Title: Ã€ la recherche du temps perdu - Templeton Frontiers Colloquia

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Abstract: Augustine of Hippo declared he knew what time is until someone asked him. After 16 centuries we still largely ignore the true essence of time, but we made definite progress in studying its properties. The most striking, and somewhat intuitively (and tragically) obvious one is the irreversibility of its flow. And yet, our fundamental theories are time-reversal invariant, they do not distinguish between past and future. This is usually accounted for by assuming an immensely special initial condition of the Universe, dressed with statistical arguments. In this talk, I will show how an irreversible behaviour, and with it a growth of complexity and information, can emerge from time-reversal invariant laws, without assuming any special initial condition and without thermodynamical arguments. This phenomenon is present in General Relativity, our most advanced theory of the Universe. This irreversibility will also allow me to propose a solution of another puzzle associated with time, namely the incompatibility between the quantum and the general-relativistic notions of time.

À LA RECHERCHE DU TEMPS PERDU

Flavio Mercati

Perimeter Institute for Theoretical Physics

arXiv:1302.6264 - arXiv:1310.5167

in collaboration with

Julian Barbour - University of Oxford

&

Tim Koslowski - University of New Brunswick

WHAT'S WRONG WITH TIME?

- GR time $\neq$ QFT time $\rightarrow$ $\begin{cases} \text{Many-fingered time} \\ \text{Frozen-formalism problem} \end{cases}$

- Past $\neq$ Future + Time-reversal symmetric laws $\rightarrow$ Arrow of Time problem

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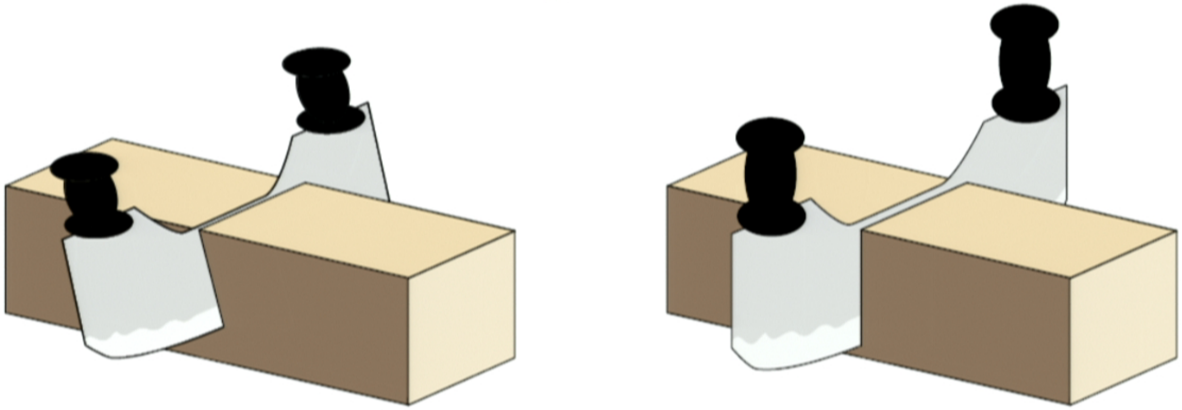
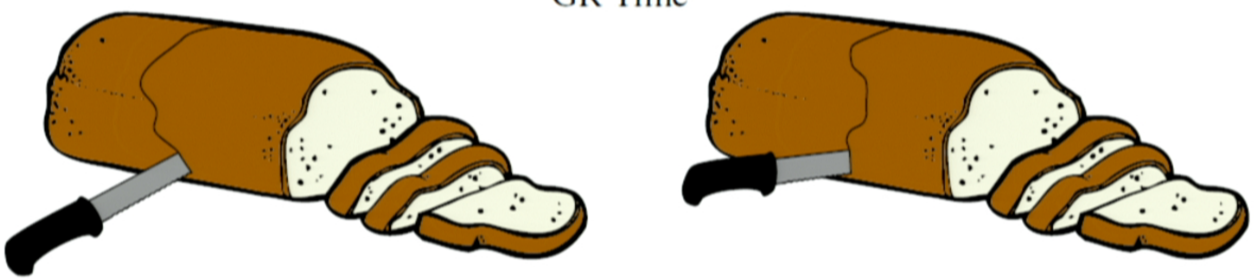
'MANY-FINGERED' TIME

