

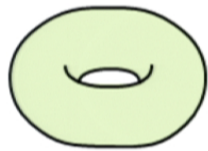
Title: Chiral spin liquid and emergent anyons in a Kagome lattice Mott insulator

Date: Feb 13, 2014 09:00 AM

URL: <http://pirsa.org/14020122>

Abstract: Topological phases in frustrated quantum spin systems have fascinated researchers for decades. One of the earliest proposals for such a phase was the chiral spin liquid put forward by Kalmeyer and Laughlin in 1987 as the bosonic analogue of the fractional quantum Hall effect. Elusive for many years, recent times have finally seen a number of models that realize this phase. However, these models are somewhat artificial and unlikely to be found in realistic materials. Here, we take an important step towards the goal of finding a chiral spin liquid in nature by examining a physically motivated model for a Mott insulator on the Kagome lattice with broken time-reversal symmetry. We first provide a theoretical justification for the emergent chiral spin liquid phase in terms of a network model perspective. We then present an unambiguous numerical identification and characterization of the universal topological properties of the phase, including ground state degeneracy, edge physics, and anyonic bulk excitations, by using a variety of powerful numerical probes, including the entanglement spectrum and modular transformations.

Emergence in complex systems



$$\vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

***Chiral spin liquid
and
emergent anyons
in a Kagome lattice Mott insulator***



Lukasz Cincio and Guifre Vidal

PERIMETER  INSTITUTE FOR THEORETICAL PHYSICS

with support from

JOHN TEMPLETON
FOUNDATION

OUTLINE

- Anyon hunter's wish list:

Ground state degeneracy

S and T matrices

Edge spectrum

- Hubbard model with magnetic field on Kagome

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➔
$$H = J_{HB} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + h_z \sum_i S_i^z + J_\chi \sum_{i,j,k \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) + \dots$$

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- Chiral spin liquid and emergent anyons

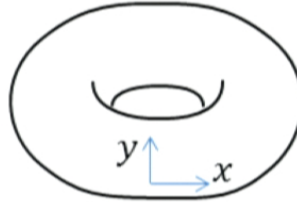
(Lukasz)

Ground state degeneracy
S and T matrices
Edge spectrum

Anyon hunter's wish list

- Ground state degeneracy on the torus

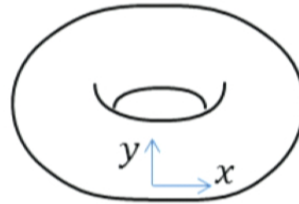
X.-G. Wen, 1989



Anyon hunter's wish list

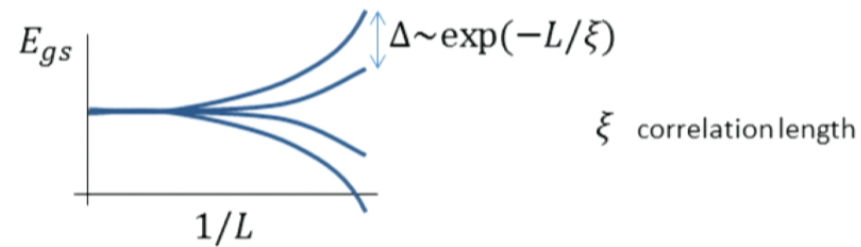
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example: toric code has 4 ground states

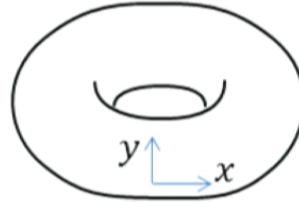
Finite size L breaks ground state degeneracy



Anyon hunter's wish list

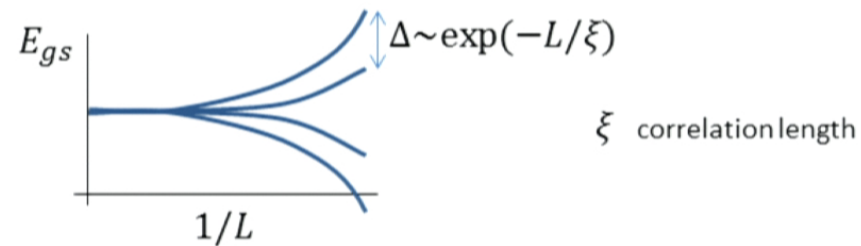
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In particular, if $L_x \gg L_y \gg \xi$

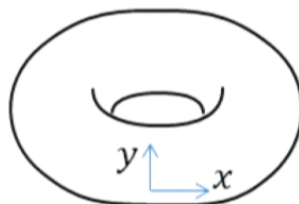
each "ground state" has a well-defined anyon flux i in x-direction



Anyon hunter's wish list

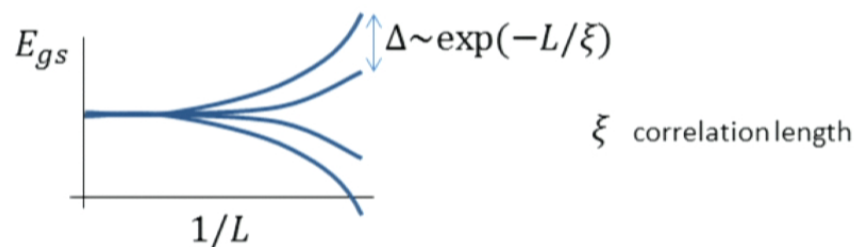
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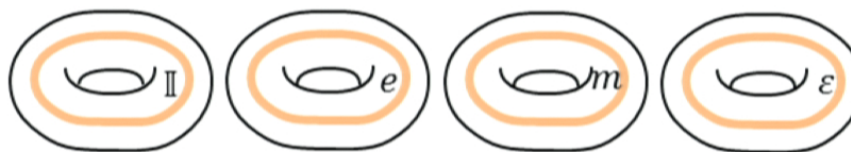
Finite size L breaks ground state degeneracy



In particular, if $L_x \gg L_y \gg \xi$

each "ground state" has a well-defined anyon flux i in x-direction

example: toric code $i = \mathbb{I}, e, m, \varepsilon$



Anyon hunter's wish list

- S and T matrices (generators of modular group)

Suppose we could create quasi-particles and exchange them,



self statistics

$$|\Psi\rangle \rightarrow e^{i\phi} |\Psi\rangle \text{ anyon}$$

$$|\Psi\rangle \rightarrow |\Psi\rangle \quad \text{boson}$$

$$|\Psi\rangle \rightarrow -|\Psi\rangle \quad \text{fermion}$$



mutual statistics

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topological T matrix

$$T_{ii} = \frac{1}{d_i} \text{ } i \text{ } \text{ (loop diagram) }$$

$$T = e^{-i\frac{2\pi}{24}c} \begin{bmatrix} \theta_{\mathbb{I}} & & & \\ & \theta_e & & \\ & & \theta_m & \\ & & & \theta_{\varepsilon} \end{bmatrix}$$

topological spin θ_i
topological central charge c

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topological spin θ_i

topological central charge c

$$T = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix}$$

$$c = 0$$



$$\theta_{\varepsilon} = -1$$

$$|\Psi\rangle \rightarrow -|\Psi\rangle$$



Anyon hunter's wish list

- S and T matrices (generators of modular group)

Suppose we could create quasi-particles and exchange or braid them



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topological S matrix

$$S_{ij} = \frac{1}{D} \text{ (diagram of two circles labeled } i \text{ and } j \text{ with arrows indicating braiding) }$$

$$S = \frac{1}{D} \begin{bmatrix} d_{\parallel} & d_e & d_m & d_{\varepsilon} \\ d_e & x & x & x \\ d_m & x & x & x \\ d_{\varepsilon} & x & x & x \end{bmatrix}$$

quantum dimensions d_i

total quantum dimension $D \equiv \sqrt{\sum_i d_i^2}$

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$$S = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

quantum dimensions $d_i = 1$

total quantum dimension $D = 2$

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$$2S_{em} = -1$$

$$|\Psi\rangle \rightarrow -|\Psi\rangle$$

fusion rules $i \times j \rightarrow k$

$$N_{ij}^k = \sum_r \frac{S_{im} S_{jm} S_{mk}^*}{S_{1m}}$$

Verlinde formula

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$$m \times e \rightarrow \varepsilon$$

