

Title: A unification of symmetric Z2 spin liquids on kagome lattice

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URL: <http://pirsa.org/14020118>

Abstract: <span>While there is mounting numerical evidence for a gapped Z2 spin liquid in the kagome Heisenberg model, a complete characterization of this topological phase remains to be accomplished. A defining property, the projective symmetry group (PSG) which fixes how the emergent excitations of the spin liquid phase transform under symmetry, remains to be determined. Following a Chern-Simons field theory, we show how PSG determines measurable properties of a Z2 spin liquid, such as the existence of symmetry protected gapless edge states. This fact enables us to unify two distinct types of projected wavefunctions for Z2 spin liquids: the Schwinger-boson states and the fermionic spinon states. We also provide concrete predictions for identifying the spin liquid ground state on the kagome lattice.</span>

# Unification of $Z_2$ spin liquids on kagome lattice

Yuan-Ming Lu

UC Berkeley and Lawrence Berkeley National Laboratory



Work in collaboration with

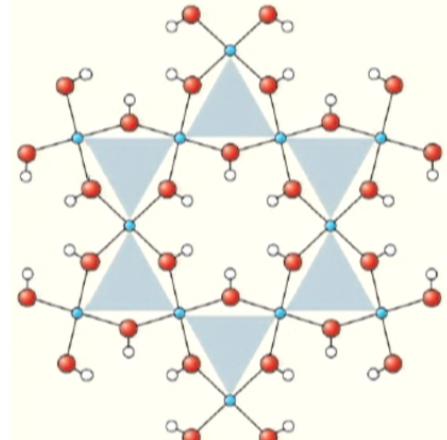
Gil Young Cho

Berkeley->UIUC

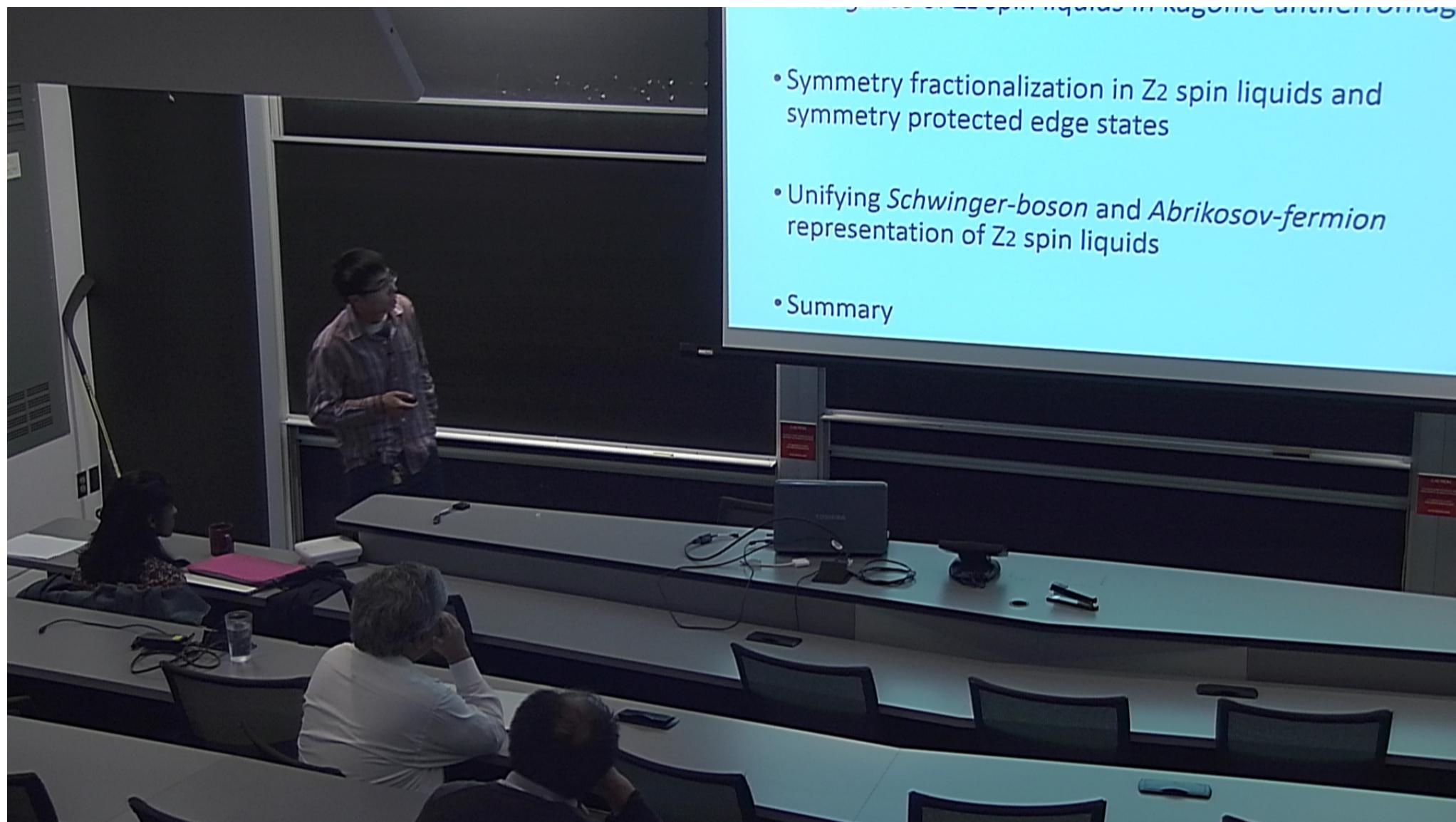


Ashvin Vishwanath

Berkeley



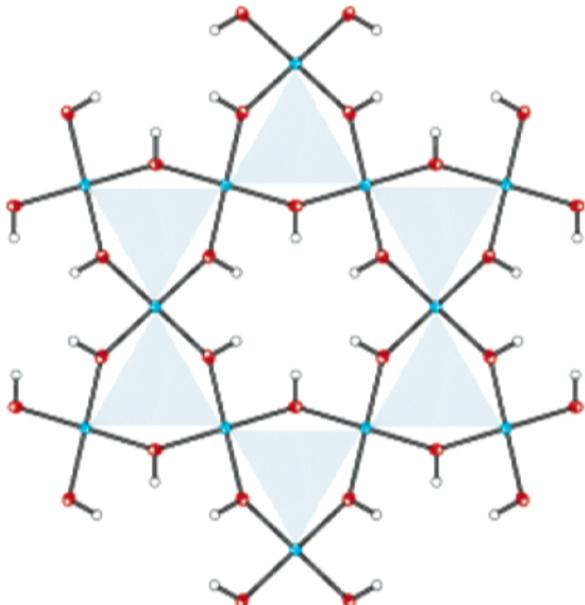
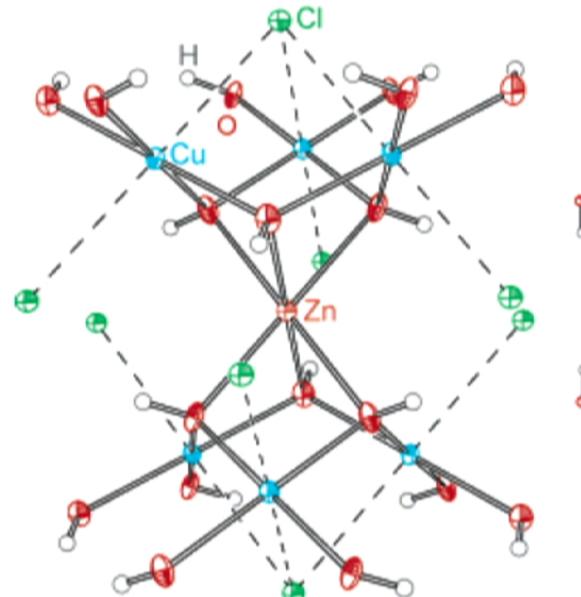
- Symmetry fractionalization in Z<sub>2</sub> spin liquids and symmetry protected edge states
- Unifying *Schwinger-boson* and *Abrikosov-fermion* representation of Z<sub>2</sub> spin liquids
- Summary



# Outline

- *Emergence* of  $Z_2$  spin liquids in kagome antiferromagnets
- Symmetry fractionalization in  $Z_2$  spin liquids and symmetry protected edge states
- Unifying *Schwinger-boson* and *Abrikosov-fermion* representation of  $Z_2$  spin liquids
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# *Herbertsmithite* $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$



Shores et al, *J. Am. Chem. Soc.* (2005)

$J \sim 17\text{meV}=200\text{K}$ ,  
yet no magnetic order  
down to 0.05 K !

