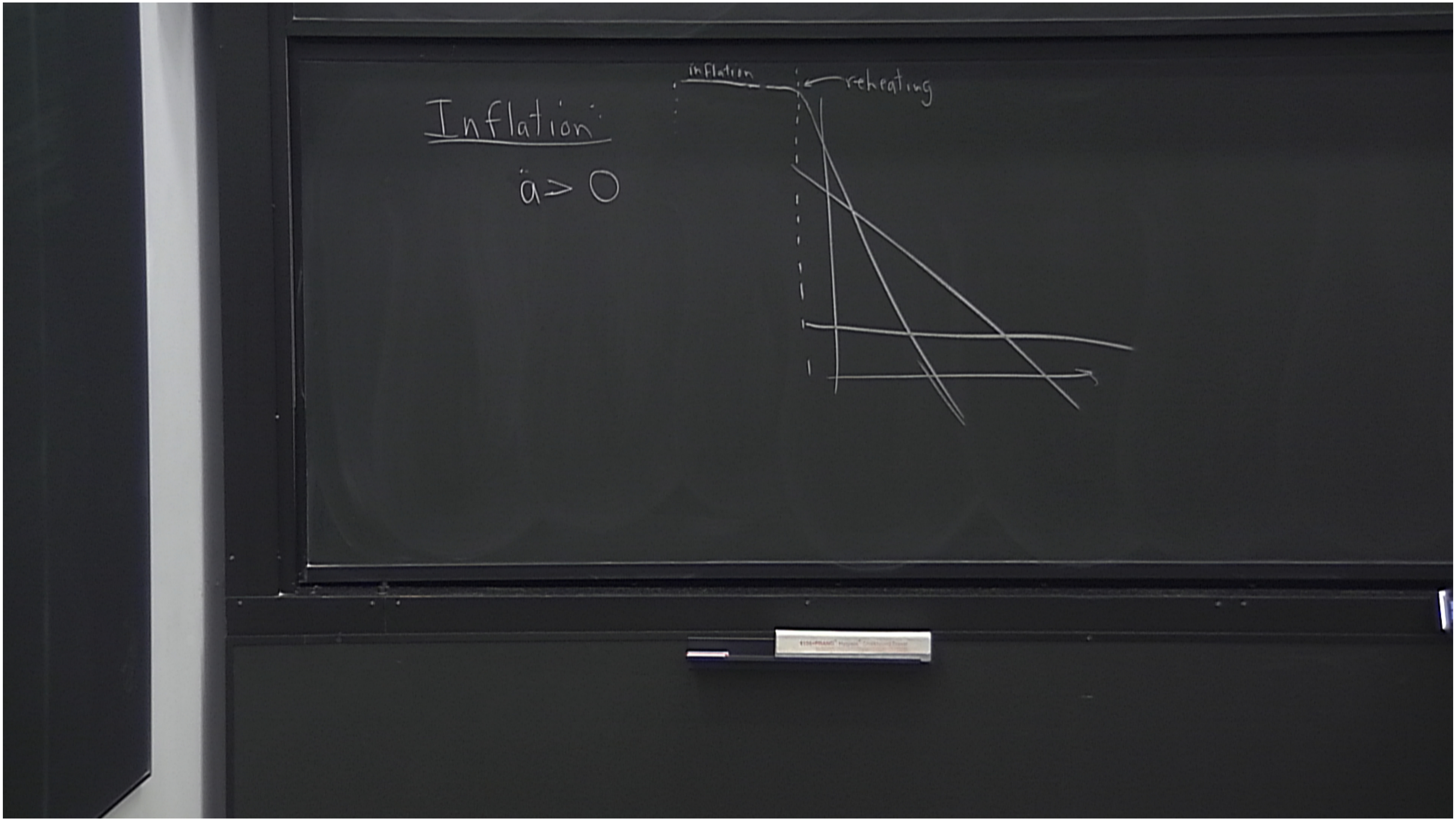
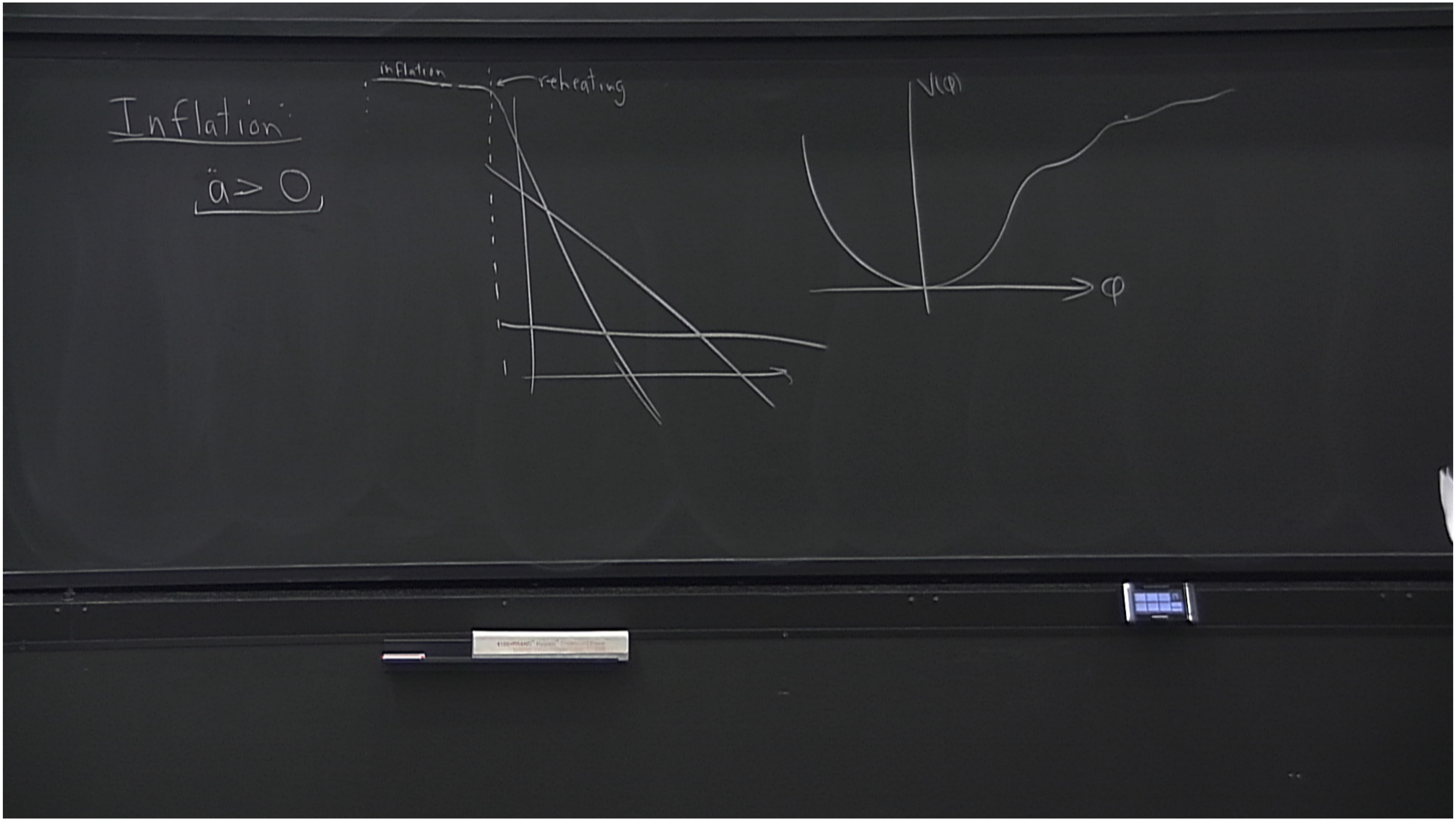
Title: 13/14 PSI - Cosmology Review - Lecture 13

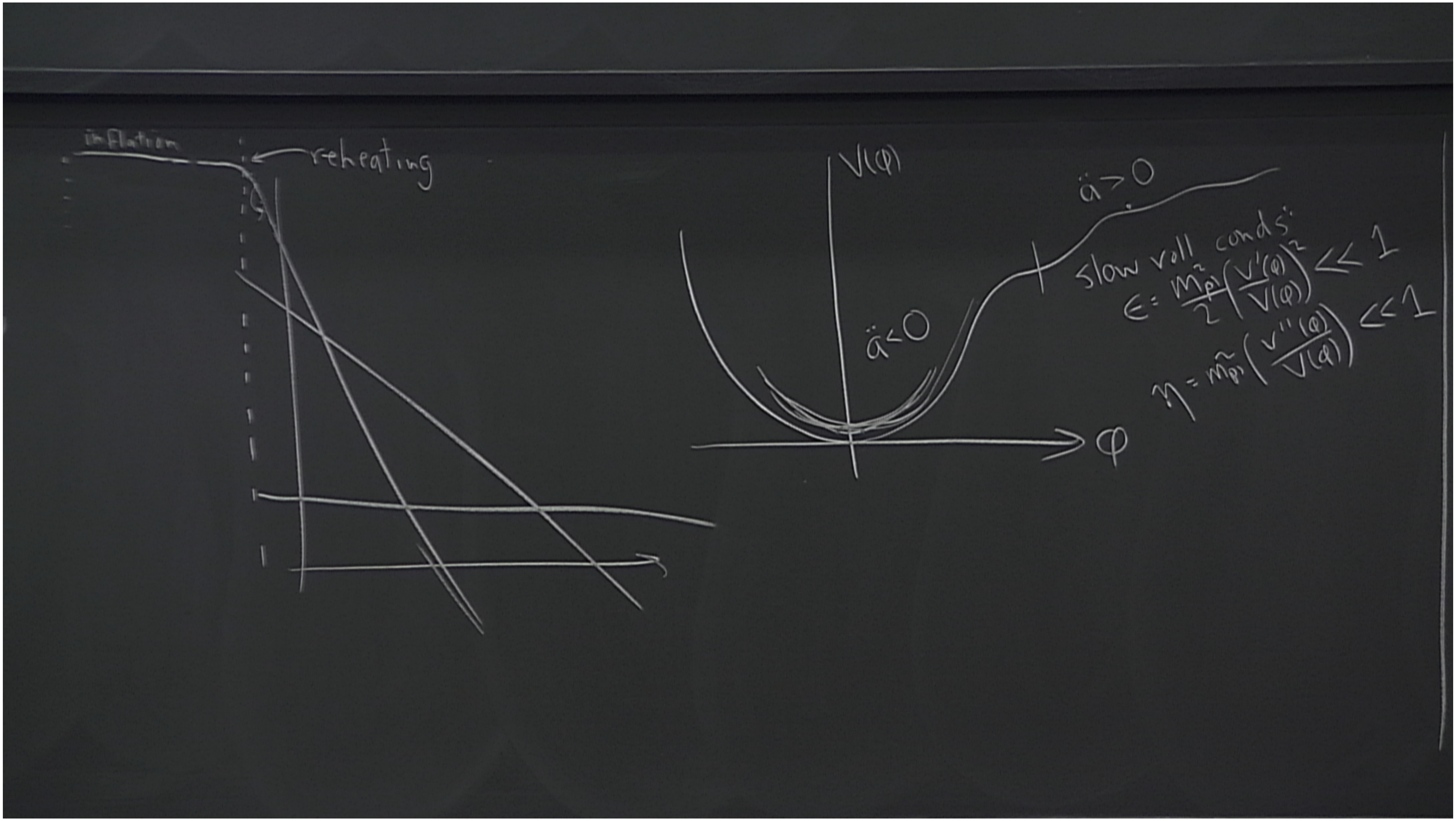
Date: Feb 14, 2014 11:30 AM

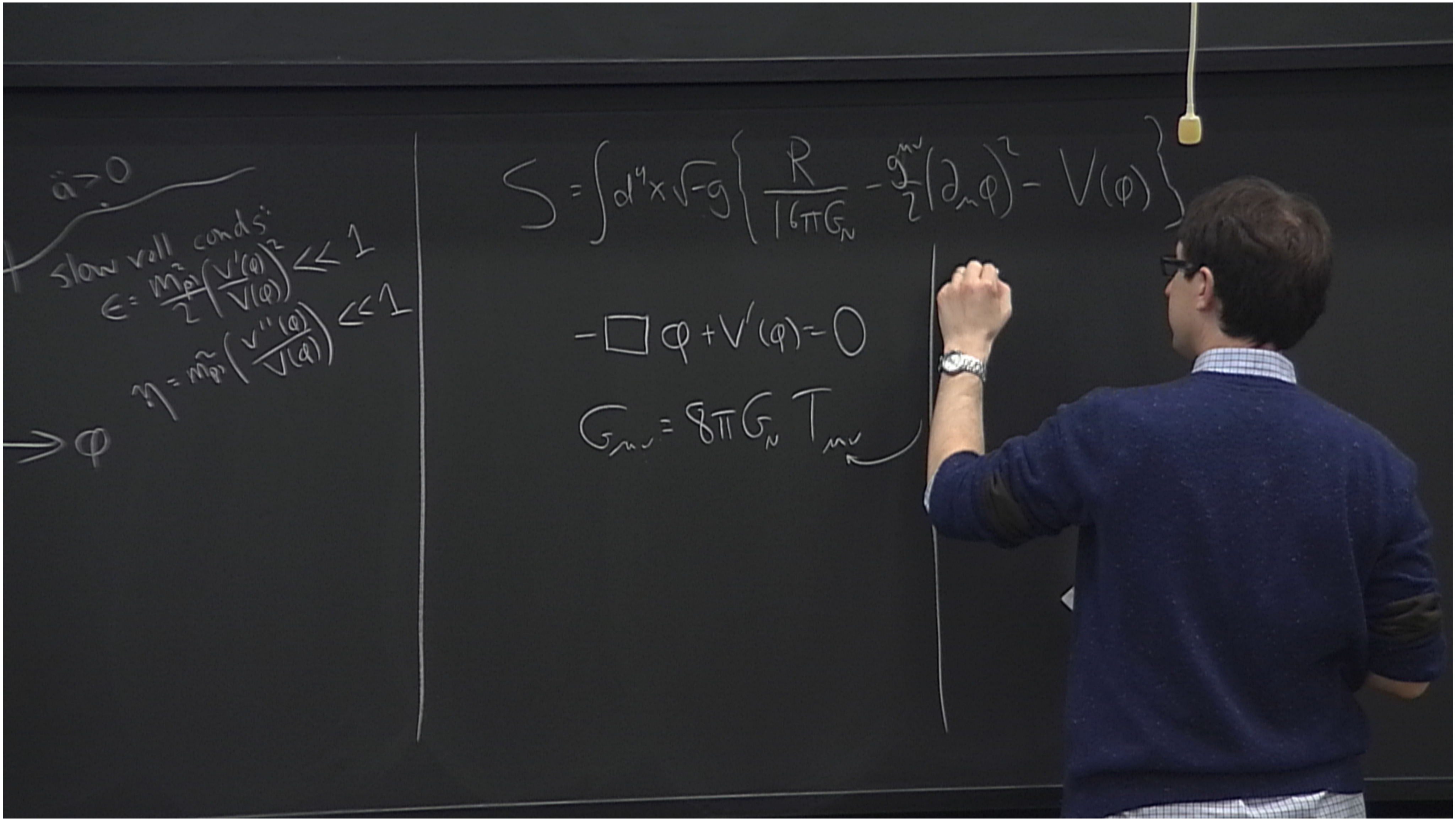
URL: http://pirsa.org/14020102

Abstract:

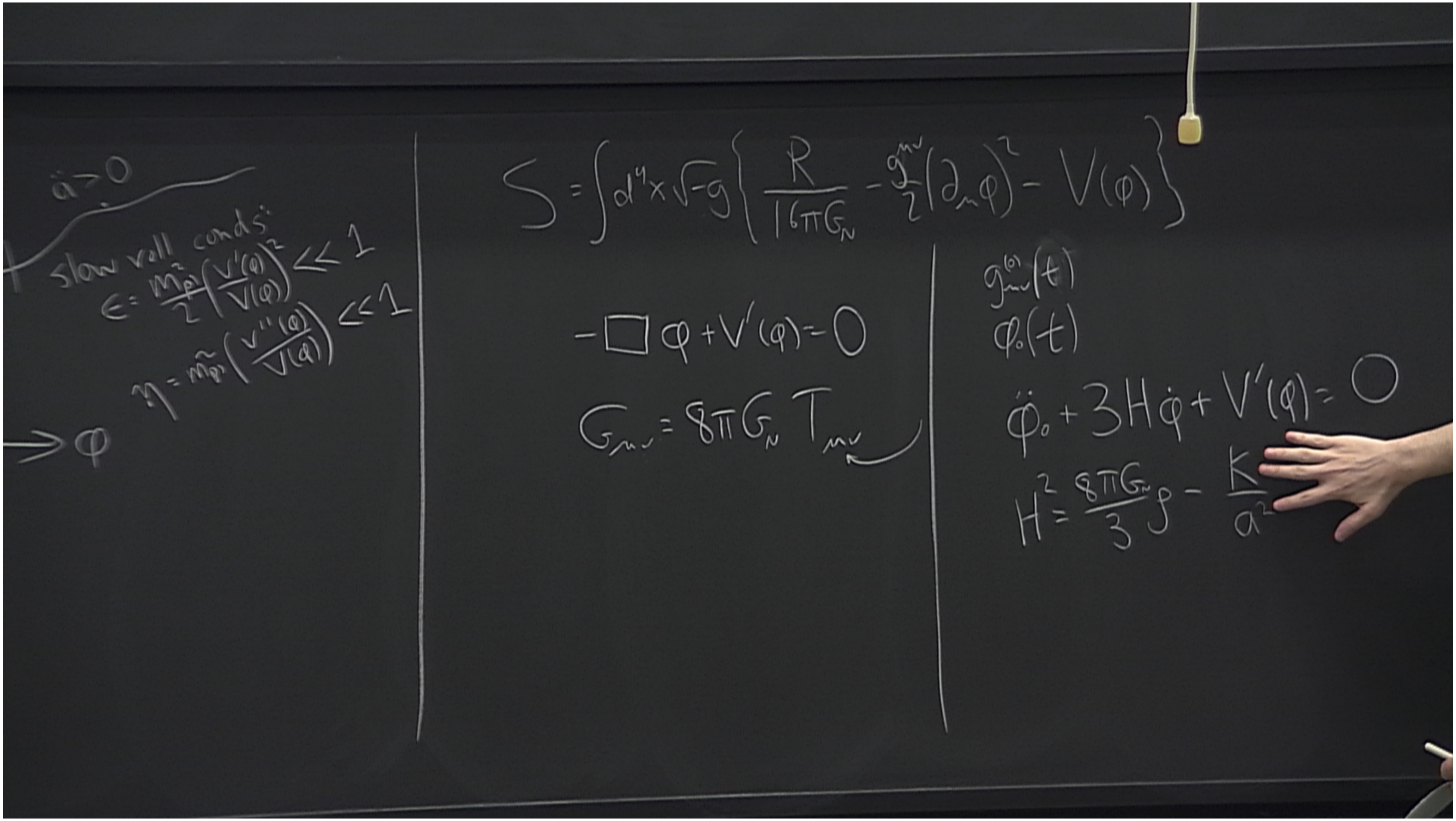

Inflation:
ä > 0
inflation
reheating


Inflation:
ä > 0
inflation
reheating
V(φ)
φ

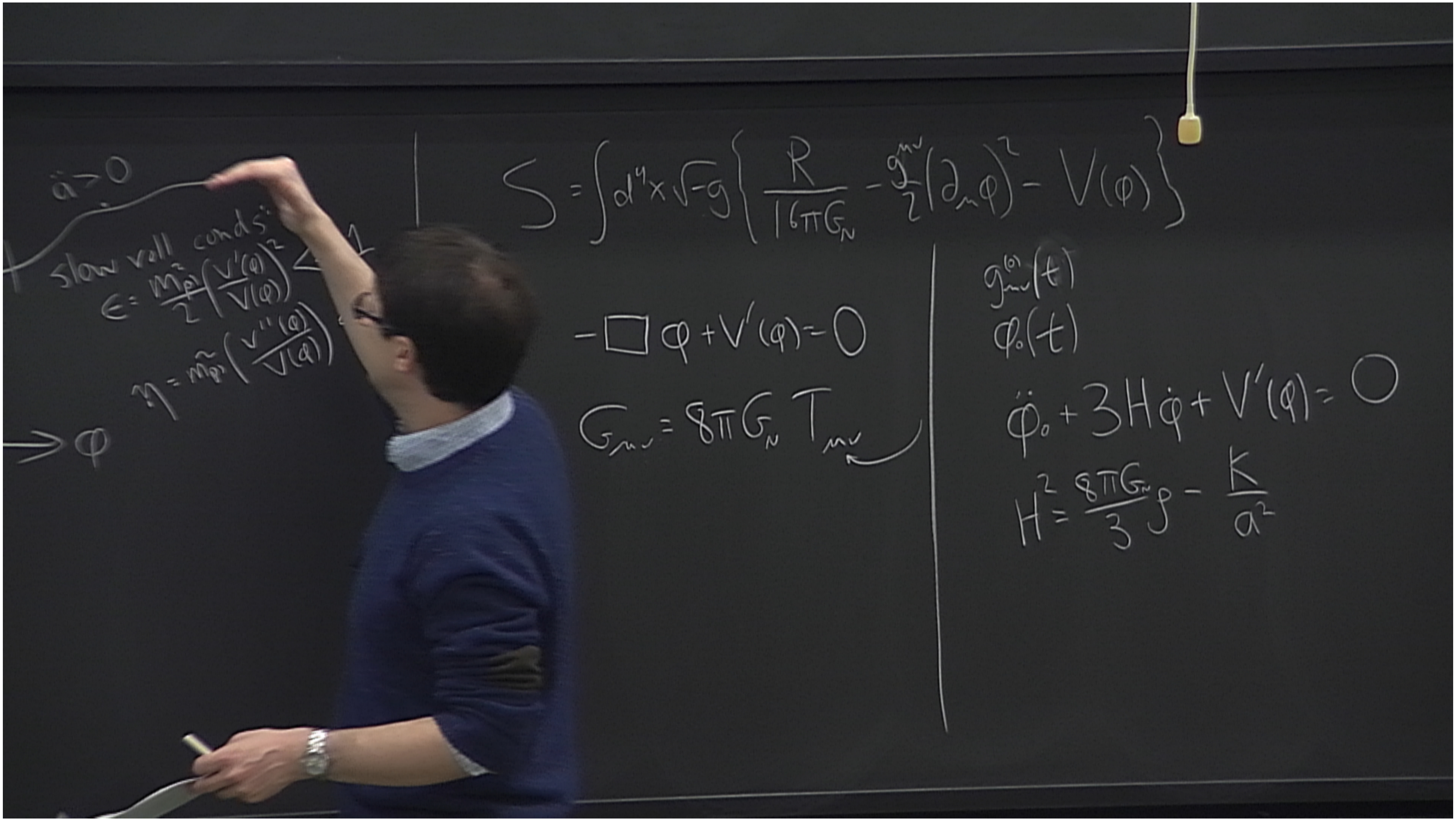

inflation
reheating
V(φ)
ä > 0
ä < 0
φ
slow roll conds:
$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{V'(\varphi)}{V(\varphi)}\right)^2 \ll 1$
$\eta = M_{pl}^2\left(\frac{V''(\varphi)}{V(\varphi)}\right) \ll 1$



$\ddot{a} > 0$
slow roll conds:
$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{V'(\varphi)}{V(\varphi)}\right)^2 \ll 1$
$\eta = M_{pl}^2\left(\frac{V''(\varphi)}{V(\varphi)}\right) \ll 1$
$\Rightarrow \varphi$
$S = \int d^4x \sqrt{-g}\left\{\frac{R}{16\pi G_N} - \frac{g^{\mu\nu}}{2}(\partial_\mu\varphi)^2 - V(\varphi)\right\}$
$-\Box\varphi + V'(\varphi) = 0$
$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$



$\ddot{a} > 0$
slow roll conds:
$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{V'(\varphi)}{V(\varphi)}\right)^2 \ll 1$
$\eta = M_{pl}^2\left(\frac{V''(\varphi)}{V(\varphi)}\right) \ll 1$
$\Rightarrow \varphi$
$S = \int d^4x \sqrt{-g}\left\{\frac{R}{16\pi G_N} - \frac{g^{\mu\nu}}{2}(\partial_\mu\varphi)^2 - V(\varphi)\right\}$
$-\Box\varphi + V'(\varphi) = 0$
$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$g^{(0)}_{\mu\nu}(t)$
$\varphi_0(t)$
$\ddot{\varphi}_0 + 3H\dot{\varphi} + V'(\varphi) = 0$
$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$


ä > 0
slow roll conds:
ϵ = M_pl^2/2 (V'(φ)/V(φ))^2 ≪ 1
η = M_pl^2 (V''(φ)/V(φ))
⇒ φ
S = ∫d^4x √-g { R/16πG_N - g^μν/2 (∂_μ φ)^2 - V(φ) }
-□φ + V'(φ) = 0
G_μν = 8πG_N T_μν
g^(0)_μν(t)
φ_0(t)
φ̈_0 + 3Hφ̇ + V'(φ) = 0
H^2 = 8πG_N/3 ρ - K/a^2

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G_N} - \frac{g^{\mu\nu}}{2} (\partial_\mu \varphi)^2 - V(\varphi) \right\}$$

$$-\Box \varphi + V'(\varphi) = 0$$

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

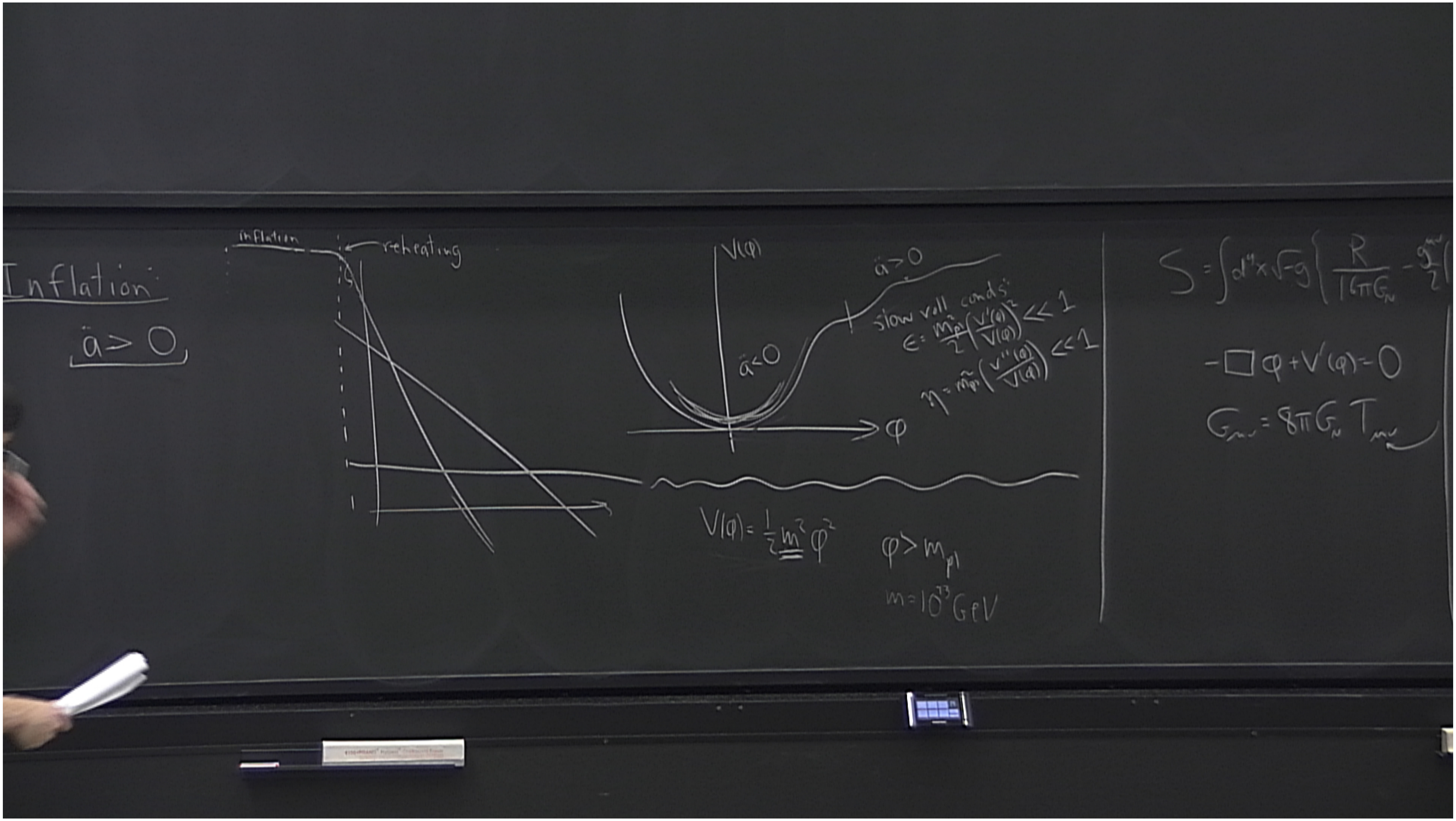
$$g^{(0)}_{\mu\nu}(t)$$

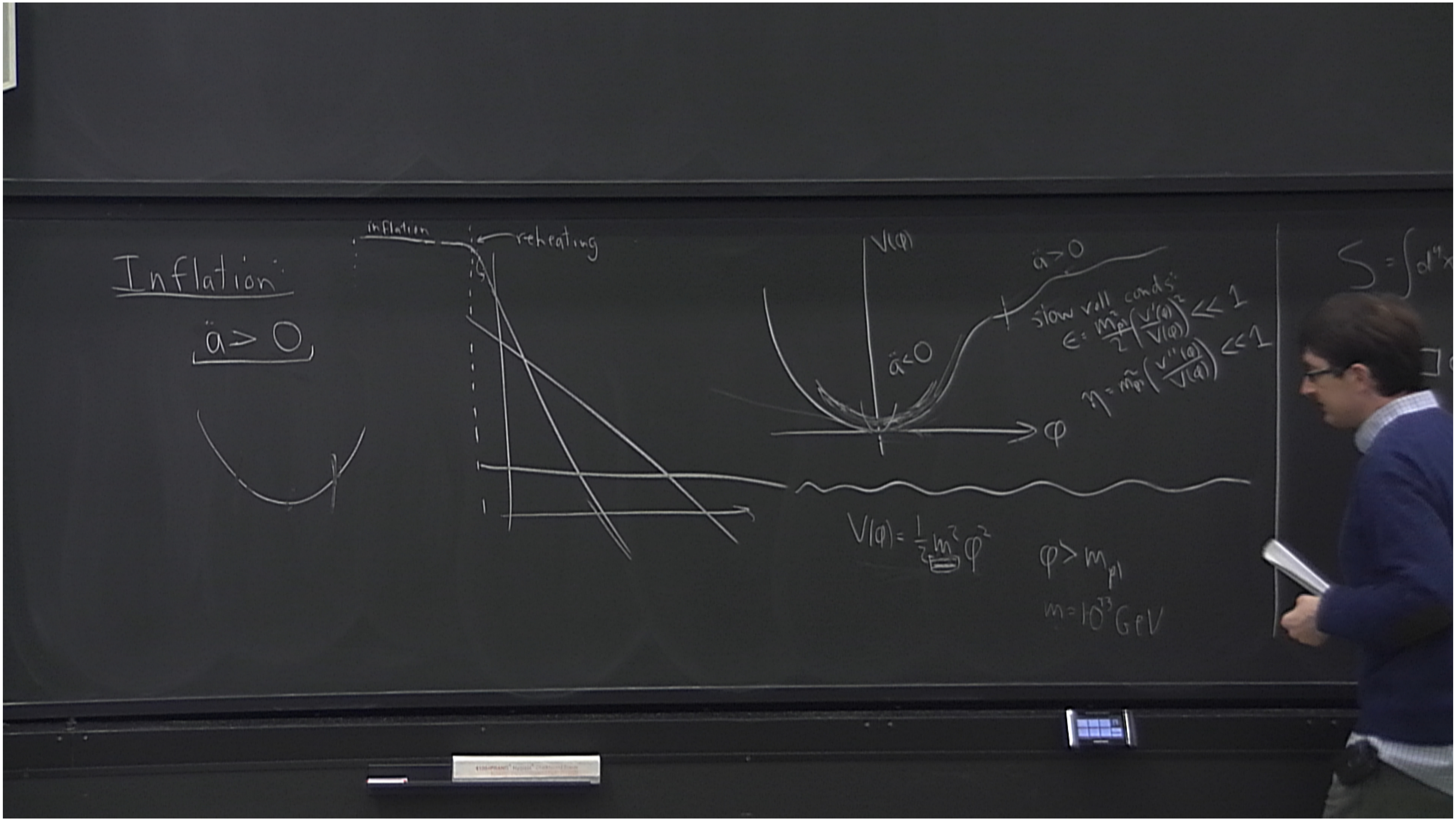
$$\varphi_0(t)$$

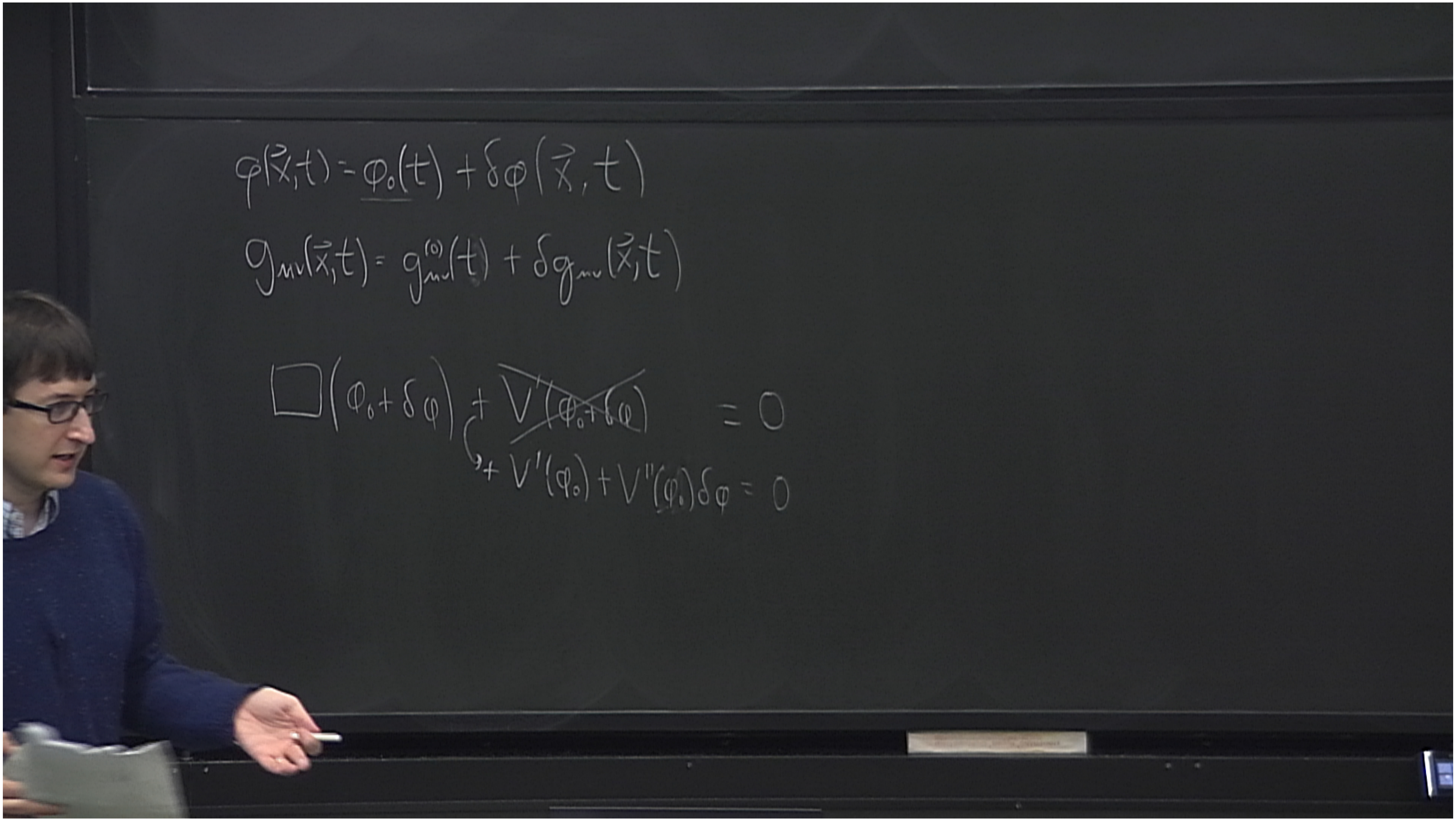
$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

$$\ddot{\varphi}_0 + 3H\dot{\varphi} + V'(\varphi) = 0$$

$$H^2 = \frac{8\pi G_N}{3}\rho$$


Inflation:
ä > 0
inflation
reheating
V(φ)
ä > 0
ä < 0
slow roll conds:
<< 1
<< 1
φ
V(φ) = ½ m² φ²
φ > m_pl
m = 10^13 GeV
-□φ + V′(φ) = 0
G_μν = 8πG_N T_μν


