Title: A symmetry-respecting topologically-ordered surface phase of 3d electron topological insulators.

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Abstract: A 3d electron topological insulator (ETI) is a phase of matter protected by particle-number conservation and time-reversal symmetry. It was previously believed that the surface of an ETI must be gapless unless one of these symmetries is broken. A well-known symmetry-preserving, gapless surface termination of an ETI supports an odd number of Dirac cones. In this talk, I will show that in the presence of strong interactions, an ETI surface can actually be gapped and symmetry preserving, at the cost of carrying an intrinsic two-dimensional topological order. I will argue that such a topologically ordered phase can be obtained from the surface superconductor by proliferating the flux 2hc/e vortex. The resulting topological order consists of two sectors: a Moore-Read sector, which supports non-Abelian charge e/4 anyons, and an Abelian anti-semion sector, which is electrically neutral. The time-reversal and particle number symmetries are realized in this surface phase in an "anomalous" way: one which is impossible in a strictly 2d system. If time permits, I will discuss related results on topologically ordered surface phases of 3d topological superconductors.

Topologically ordered surface of 3d electron topological insulators

Max Metlitski

Kavli Institute for Theoretical Physics

Perimeter Institute for Theoretical Physics,
February 12, 2014

Matthew Fisher

Charles Kane

MM, C. L. Kane, M.P.A. Fisher, arXiv:1306.3286

Related work:

Chong Wang, Andrew C. Potter, T. Senthil, arXiv:1306.3223

Parsa Bonderson, Chetan Nayak, Xiao-Liang Qi, arXiv:1306.3230

Xie Chen, Lukasz Fidkowski, Ashvin Vishwanath, arXiv:1306.3250

Plan

- Introduction to symmetry protected topological phases
- Review of 3d electron topological insulators
- Topologically ordered surface of electron topological insulators
- Extensions to 3d topological superconductors

  (with X. Chen, L. Fidkowski and A. Vishwanath)

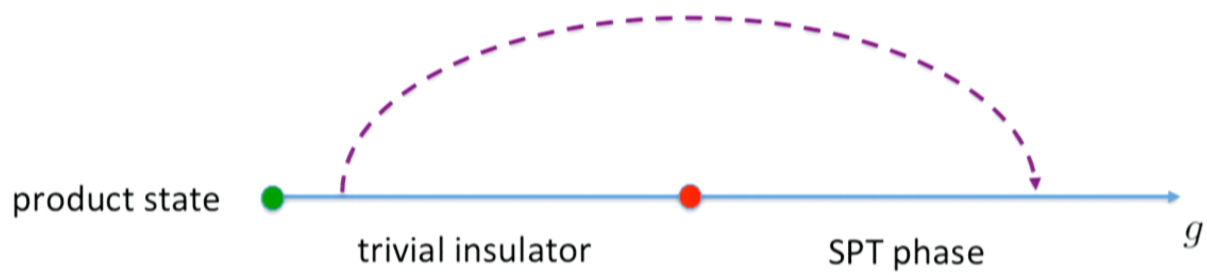
Symmetry Protected Topological Phases (SPTs)

- Gapped phases, protected by symmetry group *G*
- In the absence of symmetry – connected to vacuum

no *"intrinsic"* topological order:

- no degeneracy on a torus
- no excitations with fractional statistics/quantum numbers
- no long-range entanglement

(unlike e.g. Fractional Quantum Hall states)



X. Chen, Z.-C. Gu, X.-G. Wen (2011)

SPTs: life on the edge

- SPTs possess protected edge states

Bulk	Edge
1d	gapless
2d	gapless or spontaneously breaks symmetry
3d	gapless, spontaneously breaks symmetry or supports intrinsic 2d topological order

A. Vishwanath and T. Senthil (13)

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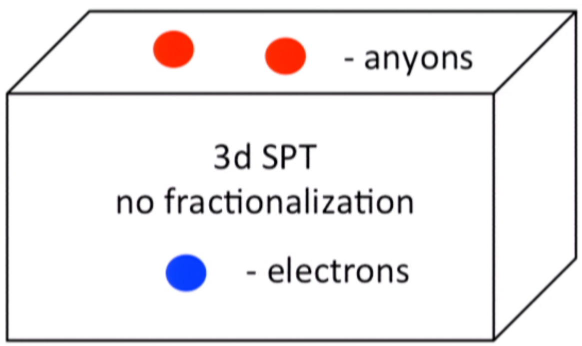
A. Vishwanath and T. Senthil (13)



SPT example: 3d electron TI

- SPT protected by particle number conservation U(1) and time-reversal $\mathcal{T}$

$$U(1): \quad c_\sigma \rightarrow e^{i\alpha} c_\sigma$$

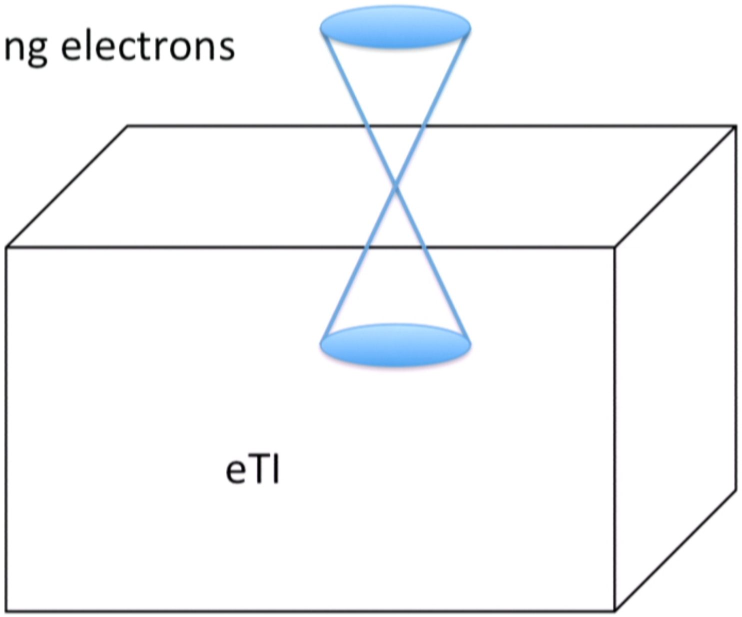
$$\mathcal{T}: \quad c_\sigma \rightarrow \epsilon_{\sigma\sigma'} c_{\sigma'}, \quad \mathcal{T}^2 = -1$$