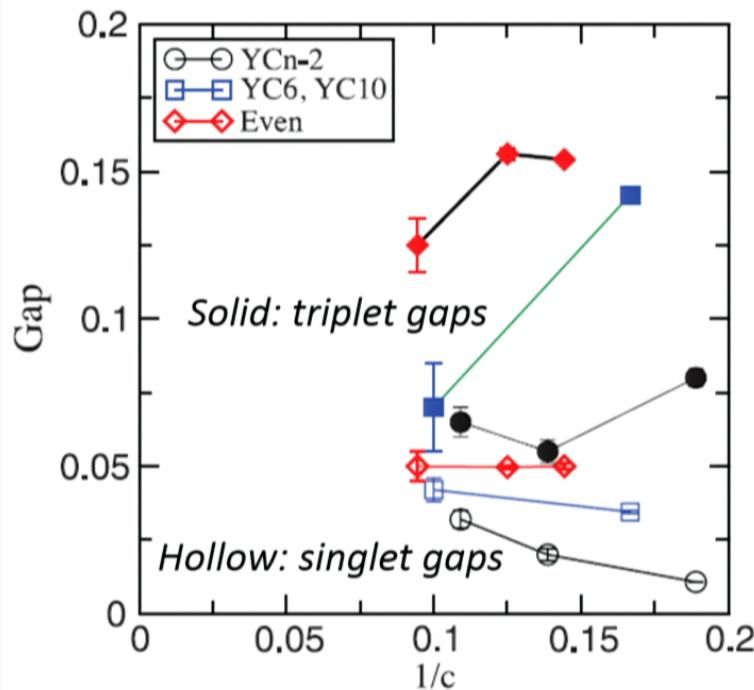
Helton et al, *PRL* (2007)

Minimal model: *Mott insulator of spin-1/2 on Cu<sup>2+</sup> ions*

$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

# Numeric evidence of a gapped spin liquid

DMRG study on various cylinders



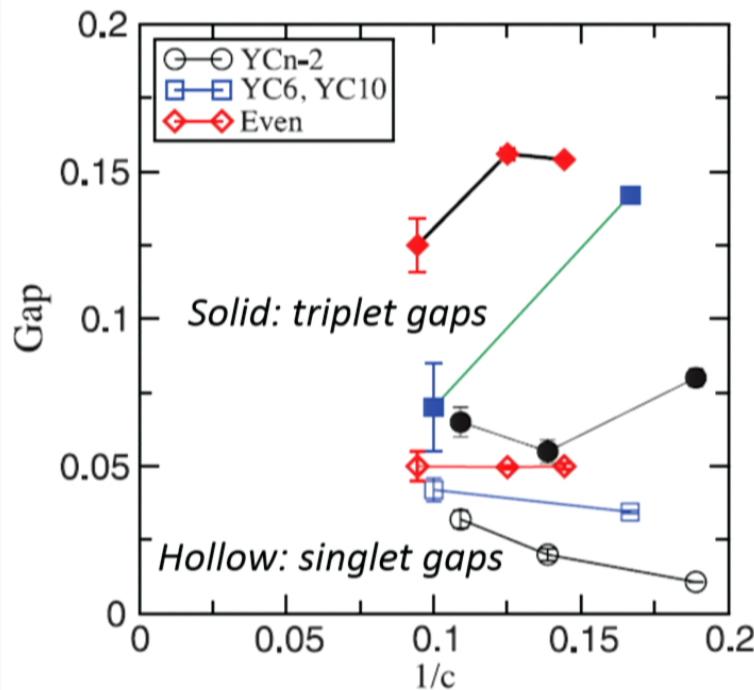
Yan, Huse, White,  
*Science* (2011)

- Not a metal (*Mott insulator, 3 spin-1/2's per u.c.*)
- No magnetic order (*spin rotational symmetric*)
- No density wave (*translational symmetric*)
- No gapless excitations (*triplet and singlet gaps*)

*What is it?*

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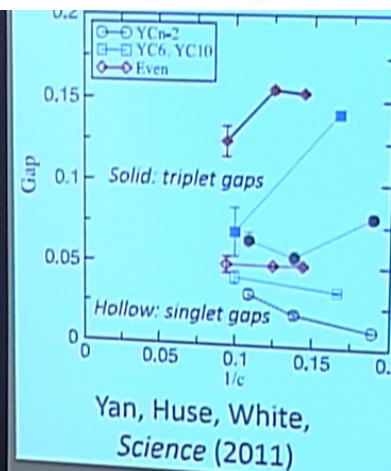
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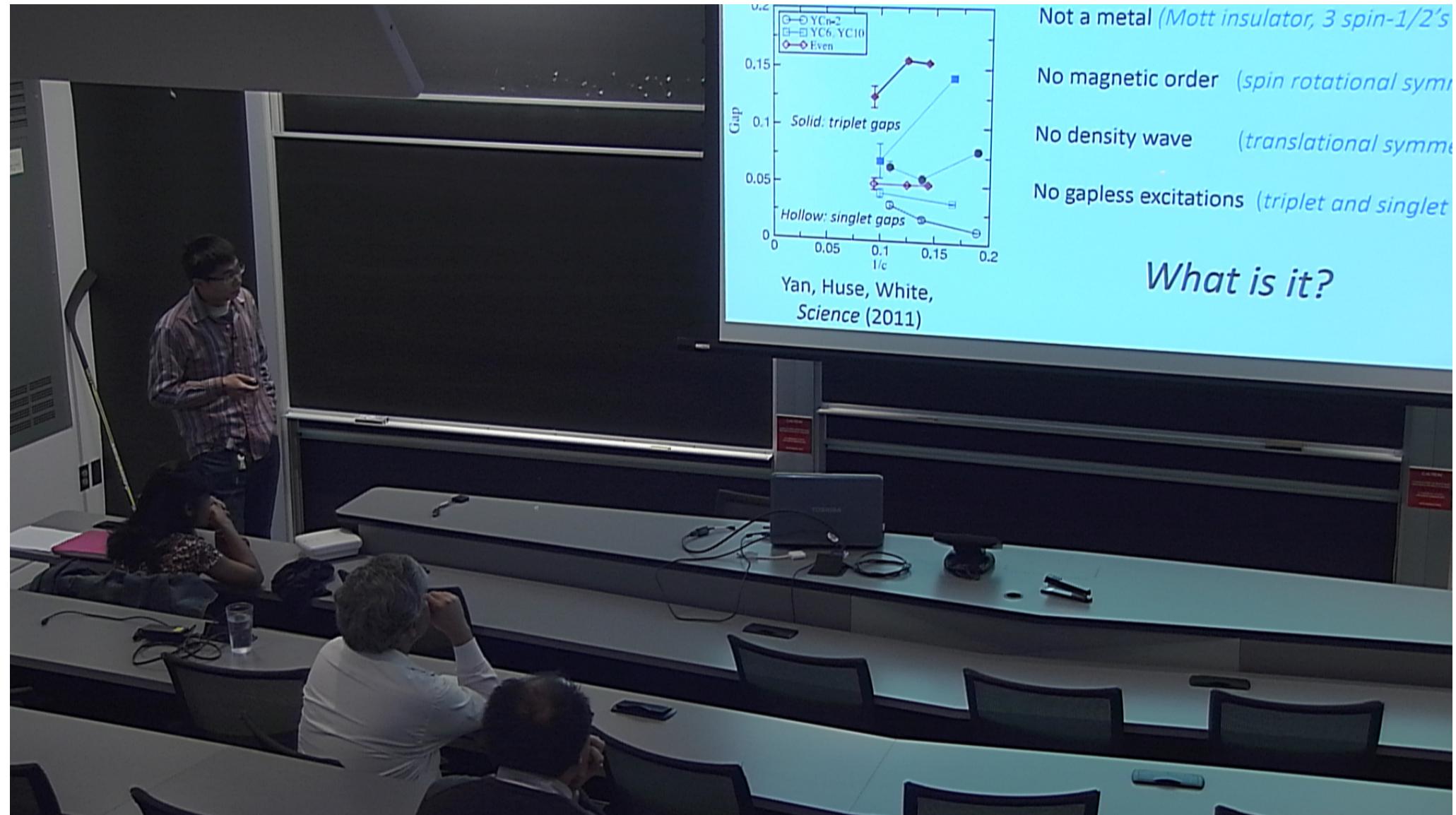
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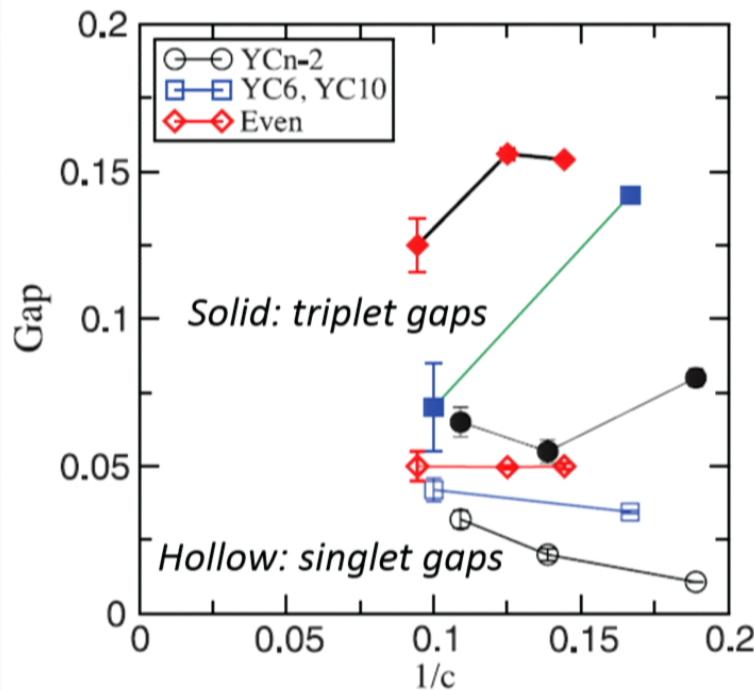
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# Numeric evidence of a gapped spin liquid

