Title: Topological response in gapless systems: from Weyl semimetals to metallic ferromagnets

Date: Feb 11, 2014 09:45 AM

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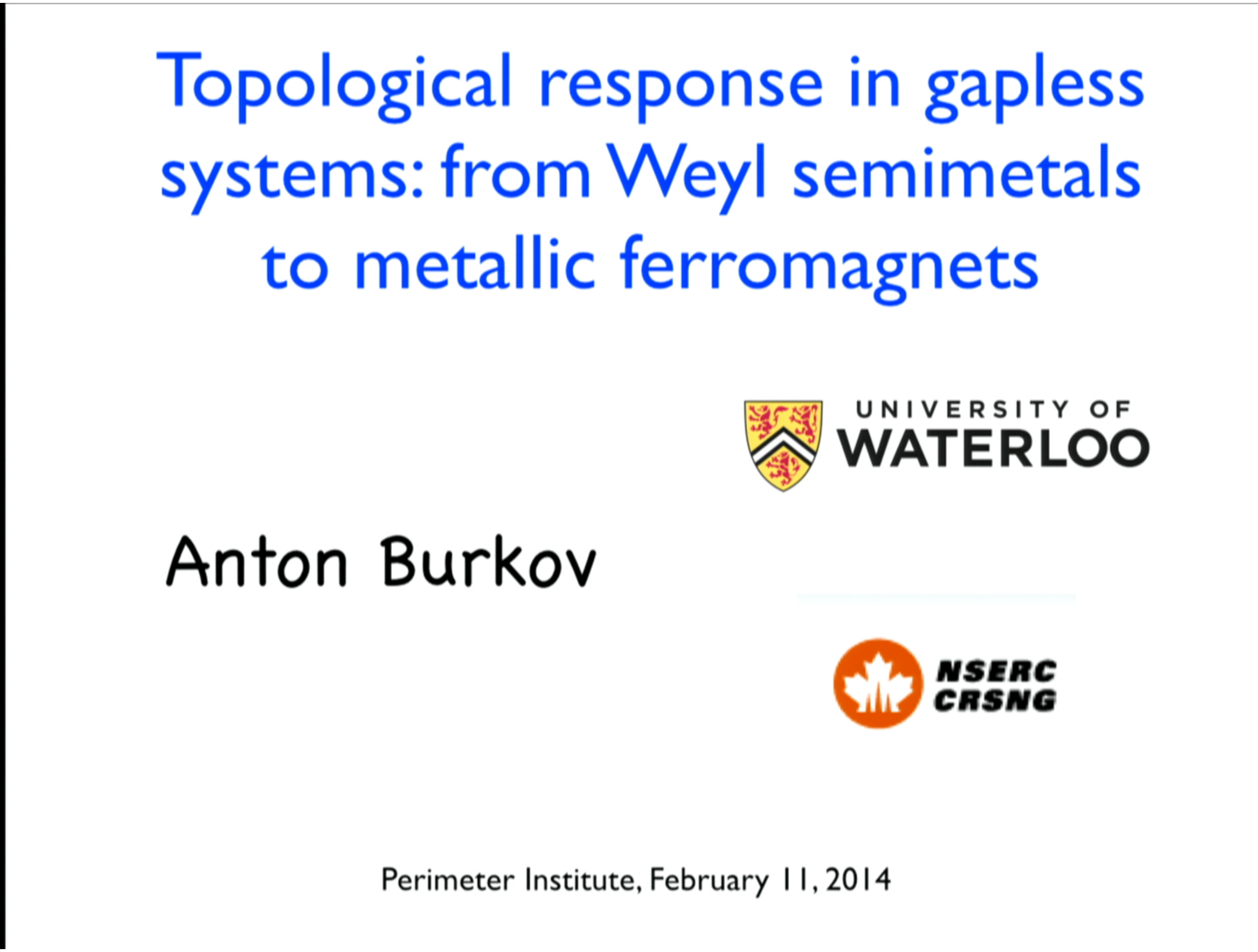
Abstract: Standard picture of a topologically-nontrivial phase of matter is an insulator with a bulk energy gap, but metallic surface states, protected by the bulk gap. Recent work has shown, however, that certain gapless systems may also be topologically nontrivial, in a precise and experimentally observable way. In this talk I will review our work on a class of such systems, in which the nontrivial topological properties arise from the existence of nondegenerate point band-touching nodes (Weyl nodes) in their electronic structure. Weyl nodes generally exist in any three-dimensional material with a broken time-reversal or inversion symmetry. Their effect is particularly striking, however, when the nodes coincide with the Fermi energy and no other states at the Fermi energy exist. Such "Weyl semimetals" have vanishing bulk density of states, but have gapless metallic surface states with an open (unlike in a regular two-dimensional metal) Fermi surface ("Fermi arc"). I will discuss our proposal to realize Weyl semimetal state in a heterostructure, consisting of alternating layers of topological and ordinary insulator, doped with magnetic impurities. I will further show that, apart from Weyl semimetals, even such "ordinary" materials as common metallic ferromagnets, in fact also possess Weyl nodes in the electronic structure, leading to the appearance of chiral Fermi-arc surface states and the corresponding contribution to their intrinsic anomalous Hall conductivity.


Topological response in gapless systems: from Weyl semimetals to metallic ferromagnets
UNIVERSITY OF WATERLOO
Anton Burkov
NSERC CRSNG
Perimeter Institute, February 11, 2014

