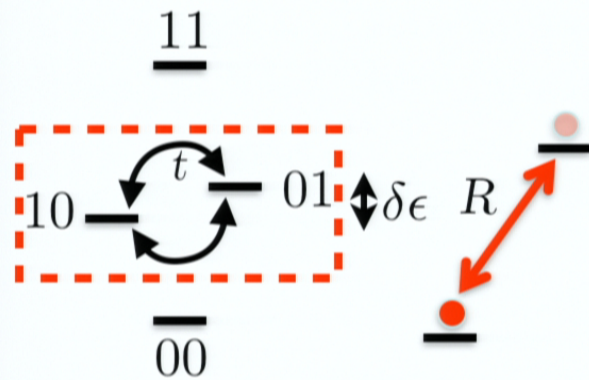
For $d \geq \alpha$, $N(R)$ diverges as $R \rightarrow \infty$

No localization!

P. W. Anderson, Phys. Rev. (1958)

Long-range Interactions

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Resonant pair at scale R

A. Burin, cond-mat/0611387, 0707.2596

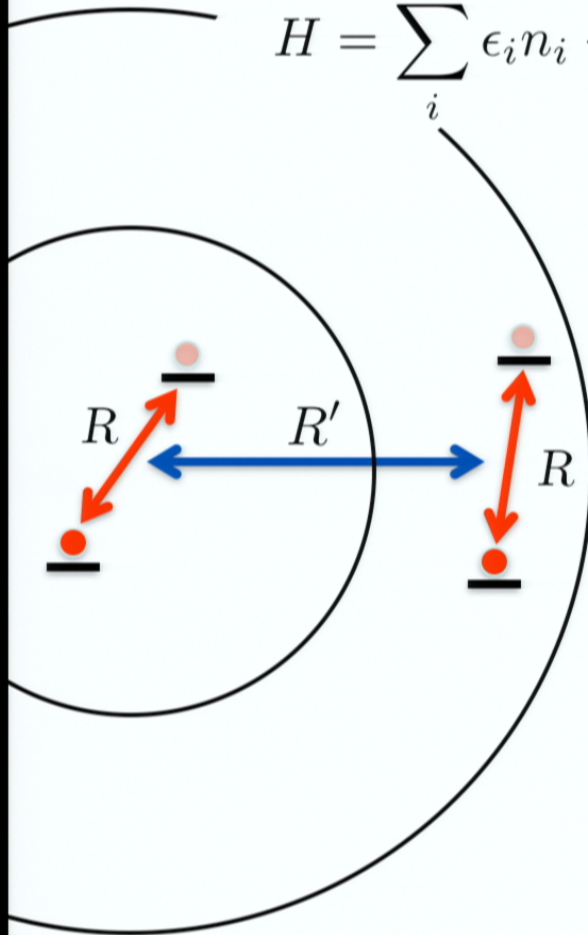
Pairs of pairs

$$H = \sum_i \epsilon_i n_i - \sum_{ij} \frac{t_{ij}}{R_{ij}^\alpha} a_i^\dagger a_j + \sum_{ij} \frac{V_{ij}}{R_{ij}^\beta} n_i n_j$$

Number of resonant pairs of pairs:

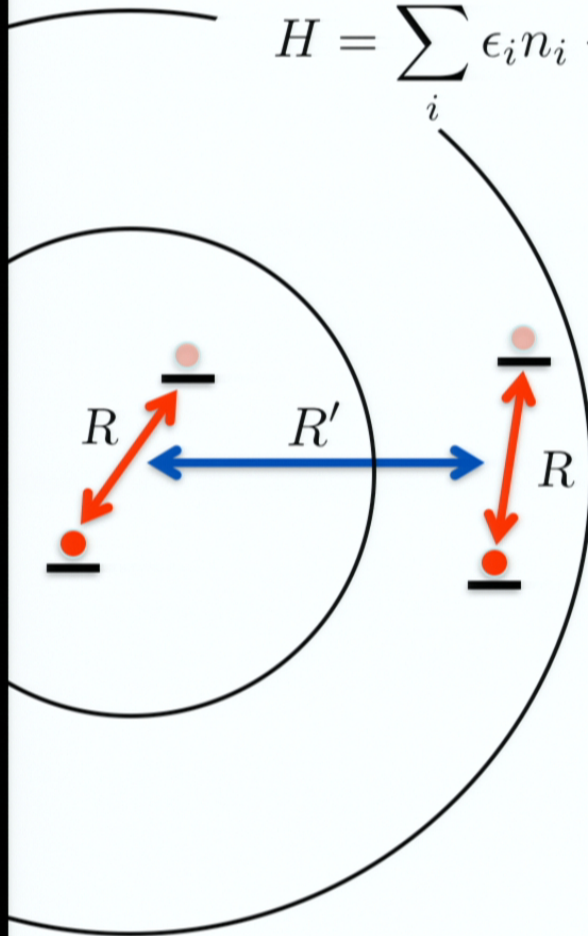
$$N_2(R, R') \sim (\rho N(R) R'^d) \cdot \frac{V/R'^\beta}{t/R^\alpha}$$

If number diverges with R' , delocalizes energy.



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Two useful limits:

$$R \text{ fixed, } R' \rightarrow \infty$$

$$R, R' \rightarrow \infty \text{ with } \frac{V/R'^\beta}{t/R^\alpha} \text{ fixed}$$

Isotropy

$$H = \sum_i \epsilon_i n_i - \sum_{ij} \frac{t_{ij}}{R_{ij}^\alpha} a_i^\dagger a_j + \sum_{ij} \frac{V_{ij}}{R_{ij}^\beta} n_i n_j$$

