

Title: Cosmological constraints on complex scalar-field dark matter

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Abstract: The nature of dark matter is a fundamental problem in cosmology and particle physics. Many particle candidates have been devised over the course of the last decades, and are still at stake to be soon discovered or rejected. However, astronomical observations, in conjunction with the phenomenological efforts in astrophysical modeling, as well as in particle theories to explain them, have helped to pin down several key properties which any successful candidate has to have. In this talk, I will explore the possibility that dark matter is described by a complex scalar field (SFDM), while the other cosmic components are treated in the usual way, assuming a cosmological constant for the dark energy. We will see that the background evolution of a Universe with SFDM and a cosmological constant (LSFDM) complies with the concordance LCDM model, if the model parameters of the SFDM Lagrangian, mass and repulsive 2-particle self-interaction coupling strength, are properly constrained by observations of the cosmic microwave background and Big Bang nucleosynthesis (BBN). However, not only does LSFDM lead to non-standard expansion histories prior to BBN, it also exemplifies differences at small scales, which could help to resolve the discrepancies found between LCDM and certain galaxy observations. I will highlight the differences between complex SFDM and dark matter described by real fields, as for instance axion-like particles. If time permits, I will also talk about possible implementations of SFDM in the very early Universe, in the wake of its inflationary phase.



Cosmological Constraints on Complex Scalar Field Dark Matter

(arXiv:1310.6061)

Tanja Rindler-Daller

Dep.of Physics & Michigan Center for Theoretical Physics
U. Michigan, Ann Arbor

with Bohua Li & Paul R. Shapiro

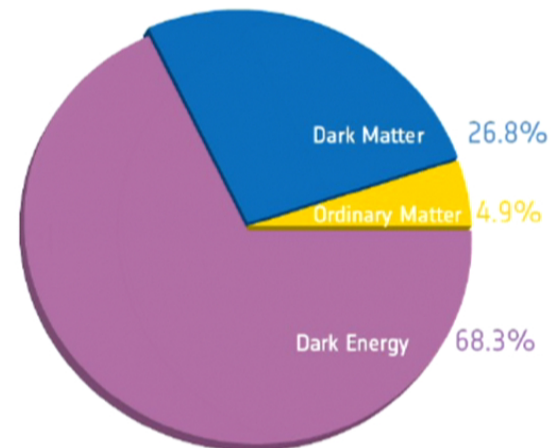
Dep.of Astronomy & Texas Cosmology Center
U. Texas, Austin

Perimeter Institute, Seminar on Cosmology & Gravitation, Feb 4th, 2014

Cosmic inventory at the present epoch

- Dark Energy: cosmological constant (?) $w = P/\rho = -1$
- Dark Matter: cold, “dust-like” $w = 0$
- Radiation (photons + SM neutrinos): $w = 1/3$
- Baryons: cold, “dust-like” $w = 0$

After PLANCK: 6-parameter base
 Λ CDM model remains a best-fit model
“concordance” model of modern
cosmology



„Concordance“: HOWEVER →

If general relativity remains unchallenged:

***What is* Dark Matter and Dark Energy ?**

→ all particle candidates of DM are beyond the standard model and none of them detected

→ DE and its equ. of state even more weird, but observations are currently fine with a cosmological constant Λ (...and its own difficulties)

A unified theory of all fundamental forces should eventually predict DM & DE

DM phenomenology

- Particle theories beyond the SM do predict DM particle candidates
 - WIMPs, axions, sterile neutrinos,....
- Astronomical observations and astrophysical modeling constrain macroscopic properties of DM
 - e.g. ruling out neutrinos as a major DM component
- if in conflict with each other → particle theories try to implement more phenomenology without challenging successes
 - e.g. endowing mediator forces to allow self-interacting DM with masses comparable to WIMPs

If in conflict with each other ...?

The primordial / thermal freeze-in temperature of DM particles eventually determines the minimum size of structures in gravitational (virial) equilibrium

→ observations of the cosmic web & dwarf galaxies require non-relativistic DM - cold dark matter (CDM) without “pressure” on large scales

Collisionless (i.e. non-interacting) CDM along with a nearly scale-independent primordial power spectrum (from inflation) provides a

well-accepted scenario for structure formation:

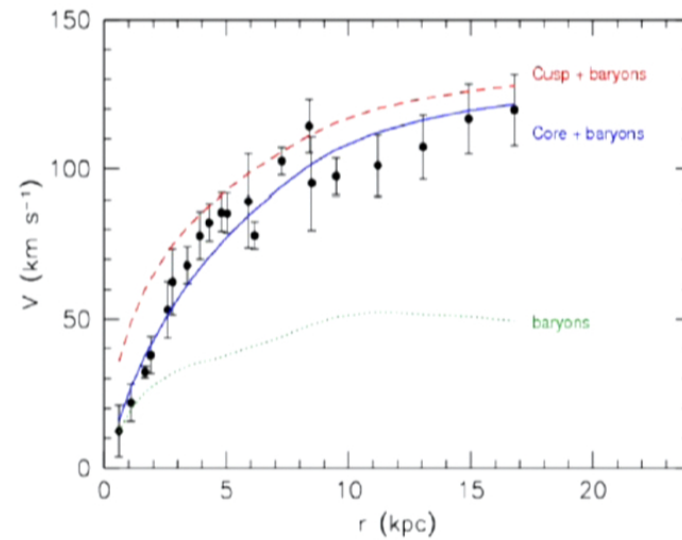
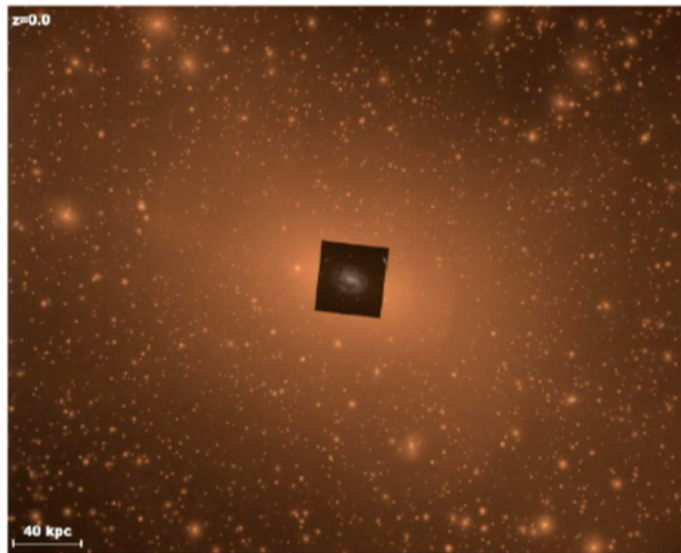
--- the hierarchical clustering of DM fluctuations and the infall of baryons into the CDM potential wells after recombination to form eventually galaxies

→ this story is in good agreement with many observational constraints, including Cosmic Microwave Background (CMB) and large-scale structure surveys

Improved observations and modeling reveal the 'devil' in the details

cosmological N-body simulations:

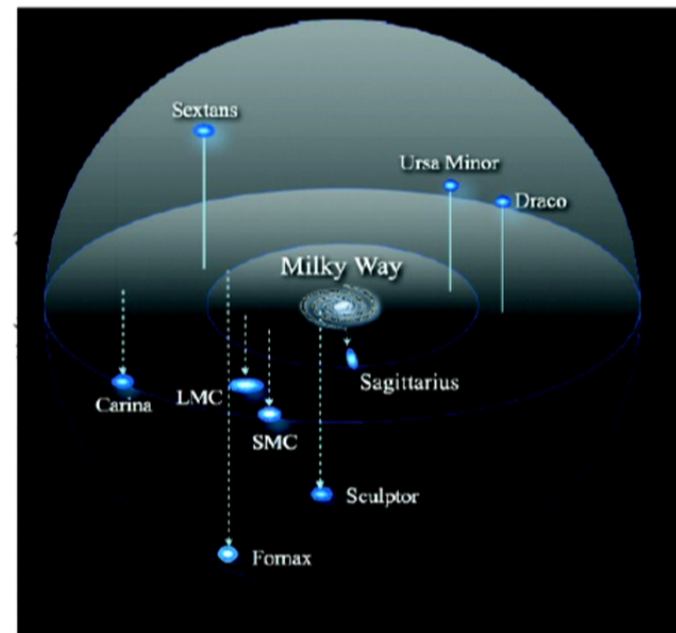
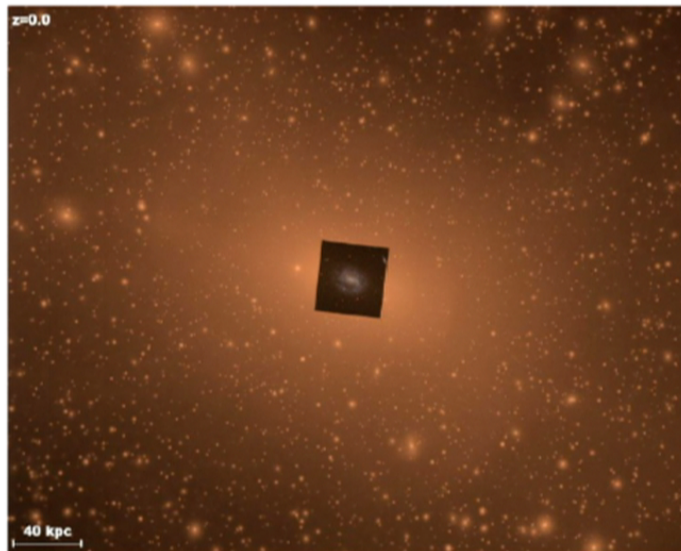
discrepancies between theory and observations on galaxy scales:
predicted density cusps versus observed density cores in galactic
velocity profiles, esp. in DM-dominated galaxies