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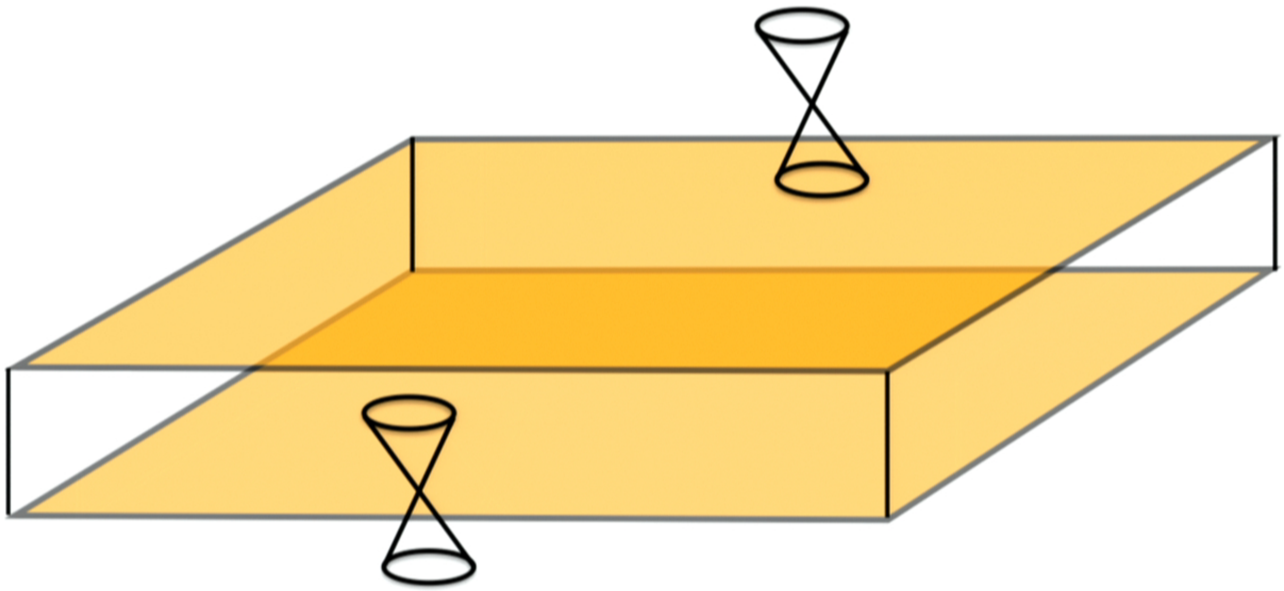
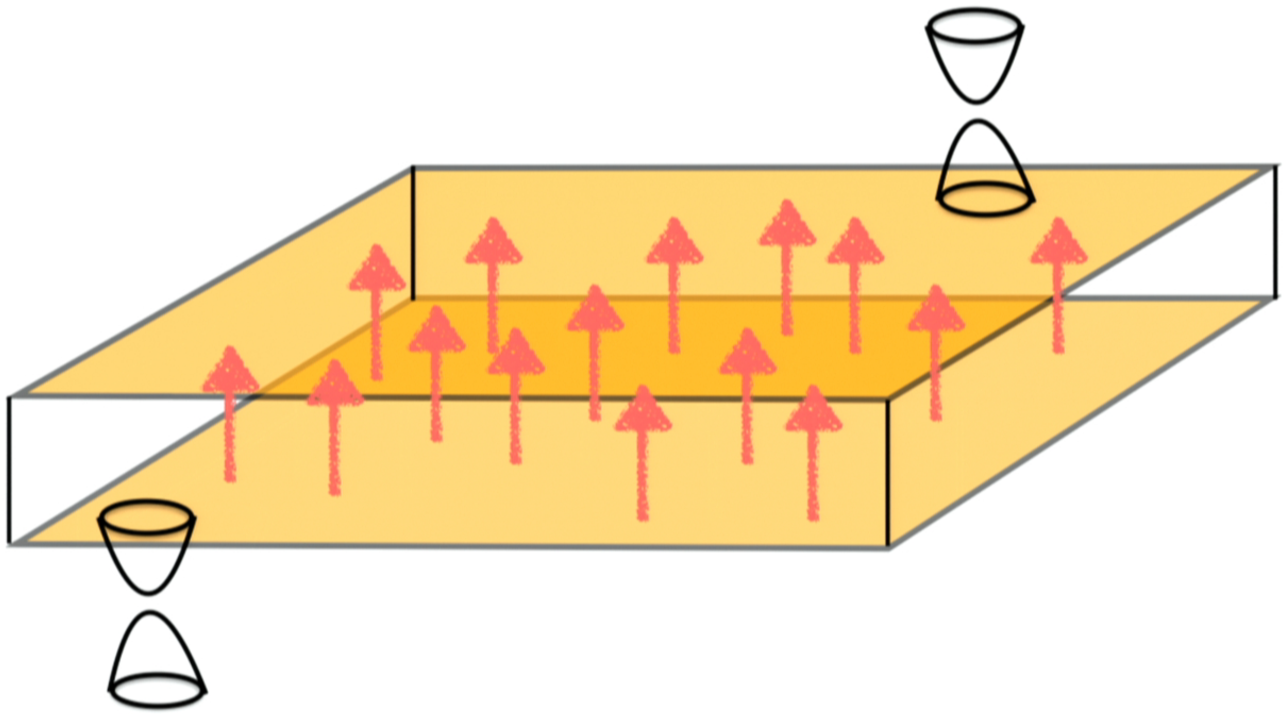
Quantum Anomalous Hall Insulator