SR/QFT Time



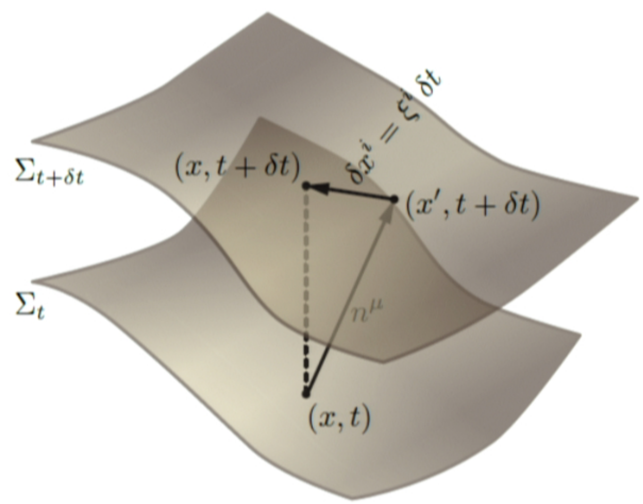
GR Time



Arnowitt–Deser–Misner formalism

$$^{(4)}g_{\mu\nu}(x,t) \to \left\{ g_{ij}(x,t),\, \xi^k(x,t),\, N(x,t) \right\}$$

$$\begin{pmatrix} ^{(4)}g_{00} & ^{(4)}g_{0k} \\ ^{(4)}g_{k0} & ^{(4)}g_{ij} \end{pmatrix} = \begin{pmatrix} g_{ij}\xi^i\xi^j - N^2 & \xi_k \\ \xi_k & g_{ab} \end{pmatrix}$$



ARNOWITT–DESER–MISNER FORMALISM

$$S_{\text{EH}} = \int d^4x \sqrt{g} \left(N\, R + K_{ij} K^{ij} - K^2 \right) , \qquad K_{ij} = \frac{1}{2N} \left(\pounds_\xi g_{ij} - \frac{dg_{ij}}{dt} \right)$$

where $\pounds_\xi g_{ij} = \nabla_i \xi_j + \nabla_j \xi_i$. No time derivatives of $N,\ \xi^i$ $\Rightarrow$ Lagrange multipliers

Canonical momenta:

$$p^{ij} = \frac{\delta \mathscr{L}}{\delta \left(\frac{dg_{ij}}{dt} \right)} = \sqrt{g} \left(K\, g^{ij} - K^{ij} \right) , \qquad \{ g_{ij}(x), p^{kl}(y) \} = \delta^{ij}_{kl}\, \delta(x-y)$$

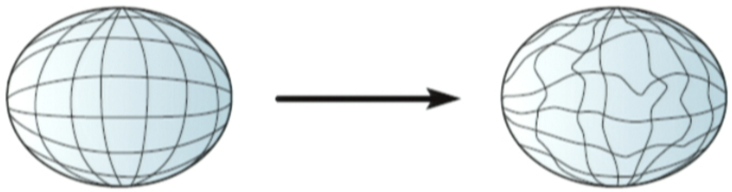
$$S_{\text{EH}} = \int dt \int d^3x \left\{ N \left[\sqrt{g} R - \frac{1}{\sqrt{g}} \left(p^{ij} p_{ij} - \frac{1}{2} p^2 \right) \right] - \xi_i 2 \nabla_j p^{ij} \right\}$$

Constraints:

$$\frac{\delta \mathscr{L}}{\delta N} = \sqrt{g} R - \frac{1}{g} \left(p^{ij} p_{ij} - \frac{1}{2} p^2 \right) \approx 0 , \qquad \frac{\delta \mathscr{L}}{\delta \xi_i} = -2 \nabla_j p^{ij} \approx 0$$

THE DIFFEOMORPHISM CONSTRAINT

$\mathscr{H}^i = -2\nabla_j p^{ij}$ generates point transformations $\left(g'_{ij} = g_{ij} + \pounds_\xi g_{ij}\right)$ 3-diffeomorphisms, or coordinate transformations.



Clear geometrical interpretation.

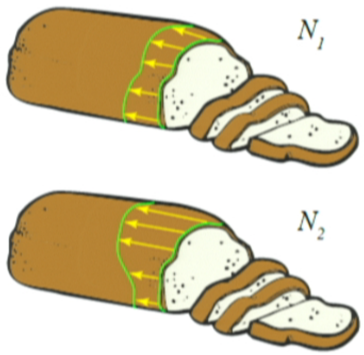
Easy(-er) to implement at the quantum level: $\Psi[g_{ij} + \pounds_\xi g_{ij}] = \Psi[g_{ij}]$.

THE HAMILTONIAN CONSTRAINT

$\mathscr{H} = \sqrt{g}\,R - \frac{1}{\sqrt{g}}\left(p^{ij}p_{ij} - \frac{1}{2}p^2\right)$ mixes g_{ij}'s and momenta $\left(g'_{ij} = F[g_{ij}, p^{ij}]\right)$

No geometrical interpretation.