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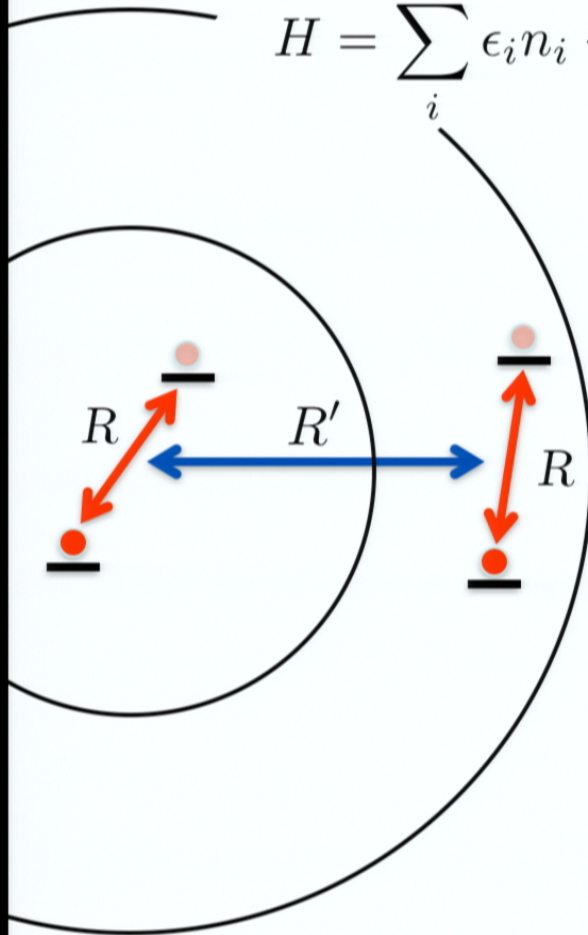
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Multipole expansion for isotropic interactions

$$“\beta \rightarrow \beta + 2”$$

More precisely: $\frac{V}{R'^\beta} \rightarrow V \frac{R^2}{R'^{\beta+2}}$



Mixed Power Law Delocalization

$$H = \sum_i \epsilon_i n_i - \sum_{ij} \frac{t_{ij}}{R_{ij}^\alpha} a_i^\dagger a_j + \sum_{ij} \frac{V_{ij}}{R_{ij}^\beta} n_i n_j$$

TABLE I. Critical dimensions for MBL with power laws

	Unmixed $\alpha = \beta$ [6]	Anisotropic $\beta < \alpha$	Isotropic $\beta < \alpha$
Hopping	$d < \alpha$	$d < \alpha$	$d < \alpha$
Small Pairs	$d < \beta$	$d < \beta$	$d < \beta + 2$
Extended Pairs	$d < \beta/2$	$d < \frac{\alpha\beta}{\alpha+\beta}$	$d < \frac{\alpha(\beta+2)}{\alpha+\beta+4}$

Mixed Power Law Delocalization

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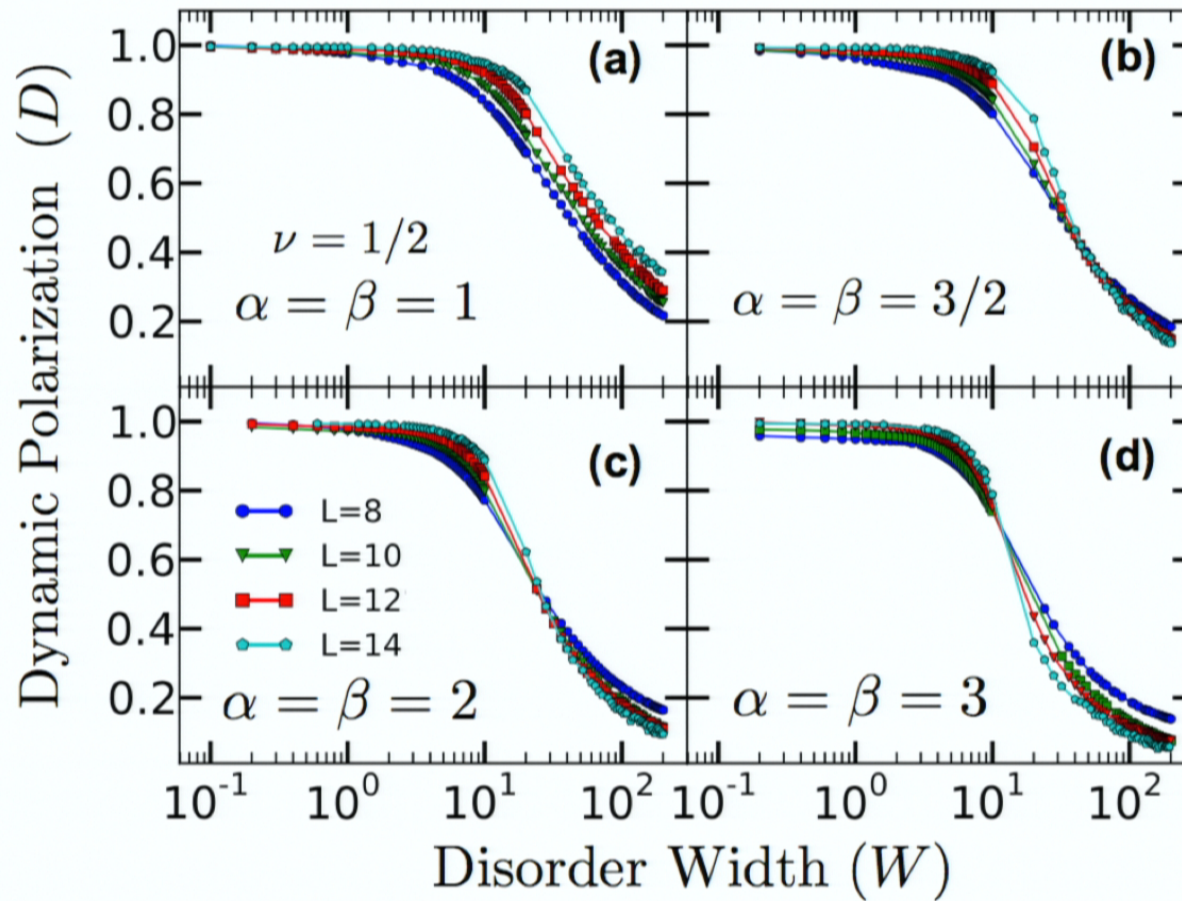
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TABLE I. Critical dimensions for MBL with power laws