- Fractional statistics  
Wen, *Int. J. Mod. Phys.* (1990)
- Ground states' degeneracy on torus  
Wen, *Int. J. Mod. Phys.* (1990)
- Long-range many-body entanglement  
Kitaev, Preskill, *PRL* (2006)  
Levin, Wen, *PRL* (2006)

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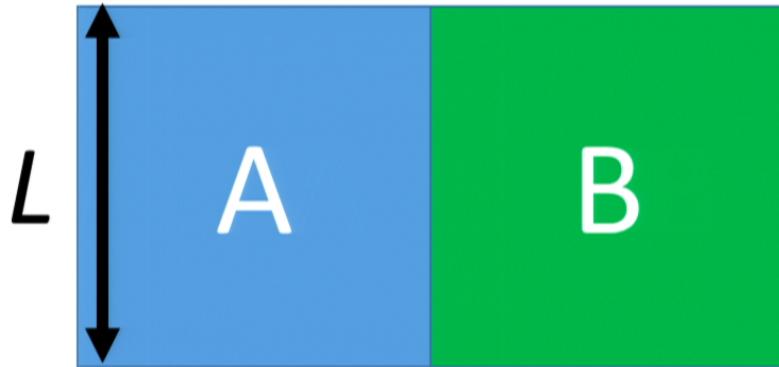
No gapless excitations (*triplet and singlet gaps*)

***Must have topological order!***

Hastings, *PRB* (2004), *EPL* (2005); Oshikawa, *PRL* (2000)

(*Generalization of Lieb-Schultz-Mattis thm. in 1d*)

# Why Z2 spin liquid?



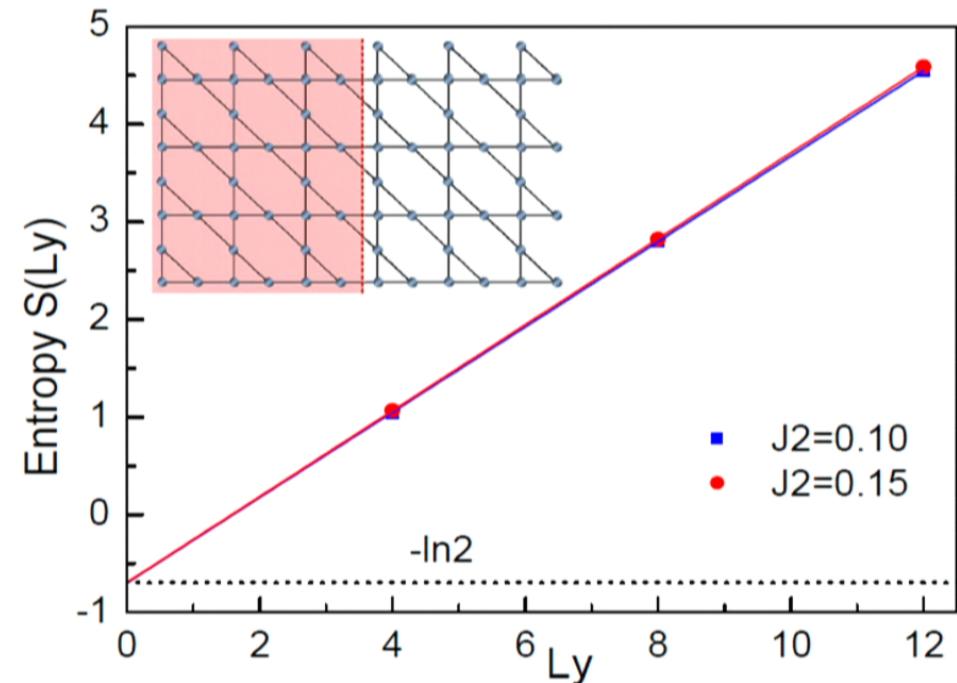
$$\rho_A = \text{Tr}_B(|g.s.\rangle\langle g.s.|)$$

$$S_{ent} \equiv \text{Tr}(\rho_A \log \rho_A) = \alpha L - \gamma$$

Topological entanglement entropy

$\gamma = \ln 2 = 0.693$  only for Z2 spin liquid (and double semion model)

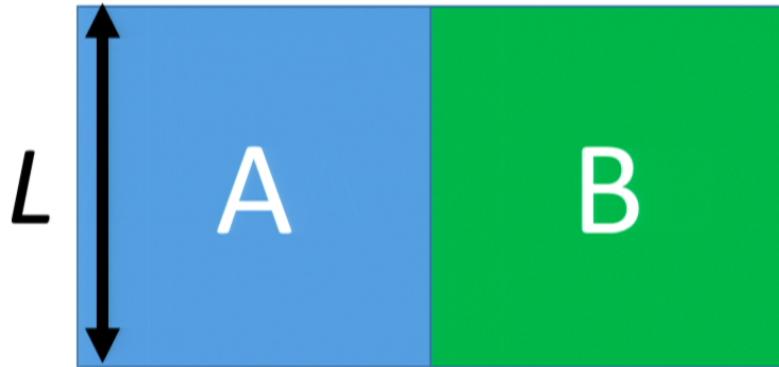
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Jiang, Wang, Balents, *Nat. Phys.* (2012)

Depenbrock, McCulloch, Schollwoeck, *PRL* (2012)

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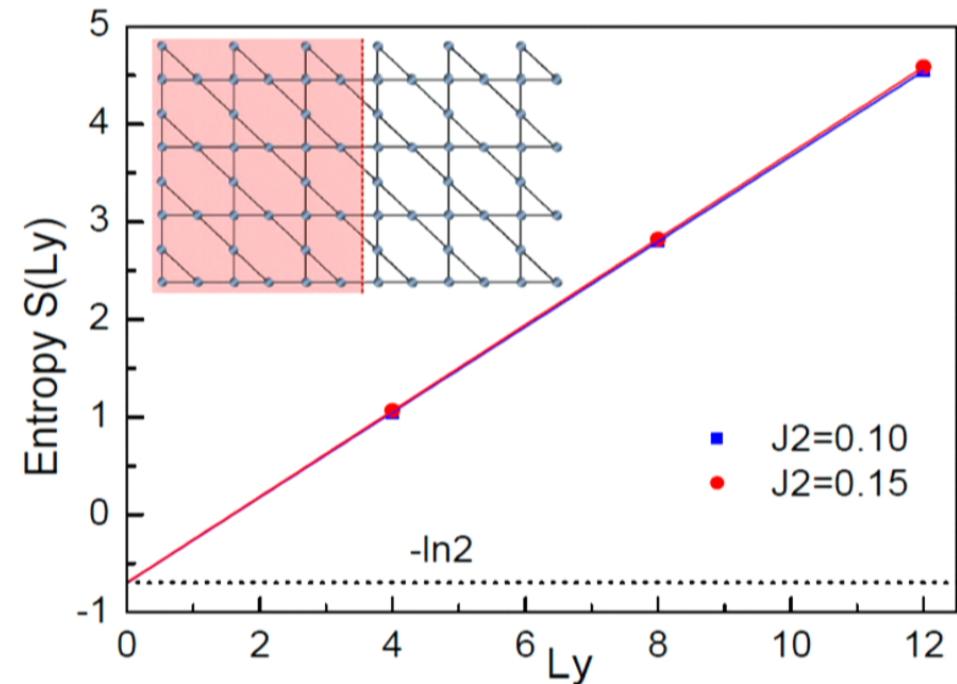
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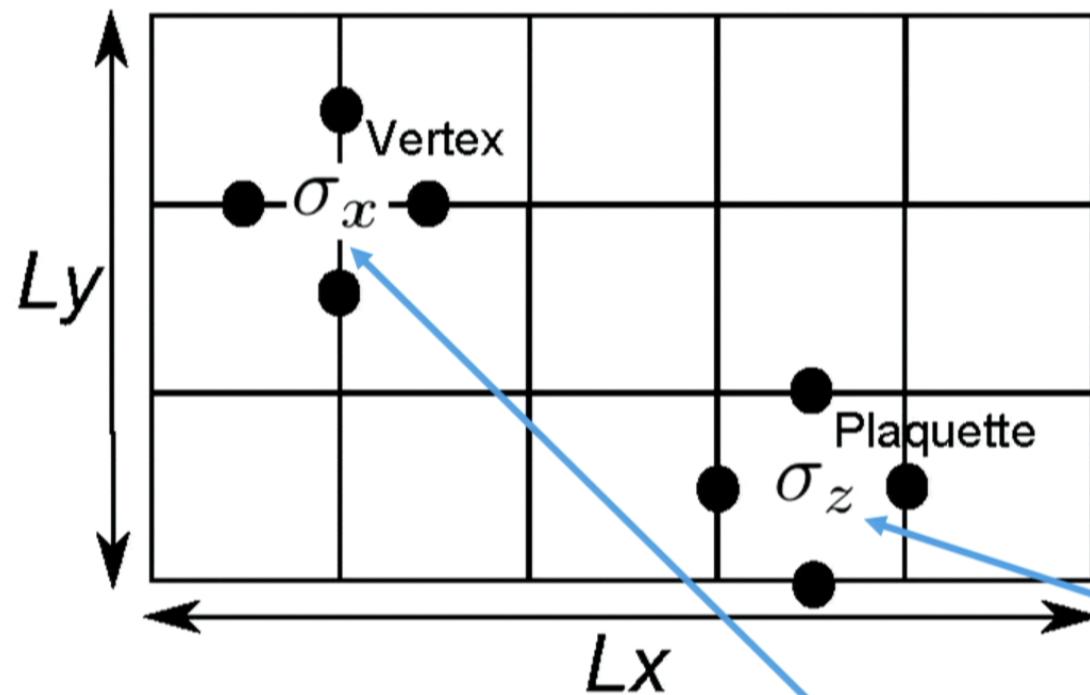
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# Toric code: simplest Z<sub>2</sub> spin liquid (Kitaev 1997)



