Title: Many-body localization: Local integrals of motion, area-law entanglement, and quantum dynamics

Date: Feb 12, 2014 09:45 AM

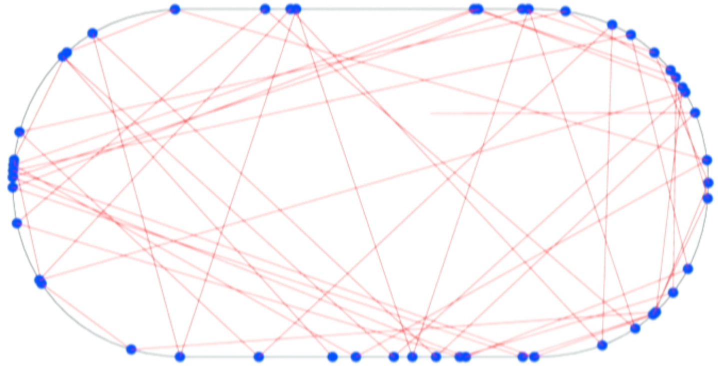
URL: http://pirsa.org/14020127

Abstract: We demonstrate that the many-body localized phase is characterized by the existence of infinitely many local conservation laws. We argue that many-body eigenstates can be obtained from product states by a sequence of nearly local unitary transformation, and therefore have an area-law entanglement entropy, typical of ground states. Using this property, we construct the local integrals of motion in terms of projectors onto certain linear combinations of eigenstates [1]. The local integrals of motion can be viewed as effective quantum bits which have a conserved z-component that cannot decay. Thus, the dynamics is reduced to slow dephasing between distant effective bits. For initial product states, this leads to a characteristic slow power-law decay of local observables, which is measurable experimentally, as well as to logarithmic in time growth of entanglement entropy [2,3]. We support our findings by numerical simulations of random-field XXZ spin chains. Our work shows that the many-body localized phase is locally integrable, reveals a simple entanglement structure of eigenstates, and establishes the laws of dynamics in this phase.

[1] M. Serbyn, Z. Papic, D. A. Abanin, Phys. Rev. Lett. 111, 127201 (2013).
[2] Jens H. Bardarson, Frank Pollmann, and Joel E. Moore, Phys. Rev. Lett. 109, 017202 (2012).
[3] M. Serbyn, Z. Papic, D. A. Abanin, Phys. Rev. Lett. 110, 260601 (2013)

Ergodicity and its breaking

System explores full phase space



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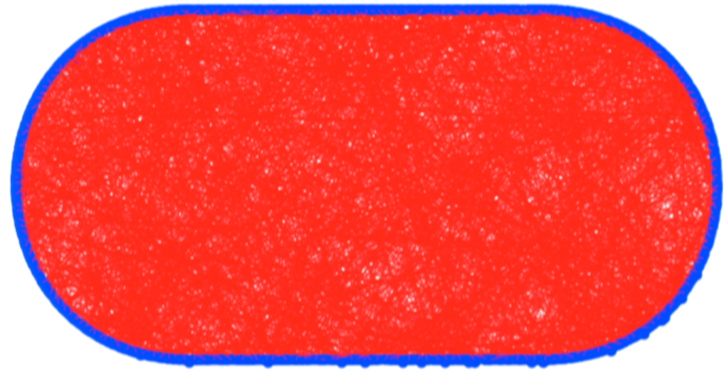

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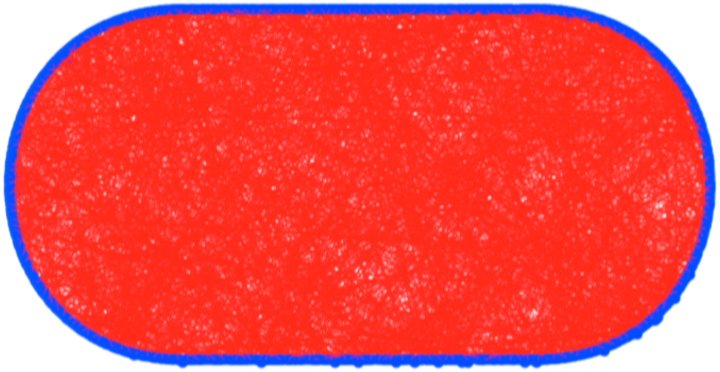
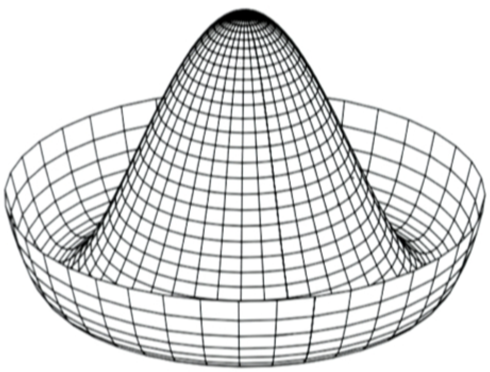


Ergodicity breaking

In phase transitions



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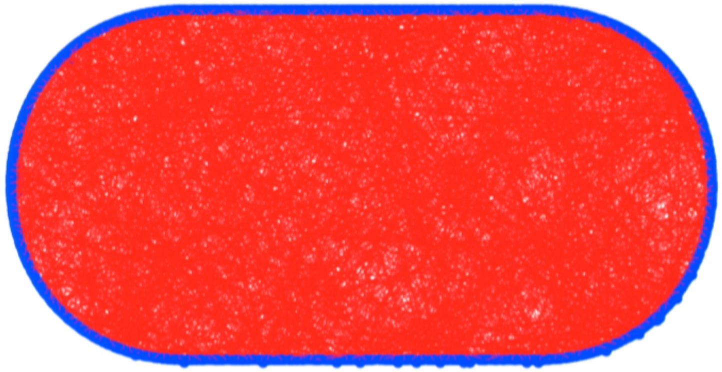

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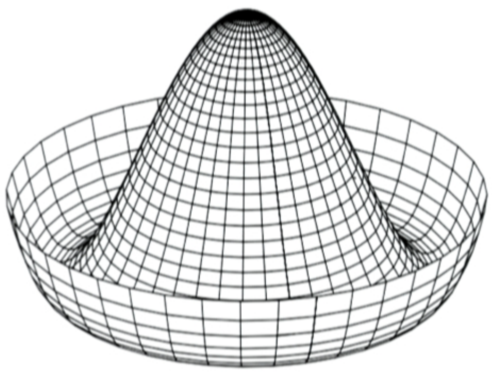