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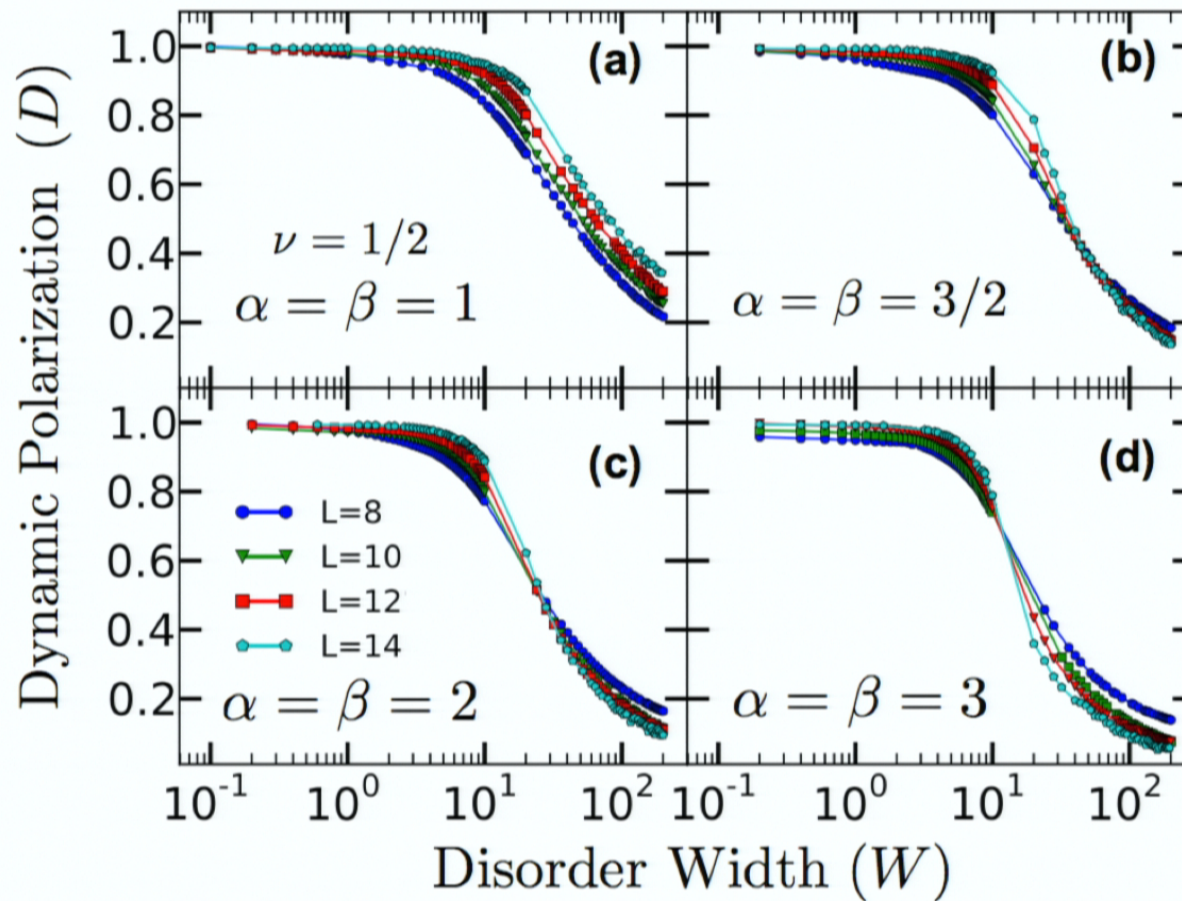
Polarization Decay in $d=1$

Decay of initial inhomogeneous (long-wavelength) number density

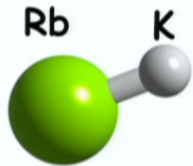


Polarization Decay in $d=1$

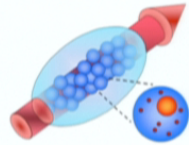
Decay of initial inhomogeneous (long-wavelength) number density



Realizing MBL with Dipoles



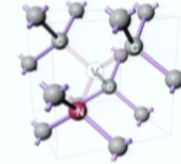
polar
molecules



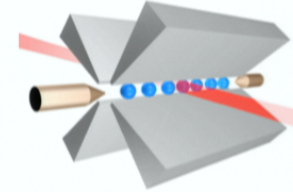
Rydberg
atoms



magnetic
atoms
(e.g. Dy)