$H_{\text{tot}} = \int d^3x\,(N\,\mathscr{H} + \xi_i\,\mathscr{H})$ generates time evolution by different amounts at different places, as determined by $N(x,t)$.



Quantum version: $\sqrt{g}\,R(x)\,\Psi[g] - \frac{\hbar^2}{\sqrt{g}}\left(g_{ik}g_{jl} - \frac{1}{2}g_{ij}g_{kl}\right) : \frac{\delta^2\Psi[g]}{\delta g_{ij}(x)\,\delta g^{kl}(x)} := 0,$

'Wheeler–DeWitt equation'. Very hard to solve or to make sense of.

The Problem of Observables

P.A.M. Dirac: the (classical) observables of a theory with gauge redundancies should Poisson-commute with the constraints (be invariant under the gauge symmetries).

Uncontroversial for Diffeomorphism constraint
(observables should be diffeo invariant).

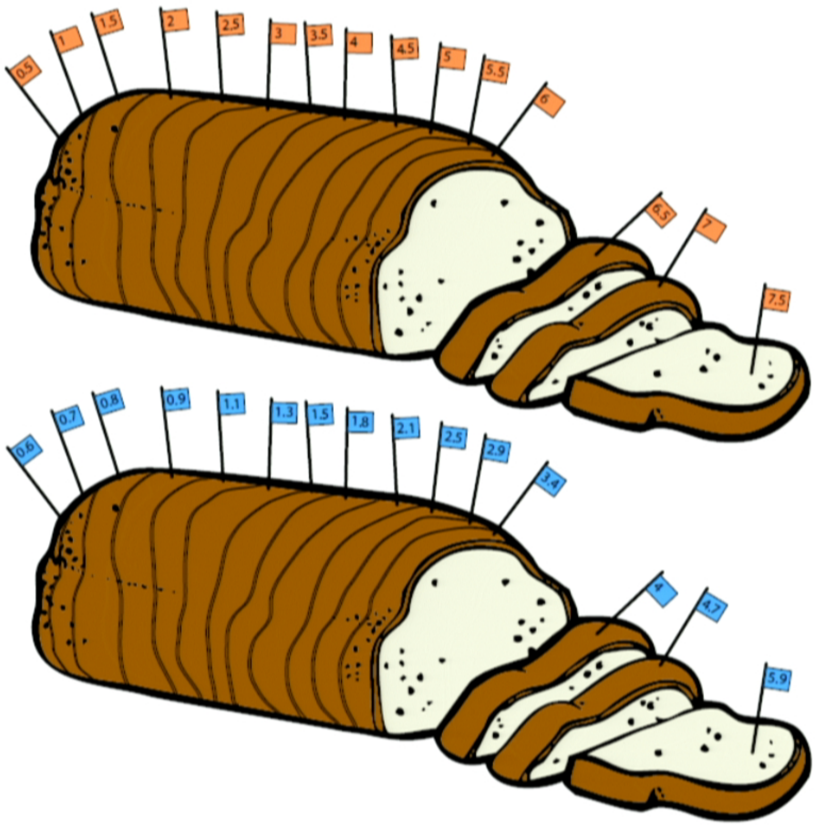
But the Hamiltonian constraint also generates the evolution
(its observables would be "perennials": constants of motion).

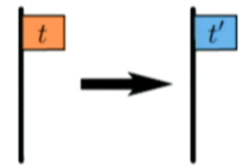
K. Kuchař: Observables should only commute with *linear* constraints.

...but then there are too many (3 degrees of freedom per point).
Don't we expect gravitational waves to have 2 polarizations?

THE 'FROZEN FORMALISM' PROBLEM

Even after fixing the foliation a *global* redundancy remains:





$$t' = t'(t)\,,$$

$$\frac{\partial t'}{\partial t} > 0\,.$$

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Reparametrization invariance

Reparametrization-invariant theories have vanishing canonical Hamiltonian

$$S = \int dt\, \mathscr{L}(q_i, \dot{q}_j) \quad \text{with } \mathscr{L} \text{ homogeneous in } \dot{q}\,,$$

$$\mathscr{L}(\alpha\, \dot{q}_j) = \alpha\, \mathscr{L}(\dot{q}_j) \quad \Rightarrow \quad \sum_i \dot{q}_i \frac{\delta \mathscr{L}}{\delta \dot{q}_i} = \mathscr{L}\,,$$

(Euler's homogeneous function theorem)

$$p^i = \frac{\delta \mathscr{L}}{\delta \dot{q}_i} \quad \Rightarrow \quad H = \sum_i p^i \dot{q}_i - \mathscr{L}(q_j, \dot{q}_k) = 0\,.$$

$H = 0$ is a *primary constraint*, consequence of the form of $p^i = p^i(q, \dot{q})$.