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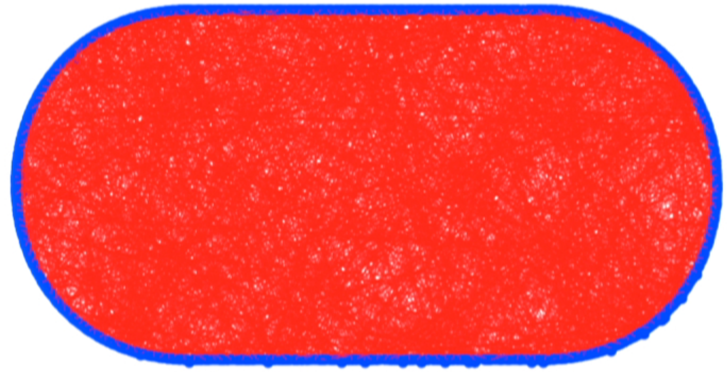

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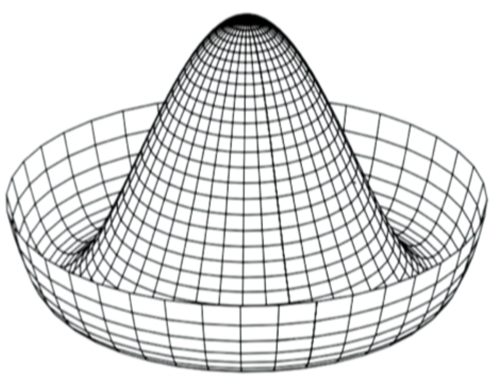
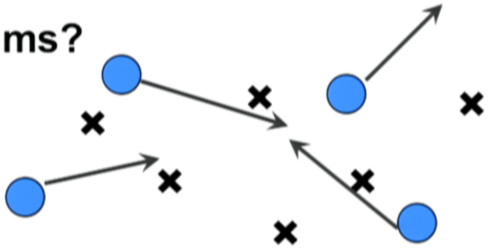


In bear habitats

Many-body localization problem

When/how does ergodicity break in many-body systems?

Do interactions destroy localization?



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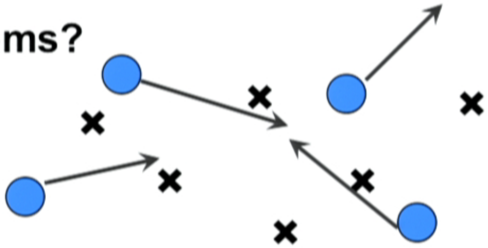

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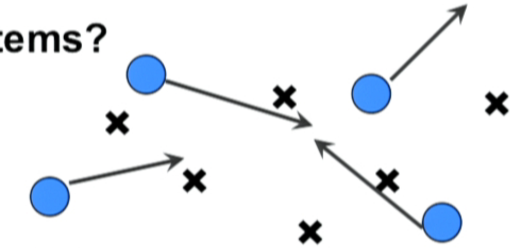

YES

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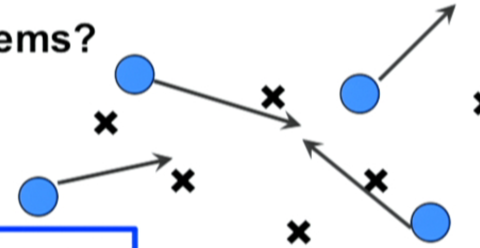

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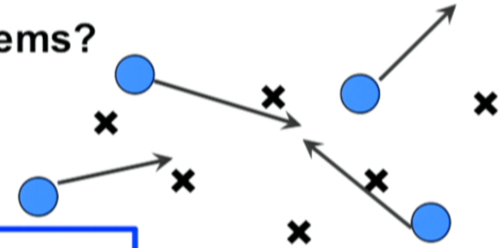

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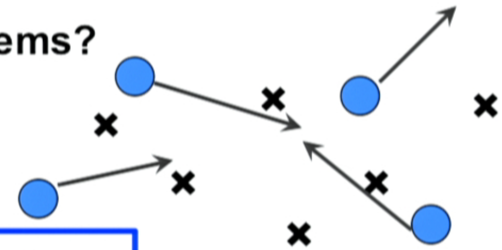


YES

NO

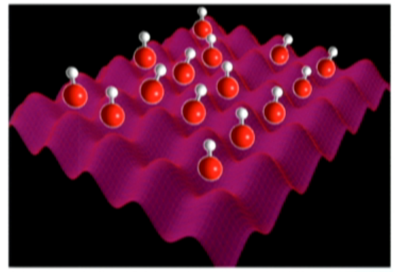
New experimental systems

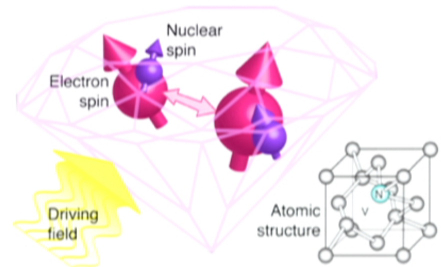
Isolated & quantum-coherent. Tunable interactions and disorder

-Cold atoms, optical lattices

-Polar molecules

-Spin systems (NV-centers in diamond)





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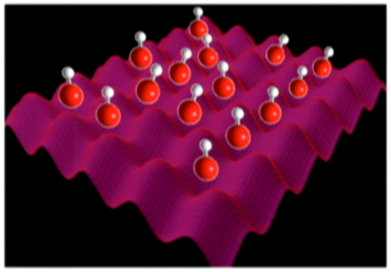
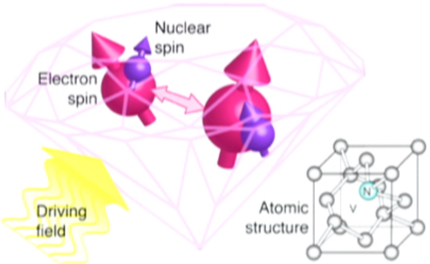
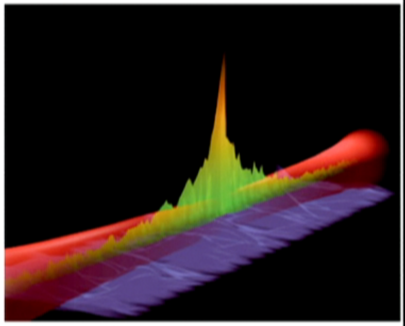




REPORTS

Three-Dimensional Anderson Localization of Ultracold Matter

S. S. Kondov, W. R. McGehee, J. J. Zirbel, B. DeMarco*



EXP: Ecole Normale, Florence, Urbana

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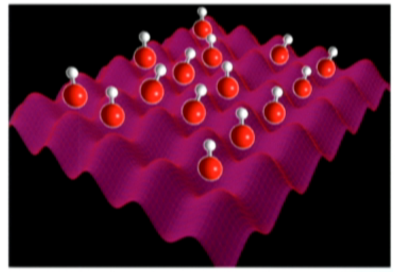