Inflation:
ä > 0
inflation
reheating
V(φ)
ä>0
ä<0
slow roll conds:
<< 1
<< 1
φ
V(φ) = 1/2 m² φ²
φ > m_pl
m = 10^13 GeV

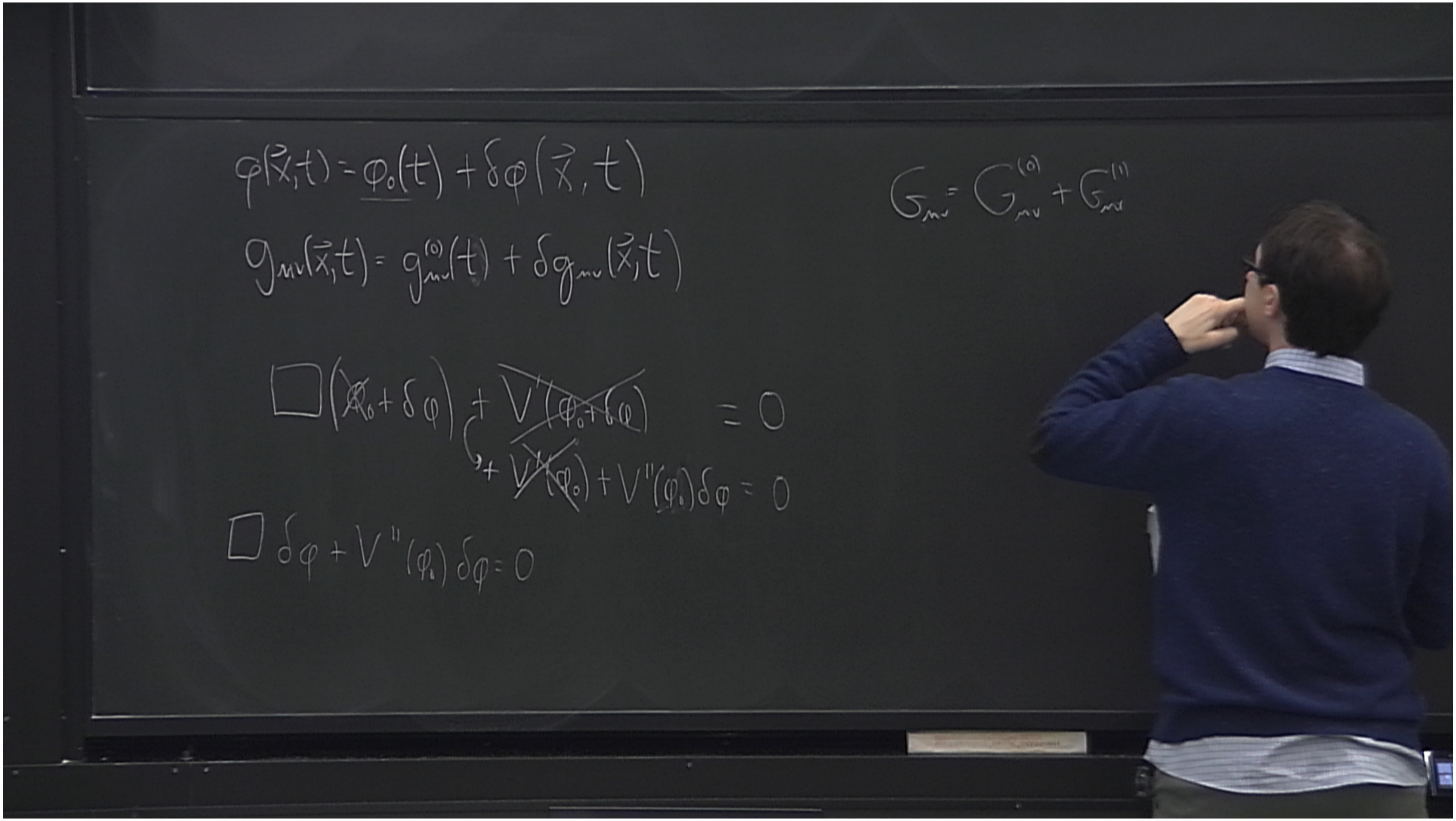

$\varphi(\vec{x},t) = \varphi_0(t) + \delta\varphi(\vec{x},t)$
$g_{\mu\nu}(\vec{x},t) = g^{(0)}_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$
$\Box(\varphi_0 + \delta\varphi) + V'(\varphi_0 + \delta\varphi) = 0$
$+ V'(\varphi_0) + V''(\varphi_0)\delta\varphi = 0$

$$\varphi(\vec{x},t) = \varphi_0(t) + \delta\varphi(\vec{x},t)$$

$$g_{\mu\nu}(\vec{x},t) = g^{(0)}_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$$

$$\Box(\varphi_0 + \delta\varphi) + V'(\varphi_0 + \delta\varphi) = 0$$

$$+ V'(\varphi_0) + V''(\varphi_0)\delta\varphi = 0$$


$\varphi(\vec{x},t) = \varphi_0(t) + \delta\varphi(\vec{x},t)$
$g_{\mu\nu}(\vec{x},t) = g^{(0)}_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$
$G_{\mu\nu} = G^{(0)}_{\mu\nu} + G^{(1)}_{\mu\nu}$
$\Box(\varphi_0 + \delta\varphi) + V'(\varphi_0 + \delta\varphi) = 0$
$+ V'(\varphi_0) + V''(\varphi_0)\delta\varphi = 0$
$\Box\delta\varphi + V''(\varphi_0)\delta\varphi = 0$

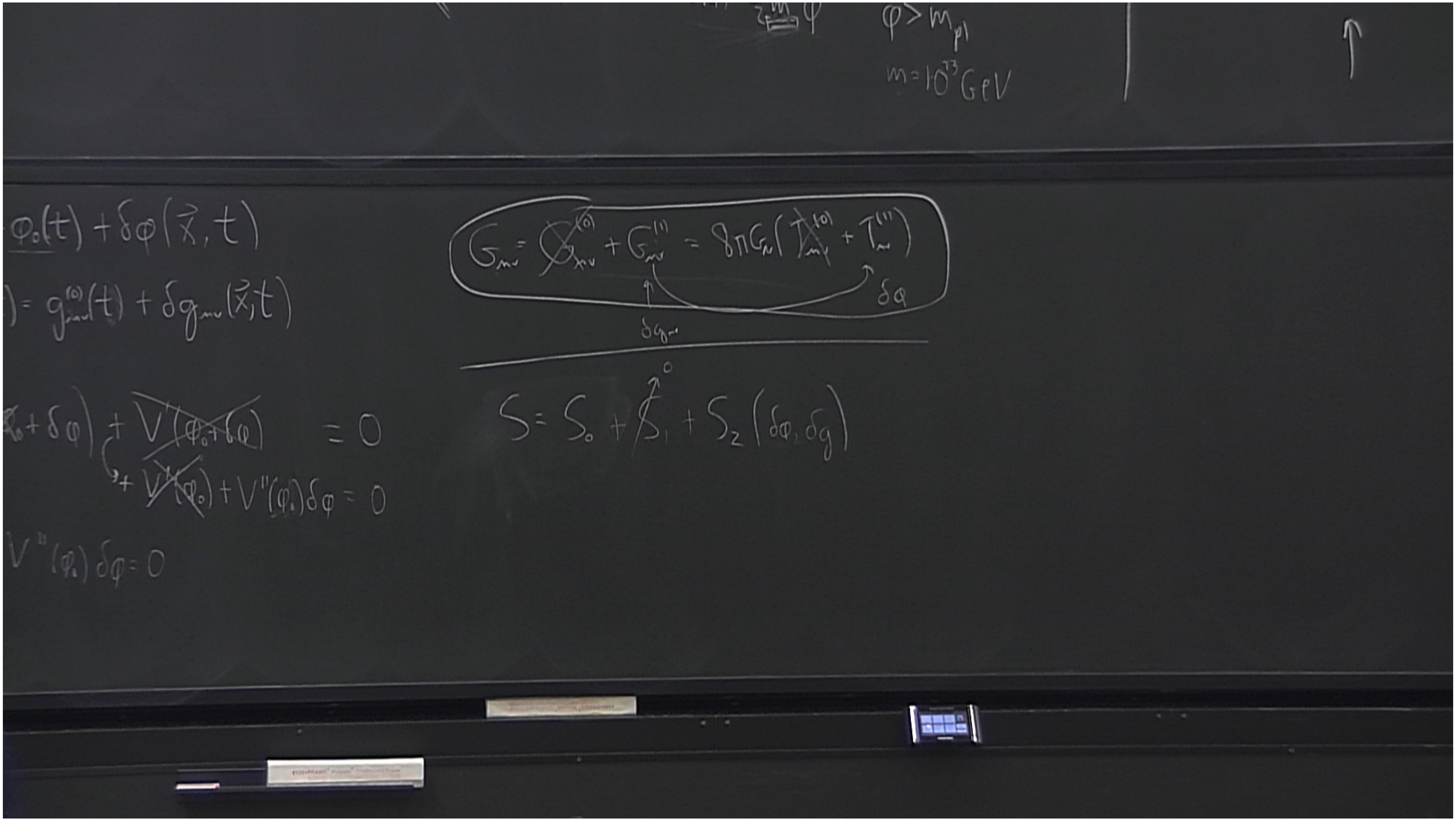
$$\varphi(\vec{x},t) = \underline{\varphi_0(t)} + \delta\varphi(\vec{x},t)$$

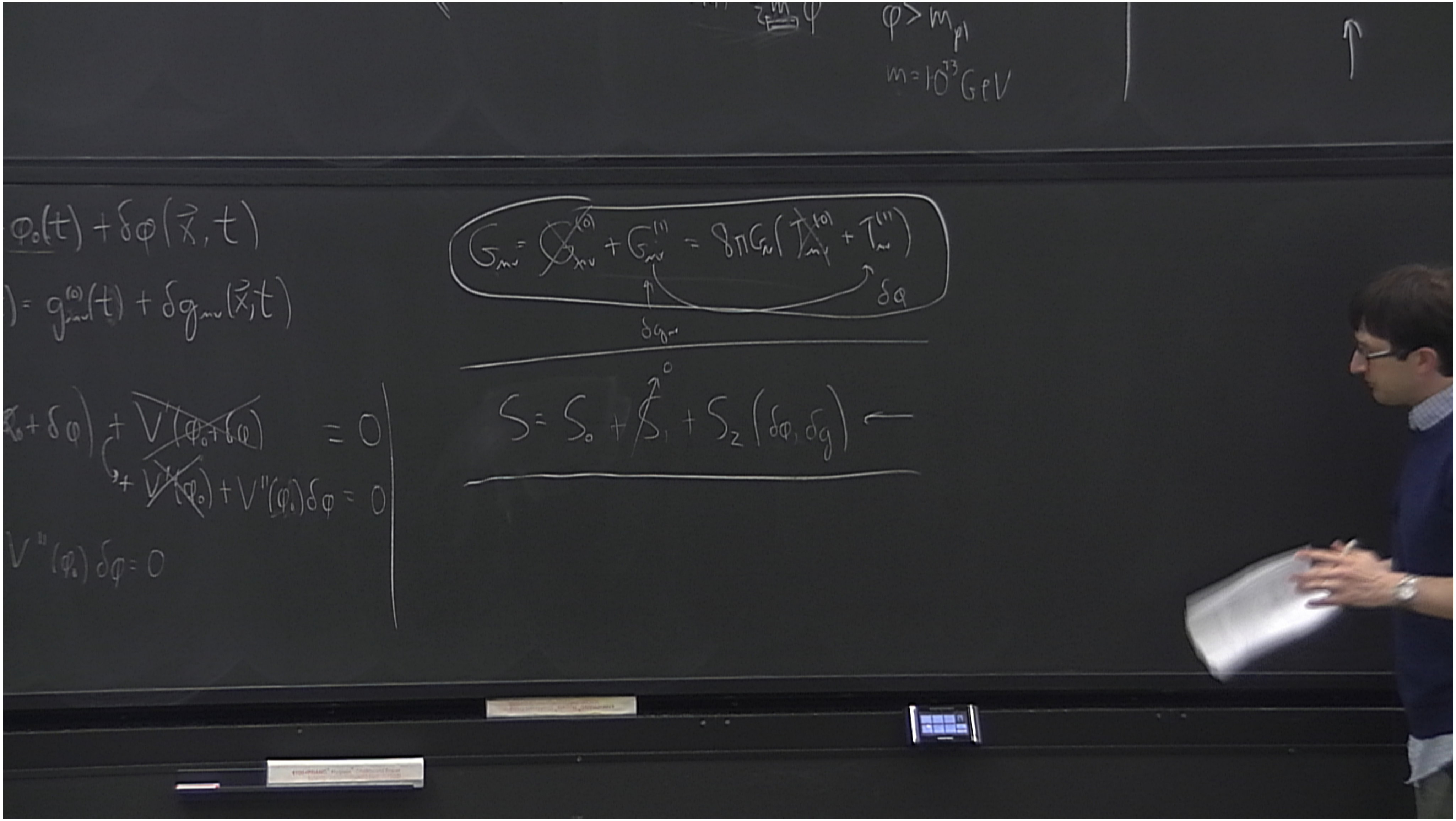
$$g_{\mu\nu}(\vec{x},t) = g^{(0)}_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$$

$$\Box(\varphi_0 + \delta\varphi) + V'(\varphi_0 + \delta\varphi) = 0$$

$$+ V'(\varphi_0) + V''(\varphi_0)\,\delta\varphi = 0$$

$$\Box\,\delta\varphi + V''(\varphi_0)\,\delta\varphi = 0$$


φ > M_pl
m = 10^13 GeV
φ0(t) + δφ(x, t)
= g^(0)_μν(t) + δg_μν(x, t)
G_μν = G^(0)_μν + G^(1)_μν = 8πG_N (T^(0)_μν + T^(1)_μν)
δg_μν
δφ
S = S_0 + S_1 + S_2 (δφ, δg)
= 0
+ V'(φ0) + V''(φ0) δφ = 0
V''(φ0) δφ = 0


φ > m_pl
m = 10^-33 GeV
φ₀(t) + δφ(x, t)
= g^(0)_μν(t) + δg_μν(x, t)
G_μν = G^(0)_μν + G^(1)_μν = 8πG_N(T^(0)_μν + T^(1)_μν)
δg_μν
δφ
S = S₀ + S₁ + S₂(δφ, δg)
V''(φ₀) δφ = 0

$$G_{\mu\nu} = \cancel{G^{(0)}_{\mu\nu}} + G^{(1)}_{\mu\nu} = 8\pi G_N \left(\cancel{T^{(0)}_{\mu\nu}} + T^{(1)}_{\mu\nu} \right)$$

$\delta g_{\mu\nu}$ $\qquad$ $\delta\varphi$

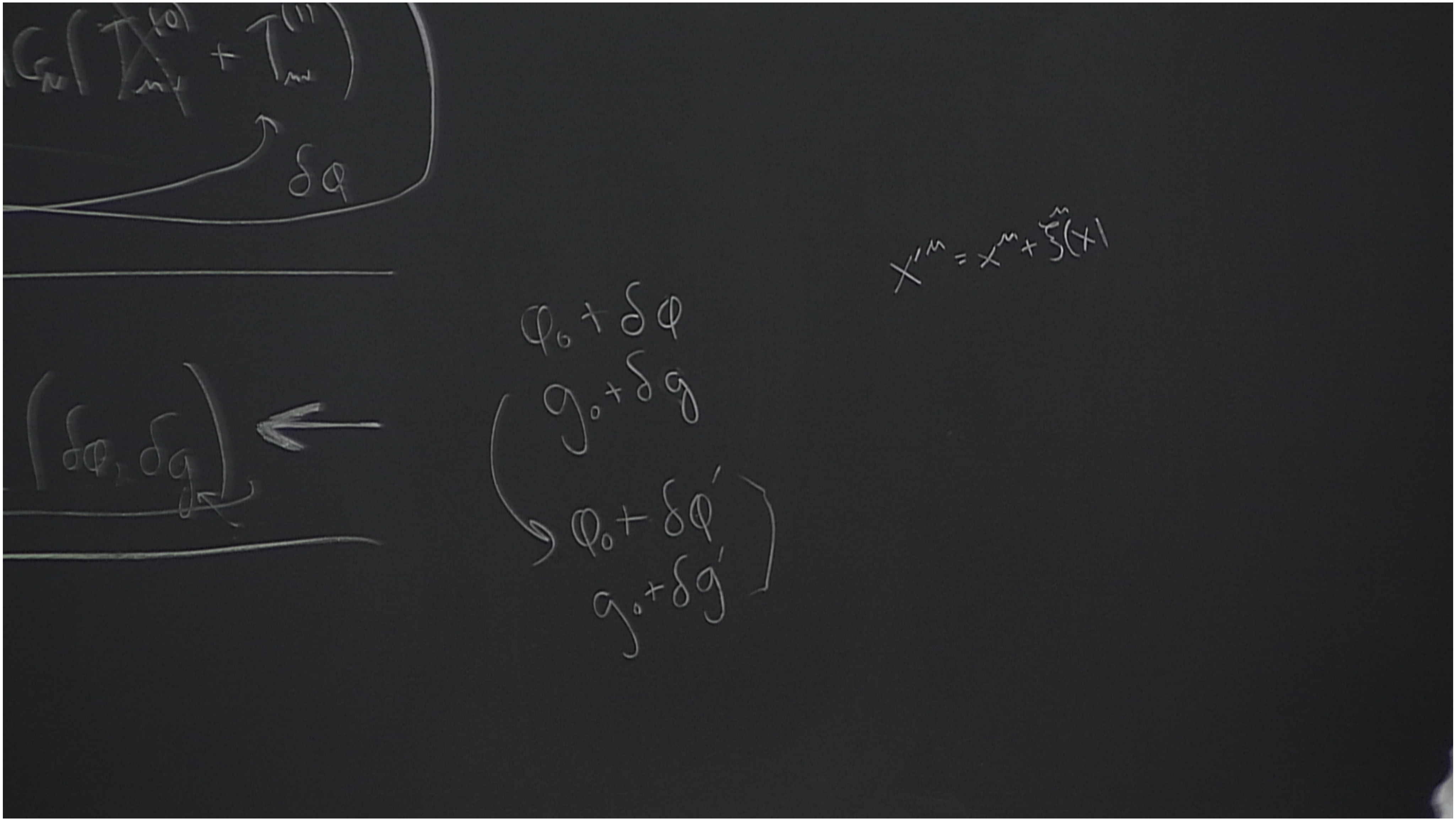
$$S = S_0 + \cancel{S_1}^{\,0} + S_2(\delta\varphi, \delta g) \leftarrow$$

