Title: Geometry of topological matter: some examples

Date: Feb 12, 2014 09:00 AM

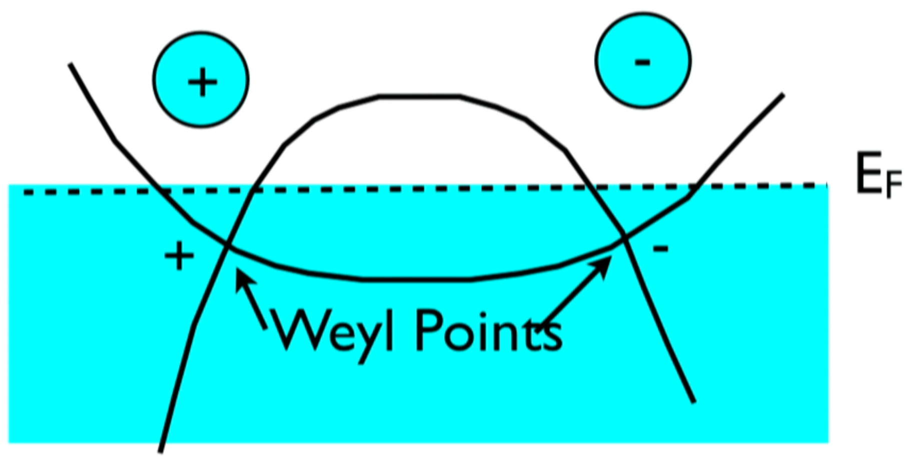
URL: http://pirsa.org/14020117

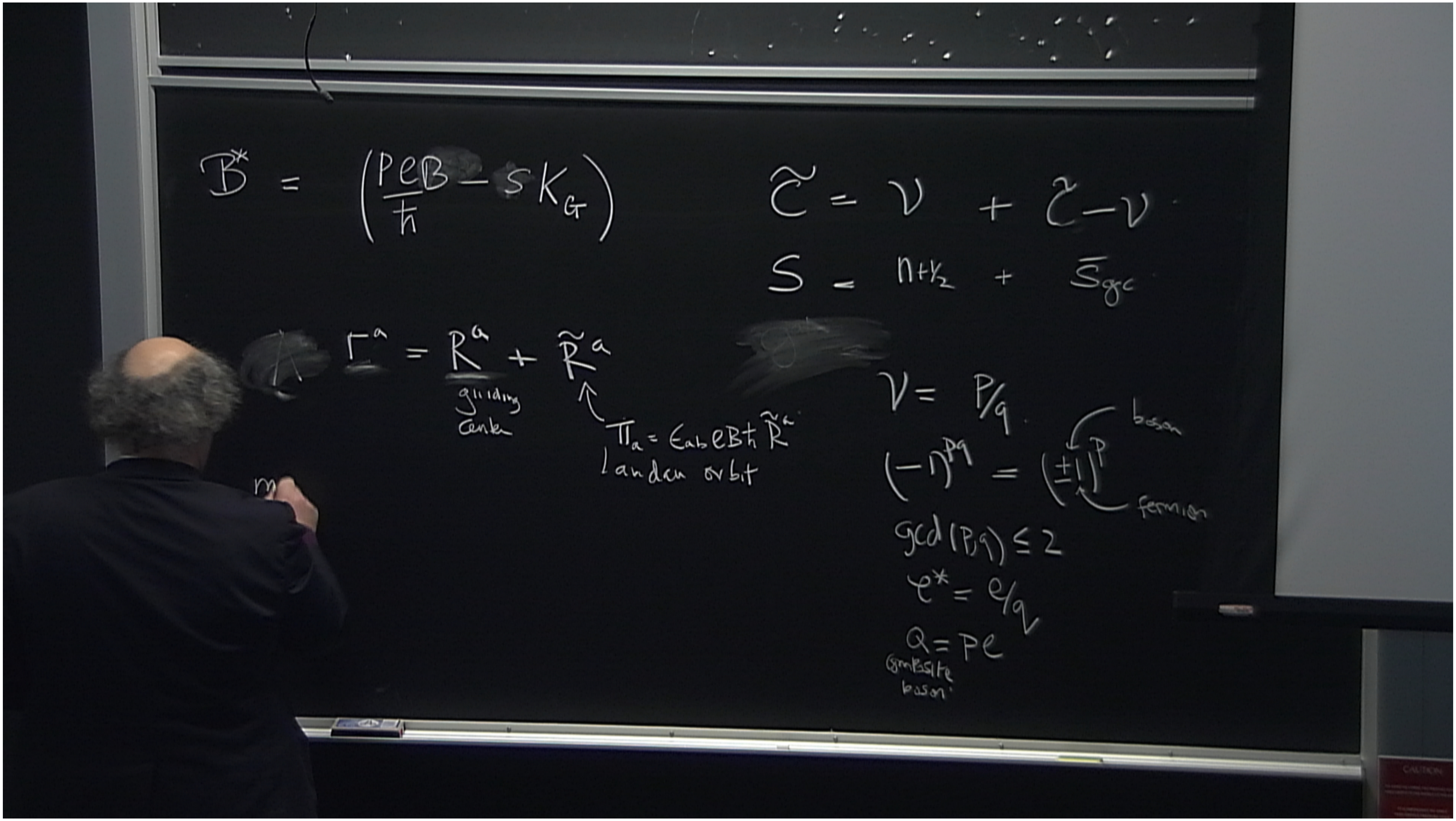
Abstract: I will look at two cases of the interplay of geometry (curvature) and topology:
(1) 3D Topological metals: how to understand their surface "Fermi arcs" in terms of their emergent conservation laws and the Streda formula for the non-quantized anomalous Hall effect.
 (2) The Hall viscosity tensor in the FQHE as a local field, and its Gaussian-curvature response that allows local compression or expansion of the fluid to accommodate substrate inhomogeneity.

