Title: Interacting electronic topological insulators in three dimensions

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Abstract: I will review recent progress in describing interacting electronic topological insulators/superconductors in three dimensions. The focus will be on Symmetry Protected Topological (SPT) phases of electronic systems with charge conservation and time reversal. I will argue that the well known Z2 classification of free fermion insulators with this important symmetry generalizes to a Z2^3 classification in the presence of interactions. I will describe the experimental fingerprints and other physical properties of these states. If time permits, I will describe results on the classification and properties of 3d electronic SPT states with various other physically relevant symmetries.

Interacting electronic topological insulators in 3 dimensions

T. Senthil (MIT)

Chong Wang, Andrew C. Potter, and T. Senthil, Science (2014).

Chong Wang, T. Senthil, arxiv:1401.1142

Collaborators

Chong Wang,
grad student @ MIT

Andrew Potter,
grad student @ MIT ==>
Berkeley

Other related collaborations: A. Vishwanath, Cenke Xu, Michael Levin, N. Regnualt

Topological insulators 1.0

Free electron band theory:
two distinct insulating phases of electrons in the presence of time reversal symmetry.

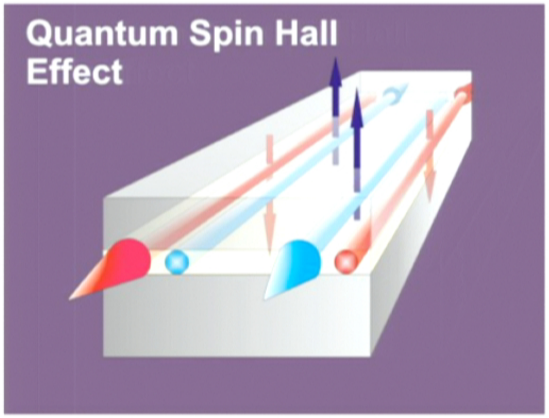
(i) Conventional Band Insulator

(ii) Topological Band Insulator (TBI)

In 3d the TBI requires spin-orbit coupling which destroys conservation of all components of spin (but preserves time reversal).

Properties of topological insulators

Usual characterization: Non-trivial surface states with gapless excitations protected by some symmetry



2d: `helical edge modes'

3d: odd number of Dirac cones

Topological insulators 2.0

Strongly correlated topological insulators

Interaction dominated phases as topological insulators?

Move away from the crutch of free fermion Hamiltonians and band topology.

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Some questions about interacting topological insulators

1. Are there new phases that have no non-interacting counterpart?

2. Physical properties?

3. Experimental realization?

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First refine the question

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How to generalize 3d topological insulators to interacting electrons?

Some simple guidelines.

1. Same realistic symmetry as Topological Band Insulator (TBI):
charge conservation, time reversal

2. TBI phases have simple bulk excitations - no fractional quantum numbers or exotic statistics
Precise: ``short range entangled'' phases.

How to generalize 3d topological insulators to interacting electrons?

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charge conservation, time reversal

2. TBI phases have simple bulk excitations - no fractional quantum numbers or exotic statistics
Precise: ``short range entangled'' phases.

Minimal interacting generalization of TBI:

- interacting systems with same symmetry as TBI but with a bulk gap and no exotic bulk excitations, i.e, with short range entanglement.

If either of these restrictions is removed there will be a richer set of answers but this is the `minimum' generalization that is also practical.

Aside: short versus long range entangled phases

``Exotic'' gapped Phases of Matter

- phases with ``intrinsic'' topological quantum order, fractional quantum numbers (eg, fractional quantum Hall state, gapped quantum spin liquids)

Emergent non-local structure in ground state wavefunction:

Characterize as ``long range quantum entanglement''

Topological band insulators do not have ``intrinsic'' topological order and are ``short range entangled''.

Some questions about interacting topological insulators

1. Are there new phases that have no non-interacting counterpart?

First refine the question

2. Physical properties?

3. Experimental realization?

Refined question: Phases and properties of interacting `short range entangled' electronic topological insulators with `realistic' symmetry (charge conservation, time reversal) ?

Terminology: ``Symmetry Protected Topological'' (SPT) phases.