$\varphi_0 + \delta\varphi$

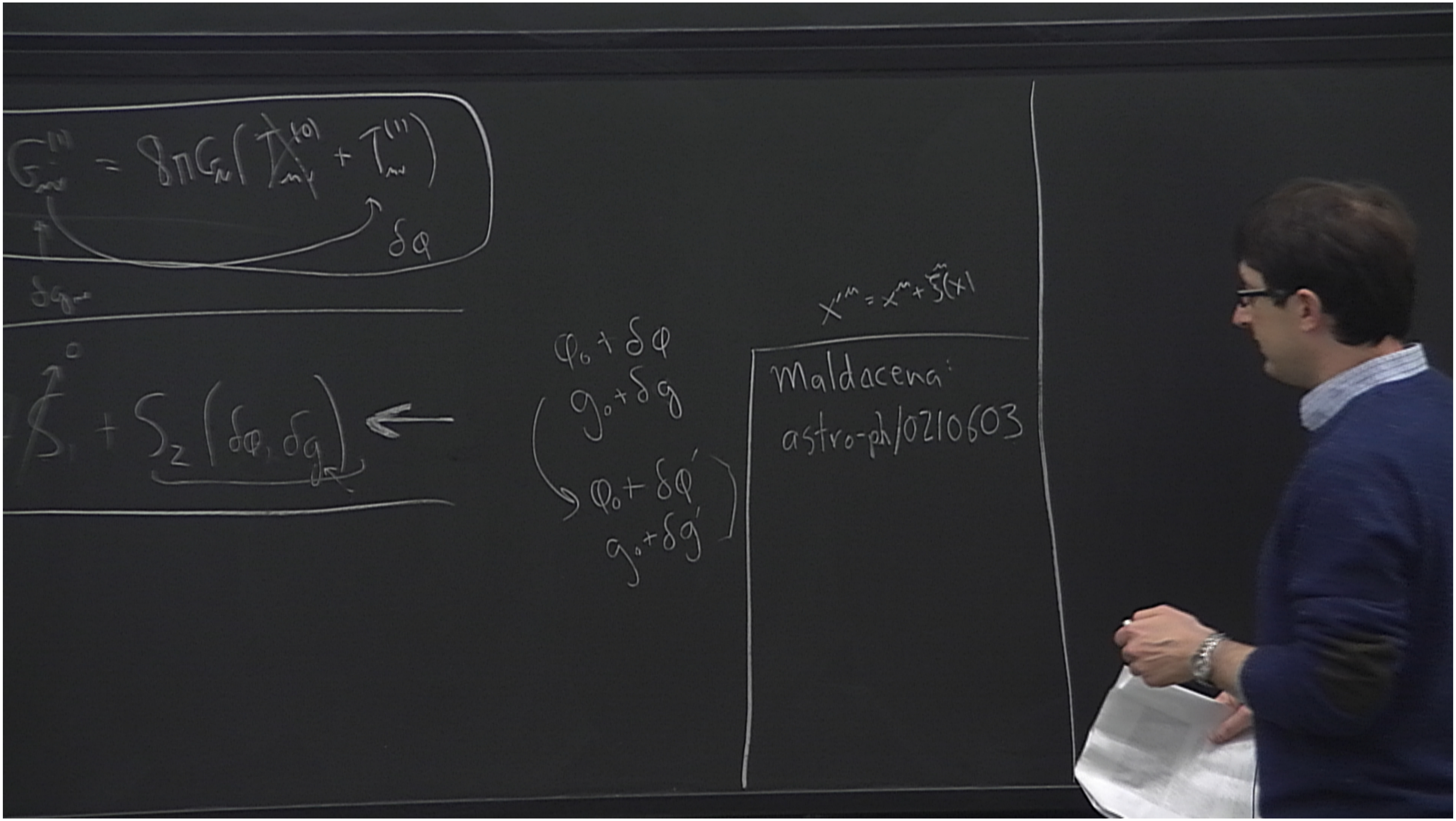
$g_0 + \delta g$

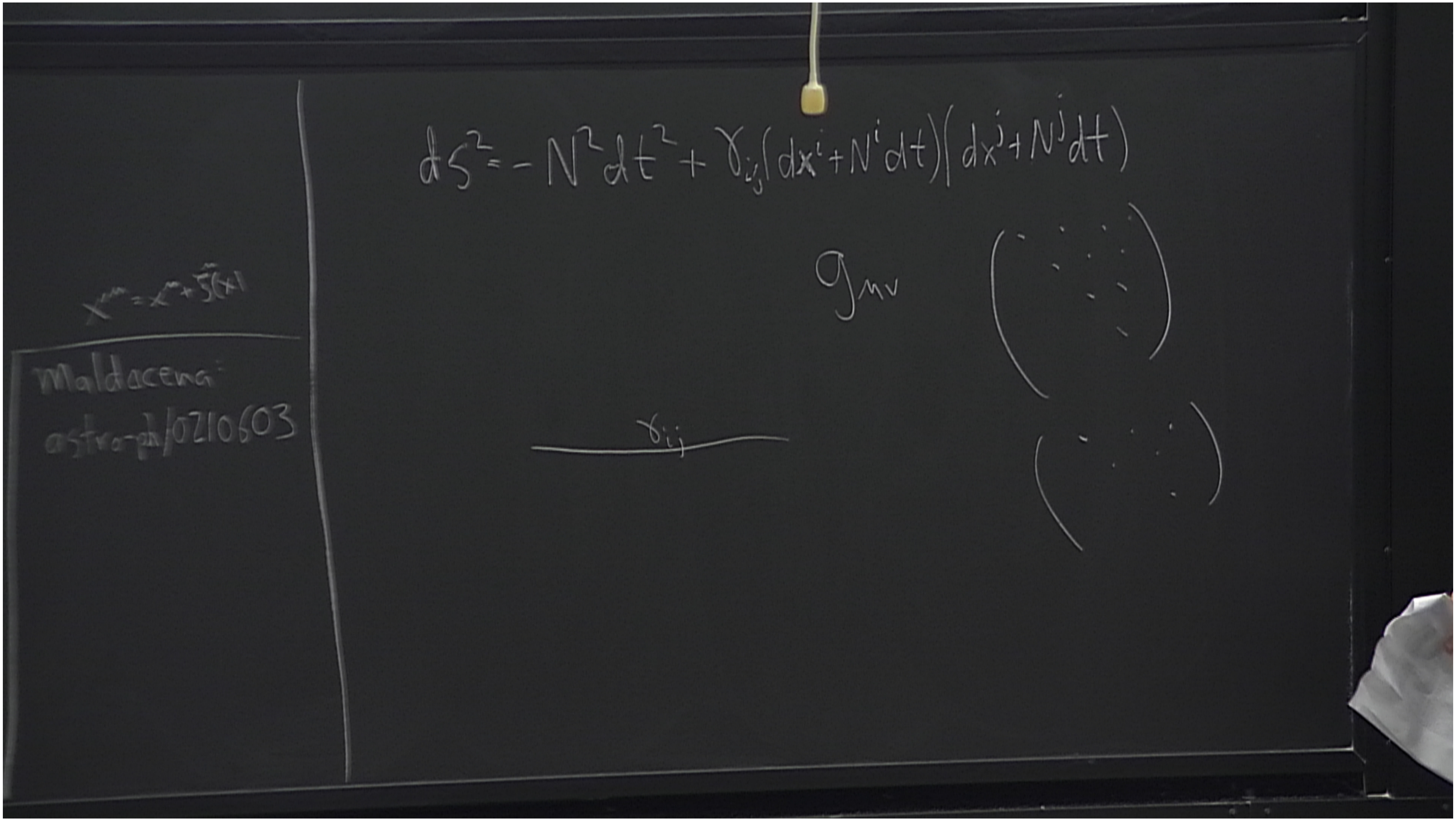
$= 0$

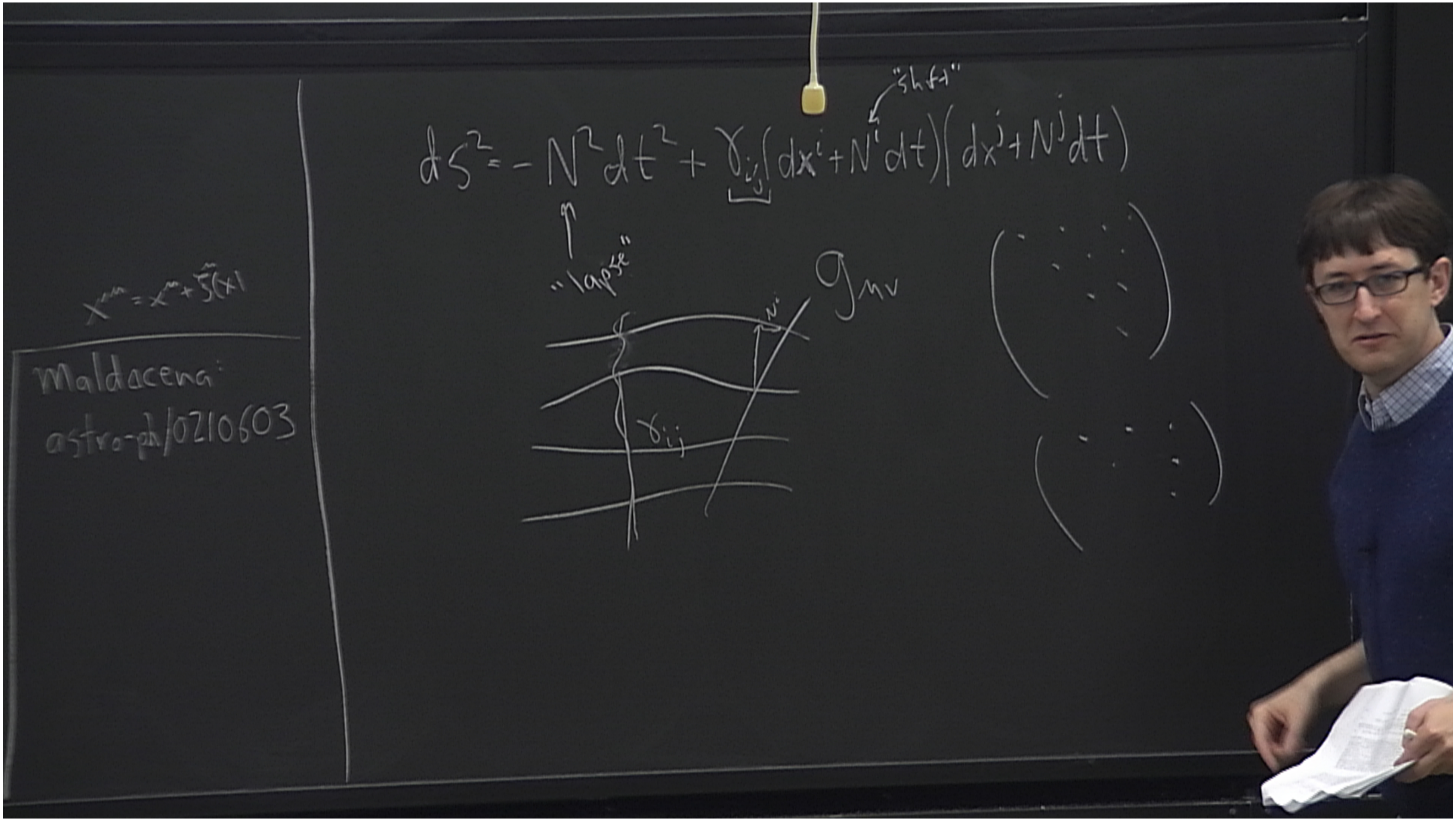
$\delta\varphi = 0$

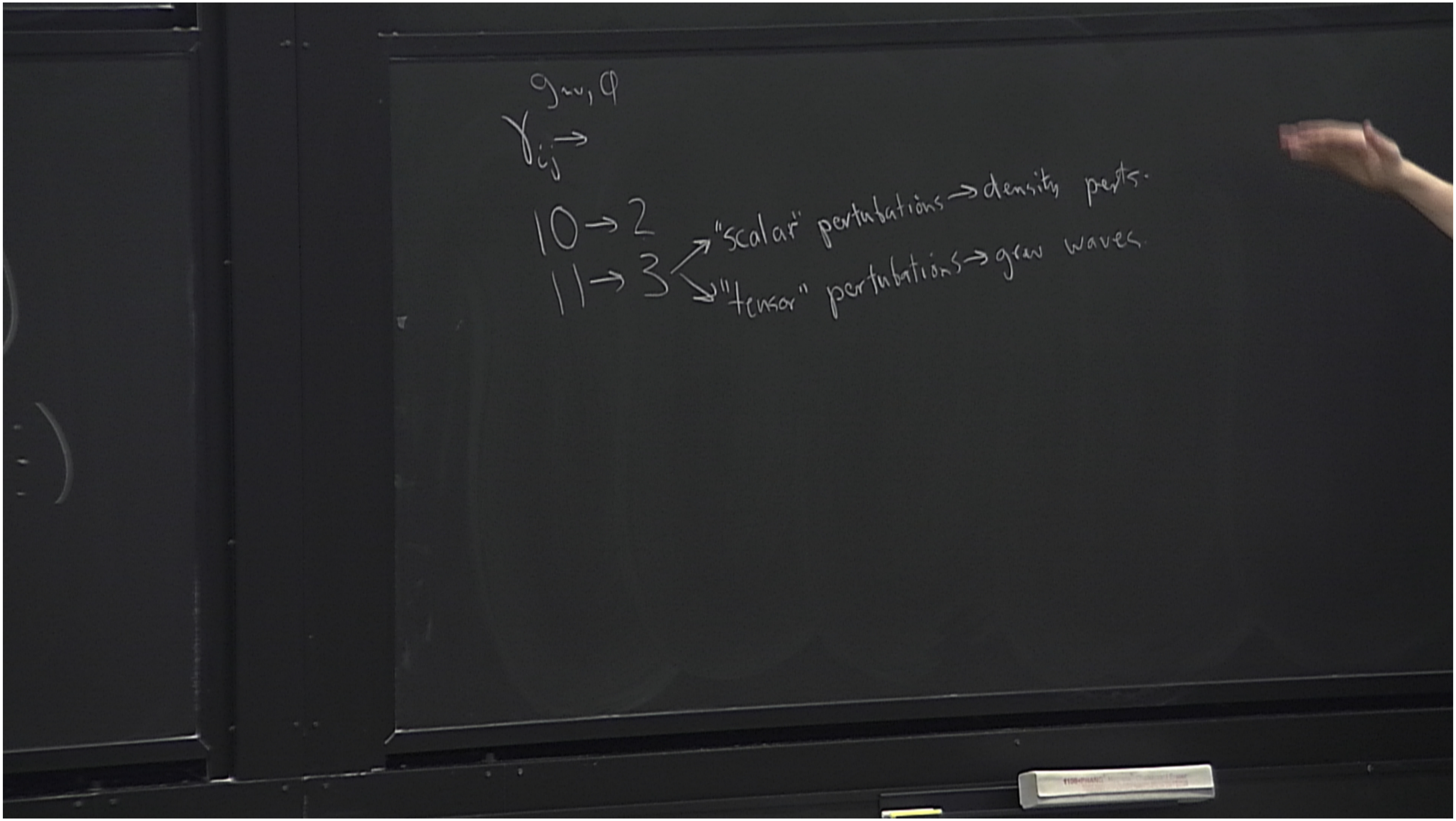


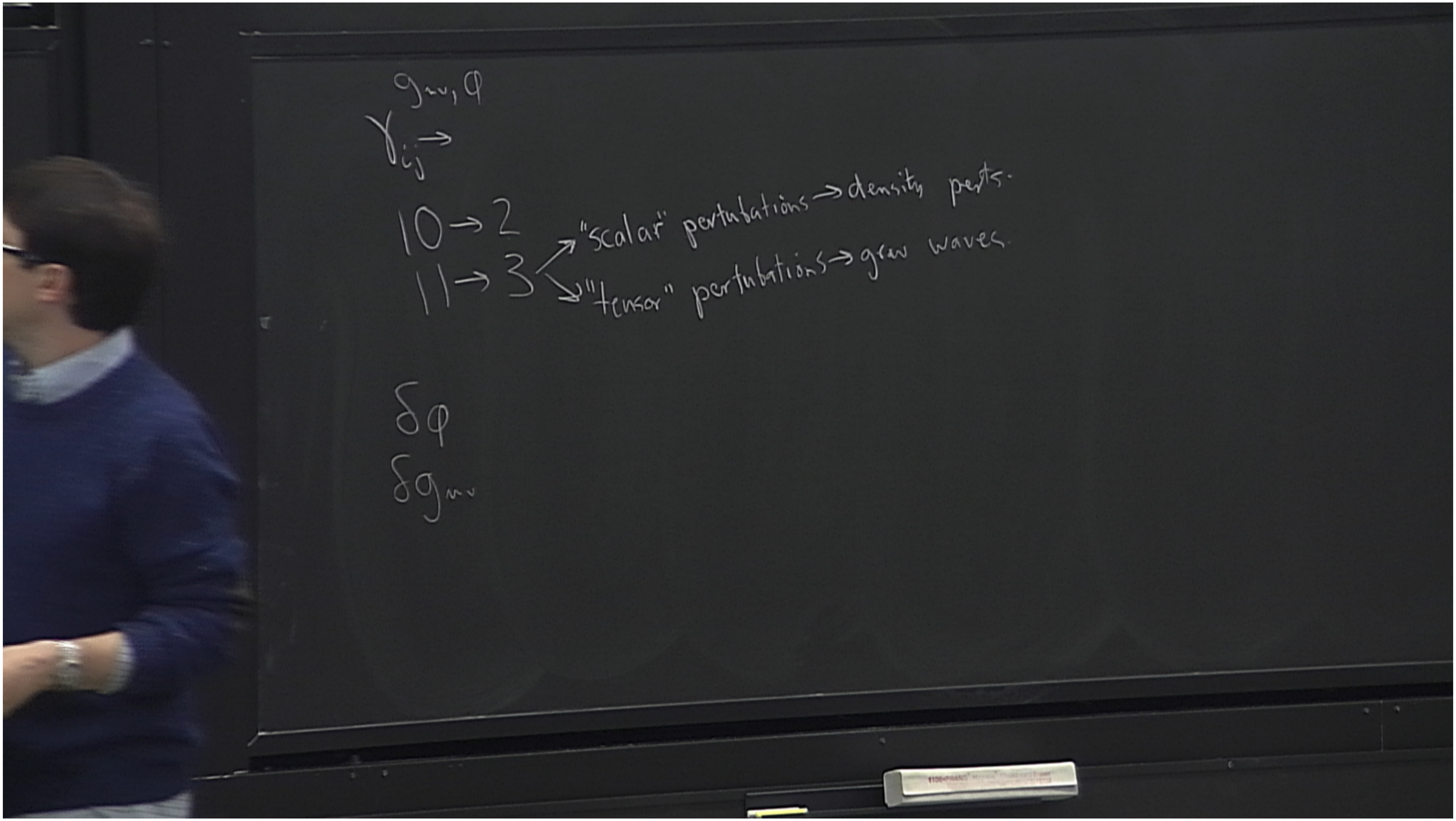
$G_N (T^{(0)}_{\mu\nu} + T^{(1)}_{\mu\nu})$
$\delta\varphi$
$(\delta\varphi, \delta g)$
$\varphi_0 + \delta\varphi$
$g_0 + \delta g$
$\varphi_0 + \delta\varphi'$
$g_0 + \delta g'$
$x'^\mu = x^\mu + \xi^\mu(x)$


Maldacena:
astro-ph/0210603

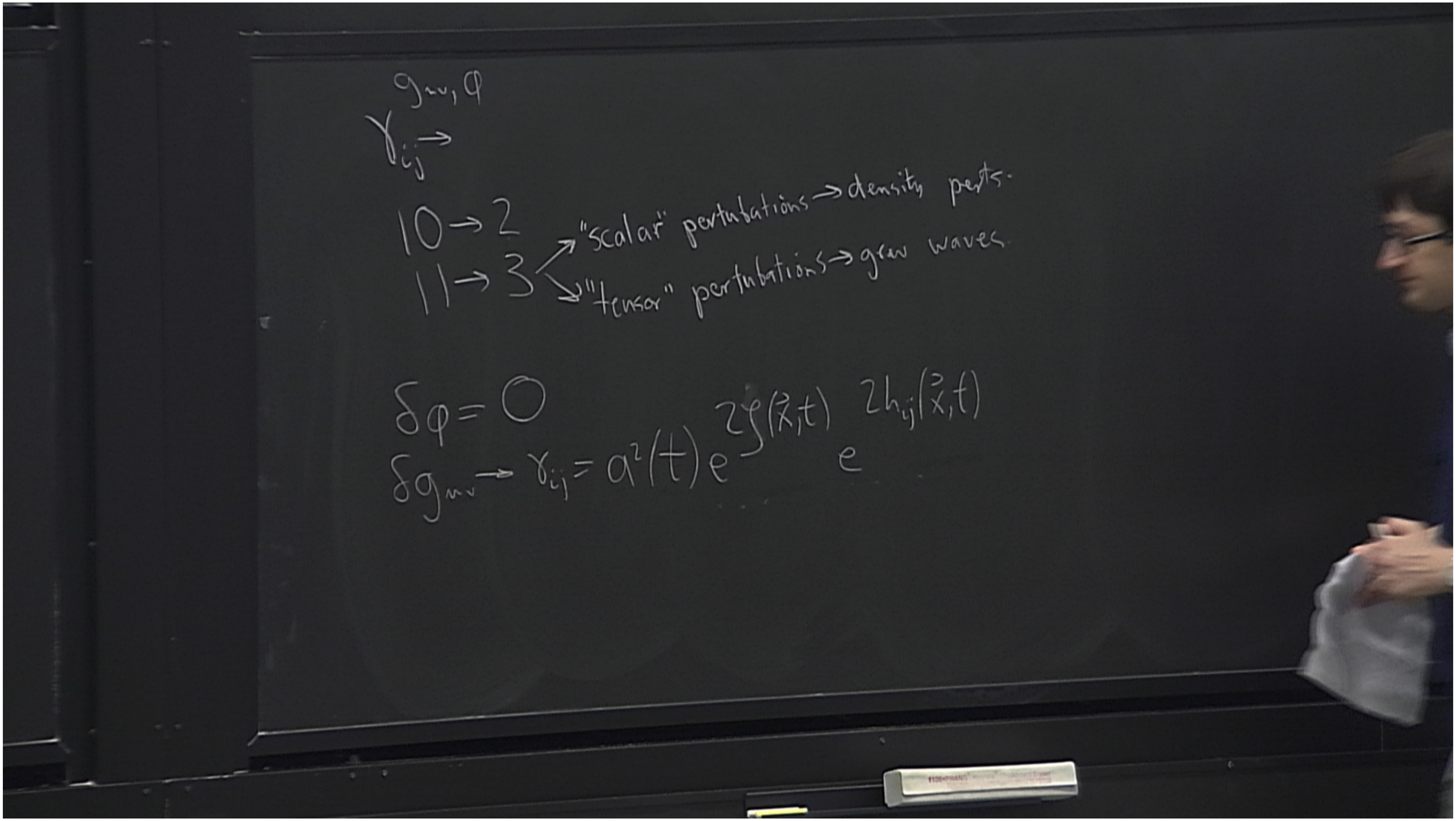

$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$g_{\mu\nu}$
γ_{ij}
Maldacena
astro-ph/0210603