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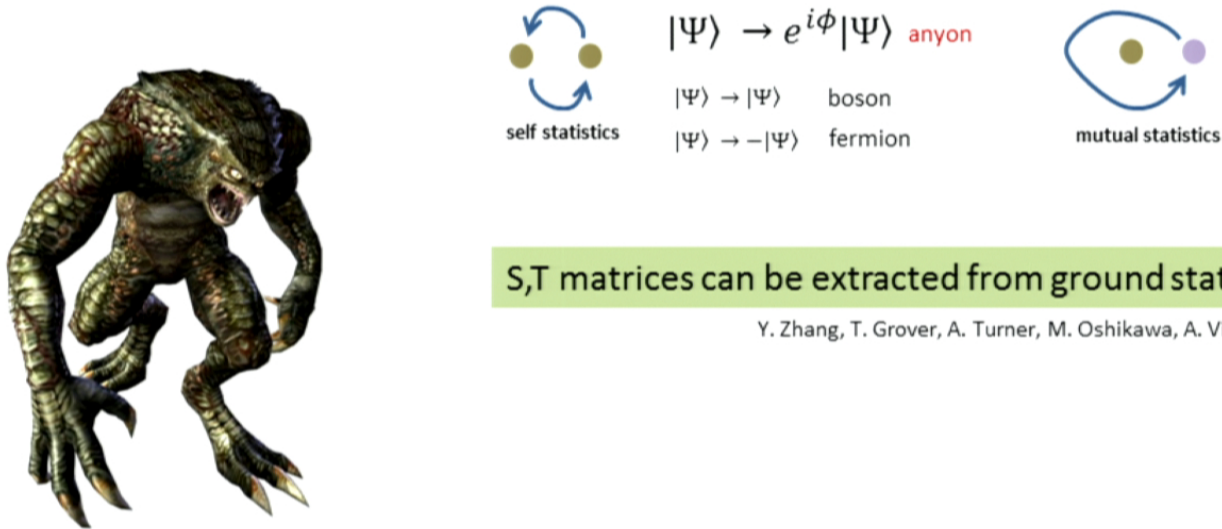
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Verlinde formula

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S,T matrices can be extracted from ground state overlaps

Y. Zhang, T. Grover, A. Turner, M. Oshikawa, A. Vishwanath, PRB 2012

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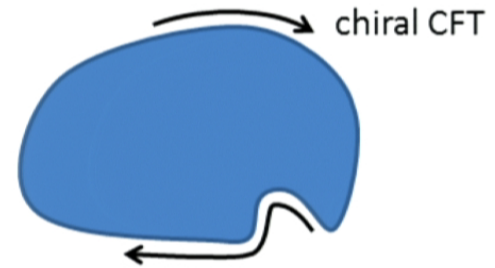
example: on square lattice

$$S_{ij} = \left\langle \text{Diagram } i \mid \text{Diagram } j \right\rangle$$

Anyon hunter's wish list

- Edge theory

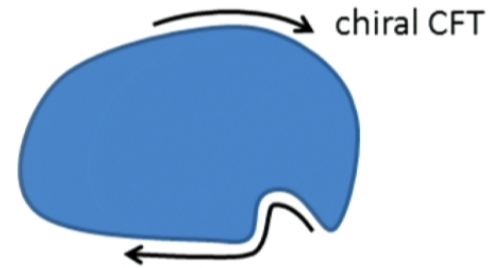
Identify the protected
gapless edge state



Anyon hunter's wish list

- Edge theory

Identify the protected gapless edge state



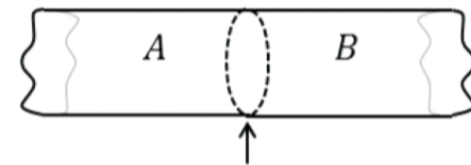
H. Li, F. D. M. Haldane, PRL 2008
X.-L. Qi, H. Katsura, A. W. W. Ludwig, PRL 2012

B. Swingle, T. Senthil, PRB 2012
A. Chandran, V. Khemani, S.L. Sondhi, arXiv 2013



physical edge

effective edge
Hamiltonian H_{edge}



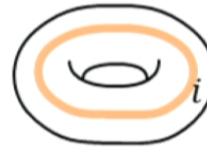
entanglement cut

density matrix
for part A $\rho_A = e^{-H_{entanglement}}$

We wish
we could:



obtain a complete set of ground states on a torus



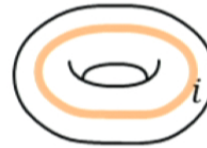
and then extract

- S,T modular matrices (topological excitations)
- entanglement spectrum (edge theory)

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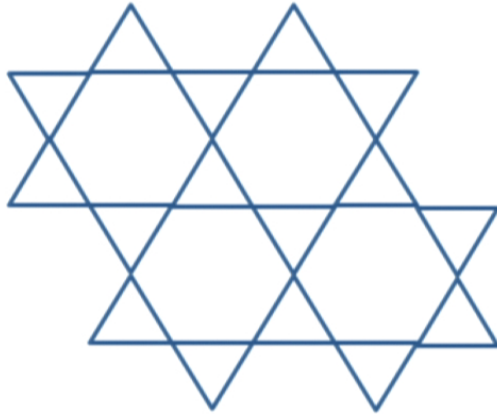


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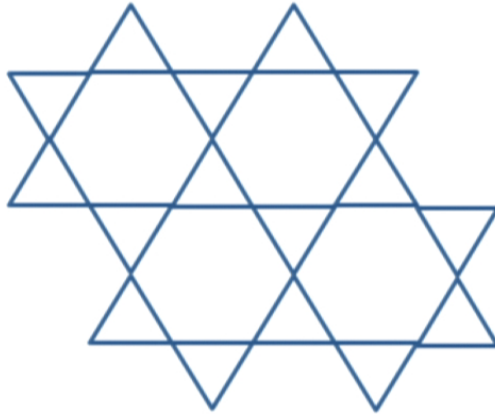


Hubbard model with magnetic field on the Kagome lattice



$$H = - \sum_{\langle i,j \rangle, \sigma} (t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + t_{ij}^* c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{h_z}{2} \sum_i (n_{i\uparrow} - n_{i\downarrow})$$

Hubbard model with magnetic field on the Kagome lattice



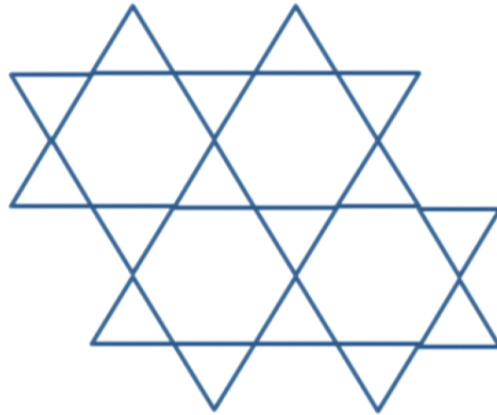
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effect of
magnetic field:

- Zeeman term h_z
- flux Φ
through each
elementary triangle

$$t_{ij} t_{jk} t_{ki} = t^3 \exp(i\Phi)$$

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effect of
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- Zeeman term h_z

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through each
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$$t_{ij} t_{jk} t_{ki} = t^3 \exp(i\Phi)$$

