Title: Introduction to Effective Field Theories - Lecture 16

Date: Mar 07, 2014 02:30 PM

URL: http://pirsa.org/14020095

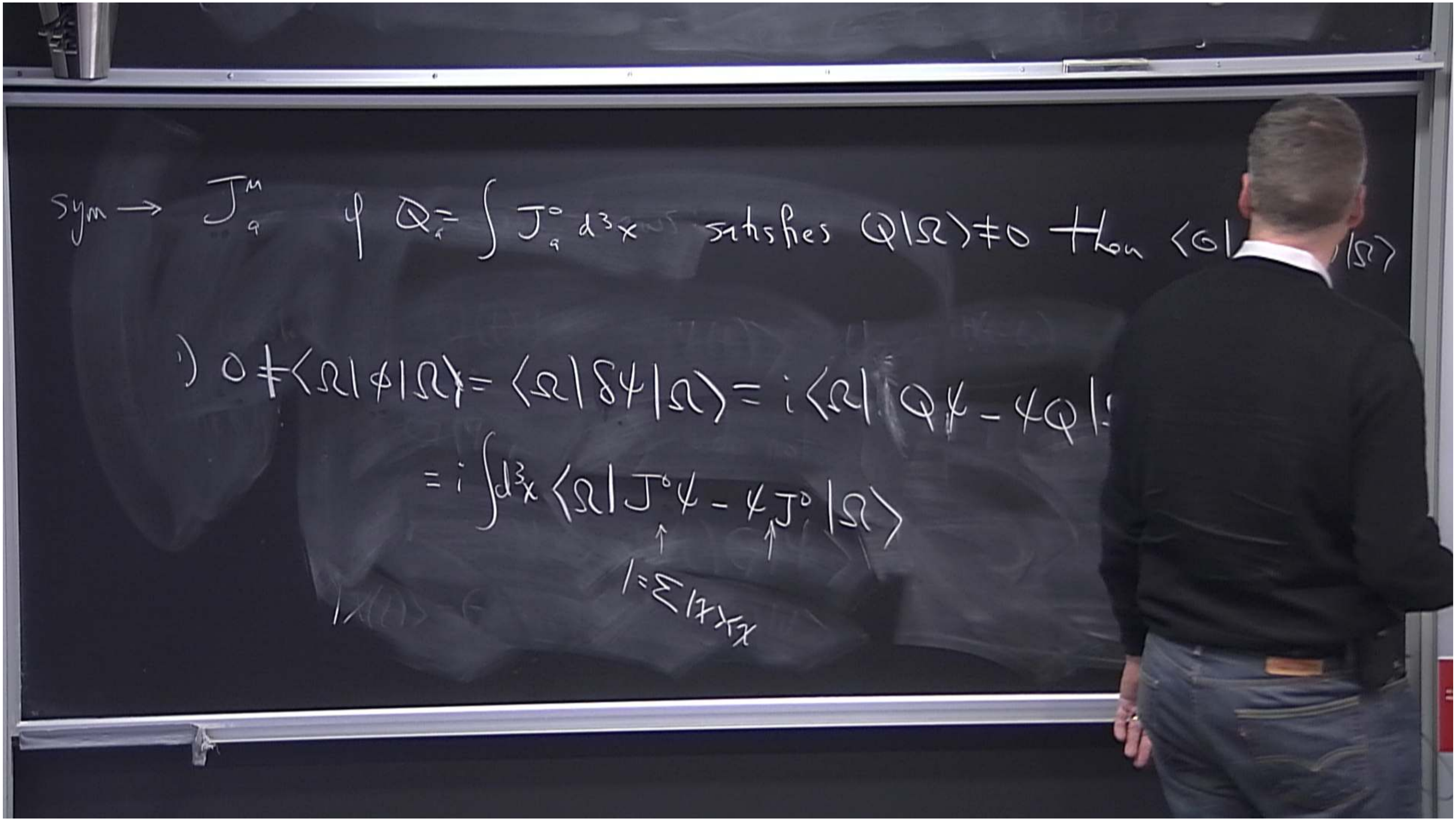
Abstract:

sym $\rightarrow$ J^μ_a if $Q_a = \int J^0_a \, d^3x$ satisfies $Q|\Omega\rangle \neq 0$ then $\langle G| J^\mu_a(x)|\Omega\rangle \neq 0$.

i) $0 \neq \langle\Omega|\phi|\Omega\rangle = \langle\Omega|\delta\psi|\Omega\rangle = i\langle\Omega| Q\psi - \psi Q|\Omega\rangle$