- exist already for non-interacting electrons
- gapped bulk
- surface: single Dirac cone

L. Fu, C.L. Kane, E.J. Mele (07)

- Surface cannot be realized strictly in 2d

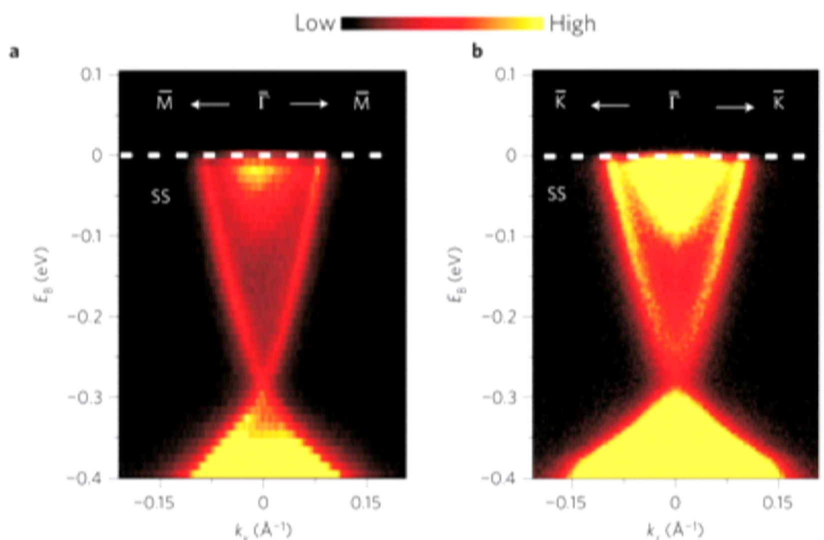
A. N. Redlich (84)
M. Mulligan and F.J.Burnell (13)



3d electron topological insulator

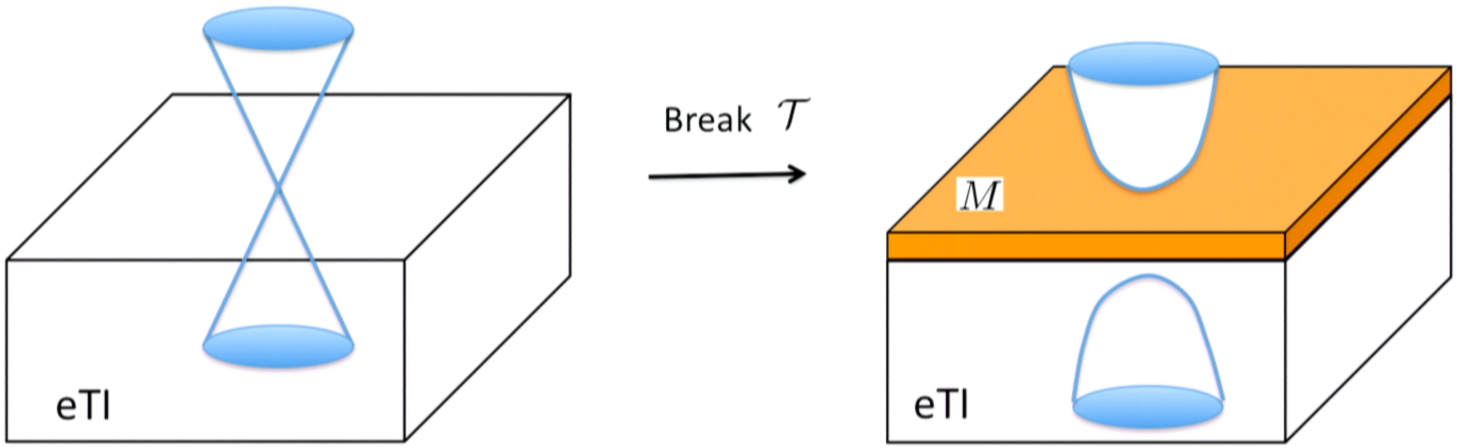
- Experimental realization: $Bi_{1-x}Sb_x$, Bi_2Se_3, Bi_2Te_3