Plan of talk

1. Lightning review of free fermion topological insulators in 3d

2. Lightning review of bosonic topological insulators in 3d

3. 3d interacting electron TIs

- new Z_2^3 classification

- description of the new interacting TIs

4. 3d fermionic interacting TI/TSc with other symmetries: Beyond the 10-fold way.

5. Toward materials

Review: free fermion 3d topological insulators

Characterize by

1. presence/absence of non-trivial surface states

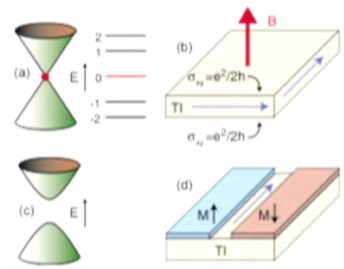
2. EM response

Surface states: Odd number of Dirac cones

Trivial gapped/localized insulator not possible at surface so long as T-reversal is preserved (even with disorder)

Review: free fermion topological insulators

EM response: Surface quantum Hall Effect



If surface gapped by B-field/proximity to magnetic insulator, surface Hall conductance

$$\sigma_{xy} = \left(n + \frac{1}{2}\right)\frac{e^2}{h}$$

Domain wall between opposite T-breaking regions: chiral edge mode of 2d fermion IQHE

Review: Free fermion topological insulators
Axion Electrodynamics

Qi, Hughes, Zhang, 09
Essin, Moore, Vanderbilt, 09

EM response of *any* 3d insulator

$$\mathcal{L}_{eff} = \mathcal{L}_{Max} + \mathcal{L}_{\theta} \quad (1)$$
$$\mathcal{L}_{\theta} = \frac{\theta}{4\pi^2}\vec{E}.\vec{B}$$

Under $\mathcal{T}$-reversal, $\theta \to -\theta$.
Periodicity $\theta \to \theta + 2\pi$: only $\theta = n\pi$ consistent with $\mathcal{T}$-reversal.
Domain wall with $\theta = 0$ insulator: Surface quantum Hall effect

$$\sigma_{xy} = \frac{\theta}{2\pi}$$

Free fermion TI: $\theta = \pi$.
Interpretation of periodicity:
$\theta \to \theta + 2\pi$: deposit a 2d fermion IQHE at surface.
Not a distinct state.

Consequences of axion response: Witten effect

External magnetic monopole in EM field:

θ term => monopole has electric charge $\theta/2\pi$.

(``Witten dyons'').

$$\begin{aligned} \mathcal{L}_\theta &= \frac{\theta}{4\pi^2}\vec{E}.\vec{B} \\ &= -\frac{\theta}{4\pi^2}\vec{\nabla}A_0.\vec{B} + \ldots. \\ &= \frac{\theta}{4\pi^2}A_0\vec{\nabla}.\vec{B} \end{aligned}$$

A very useful detour: Bosonic topological insulators

Useful stepping stone to interacting fermionic TIs.

Many new and useful non-perturbative ideas to discuss topological insulator/SPT states.

Old example: Haldane spin-1 chain
(Symmetry protected dangling spin-1/2 edge states).

d > 1: Progress in classification
1. Group Cohomology (Chen, Gu. Liu, Wen, 2011)
2. Chern-Simons approach in d =2 (Lu, Vishwanath, 2012).

Here we will need some physical ideas on d = 3 boson TI/SPTs.

Physics of 3d boson topological insulators

Vishwanath, TS, 2012

1. Quantized magneto-electric effect (eg: axion angle $\theta = 2\pi, 0$)

2. Emergent exotic (eg: fermionic, Kramers or both) vortices at surface, ...

3. Related exotic bulk monopole of external EM field (fermion, Kramers, or both) (Wang, TS, 2013; Metlitski, Kane, Fisher, 2013).

Explicit construction in systems of coupled layers: Wang, TS (2013).

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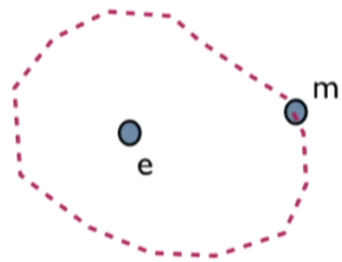
Surface topological order of 3d SPTs

3d SPT surface can have intrinsic topological order though bulk does not (Vishwanath, TS, 2012).