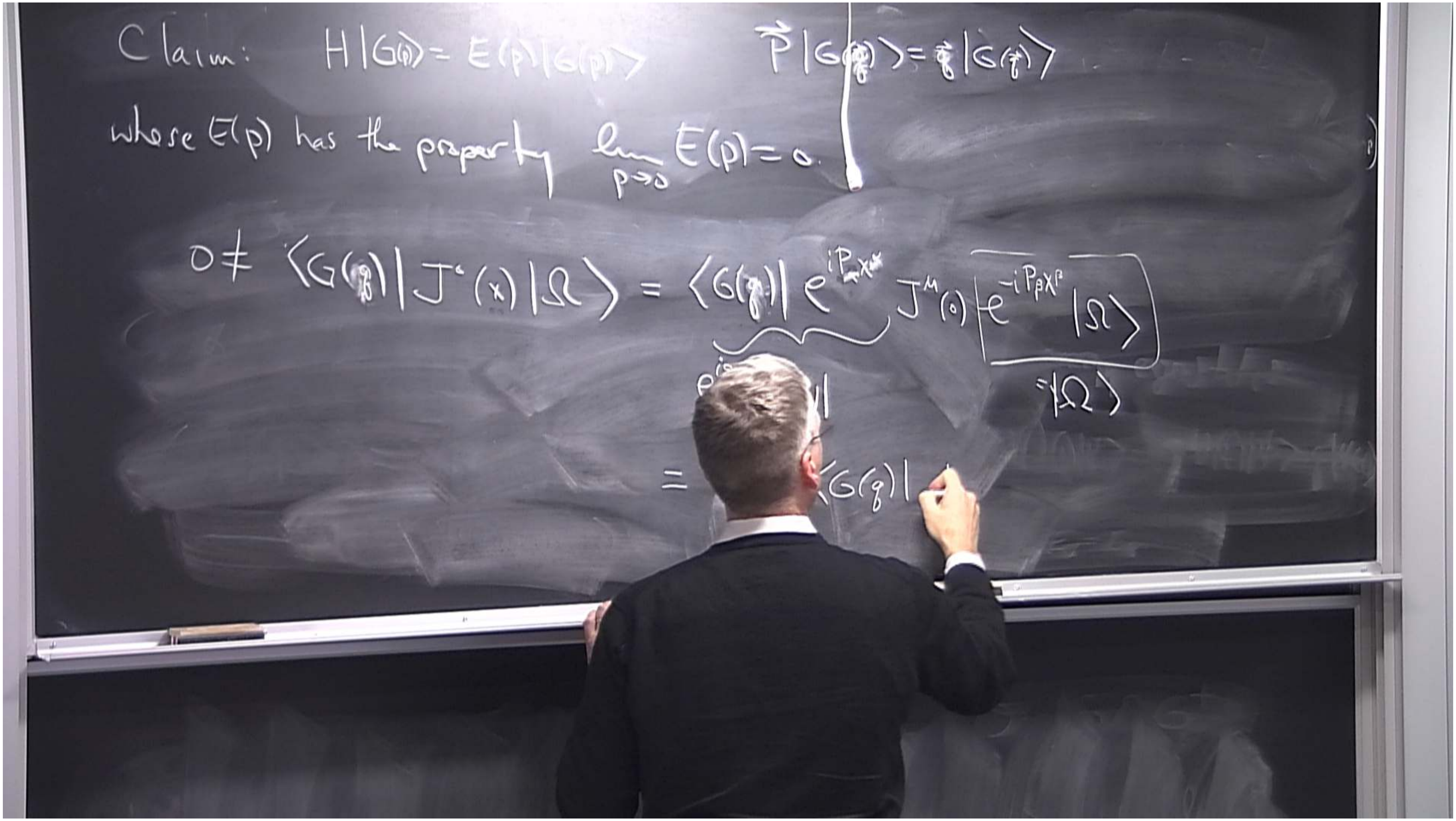
$= i\int d^3x \, \langle\Omega|J^0\psi - \psi J^0|\Omega\rangle$