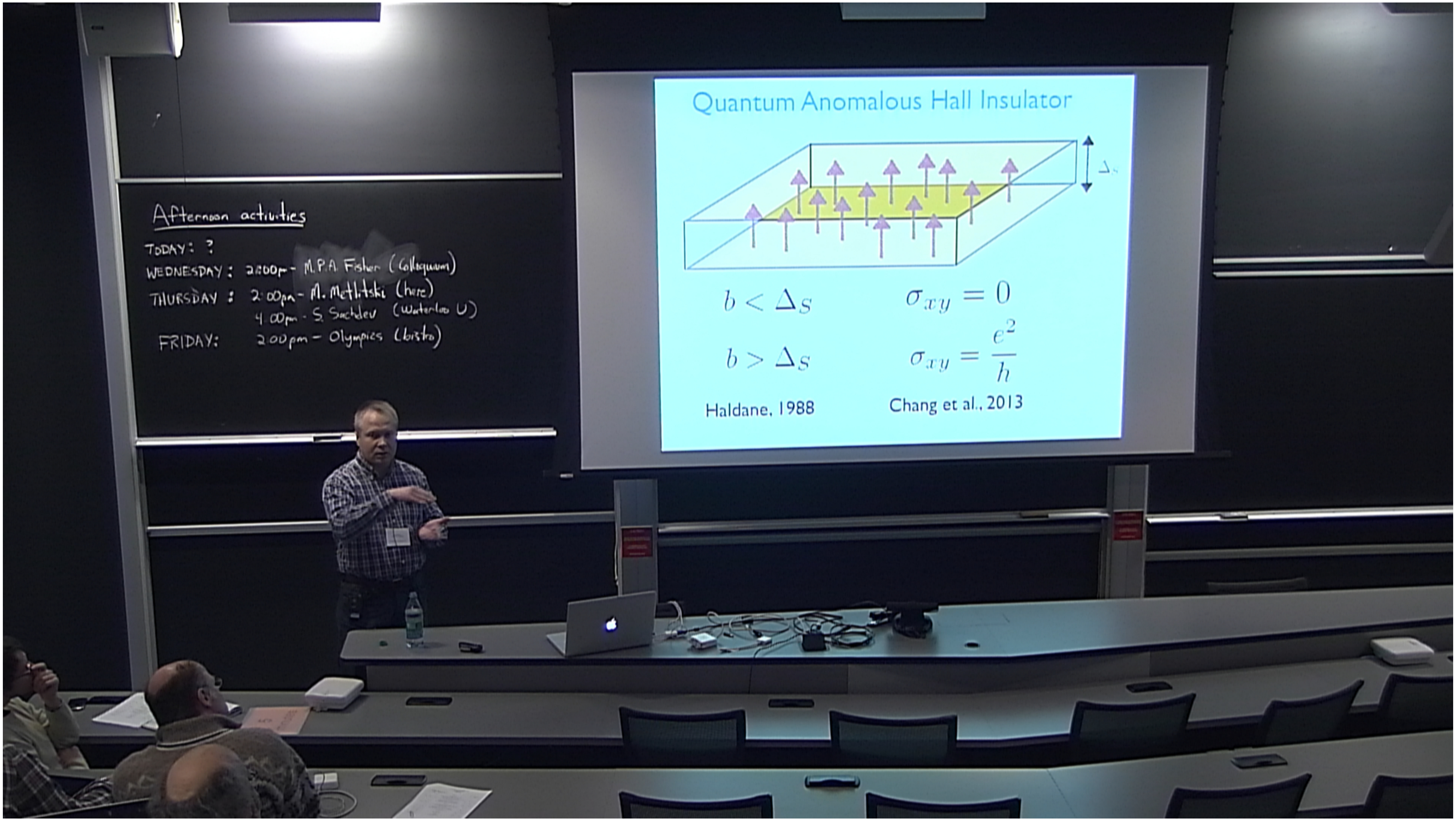


Thin film of 3D TI: gapless Dirac surface states protected by TRS

Quantum Anomalous Hall Insulator



Break TRS by doping with magnetic impurities


Quantum Anomalous Hall Insulator
$b < \Delta_S$
$\sigma_{xy} = 0$
$b > \Delta_S$
$\sigma_{xy} = \frac{e^2}{h}$
Haldane, 1988
Chang et al., 2013
Afternoon activities
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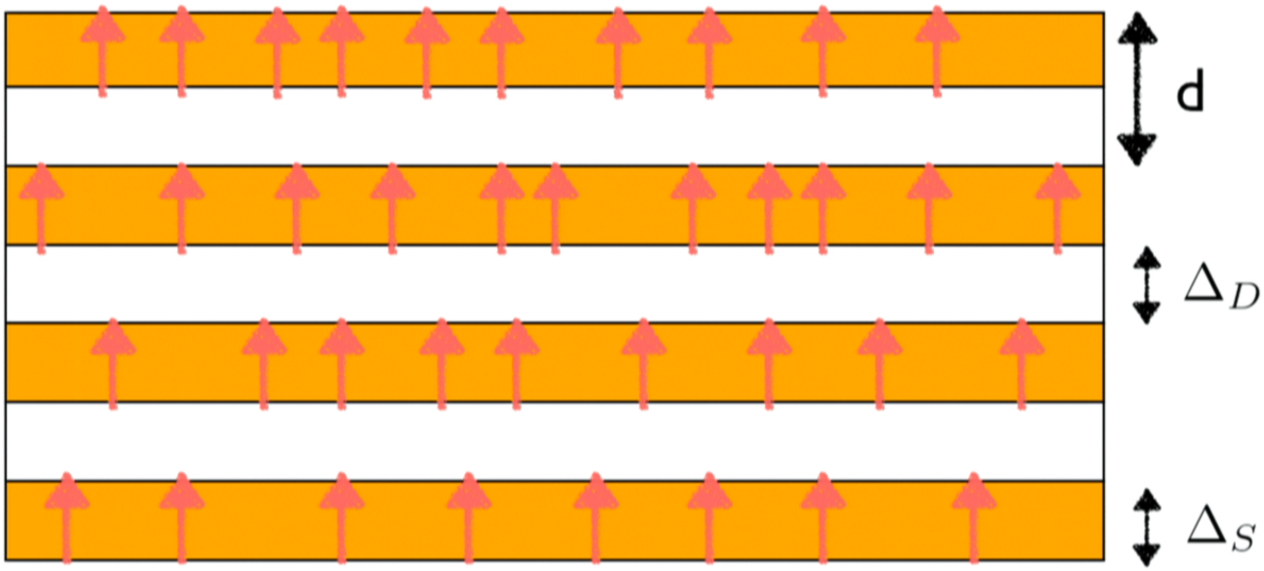
From 2D QAH insulator to 3D Weyl semimetal

- Stack of 2D QAH insulators, separated by ordinary insulator spacers.



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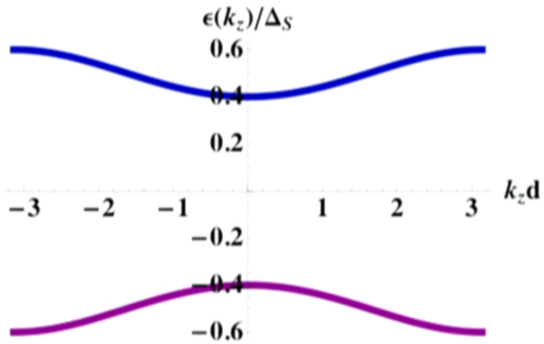

$$b > \Delta_S + \Delta_D$$

$$\sigma_{xy} = \int_{-G/2}^{G/2} \frac{dk_z}{2\pi} \frac{e^2}{h} = \frac{e^2}{h} \frac{G}{2\pi}$$

- "Trivial" generalization of IQHE to 3D.

Kohmoto, Halperin, Wu, 1992

From 2D QAH insulator to 3D Weyl semimetal

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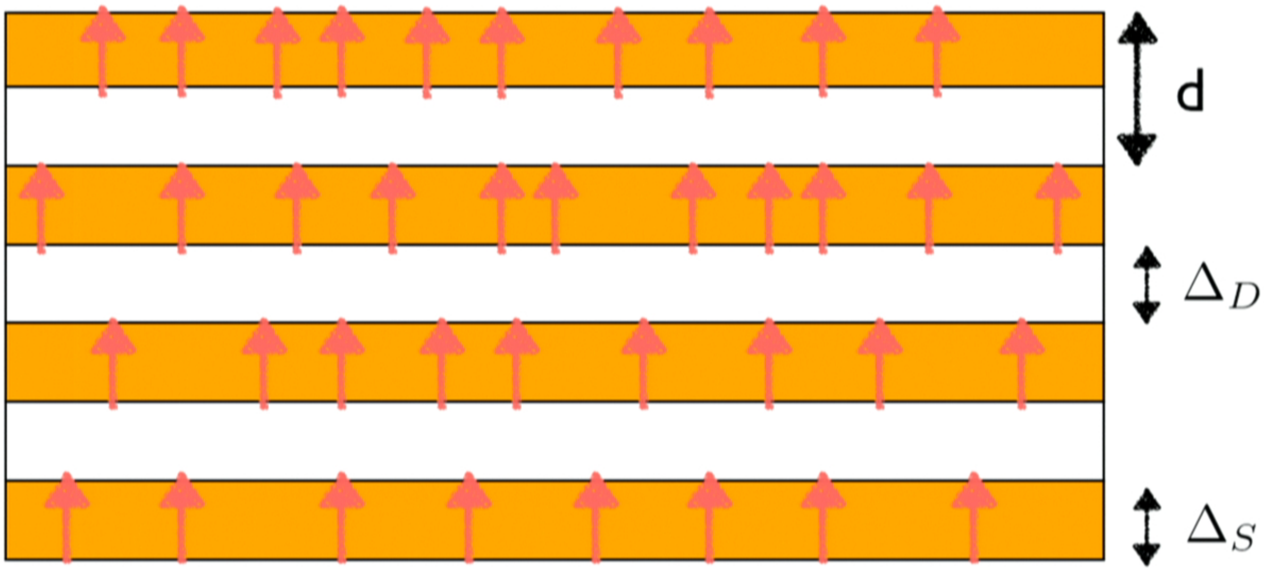

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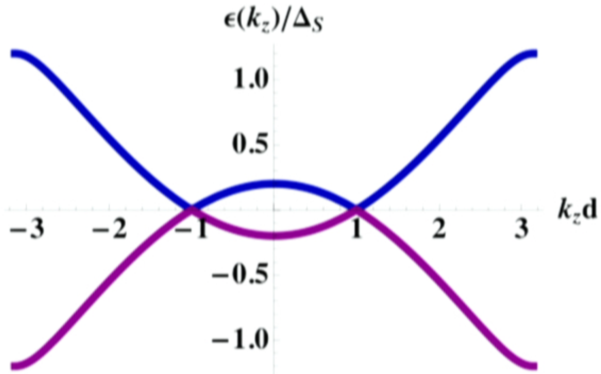


$$|\Delta_S - \Delta_D| < b < \Delta_S + \Delta_D$$

$$\sigma_{xy} = \frac{e^2}{h}\frac{K}{2\pi}$$

- "Nontrivial" generalization of IQHE to 3D.