Improved observations and modeling reveal the 'devil' in the details

cosmological N-body simulations:

discrepancies between theory and observations on galaxy scales:
hundreds of satellite galaxies predicted, while all of the bigger ones
already known



Dark Matter candidates (rough guide):

- WIMPs: **CDM**
 - .) lightest supersymmetric particle; $m \gtrsim (1-1000) \text{ GeV}$
 - .) lightest Kaluza-Klein particle; $m \sim \text{TeV}$
- QCD axion: **CDM**

pseudo-Nambu-Goldstone particle as a solution to the CP problem of the strong force; $m \sim (10^{-6} - 10^{-3}) \text{ eV}$
- sterile neutrinos: **WDM**

right-handed brethren to neutrinos in order to explain their mass; $m \approx \text{keV}$ to be WDM

Dark Matter candidates (rough guide): cont.

- adding DM phenomenology („hidden sector“):
self-interacting DM; asymmetric DM; composite DM ($m \gtrsim \text{GeV}$)
→ CDM but with notable differences to explain small-scale discrepancies
- **ultra-light bosons** (some 'axion-like', some not): $m \sim (10^{-33} - 10^{-15}) \text{ eV}/c^2$
(described by *scalar fields*, even on galactic scales)
- Mixed DM models: CDM + WDM; CDM + axions;

- ultra-light DM bosons described by (coherent) scalar field SF, even on galactic scales !
 - a quadratic term in the SF potential makes them CDM
 - provide natural cutoff scale for clustering in the Universe

e.g. $m \sim 10^{-22} \text{ eV}/c^2 \leftrightarrow R \sim \lambda_{\text{deB}} \sim 1 \text{ kpc}$
(no self-interaction)

$m \gg 10^{-22} \text{ eV}/c^2 \leftrightarrow \lambda_{\text{deB}} \ll R \sim 1 \text{ kpc}$
(high self-interaction 'pressure')

→ attracts attention of astrophysicists

Astrophysics literature on scalar field dark matter

(excluding the QCD axion)

Khlopov, Malomed & Zeldovich (1985); Tkachev (1986); Ratra & Peebles(1988)
Press, Ryden & Spergel (1990); Sin (1994); Schunck (1994); Lee & Koh (1995)
Vilenkin and Peebles (1999); Hu, Barkana & Gruzinov (2000)
Goodman (2000); Peebles (2000); Riotto & Tkachev (2000)
Guzmán & Matos (2000); Barcelo, Liberati & Visser (2001)
Matos & Ureña-López (2001); Boyle, Caldwell & Kamionkowski (2002)
Guzmán & Ureña-López (2003); Arbey, Lesgourges & Salati (2002, 2003)
Short & Coles (2006); Fukuyama, Morikawa & Tatekawa (2008)
Woo & Chiueh (2009), Ureña-López (2009); Lee & Lim (2010)
Arvanitaki et al. (2010); Marsh & Ferreira (2010); Marsh (2011)
Suárez & Matos (2011); Harko (2011); Chavanis (2011);
TRD & Shapiro (2010, 2012, 2014); Slepian & Goodman (2012)
Diez-Tejedor et al (2013); Marsh & Silk (2014);.....

Complex Scalar Field Dark Matter (SFDM)

Complex scalar field $\psi = |\psi| e^{i\theta}$

$$\mathcal{L} = \frac{\hbar^2}{2m} g^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi - V(\psi) \quad V(\psi) = \frac{1}{2} m c^2 |\psi|^2 + \frac{\lambda}{2} |\psi|^4$$

units: $[\mathcal{L}] = [\text{eV}/\text{cm}^3]$, $[\psi] = \text{cm}^{-3/2}$, $\lambda = \hat{\lambda} \frac{\hbar^3}{m^2 c}$

Complex field obeys

U(1)-symmetry, particle number conserved \rightarrow no self-annihilation !

2-body repulsive interactions only

first Born approx: $\lambda \geq 0$ is an energy-independent coupling constant

Fundamental SFDM parameters: m and λ

Equations of motion

Klein-Gordon equation for the scalar field ψ

$$g^{\mu\nu} \partial_\mu \partial_\nu \psi - g^{\mu\nu} \Gamma_{\mu\nu}^\sigma \partial_\sigma \psi + \frac{m^2 c^2}{\hbar^2} \psi + \frac{2\lambda m}{\hbar^2} |\psi|^2 \psi = 0$$

Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

KG equation in FRW for homogeneous, complex SFDM

$$\psi = |\psi|e^{i\theta}$$

$$\frac{\hbar^2}{2mc^2}(\partial_t^2\psi + 3H\partial_t\psi) + V'(|\psi|) = 0$$

in terms of amplitude and phase:

$$\partial_t^2|\psi| - |\psi|(\partial_t\theta)^2 + 3H\partial_t|\psi| + \frac{2mc^2}{\hbar^2}V'(|\psi|) = 0$$

$$2\partial_t|\psi|\partial_t\theta + |\psi|\partial_t^2\theta + 3H|\psi|\partial_t\theta = 0$$

$$\partial_t(a^3|\psi|^2\partial_t\theta) = 0 \iff a^3|\psi|^2\partial_t\theta = \text{const.}$$

conserved DM number: 'charge density'

“Spintessence” limit

(Boyle, Caldwell & Kamionkowski, 2002)

the phase (angular mode) carries the major oscillation behavior,
while the time-dependence of the amplitude (radial mode) is much
smoother:

always assume that $\frac{\partial_t |\psi|}{|\psi|} \ll \partial_t \theta$

Friedmann equation

$$H^2(t) \equiv \left(\frac{da/dt}{a} \right)^2 = \frac{8\pi G}{3c^2} [\bar{\rho}_r(t) + \bar{\rho}_b(t) + \bar{\rho}_\Lambda(t) + \bar{\rho}_{\text{SFDM}}(t)]$$

radiation: $\bar{\rho}_r(t) = \frac{\Omega_r \rho_{0,crit}}{a^4}$ baryons: $\bar{\rho}_b(t) = \frac{\Omega_b \rho_{0,crit}}{a^3}$

cosmological constant: $\bar{\rho}_\Lambda(t) = \Omega_\Lambda \rho_{0,crit}$ SFDM: $\bar{\rho}_{\text{SFDM}}(t)$

critical energy density at the present epoch: $\rho_{0,crit} = \frac{3H_0^2 c^2}{8\pi G}$

Homogeneous background Universe

Scalar field of SFDM depends only on time

Energy-momentum tensor is diagonal \rightarrow perfect fluid description

$$(T_{\mu\nu})_{\text{SFDM}} = (\bar{\rho}_{\text{SFDM}} + \bar{p}_{\text{SFDM}})u_{\mu}u_{\nu}/c^2 - g_{\mu\nu}\bar{p}_{\text{SFDM}}$$

$$\bar{\rho}_{\text{SFDM}} = (T_0^0)_{\text{SFDM}} = \frac{\hbar^2}{2mc^2}|\partial_t\psi|^2 + \frac{1}{2}mc^2|\psi|^2 + \frac{\lambda}{2}|\psi|^4$$

$$\bar{p}_{\text{SFDM}} = -(T_i^i)_{\text{SFDM}} = \frac{\hbar^2}{2mc^2}|\partial_t\psi|^2 - \frac{1}{2}mc^2|\psi|^2 - \frac{\lambda}{2}|\psi|^4$$

KG equation of motion $\rightarrow \frac{\partial\bar{\rho}_{\text{SFDM}}}{\partial t} + \frac{3da/dt}{a}(\bar{\rho}_{\text{SFDM}} + \bar{p}_{\text{SFDM}}) = 0$

Basic behavior of scalar fields: oscillation over time, characterized by its changes in phase θ , and oscillation angular frequency $\omega = \partial_t \theta$