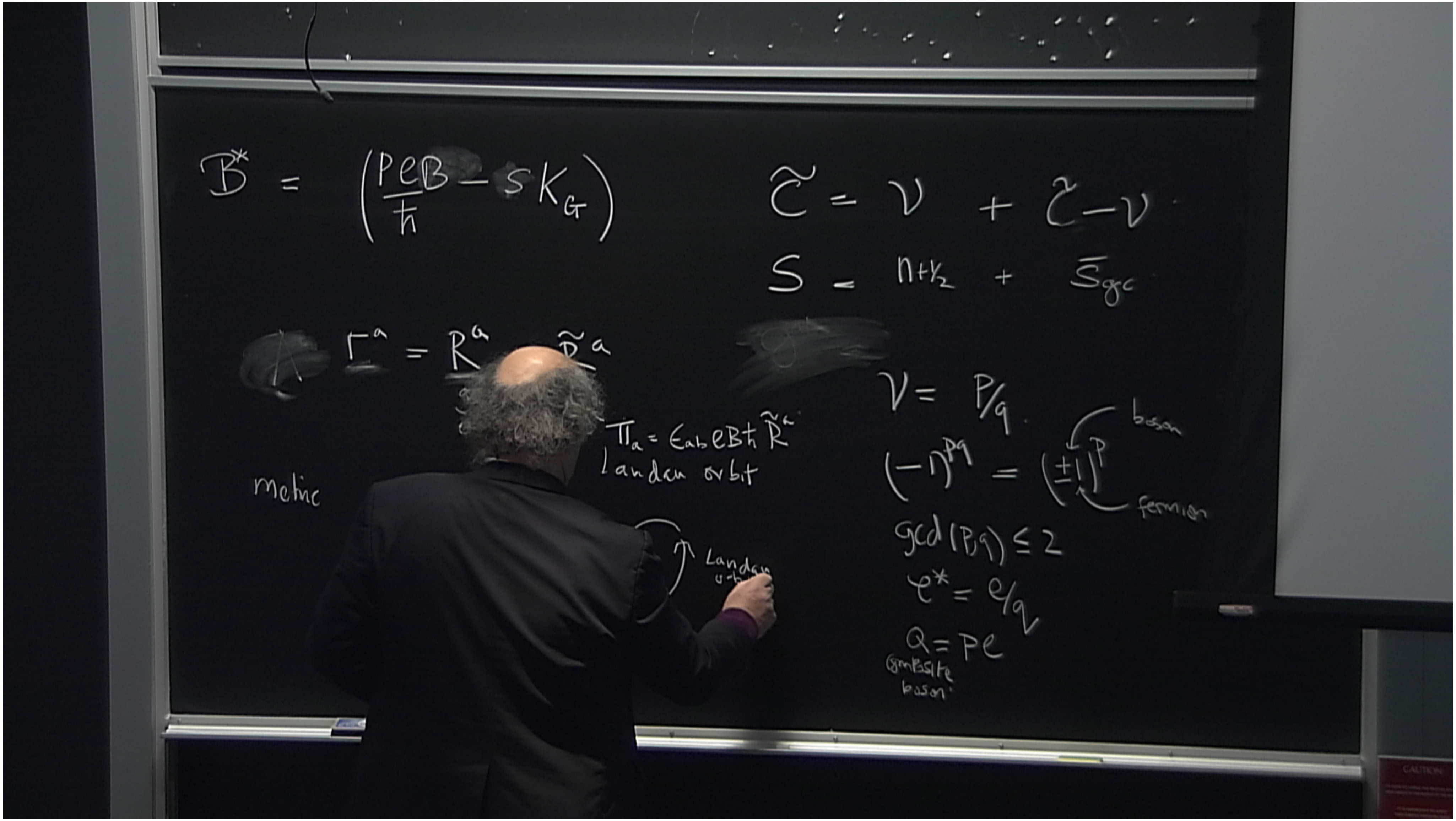
Topological Metals

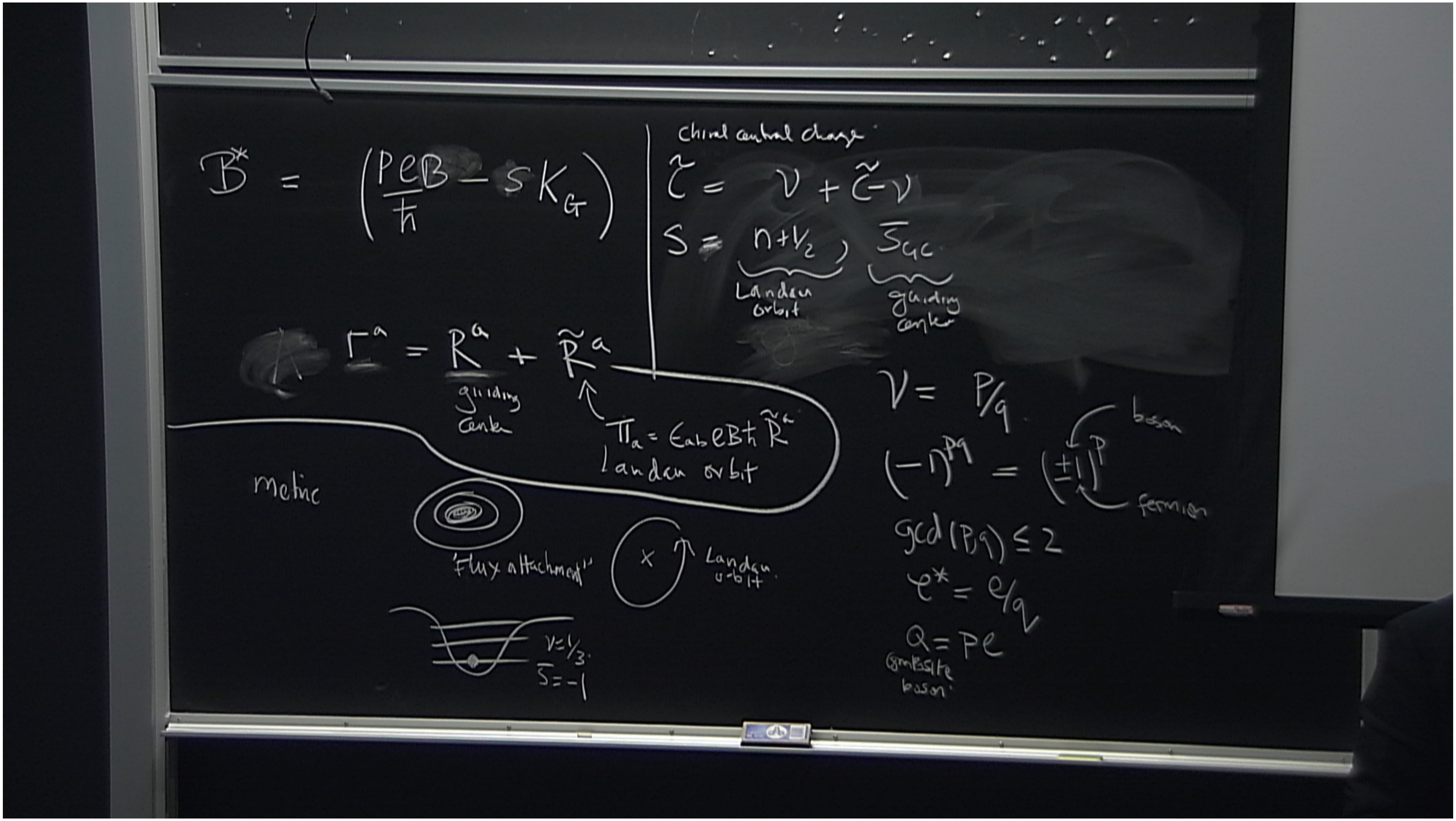
- The Fermi surface of 3D metals can break up into topologically disconnected sheets
- A sheet of the the Fermi surface of a 3D metal with spin-orbital coupling can have a non-zero Chern number (total Chern number of all Fermi surface sheets must vanish)