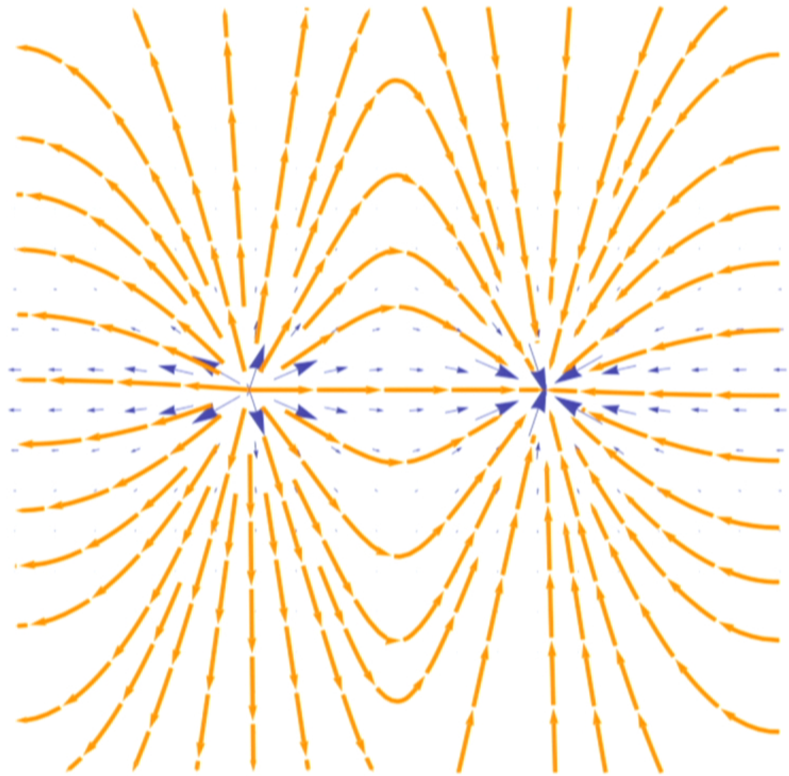
Wan et al., 2011 AAB & Balents, 2011

Weyl nodes

- Weyl nodes are "magnetic monopoles" in momentum space

Monopole sources of Berry curvature:

$$\mathbf{\Omega} = \pm \frac{\mathbf{k}}{2k^3}$$



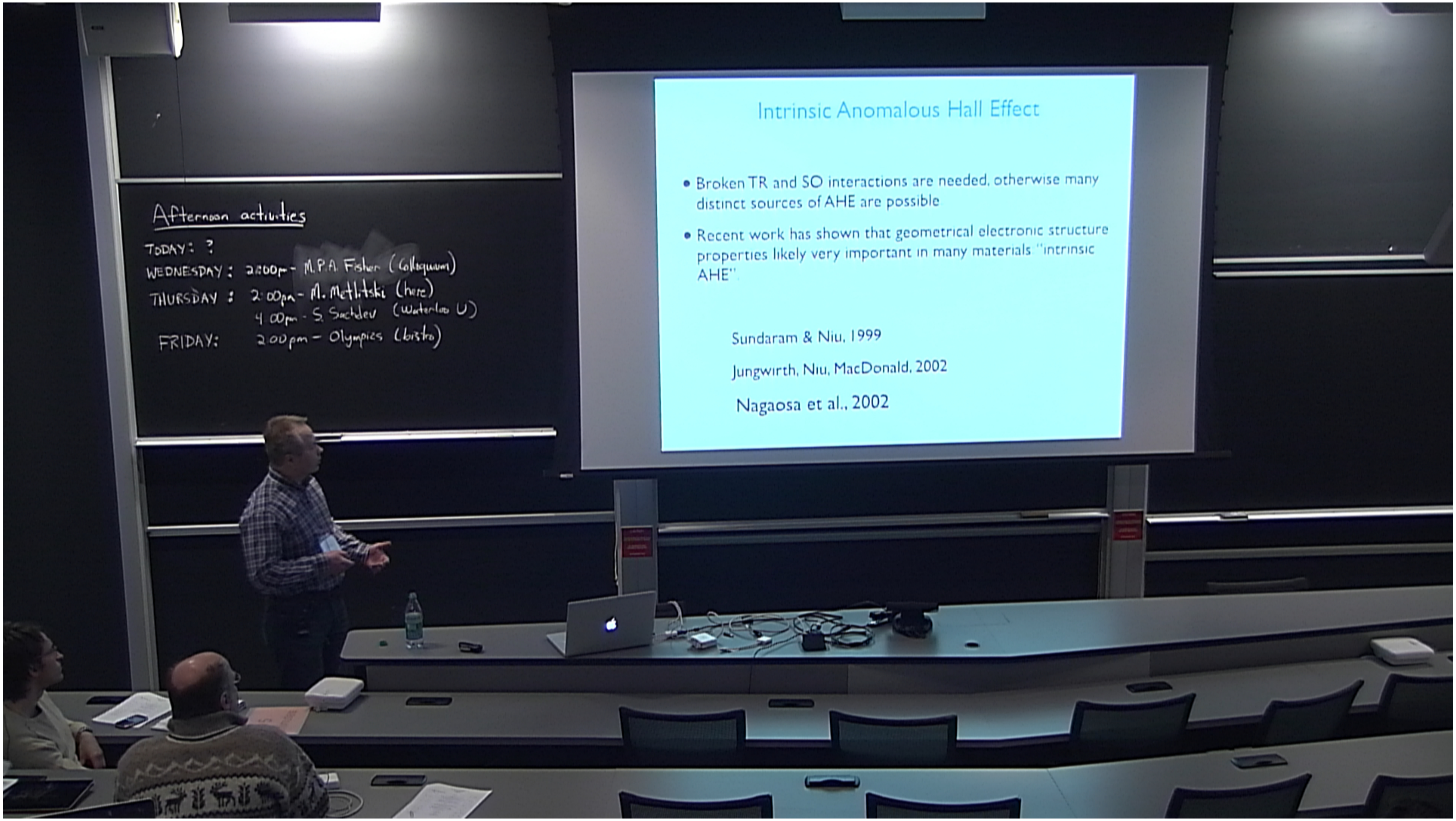
Intrinsic Anomalous Hall Effect

- Broken TR and SO interactions are needed, otherwise many distinct sources of AHE are possible.
- Recent work has shown that geometrical electronic structure properties likely very important in many materials: "intrinsic AHE".

Sundaram & Niu, 1999

Jungwirth, Niu, MacDonald, 2002

Nagaosa et al., 2002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Intrinsic Anomalous Hall Effect

- Intrinsic anomalous Hall conductivity is given by the integral of the anomalous velocity over all occupied states:

$$\sigma_{xy} = \frac{e^2}{\hbar} \sum_n \int \frac{d^3k}{(2\pi)^3} n_F(\epsilon_{n\mathbf{k}}) \Omega^z_{n\mathbf{k}}$$

- This formula may be misleading: it appears to suggest that the anomalous Hall conductivity is a thermodynamic equilibrium property.

Doped Weyl semimetal

- Multilayer heterostructure model is not only the simplest model of WS.
- It is the simplest model of a 3D metallic ferromagnet with SO interactions, when the Fermi level is doped away from the Weyl nodes.