- Fast oscillation regime („oscillation“): $\omega / H \gg 1$

$$\omega = \frac{mc^2}{\hbar} \sqrt{1 + \frac{2\lambda}{mc^2} |\psi|^2}$$

$$\langle \bar{\rho} \rangle = mc^2 \langle |\psi|^2 \rangle + \frac{3}{2} \lambda \langle |\psi|^4 \rangle \approx mc^2 \langle |\psi|^2 \rangle + \frac{3}{2} \lambda \langle |\psi|^2 \rangle^2,$$

$$\langle \bar{p} \rangle = \frac{1}{2} \lambda \langle |\psi|^4 \rangle \approx \frac{1}{2} \lambda \langle |\psi|^2 \rangle^2.$$

→ **equation of state** $\langle \bar{p} \rangle = \frac{m^2 c^4}{18\lambda} \left(\sqrt{1 + \frac{6\lambda \langle \bar{\rho} \rangle}{m^2 c^4}} - 1 \right)^2$

(1) CDM-like phase: non-relativistic $\langle \bar{w} \rangle = 0$

$$\frac{3}{2}\lambda\langle|\psi|^2\rangle^2 \ll mc^2\langle|\psi|^2\rangle \quad \langle \bar{p} \rangle \approx \frac{\lambda}{2m^2c^4}\langle \bar{\rho} \rangle^2 \approx 0.$$

→ evolves like CDM $\langle \bar{\rho} \rangle \propto a^{-3}, \quad a \sim t^{2/3}$

(2) Radiation-like phase: relativistic $\langle \bar{w} \rangle = 1/3$

$$\frac{3}{2}\lambda\langle|\psi|^2\rangle^2 \gg mc^2\langle|\psi|^2\rangle \quad \langle \bar{p} \rangle \approx \frac{1}{3}\langle \bar{\rho} \rangle \approx \frac{1}{2}\lambda\langle|\psi|^2\rangle^2$$

→ evolves like radiation $\langle \bar{\rho} \rangle \propto a^{-4}, \quad a \sim t^{1/2}$

NOTE: SFDM without self-interaction ($\lambda = 0$) does *not* undergo this radiation-like phase !

Basic behavior of scalar fields: oscillation over time, characterized by its changes in phase θ , and oscillation angular frequency $\omega = \partial_t \theta$

- Fast oscillation regime („oscillation“): $\omega / H \gg 1$

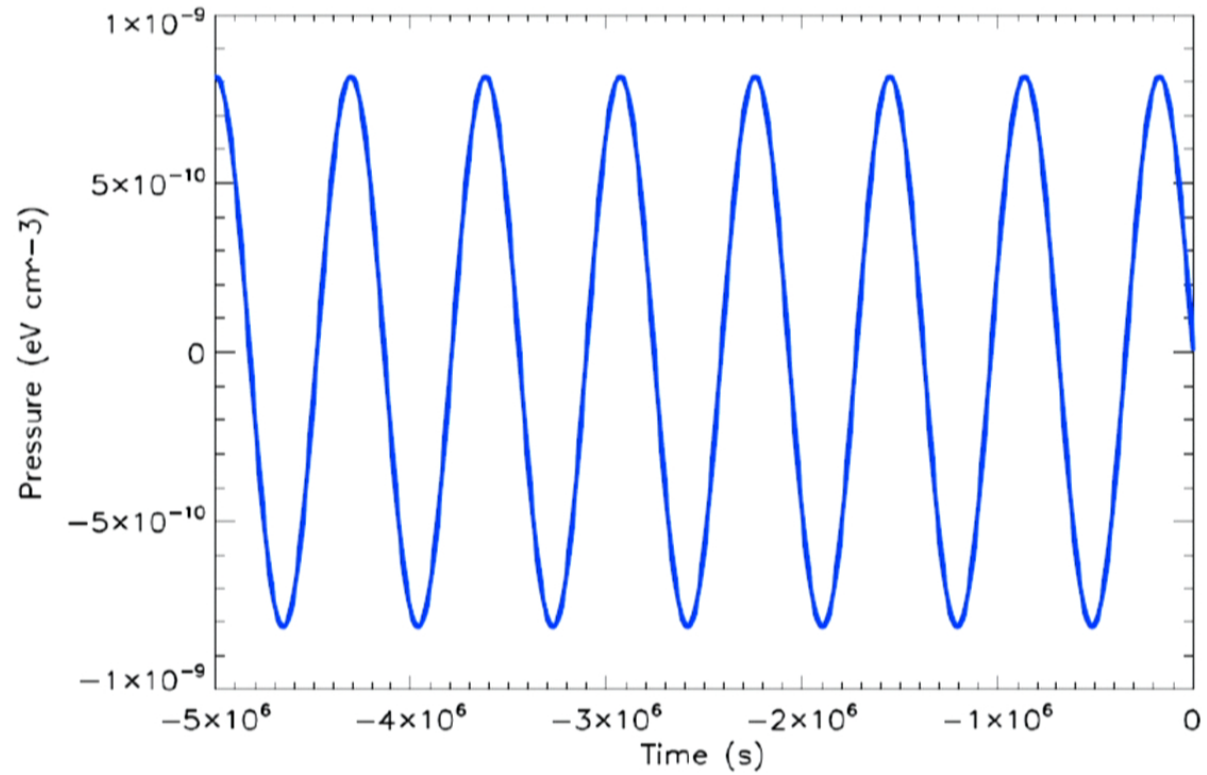
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Fast oscillation regime („ oscillation “): $\omega / H \gg 1$



- Slow oscillation regime („roll“): $\omega / H \ll 1$

(1) Stiff phase: equation of state of ‚stiff matter‘
relativistic limit ($w = 1$)

$$\bar{p} \approx \bar{\rho} \approx \frac{\hbar^2}{2mc^2} |\partial_t \psi|^2$$

→ evolves as $\langle \bar{\rho} \rangle \propto a^{-6}$, $a \sim t^{1/3}$ „kination“ (Joyce, 1997)

NOTE: this phase cannot be avoided for complex scalar fields, since the kinetic energy term due to the conserved charge will dominate over all the other terms, if that conserved charge is fixed by the current DM density

Take the same cosmic inventory as the basic Λ CDM model, except that **CDM is replaced by SFDM** \rightarrow **Λ SFDM**

Cosmological parameters from **Planck results XVI (2013)**:

	Basic		Derived	
$\Omega_m = \Omega_b + \Omega_c$	h	0.673	$\Omega_m h^2$	0.14187
$\Omega_\Lambda = 1 - \Omega_m - \Omega_r$	$\Omega_b h^2$	0.02207	$\Omega_r h^2$	4.184×10^{-5}
	$\Omega_c h^2$	0.1198	z_{eq}	3390
	T_{CMB}/K	2.7255	Ω_Λ	0.687

assuming SM
neutrinos are
massless

TABLE I. Cosmological parameters. The values in the left column ('Basic') are quoted from the Planck collaboration: central values of the 68% confidence limits for the base Λ CDM model with Planck+WP+highL data, see Table 5 in [5]. We calculate those in the right column ('Derived').

Evolution of Λ SFDM

- Solve Friedmann equ. coupled with equ. Of motion and the EOS, by integrating from the present-day backwards to the point when $\omega/H = 200$ (i.e. still well into the fast-oscillation regime, where we average) at a $\sim 10^{-7}$ (dep. on parameters)
'Late-time solution': its initial conditions are from the Table (Planck data)
- At earlier times up to the Big Bang, solve the system exactly (i.e. **no** averaging over oscillation periods)
'Early-time solution': integration starts where we cease to apply the fast-oscillation approximation at $\omega/H = 200$, back to the Big Bang, in a way that it matches to the late-time solution

Can do that

for different choices of SFDM mass m and coupling strength λ

The other cosmic components are handled in the usual way.