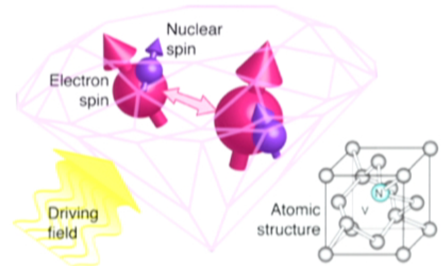

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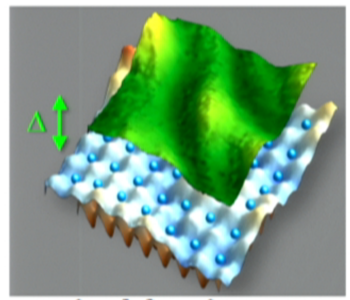
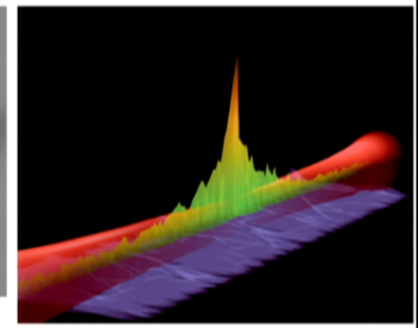




Interplay of disorder and interactions in an optical lattice Hubbard model

S. S. Kondov, W. R. McGehee, B. DeMarco[1]

(Dated: May 28, 2013)

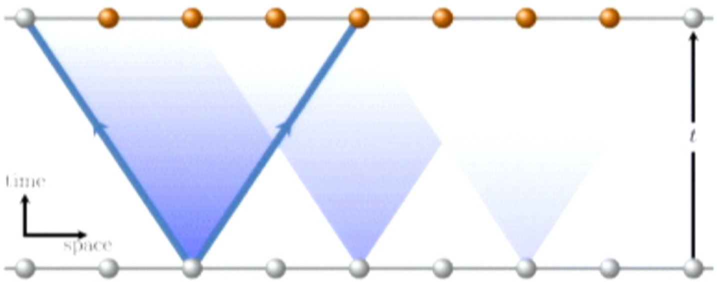
EXP: Ecole Normale, Florence, Urbana

Studying many-body localization experimentally now possible

Entanglement propagation in ergodic systems

Delocalized systems

Light-cone-like spreading of correlations



Lieb,Robinson'72, Hastings'04, Calabrese,Cardy'05

Kim, Huse'13

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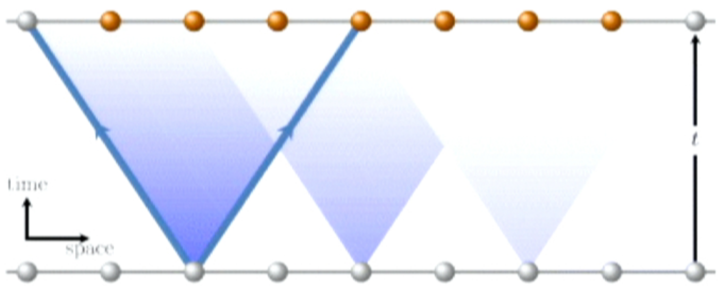
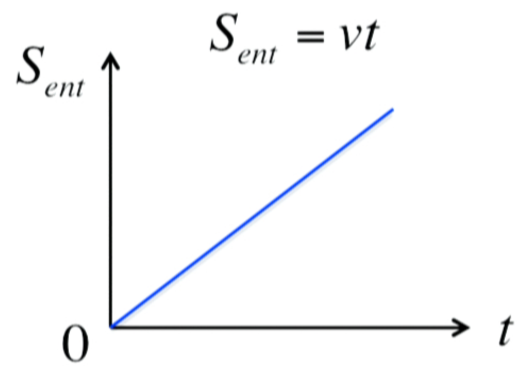


Lieb,Robinson'72, Hastings'04, Calabrese,Cardy'05
Kim, Huse'13

Initial product state

Linear growth of entanglement entropy



(even when transport is diffusive)

Entanglement propagation in ergodic systems

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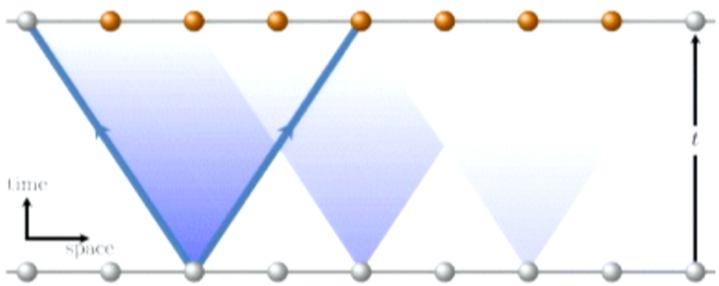

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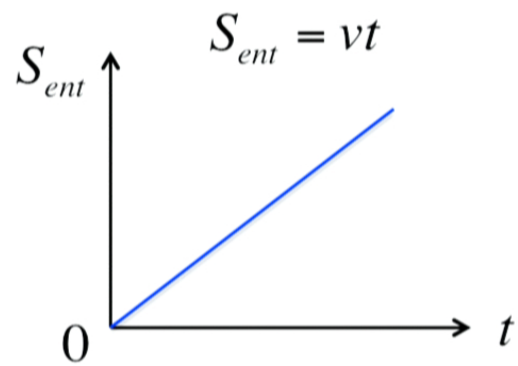

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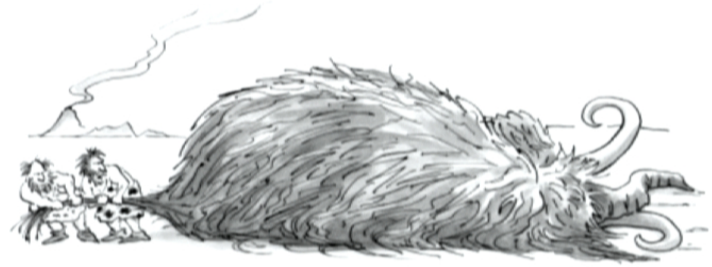
Experimental observation: Cold atoms Cheneau et al'12 Trapped ions Jurcevic et al'14

A quantum many-body system

Condensed matter physicists

A quantum many-body system

Condensed matter physicists



System in a **ground state**

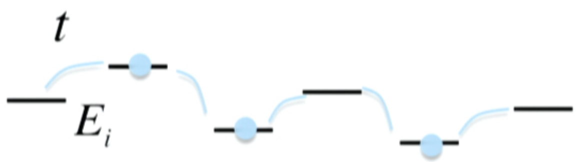
But there is much more…

This talk: Highly excited many-body states

A simple model of many-body localization

Jordan-Wigner

Spinless interacting 1D fermions $\approx$ Random-field XXZ spin-1/2 chain



$$H = \sum_i E_i c_i^+ c_i + t\sum_i c_i^+ c_{i+1} + h.c. + V\sum_i n_i n_{i+1}$$

$$H = \sum_i h_i S_i^z + J_\perp \sum_i (S_i^+ S_{i+1}^- + h.c) + J_z \sum_i S_i^z S_{i+1}^z$$