Bi_2Se_3



Y. Xia, D. Qian, D. Hsieh et al (09)

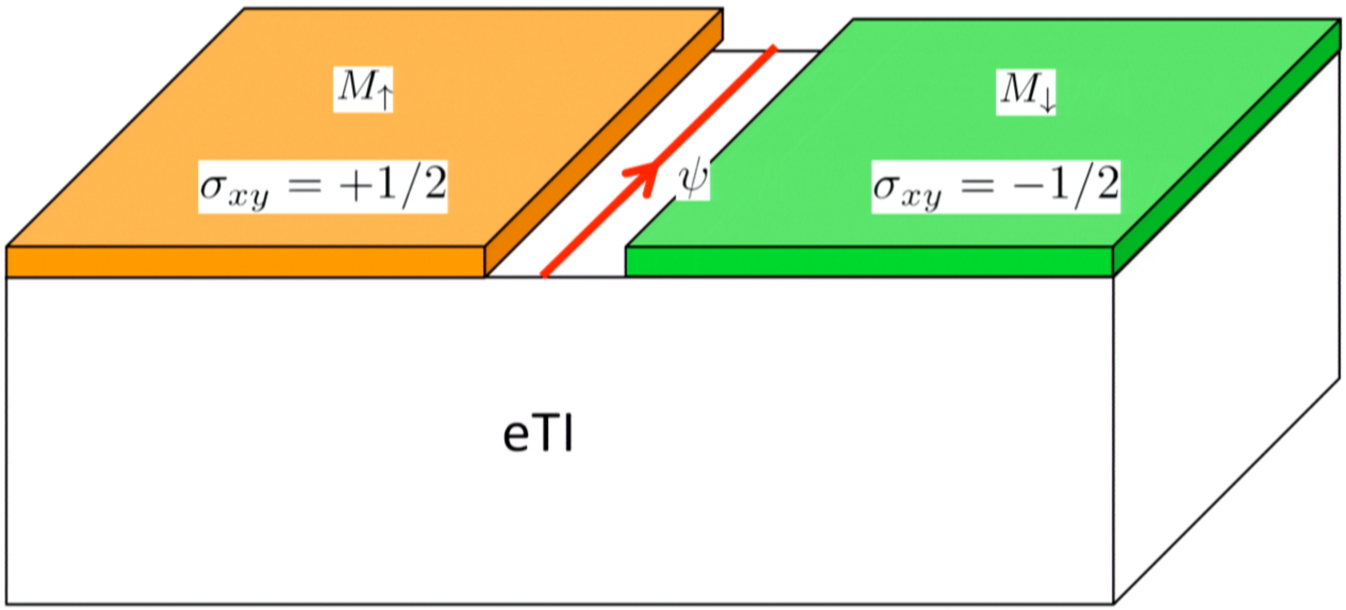
T-breaking surface phase of an eTI



- Fully gapped state with no intrinsic topological order (only electron excitations with q = 1)
- $J_\mu = \frac{\sigma_{xy}}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$ $\qquad\qquad \sigma_{xy} = \pm 1/2$

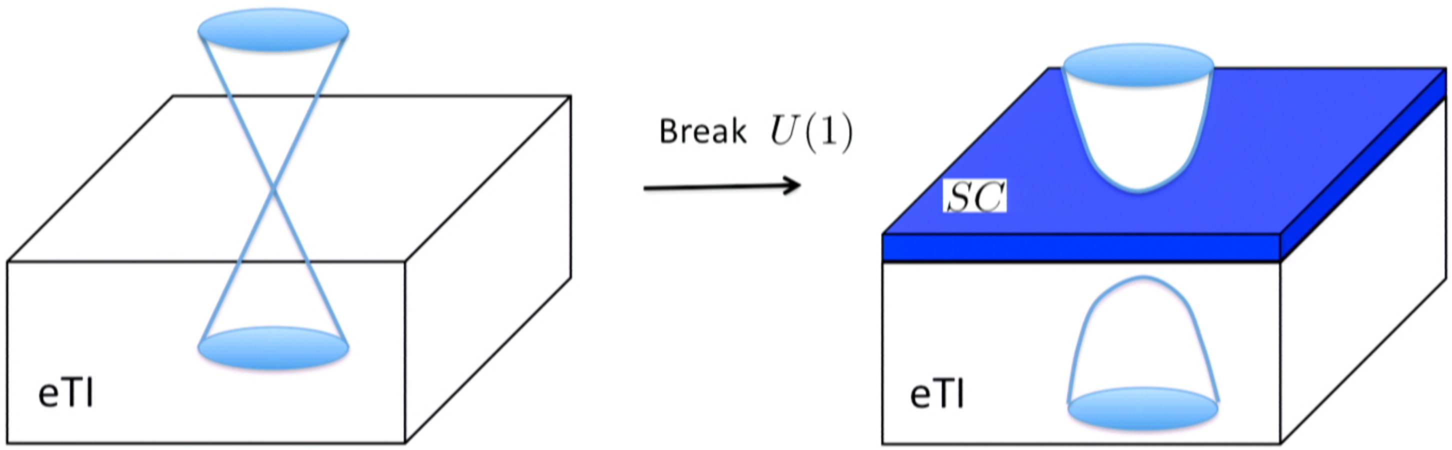
L. Fu and C. L. Kane (07)

T-breaking surface of an eTI: domain wall



Chiral fermion ψ (c = 1),
like ν = 1 IQH edge

Superconducting surface phase of an eTI



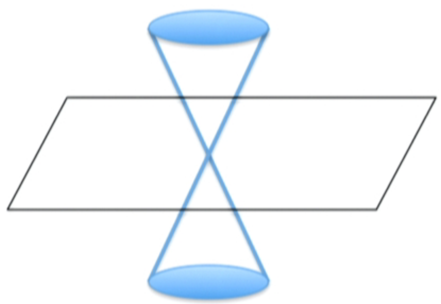
- Excitations: - gapped Bogolioubov quasiparticles f_σ
  - superfluid Goldstone mode
  - superconducting vortices

- π – flux (hc/2e) vortices support Majorana zero-modes Fu, Kane (08)
  - (projective) non-Abelian statistics

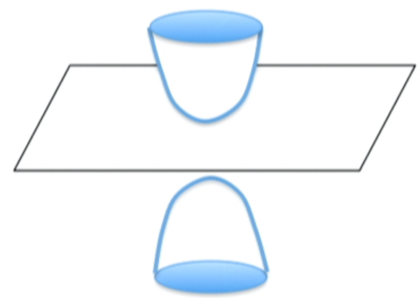
- Incompatible with $\mathcal{T}$ strictly in 2d
  - Majorana modes on π fluxes in 2d ⟶ p+ip SC ⟶ 

Topological insulator surface

i) gapless



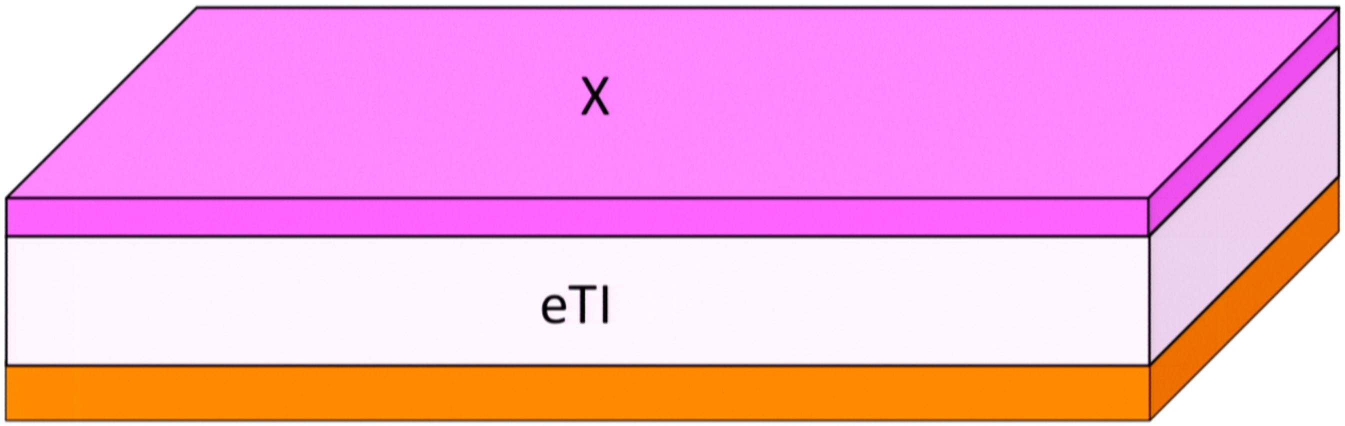
ii) symmetry broken: $\mathcal{T}$ or $U(1)$



iii) Intrinsic surface topological order?