Fiducial SFDM Model

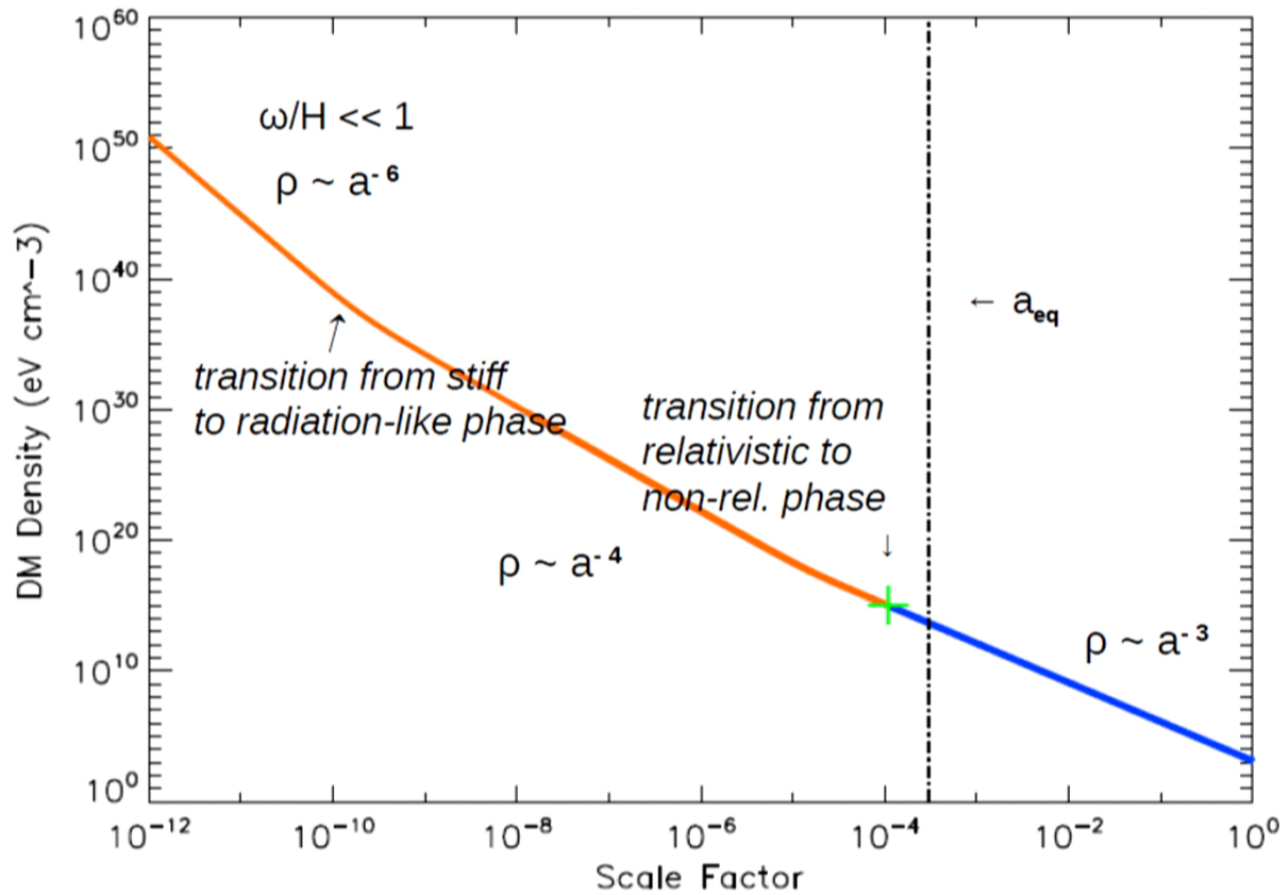
$$(m, \lambda)_{\text{fiducial}} = (3 \times 10^{-21} \text{ eV}/c^2, 1.8 \times 10^{-59} \text{ eV cm}^3)$$

$$\lambda/(mc^2)^2 = 2 \times 10^{-18} \text{ eV}^{-1} \text{ cm}^3$$

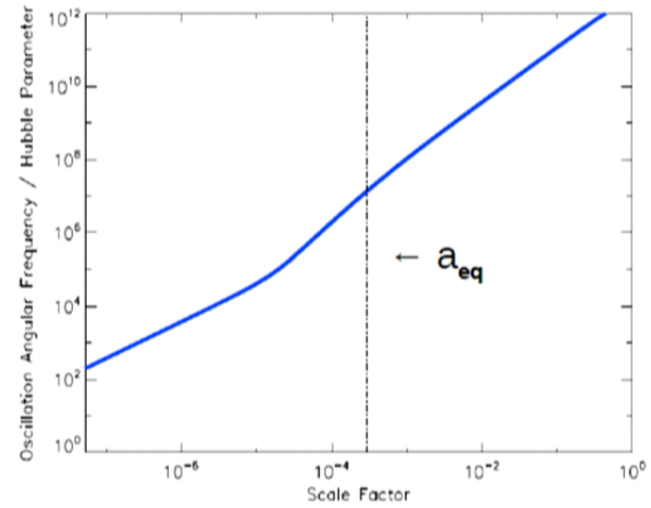
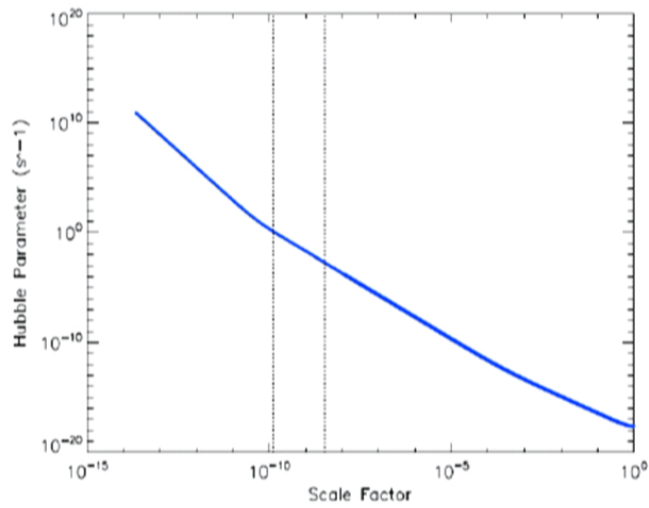
in natural units: $\hat{\lambda}_{\text{fiducial}} \simeq 10^{-83}$