Counting g.s. degeneracy on torus:

$2^*L_x^*L_y$  spins

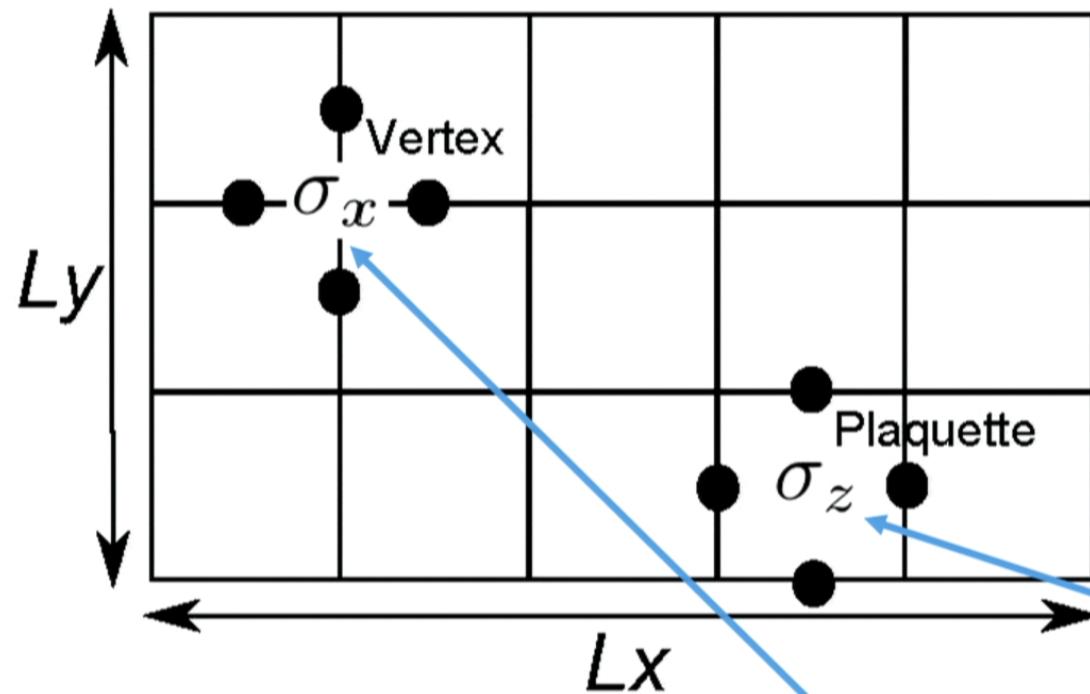
$L_x^*L_y$  vertices +  $L_x^*L_y$  plaquettes

2 global redundancies

$\Rightarrow 2^2=4$  fold g.s. degeneracy  
on torus

$$H_{t.c.} = -J_v \sum_{vertex v} \left( \prod_{i \in v} \sigma_x^{i,v} \right) - J_p \sum_{plaquette p} \left( \prod_{j \in p} \sigma_z^{j,p} \right)$$

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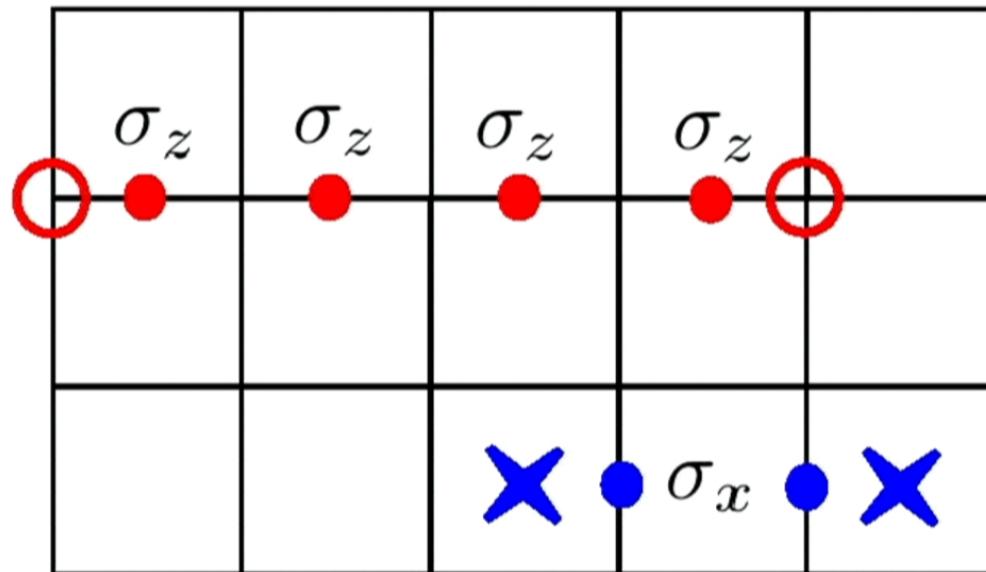
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# Emergent fractional statistics (anyons)



"Electric charge"  $b$



"Magnetic vortex"  $v$

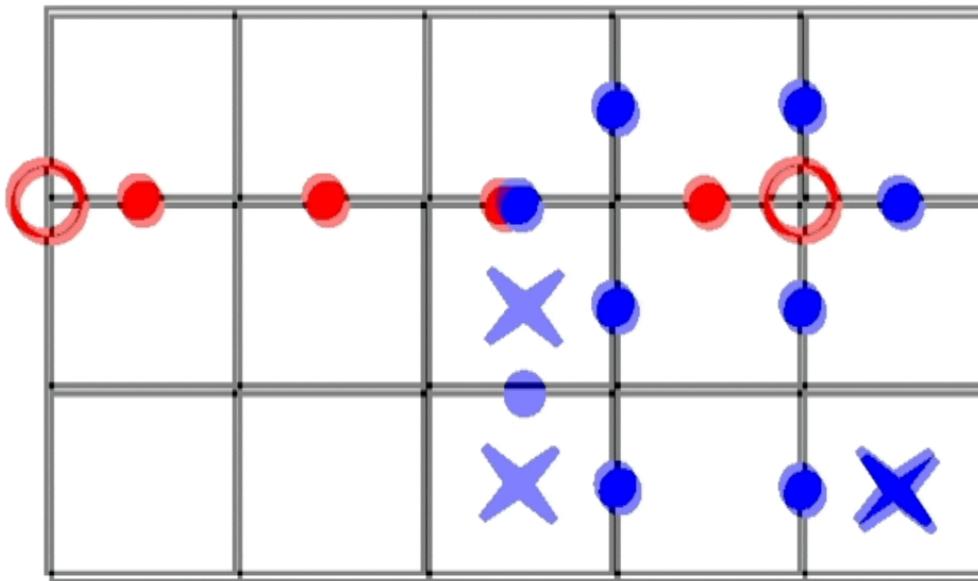


$$\text{Fermion } f = b \times v$$

*Mutual semion statistics: (-1)*

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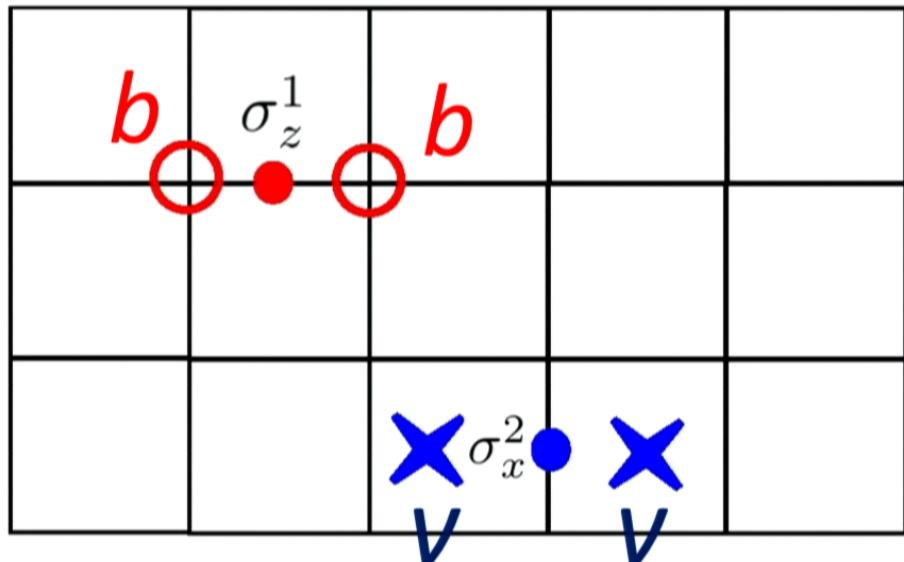
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## Symmetry fractionalization:

*Anyons can carry fractional symmetry quantum numbers !*



$$S_x \sigma_z^1 |G\rangle = -\sigma_z^1 S_x |G\rangle$$

$$S_x b S_x^{-1} = \boxed{\pm i} \cdot b$$

$$S_x \sigma_x^2 |G\rangle = \sigma_x^2 S_x |G\rangle$$

$$S_x v S_x^{-1} = \boxed{\pm v}$$

D2 spin rotation sym.  $S_x \equiv \prod_i \sigma_x^i$

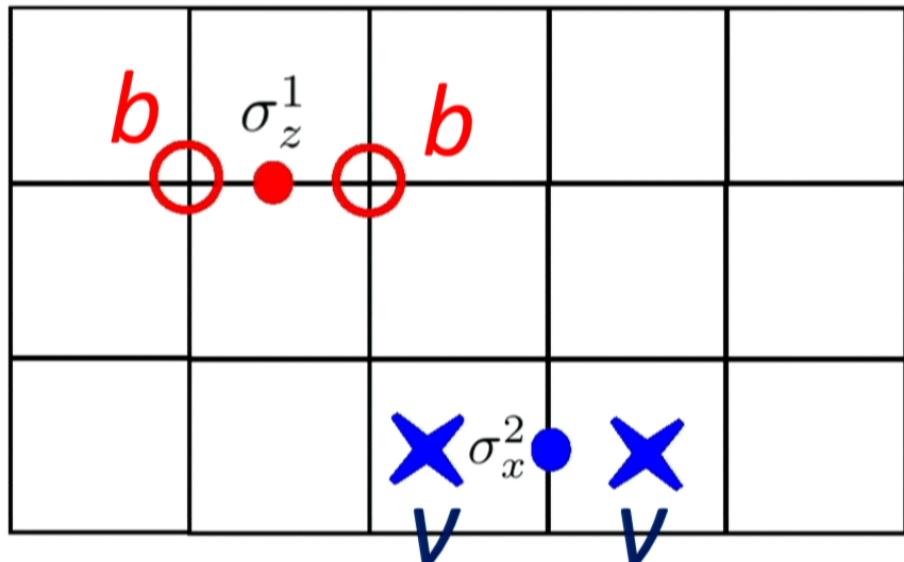
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$$S_y = S_x S_z$$

## Symmetry fractionalization:

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Anyon type	<b>b</b>	<b>v</b>	<b>f</b>
$(S_x)^2$	-1	+1	-1
$(S_z)^2$	+1	-1	-1
$(S_y)^2$	-1	-1	+1

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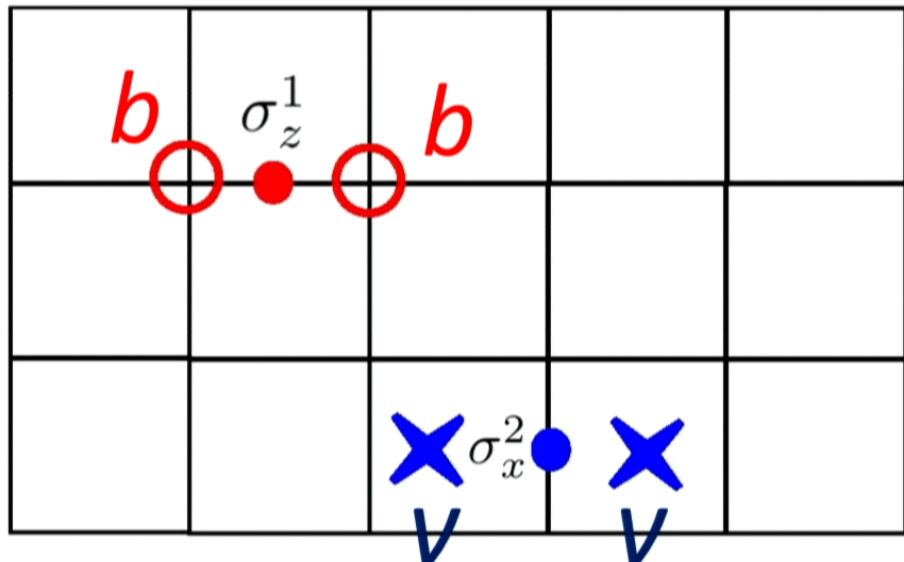
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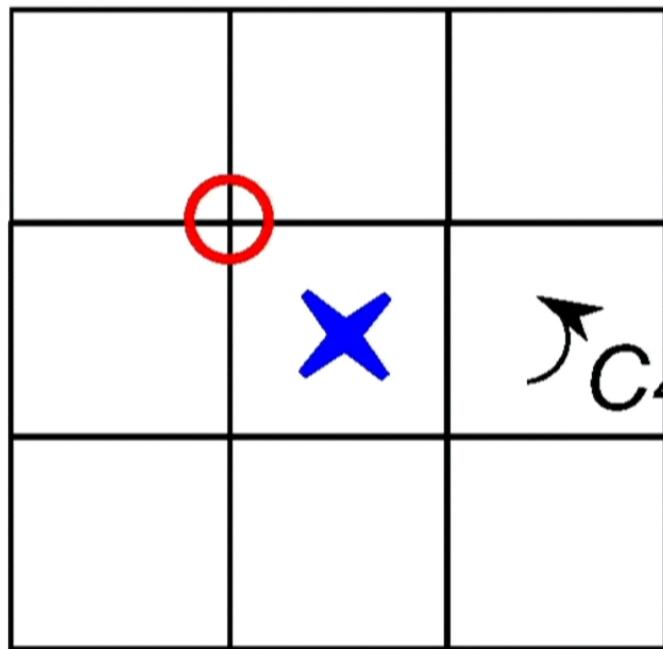
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# Crystal symmetry fractionalization

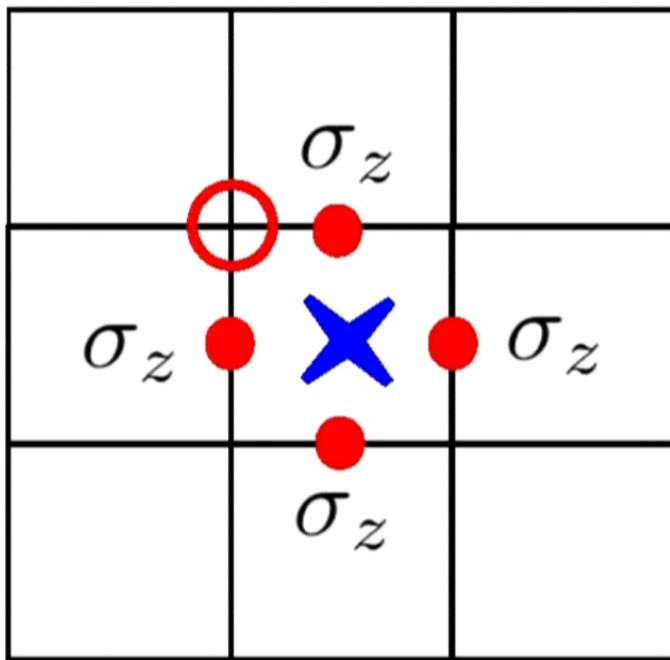


“Electric charge”  $b$

“Magnetic vortex”  $v$

Fermion  $f = b \times v$

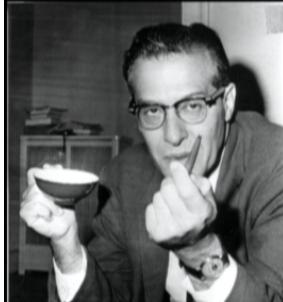
# Crystal symmetry fractionalization



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Auerbach, Arovas, *PRB* (1988)

$$\vec{S}_{\mathbf{r}} = \frac{1}{2} \sum_{\alpha, \beta = \uparrow/\downarrow} b_{\mathbf{r}, \alpha}^\dagger \vec{\sigma}_{\alpha, \beta} b_{\mathbf{r}, \beta}$$

$$\hat{H}_{MF}^b = \sum_{\mathbf{x}, \mathbf{y}} \sum_{\alpha, \beta} A_{\mathbf{x}, y} b_{\mathbf{x}, \alpha}^\dagger b_{\mathbf{y}, \alpha} + B_{\mathbf{x}, y} b_{\mathbf{x}, \alpha} \epsilon^{\alpha \beta} b_{\mathbf{y}, \beta} + h.c.$$

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$$|RVB\rangle = \prod_{\mathbf{r}} \hat{P}_{n_{\mathbf{r}}=1} |MF\rangle$$

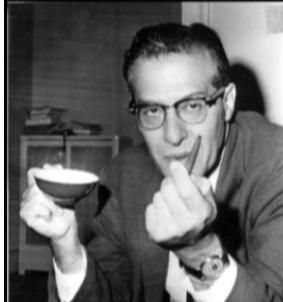
**|MF>: Singlet pair superfluid of *b* vs. Singlet superconductor of *f***

