Title: Fractional Quantum Hall Effect in a curved space

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Abstract: We developed a general method to compute the correlation functions of FQH states on a curved space. The computation features the gravitational trace anomaly and reveals geometric properties of FQHE. Also we highlight a relation between the gravitational and electromagnetic response functions. The talk is based on the recent paper with T. Can and M. Laskin.

FQH in a Curved Space:

Gravitational Anomaly and Electromagnetic Response

P. Wiegmann

based on the recent paper arXiv:1402.1531 with

Tankut Can and Misha Laskin

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Response to local transformations vs global transformation

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- Hall conductance σ_{xy} - response to homogeneous flux transformation (Laughlin, 1983);
- Anomalous (odd) viscosity η - response to homogeneous metric transformation (Avron, Seiler, Zograf, 1995);

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- Momentum dependence of $\sigma_{xy}(k)$;
- Momentum dependence of the structure function $s(k) = \langle \rho_k \rho_{-k} \rangle_c / \rho_0$
- Momentum dependence of anomalous viscosity $\eta(k)$

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Main Results

Only Laughlin’s states (for now), a comment on Pffafian state.

Density on a curved space

- Number of particles could be placed on the surface (Wen & Zee, J. Frohlich)

$$N_\nu = \nu N_\phi + \frac{\chi}{2}$$

- Density

$$\boxed{\rho = \rho_0 + \tfrac{1}{8\pi} R - \tfrac{b}{8\pi}(-l^2 \Delta_g) R}$$

$$\boxed{b = \tfrac{1}{3} + \tfrac{\nu - 1}{4\nu}}$$

- The coefficient b is controlled by the gravitational anomaly

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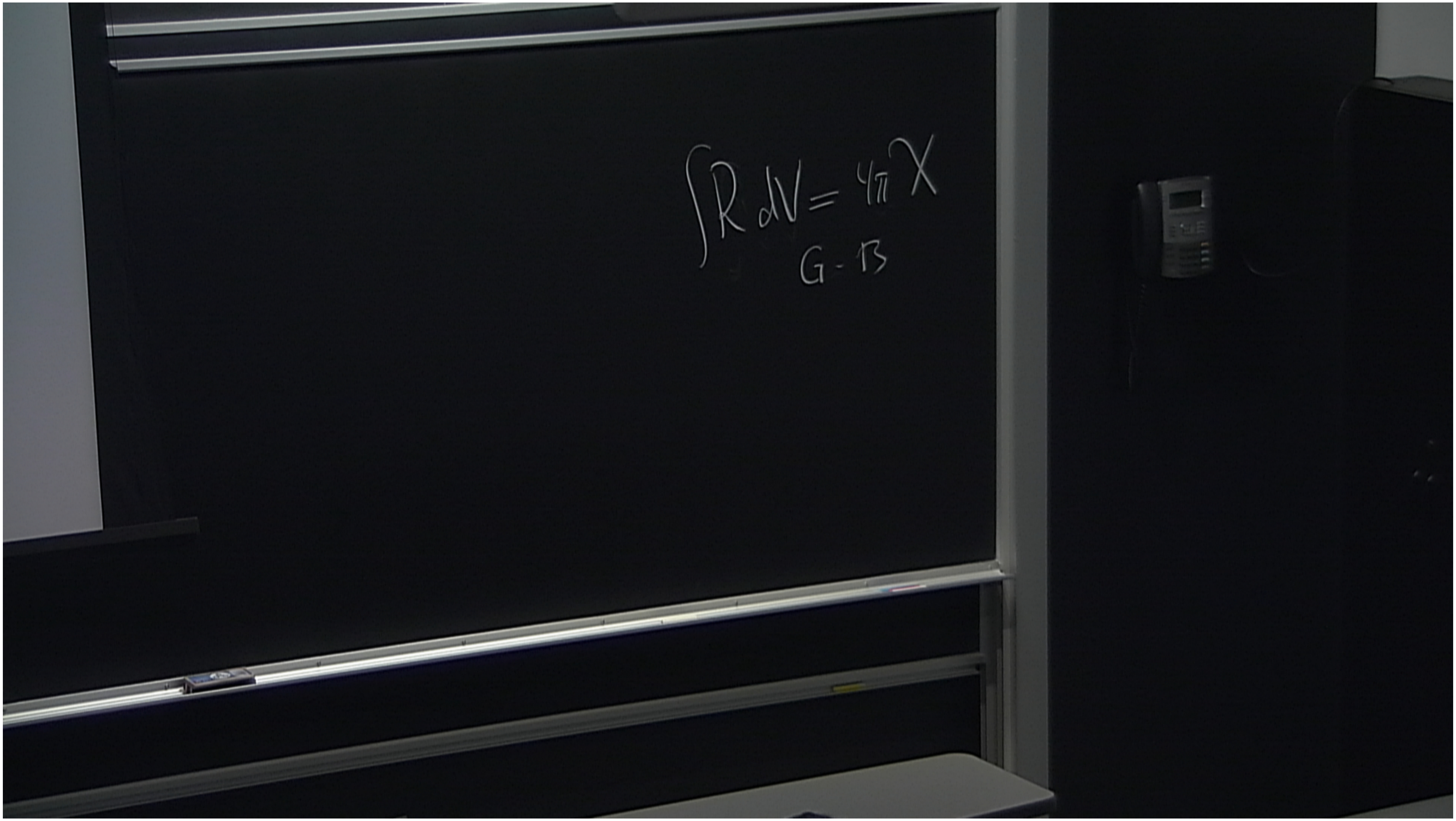