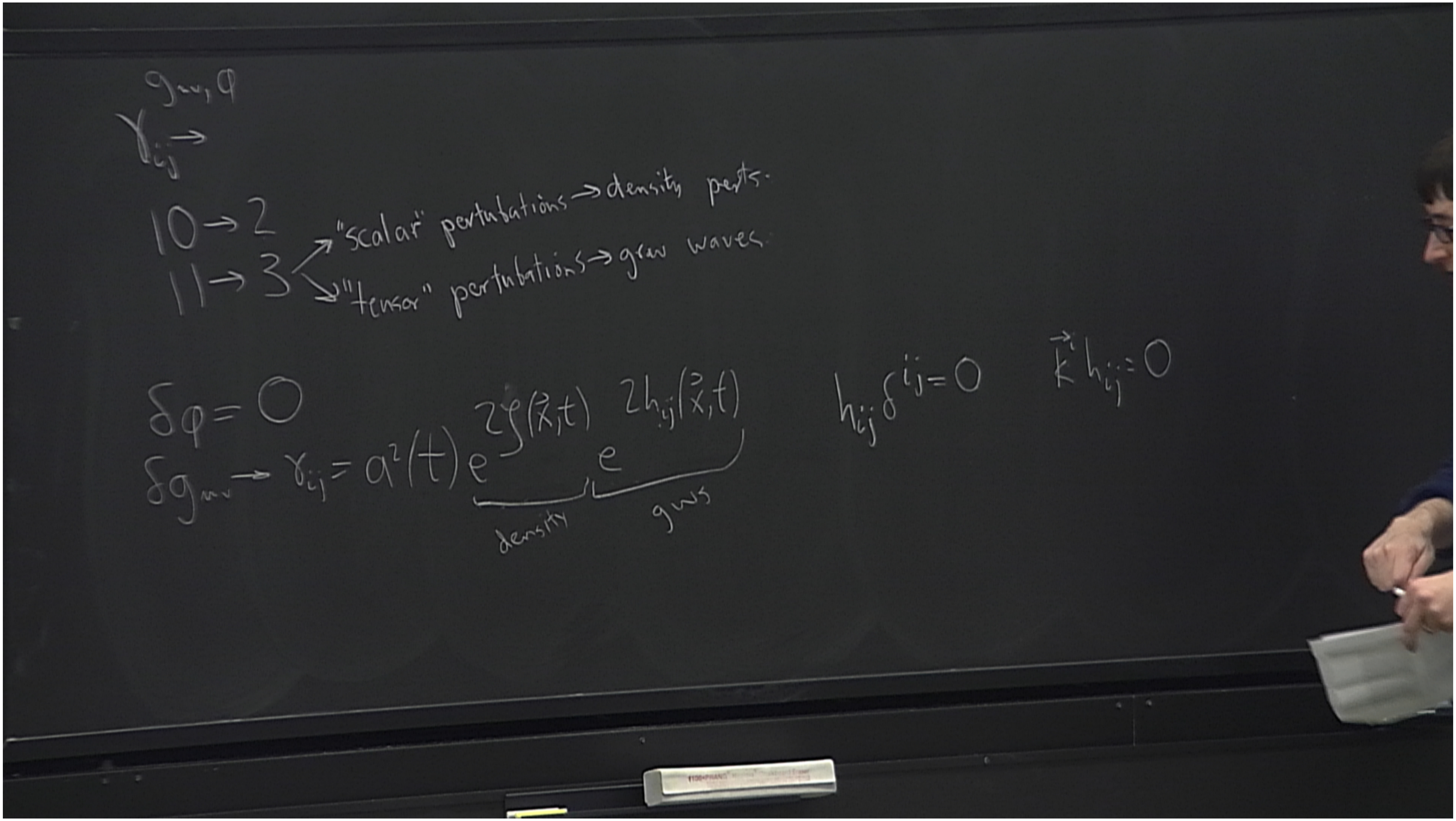

"shift"
$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
"lapse"
$g_{\mu\nu}$
γ_{ij}
N^i
$x'^\mu = x^\mu + \xi^\mu(x)$
Maldacena:
astro-ph/0210603


$g_{\mu\nu}, \varphi$
$\gamma_{ij} \rightarrow$
$10 \rightarrow 2$
$11 \rightarrow 3$
"scalar" perturbations → density perts.
"tensor" perturbations → grav waves.

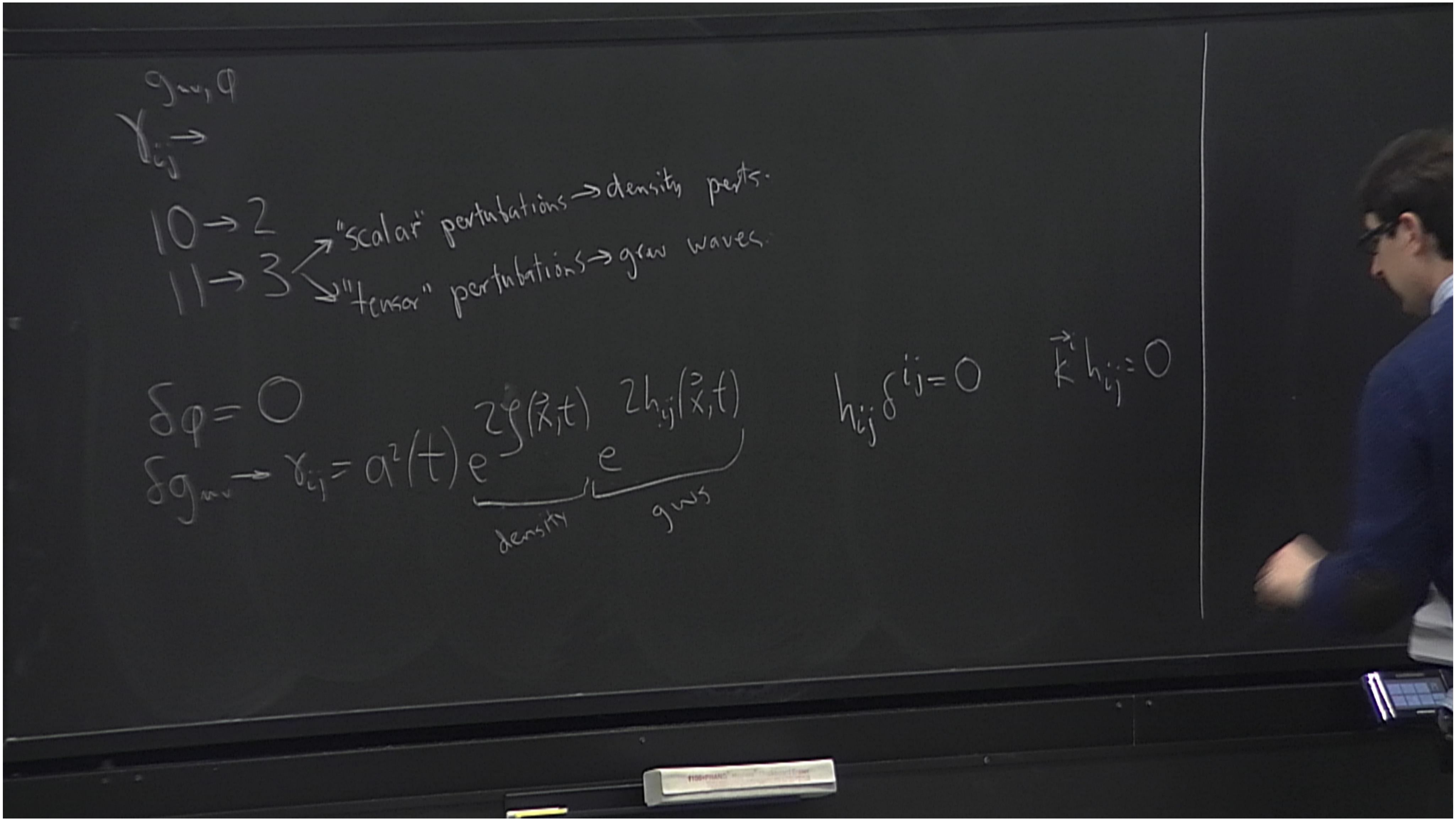


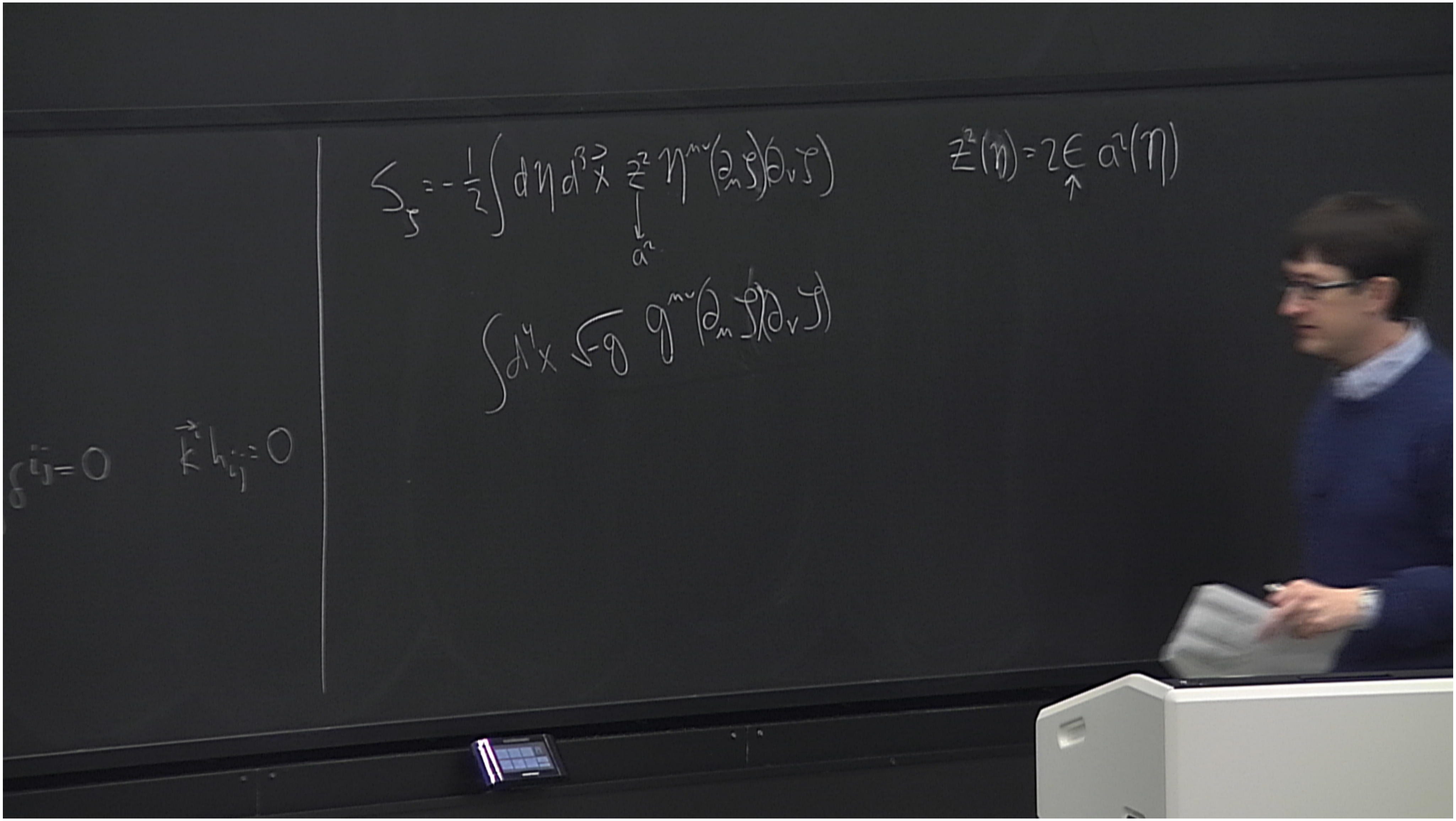
$g_{\mu\nu}, \varphi$
$\gamma_{ij} \rightarrow$
$10 \rightarrow 2$
$11 \rightarrow 3$
→ "scalar" perturbations → density perts.
→ "tensor" perturbations → grav waves.
$\delta\varphi$
$\delta g_{\mu\nu}$