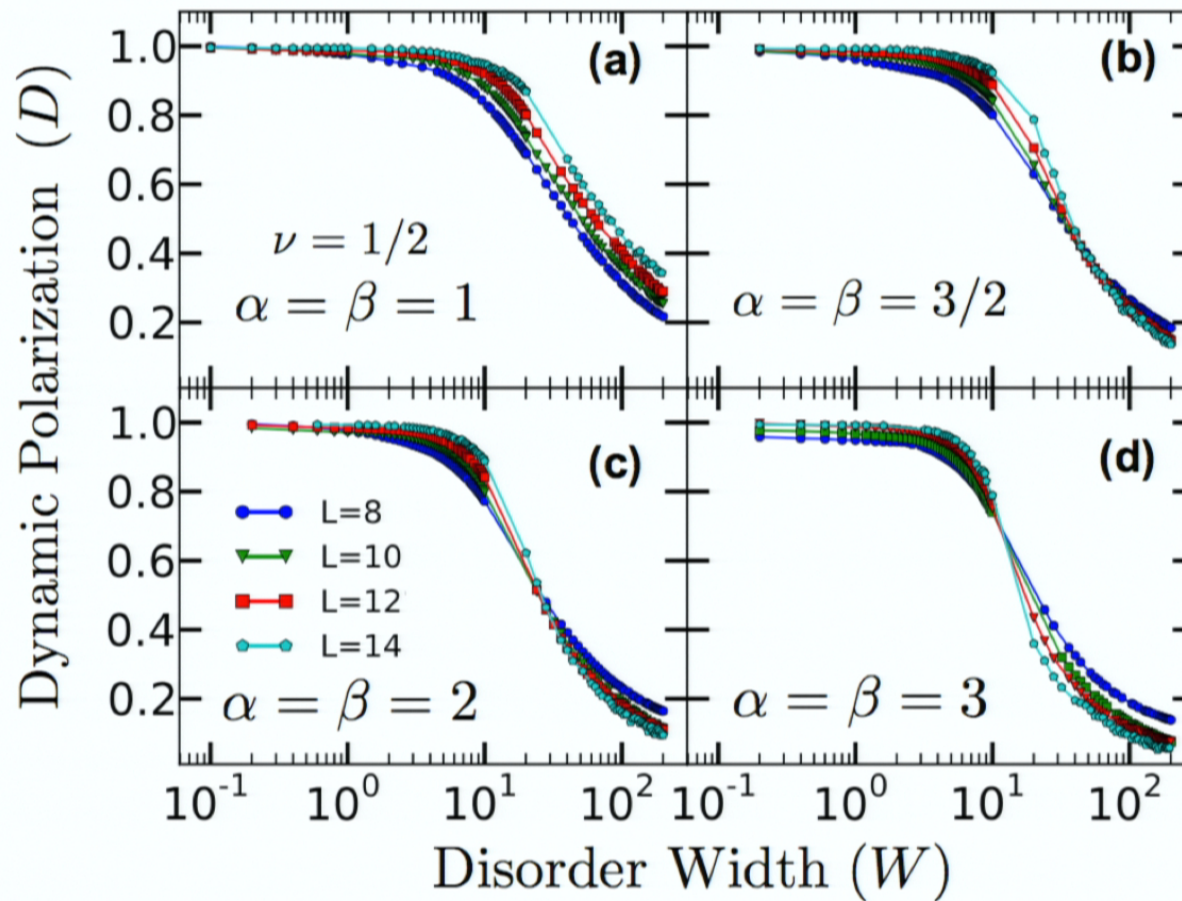
NV centers



Trapped
ions

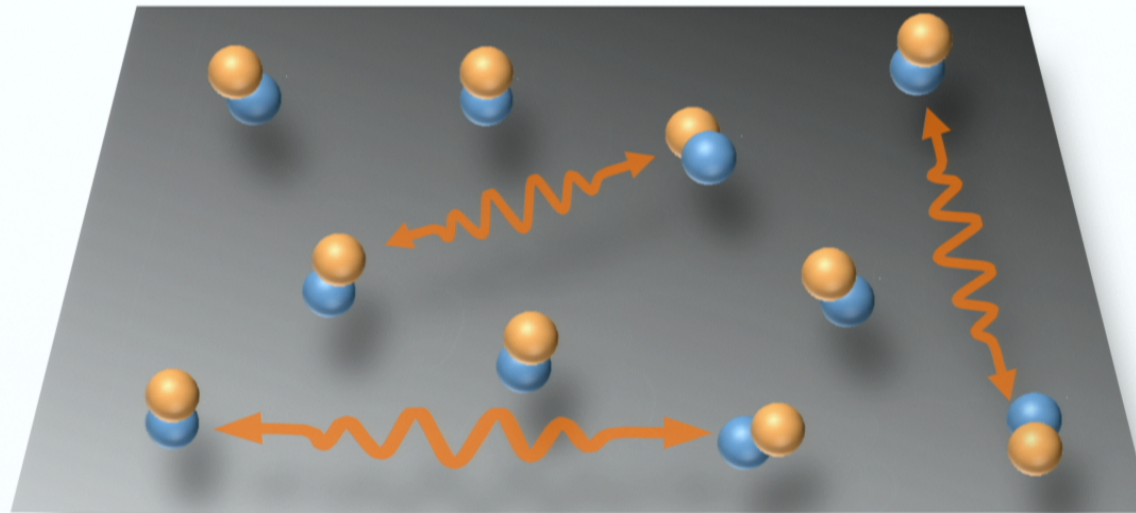
Polarization Decay in $d=1$

Decay of initial inhomogeneous (long-wavelength) number density



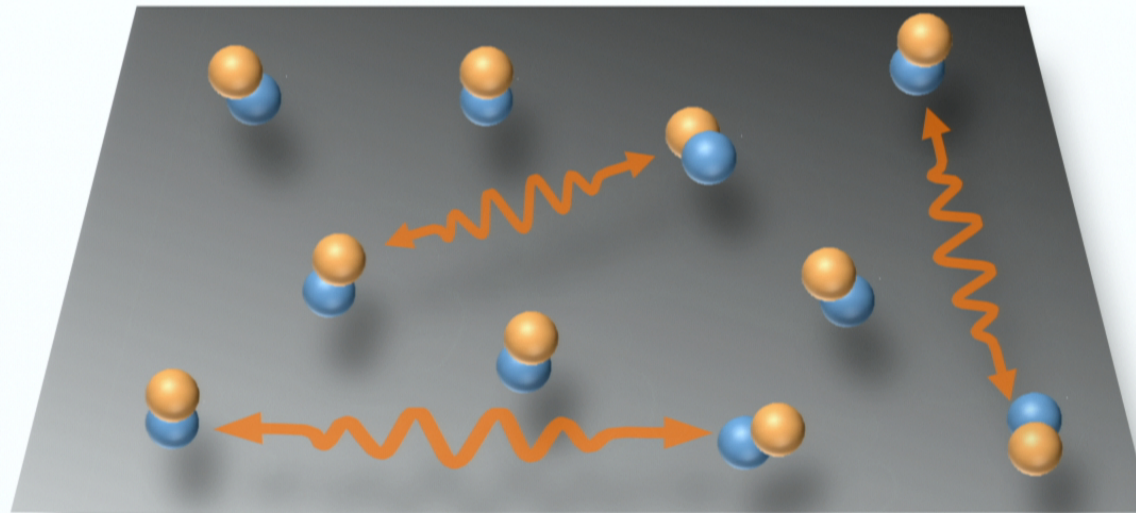
Dipolar Molecules

$$H_{dd} = \frac{1}{2} \sum_{i \neq j} \frac{\kappa}{r_{ij}^3} [\mathbf{d}_i \cdot \mathbf{d}_j - 3(\mathbf{d}_i \cdot \hat{\mathbf{r}}_{ij})(\mathbf{d}_j \cdot \hat{\mathbf{r}}_{ij})]$$



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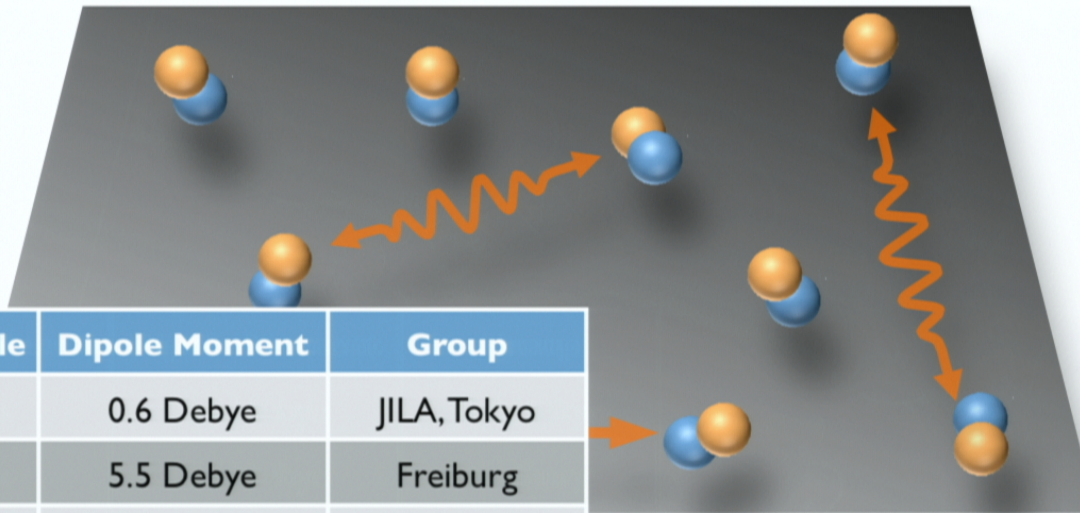
Cubic Perovskite LaMnO_3
 T. O. Dohmae $\text{La}_2\text{FeMoO}_8$
 VO_2 CrO_2
 NiO
 TMH FeI_2 NiGa_2
 CrN
 $\text{Hf}_2\text{Ru}_2\text{O}_7$ Li_2RuO_4