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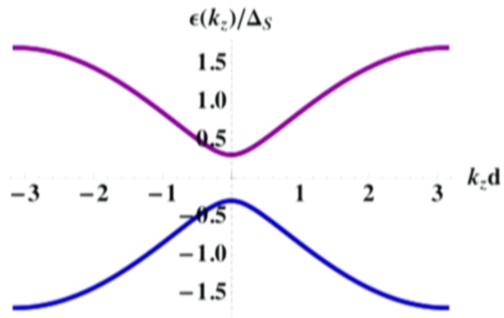
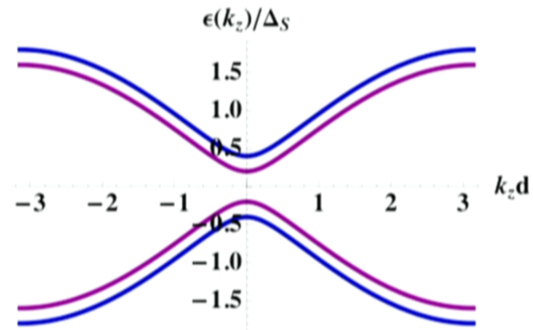
Doped Weyl semimetal

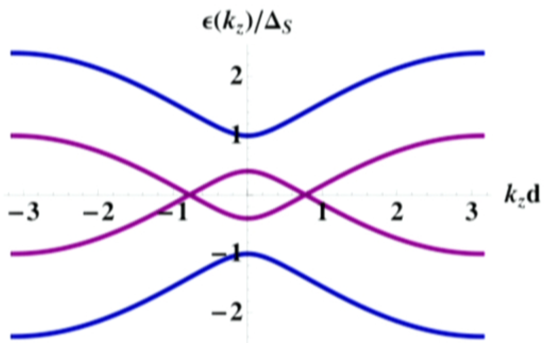
$$b = 0$$



$$0 < b < |\Delta_S - |\Delta_D||$$



$$|\Delta_S - |\Delta_D|| < b < \Delta_S + |\Delta_D|$$



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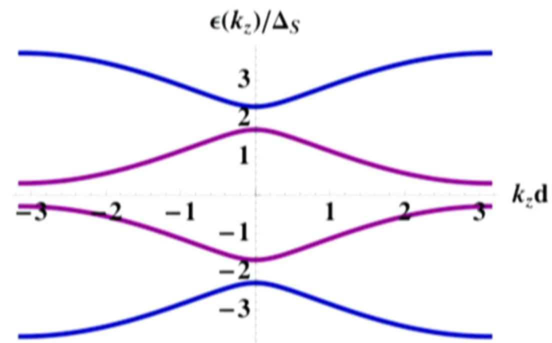

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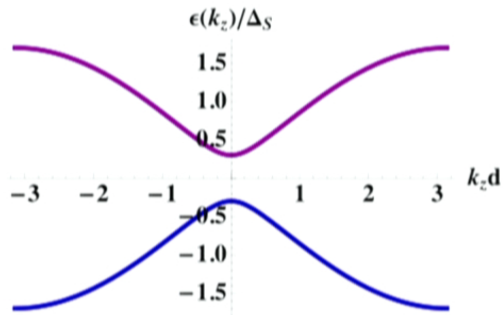

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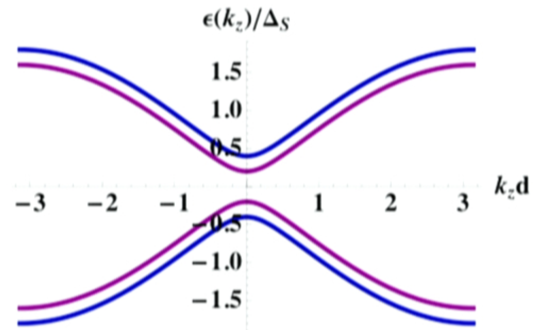

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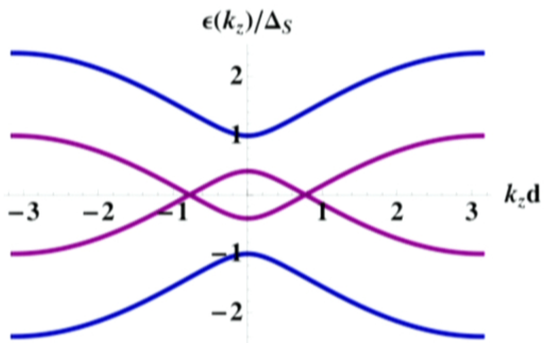

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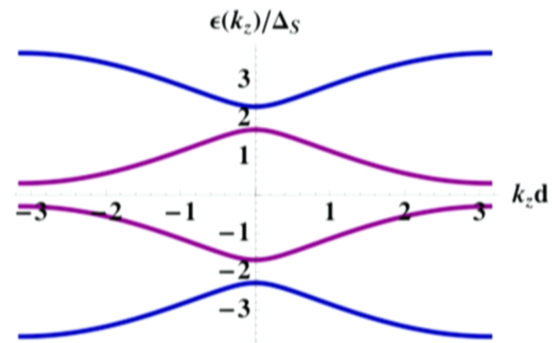

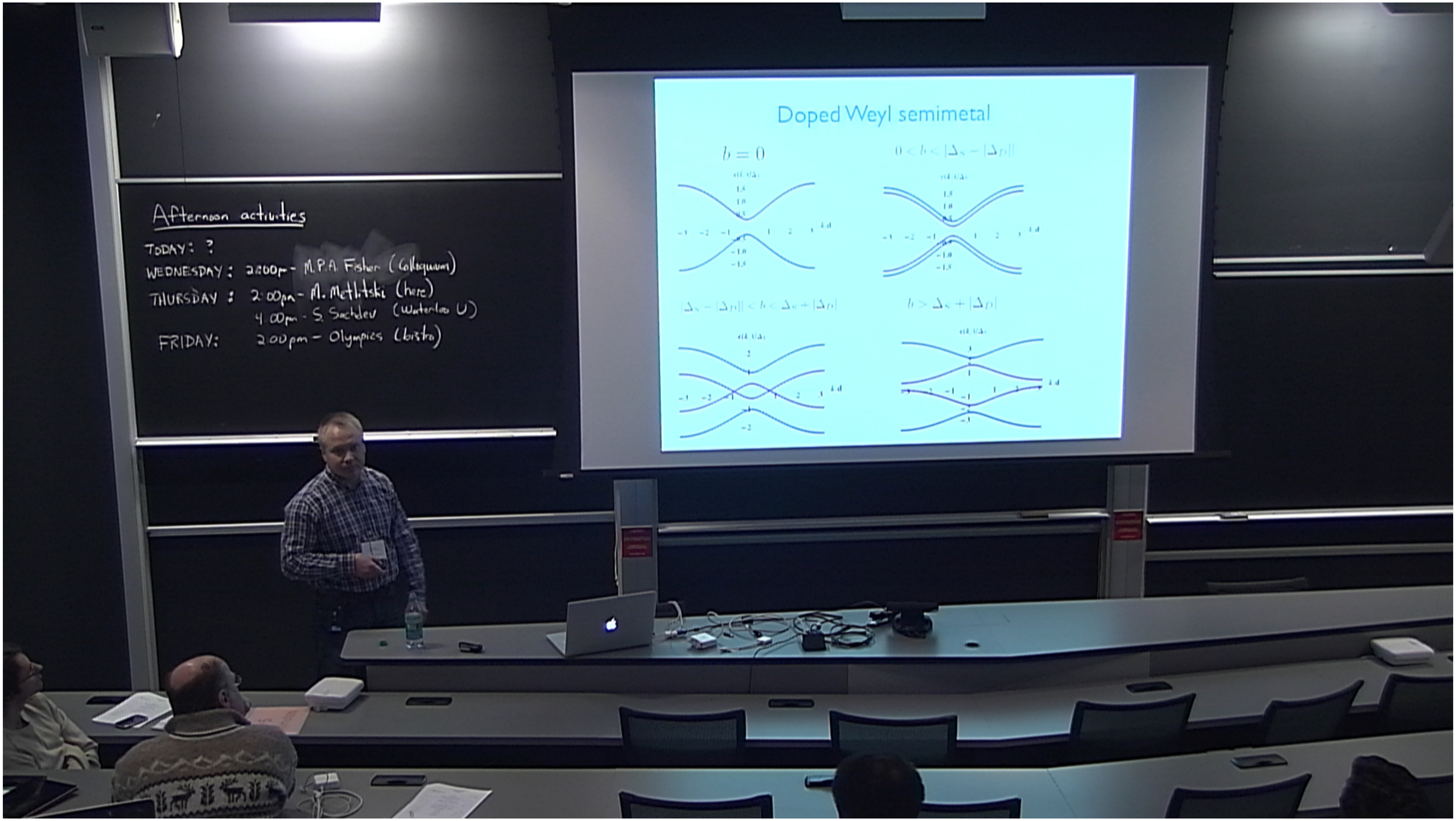
Doped Weyl semimetal
b = 0
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Doped Weyl semimetal