$g_{\mu\nu}, \varphi$
$\gamma_{ij} \to$
$10 \to 2$
$11 \to 3$
"scalar" perturbations → density perts.
"tensor" perturbations → grav waves.
$\delta\varphi = 0$
$\delta g_{\mu\nu} \to \gamma_{ij} = a^2(t)\, e^{2\zeta(\vec{x},t)}\, e^{2h_{ij}(\vec{x},t)}$

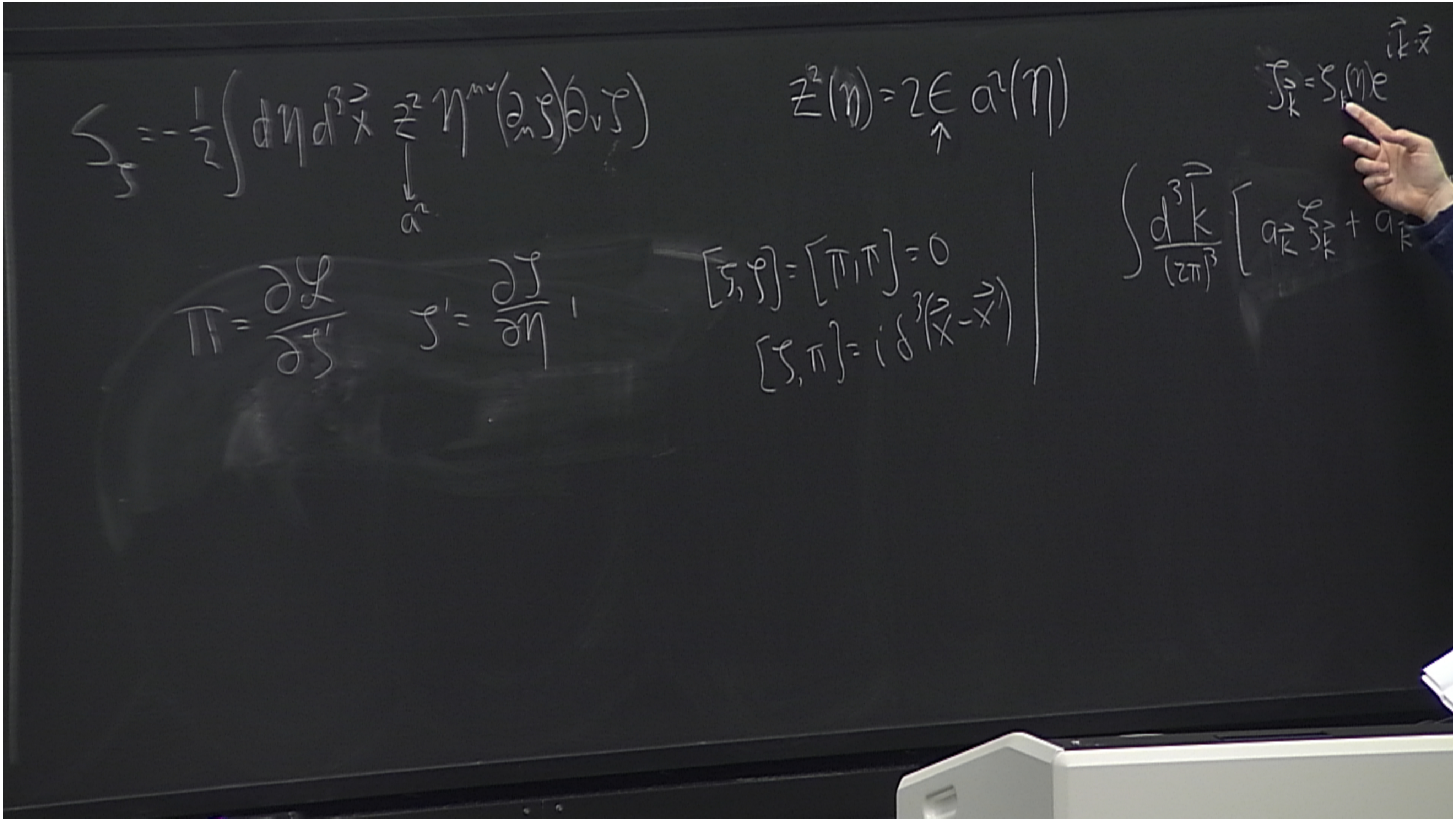


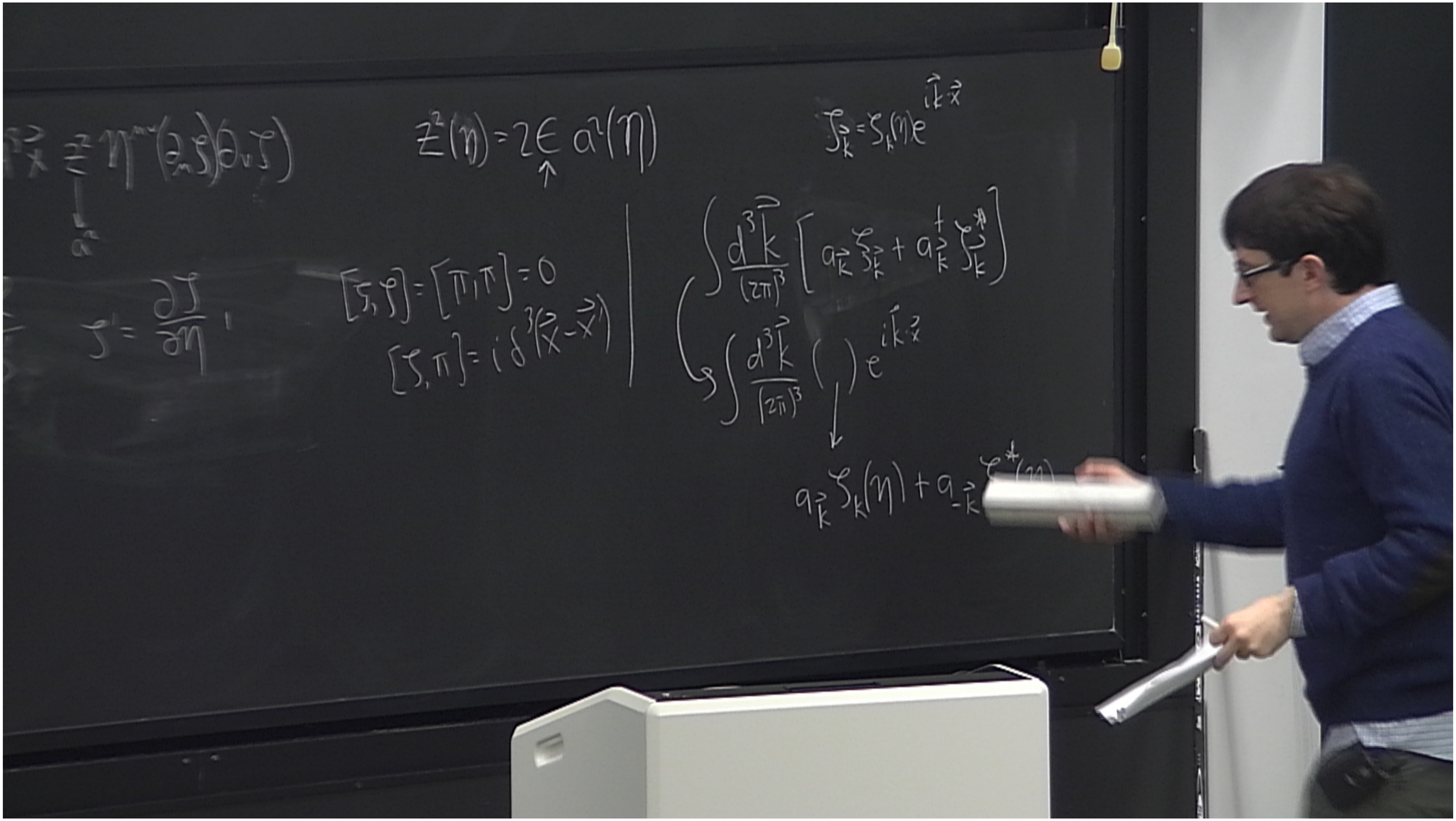
$g_{\mu\nu}, \varphi$
$\gamma_{ij} \rightarrow$
$10 \rightarrow 2$
$11 \rightarrow 3$
"scalar" pertubations → density perts.
"tensor" pertubations → grav waves.
$\delta\varphi = 0$
$\delta g_{\mu\nu} \rightarrow \gamma_{ij} = a^2(t)\, e^{2\zeta(\vec{x},t)}\, e^{2h_{ij}(\vec{x},t)}$
density
gws
$h_{ij}\delta^{ij} = 0$
$\vec{k}^i h_{ij} = 0$

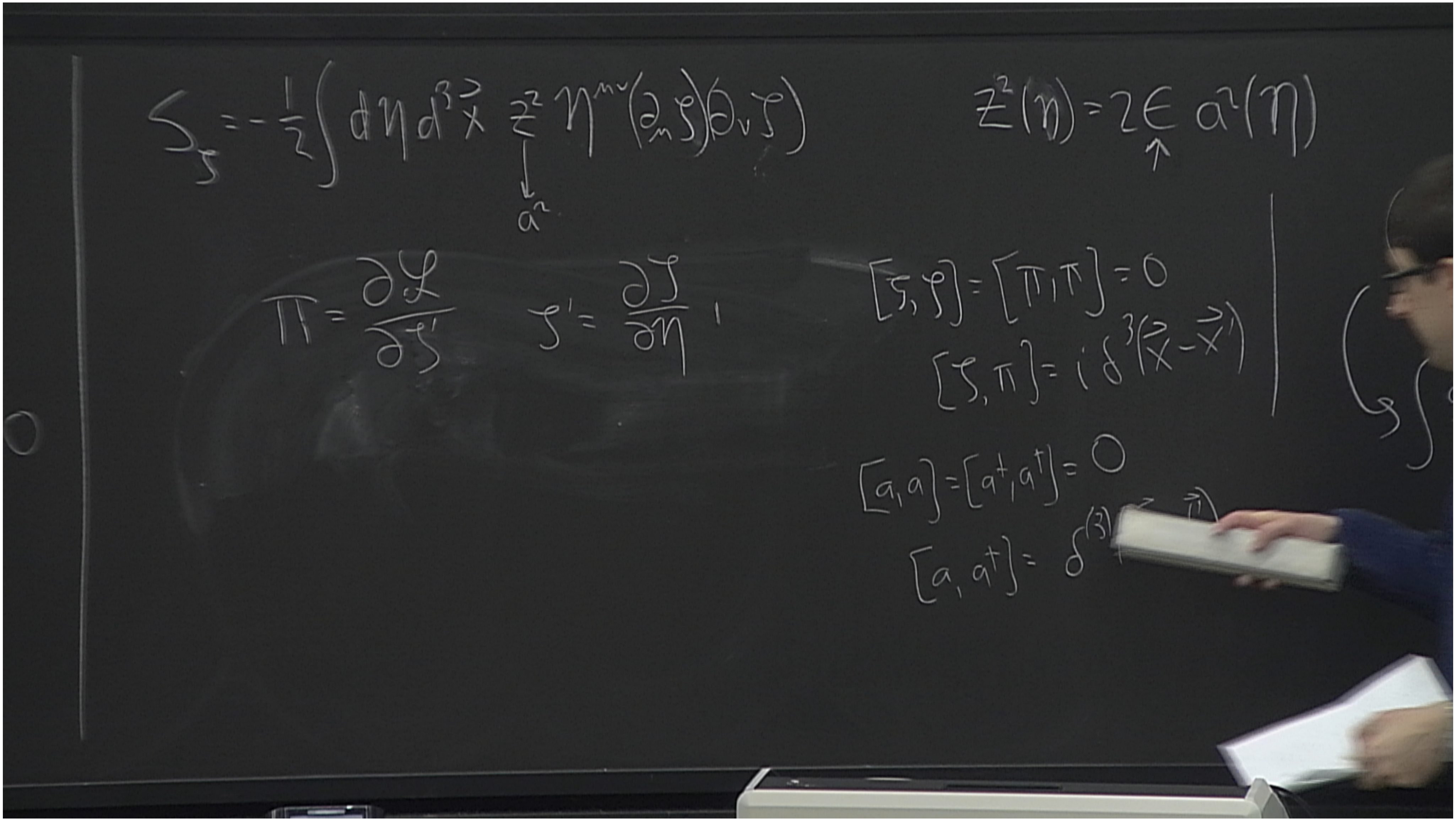

$g_{\mu\nu}, \varphi$
$\gamma_{ij} \to$
$10 \to 2$
$11 \to 3$
"scalar" perturbations → density perts.
"tensor" perturbations → grav waves.
$\delta\varphi = 0$
$\delta g_{\mu\nu} \to \gamma_{ij} = a^2(t) e^{2\zeta(\vec{x},t)} e^{2h_{ij}(\vec{x},t)}$
density
gws
$h_{ij}\delta^{ij} = 0$
$\vec{k}^i h_{ij} = 0$



$S_\zeta = -\frac{1}{2}\int d\eta\, d^3\vec{x}\, z^2 \eta^{\mu\nu}(\partial_\mu \zeta)(\partial_\nu \zeta)$
a^2
$\int d^4x \sqrt{-g}\, g^{\mu\nu}(\partial_\mu \zeta)(\partial_\nu \zeta)$
$z^2(\eta) = 2\epsilon\, a^2(\eta)$
$\delta^{ij} = 0$
$\vec{k}\, h_{ij} = 0$


$S_\zeta = -\frac{1}{2}\int d\eta\, d^3\vec{x}\, z^2 \eta^{\mu\nu}(\partial_\mu \zeta)(\partial_\nu \zeta)$
$z^2 \to a^2$
$z^2(\eta) = 2\epsilon\, a^2(\eta)$
$\zeta_{\vec{k}} = \zeta_k(\eta) e^{i\vec{k}\cdot\vec{x}}$
$\pi = \frac{\partial \mathcal{L}}{\partial \zeta'}$ $\zeta' = \frac{\partial \zeta}{\partial \eta}$
$[\zeta, \zeta] = [\pi, \pi] = 0$
$[\zeta, \pi] = i\,\delta^3(\vec{x} - \vec{x}')$
$\int \frac{d^3\vec{k}}{(2\pi)^3}\left[a_{\vec{k}} \zeta_{\vec{k}} + a_{\vec{k}}\right.$


$z^2(\eta) = 2\epsilon\, a^2(\eta)$
$\zeta_{\vec{k}} = \zeta_k(\eta) e^{i\vec{k}\cdot\vec{x}}$
$[\zeta, \zeta] = [\pi, \pi] = 0$
$[\zeta, \pi] = i\delta^3(\vec{x} - \vec{x}')$
$\zeta' = \frac{\partial \zeta}{\partial \eta}$
$\int \frac{d^3\vec{k}}{(2\pi)^3} [a_{\vec{k}} \zeta_{\vec{k}} + a^\dagger_{\vec{k}} \zeta^*_{\vec{k}}]$
$\int \frac{d^3\vec{k}}{(2\pi)^3} (\ ) e^{i\vec{k}\cdot\vec{x}}$
$a_{\vec{k}} \zeta_k(\eta) + a_{-\vec{k}}$



$S_\zeta = -\frac{1}{2}\int d\eta\, d^3\vec{x}\; z^2\, \eta^{\mu\nu}(\partial_\mu \zeta)(\partial_\nu \zeta)$
a^2
$z^2(\eta) = 2\epsilon\, a^2(\eta)$
$\Pi = \frac{\partial \mathcal{L}}{\partial \zeta'}$
$\zeta' = \frac{\partial \zeta}{\partial \eta}$
$[\zeta, \zeta] = [\Pi, \Pi] = 0$
$[\zeta, \Pi] = i\,\delta^3(\vec{x} - \vec{x}')$
$[a, a] = [a^\dagger, a^\dagger] = 0$
$[a, a^\dagger] = \delta^{(3)}$

$$S_\zeta = -\frac{1}{2}\int d\eta\, d^3\vec{x}\; z^2\, \eta^{\mu\nu}(\partial_\mu \zeta)(\partial_\nu \zeta)$$