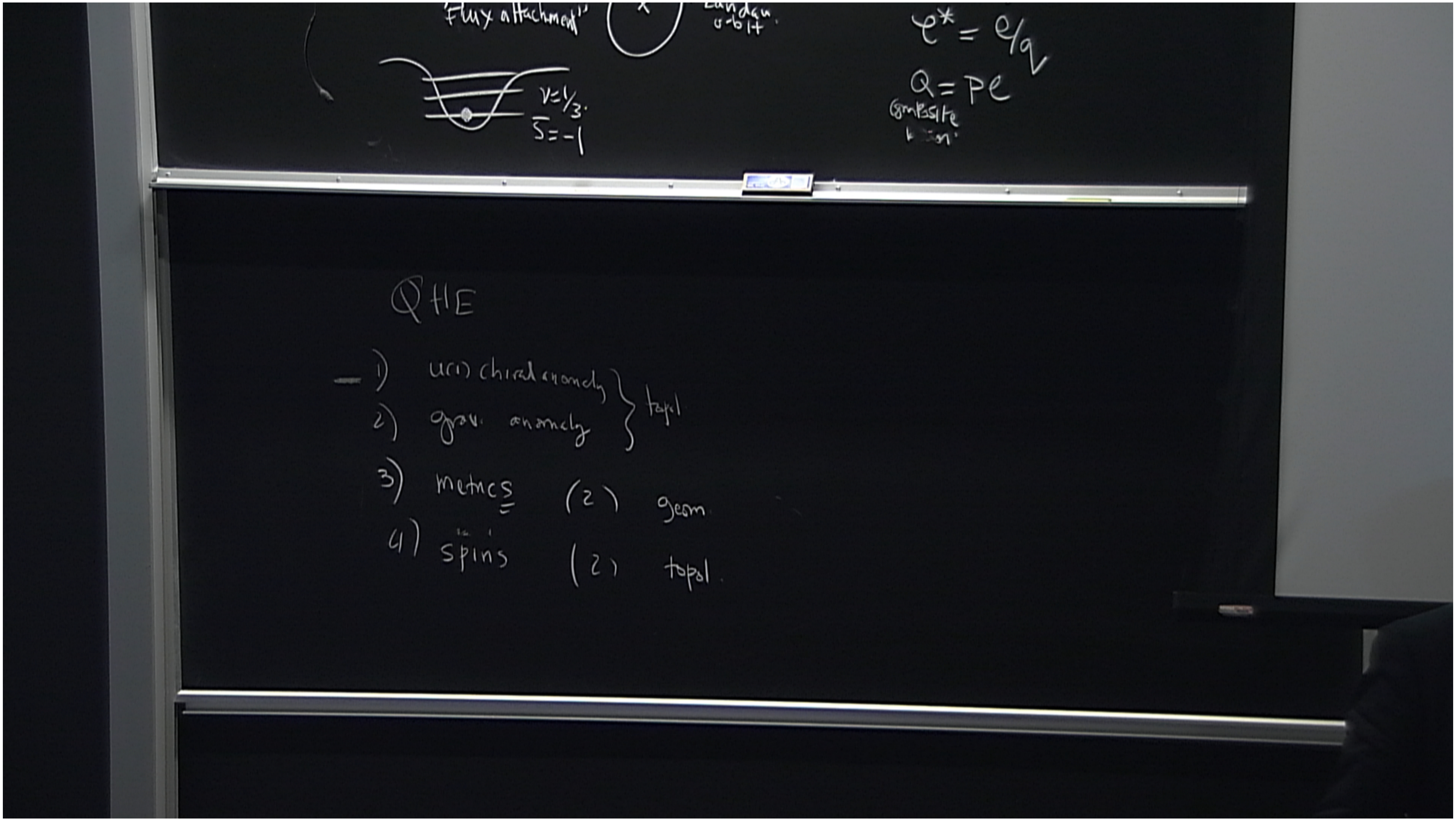
Weyl points are monopole sources/sinks of Berry curvature flux




$B^* = \left(\frac{peB}{\hbar} - sK_G\right)$
$\tilde{C} = \nu + \tilde{C} - \nu$
$S = n + \frac{1}{2} + \bar{S}_{gc}$
$\Gamma^a = R^a + \tilde{R}^a$
guiding center
$\Pi_a = \epsilon_{ab} eB\hbar \tilde{R}^a$
Landau orbit
$\nu = p/q$
boson
$(-1)^{pq} = (\pm 1)^p$
fermion
$\gcd(p,q) \leq 2$
$e^* = e/q$
$Q = pe$
composite boson


metric
Landau orbit
Landau orbit
boson
fermion
gcd (p,q) ≤ 2
composite boson


B* = (peB/ħ − s K_G)
chiral central charge
c̃ = ν + c̃ − ν
s = n + 1/2   s̄_GC
Landau orbit
guiding center
Γ^a = R^a + R̃^a
guiding center
Π_a = ε_ab eBħ R̃^a
Landau orbit
metric
"Flux attachment"
Landau orbit
ν = 1/3
s̄ = −1
ν = p/q
(−1)^pq = (±1)^p
boson
fermion
gcd(p,q) ≤ 2
e* = e/q
Q = pe
composite boson


"Flux attachment"
Landau orbit.
e* = e/q
Q = pe
ν=1/3.
S̄=-1
QHE
1) U(1) chiral anomaly
2) grav. anomaly
topol
3) metrics (2) geom.
4) spins (2) topol.

- (Intrinsic) Anomalous Hall effect
- Hall effect due to broken time-reversal symmetry, but not from Lorentz force
- Modern form of Karplus-Luttinger formula

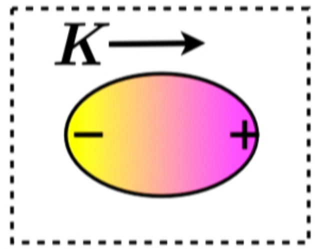
$$\sigma_H^{ab} = \frac{e^2}{\hbar} \int_{\mathrm{BZ}} \frac{d^3\boldsymbol{k}}{(2\pi)^3} \sum_n n_n(\boldsymbol{k}) \mathcal{F}_n^{ab}(\boldsymbol{k})$$

occupation factor

Berry curvature (antisymmetric)

$$\boldsymbol{K} = \boldsymbol{G} + \frac{1}{2\pi}\left(\sum_{\alpha}\int_{\mathrm{FS}_\alpha} \boldsymbol{k}_F \mathcal{F}_n dA + \sum_{\alpha,i}(\Delta \boldsymbol{G}_i)\oint \mathcal{A} dk\right)$$