Use disorder as tuning parameter $h_i \in [-W; W]$

A simple model of many-body localization

Jordan-Wigner

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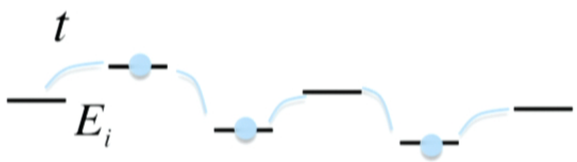

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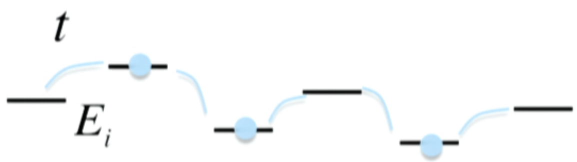

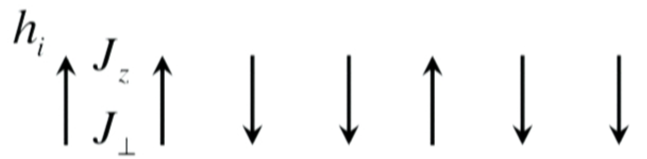

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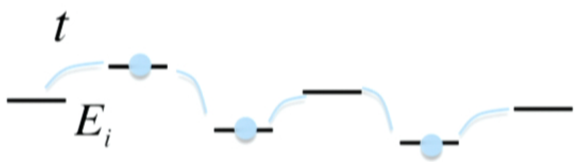

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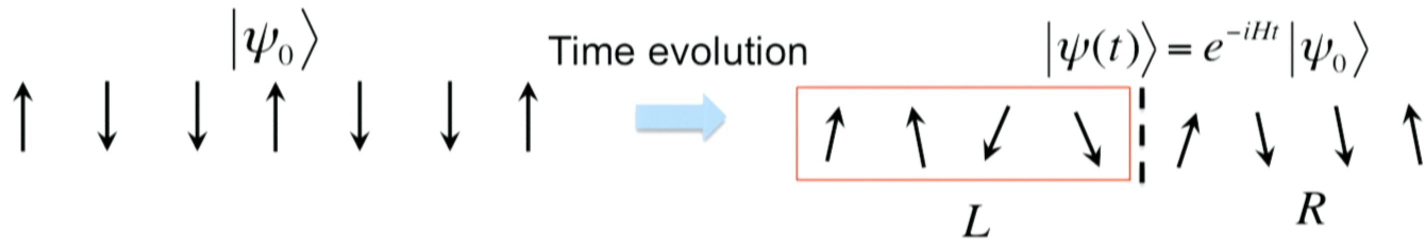
Use disorder as tuning parameter $h_i \in [-W;W]$

Many-body localization at strong disorder (numerics)

Oganesyan, Huse'07, Prosen et al'08, Pal, Huse'10, Monthus, Garel'10,..

Entanglement propagation in localized systems



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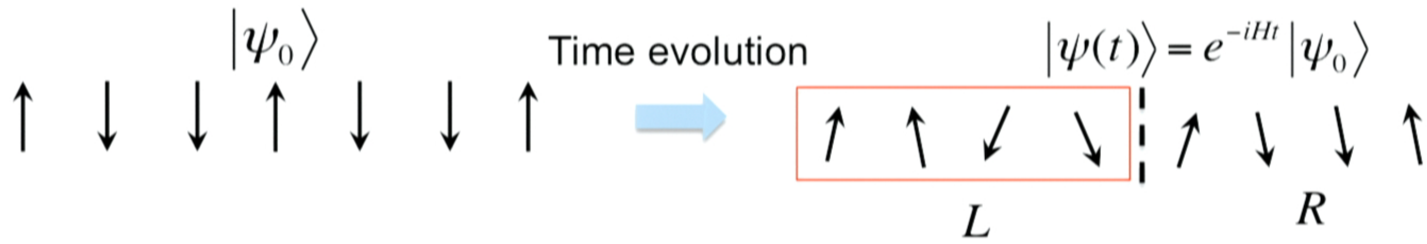
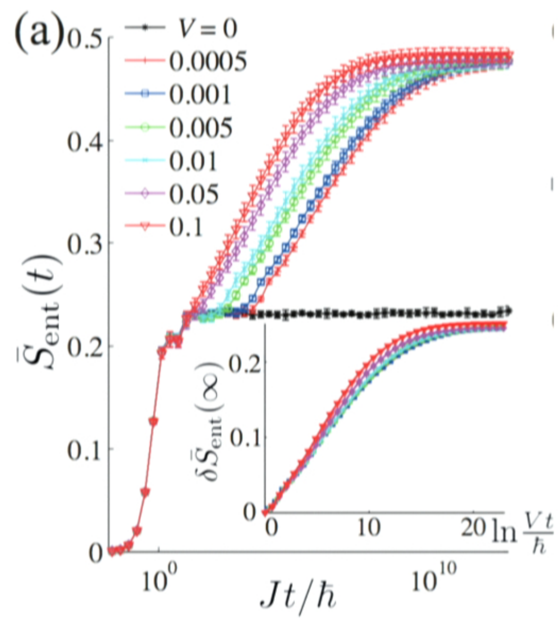


-Anderson-localized: $S_{ent}(t) \le const$

-Many-body localized: slow growth of entanglement

$$S_{ent}(t) \propto \log t$$



Bardarson, Pollmann, Moore'12

Entanglement propagation in localized systems

$|\psi_0\rangle$ Time evolution $|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle$