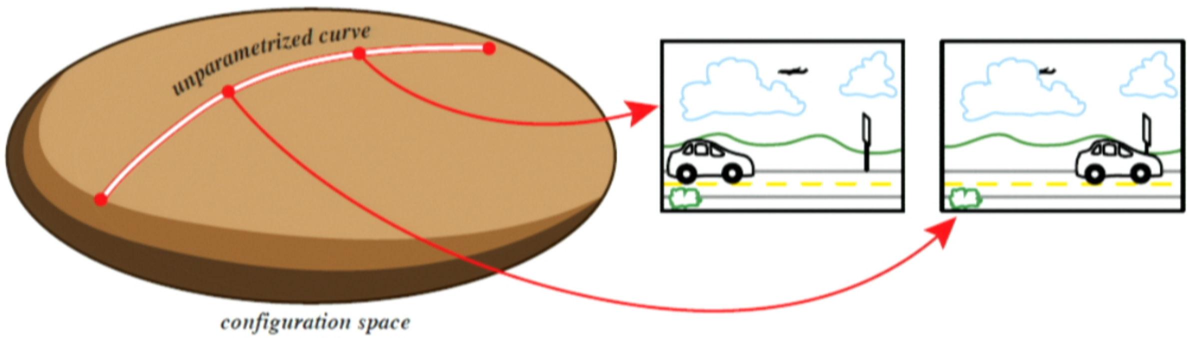
The quantization of a reparametrization-invariant theory gives a *time-independent* Schroedinger equation

$$\hat{H}\, \psi(q) = 0\,.$$

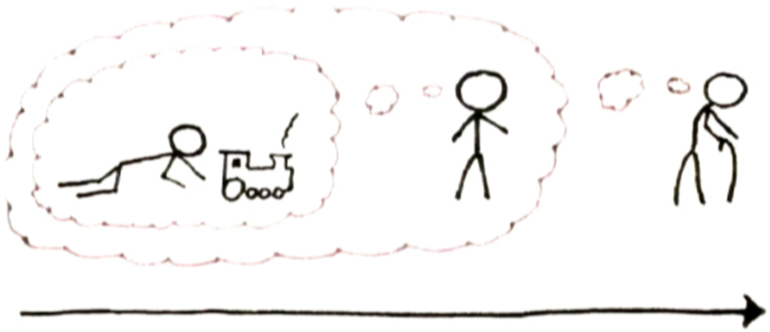
The Wheeler-DeWitt equation is like that (but *local*, with $\hat{H} = \hat{H}(x)$).

TEMPORAL RELATIONALISM

E. Mach: "It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things."



THE ARROW OF TIME PROBLEM



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The laws of physics are time-reversal symmetric, but our universe has a pronounced arrow of time.

Normally explained by assuming the universe started in an ultra-low entropy state.

THE THERMODYNAMICAL ARROW OF TIME

we often see this:

$\longrightarrow$

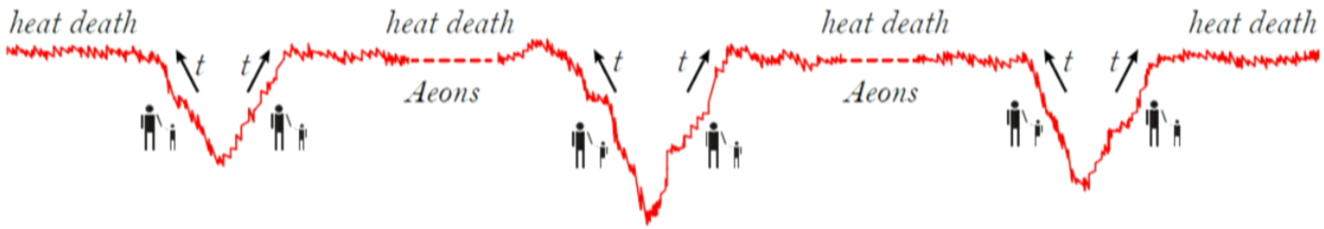
but seldom this:

$\not\longrightarrow$

the explanation is the Second Law of Thermodynamics
and the fact that pots have lower entropy than crack(ed)pots.