Auerbach, Arovas (1988), Sachdev (1992),  
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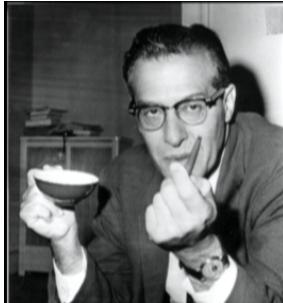
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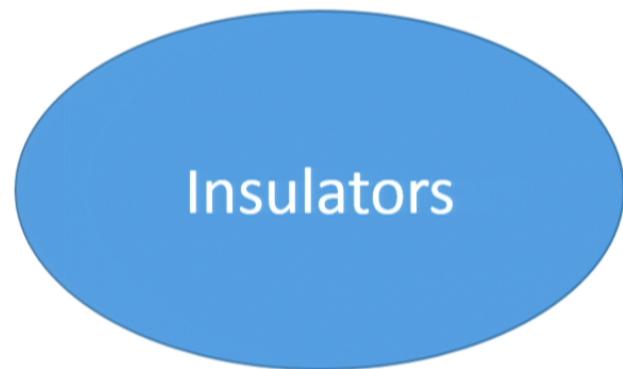
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# Symmetry meets topological order

Metals



*U(1) charge conservation*

# Symmetry meets topological order

Symmetry protected topological  
(SPT) phases

Chen, Gu, Liu, Wen (2011)



Metals

Gapless spin liquids...

Z<sub>2</sub> spin liquids  
(Z<sub>2</sub> gauge theory)

*Time reversal symmetry*

+

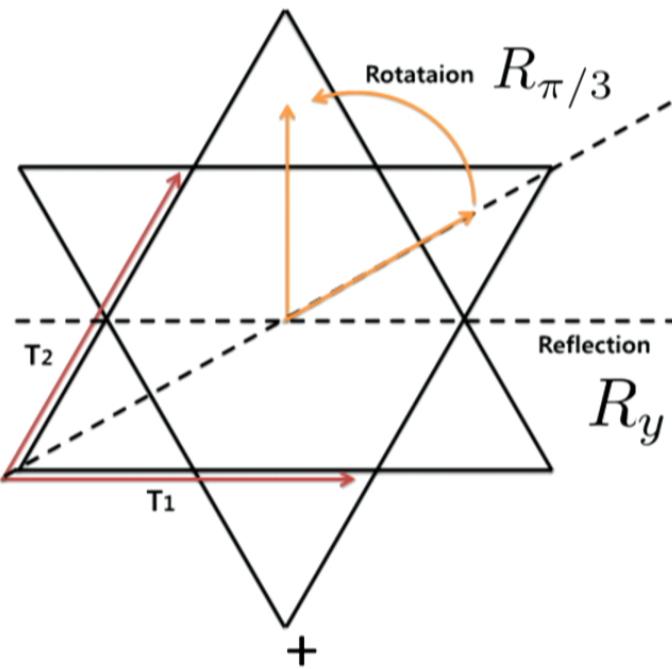
*U(1) charge conservation*

# Unify symmetric Z2 SLs on kagome lattice?

Schwinger-bosons:

8 states

Fa Wang, Vishwanath (2006)



Abrikosov-fermions

20 states

YML, Ran, Lee (2011)

Time reversal + SO(3) spin rotation

**Projective symmetry group (PSG) characterizes symmetry fractionalization**

Xiao-Gang Wen (2002)

# PSGs of $b$ & $v$ -> PSGs of $f$

Anyon type	Bosonic spinon $b$	Vison (bosonic) Senthil, Fisher (2000)	Fermionic spinon $f$
$T^2$	-1	+1	-1
$T_1^{-1}T_2^{-1}T_1T_2$	$(-1)^{p_1}$	-1	$(-1)^{p_1+1}$
$(R_y)^2$	$a$	$b$	$ab$
$(R_{\pi/3})^6$	$c$	$d$	$-cd$
		$b \times f = v, \quad b \times v = f, \quad f \times v = b,$	
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$Z_2$  gauge theory  
underlies RVB  
( $Z_2$  S.L.) states

Wen (1991)  
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# Schwinger-boson states cannot have symmetry protected edge states

$$\hat{H}_{MF}^b = \sum_{\mathbf{x}, \mathbf{y}} \sum_{\alpha, \beta} A_{\mathbf{x}, y} b_{\mathbf{x}, \alpha}^\dagger b_{\mathbf{y}, \alpha} + B_{\mathbf{x}, y} b_{\mathbf{x}, \alpha} \epsilon^{\alpha \beta} b_{\mathbf{y}, \beta} + h.c.$$

Vacuum

Bulk of  
Z2 spin liquid

Solve BdG equations  
of boson pair superfluid  
---> no gapless edge states!

$\mu = +\infty$

$\mu > |B|$

$E \sim \sqrt{\mu^2 - |B|^2}$

YML, Cho, Vishwanath, to appear

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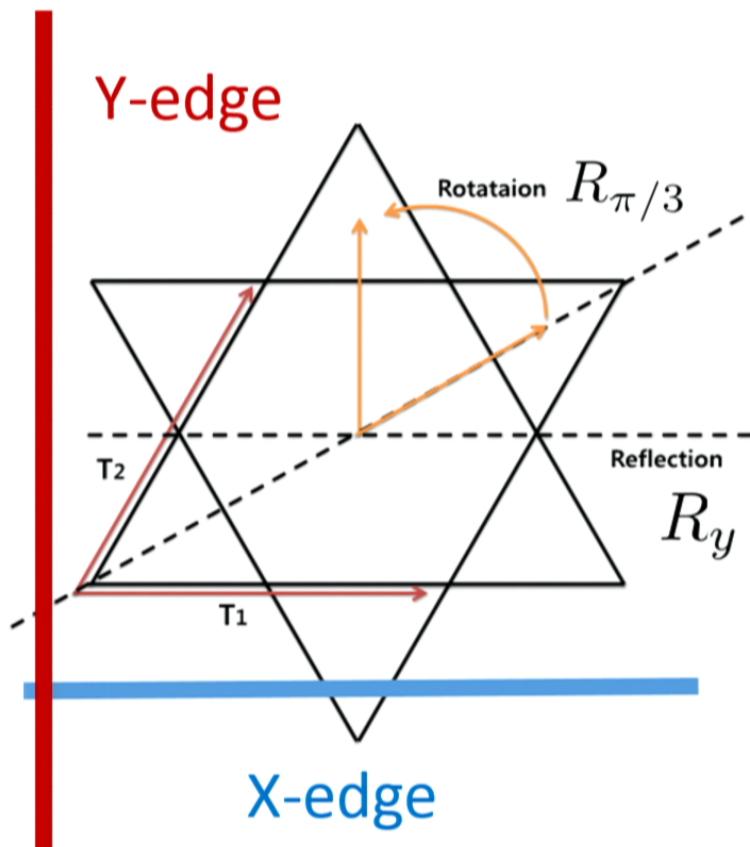
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# Trivial PSGs for visons on the edge of Schwinger-boson states!