$\int R \, dV = 4\pi \chi$
G - B

Gravitational response and anomalous viscosity

Having

$$\langle\rho\rangle = \rho_0 + \frac{1}{8\pi}R - \frac{b}{8\pi}(-l^2\Delta_g)R.$$

The response of the density to to gravity

$$\boxed{\eta = (\rho_0 l^2)^{-1}\frac{\delta\rho}{\delta R}\Big|_{R=0}}$$

is anomalous viscosity

$$\eta(q) = \frac{1}{4\nu}(1 - bq^2 + \mathcal{O}(q^4)), \qquad q = kl.$$

Gravitational response and electromagnetic response

Special properties (local symmetries) of the Lowest Landau level are coded by the general relation between responses:

- A general relation connecting the gravitational response and the structure functions

$$\frac{q^4}{2}\eta(q) = -\frac{q^2}{2} + \left(1 + \frac{q^2}{2}\right)s(q), \quad q = kl.$$

$$s(k) = \langle \rho_k \rho_{-k} \rangle_c / \rho_0$$

Gradient expansion

- Gravitational response

$$\eta(q) = \frac{1}{4\nu}(1 - \left(\frac{1}{3} + \frac{\nu - 1}{4\nu}\right) q^2 + \mathcal{O}(q^4)), \qquad q = kl.$$

- Structure function $\quad s(k) = \langle \rho_k \rho_{-k} \rangle_c / \rho_0$

$$s(q) = \frac{1}{2} q^2 + s_2 q^4 + s_3 q^6 + O(q^8)$$

$$\boxed{s_2 = (\nu^{-1} - 2)/8, \qquad s_3 = (3\nu^{-1} - 4)(\nu^{-1} - 3)/96}$$

- Hall conductance

$$\sigma_{xy}(q) = \sigma_{xy}(0) \left(1 + 2 s_2 q^2 + 2 s_3 q^4 + \ldots\right)$$

Gaussian fields

All these results are coming from the correlation function of the potential

$$-\Delta_g \varphi = 4\pi \nu^{-1} \rho$$

Their correlation function is Gaussian

$$\langle \varphi(1)\varphi(2)\rangle_c = \nu^{-1} \begin{cases} G(1,2) & \text{at large separation} \\ G_R(1,2) & \text{at short distances.} \end{cases}$$