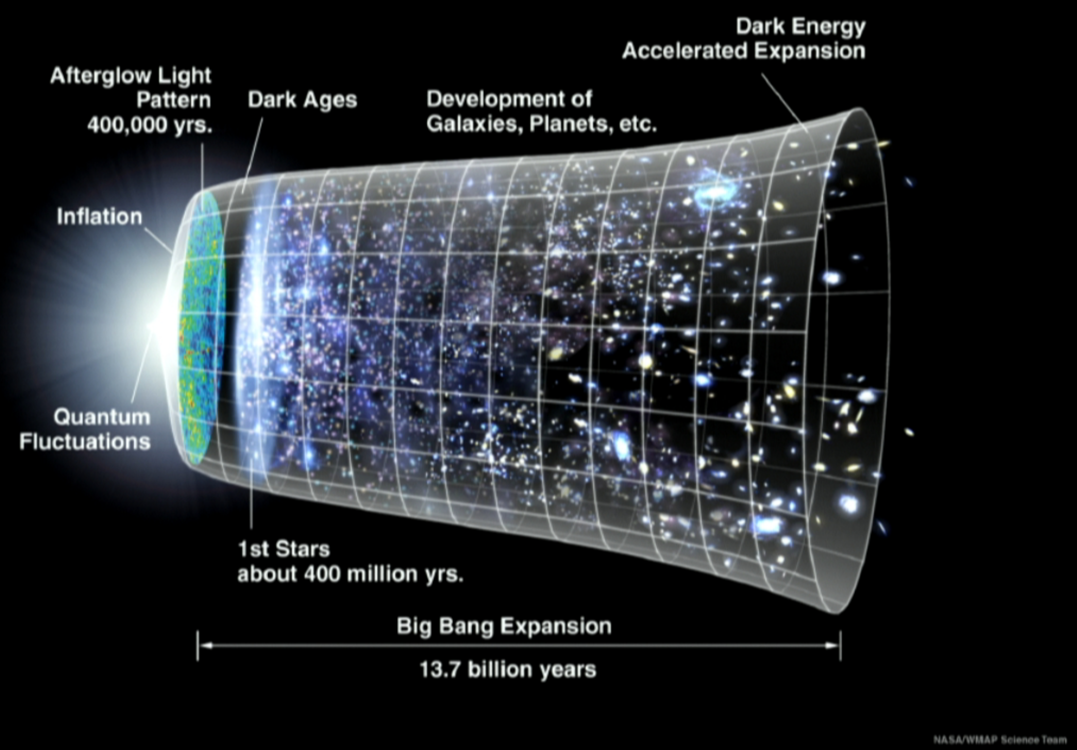
Poincaré recurrence

Boltzmann's resolution of time-reversal symmetry puzzle by Poincaré recurrence:



Boltzmann: The direction of entropy growth defines the direction of time (to heat death)

Eternal recurrence of 'one-past–two-futures' scenario


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
NASA/WMAP Science Team

SHAPE DYNAMICS

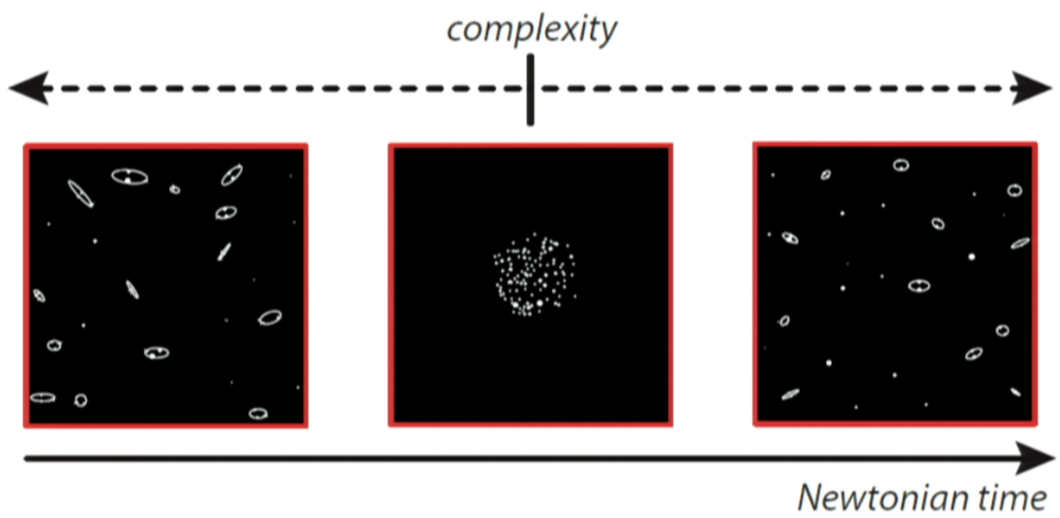
Describes the Universe as a sequence of *shapes* (3D geometries without scale)

Recovers GR and spacetime in a 'CMC' foliation: Constant Mean extrinsic Curvature
the local spatial volume $\sqrt{g}$ expands or contracts by the same amount everywhere.