- Half filling $\langle n \rangle = 1$
- Perturbation theory in t/U (large U limit)

O.I. Motrunich, PRB 2006

$$H = J_{HB} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + h_z \sum_i S_i^z + J_\chi \sum_{i,j,k \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) + \dots$$

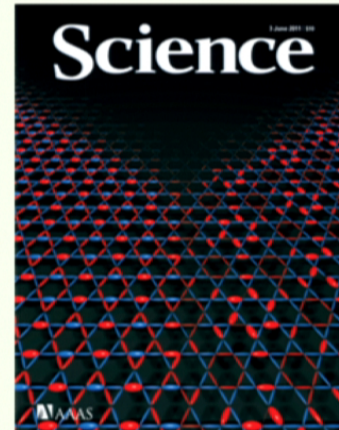
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$J_{HB} \sim \frac{t^2}{U}$
 $J_\chi \sim \Phi \frac{t^3}{U^2}$
 subleading

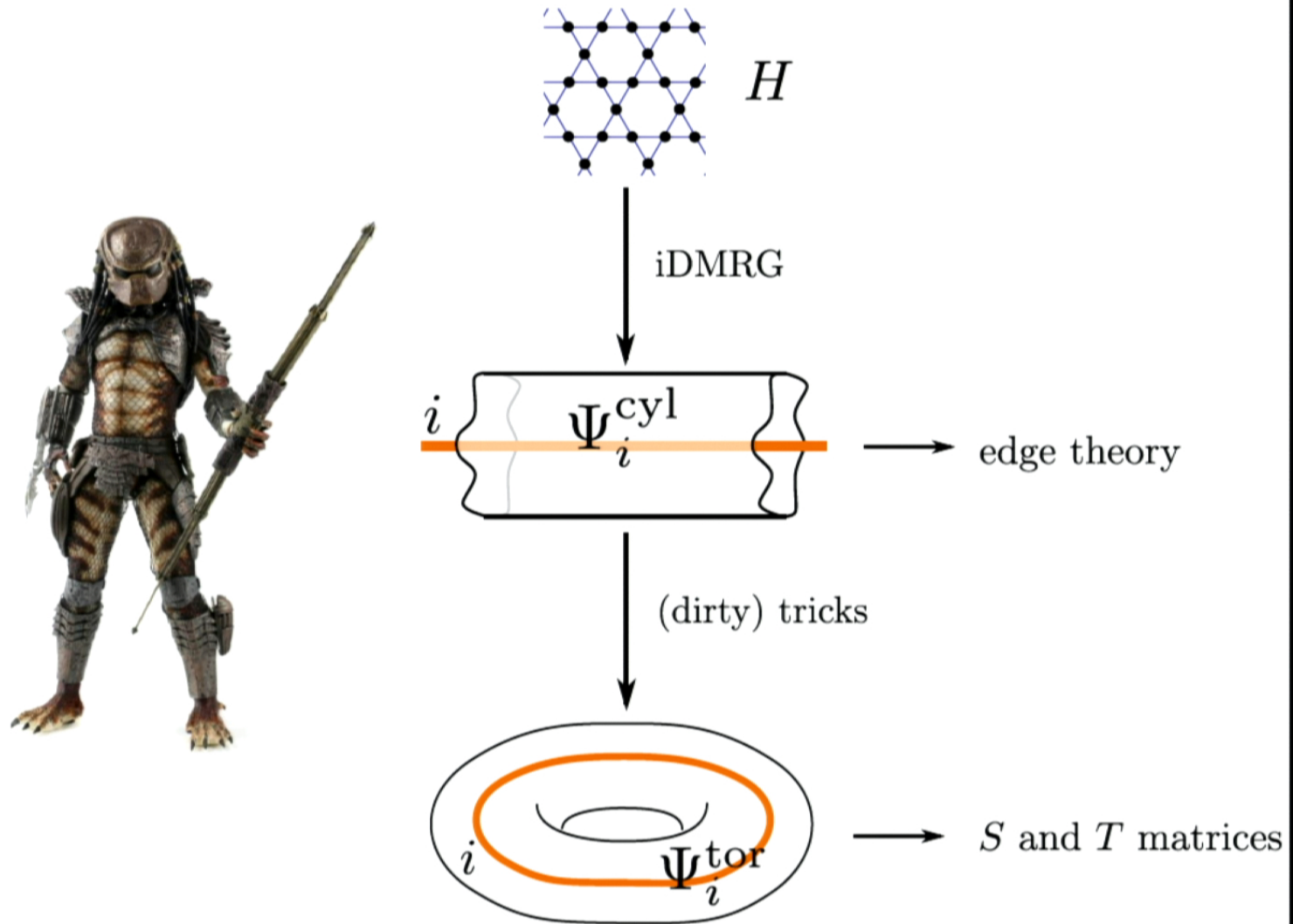
$$h_z = 0, \Phi = 0$$

$$H = \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

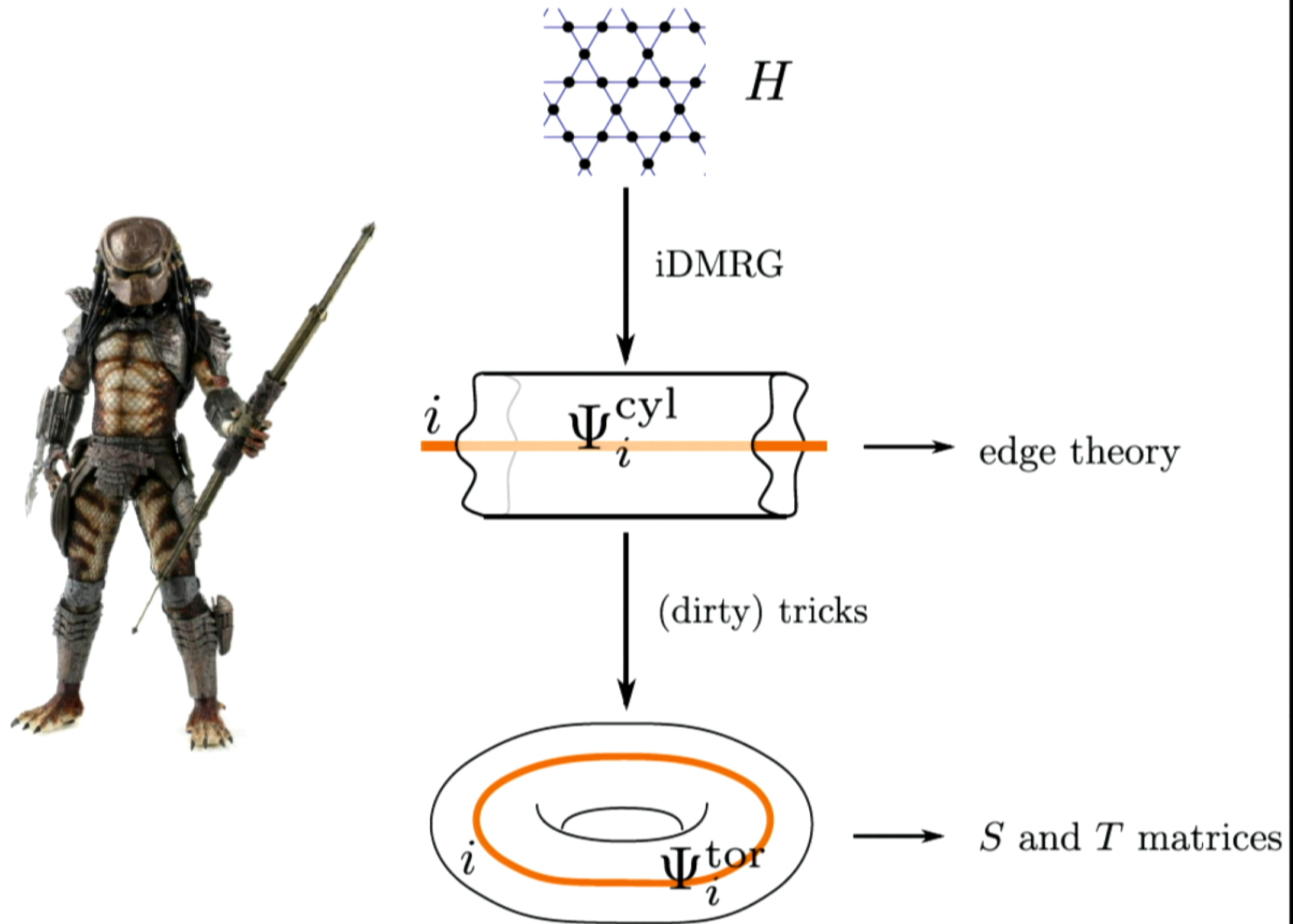
“Spin-liquid ground state of the
 $s = 1/2$ Kagome Heisenberg Antiferromagnet”
 S. Yan, D. A. Huse, S. R. White, *Science* (2011)