-Anderson-localized: $S_{ent}(t) \le const$

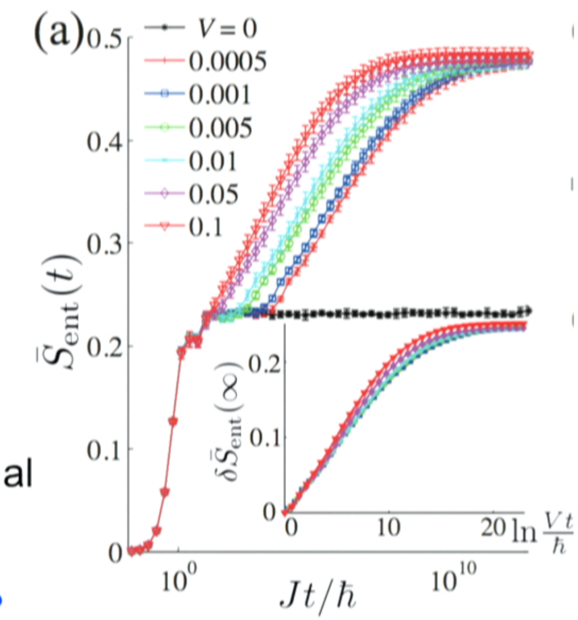
-Many-body localized: slow growth of entanglement

$$S_{ent}(t) \propto \log t$$

-"Glassy" dynamics, extremely long time scales

-Entanglement <u>extensive in system size,</u> non-thermal

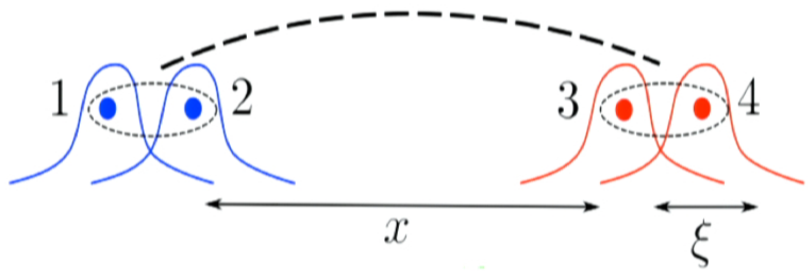
Ergodicity restored? Slow particle transport??



Bardarson, Pollmann, Moore'12

The mechanism of entanglement growth: Toy model

$$|\psi_0\rangle = \frac{1}{2}(c_1^+ + c_2^+)(c_3^+ + c_4^+)|0\rangle$$



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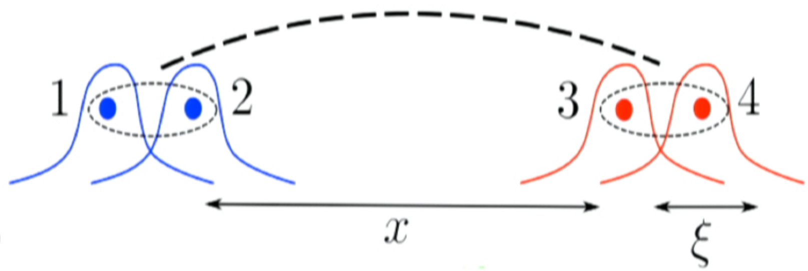

Assume weak interactions

Eigenstate $|\alpha\beta\rangle = c_\alpha^+ c_\beta^+ |0\rangle + O(e^{-x/\xi})$

Energy: $E_{\alpha\beta} = E_\alpha + E_\beta + \boxed{C_{\alpha\beta} V e^{-x/\xi}}$

Reduced density matrix

$$\rho(t) = \frac{1}{2}\begin{bmatrix} 1 & \cos\omega t \\ \cos\omega t & 1 \end{bmatrix}$$

$$\omega \sim \frac{V}{\hbar} e^{x/\xi}$$

Serbyn, Papic, Abanin PRL '13

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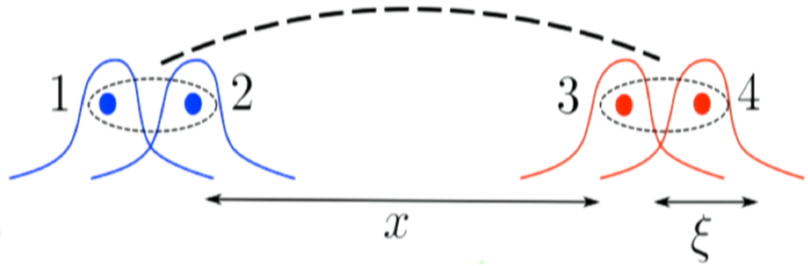

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$$\omega \sim \frac{V}{\hbar} e^{x/\xi} \qquad t_{deph} \sim \frac{\hbar}{V} e^{x/\xi}$$

Interaction-induced dephasing → entanglement generation

Serbyn, Papic, Abanin PRL '13

Case of many particles

Hypothesis: Eigenstates at **small *V*** are "close"* to non-interacting eigenstates

Non-interacting $c^{+}_{\alpha_1} c^{+}_{\alpha_2} \ldots c^{+}_{\alpha_i} \ldots c^{+}_{\alpha_N} \left|0\right\rangle$

Interacting $\left|\{\alpha\}\right\rangle = \left|\alpha_1 \alpha_2 \ldots \alpha_i \ldots \alpha_N\right\rangle$

Energy: perturbation theory in *V*

$$E_{\{\alpha\}} = \sum E_{\alpha_i} + V \sum C_{\alpha_i \alpha_j} e^{-\frac{|R_i - R_j|}{\xi}} + \ldots$$

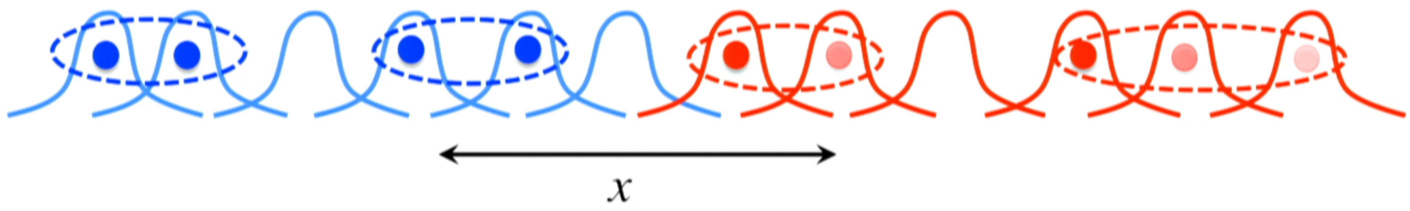
1-body energy | 2-body interactions | 3,4…-body interactions

Interactions of far-away particles are exponentially small

The laws of entanglement growth

Serbyn, Papic, Abanin PRL'13

Initial product state is a superposition of many eigenstates



The laws of entanglement growth

Serbyn, Papic, Abanin PRL'13

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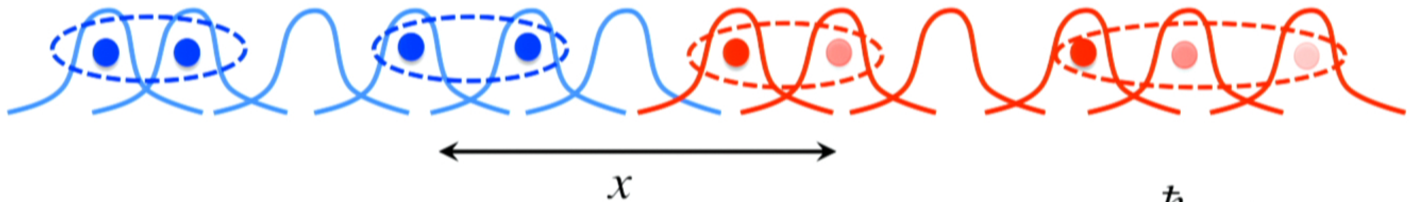