$z^2 \to a^2$

$$z^2(\eta) = 2\epsilon\, a^2(\eta)$$

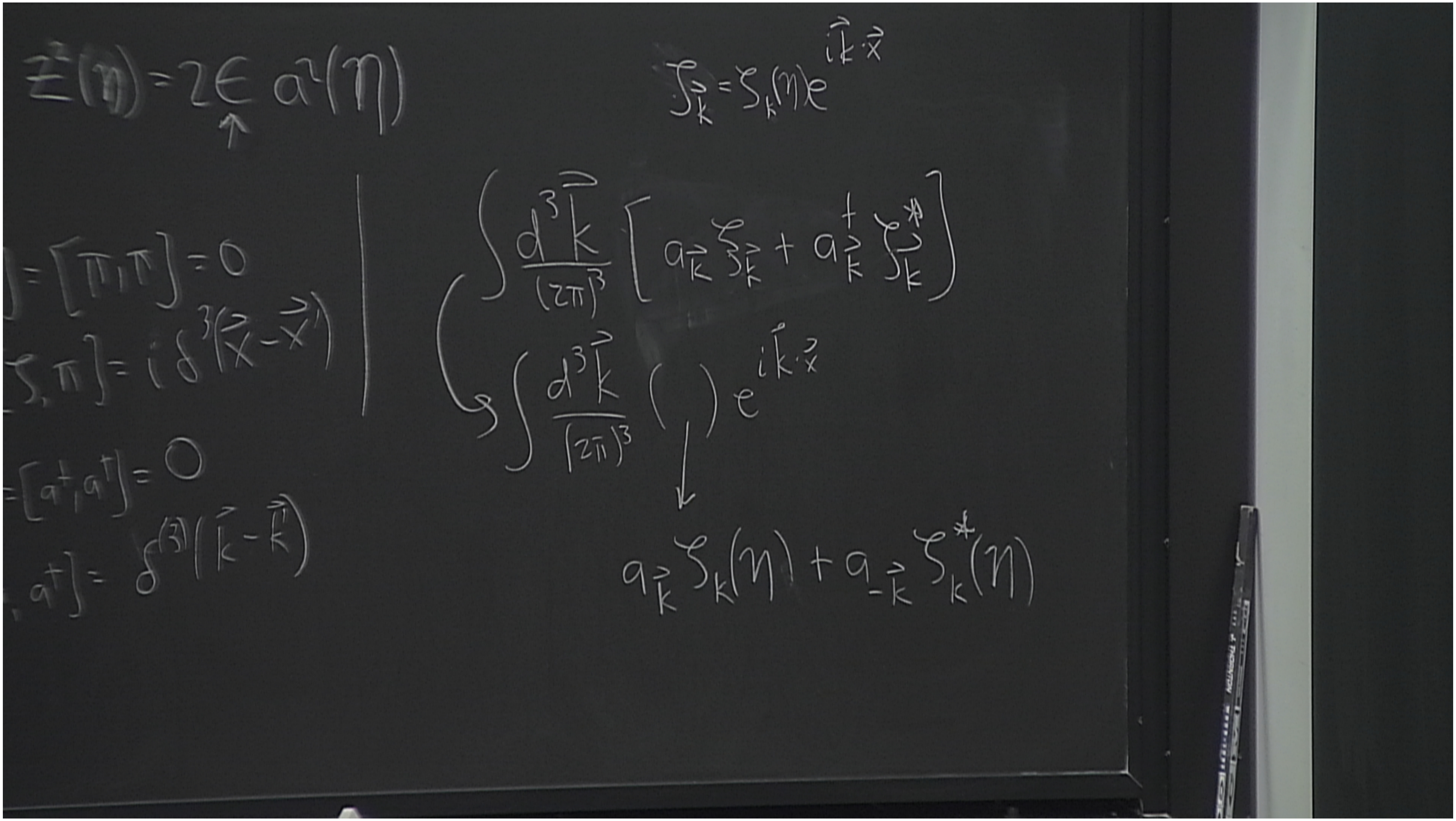
$$\Pi = \frac{\partial \mathcal{L}}{\partial \zeta'} \qquad \zeta' = \frac{\partial \zeta}{\partial \eta}$$

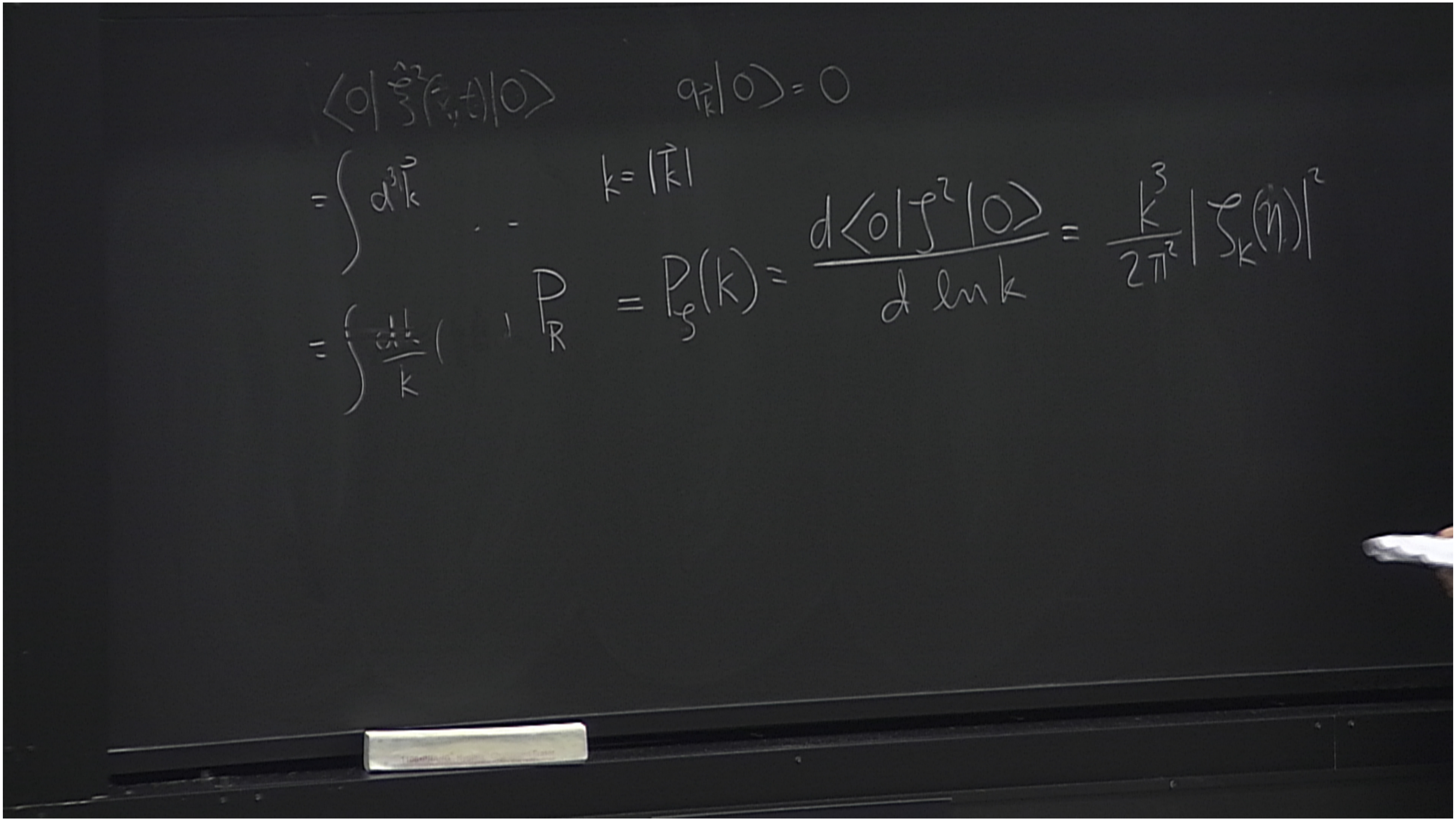
$$[\zeta, \zeta] = [\Pi, \Pi] = 0$$

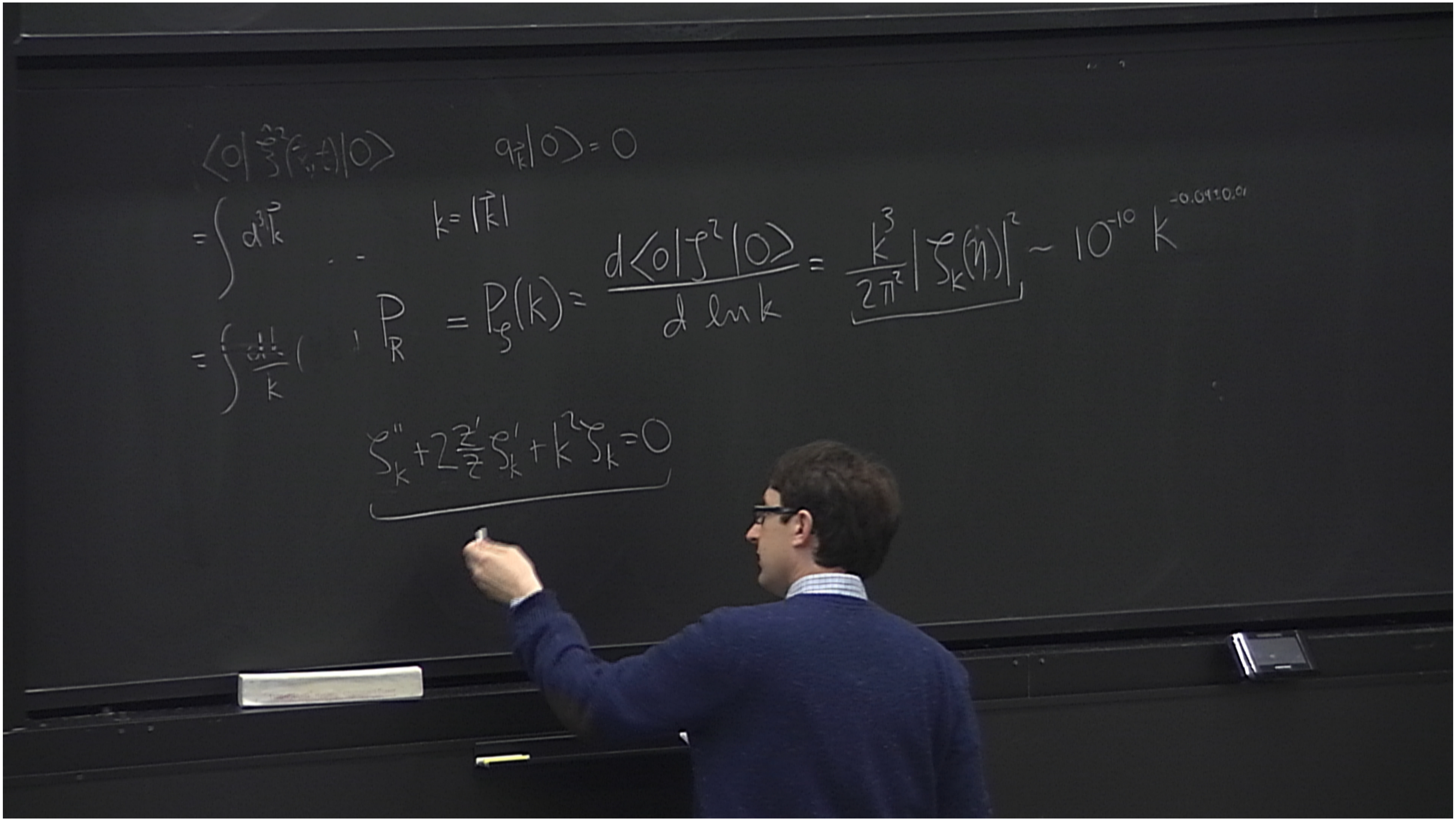
$$[\zeta, \Pi] = i\,\delta^3(\vec{x} - \vec{x}')$$

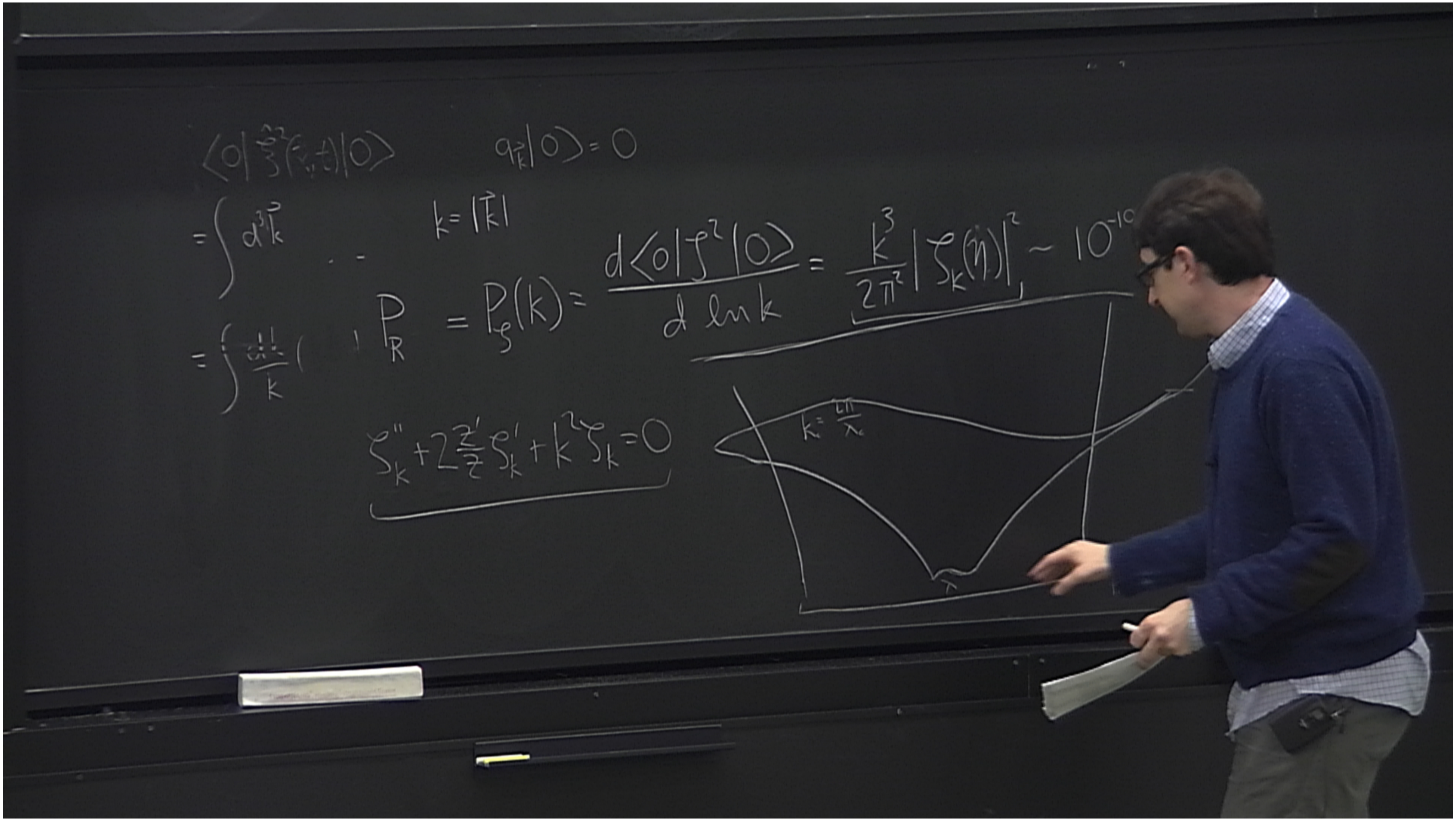
$$[a, a] = [a^\dagger, a^\dagger] = 0$$

$$[a, a^\dagger] = \delta^{(3)}(\vec{k} - \vec{k}')$$

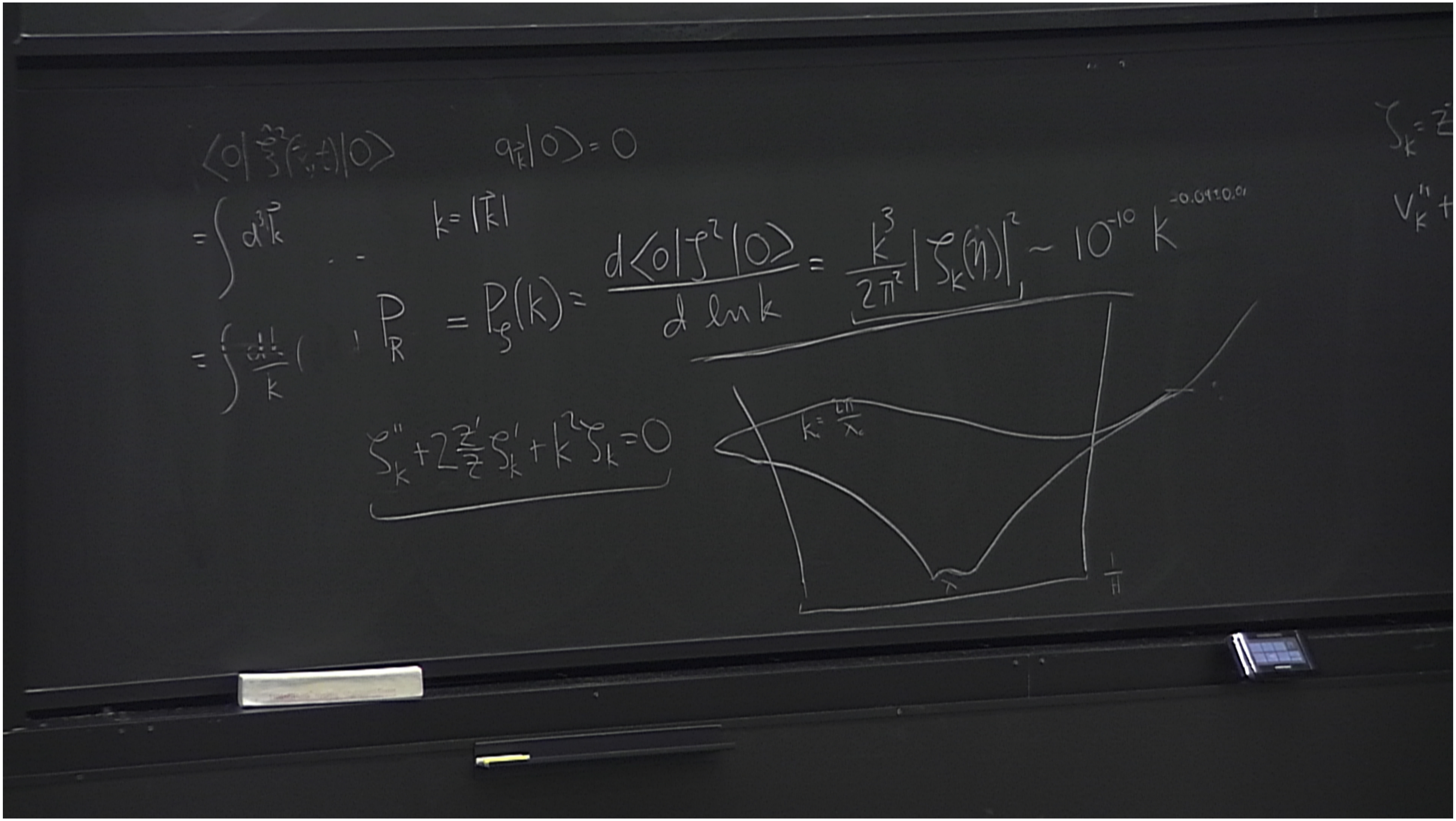

z^2(η) = 2ε a^2(η)
ζ_k = ζ_k(η) e^{i k·x}
= [π, π] = 0
ζ, π] = i δ^3(x − x')
= [a^†, a^†] = 0
, a^†] = δ^(3)(k − k')
∫ d^3k/(2π)^3 [a_k ζ_k + a_k^† ζ_k^*]
∫ d^3k/(2π)^3 ( ) e^{i k·x}
a_k ζ_k(η) + a_{−k} ζ_k^*(η)