- Many-fingered time problem absent: work in a preferred foliation.
- Hamiltonian constraint solved before quantization: no Wheeler-DeWitt equation.
- Observables are simple: conformally and diffeo invariant quantities. 2 dofs per point.
- No frozen formalism problem if one uses the rate of expansion as a time.

SD description of Newtonian Universe

N point particles interacting with Newton's potential
No extraneous frame or scale $\vec{J} = 0, \quad \vec{P} = 0, \quad E_{tot} = 0,$



A 'one-past–two-futures' scenario: the two sides look like an expanding universe

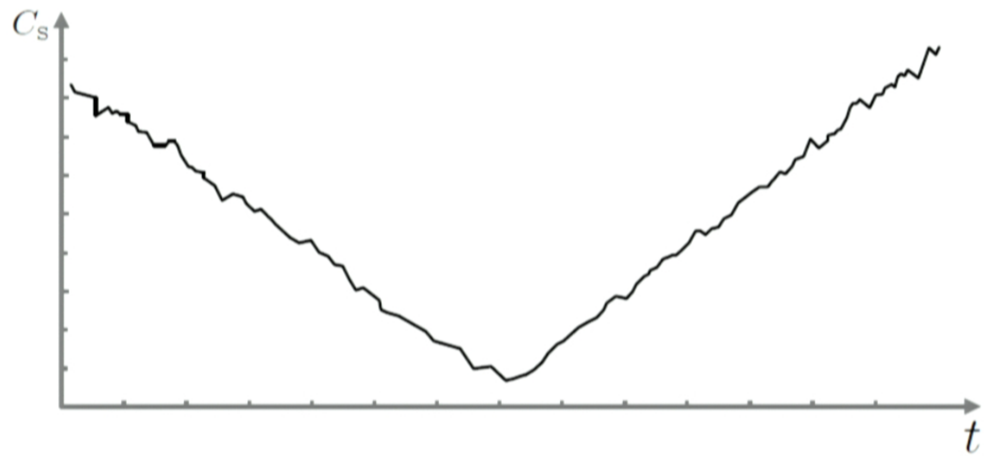
The explanation of this involves discarding the overall scale of the system and describing everything in terms of the *shapes* of the universe.

COMPLEXITY MEASURE

$$-V_{\text{Newton}}/m_{\text{tot}}^2 = \frac{1}{m_{\text{tot}}^2}\sum_{a<b}\frac{m_a m_b}{r_{ab}} = \frac{1}{\ell} \quad \rightarrow \quad \text{'mean harmonic length' } \ell$$

$$I_{\text{cm}}/m_{\text{tot}} = \frac{1}{m_{\text{tot}}^2}\sum_{a<b} m_a m_b\, r_{ab} = L^2 \quad \rightarrow \quad \text{'root mean square length' } L$$

'Complexity' $\boxed{C_{\text{S}} = \frac{L}{\ell}}$ a sensitive measure of clustering



Dynamical Similarity

$$V_{\text{Newton}} = -\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_1 m_2}{r_{23}} + \ldots\right)$$ is homogeneous of degree $k = -1$

$V(\alpha\, \mathbf{r}_a) \to \alpha^k\, V(\mathbf{r}_a)$ $\quad \mathbf{r}_a \to \alpha\, \mathbf{r}_a \quad t \to \alpha^{1-\frac{k}{2}} t$ rescalings are reparametrizations

Lagrange-Jacobi relation

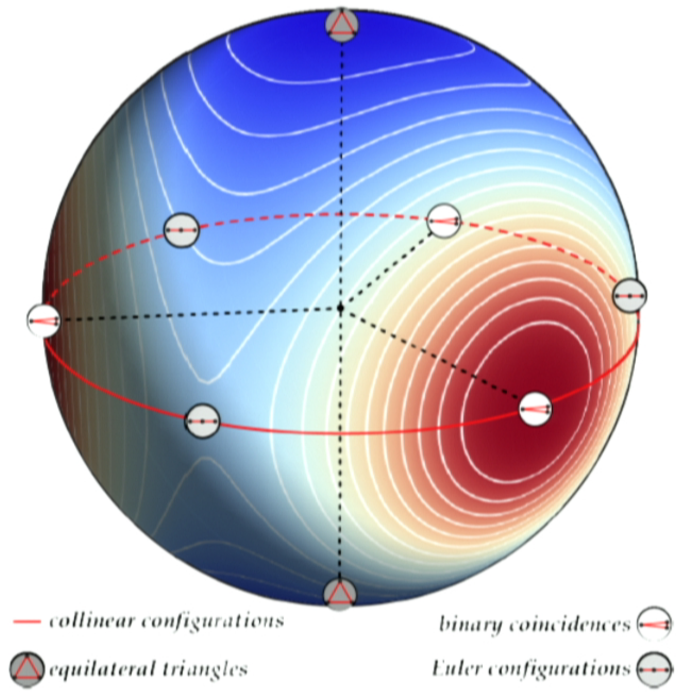
$$\frac{dI_{\text{cm}}}{dt} = 2\,D, \qquad D = \sum_a \mathbf{r}_a \cdot \mathbf{p}^a$$ is the **dilatational momentum**