for comparison $\hat{\lambda}_{\text{QCDaxion}} \simeq 10^{-57}$

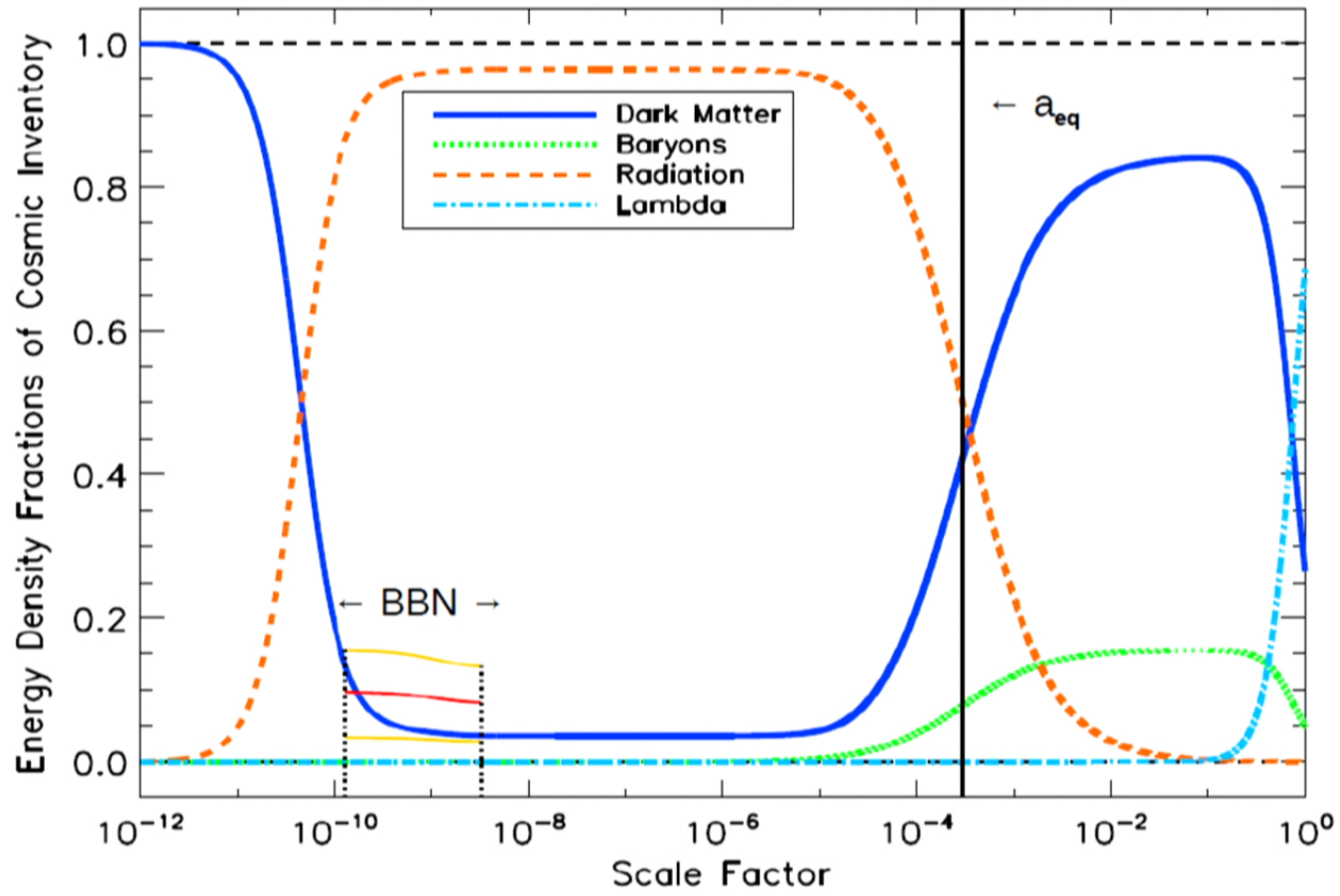
Evolution of SFDM energy density



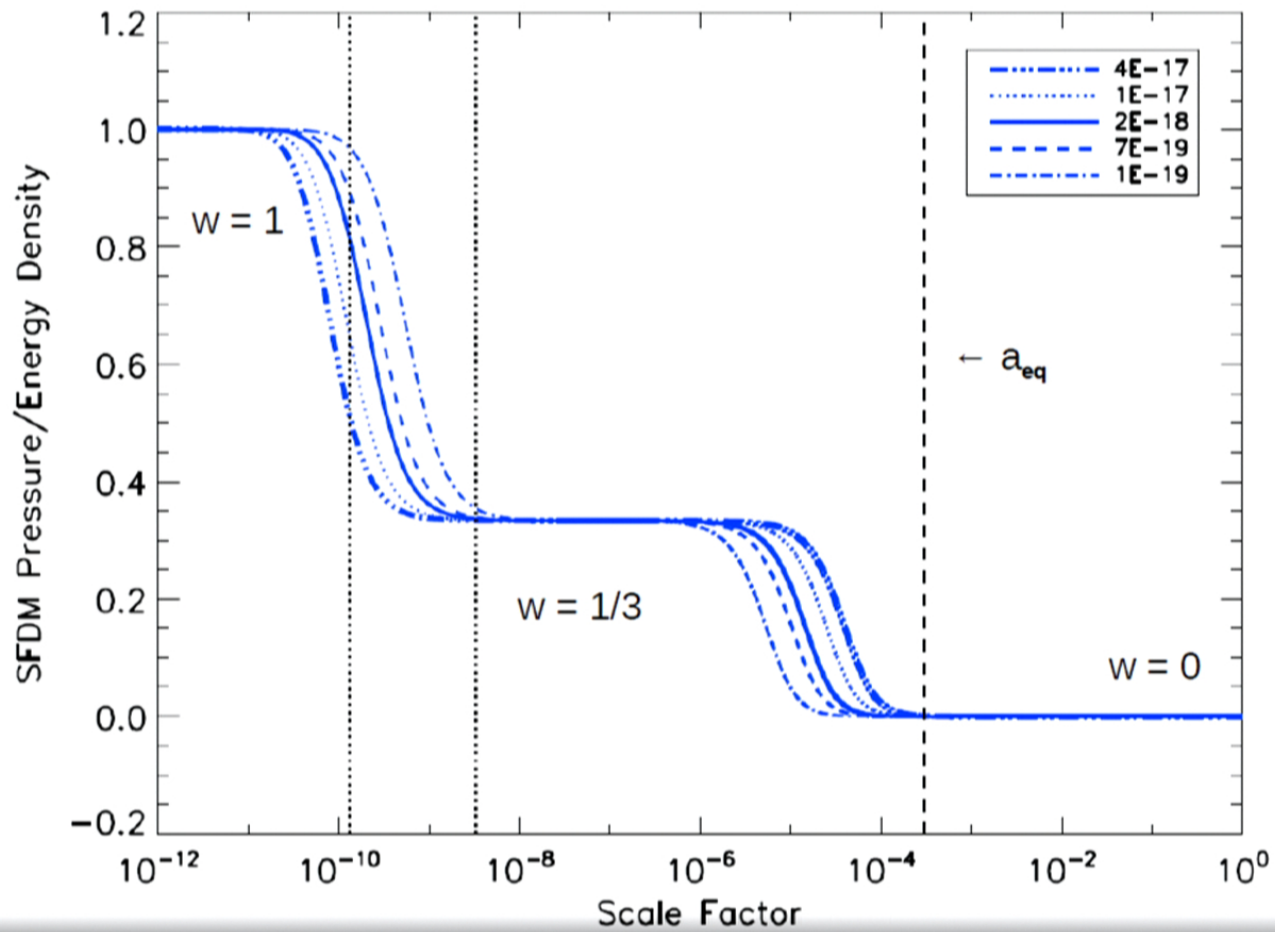
Evolution of H and ω/H



Evolution of Λ SFDM



The larger $\lambda/(mc^2)^2$, the longer lasts the radiation-like phase



Constraints on SFDM from the CMB

redshift of **matter-radiation equality** z_{eq} :

$$1 + z_{\text{eq}} = \frac{\Omega_b h^2 + \Omega_c h^2}{\Omega_r h^2}$$

require SFDM to be fully non-relativistic at z_{eq}

i.e. the transition from the relativistic phase (‘radiation-like’) to the non-relativistic phase (‘CDM-like’) must happen early enough such that SFDM is cold at z_{eq}

→ *constraint on the ratio* $\lambda/(mc^2)^2$

$$\lambda/(mc^2)^2 \leq 4 \times 10^{-17} \text{ eV}^{-1} \text{ cm}^3 \text{ for a chosen threshold of } \langle \bar{w} \rangle = 0.001$$

Constraints on SFDM from BBN

effective number of relativistic degrees of freedom / neutrinos: N_{eff}

in Λ CDM with SM neutrinos only: $N_{\text{eff,standard}} = 3.046$

in Λ SFDM: if SFDM is relativistic during BBN

→ contributes to N_{eff} as an extra relativistic component

$$\Delta N_{\text{eff}} \equiv N_{\text{eff}} - N_{\text{eff,standard}}$$

Therefore, constraints on N_{eff} from BBN allow control on SFDM parameters.

in Λ SFDM: ΔN_{eff} caused by SFDM is *changing with time* !

→ must study the evolution of N_{eff} throughout BBN

Important 2 stages in standard BBN:

- beginning of neutron/proton freeze-out around $T_{\text{n/p}} = 1.293 \text{ MeV}$: $a_{\text{n/p}}$
- beginning of nuclei production (D) around $T_{\text{nuc}} \sim 0.07 \text{ MeV}$: a_{nuc}

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Constraints on SFDM from BBN

effective number of relativistic degrees of freedom / neutrinos: N_{eff}

SFDM is the only source for ΔN_{eff} , i.e. infer N_{eff} during BBN from

$$\frac{\Delta N_{\text{eff}}}{N_{\text{eff,standard}}} = \frac{\bar{\rho}_{\text{SFDM}}}{\bar{\rho}_{\nu}}$$

and compare the N_{eff} obtained this way to the measured value
(which is constant over time)

We impose a conservative constraint: the N_{eff} during BBN *be all the time* within the **1 σ confidence limits** of

$$N_{\text{eff}} = 3.71^{+0.47}_{-0.45} \quad \text{or} \quad \Delta N_{\nu} = 0.66^{+0.47}_{-0.45} \quad (\text{Steigman 2012})$$