- “Dipole moment of Fermi-Surface Berry curvature in the Brillouin zone



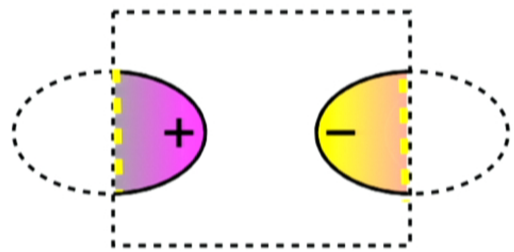
- gauge invariance

$$\boldsymbol{k}_F \mapsto \boldsymbol{k}_F + e\boldsymbol{A}/\hbar$$

$$\boldsymbol{K} \mapsto \boldsymbol{K} + \frac{1}{2\pi}\sum_{\alpha}\int_{\mathrm{FS}_\alpha} \mathcal{F} dA$$

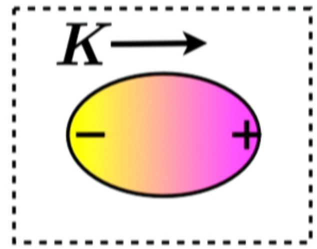
sum of Chern numbers must vanish

BZ boundary term ensures these two choices of BZ give same K



$$\boldsymbol{K} = \boldsymbol{G} + \frac{1}{2\pi}\left(\sum_{\alpha}\int_{\mathrm{FS}_\alpha} \boldsymbol{k}_F \mathcal{F}_n dA + \sum_{\alpha,i}(\Delta \boldsymbol{G}_i)\oint A dk\right)$$

- “Dipole moment of Fermi-Surface Berry curvature in the Brillouin zone



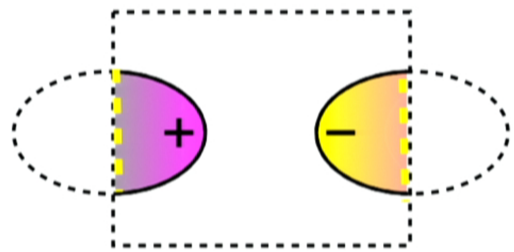
- gauge invariance

$$\boldsymbol{k}_F \mapsto \boldsymbol{k}_F + e\boldsymbol{A}/\hbar$$

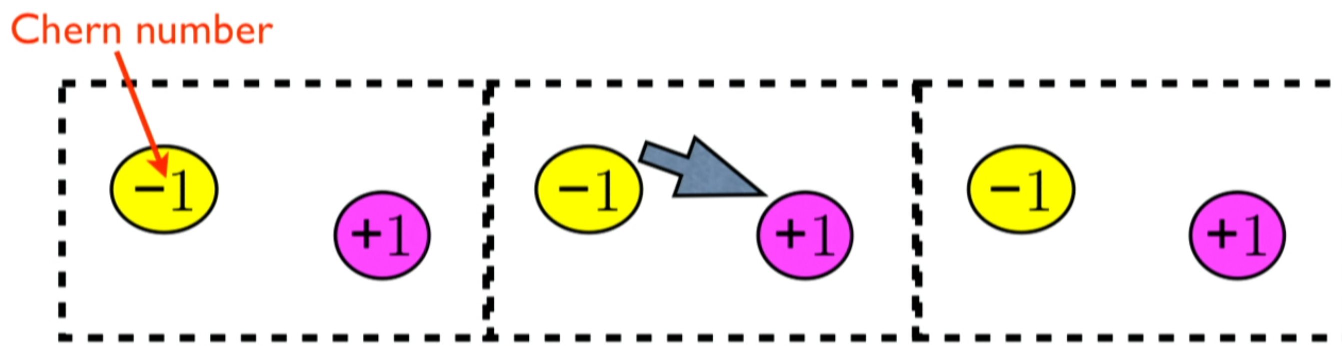
$$\boldsymbol{K} \mapsto \boldsymbol{K} + \frac{1}{2\pi}\sum_{\alpha}\int_{\mathrm{FS}_\alpha} \mathcal{F} dA$$

sum of Chern numbers must vanish

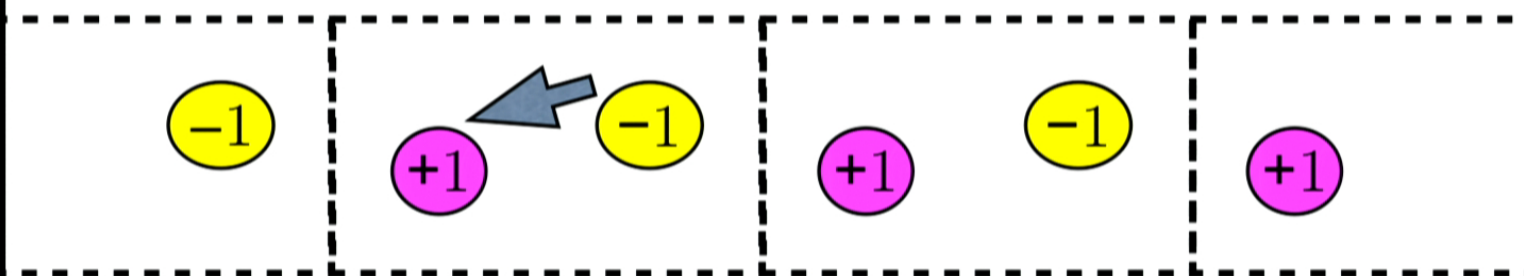
BZ boundary term ensures these two choices of BZ give same K