Phase of π

Resulting symmetry preserving gapped surface state realizes symmetry `anomalously' (cannot be realized in strict 2d; requires 3d bulk).

Targetting such `anomalous' surface topological ordered state has been fruitful in microscopic constructions of boson TIs
(Wang, TS, 2013; Burnell, Chen, Fidkowski, Vishwanath, 2013)

Electronic topological insulators
(Chong Wang, A.Potter, TS 2013)

The problem

Electronic insulators with no bulk topological order/fractionalization (``short range entangled").

Realistic Symmetry: charge conservation, T-reversal (strong spin orbit => spin not conserved).

Band theory: Z_2 classification

Beyond band theory: Strong correlations + strong spin-orbit

How many such phases for there with strong interactions?

What are their physical properties?

The answer

3d electronic insulators with charge conservation/T-reversal classified by Z_2^3 (corresponding to total of 8 distinct phases).

3 `root' phases:

Familiar topological band insulator, two new phases obtained as electron Mott insulators where spins form a spin-SPT (topological paramagnets).

Topological Insulator	**Representative surface state**	**$\mathcal{T}$-breaking transport signature**	**$\mathcal{T}$-invariant gapless super-conductor**
Free fermion TI	Single Dirac cone	$\sigma_{xy} = \frac{\kappa_{xy}}{\kappa_0} = \pm 1/2$	None
Topological paramagnet I ($eTmT$)	$\mathbb{Z}_2$ spin liquid with Kramers doublet spinon(e) and vison(m)	$\sigma_{xy} = \kappa_{xy} = 0$	$N = 8$ Majorana cones
Topological paramagnet II ($e_f m_f$)	$\mathbb{Z}_2$ spin liquid with Fermionic spinon(e) and vison(m)	$\sigma_{xy} = 0; \frac{\kappa_{xy}}{\kappa_0} = \pm 4$	$N = 8$ Majorana cones

Obtain all 8 phases by taking combinations of root phases.

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Proof

Bulk EM response

Start with EM response of *any* 3d insulator

$$\mathcal{L}_{eff} = \mathcal{L}_{Max} + \mathcal{L}_{\theta} \tag{1}$$
$$\mathcal{L}_{\theta} = \frac{\theta}{4\pi^2}\vec{E}.\vec{B}$$

Under $\mathcal{T}$-reversal, $\theta \to -\theta$.
Periodicity $\theta \to \theta + 2\pi$: only $\theta = 0, \pi$ consistent with $\mathcal{T}$-reversal.

If there are 2 distinct insulators with $\theta = \pi$, can combine to make $\theta = 0$ insulator.

=> To look for new insulators, sufficient to restrict to $\theta = 0$.

Bulk magnetic monopole

Witten effect => monopole charge $\frac{\theta}{2\pi}$.
At $\theta = 0$, monopole has charge 0.
Time reversal:

$$\begin{aligned} \mathcal{T}^{-1} m \mathcal{T} &= e^{i\alpha} m^\dagger && (1) \\ \mathcal{T}^{-1} m^\dagger \mathcal{T} &= e^{-i\alpha} m && (2) \end{aligned}$$

But can combine with (magnetic) gauge transformation to set $\alpha = 0$.
=> Monopole transforms trivially under T-reversal.
Symmetries of monopole fixed.

Remaining possibilities:
Bosonic versus fermionic statistics of the monopole.

Claim

Bosonic monopole:

Only allow for topological paramagnets* as new root states (simple proof next few slides)

Fermionic monopole:

Impossible in strictly 3d interacting topological insulators (somewhat difficult proof, see Wang, Potter, TS Science paper Supplement).

*Reminder: Topological paramagnet = electronic Mott insulator where spins form a bosonic SPT phase.

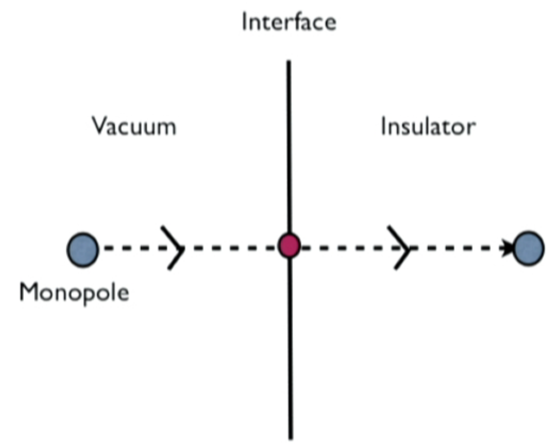
Bosonic magnetic monopole: implications for surface effective theory

Tunnel monopole from vacuum into bulk

Tunneling event leaves behind surface excitation which has charge-0 and is a boson.