- 2D Dirac fermions with kz-dependent mass.

$$H_{\pm}(\mathbf{k}) = v_F(\hat{z} \times \boldsymbol{\sigma}) \cdot \mathbf{k} + m_{\pm}(k_z)\sigma^z$$

$$m_{\pm}(k_z) = b \pm \sqrt{\Delta_S^2 + \Delta_D^2 + 2\Delta_S\Delta_D \cos(k_z d)}$$

- One of the masses changes sign at the locations of the Weyl nodes.

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Order of limits

- Total anomalous Hall conductivity is the DC limit of the interband optical Hall conductivity:

$$\sigma_{xy} = \lim_{i\Omega \to 0} \lim_{\mathbf{q} \to 0} \Pi(\mathbf{q}, i\Omega)$$

- Opposite order of limits gives a part of the total Hall conductivity, which is a thermodynamic equilibrium property:

$$\sigma_{xy}^{II} = \lim_{\mathbf{q} \to 0} \lim_{i\Omega \to 0} \Pi(\mathbf{q}, i\Omega) = e\left(\frac{\partial N}{\partial B}\right)_{\mu}$$

Fermi surface part of AHE

$$\sigma_{xy} = \lim_{i\Omega\to 0} \lim_{\mathbf{q}\to 0} \Pi(\mathbf{q}, i\Omega) \qquad \sigma_{xy}^{II} = \lim_{\mathbf{q}\to 0} \lim_{i\Omega\to 0} \Pi(\mathbf{q}, i\Omega) = e\left(\frac{\partial N}{\partial B}\right)_{\mu}$$

- The difference is a purely transport property, which vanishes in equilibrium and which can be associated with states on the Fermi surface:

$$\sigma_{xy}^{I} = \sigma_{xy} - \sigma_{xy}^{II}$$

- This is analogous to the difference between the Drude weight and the superfluid weight.

Scalapino, White, Zhang, 1993

Few details

$$\Pi(\mathbf{q}, i\Omega) = \frac{ie^2 v_F}{V} \sum_{\mathbf{k}} \frac{n_F[\xi_{s't'}(\mathbf{k})] - n_F[\xi_{st}(\mathbf{k}+\mathbf{q})]}{i\Omega + \xi_{s't'}(\mathbf{k}) - \xi_{st}(\mathbf{k}+\mathbf{q})} \times \langle z^{st}_{\mathbf{k}+\mathbf{q}} | z^{s't'}_{\mathbf{k}} \rangle \langle z^{s't'}_{\mathbf{k}} | \boldsymbol{\sigma} \cdot \hat{q} | z^{st}_{\mathbf{k}+\mathbf{q}} \rangle, \tag{11}$$

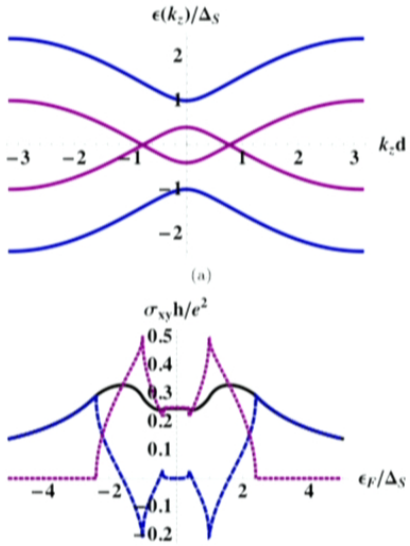
- When q is taken to zero first, only interband part of the response function survives.

- When frequency is taken to zero first, there is in addition intraband contribution, which cancels part of the interband contribution, which can be related to Fermi surface using Stokes theorem.

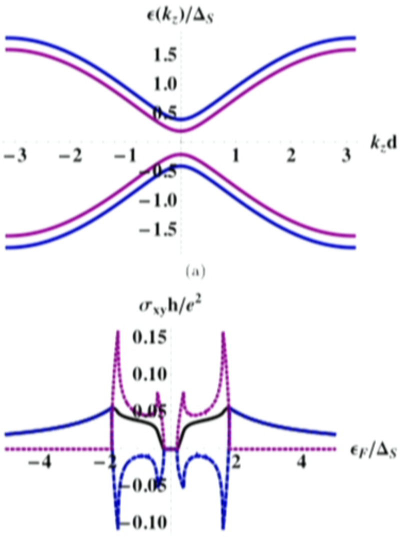
$$\Pi^{intra}(\mathbf{q}, i\Omega) = \frac{ie^2 v_F}{V} \sum_{t} \sum_{\mathbf{k}} \left. \frac{dn_F(x)}{dx} \right|_{x=\epsilon_t(\mathbf{k})-\epsilon_F} \times \langle z^{+t}_{\mathbf{k}+\mathbf{q}} | z^{+t}_{\mathbf{k}} \rangle \langle z^{+t}_{\mathbf{k}} | \boldsymbol{\sigma} \cdot \hat{q} | z^{+t}_{\mathbf{k}+\mathbf{q}} \rangle. \tag{24}$$

Role of Weyl nodes

- Weyl nodes contribute significantly to the equilibrium part of the Hall conductivity, but not to the Fermi surface part.



(a)
(b)