$$\frac{d^2 I_{\text{cm}}}{dt^2} = 4E - 2(k+2)V \quad \Longrightarrow \quad \frac{d^2 I_{\text{cm}}}{dt^2} = 4\,E - 2\,V_{\text{New}} > 0 \quad \text{if } E \geq 0$$

$\Longrightarrow \boxed{\frac{dD}{dt} > 0}$ $\quad D$ is **monotonic** $\Rightarrow$ can be used as 'time' variable

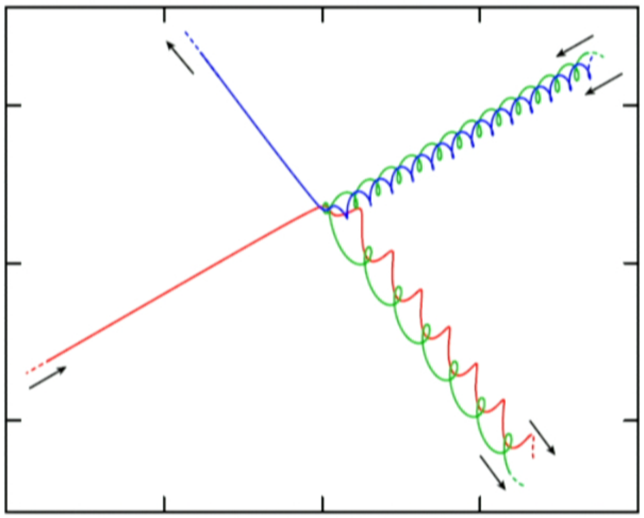
SHAPE SPACE

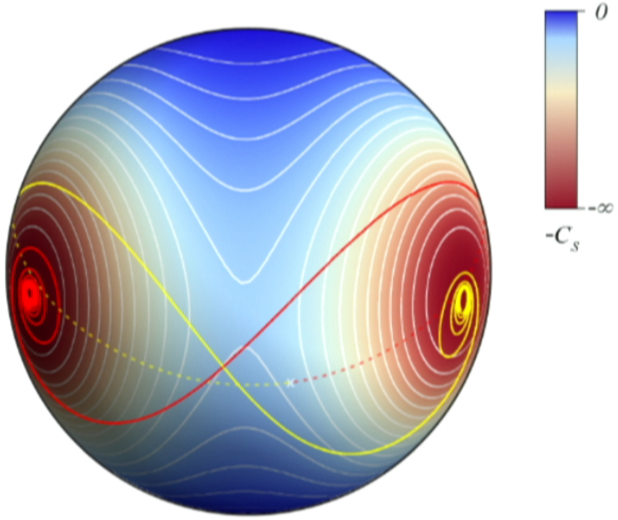
If D is used as time, its conjugate variable (the size of the system) is the Hamiltonian. What remains are the *shape* (scale-invariant) degrees of freedom, forming *shape space:*



If $N = 3$ shape space is the space of triangles. 2 internal angles characterize a triangle: shape space is 2D.