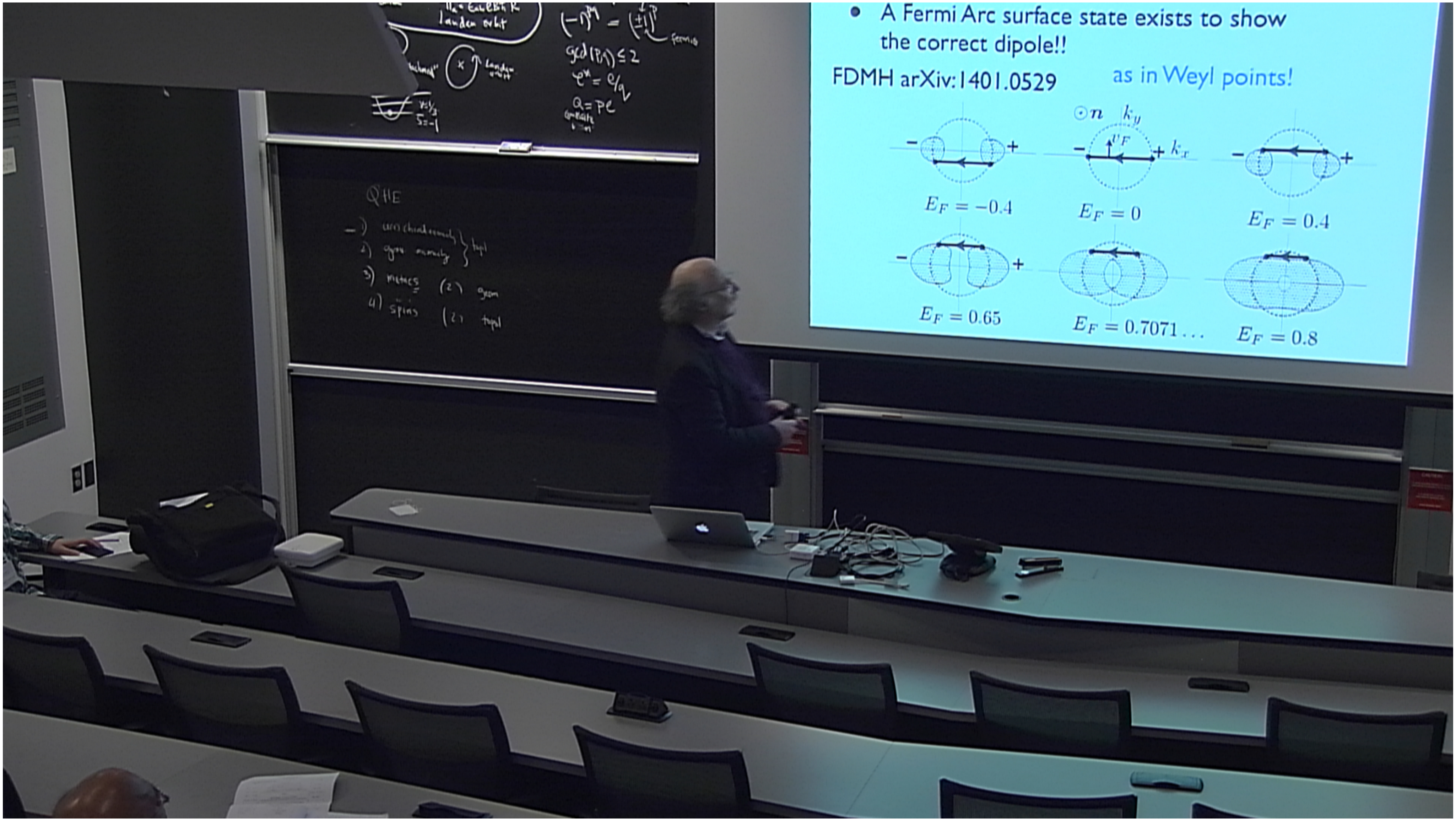


- Ambiguity in “BZ Berry dipole moment” of topological metals:



Dipoles differ by G depending on choice of BZ




A Fermi Arc surface state exists to show the correct dipole!!
FDMH arXiv:1401.0529
as in Weyl points!
$E_F = -0.4$
$E_F = 0$
$E_F = 0.4$
$E_F = 0.65$
$E_F = 0.7071\ldots$
$E_F = 0.8$

$$\Psi = \prod_{i<j}(z_i - z_j)^3 \prod_i e^{-\frac{1}{2}z_i^* z_i}$$ Laughlin 1983

- elegant wavefunction, describes topologically-ordered fluid with fractional charge fractional statistics excitations

- exact ground state of modified model keeping only short range part of coulomb repulsion

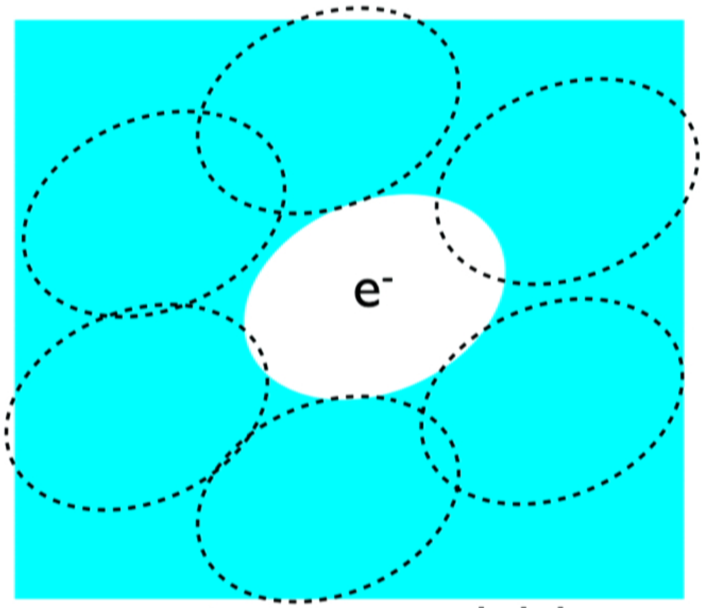
- Validity confirmed by numerical exact diagonalization

30 years later:
unanswered question:
we know it works, but why?

my answer:
hidden geometry

but no broken symmetry

- similar story in FQHE:

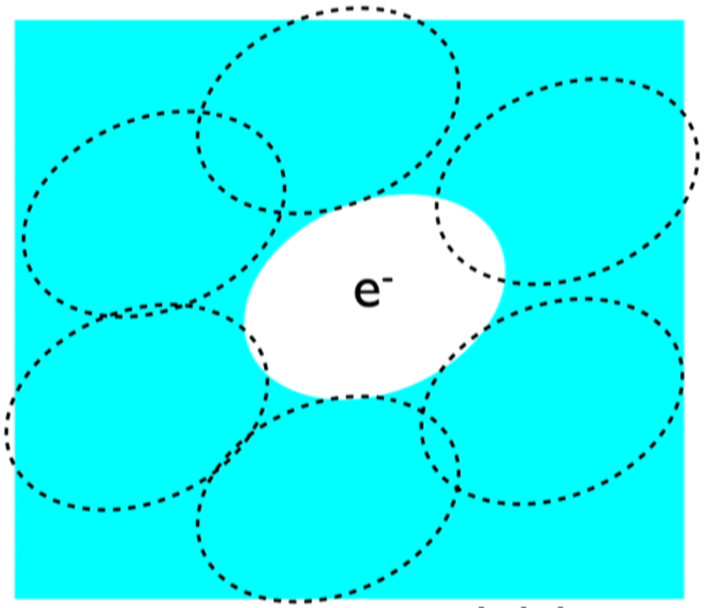


- continuum model, but similar physics to Hubbard model

- “flux attachment” creates correlation hole
- defines an emergent geometry
- potential well must be strong enough to bind electron
- new physics: Hall viscosity, geometry............