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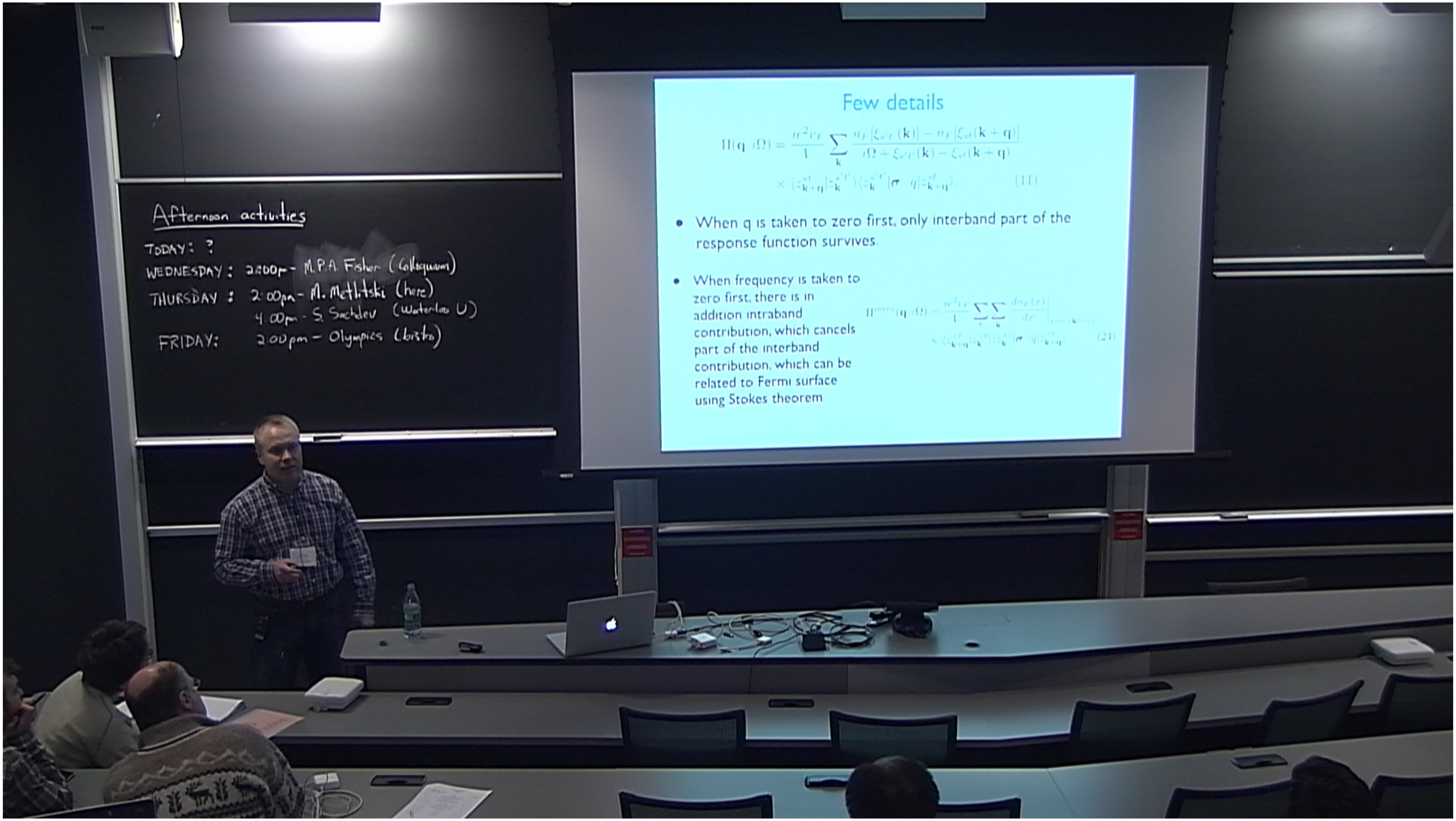

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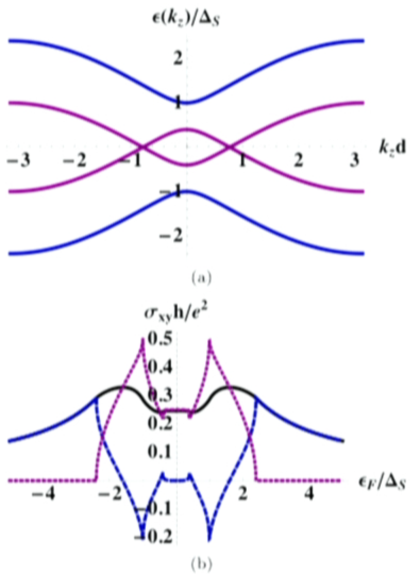
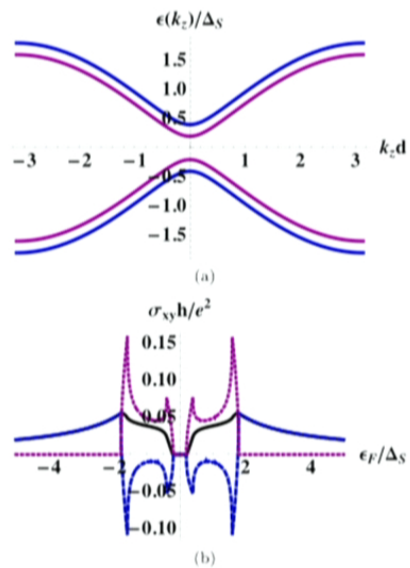
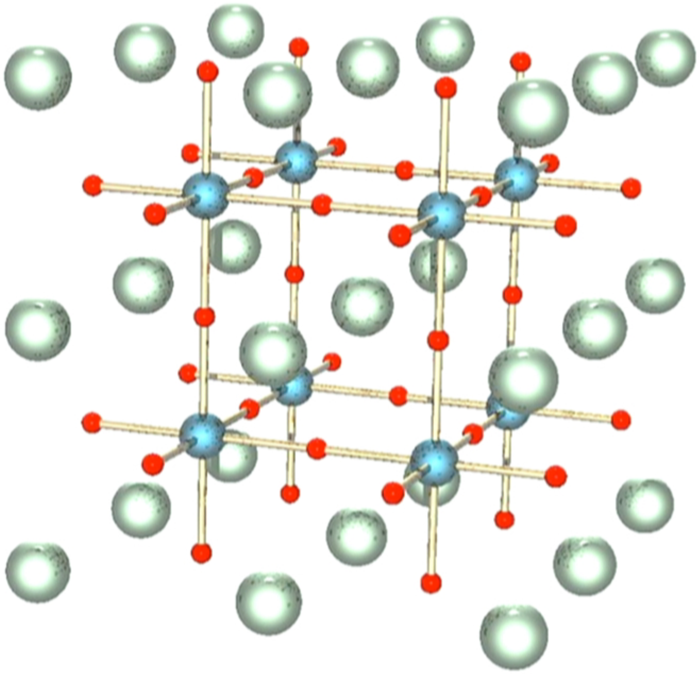




AHE in $SrRuO_3$

- Perovskite structure, typically slightly distorted, but this is not important for us.
- Relatively simple electronic structure, consisting of 6 bands, derived mostly from the t2g orbitals of Ru.