The topological order option

- If an eTI surface is neither gapless, nor symmetry broken it must be topologically ordered
- Imagine a gapped and symmetry preserving surface phase X
  - X has $\sigma_{xy} = 0$



$M : \sigma_{xy} = 1/2$

- The slab as a whole has $\sigma_{xy} = 1/2$
  - must have excitations with $q = \sigma_{xy} = 1/2$
  - X is topologically ordered!

Surface topological order of an eTI

- Does a gapped, symmetry respecting, topologically ordered surface phase of an eTI exist? - Yes!

- the topological order is non-Abelian: Moore-Read x $U(1)_{-2}$

Moore-Read – charged (like $\nu = 5/2$ FQH)
charge e/4 non-Abelian anyons

$U(1)_{-2}$ – neutral antisemion

- time-reversal is implemented non-trivially

- cannot be realized in 2d preserving both T and U(1) symmetries

MM, C. L. Kane, M.P.A. Fisher, arXiv:1306.3286

C. Wang, A. C. Potter, T. Senthil, arXiv:1306.3223

Strategy

- Start with the T-respecting superconducting surface
- "Quantum disorder" the surface SC by proliferating flux 4π vortices

  - restores the U(1) symmetry

  - preserves T

  - gives rise to a topologically ordered surface state

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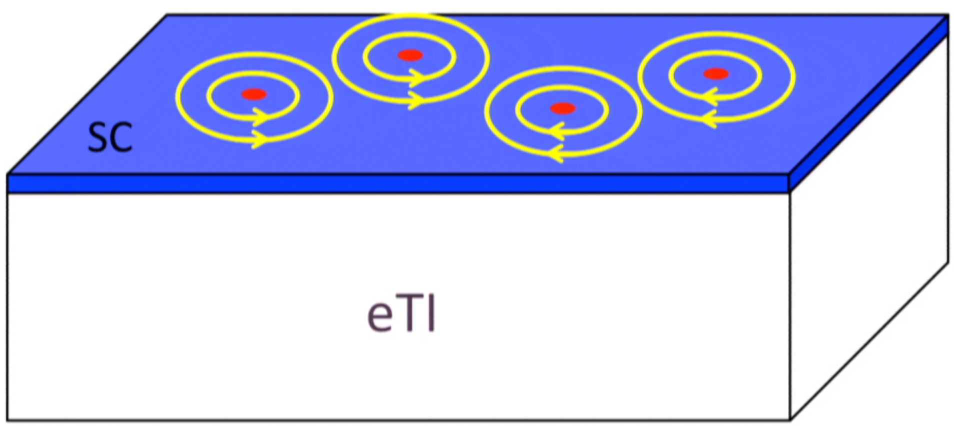
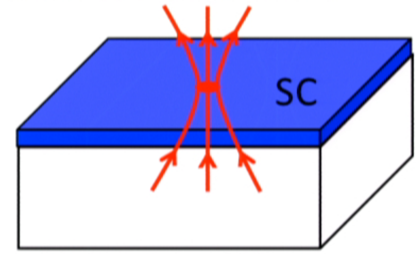
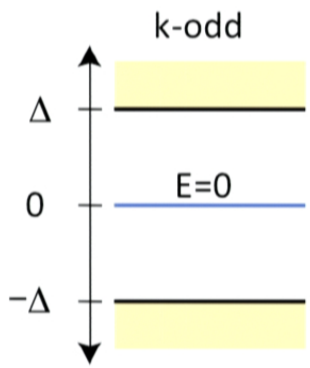


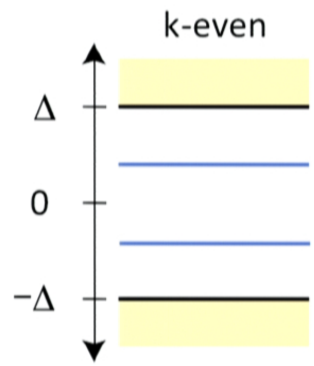
More on superconducting surface of an eTI