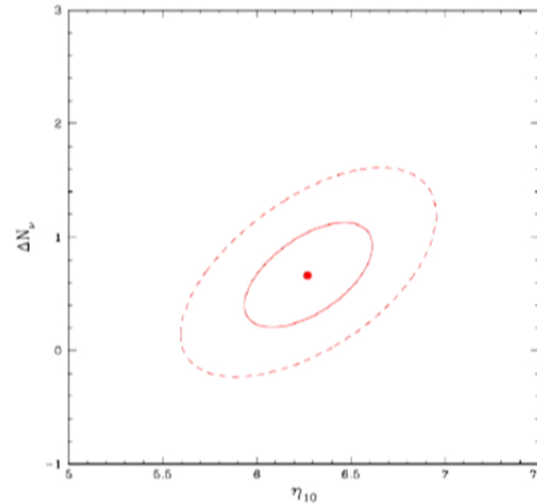
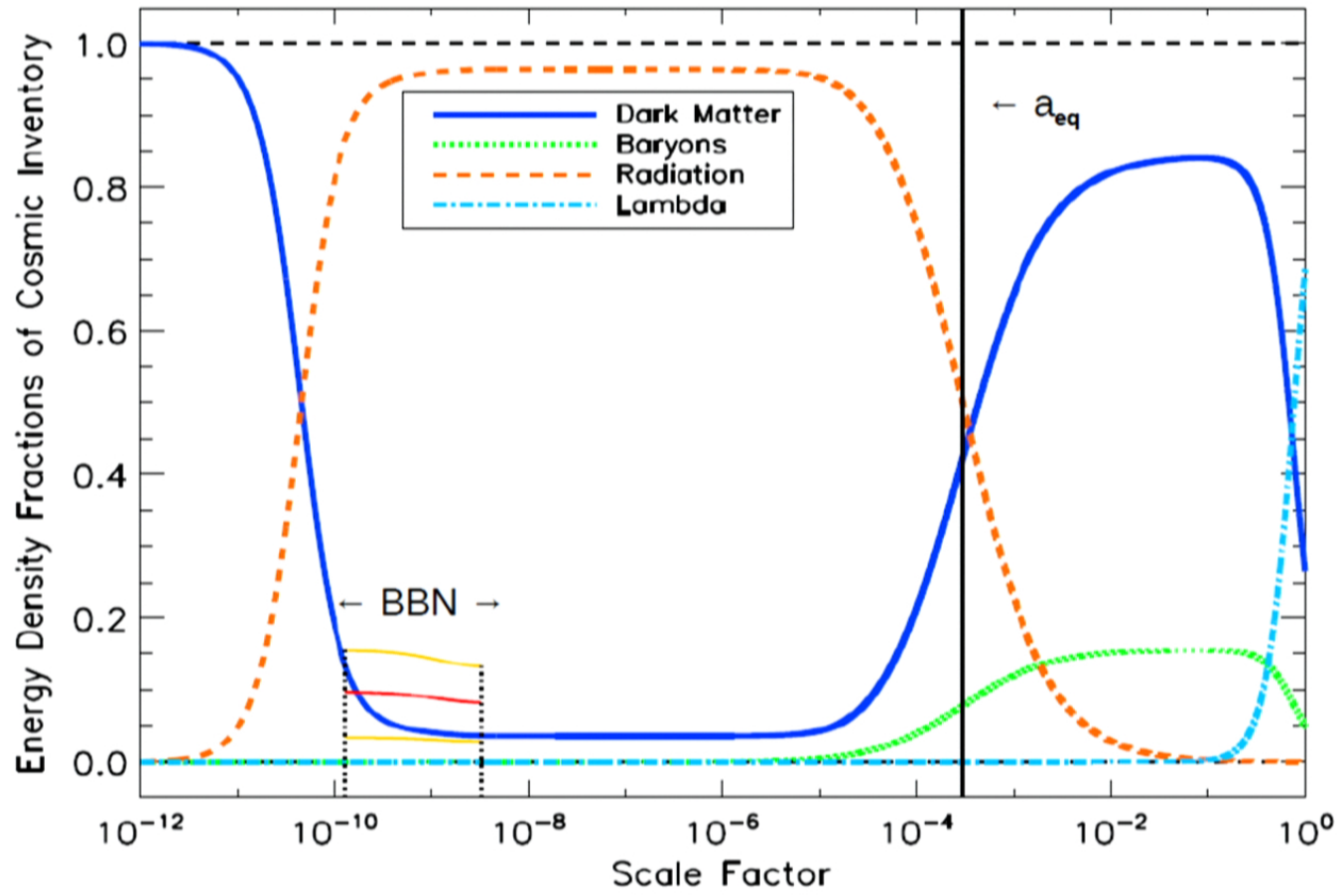
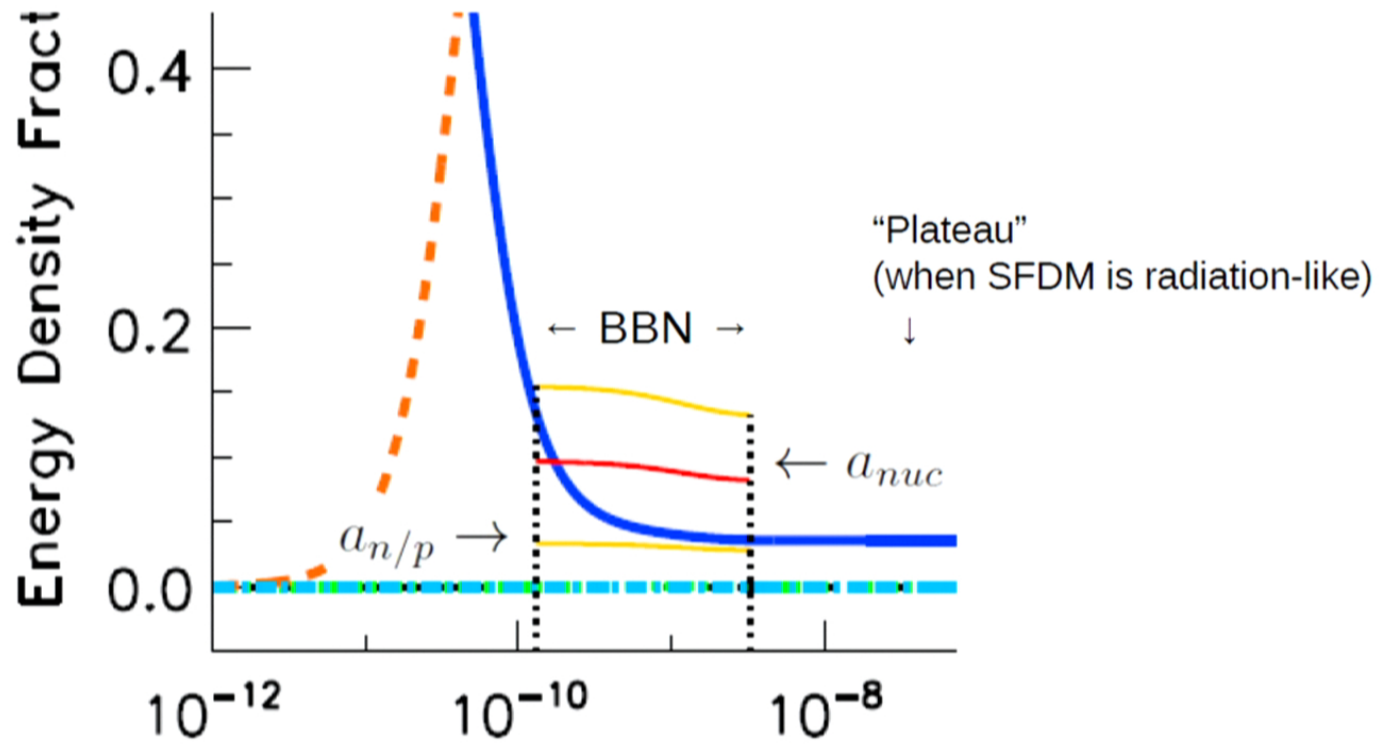


Figure 4: The BBN-inferred 68% (solid) and 95% (dashed) contours in the $\Delta N_{\nu} - \eta_{10}$ plane derived from D and ${}^4\text{He}$ assuming that $\xi = 0$.

Evolution of Λ SFDM



Evolution of Λ SFDM during BBN



Constraints on SFDM from BBN

effective number of relativistic degrees of freedom / neutrinos: N_{eff}

the relation between N_{eff} and Ω_{SFDM} is analytic during the „plateau“
(i.e. during the radiation-like phase) *if* SFDM reaches it before a_{nuc} :

$$N_{\text{eff}} = 3.71^{+0.47}_{-0.45} \quad \rightarrow \quad 0.028 \leq \Omega_{\text{SFDM,plateau}} \leq 0.132$$

the higher $\lambda/(mc^2)^2$, the higher the “plateau”

\rightarrow *constraint on $\lambda/(mc^2)^2$*

$$9.5 \times 10^{-19} \text{ eV}^{-1} \text{ cm}^3 \leq \lambda/(mc^2)^2 \leq 1.5 \times 10^{-16} \text{ eV}^{-1} \text{ cm}^3$$