Polar Molecule	Dipole Moment	Group
KRb	0.6 Debye	JILA, Tokyo
LiCs	5.5 Debye	Freiburg
RbCs	1.2 Debye	Yale, Innsbruck
NaK	2.8 Debye	MIT
LiK	3.6 Debye	MIT
SrRb	1.7 Debye	Innsbruck

Loaded in 1D Optical Lattice (2011)
 Loaded in 3D Optical Lattice (2012)
 Observed dipole-dipole exchange (2013)

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Loaded in 1D Optical Lattice (2011)

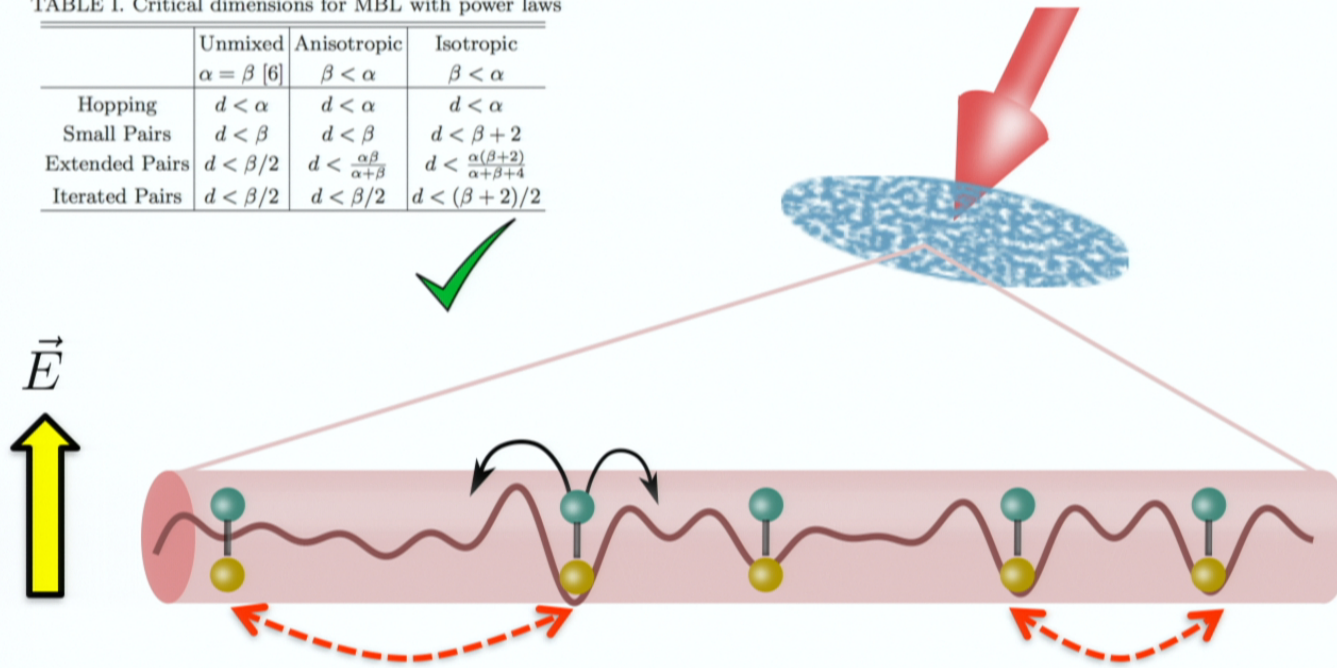
Loaded in 3D Optical Lattice (2012)

Observed dipolar spin exchange (2013)

Hopping Molecules in 1D

TABLE I. Critical dimensions for MBL with power laws

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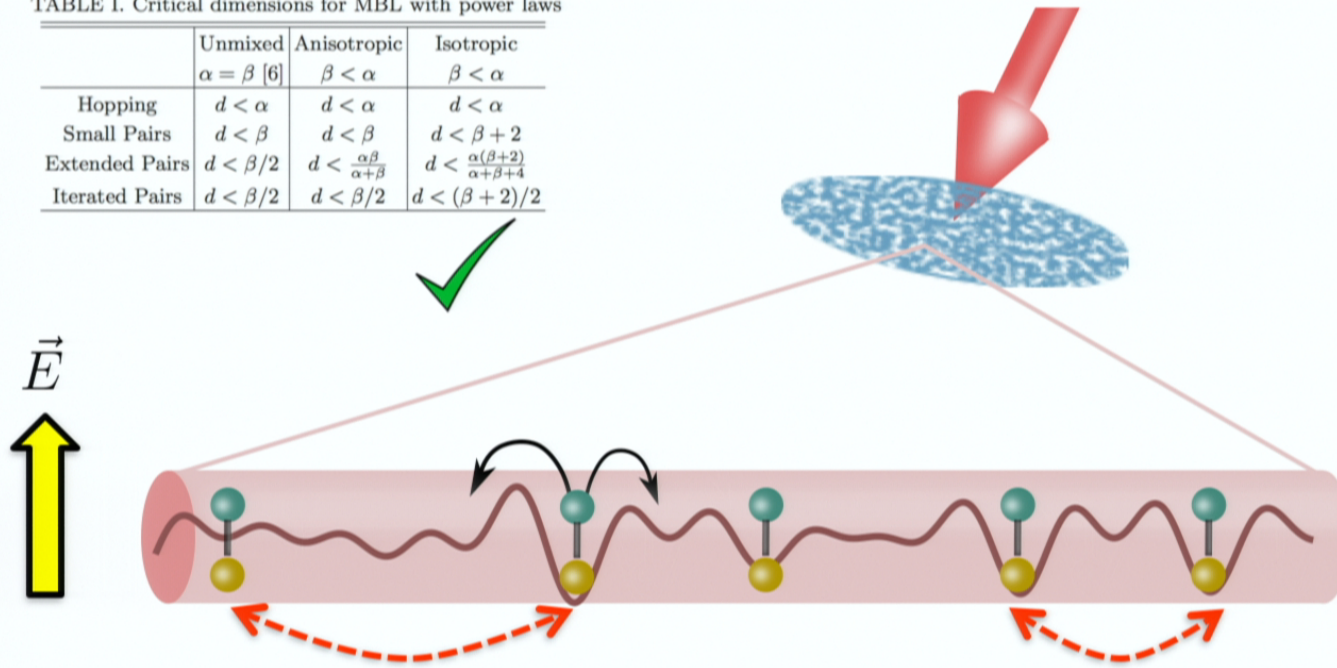