- Vorticity $k \quad \rightarrow \quad \Phi = \pi k$





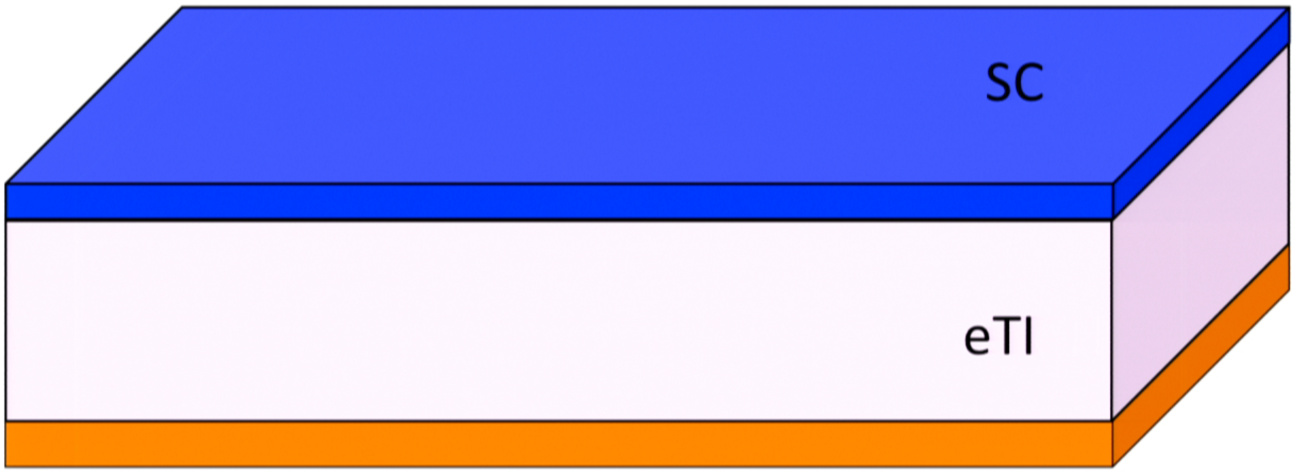
Majorana zero mode
non-Abelian



No zero mode
Abelian

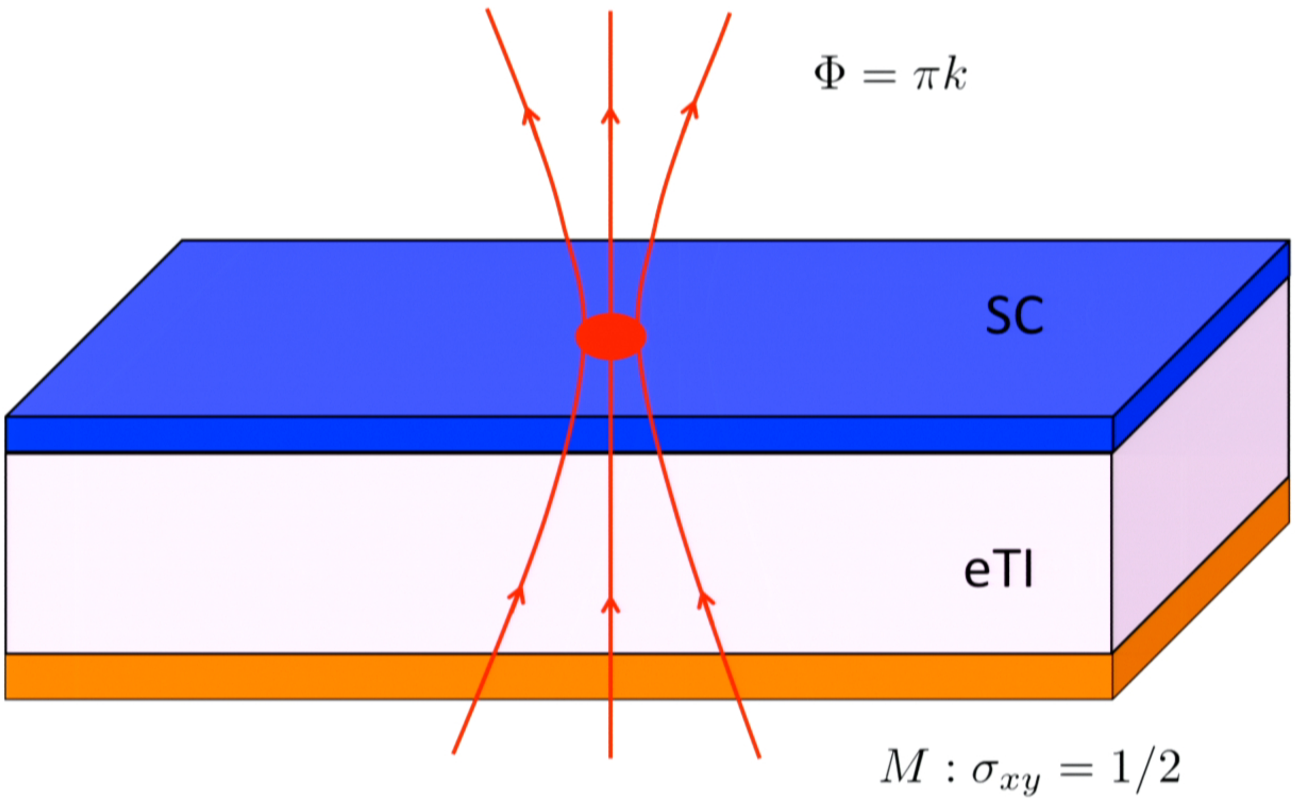
- Logarithmic interactions between vortices: Abelian part of statistics ill-defined
- Introduce dynamical EM field – statistics well-defined