THE GENERIC 3-BODY SOLUTION

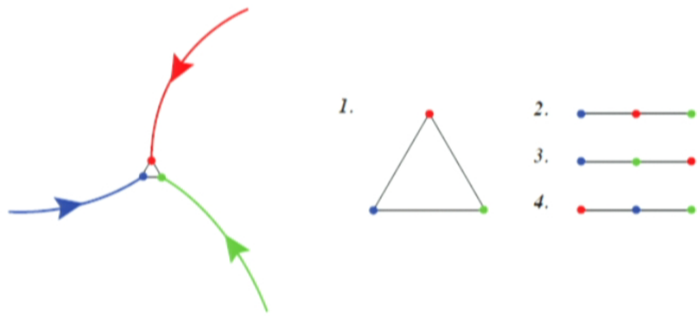




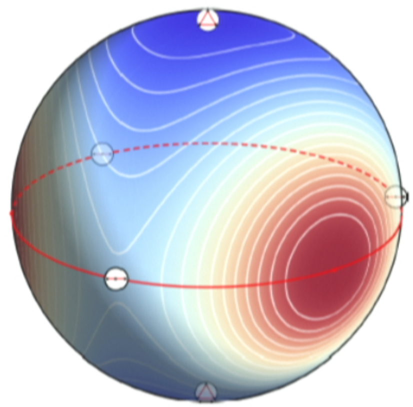
In the SD description, $\boxed{V_S = -C_S}$ acts as a potential on shape space and the dynamics appears dissipative (therefore C_S grows secularly)

THE MEASURE-ZERO 3-BODY SOLUTIONS

A *central collision* (simultaneous collision of the 3 particles) can end up only in one of three special shapes:



These are the equilateral triangle and the *Euler configurations*, which are the only *stationary points* of the complexity function C_s



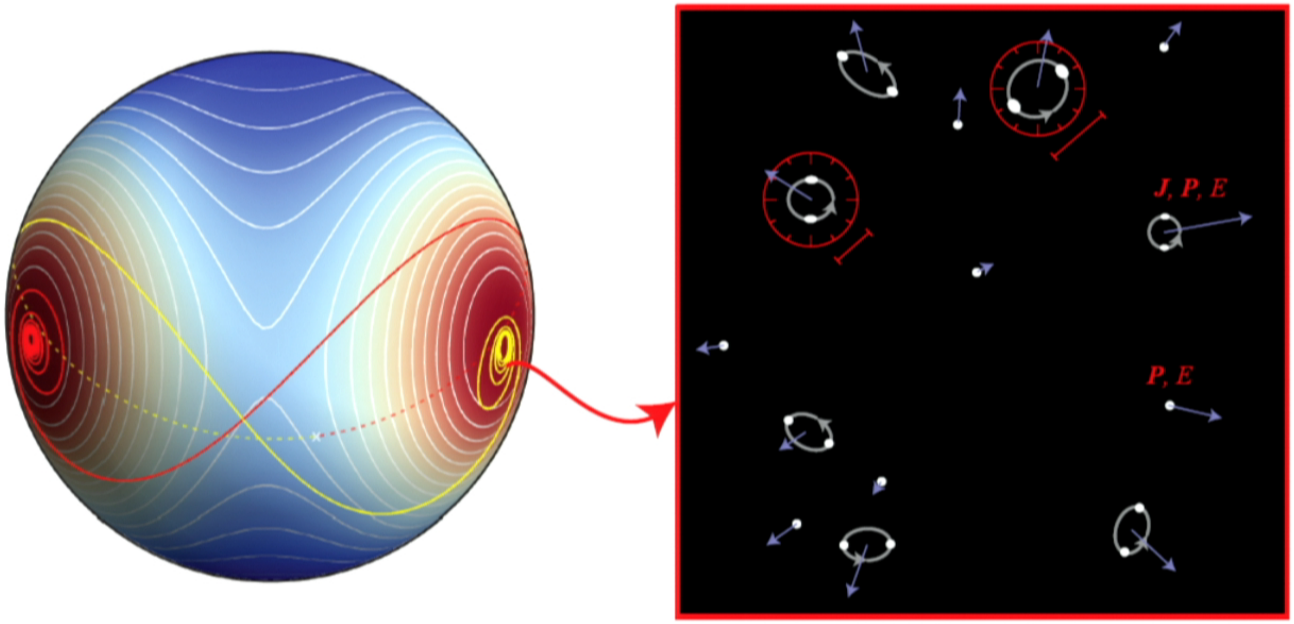
THE EMERGENCE OF RODS AND CLOCKS

"It is striking that the theory (except for four-dimensional space) introduces two kinds of physical things, i.e. (1) measuring rods and clocks, (2) all other things, e.g., the electromagnetic field, material point, etc. This, in a certain sense, is inconsistent; strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations... not, as it were, as theoretically self-sufficient entities."

[A. Einstein, autobiographical notes, 1949]

The Emergence of Rods and Clocks

In the N-body toy model we have a realization of Einstein's theory of rods and clocks.



The Kepler pairs keep mutual congruence like good rods,
their revolutions are mutually isochronous like good clocks.

SUMMARY

- A hint that the arrow of time is explained solely by the form of the dynamical law and not a special initial condition. Established for the N-body problem. Remarkable that the simplest dimensionless measure of complexity is the gravitational shape potential.

- Growth of complexity is accompanied by emergence of rods and clocks.