ANYON HUNTER'S WEAPON

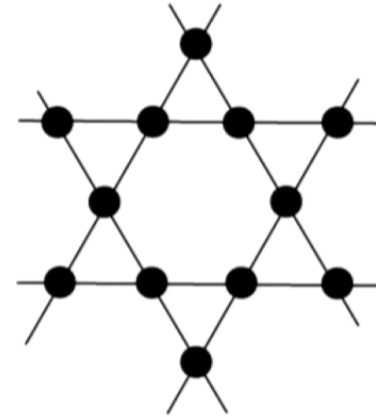


ANYON HUNTER'S WEAPON



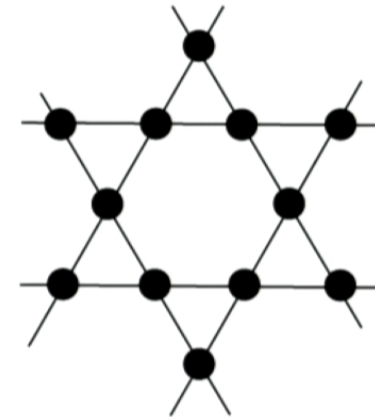
MODEL

$$\begin{aligned} H &= \cos \theta \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \\ &+ \sin \theta \sum_{i,j,k \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) \\ &+ h_z \sum_i S_i^z \end{aligned}$$

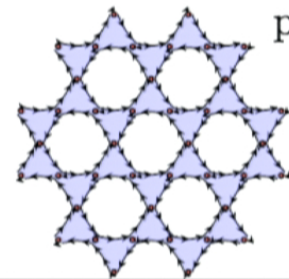
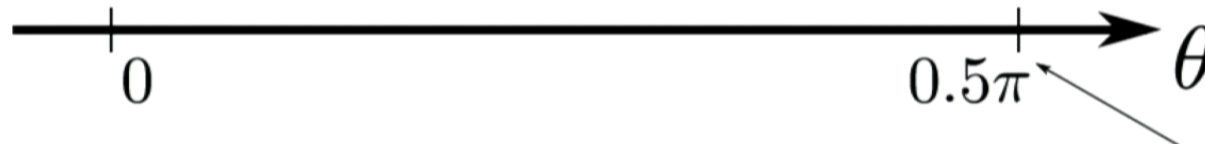


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 \end{aligned}$$



- so far ($h_z = 0$):



purely chiral point

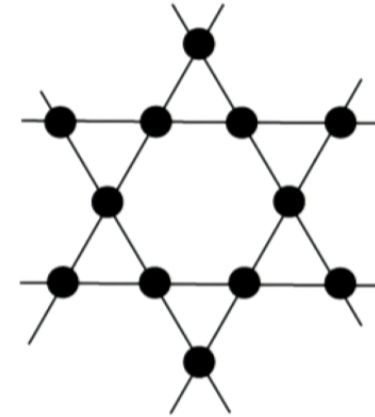
$$H = \sum_{i,j,k \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

speculation:
chiral spin liquid

network model
argument

MODEL

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 H = & \cos \theta \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \\
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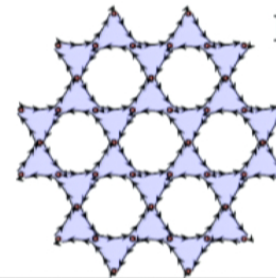
• so far ($h_z = 0$):

CSL



$H = \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$
 possible $\mathbb{Z}_2$ spin liquid

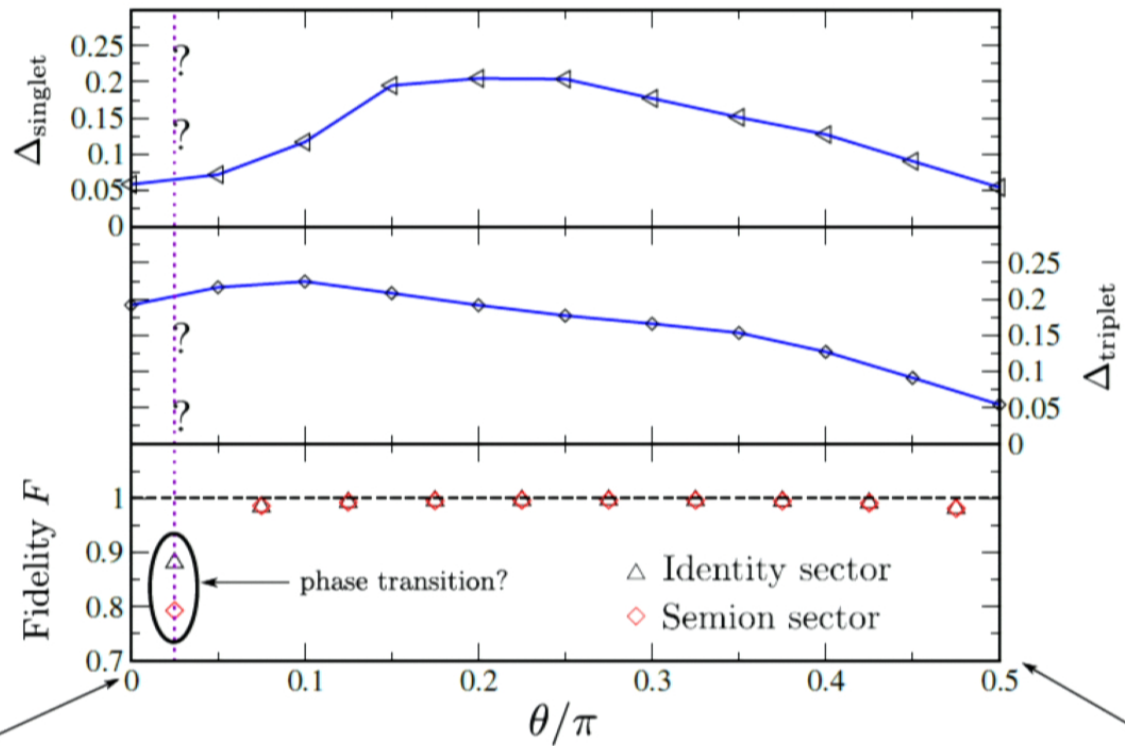
S. Yan, D. A. Huse, S. R. White, Science (2011)
 H.-C. Jiang, Z. Wang, L. Balents, Nature Physics (2012)
 S. Depenbrock, I. P. McCulloch, U. Schollwöck, PRL (2012)



purely chiral point:

$H = \sum_{i,j,k \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$
 speculation:
 chiral spin liquid
 network model
 argument