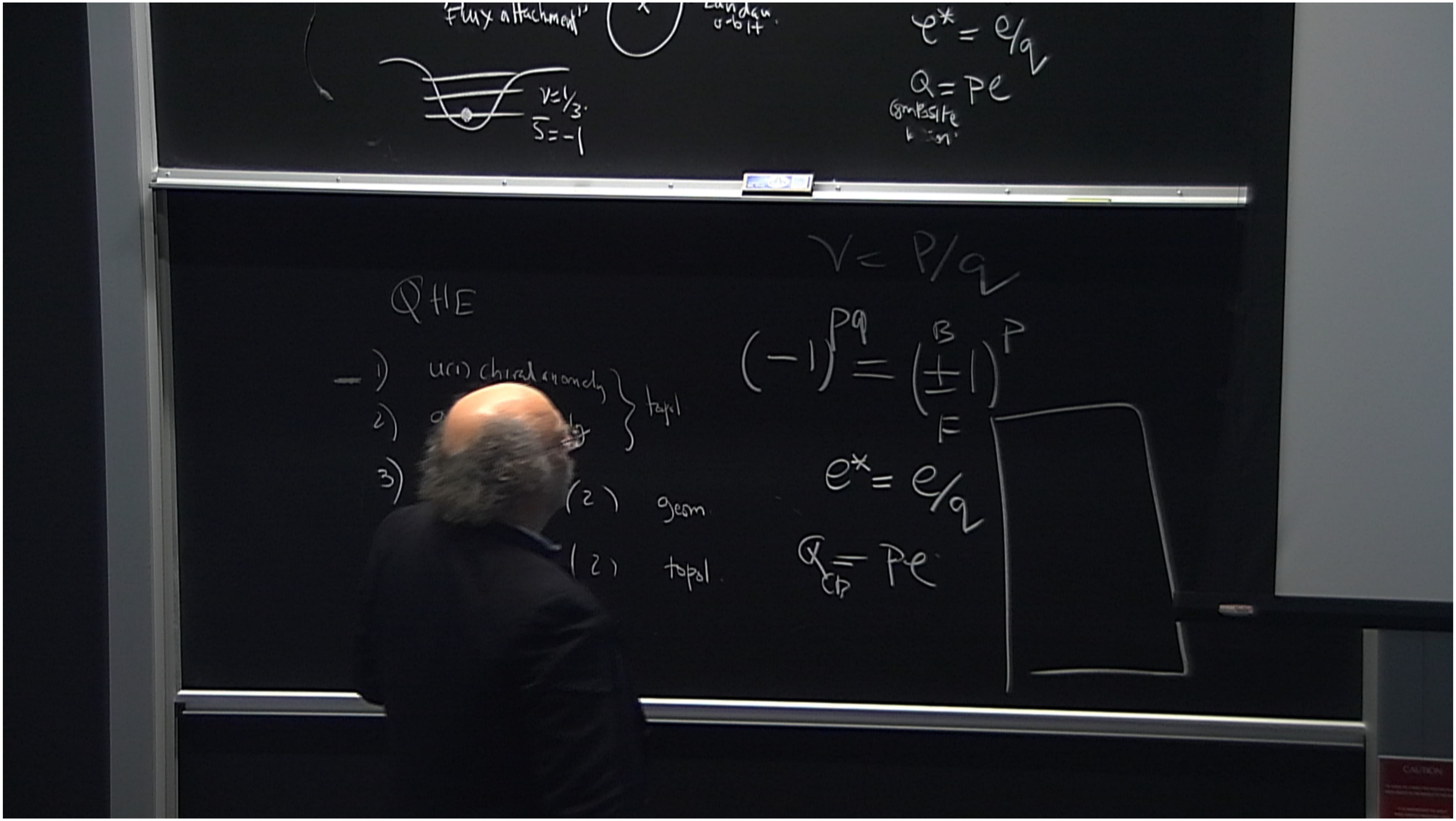
but no broken symmetry

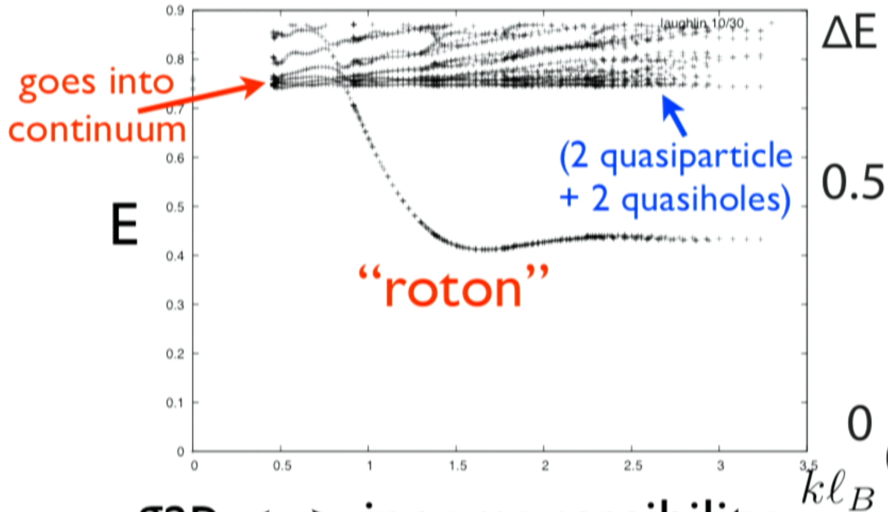
- similar story in FQHE:

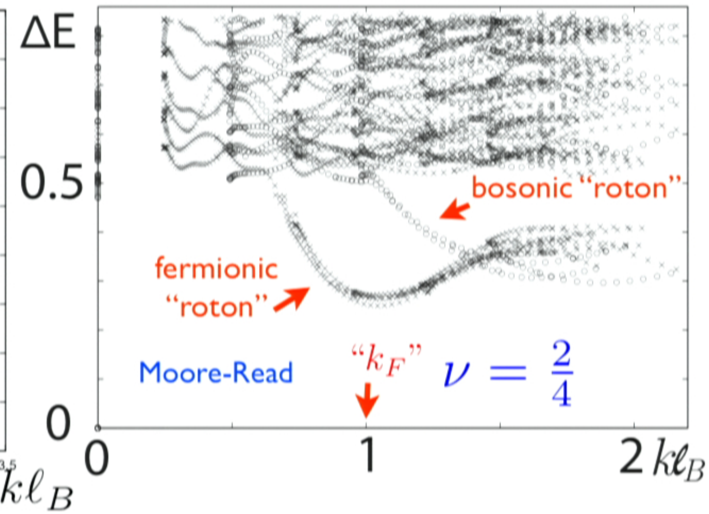


- continuum model, but similar physics to Hubbard model

- "flux attachment" creates correlation hole
- defines an emergent geometry
- potential well must be strong enough to bind electron
- new physics: Hall viscosity, geometry............