A convenient surface termination- a surface superconductor*

Monopole tunneling leaves behind hc/e vortex which is a boson (and transforms trivially into a hc/e antivortex under T-reversal).



*More details : see Appendix of Wang, Potter, TS, Science 2014.

Symmetry preserving surface

Disorder the superconducting surface:

Condense the bosonic hc/e vortex.

Result is a symmetry preserving insulating surface with ``intrinsic'' topological order.

hc/e vortex condensate => Charge quantized in units of e.

=> Surface TQFT: every topological sector can be made neutral
(integer charge => bind physical electrons to make neutral).

Surface TQFT = (1, ε, ...) x (1,c) = (Neutral boson TQFT) x (1,c)

=> Bulk SPT order is same as for neutral boson SPT (supplemented by physical electron).

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Neutral boson SPTs in electronic systems

From electrons form neutral composites which are bosons, and let these bosons form a boson SPT.

Describe as electron Mott insulators where spins form an SPT.

Strong spin-orbit => spin system only has T-reversal symmetry.

``Topological paramagnets" protected by time reversal symmetry.

Classification of 3d electron TI:

(band insulator classification) x (such topological paramagnets classification)

= Z_2 x (such topological paramagnets classification)

Time reversal invariant Topological paramagnets

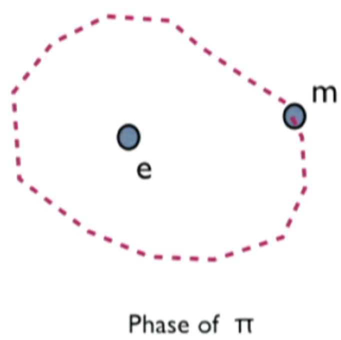
Classified by Z_2^2 with two root states conveniently characterized by surface topological order (Vishwanath, TS, 2012, Wang, TS, 2013) of deconfined Z_2 gauge theory.

1. Topological paramagnet-I

Z_2 topological order where both e and m are Kramers

2. Topological paramagnet-II (beyond `cohomology')

``All fermion'' Z_2 topological order where all topological particles are fermions



Classification of 3d electron TI

(Band insulator classification) x (T-reversal symmetric topological paramagnets classification)

= Z_2 x (such topological paramagnets classification)

= $Z_2 \times Z_2^2$ = Z_2^3

Physical characterization of the 8 interacting 3d TIs

4 insulators with $\theta = 0$: Trivial, 3 topological paramagnets

4 insulators with $\theta = \pi$: Topological band insulator and its combinations with the 3 topological paramagnets

How to tell in experiments?

Symmetry preserving surface topological order?

Conceptually powerful characterization of surface but not very practical

Alternate: Break symmetry at surface to produce a simple state without topological order (eg: deposit ferromagnet or superconductor)

Unique experimental fingerprint!

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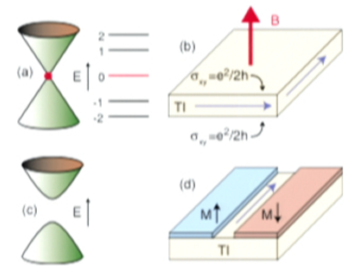
Hall transport signatures of interacting TIs

Break T explicitly to get a `trivial' surface:

Surface electrical Hall conductivity:
Band TI: σ_{xy} = 1/2 (related to $\theta = \pi$)
Topological paramagnets: σ_{xy} = 0



Surface thermal Hall conductivity*:
Band TI: κ_{xy} = 1/2
Topological Paramagnet-I: κ_{xy} = 0
Topological Paramagnet-II: κ_{xy} = 4

*(mod 8)

Understanding induced quasiparticle nodes

Argue in reverse:

Start with surface SC theory with 4 Dirac nodes:

$$\mathcal{L}_{free} = \sum_{i=1}^{4} \psi_i^\dagger (p_x \sigma^x + p_y \sigma^z) \psi_i, \tag{1}$$

with time-reversal acting as

$$\mathcal{T} \psi_i \mathcal{T}^{-1} = i\sigma_y \psi_i^\dagger. \tag{2}$$

Time reversal symmetric free fermion perturbations cannot gap the nodes (same as surface of certain free fermion topological SC).