$$H = -t \sum_{\langle ij \rangle} a_i^\dagger a_j + V \sum_{i \neq j} \frac{n_i n_j}{|i - j|^3} + \sum \mu_i n_i$$

Hopping Molecules in 1D

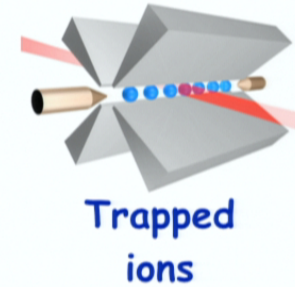
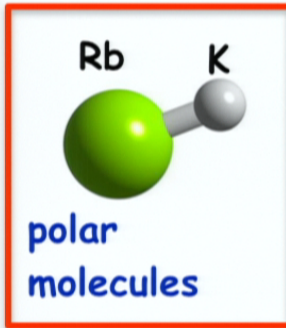
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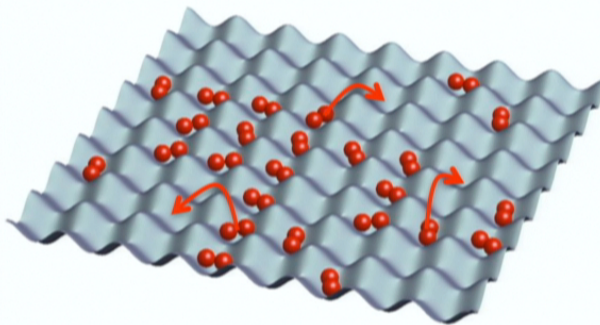


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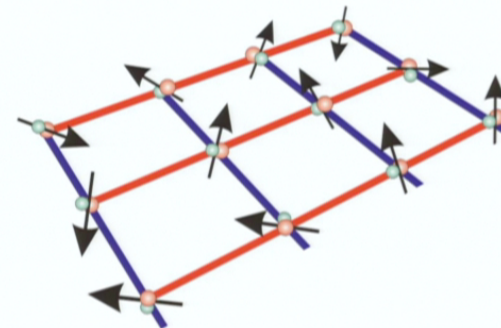
Realizing MBL with Dipoles