G is a Green function of the Laplace-Beltrami operator

$$\Delta_g G(1,2) = -4\pi \delta^{(2)}(1,2)$$

Regularized Green function

$$G_R(1,2) = G(1,2) + 2\log d(1,2)$$

$d(1,2)$ - geodesic distance

Setting

- Holomorphic coordinates

$$ds^2 = \sqrt{g} dz d\bar{z}$$

- Scalar curvature reads

$$R = -\Delta_g \log \sqrt{g}$$

- Laplace-Beltrami operator takes the form

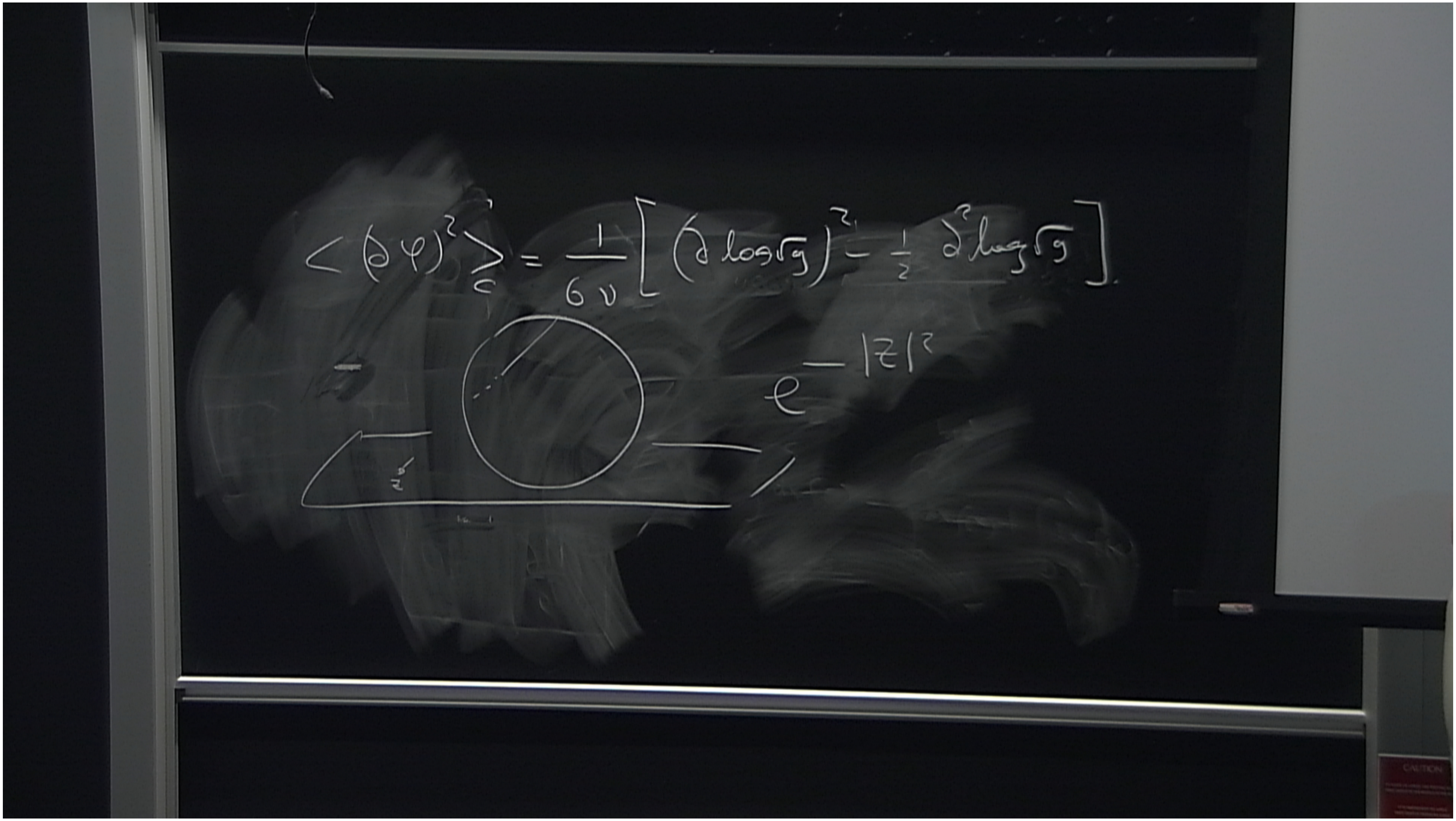
$$\Delta_g = (4/\sqrt{g})\partial\bar{\partial}$$

- Kähler potential K, defined through the equation

$$\partial\bar{\partial}K = \sqrt{g}$$

- magnetic field

$$\nabla \times \mathbf{A} = B\sqrt{g}$$


⟨(∂φ)²⟩_c = 1/(6ν) [(∂ log √g)² − ½ ∂² log √g]
e^{−|z|²}

Lowest Landau Level

- states annihilated by the anti-holomorphic momentum

$$\bar{\Pi}\psi_n = (-i\hbar\bar{\partial} - e\bar{A})\psi_n$$

- Solutions

$$\psi_n(z) = s_n(z)e^{-K(z,\bar{z})/4l^2}$$

- Holomorphic sections

$$\bar{\partial} s_n = 0$$

- Riemann-Roch theorem: number of normalized solutions is

$$N = N_\phi + \chi/2$$

- Filled level

$$\det[\psi_n(z_i)] \propto \prod_{i<j}^{N}(z_i - z_j)^\beta e^{-\frac{1}{4l^2}\sum_i^N K(z_i,\bar{z}_i)}$$