Inferring flux-tube statistics: slab trick

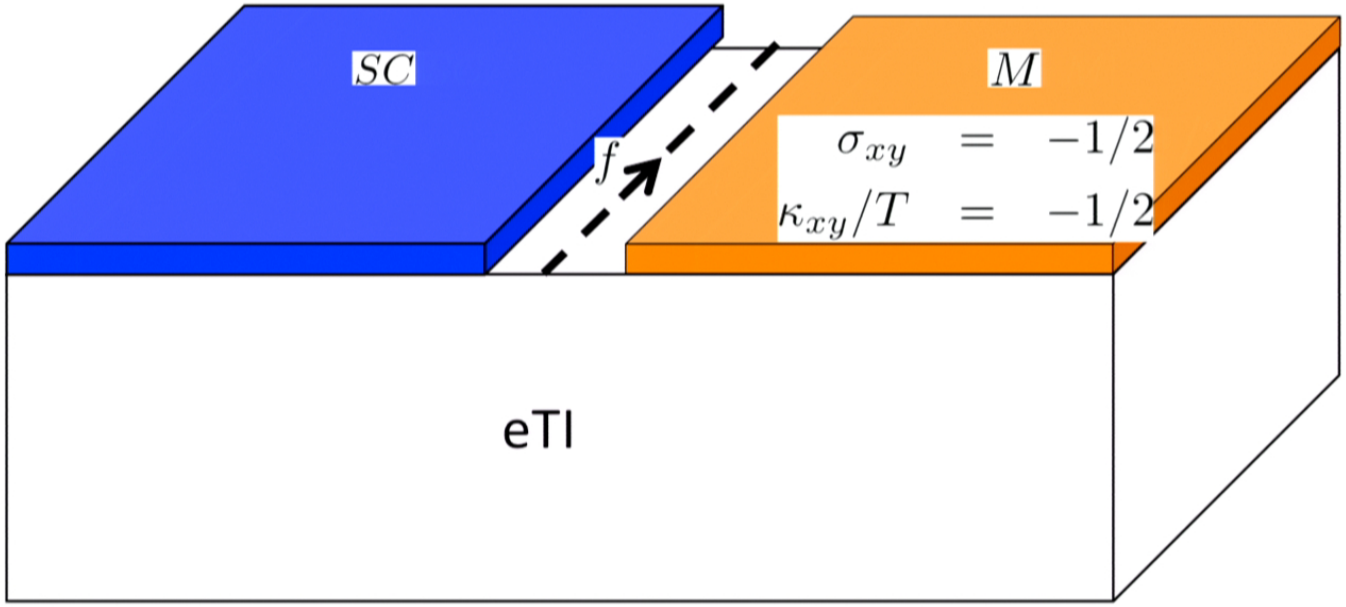


$M : \sigma_{xy} = 1/2$

Inferring flux-tube statistics: slab trick

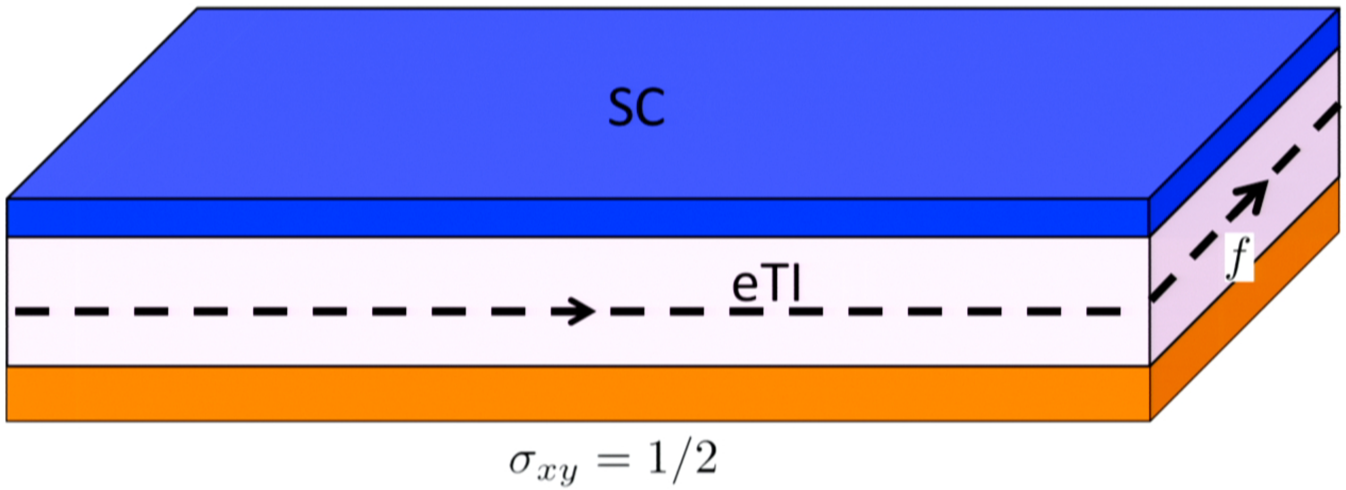


SC – T-broken interface



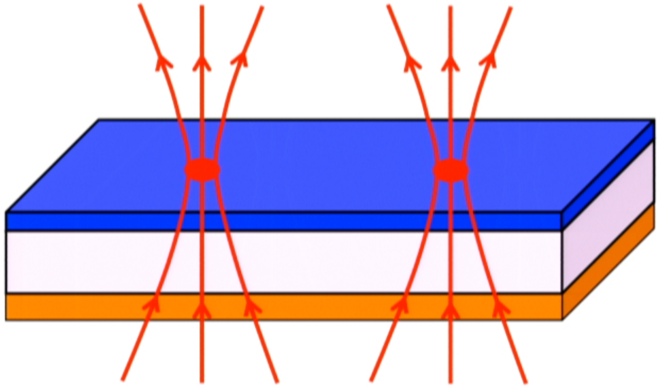
- Chiral Majorana mode f (c = 1/2) on the 1d interface

Slab as a 2d system



- Chiral Majorana mode f (c = 1/2) on the 1d edge
- As a 2d system – a p+ip superconductor

Flux-tube statistics in a p+ip SC



- Ising anyon model

	1	f	σ
d	1	1	$\sqrt{2}$
θ	1	-1	$e^{\pi i/8}$

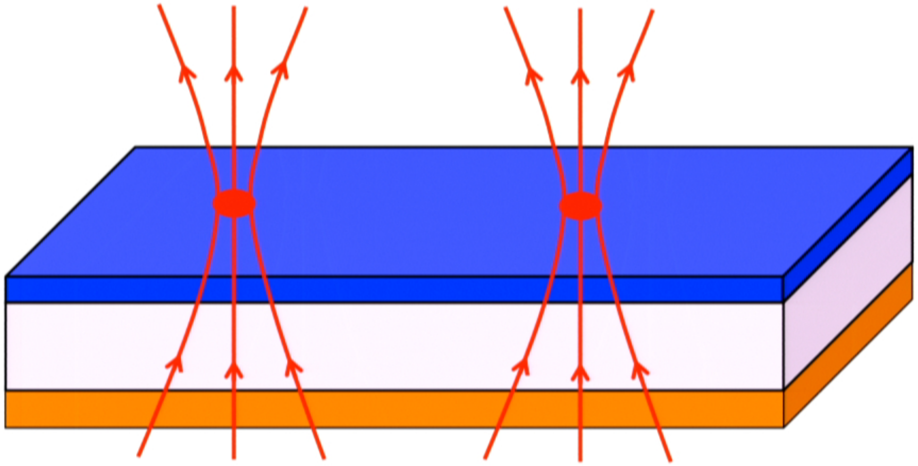
- Fusion rules: $\sigma \times \sigma = 1 + f, \quad \sigma \times f = \sigma, \quad f \times f = 1$
- Flux tubes, vorticity $k \quad \rightarrow \quad \Phi = \pi k$