Degrees of freedom a distance x away get etangled at $t_{deph} \sim \frac{\hbar}{V} e^{x/\xi}$

$$S_{ent}(t) = C\xi \log \frac{Vt}{\hbar}$$

C depends on diagonal entropy of initial state

The laws of entanglement growth

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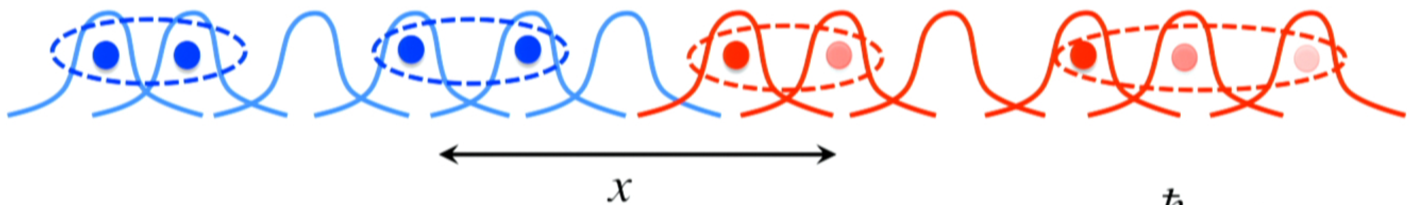

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$$S_{ent}(t) = C\xi \log \frac{Vt}{\hbar}$$

C depends on diagonal entropy of initial state

Saturated value: set by diagonal entropy

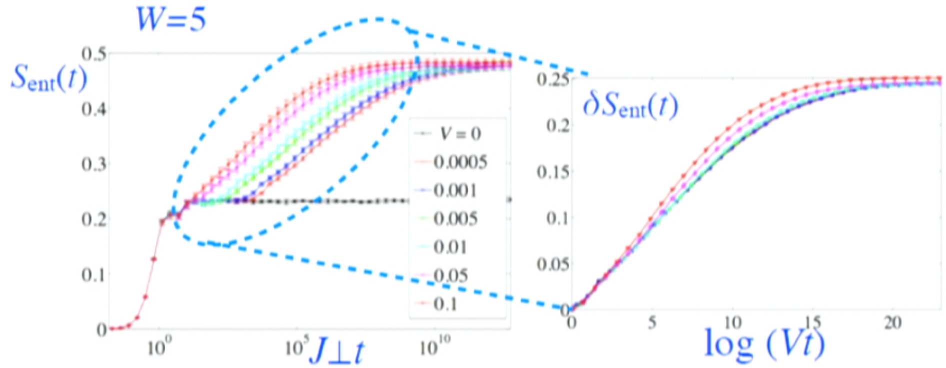
$$S_{ent}(\infty) = S_{diag} << S_{Thermal}$$

Predicted: propagation speed, dependence on disorder, interactions, initial state

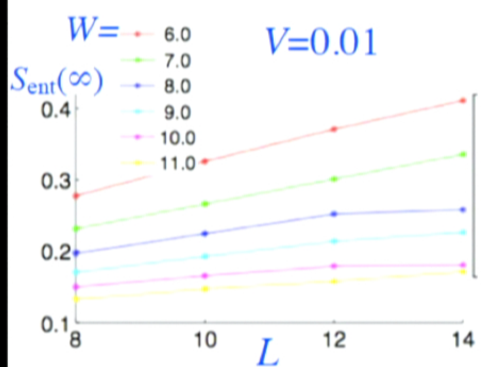
Confirmed by numerics

Numerical analysis

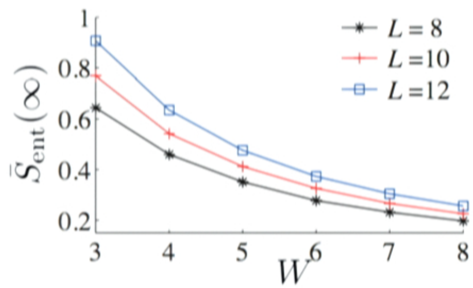
$$S_{ent}(t) \propto \xi \log \frac{Vt}{\hbar}$$



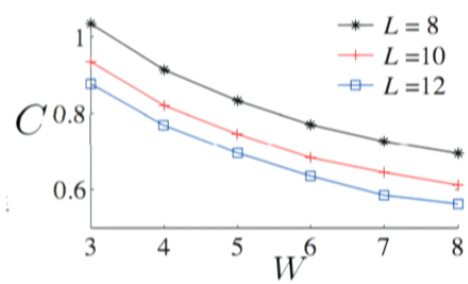
Interaction dependence



Saturated entanglement extensive in system size



Disorder dependence



$$C = \frac{S_{ent}(\infty)}{S_{diag}}$$

Localized phase at **strong interactions (e.g., spins)**?
Is entanglement growth universal?

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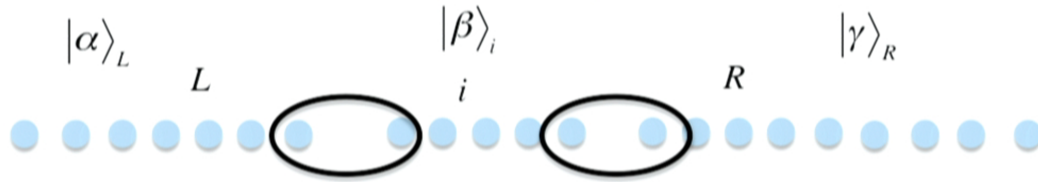
Is entanglement growth universal?

YES!
The key: MBL phase is integrable. Integrals of motion are local.