Constraints on SFDM from BBN

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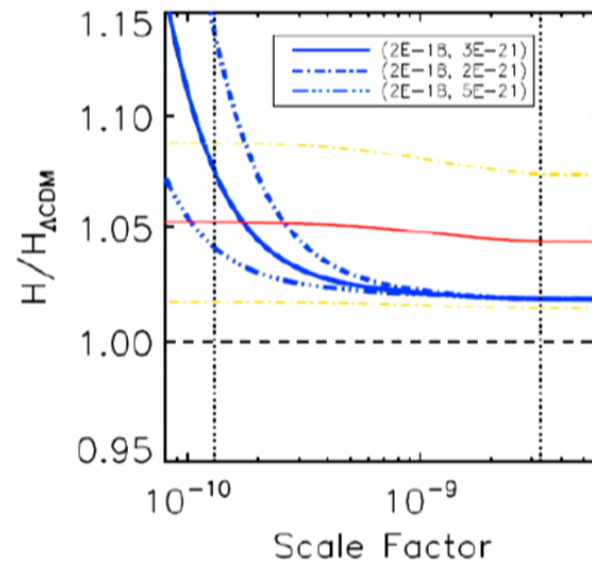
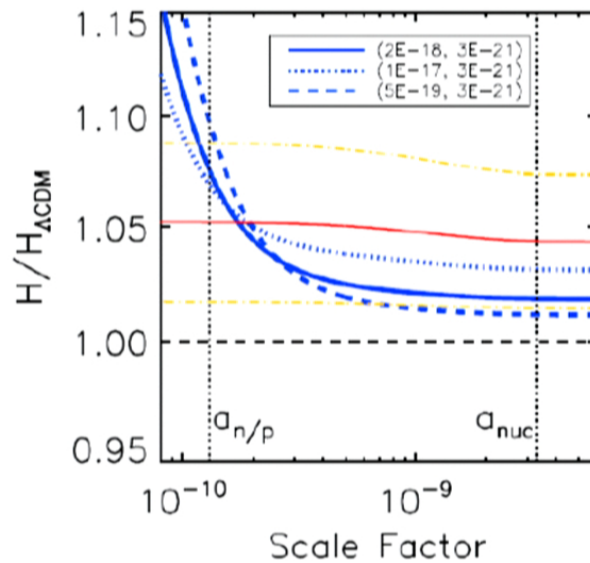
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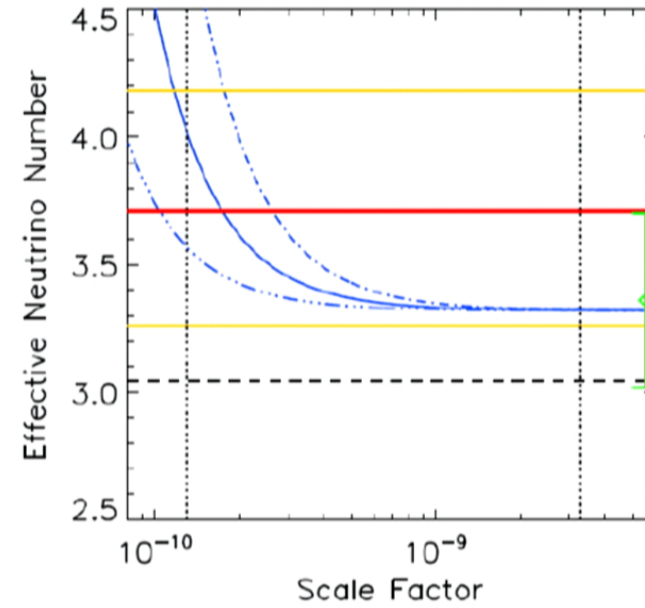
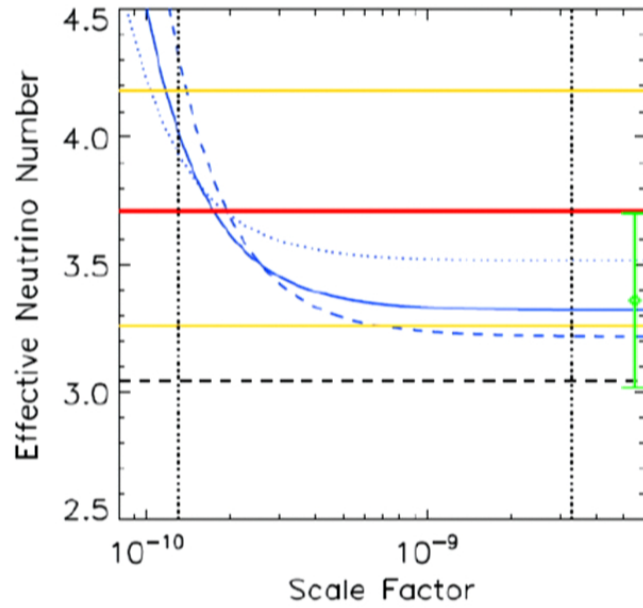
Constraints on SFDM from BBN

Meanwhile, earlier at $a_{n/p}$, the transition from the stiff to the radiation-like phase may not have finished and the value of N_{eff} can be higher than at the plateau – this is a function of *both* $\lambda/(mc^2)^2$ and m