GAPS AND FIDELITY



Heisenberg point

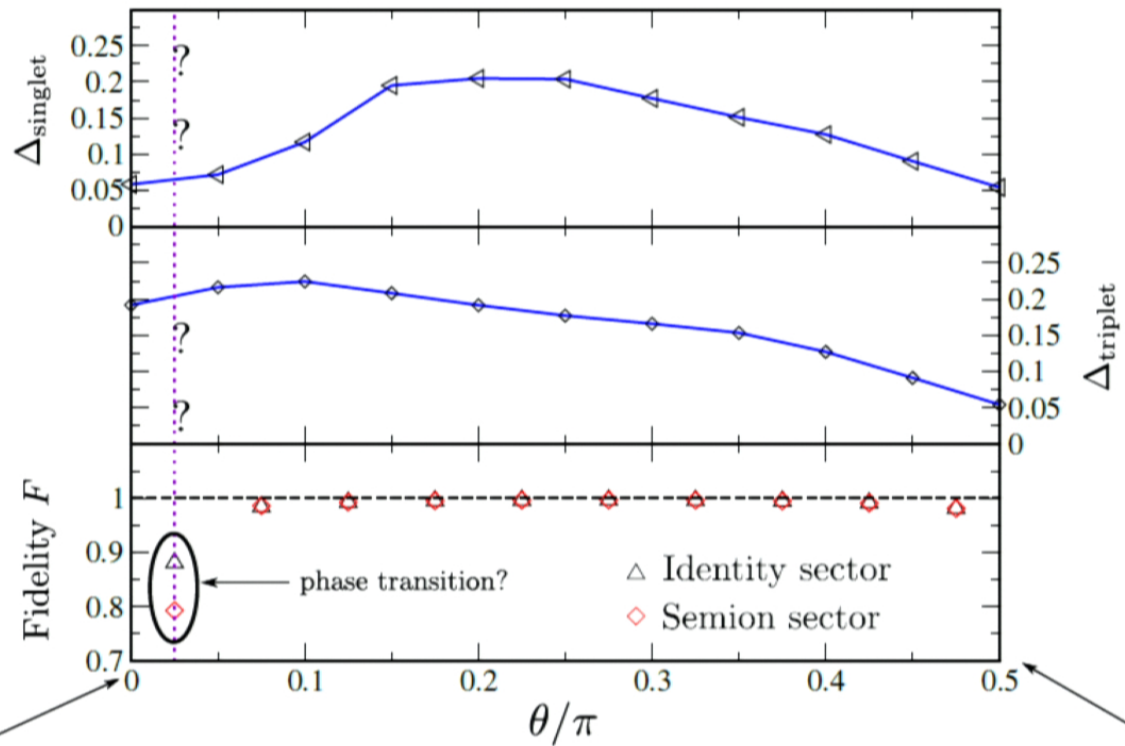
$$H = \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

- Fidelity: $F(\theta) = \langle \Psi_a(\theta - \epsilon) | \Psi_a(\theta + \epsilon) \rangle$

pure chiral point

$$H = \sum_{i,j,k \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

GAPS AND FIDELITY



Heisenberg point

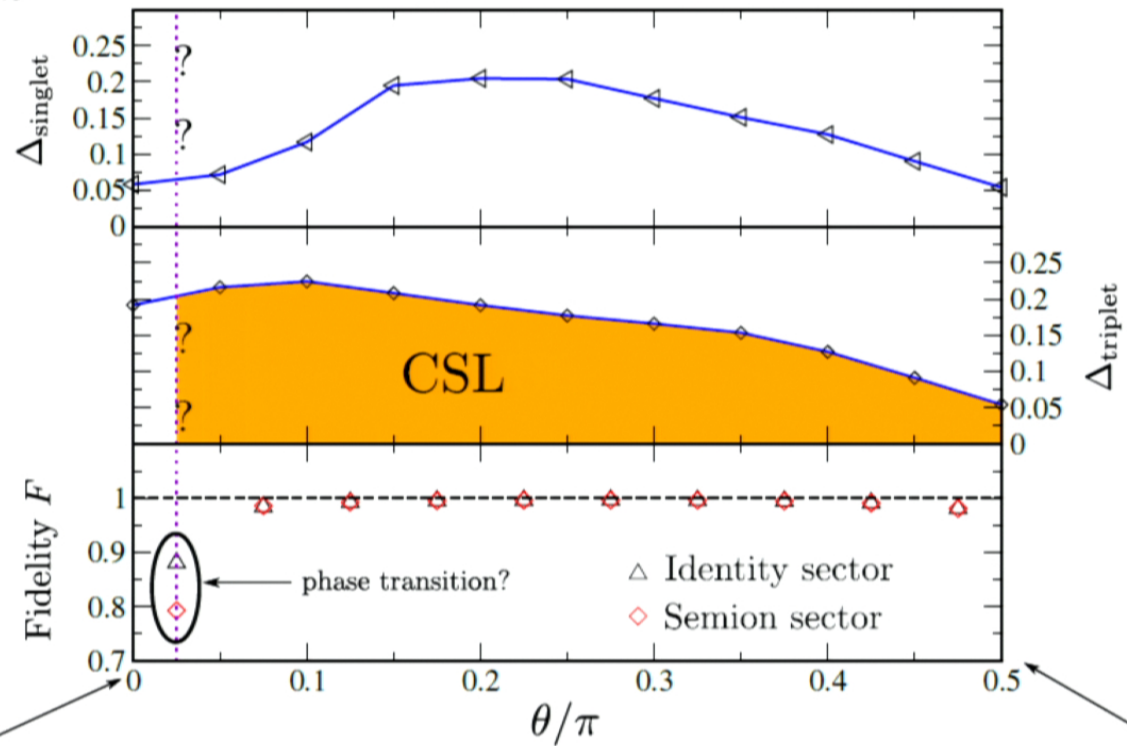
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pure chiral point