Chen, Bergman, AAB, 2013

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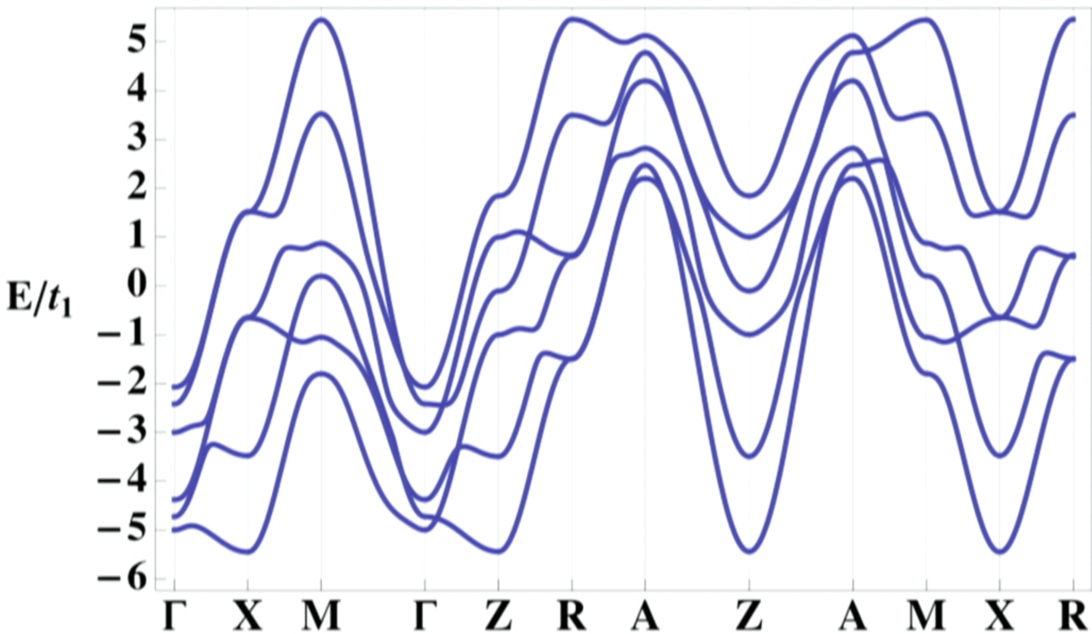
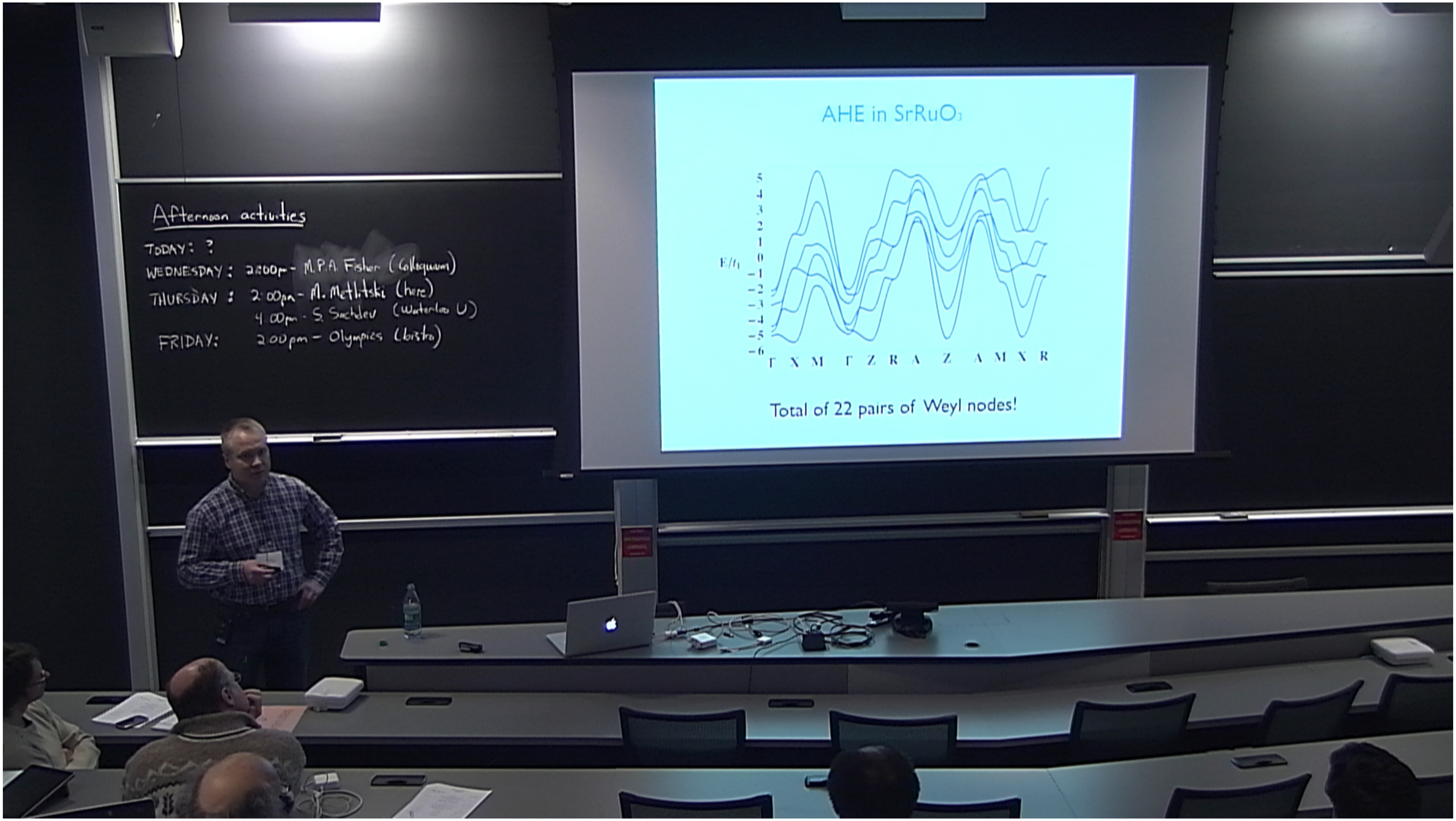


Total of 22 pairs of Weyl nodes!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AHE in SrRuO3
E/t1
Γ X M Γ Z R A Z A M X R
Total of 22 pairs of Weyl nodes!

Conclusions

- Weyl semimetal provides a nontrivial generalization of IQHE to 3 dimensions.
- Similar physics occurs in any 3D metallic ferromagnet: anomalous Hall conductivity may be separated into a thermodynamic equilibrium part, and transport part. Weyl nodes contribute to the equilibrium part.
- Important issue for the future: role of impurity scattering (side-jump must significantly affect the Fermi surface part, but not the equilibrium part of the anomalous Hall conductivity).

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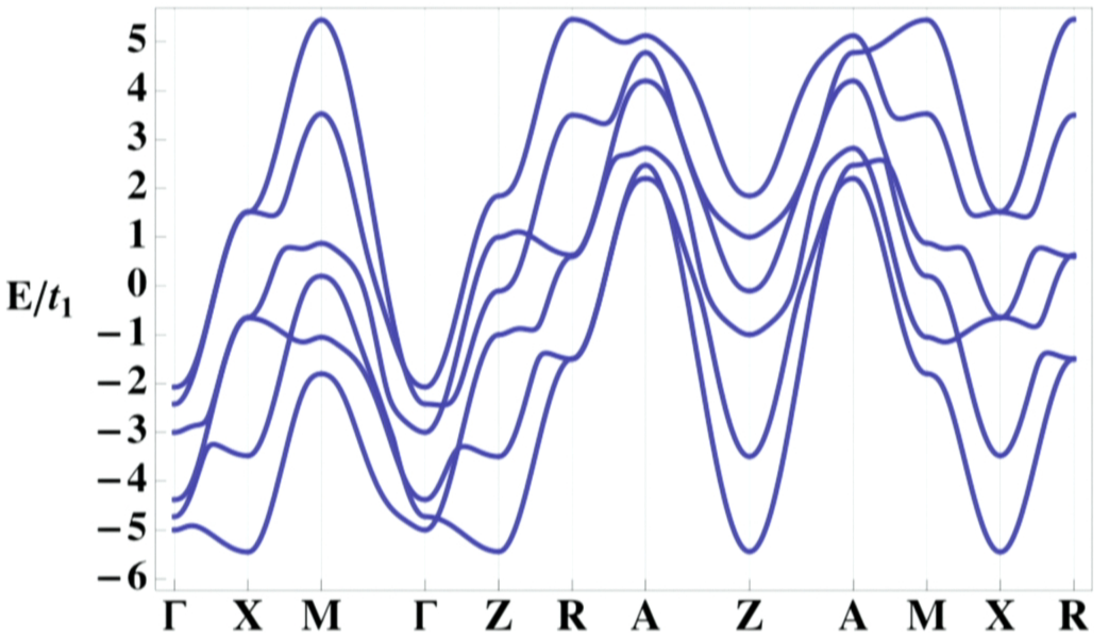

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