Constructing integrals of motion



A local integral of motion in terms of projectors onto eigenstates

$$\hat{P}_i(\beta) = \sum_{\alpha,\gamma} |\alpha\beta\gamma\rangle\langle\alpha\beta\gamma|$$

$$|\alpha\beta\gamma\rangle = \hat{O}_{Ri}\hat{O}_{Li}\,|\alpha\rangle_L \otimes |\beta\rangle_i \otimes |\gamma\rangle_R$$

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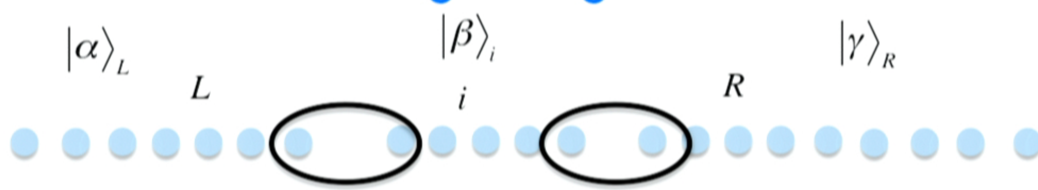

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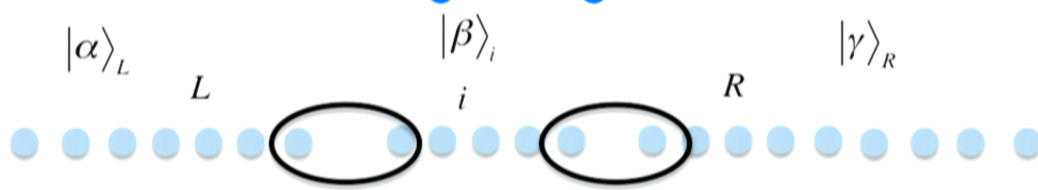

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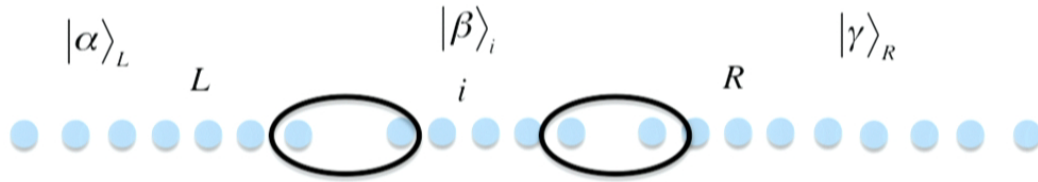

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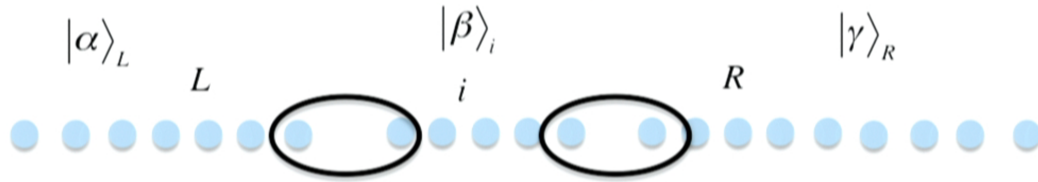

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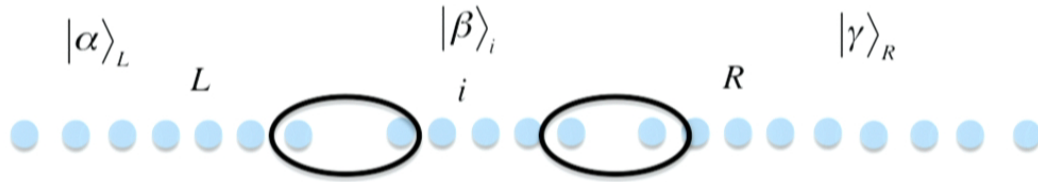
These operators are local. Integrals of motion by construction.

Constructing integrals of motion



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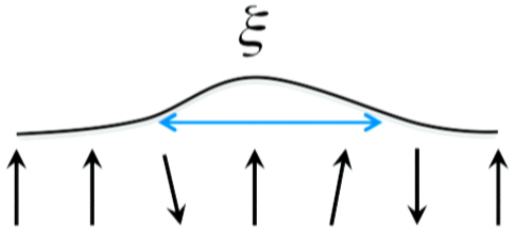
Universal Hamiltonian of MBL phase

"Effective" spins ½ τ_z^i with conserved z-projection (defined via projectors)

$$[\tau_z^i, H] = 0$$



τ_z^i support in a region of size ~ ξ

Hamiltonian depends only on τ_z^i

$$H = \sum_i H_i \tau_z^i + \sum_{ij} H_{ij} \tau_z^i \tau_z^j + \sum_{ijk} H_{ijk} \tau_z^i \tau_z^j \tau_z^k + ..$$

$$H_{ij} \propto \exp(-|i-j|a/\xi)$$

Universal dynamics and entanglement growth

-No relaxation → no thermalization

$$\left\langle \tau_z^i(t) \right\rangle = Const$$

-Interaction-induced dephasing

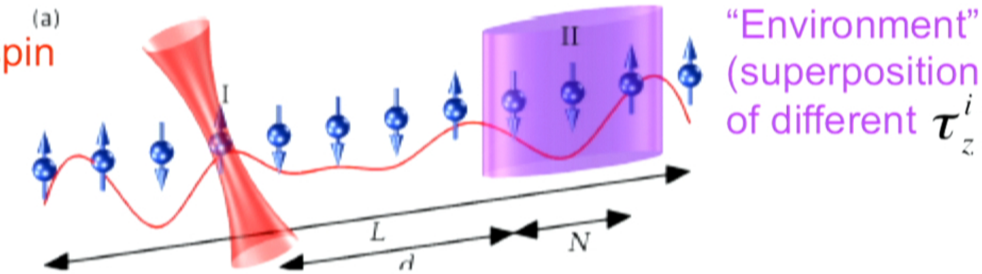
$$\left\langle \tau_x^i(t) \right\rangle \to 0 \quad t \to \infty$$

-Steady non-thermal state

-Universal logartihmic growth of entanglement entropy (hard to measure)

-Any local observable would show oscillations and a power-law decay (easy to measure)

Interferometric signatures of many-body localization

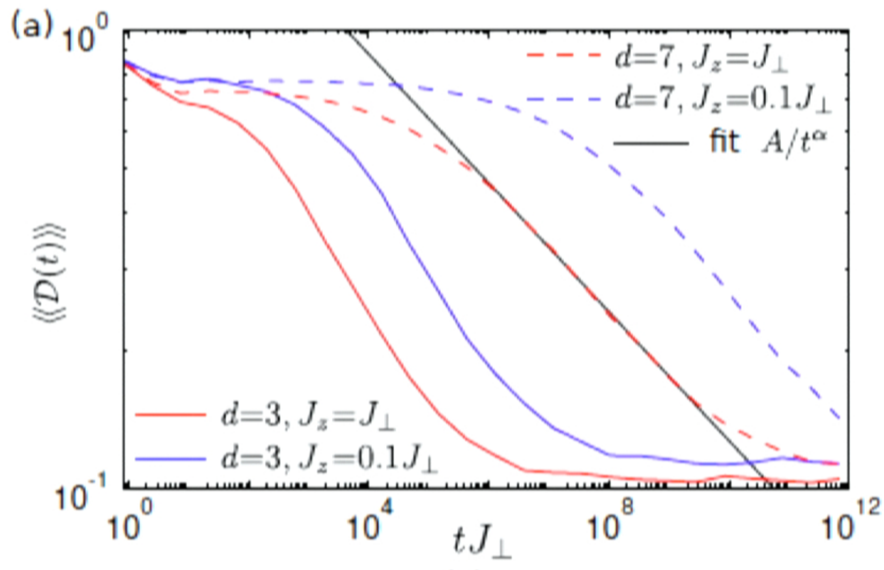


Modified spin-echo

Dephasing → power-law spin-echo decay $\bar{\mathcal{D}}(t) \sim t^{-\xi \ln 2}$

A way to directly probe MBL

in spin systems, cold atoms



Serbyn, Papic, Abanin+Lukin, Demler'14

The structure of many-body localized states

-Eigenstates obtained from product states by a sequence of local unitaries (quantum circuit of finite depth)

-Excited MBL eigenstates obey area law-entanglement
(similar to ground states of gapped systems!)