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$\theta-h_z$ PHASE DIAGRAM



Heisenberg point

$$H = \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

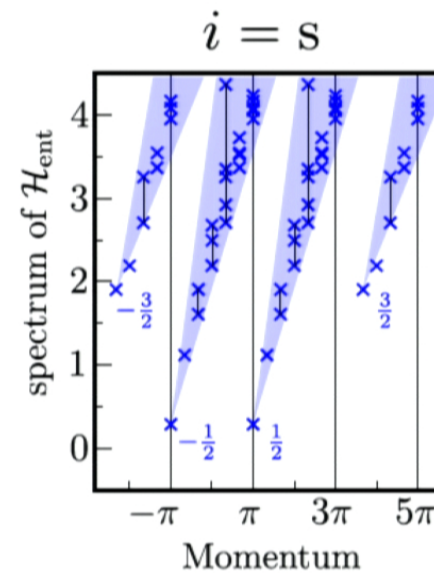
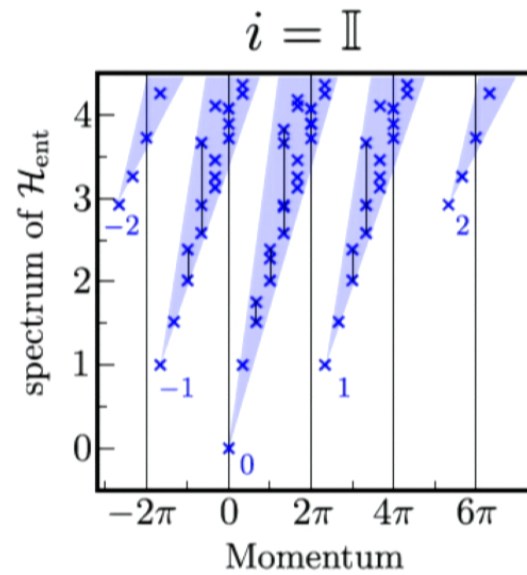
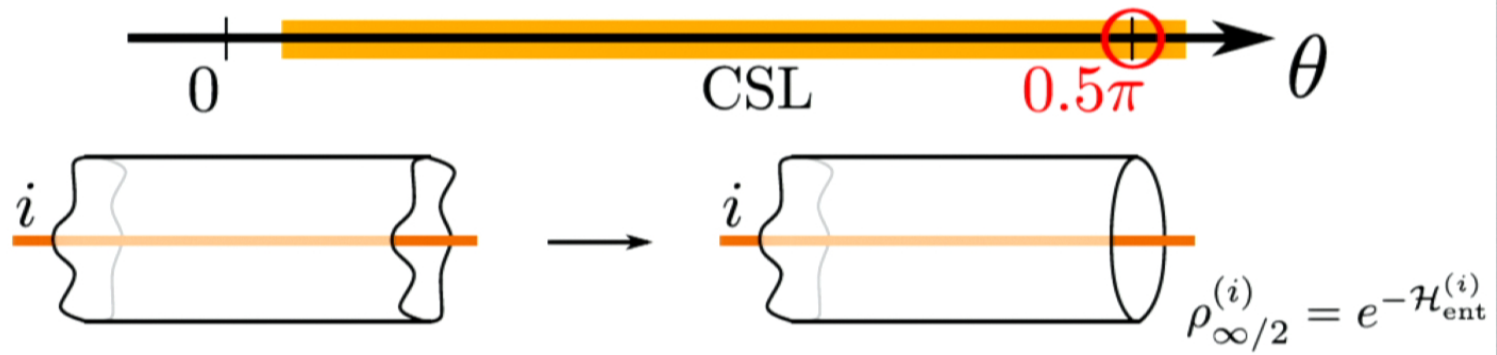
• Fidelity: $F(\theta) = \langle \Psi_a(\theta - \epsilon) | \Psi_a(\theta + \epsilon) \rangle$

• $h_c \geq \Delta_{\text{triplet}}$

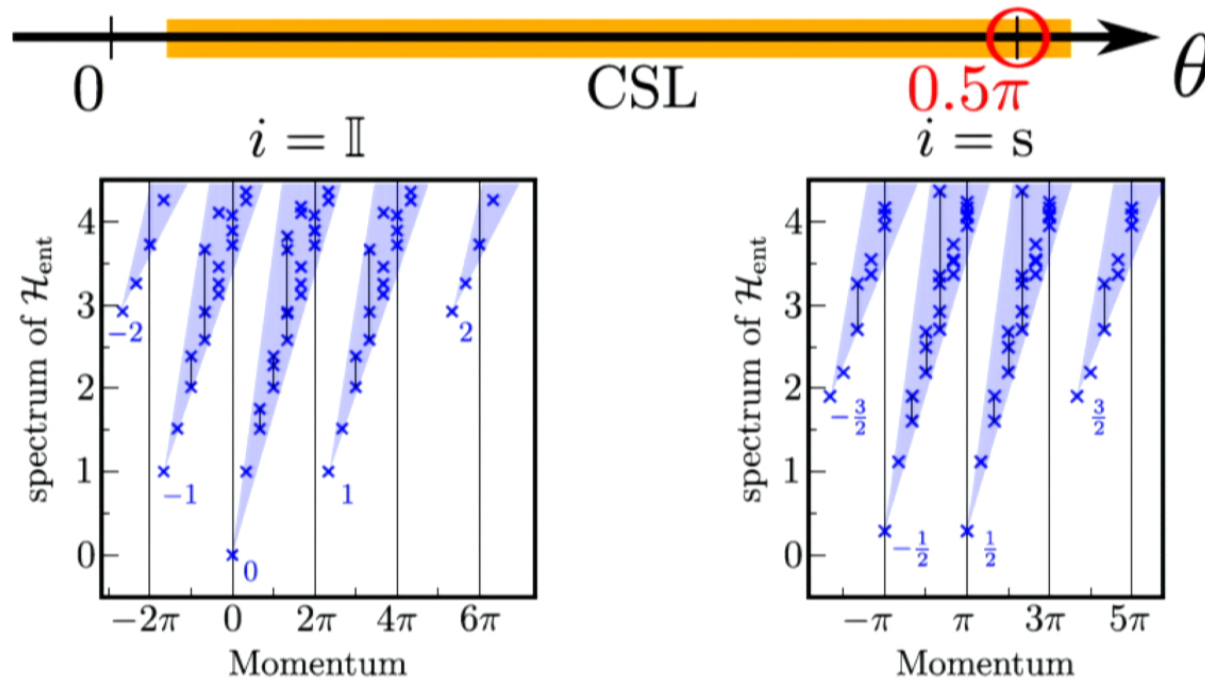
pure chiral point

$$H = \sum_{i,j,k \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

ENTANGLEMENT SPECTRUM



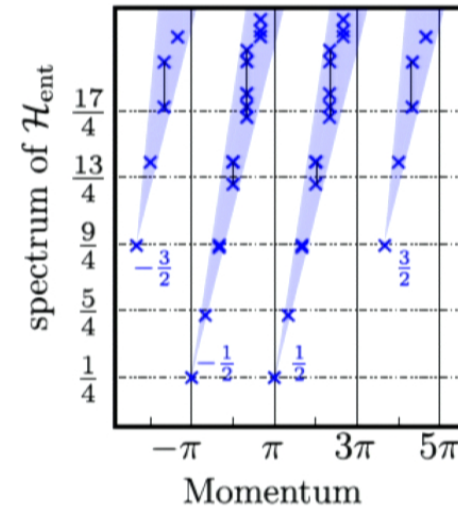
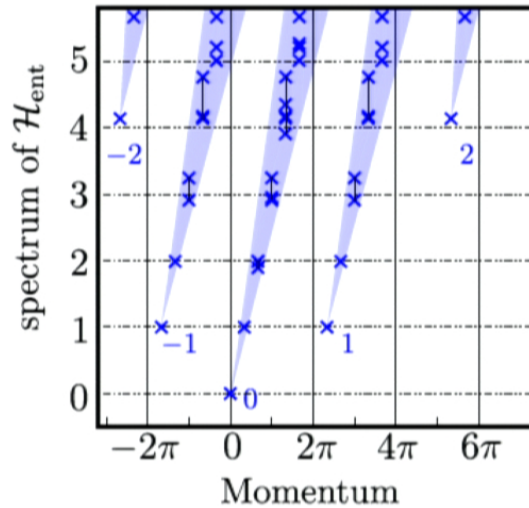
ENTANGLEMENT SPECTRUM



identification: chiral $\text{SU}(2)_1$ Wess-Zumino-Witten CFT

- sequence of degeneracies in momentum: 1-1-2-3-5-...
- $\text{SU}(2)$ multiplets
- **integer** reps. of the spin (identity) and **half-integer** (semion)

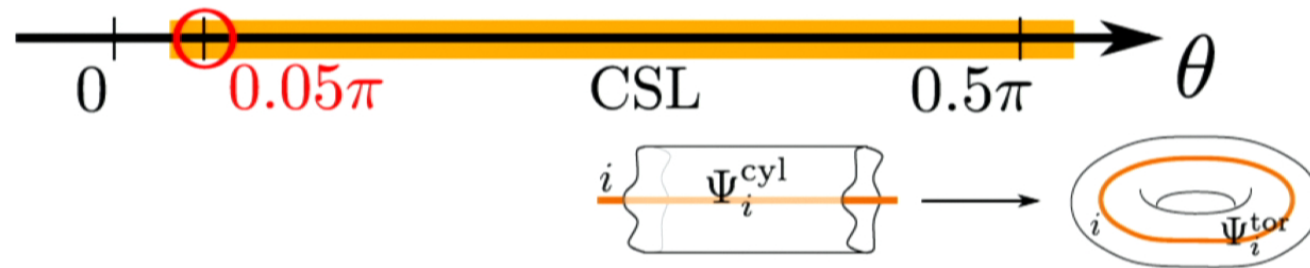
ENTANGLEMENT SPECTRUM



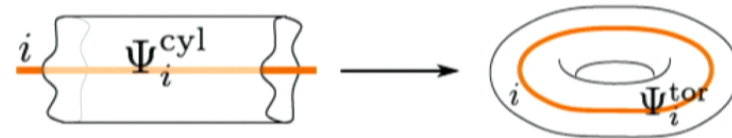
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S AND T MATRICES