→ constraint on $\lambda/(mc^2)^2$ and m

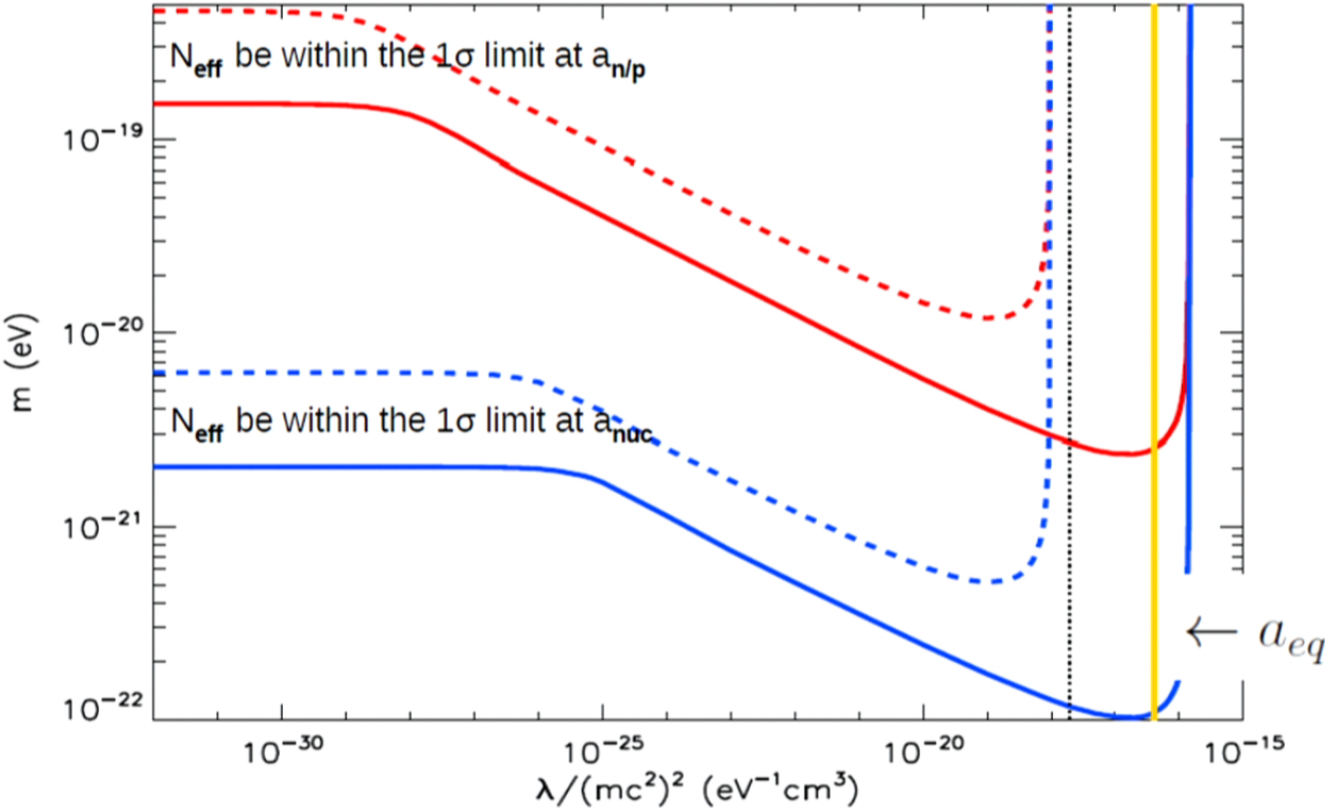


Constraints on SFDM from BBN

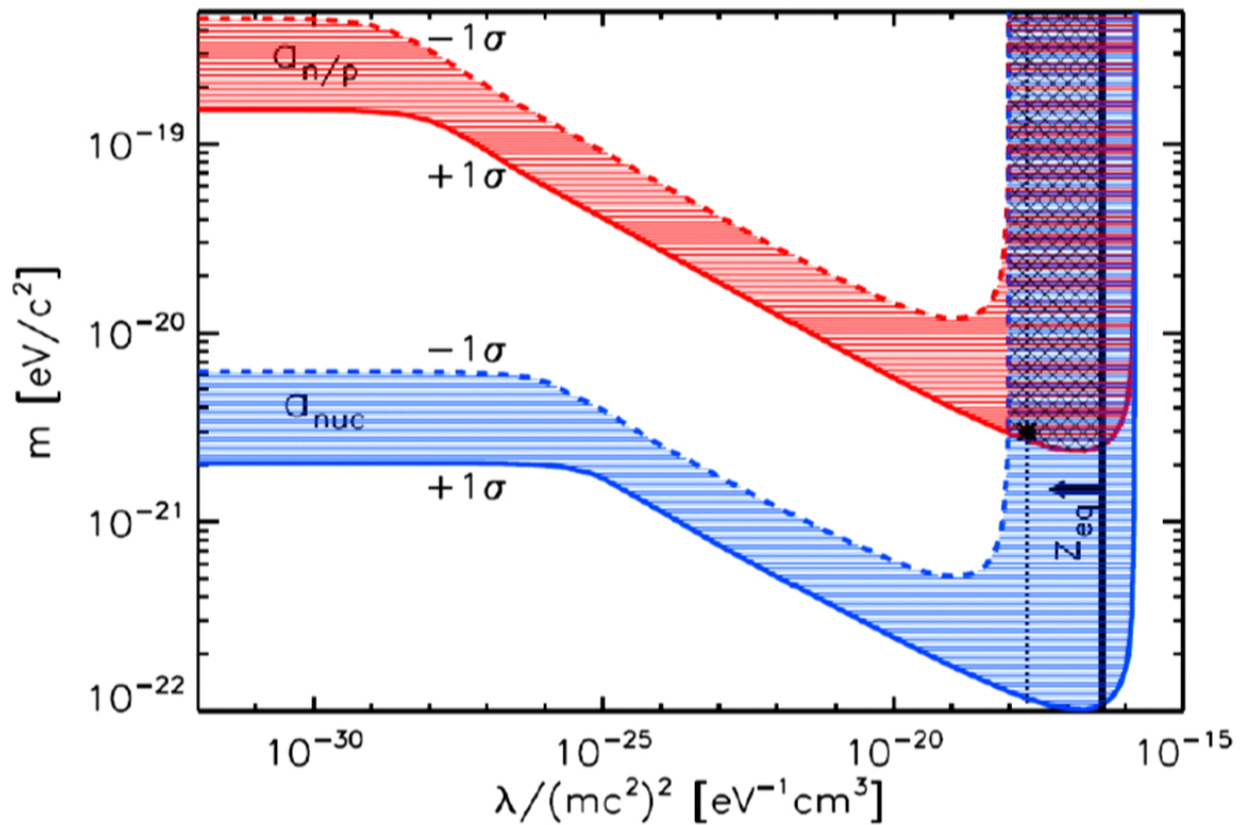


compare: $N_{\text{eff}} = 3.36 \pm 0.34$ (green bar) from CMB alone (Planck+WP+highL)

Parameter space for SFDM

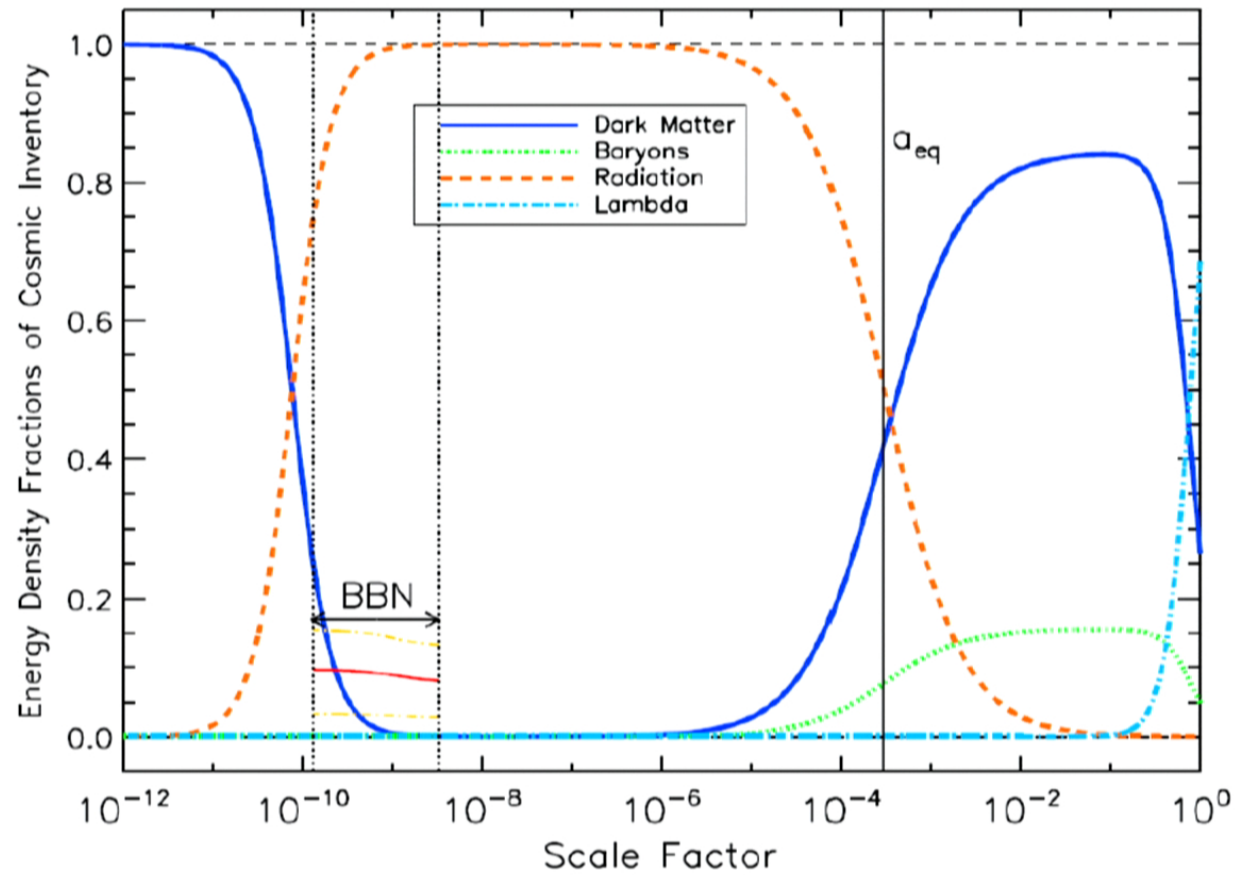


Allowed parameter space for SFDM



Models with $\lambda = 0$ (Fuzzy Dark Matter), as the only component, are disfavored within 1σ . Also, need high enough mass m !

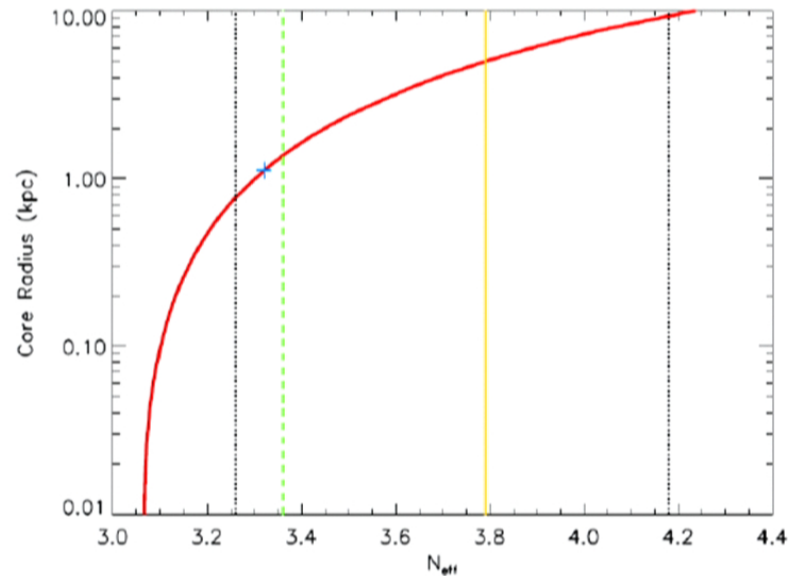
Evolution of Λ SFDM *without* self-interaction



N_{eff} and the minimum size of halos

- N_{eff} during the radiation-like phase of SFDM is solely determined by $\lambda/(mc^2)^2$
- The radius R of the smallest virialized object (core radius of SFDM halo) is also determined solely by $\lambda/(mc^2)^2$ for SFDM with $R \gg \lambda_{\text{deB}}$

$$R = \pi c^2 \sqrt{\frac{\lambda}{4\pi G (mc^2)^2}}$$



- higher N_{eff} implies stronger self-interaction pressure,
hence larger minimum scale for Dark Matter structure !

Derived Bounds:

- DM mass: $m \geq 2.4 \times 10^{-21} \text{eV}/c^2$

- DM self-interaction:

$$9.5 \times 10^{-19} \text{eV}^{-1} \text{cm}^3 \leq \lambda / (mc^2)^2 \leq 4 \times 10^{-17} \text{eV}^{-1} \text{cm}^3.$$

- DM halo core size:

$$0.75 \text{ kpc} \leq R_{\text{core}} \leq 5.2 \text{ kpc}$$

Conclusions

- Complex SFDM is a good dark matter candidate so far → but work in progress whether that remains true for structure formation
- We constrained the allowed parameter space severely (compared to previous literature), even by considering only the evolution of the background universe
- Nevertheless, there remains a semi-infinite stripe in the parameter space which is in accordance with current observations, including parameter sets which are able to resolve the small-scale problems of CDM
- The currently favored value of $N_{\text{eff}} > N_{\text{eff,standard}}$ from BBN and CMB would rule out the possibility of complex Fuzzy Dark Matter (as the only DM component)
- Complex SFDM with self-interaction provides a natural explanation of why N_{eff} during BBN is higher than that inferred from the CMB from Planck data