  - odd k: σ

  - even k: $1, f$

A. Kitaev, (06)

Peeling off the bottom surface



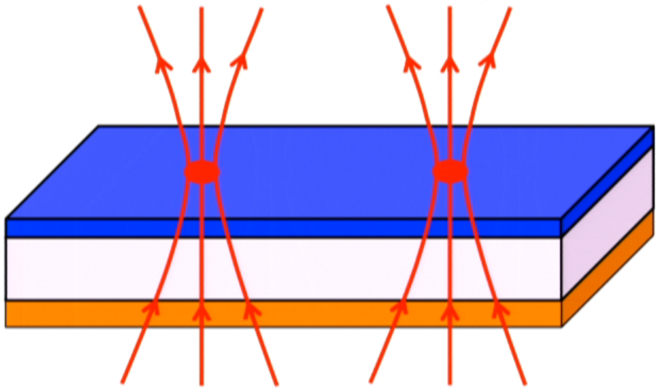
$$S_{\text{slab}} = S_{\text{Ising}}[j_v, j_f]$$

$$S_{\text{slab}} = S_{\text{top}}[j_v, j_f] + S_{\text{bulk}}[A] + S_{\text{bottom}}[A]$$

$$S_{\text{bulk}}[A] \sim \int d^3x d\tau F_{\mu\nu}^2$$

$$S_{\text{bottom}} = -\frac{ik}{4\pi} \int_{\text{bott}} d^2x d\tau \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda, \qquad k = \sigma_{xy} = 1/2$$

Peeling off the bottom surface



$$S_{\rm top} = S_{\rm Ising}[j_v, j_f] + \frac{i}{8\pi}\int d^2x d\tau \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda, \qquad j^v_\mu = \frac{1}{\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$$

$$S_{\rm top} = S_{\rm Ising}[j_v, j_f] + \int d^2x d\tau \left(\frac{-8i}{4\pi}\epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda - i a_\mu j^v_\mu\right)$$

- Intrinsic flux-tube statistics described by Ising x $U(1)_{-8}$ theory

  - extra $e^{-\pi i/8}$ Abelian phase in self-statistics of π-fluxes

Flux-tube statistics

- Intrinsic flux-tube statistics described by a subset of

$$\text{Ising} \times U(1)_{-8}$$

$$\{1, \sigma, f\} \quad \times \quad e^{ik\alpha}, \qquad \Phi = \pi k$$

- Allowed anyons: $\sigma e^{ik\alpha}$, $\quad \text{k} - \text{odd}$

  $e^{ik\alpha},\ f e^{ik\alpha}, \quad \text{k} - \text{even}$

- Reminiscent of Moore-Read state (subset of $\text{Ising} \times U(1)_{+8}$)

- Statistical properties invariant under $\quad k \to k + 8$

  - compare 2d: $\quad k \to k + 2$

Flux-tube statistics

- Intrinsic flux-tube statistics described by a subset of

$$\text{Ising} \times U(1)_{-8}$$

$$\{1, \sigma, f\} \qquad \times \qquad e^{ik\alpha}, \qquad \Phi = \pi k$$

- Allowed anyons: $\sigma e^{ik\alpha}$, $\quad \text{k} - \text{odd}$

  $e^{ik\alpha}$, $f e^{ik\alpha}$, $\quad \text{k} - \text{even}$

- Topological spins:

$U(1)_{-8}$ (vorticity)

Ising	0	1	2	3	4	5	6	7
1	1		$-i$		1		$-i$	
σ		1		-1		-1		1
f	-1		i		-1		i	

Vortex condensation

- Cannot condense vortices with Majorana modes (k – odd)
- Can only condense vortices, which are self-bosons
  - smallest vortex: $e^{4i\alpha},\ (\Phi = 4\pi)$

    subtlety: mutual semion with k-odd vortices

$U(1)_{-8}$ (vorticity)

Ising	0	1	2	3	4	5	6	7
1	1		$-i$		1		$-i$	
σ		1		-1		-1		1
f	-1		i		-1		i	

8π vortex condensation

- Begin by condensing $e^{8i\alpha}, \ (\Phi = 8\pi)$
  - statistically trivial
- Gapped, T-invariant, topologically-ordered insulator
- Vortices survive as *electrically neutral* anyons:

$$\left.\begin{array}{ll} \sigma e^{ik\alpha}, & k = 1,3,5,7 \\ e^{ik\alpha}, \ f e^{ik\alpha}, & k = 0,2,4,6 \end{array}\right\rangle \in \text{Ising} \times U(1)_{-8}$$

- In addition, charge 1/4 boson: $e^{i\phi}$ - vortex in a vortex

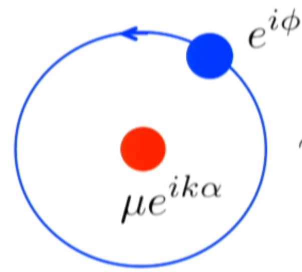
$\langle cc \rangle, \ q = 2 \quad \longrightarrow$  $\Phi = \dfrac{2\pi}{q} = \pi$

 $\Phi = 8\pi \quad \longrightarrow \quad \bullet \ e^{i\phi} \qquad q = \dfrac{2\pi}{\Phi} = \dfrac{1}{4}$