Laughlin wave-function

$$\Psi(z_1,...,z_N) = \frac{1}{\sqrt{Z[g]}} \prod_{i<j}^{N} (z_i - z_j)^{\beta} e^{-\frac{1}{4l^2}\sum_i^N K(z_i,\bar{z}_i)},$$

Generating functional

$$Z[g] = \int |\Psi|^2 \prod_i \sqrt{g(z_i)}\, d^2 z_i = \int \prod_{i<j}^{N} |z_i - z_j|^{2\beta} \prod_i^N e^{W(z_i,\bar{z}_i)} d^2 z_i$$

$$W = -\frac{1}{2l^2} K + \log\sqrt{g}.$$

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Generating functional and connection between various responses

$$Z[g] = \int \prod_{i<j}^{N} |z_i - z_j|^{2\beta} \prod_{i}^{N} e^{W(z_i,\bar{z}_i)} d^2 z_i$$

$$W = -\frac{1}{2l^2} K + \log\sqrt{g}, \quad \Delta W = \frac{2}{l^2}\sqrt{g}\left(1 - \frac{l^2}{2} R\right)$$

Correlation functions

$$\langle \rho \rangle = \frac{\delta \log Z}{\delta W}, \quad \langle \rho(1)\rho(2)\rangle_c = \frac{\delta^2 \log Z}{\delta W(1) \delta W(2)}$$

$$\eta \propto \frac{\delta \rho}{\delta R} = \frac{\delta^2 \log Z}{\delta W \delta R}$$

Gravitational response and electromagnetic response

- A general relation connecting the gravitational response and the structure functions

$$\frac{q^4}{2}\eta(q) = -\frac{q^2}{2} + \left(1 + \frac{q^2}{2}\right)s(q), \quad q = kl.$$

$$\eta = \left(\frac{2\pi}{\nu}\right) \cdot \frac{\delta\rho}{\delta R}$$

All is summarized in $1/N_\phi$ - expansion of the **generating functional**

$$Z[g] = \int |\Psi|^2 \prod_i \sqrt{g(z_i)}\, d^2 z_i$$

$$\Psi = \prod_{i<j}^{N} (z_i - z_j)^\beta e^{-\frac{1}{4l^2}\sum_i^N K(z_i,\bar{z}_i)},$$

$$\log \frac{Z[g]}{Z[g_{sphere}]} = N_\phi^2 A^{(2)}[g] + N_\phi A^{(1)}[g] + A^{(0)}[g],$$

$$A^{(2)} = -\frac{\pi}{2}\frac{\nu}{V^2}\int K dV, \quad dV = \sqrt{g} dz d\bar{z}.$$

$$A^{(1)} = \frac{1}{2V}\int \log\sqrt{g}\, dV,$$

$$A^{(0)} = \frac{1}{16\pi}\left(\frac{1}{3} - \frac{\nu - 1}{2\nu}\right)\int \log\sqrt{g} R\, dV.$$

$A^{(2)}$ and $A^{(1)}$ - "gravitational Wess-Zumino".

$A^{(0)}$ - Polyakov's Liouville action of quantum gravity.

$$-\frac{1}{2}\log\frac{\det\left(-\Delta_g\right)}{\det\left(-\Delta_{g_0}\right)} = \frac{1}{96\pi}\int \log\sqrt{g} R\, dV$$

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Ward Identity (Zabrodin, P. W. 2006)

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The integral is invariant under diffeomorphisms of the integrand

$$z_i \to z_i + \frac{\epsilon}{z - z_i}$$

The integrand changes

$$0 = \left\langle \sum_i \frac{\partial_{z_i} W}{z - z_i} + \sum_{j \neq i} \frac{\beta}{(z - z_i)(z_i - z_j)} + \sum_i \frac{1}{(z - z_i)^2} \right\rangle$$

or in terms of fields

$$-2\beta \int \frac{\partial W}{z - \xi} \langle \rho \rangle \sqrt{g} d^2 \xi = \langle (\partial \varphi)^2 \rangle + (2 - \beta) \langle \partial^2 \varphi \rangle$$

where

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