But interactions can induce a gap while preserving symmetry.

Understanding induced quasiparticle nodes

Enlarge symmetry to $U(1) \times \mathcal{T}$ with $U(1)$ acting as

$$U_\theta \psi U_\theta^{-1} = e^{i\theta}\psi \tag{1}$$

Consider pairing mass that breaks $U(1)$ and $\mathcal{T}$ but preserves a combination:

$$\mathcal{L}_{gap} = i\Delta \sum_{i=1}^{4} \psi_i \sigma_y \psi_i + h.c. \tag{2}$$

Now attempt to disorder the broken symmetry => proliferate vortices in the pairing order parameter.

However vortices have zero modes which restrict which kinds can condense.

Understanding induced quasiparticle nodes

For N = 4 Dirac nodes, can condense strength-2 vortices.

Result: Z_2 topological order where e and m are both Kramers (eTmT) = surface topological order of Topological Paramagnet-I.

(Can now get rid of auxiliary $U(1)$).

Closely related to a result using other methods: Classification of interacting topological SC with time reversal by Fidkowski, Chen, Vishwanath (2013)

Other symmetries: Beyond the 10-fold way

Ideas discussed above enable us to determine stability to interactions of all the 3d free fermion topological insulators/SC represented in ``10-fold way'' and in many cases to get the full classification. (Wang, TS, arxiv:1401:1142)

SC with time reversal (class D III): Free fermion classification: Z

Interactions reduce this to Z_{16} (Fidkowski, Chen, Vishwanath (2013)).

Our arguments give elementary understanding of this result, and to generalize to other symmetries.

Results for other symmetries (Wang, TS, arxiv:1401:1142)

Symmetry class	Reduction of free fermion states	Distinct boson SPT	Complete classification
$U(1)$ only (A)	0	0	0
$U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (AII)	$\mathbb{Z}_2 \to \mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^3$
$U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = 1$ (AI)	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$
$U(1) \times \mathbb{Z}_2^T$ (AIII)	$\mathbb{Z} \to \mathbb{Z}_8$	$\mathbb{Z}_2$	$\mathbb{Z}_8 \times \mathbb{Z}_2$
$U(1) \rtimes (\mathbb{Z}_2^T \times \mathbb{Z}_2^C)$ (CII)	$\mathbb{Z}_2 \to \mathbb{Z}_2$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^5$
$(U(1) \rtimes \mathbb{Z}_2^T) \times SU(2)$	0	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^4$
$\mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (DIII)	$\mathbb{Z} \to \mathbb{Z}_{16}$	0	Z_{16} (?)
$SU(2) \times \mathbb{Z}_2^T$ (CI)	$\mathbb{Z} \to \mathbb{Z}_4$	$\mathbb{Z}_2$	$Z_4 \times Z_2$ (?)

Example: SC with spin U(1) and T-reversal (class A III)
Z classification in free fermion becomes $Z_4 \times Z_2$ with interactions.

Many new insights:

1. In some cases there is no symmetry preserving surface topological order (SC with spin SU(2) and T-reversal): ``symmetry-enforced gaplessness.

2. new parton constructions of topological paramagnets suggesting possible `realistic' model, and may be material identifications (in progress).

Summary

1. Interacting electron TIs in 3d have a Z_2^3 classification

- apart from trivial and topological band insulators, 6 new TI phases with no non-interacting counterpart.

2. Unique experimental fingerprint:

Deposit ferromagnet at surface: Quantum `anomalous' ***electrical*** and ***thermal*** Hall effect of surface.
Deposit s-wave SC at surface: Presence/absence of 4 induced gapless Dirac nodes.

3. Progress in understanding interacting 3d fermion SPTs with many symmetries.

3. Hints to important open question:

what kinds of real electronic insulators may be these new 3d TIs?