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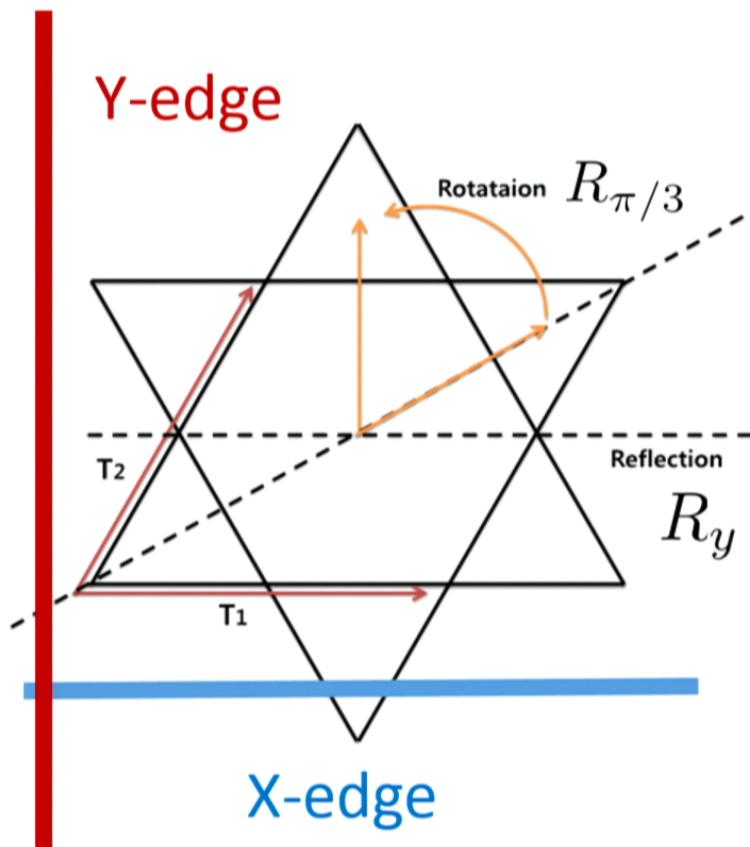
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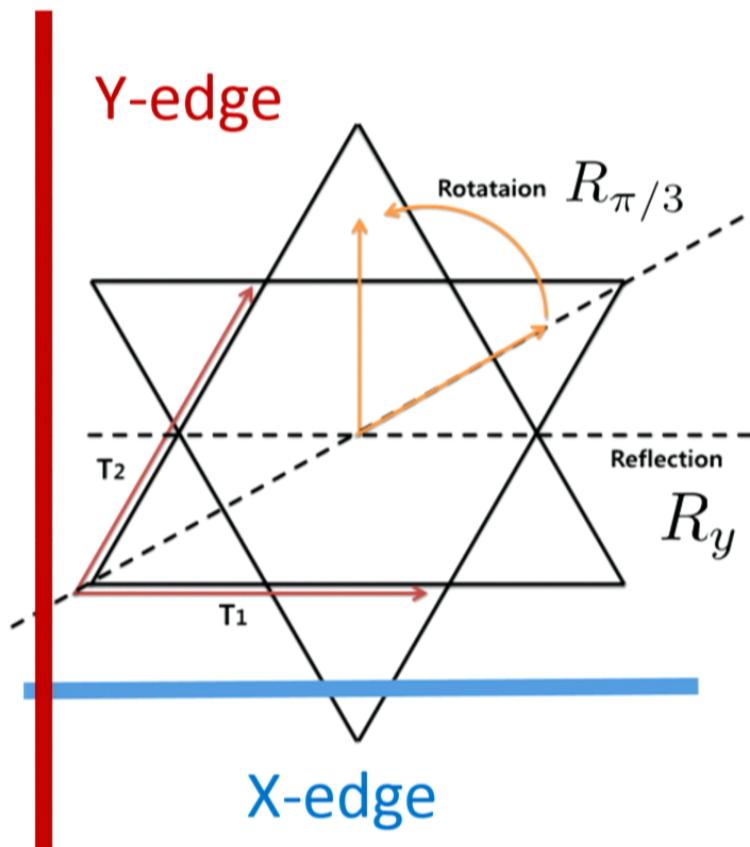
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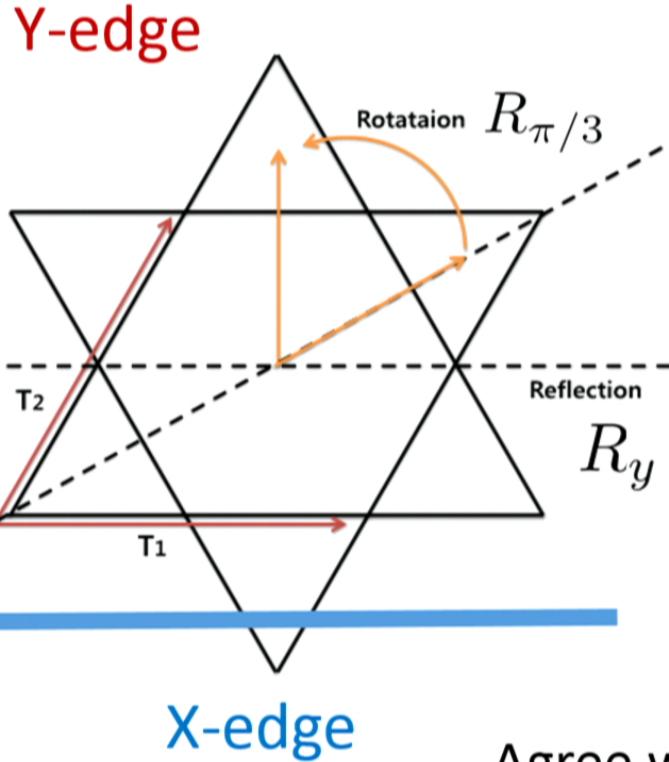
# Trivial PSGs for visons on the edge of Schwinger-boson states!



# Trivial PSGs for visons on the edge of Schwinger-boson states!



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Algebraic Identities	bosonic $b_\sigma$	fermionic $f_\sigma$	vison $v = b \times f$
$T_2^{-1}T_1^{-1}T_2T_1$	$(-1)^{p_1}$	$\eta_{12}$	-1
$T_1^{-1}R_{\pi/3}^{-1}T_2R_{\pi/3}$	1	1	1
$T_1^{-1}T_2R_{\pi/3}^{-1}T_1R_{\pi/3}$	1	1	1
$T_1^{-1}R_y^{-1}T_1R_y$	$(-1)^{p_1}$	$\eta_{12}$	-1
$T_1^{-1}T_2R_y^{-1}T_2R_y$	$(-1)^{p_1}$	$\eta_{12}$	-1
$(R_{\pi/3}R_y)^2$	$(-1)^{p_2}$	$\eta_\sigma$	1
$(R_y)^2$	$(-1)^{p_2+p_3}$	$\eta_\sigma \eta_\sigma C_6$	1
$(R_{\pi/3})^6$	$(-1)^{p_1+p_3}$	$\eta_{C_6}$	1
$T_1^{-1}T^{-1}T_1T$	1	1	1
$T_2^{-1}T^{-1}T_2T$	1	1	1
$R_y^{-1}T^{-1}R_yT$	$(-1)^{p_2}$	$\eta_{\sigma T} \eta_{C_6 T}$	1
$R_{\pi/3}^{-1}T^{-1}R_{\pi/3}T$	$(-1)^{p_3}$	$\eta_{C_6 T}$	1
$T^2$	-1	-1	1

Agree with explicit calculation: *Huh, Punk, Sachdev (2011)*

# Unification of Z2 SLs on kagome lattice

Fermionic-spinon representation[34] ( <i>f</i> SR)								Schwinger-boson rep.[23] ( <i>b</i> SR)		
#	$\eta_{12}$	$\eta_\sigma$	$\eta_{\sigma T}$	$\eta_{\sigma C_6}$	$\eta_{C_6 T}$	$\eta_{C_6}$	Label	Perturbatively gapped?	$(p_1, p_2, p_3)$	Label
1	+1	+1	+1	+1	+1	+1	$Z_2[0, 0]A$	Yes	(1,0,0)	-
2	-1	+1	+1	+1	+1	-1	$Z_2[0, \pi]\beta$	Yes	(0,0,0)	-
15	+1	-1	-1	+1	+1	+1	$Z_2[0, 0]C$	Yes	(1,1,0)	-
16	-1	-1	-1	+1	+1	-1	$Z_2[0, \pi]\delta$	No	(0,1,0)	$Q_1 = Q_2$ state

In the neighborhood of U(1) Dirac SL

(Ran, Hermele, Lee, Wen, PRL 2007)

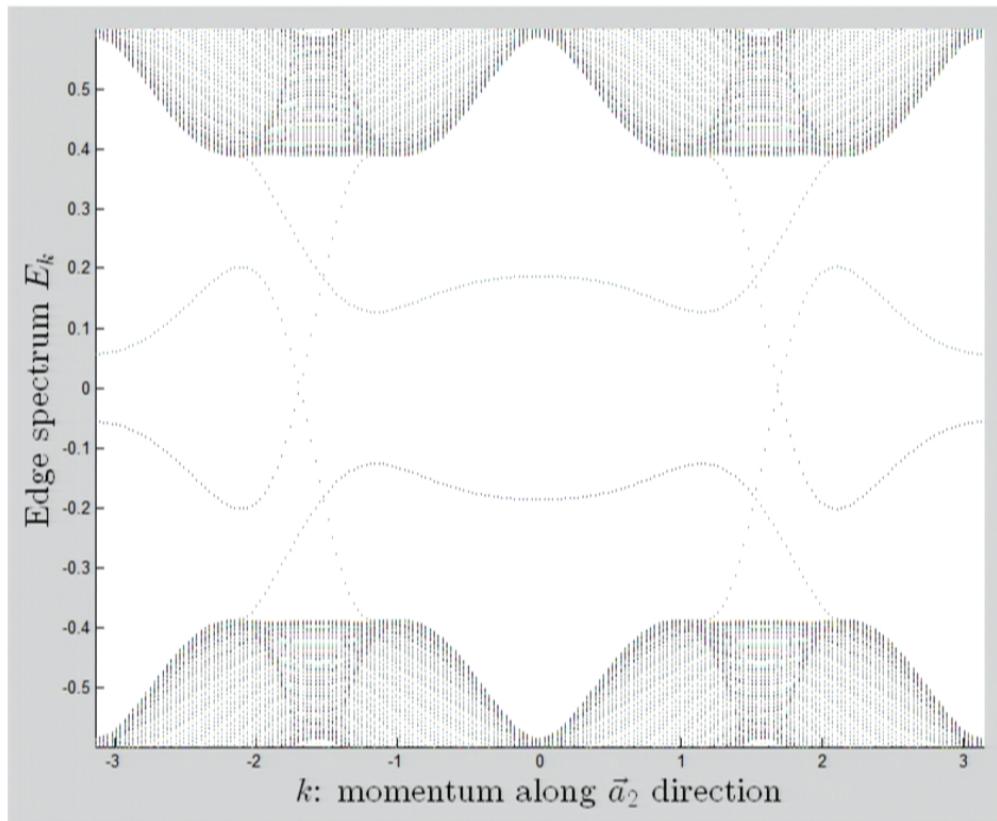
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YML, Cho, Vishwanath, to appear

# Reflection-protected gapless edge modes in Abrikosov-fermion RVB states



Topological crystalline superconductors  
of fermionic spinons

Only possible when  $R^2 = -1$   
for fermionic spinons!