Explicitly constructing integrals of motion

Effective spins: good for quantum information processing?
Yes, but how do they relate to physical spins?

Approach 1: Connect eigenstates to product states by local unitary U

$$\tau_z^i = U s_z^i U^+$$

Expand τ_z^i via physical operators

$$\tau_z^i = \boxed{C_1 s_z^i + C_2 s_z^{i+1} + C_3 s_z^{i-1}} + D_1 s_z^i s_x^{i+1} + ..$$

with A. Chandran, G. Vidal, in progress

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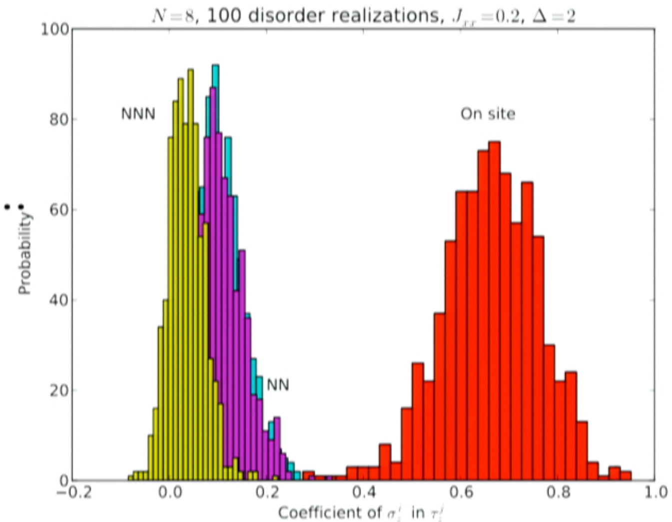


Extract statistics of couplings and properties of integrals of motion

Approach 2: Extension of a strong disorder renormalization group

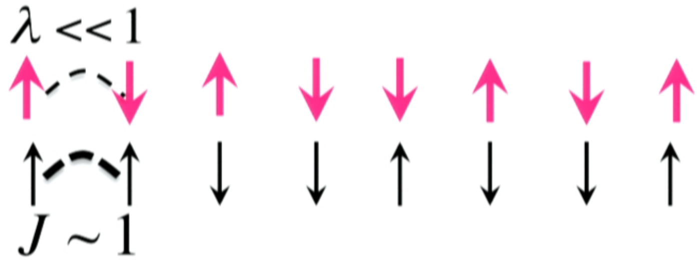
with A. Chandran, G. Vidal, in progress

Localization without quenched disorder?

(Muller'13, Huveneers, de Roeck'13, Fisher, Grover'13..)

Two coupled spin chains, "fast" σ_i^z and "slow" s_i^z

Papic, Stoudenmire, DA '14



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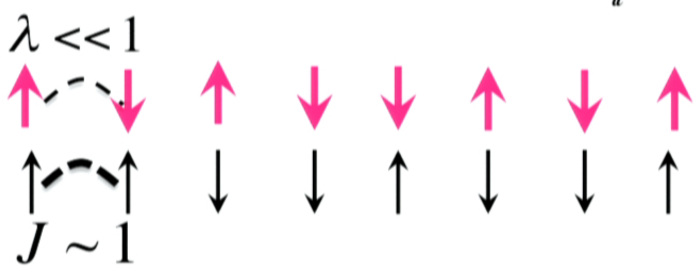
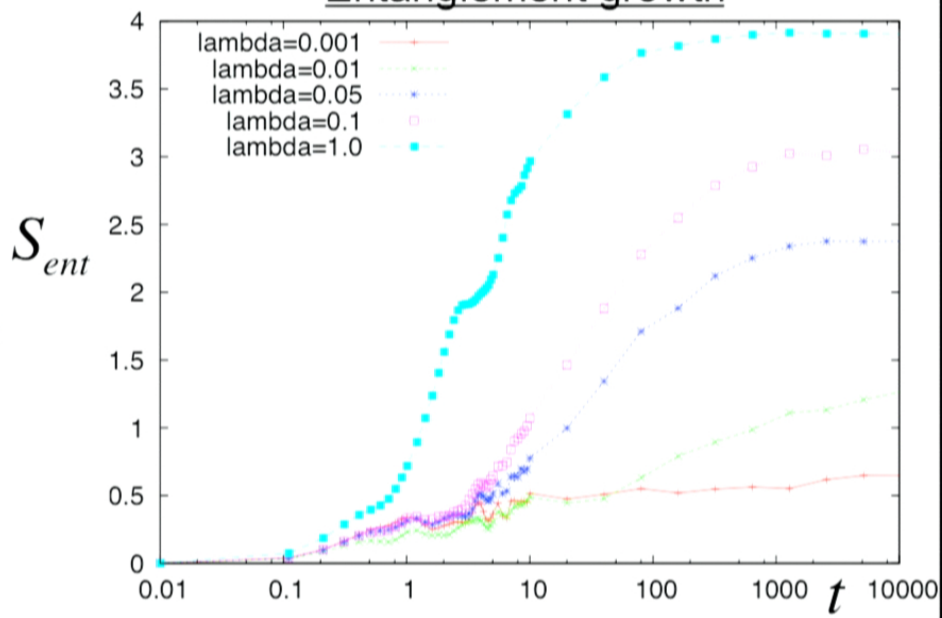


Interaction $$H_{\text{int}} = W \sum_i \sigma_i^z s_i^z$$

Localization at $\lambda << 1$?

Entanglement growth



Preliminary:

Log-growth of entanglement

Signals localization??

Summary

-Ergodicity breaking in disordered systems: local conservation laws

-Universal dynamics: slow dephasing

-Logarithmic growth of entanglement entropy

-More to come…

DETAILS:
Phys. Rev. Lett. **110**, 260601 (2013),
Phys. Rev. Lett. **111**, 127201 (2013)+To appear

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(Perimeter)

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