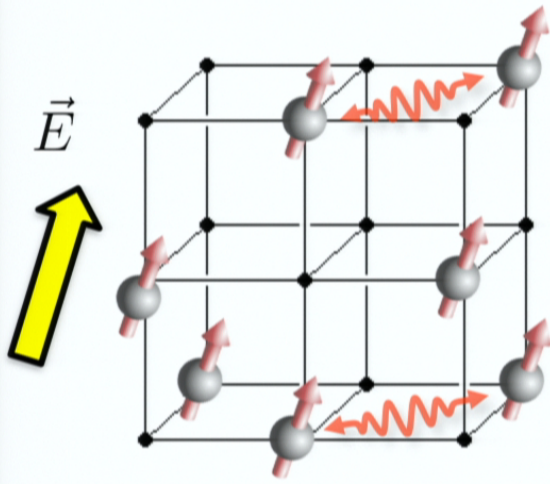
Orbital Motion



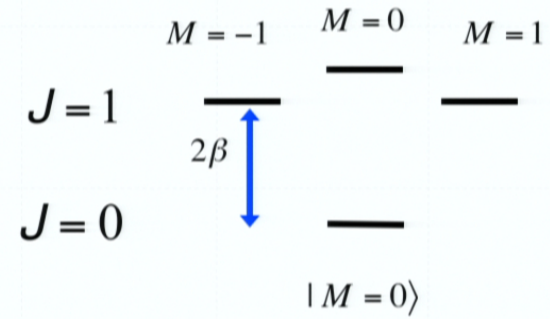
Spin Models



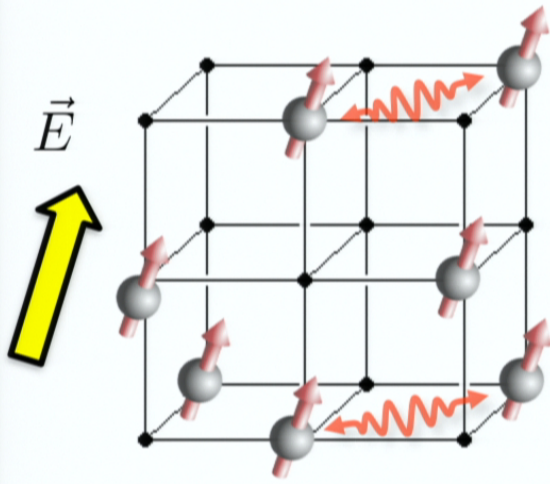
Molecular Spins in 3D



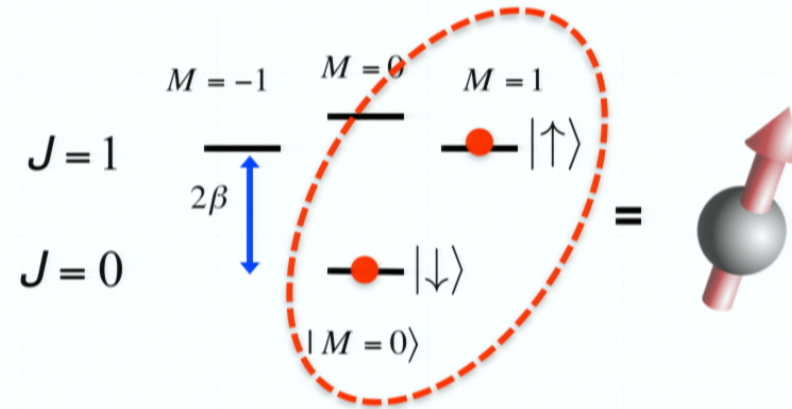
$$H = \beta \hat{J}^2 - \hat{d}_z E$$



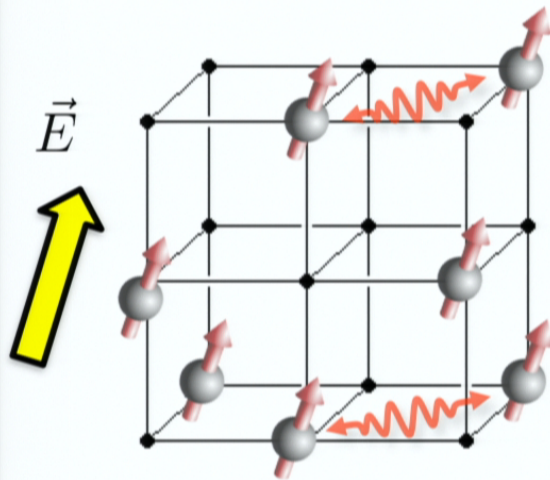
Molecular Spins in 3D



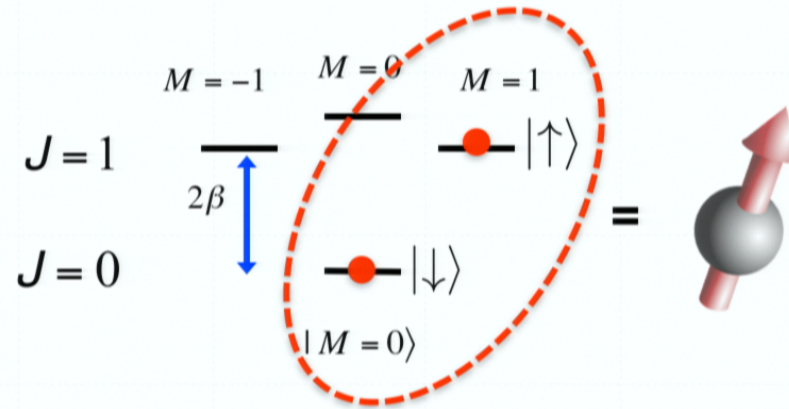
$$H = \beta \hat{J}^2 - \hat{d}_z E$$



Molecular Spins in 3D



$$H = \beta \hat{J}^2 - \hat{d}_z E$$



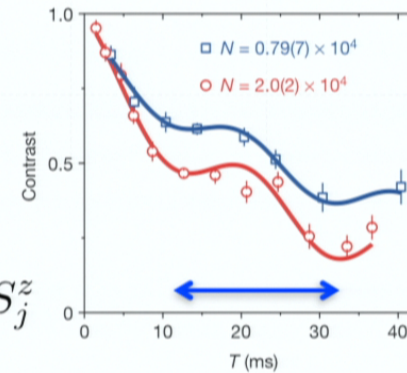
LETTER

doi:10.1038/nature12483

Observation of dipolar spin-exchange interactions with lattice-confined polar molecules

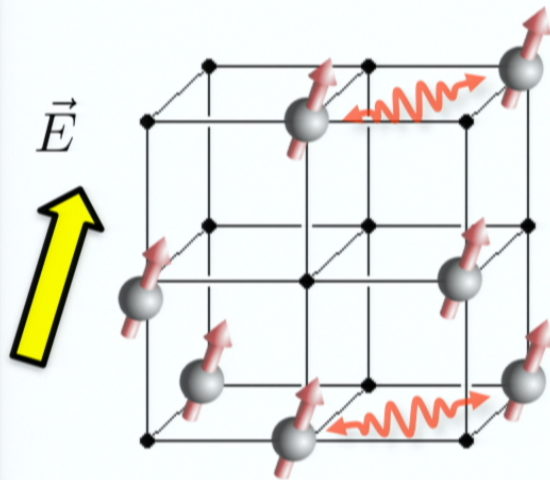
Bo Yan^{1,2}, Steven A. Moses^{1,2}, Bryce Gadway^{1,2}, Jacob P. Covey^{1,2}, Kaden R. A. Hazzard^{1,2}, Ana Maria Rey^{1,2}, Deborah S. Jin^{1,2} & Jun Ye^{1,2}

$$H = \sum \epsilon_i S_i^z + \sum \frac{t_{ij}}{R_{ij}^3} (S_i^+ S_j^- + S_i^- S_j^+) + \sum \frac{V_{ij}}{R_{ij}^3} S_i^z S_j^z$$

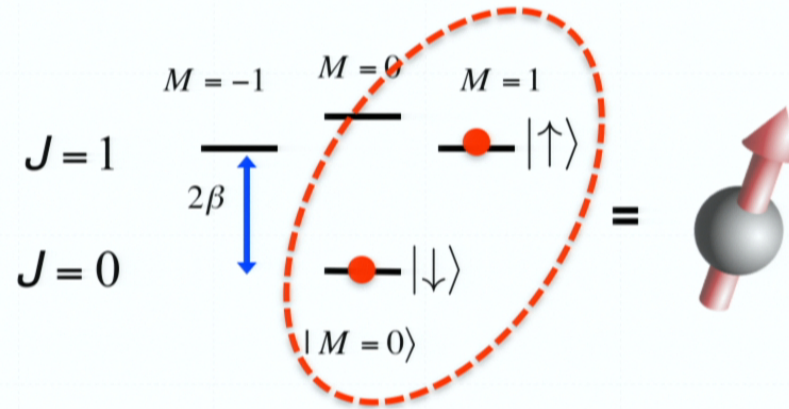


Dipolar Oscillations!