"Flux attachment"
ν=1/3
S=-1
e* = e/q
Q = pe
composite
QHE
1) U(1) chiral anomaly
topol
3)
geom
topol
ν = p/q
e* = e/q
Q = pe





gap $\longleftrightarrow$ incompressibility

Collective mode with short-range V_1 pseudopotential, 1/3 filling (Laughlin state is exact ground state in that case)

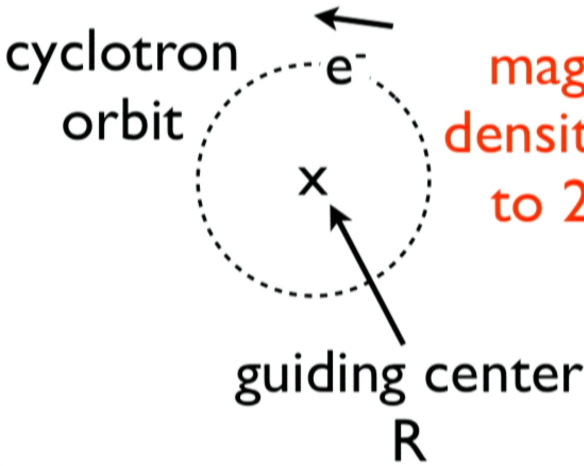
Collective mode with short-range three-body pseudopotential, 1/2 filling (Moore-Read state is exact ground state in that case)

- momentum $\hbar$k of a quasiparticle-quasihole pair is proportional to its **electric dipole moment p_e** $\hbar k_a = \epsilon_{ab} B p_e^b$

gap for electric dipole excitations is a MUCH stronger condition than charge gap: **doesn't transmit pressure!**

(origin of Virasoro algebra in FQHE ?)

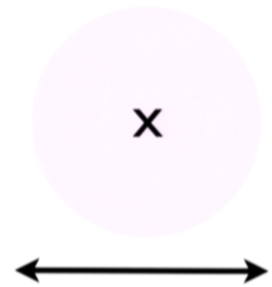
- electron in 2D Landau orbit (bound to 2D surface)



magnetic flux density B normal to 2D surface

=

Becomes a "fuzzy object" after kinetic energy is quantized

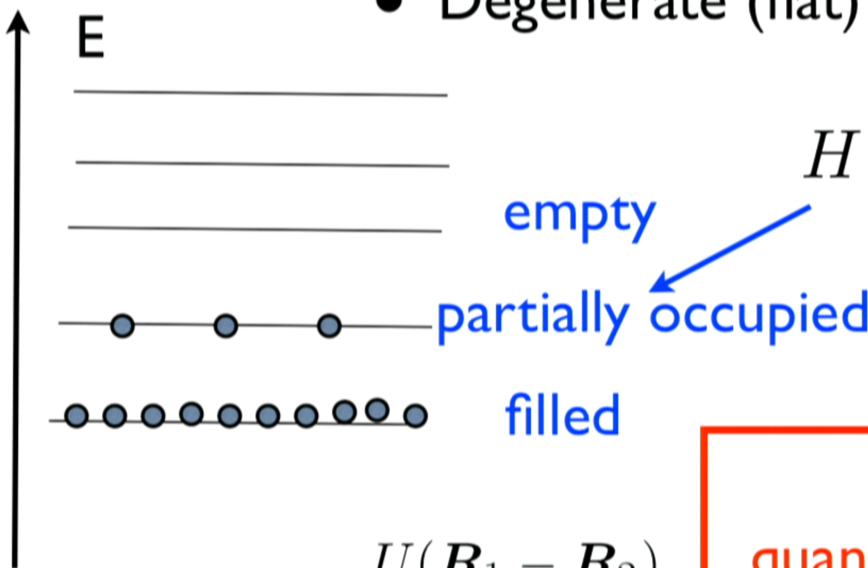


$$\ell_B = \left(\frac{\hbar}{|eB|} \right)^{\frac{1}{2}}$$

$$\boxed{[R^x, R^y] = -i\ell_B^2}$$

non-commutative geometry

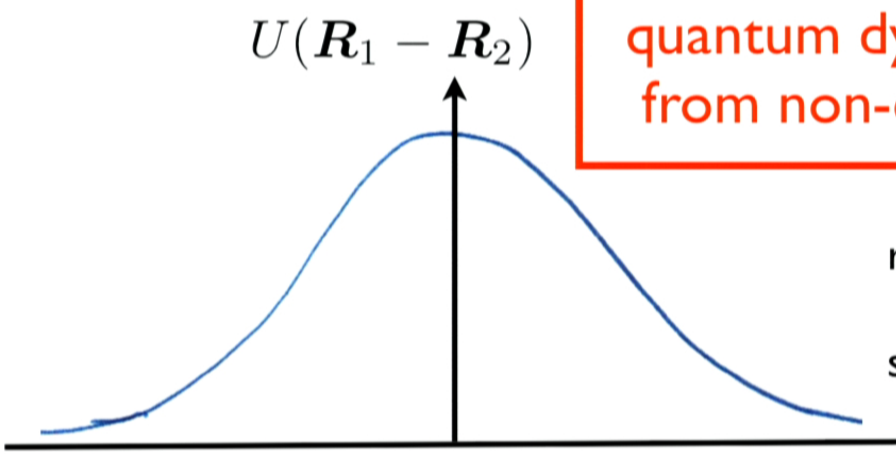
- Degenerate (flat) Landau levels



$$H = \sum_{i<j} U(\boldsymbol{R}_i - \boldsymbol{R}_j)$$

$$[R^x, R^y] = -i\ell_B^2$$

quantum dynamics comes from non-commutativity



effective Coulomb repulsion is analytic at origin because of smoothing by Landau-orbit form factor

This is the **entire** problem: nothing other than this matters!

- H has translation and inversion symmetry

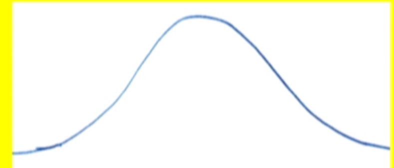
$$[(R_1^x + R_2^x), (R_1^y - R_2^y)] = 0$$

$$[H, \sum_i \boldsymbol{R}_i] = 0$$

- generator of translations and electric dipole moment!

$$[(R_1^x - R_2^x), (R_1^y - R_2^y)] = -2i\ell_B^2$$

- relative coordinate of a pair of particles behaves like a single particle

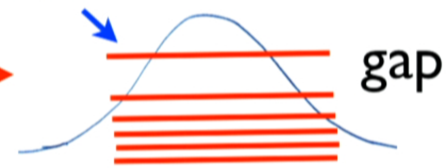


$$H = \sum_{i<j} U(\boldsymbol{R}_i - \boldsymbol{R}_j)$$

$$[R^x, R^y] = -i\ell_B^2$$

like phase-space, has Heisenberg uncertainty principle

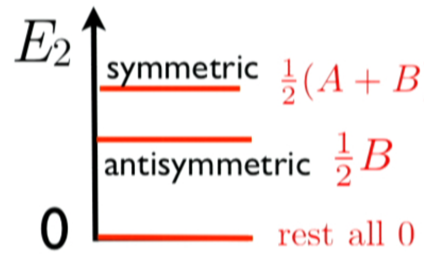
want to avoid this state



two-particle energy levels

- Solvable model! ("short-range pseudopotential")

$$\boxed{U(r_{12}) = \left(A + B\left(\frac{(r_{12})^2}{\ell_B^2}\right)\right) e^{-\frac{(r_{12})^2}{2\ell_B^2}}}$$