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$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$+ \frac{10^{-3}}{\sqrt{2}} \begin{bmatrix} -4 & -5 \\ -4 & 4 + 6i \end{bmatrix}$$

$$T = e^{-i \frac{2\pi}{24} \cdot 1} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$$\times \left(e^{i \frac{2\pi}{24} 0.012} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i 0.002\pi} \end{bmatrix} \right)$$

chiral semion
(exact)

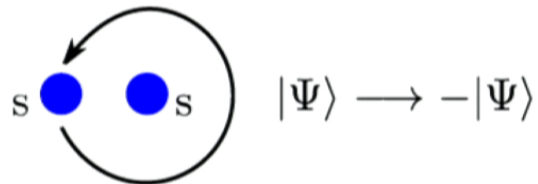
deviation

S AND T MATRICES



$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} + \frac{10^{-3}}{\sqrt{2}} \begin{bmatrix} -4 & -5 \\ -4 & 4 + 6i \end{bmatrix}$$

- quantum dimensions
 $d_{\mathbb{I}} = 1, d_s = 1$
- total quantum dimension
 $D = \sqrt{2}$
- mutual statistics



- fusion rules
 $s \times s = \mathbb{I}$



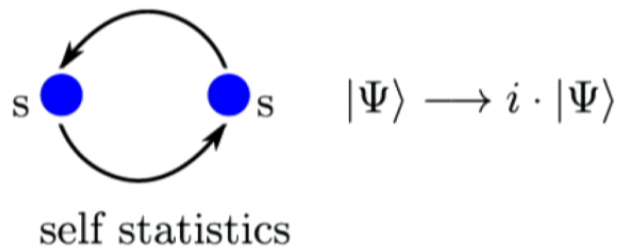
S AND T MATRICES



$$T = e^{-i\frac{2\pi}{24} \cdot 1} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \times \left(e^{i\frac{2\pi}{24} 0.012} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i0.002\pi} \end{bmatrix} \right)$$

- topological central charge
 $c = 1$
- topological spin

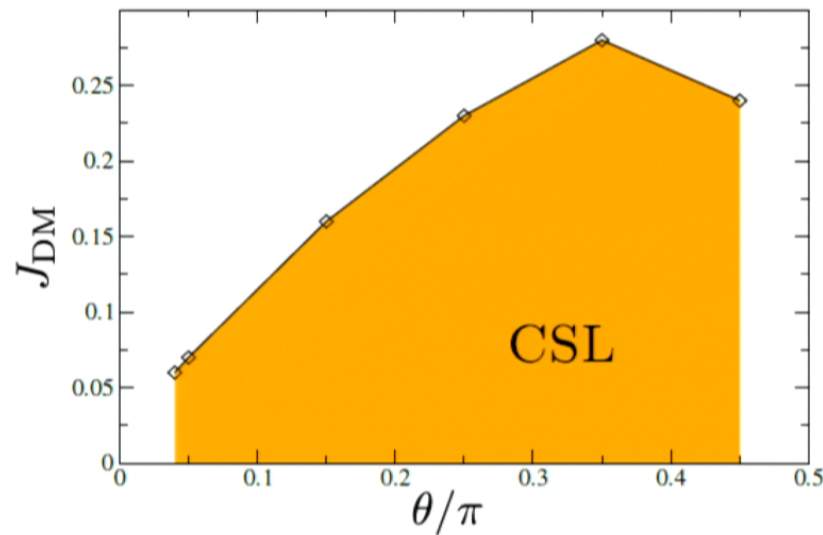
$$\theta_s = i$$



PERTURBATIONS

$$\begin{aligned}
 H &= \cos \theta \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \\
 &+ \sin \theta \sum_{i,j,k \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) + H_{\text{int}}
 \end{aligned}$$

- Dzyaloshinskii-Moriya interaction $H_{\text{int}} = J_{\text{DM}} \sum_{i < j} \hat{z} \cdot (\vec{S}_i \times \vec{S}_j)$



PERTURBATIONS

$$\begin{aligned} H &= \cos \theta \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \\ &+ \sin \theta \sum_{i,j,k \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) + H_{\text{int}} \end{aligned}$$

$$\bullet \quad H_{\text{int}} = J_{\text{NNN}} \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j \quad J_{\text{NNN}} \in [-0.1, 0.27]$$

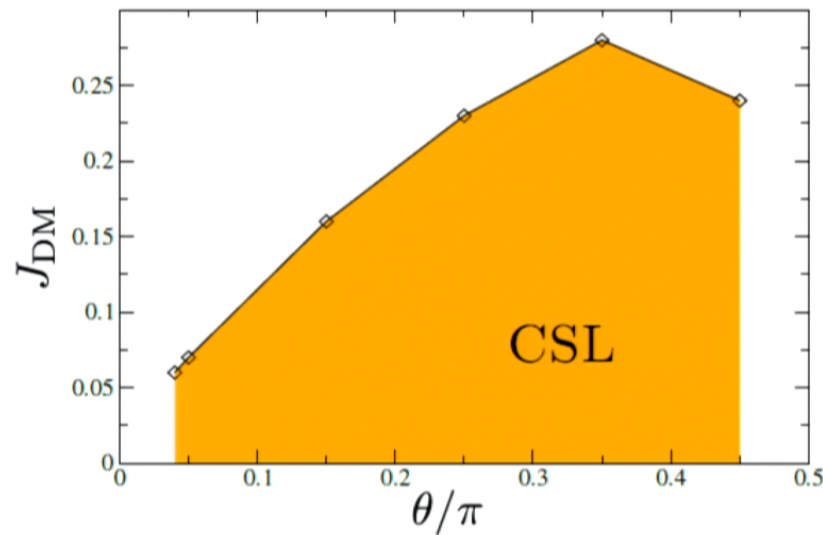
$$\bullet \quad H_{\text{int}} = J_z \sum_{\langle i,j \rangle} S_i^z S_j^z \quad J_z \in [-1.2, 0]$$

$$(\theta = 0.15\pi)$$

PERTURBATIONS

$$\begin{aligned}
 H &= \cos \theta \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \\
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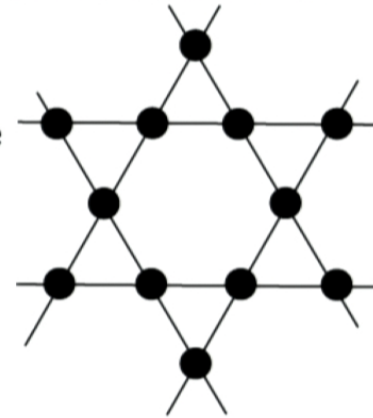
CONCLUSIONS

- Hubbard model with magnetic field on Kagome lattice

$$H = - \sum_{\langle i,j \rangle, \sigma} (t_{i,j} c_{i\sigma}^\dagger c_{j\sigma} + t_{i,j}^* c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{h_z}{2} (n_{i\uparrow} - n_{i\downarrow})$$

- t/U expansion

$$H = \cos \theta \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + \sin \theta \sum_{i,j,k \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) + h_z \sum_i S_i^z$$