Molecular Spins in 3D



$$H = \beta \hat{J}^2 - \hat{d}_z E$$



LETTER

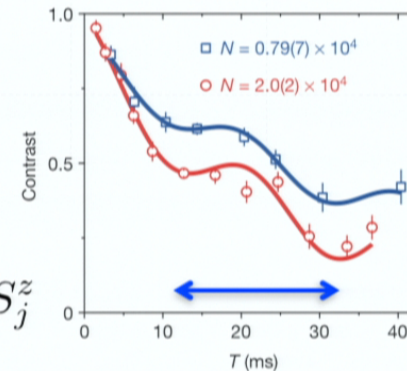
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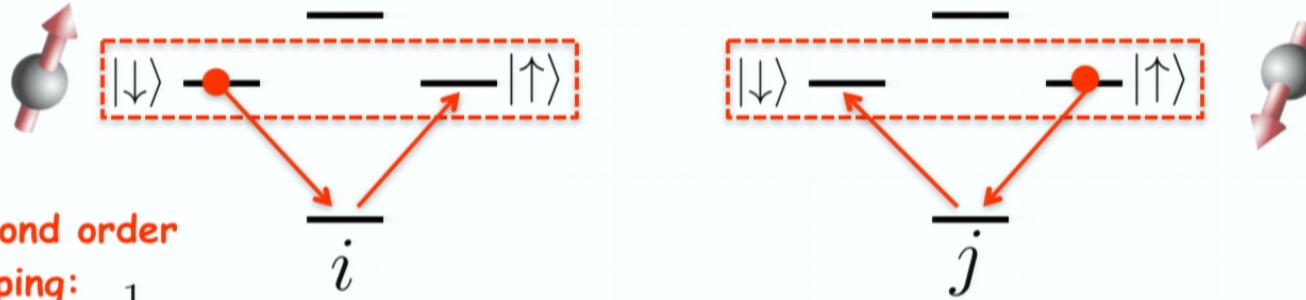
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$$\alpha = \beta = 3 \quad \times \text{MBL}$$



Dipolar Oscillations!

Controlling Dipolar Power-laws



Second order
hopping:
 $t_{eff} \sim \frac{1}{R^6}$

$$H = \sum \epsilon_i S_i^z + \sum \frac{t'_{ij}}{R_{ij}^6} (S_i^+ S_j^- + S_i^- S_j^+) + \sum \frac{V'_{ij}}{R_{ij}^3} S_i^z S_j^z$$

TABLE I. Critical dimensions for MBL with power laws

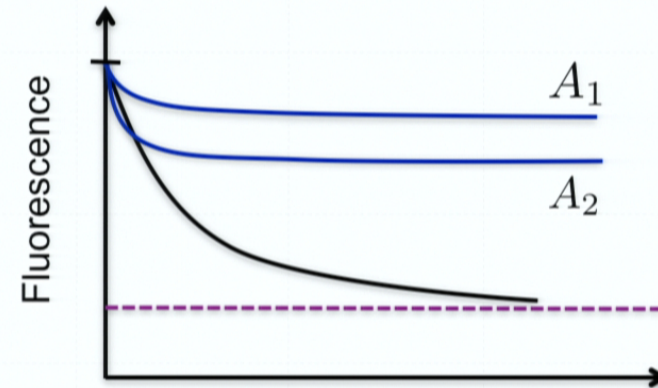
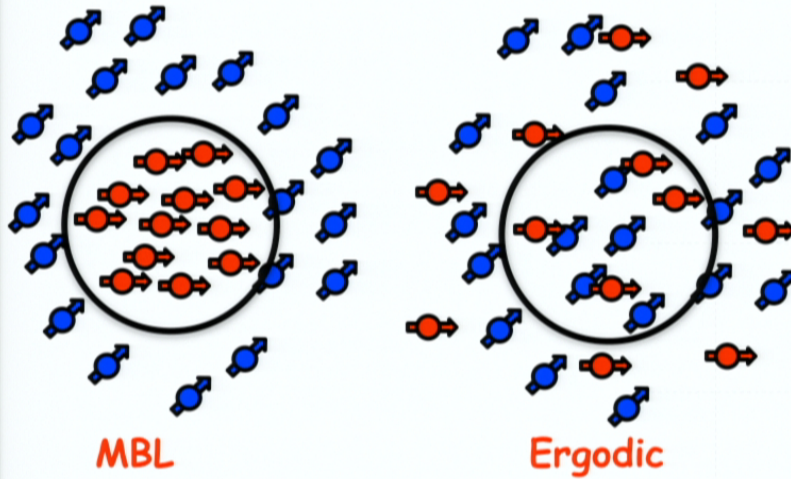
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$d_c = 2.3$

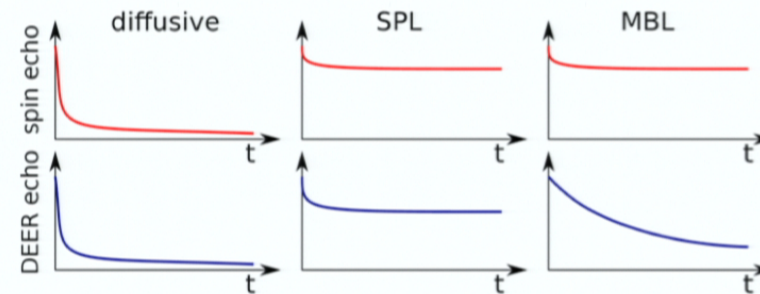
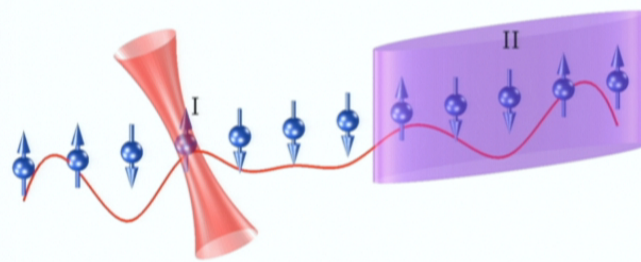


Probing Many-body Localization

1) **Absence of Diffusion:** Measure the decay of spot fluorescence



2) **Interferometry:** Generalized Spin Echo



Conclusions

Resonance counting arguments place constraints on power laws for many-body localization.

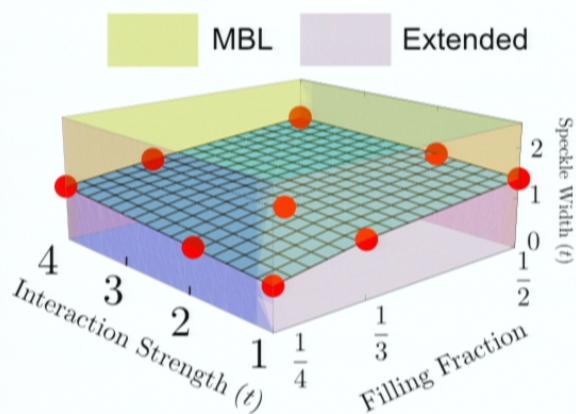


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Small scale numerics hint at localization in suitable power law systems.

Many-body localization may be realizable in near term ultracold experiments.