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Morimoto, Furusaki (2013)

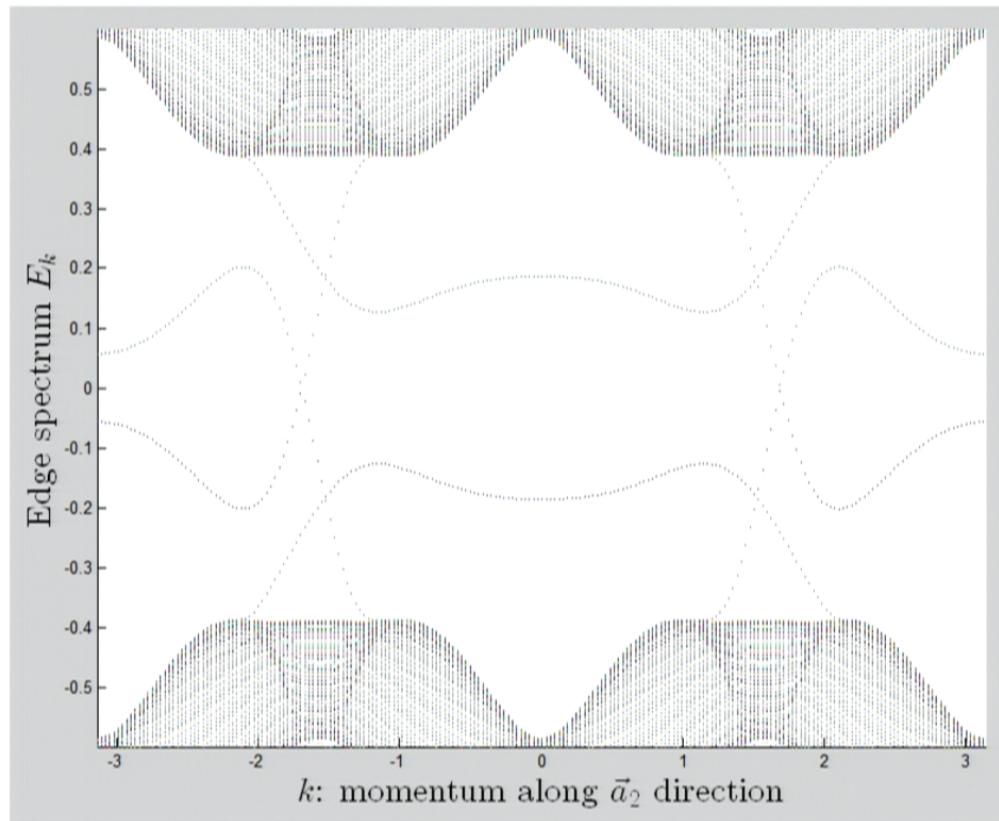
Chern-Simons theory: 4x4 K matrix

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# Conclusion and outlook

- Symmetry fractionalization must lead to symmetry protected gapless edge states in a Z2 spin liquid
- Schwinger-boson RVB states cannot have gapless edge states
- Unification of Schwinger-boson and Abrikosov-fermion RVB states
- Abrisokov-fermion RVB states can have protected edge states:  
topological crystalline (s-wave) superconductors of fermionic spinons
- Chern-Simons field theory for the bulk/edge
  
- Neighboring phases of Z2 spin liquids?
- Application to other lattices, such as square lattice (J1-J2 model)?
- Interaction effects on topological crystalline superconductors?

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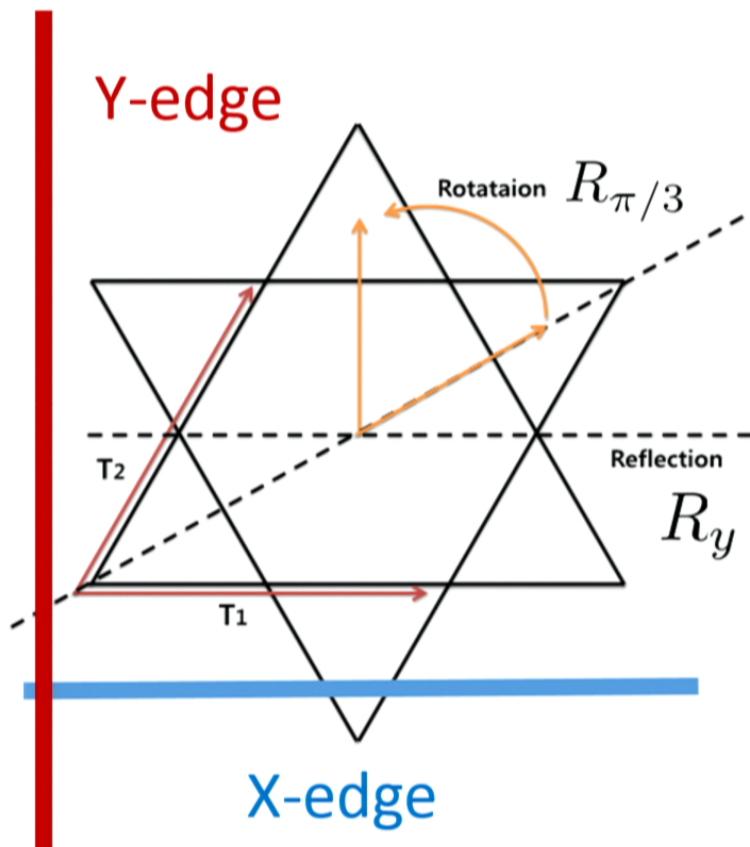
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#	$\eta_{12}$	$\eta_\sigma$	$\eta_{\sigma T}$	$\eta_{\sigma C_6}$	$\eta_{C_6 T}$	$\eta_{C_6}$	$(p_1, p_2, p_3)$	$\Lambda_s$	$u_\alpha$	$u_\beta$	$u_\gamma$	$\tilde{u}_\gamma$	Label
1	+1	+1	+1	+1	+1	+1	(1,0,0)	$\tau^2, \tau^3$	$Z_2[0, 0]A$				
2	-1	+1	+1	+1	+1	-1	(0,0,0)	$\tau^2, \tau^3$	$\tau^2, \tau^3$	$\tau^2, \tau^3$	$\tau^2, \tau^3$	0	$Z_2[0, \pi]\beta$
3	+1	+1	+1	-1	+1	-1		0	$\tau^2, \tau^3$	0	0	0	$Z_2[\pi, \pi]A$
4	-1	+1	+1	-1	+1	+1		0	$\tau^2, \tau^3$	0	0	$\tau^2, \tau^3$	$Z_2[\pi, 0]A$
5	+1	+1	+1	-1	-1	-1		$\tau^3$	$\tau^2, \tau^3$	$\tau^3$	$\tau^3$	$\tau^3$	$Z_2[0, 0]B$
6	-1	+1	+1	-1	-1	+1		$\tau^3$	$\tau^2, \tau^3$	$\tau^3$	$\tau^3$	$\tau^2$	$Z_2[0, \pi]\alpha$
7	+1	-1	+1	-1	+1	-1		0	0	$\tau^2, \tau^3$	0	0	-
8	-1	-1	+1	-1	+1	+1		0	0	$\tau^2, \tau^3$	0	0	-
9	+1	-1	+1	+1	+1	+1		0	0	0	$\tau^2, \tau^3$	0	-
10	-1	-1	+1	+1	+1	-1		0	0	0	$\tau^2, \tau^3$	0	-
11	+1	-1	+1	+1	-1	-1		0	0	$\tau^2$	$\tau^2$	0	-
12	-1	-1	+1	+1	-1	+1		0	0	$\tau^2$	$\tau^2$	0	-
13	+1	-1	-1	-1	-1	-1		$\tau^3$	$\tau^3$	$\tau^2, \tau^3$	$\tau^3$	$\tau^3$	$Z_2[0, 0]D$
14	-1	-1	-1	-1	-1	+1		$\tau^3$	$\tau^3$	$\tau^2, \tau^3$	$\tau^3$	0	$Z_2[0, \pi]\gamma$
15	+1	-1	-1	+1	+1	+1	(1,1,0)	$\tau^3$	$\tau^3$	$\tau^3$	$\tau^2, \tau^3$	$\tau^3$	$Z_2[0, 0]C$
16	-1	-1	-1	+1	+1	-1	(0,1,0)	$\tau^3$	$\tau^3$	$\tau^3$	$\tau^2, \tau^3$	0	$Z_2[0, \pi]\delta$
17	+1	-1	-1	+1	+1	-1		0	$\tau^2$	$\tau^3$	0	0	$Z_2[\pi, \pi]B$
18	-1	-1	-1	+1	+1	+1		0	$\tau^2$	$\tau^3$	0	$\tau^3$	$Z_2[\pi, 0]B$
19	+1	-1	-1	+1	-1	-1		0	$\tau^2$	0	$\tau^2$	0	$Z_2[\pi, \pi]C$
20	-1	-1	-1	+1	-1	+1		0	$\tau^2$	0	$\tau^2$	$\tau^3$	$Z_2[\pi, 0]C$

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