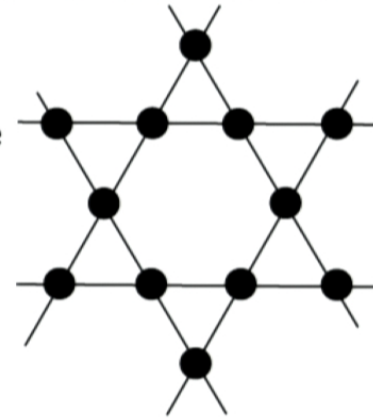
CONCLUSIONS

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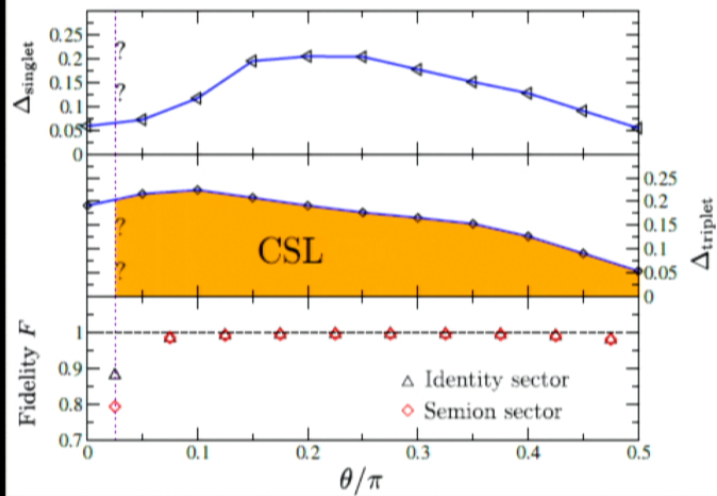
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θ - h_z phase diagram



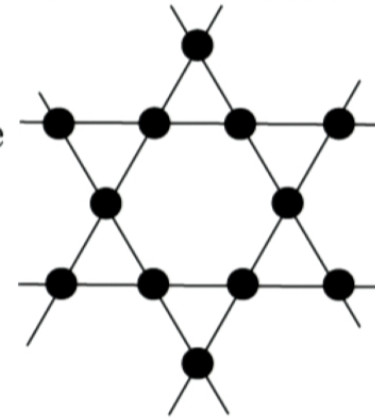
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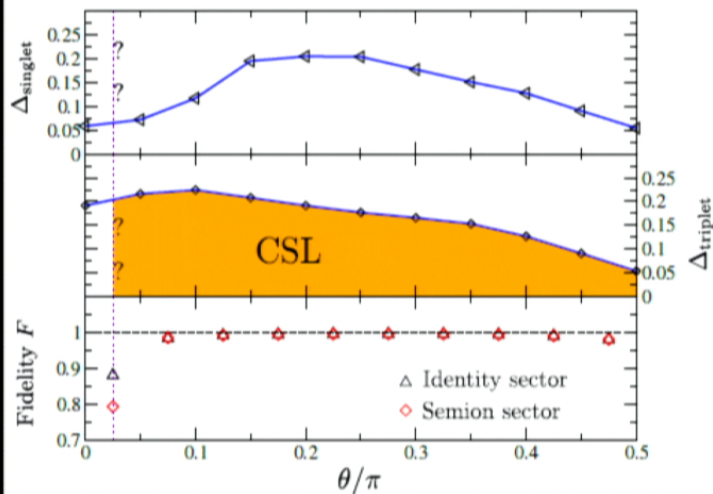
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- t/U expansion

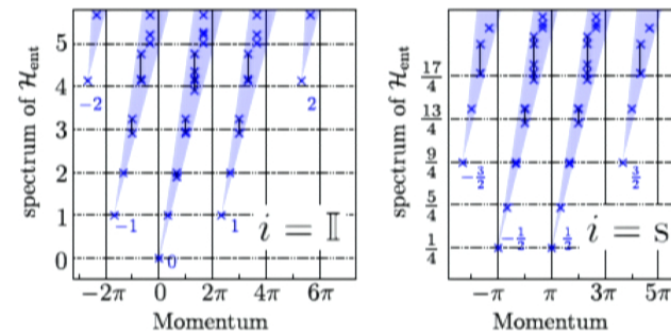
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θ - h_z phase diagram

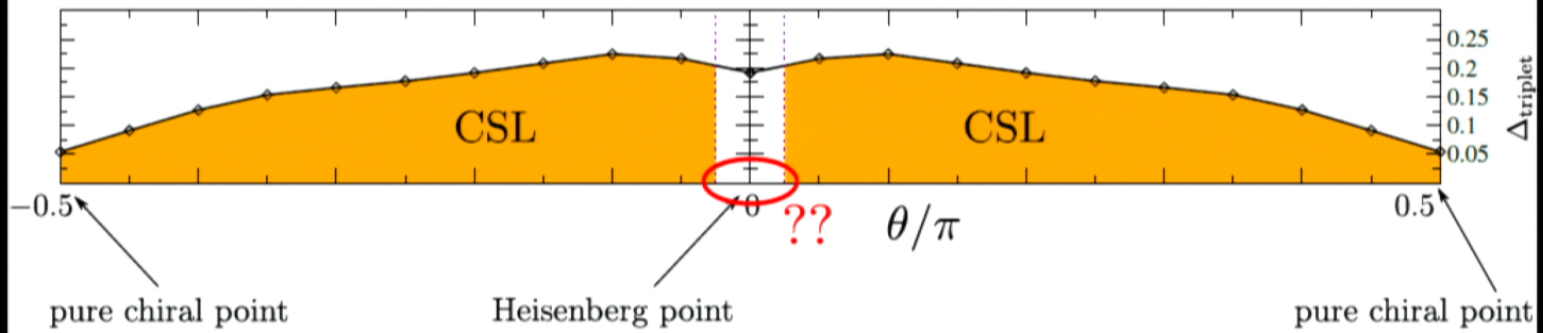


entanglement spectrum

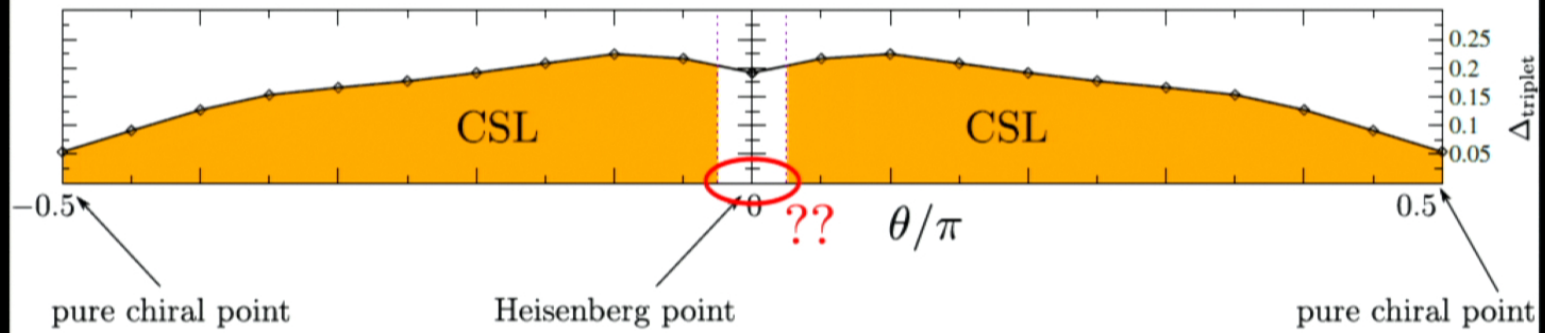


- edge theory: chiral $SU(2)_1$ WZW

WORK IN PROGRESS



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THANK YOU!