96 anyon state

- In addition, charge 1/4 boson: $e^{i\phi}$

  $e^{8i\phi}$ - Cooper pair

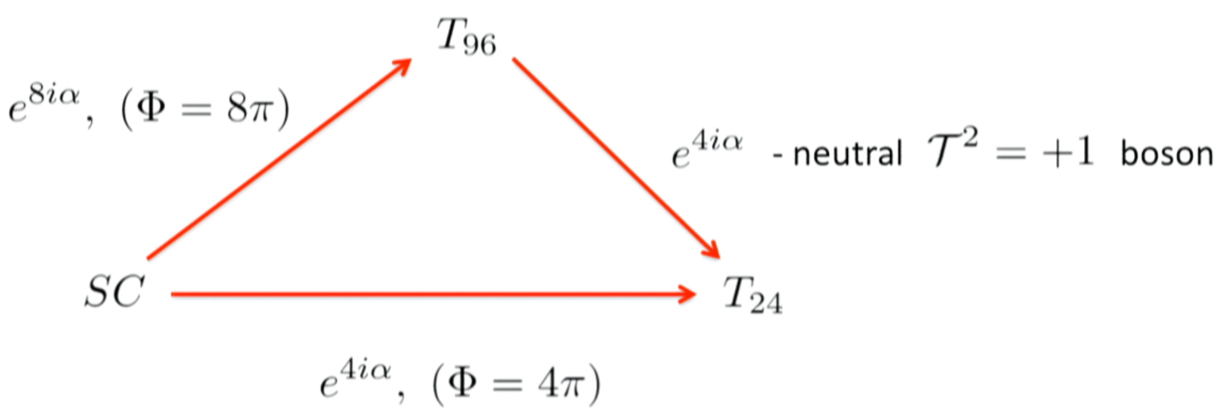


$$\sim e^{-iq\text{“}\Phi\text{”}} = e^{-i\pi k/4}$$

- 96 anyons

$$T_{96} = \begin{array}{lll} \sigma e^{im\phi} e^{ik\alpha}, & 0 \le m \le 7,\ k = 1,3,5,7, & q = m/4 \\ e^{im\phi} e^{ik\alpha},\ f e^{im\phi} e^{ik\alpha}, & 0 \le m \le 7,\ k = 0,2,4,6, & q = m/4 \end{array}$$

4π vortex condensation

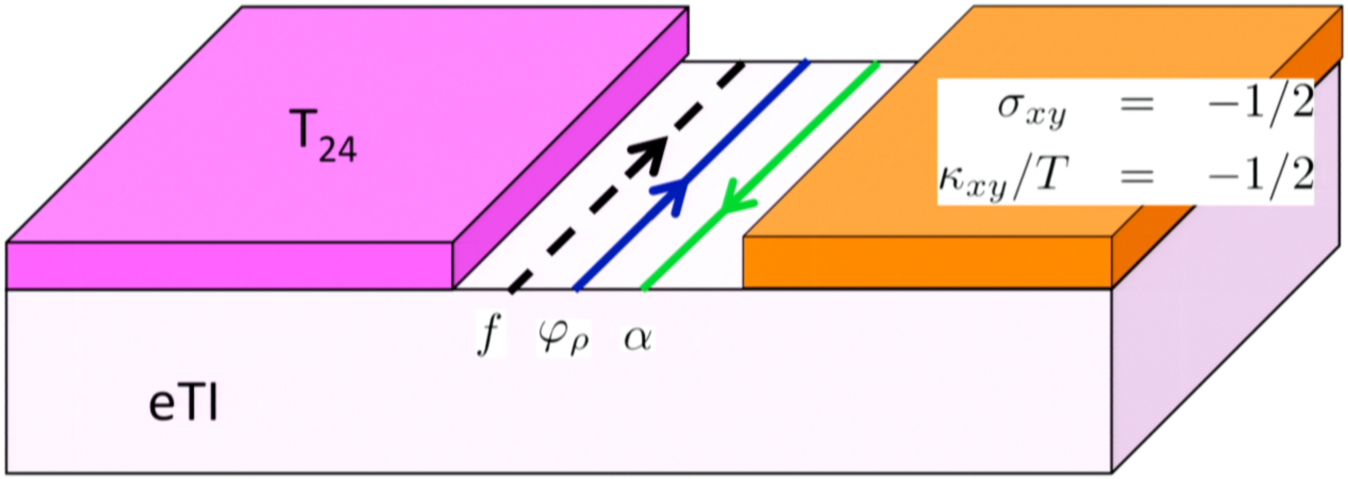


- $T_{24} = \text{Moore} - \text{Read} \times U(1)_{-2}$ - 24 anyons

$$\text{Moore} - \text{Read} = \begin{array}{lll} \sigma e^{im\varphi_\rho}, & m = 1,3,5,7, & q = m/4 \\ \{1, f\} e^{im\varphi_\rho}, & m = 0,2,4,6, & q = m/4 \end{array}$$

$$U(1)_{-2} = \quad \{1, e^{2i\alpha}\}, \qquad q = 0$$

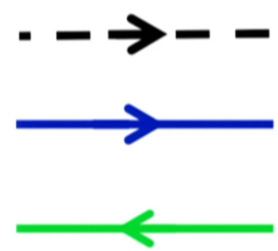
T_{24}– T-broken interface



$\text{Moore} - \text{Read}$ < Neutral Majorana mode f - c = 1/2

Charged bosonic mode φ_ρ - c = 1

$U(1)_{-2}$ - Neutral bosonic mode α - c = -1



Related work

- $\mathrm{Moore-Read} \times U(1)_{-2} \subset \mathrm{Ising} \times U(1)_8 \times U(1)_{-2}$ - 24 anyons

  $\mathrm{T-Pfaffian} \subset \mathrm{Ising} \times U(1)_{-8}$ - 12 anyons

  Parsa Bonderson, Chetan Nayak, Xiao-Liang Qi, arXiv:1306.3230

  Xie Chen, Lukasz Fidkowski, Ashvin Vishwanath, arXiv:1306.3250

- T-Pfaffian surface realized in a Walker-Wang model
  - 4 versions differing in action of $\mathcal{T}$ and $U(1)$

- Full classification: $\mathbb{Z}_2 \xrightarrow{\text{interactions}} \mathbb{Z}_2^3$

  - $\mathbb{Z}_2$ – ordinary electron TIs
  - $\mathbb{Z}_2^2$ – SPTs of neutral bosons with $\mathcal{T}$

  Chong Wang, Andrew C. Potter, T. Senthil, arXiv:1306.3238

Plan

- Introduction to symmetry protected topological phases
- Review of 3d electron topological insulators
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Conclusion

- Topologically ordered symmetry respecting surface states
  - powerful theoretical tool to characterize 3d SPTs
  - insight into allowed symmetry realization in lower d
- Topologically ordered surface state constructed for
  - electron topological insulators
  - fermion topological superconductors $\nu \in \mathbb{Z} \longrightarrow \mathbb{Z}_{16}$
- Distinguishing topological superconductor phases in the bulk?
- Realistic surface Hamiltonians?

Thank you!