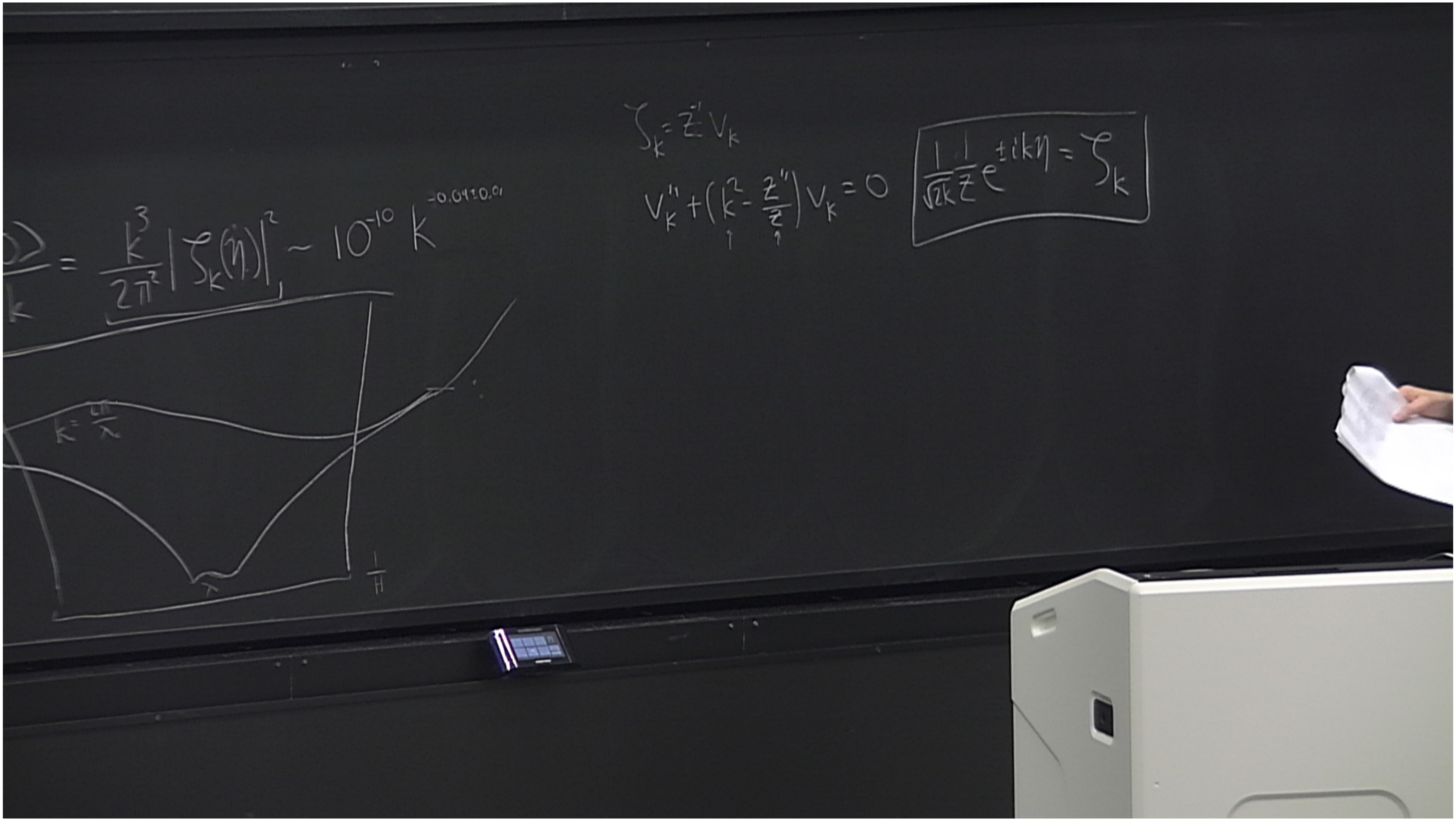

⟨0| ζ̂²(x, t) |0⟩
a_k |0⟩ = 0
= ∫ d³k . . .
k = |k|
= ∫ dk/k ( ) P_R = P_ζ(k) = d⟨0|ζ²|0⟩ / d ln k = k³/2π² |ζ_k(η)|²


k = |k|
P_R = P_ζ(k) = d⟨0|ζ²|0⟩ / d ln k = k³/2π² |ζ_k(η)|² ~ 10⁻¹⁰ k^(−0.04±0.01)
ζ_k'' + 2 z'/z ζ_k' + k² ζ_k = 0

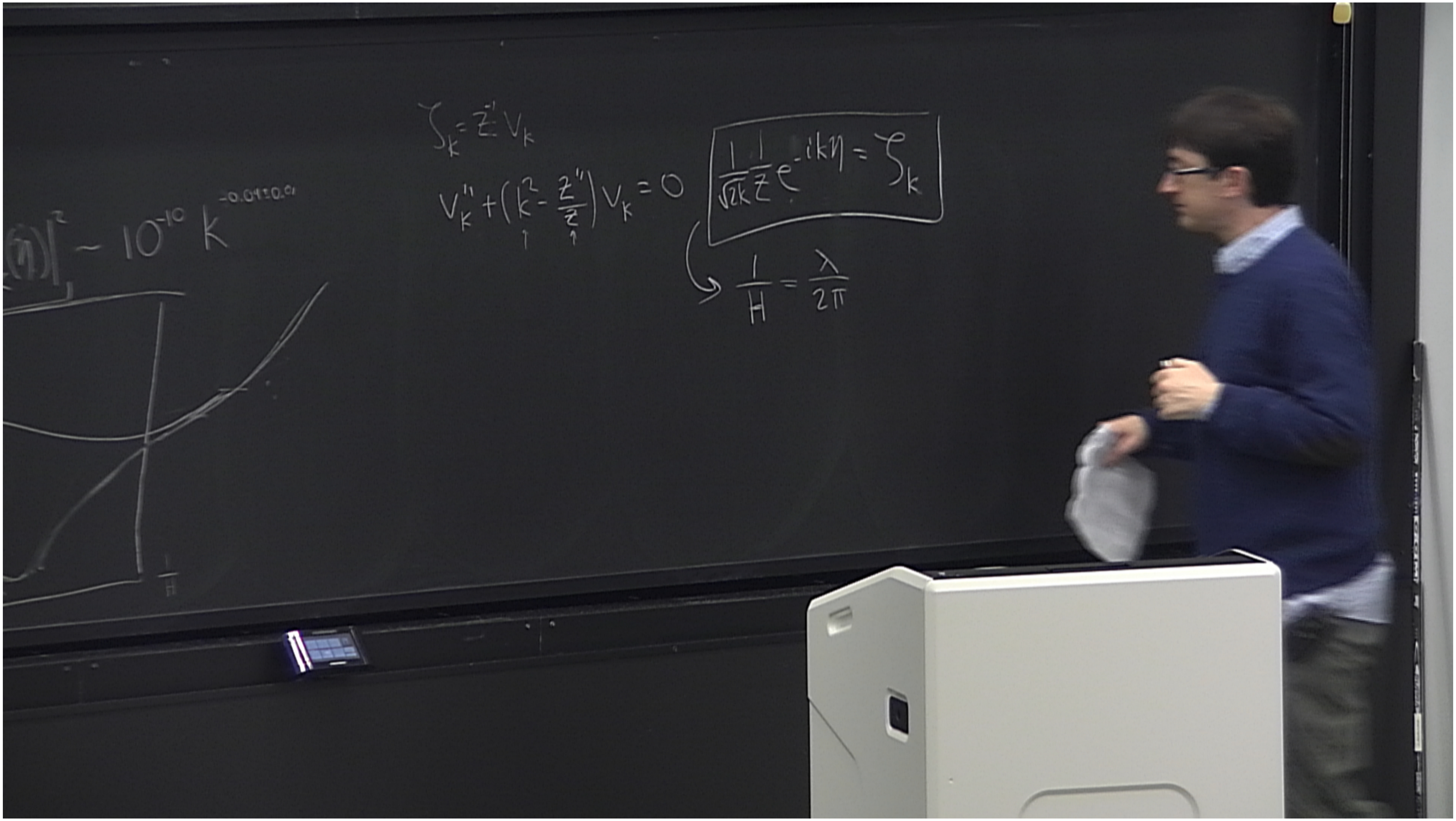


k = |k|
ζ''_k + 2 z'/z ζ'_k + k^2 ζ_k = 0

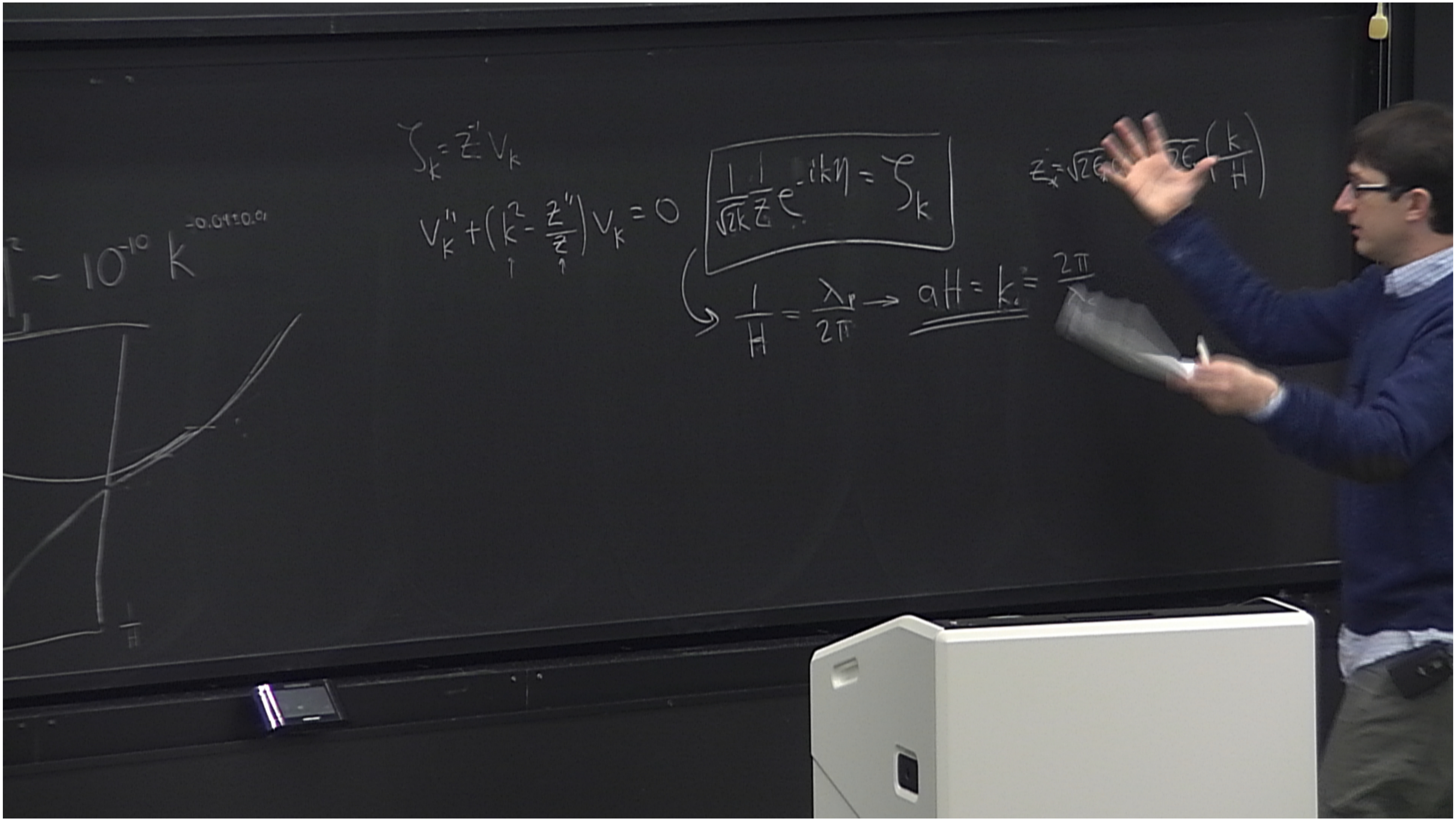

⟨0|ζ²(x, t)|0⟩
a_k|0⟩ = 0
k = |k|
= ∫ d³k
d⟨0|ζ²|0⟩ / d ln k = k³/(2π²) |ζ_k(η)|² ~ 10⁻¹⁰ k^(−0.04±0.01)
P_R = P_ζ(k) =
ζ_k'' + 2 (z'/z) ζ_k' + k² ζ_k = 0

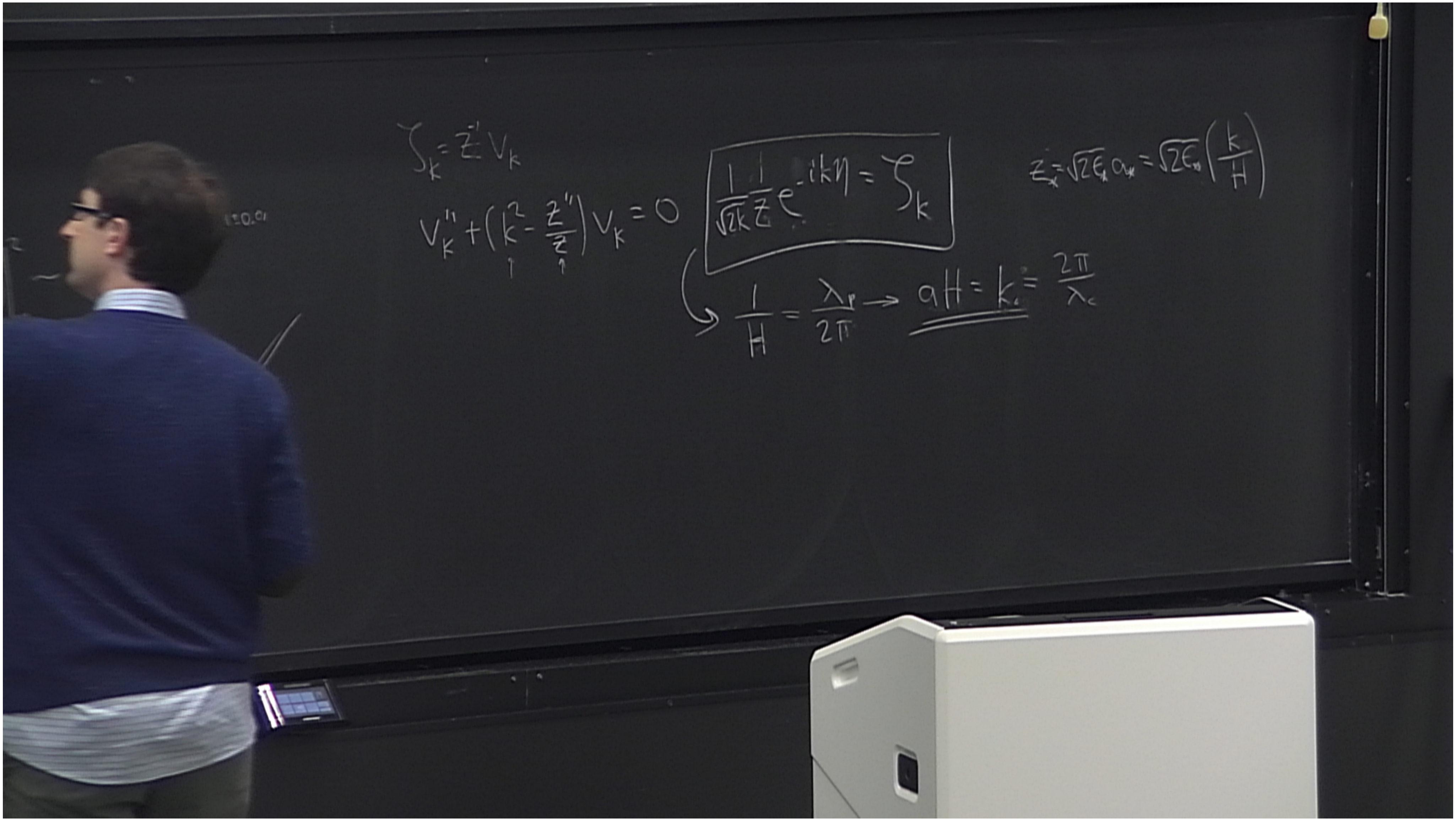


ζ_k = z^{-1} v_k
v_k'' + (k^2 - z''/z) v_k = 0
1/√(2k) 1/z e^{±ikη} = ζ_k
= k^3/(2π^2) |ζ_k(η)|^2 ~ 10^{-10} k^{-0.04±0.01}
k = 2π/λ
1/H



$\zeta_k = z^{-1} v_k$
$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$
$\frac{1}{\sqrt{2k}} \frac{1}{z} e^{-ik\eta} = \zeta_k$
$\frac{1}{H} = \frac{\lambda}{2\pi}$
$\sim 10^{-10}\, k^{-0.04 \pm 0.01}$


$\zeta_k = z^{-1} v_k$
$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$
$\frac{1}{\sqrt{2k}} \frac{1}{z} e^{-ik\eta} = \zeta_k$
$\frac{1}{H} = \frac{\lambda_p}{2\pi} \rightarrow aH = k_* = \frac{2\pi}{\lambda}$
$\sim 10^{-10} k^{-0.04 \pm 0.01}$



$\zeta_k = z^{-1} v_k$
$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$
$\frac{1}{\sqrt{2k}}\frac{1}{z} e^{-ik\eta} = \zeta_k$
$z_* = \sqrt{2\epsilon_*}\, a_* = \sqrt{2\epsilon_*}\left(\frac{k}{H}\right)$
$\frac{1}{H} = \frac{\lambda_p}{2\pi} \rightarrow aH = k_c = \frac{2\pi}{\lambda_c}$