- Laughlin state

$$|\Psi_L^m\rangle = \prod_{i<j}\left(a_i^\dagger - a_j^\dagger\right)^m |0\rangle$$

$$a_i|0\rangle = 0 \qquad a_i^\dagger = \frac{R^x + iR^y}{\sqrt{2}\ell_B}$$

$$E_L = 0 \qquad [a_i, a_j^\dagger] = \delta_{ij}$$

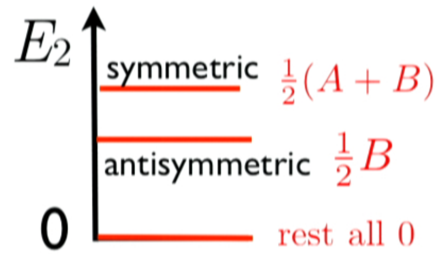
maximum density null state

- m=2: (bosons): all pairs avoid the symmetric state E_2 = ½(A+B)

- m=3: (fermions): all pairs avoid the antisymmetric state E_2 = ½B

- Solvable model! ("short-range pseudopotential")

$$\boxed{U(r_{12}) = \left(A + B\left(\frac{(r_{12})^2}{\ell_B^2}\right)\right) e^{-\frac{(r_{12})^2}{2\ell_B^2}}}$$



- Laughlin state

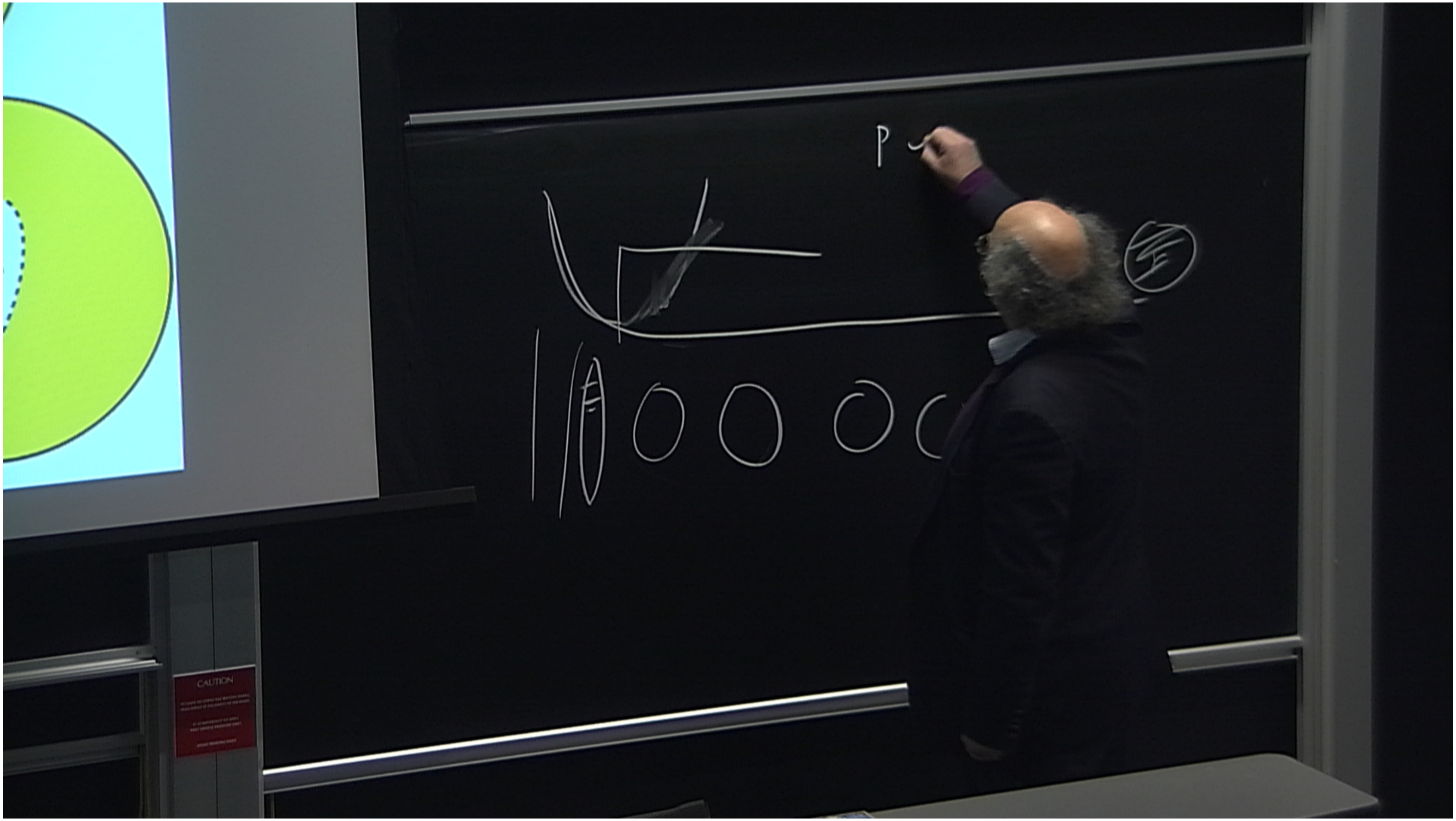
$$|\Psi_L^m\rangle = \prod_{i<j} \left(a_i^\dagger - a_j^\dagger\right)^m |0\rangle$$

$$a_i|0\rangle = 0 \qquad a_i^\dagger = \frac{R^x + iR^y}{\sqrt{2}\ell_B}$$

$$E_L = 0 \qquad [a_i, a_j^\dagger] = \delta_{ij}$$

maximum density null state

- m=2: (bosons): all pairs avoid the symmetric state $E_2 = \frac{1}{2}(A+B)$
- m=3: (fermions): all pairs avoid the antisymmetric state $E_2 = \frac{1}{2}B$



- usual suggestion: edge states of FQH states are described by 1+1d cft.
- But NO Lorentz invariance! so not cft.
- 1+1 cft requires a space-time metric

$$\cancel{ds^2 = dx^2 - v^2 dt^2}$$

universal speed of massless excitations

There is **no** universal speed of FQH edge modes!
They propagate with different speeds!

- suggested explanation (later)