$1 = \sum |x\rangle\langle x|$

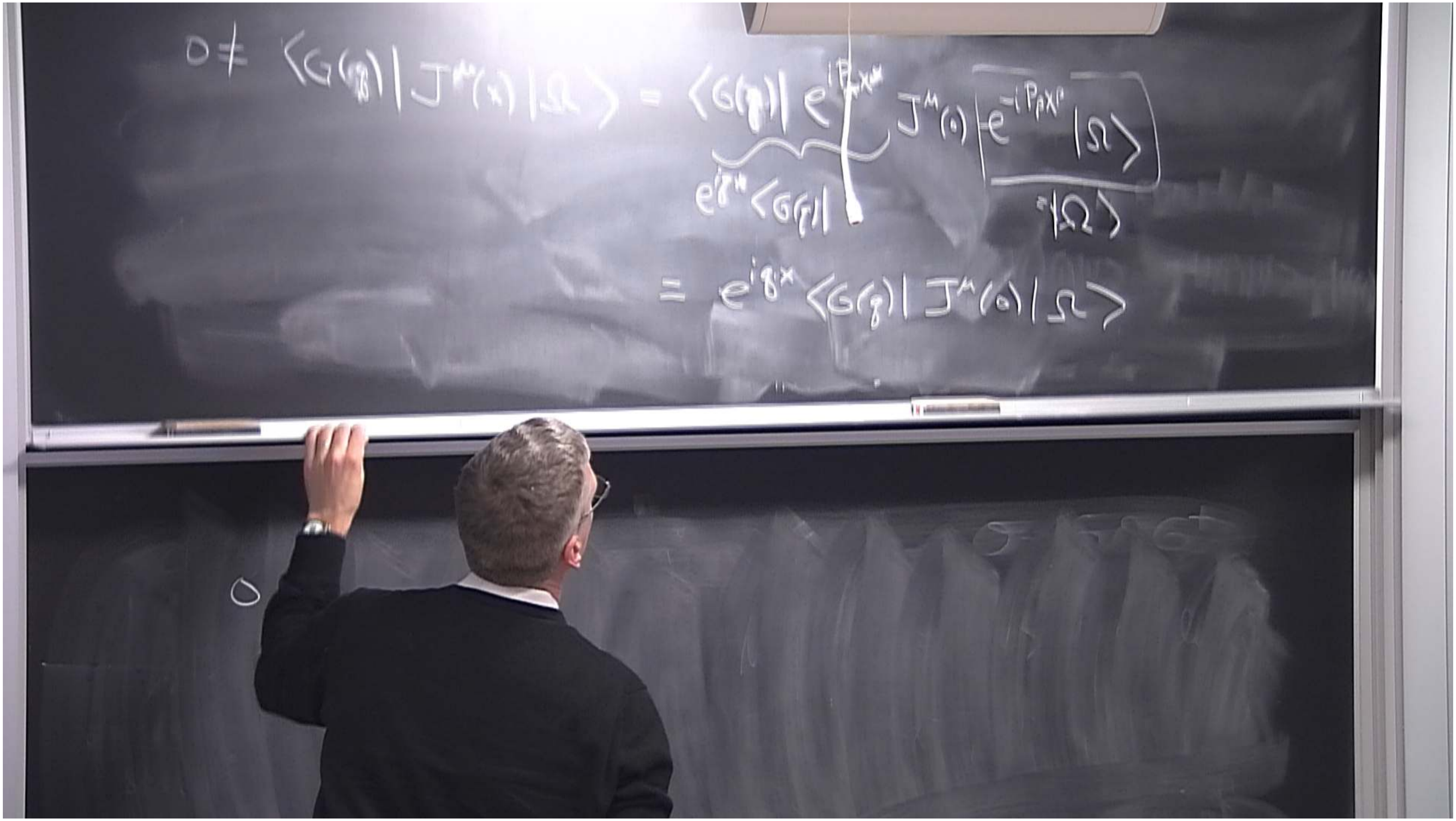

sym → J^μ_a   if Q_a = ∫ J^0_a d^3x satisfies Q|Ω⟩≠0 then ⟨Ω|
|Ω⟩
0 ≠ ⟨Ω|φ|Ω⟩ = ⟨Ω|δψ|Ω⟩ = i⟨Ω| Qψ − ψQ |
= i ∫d^3x ⟨Ω|J^0ψ − ψJ^0|Ω⟩
1 = Σ|χ⟩⟨χ|

Claim: $H|G(p)\rangle = E(p)|G(p)\rangle$

whose $E(p)$ has the property $\lim_{p\to 0} E(p) = 0$.



Claim: $H|G(p)\rangle = E(p)|G(p)\rangle$
$\vec{P}|G(q)\rangle = \vec{q}|G(q)\rangle$
whose $E(p)$ has the property $\lim_{p\to 0} E(p) = 0$.
$0 \neq \langle G(q)| J^0(x) |\Omega\rangle = \langle G(q)| e^{iP_\mu x^\mu} J^\mu(0) \boxed{e^{-iP_\rho x^\rho} |\Omega\rangle}$
$= |\Omega\rangle$
$= \langle G(q)|$


$0 \neq \langle G(q) | J^\mu(x) | \Omega \rangle = \langle G(q) | e^{iP_\mu x^\mu} J^\mu(0) e^{-iP_\rho x^\rho} | \Omega \rangle$
$e^{iq \cdot x} \langle G(q) |$
$| \Omega \rangle$
$= e^{iq \cdot x} \langle G(q) | J^\mu(0) | \Omega \rangle$

$$\partial_\mu \left[\langle G(q) | J^\mu(x) | \Omega \rangle \right] = \langle G(q) | \underbrace{\partial_\mu J^\mu(x)}_{=0 \text{ (conservation)}} | \Omega \rangle = 0$$

$$\|$$

$$i q_\mu e^{i q\cdot x} \langle G(q) | J^\mu(0) | \Omega \rangle$$

$$\lim_{\vec{q}\to 0} i q_\mu e^{i q\cdot x} \langle G(q) | J^\mu(0) | \Omega \rangle = \lim_{\vec{q}\to 0} -i E(q) e^{-iE(q)t} \underbrace{\langle G(\vec{q}) | J^0(0) | \Omega \rangle}_{\neq 0} = 0$$

$$\longrightarrow \lim_{q\to 0} E(q) = 0$$

$$\phi = \rho e^{i\theta} \qquad \phi \to e^{i\alpha}\phi \longrightarrow \begin{array}{l}\rho \to \rho \\ \boxed{\theta \to \theta + \alpha}\end{array}$$

$$E = \sqrt{\vec{q}^{\,2} + m^2}$$

$$\langle \Omega | J^0_a(x) | \Omega \rangle \neq 0.$$

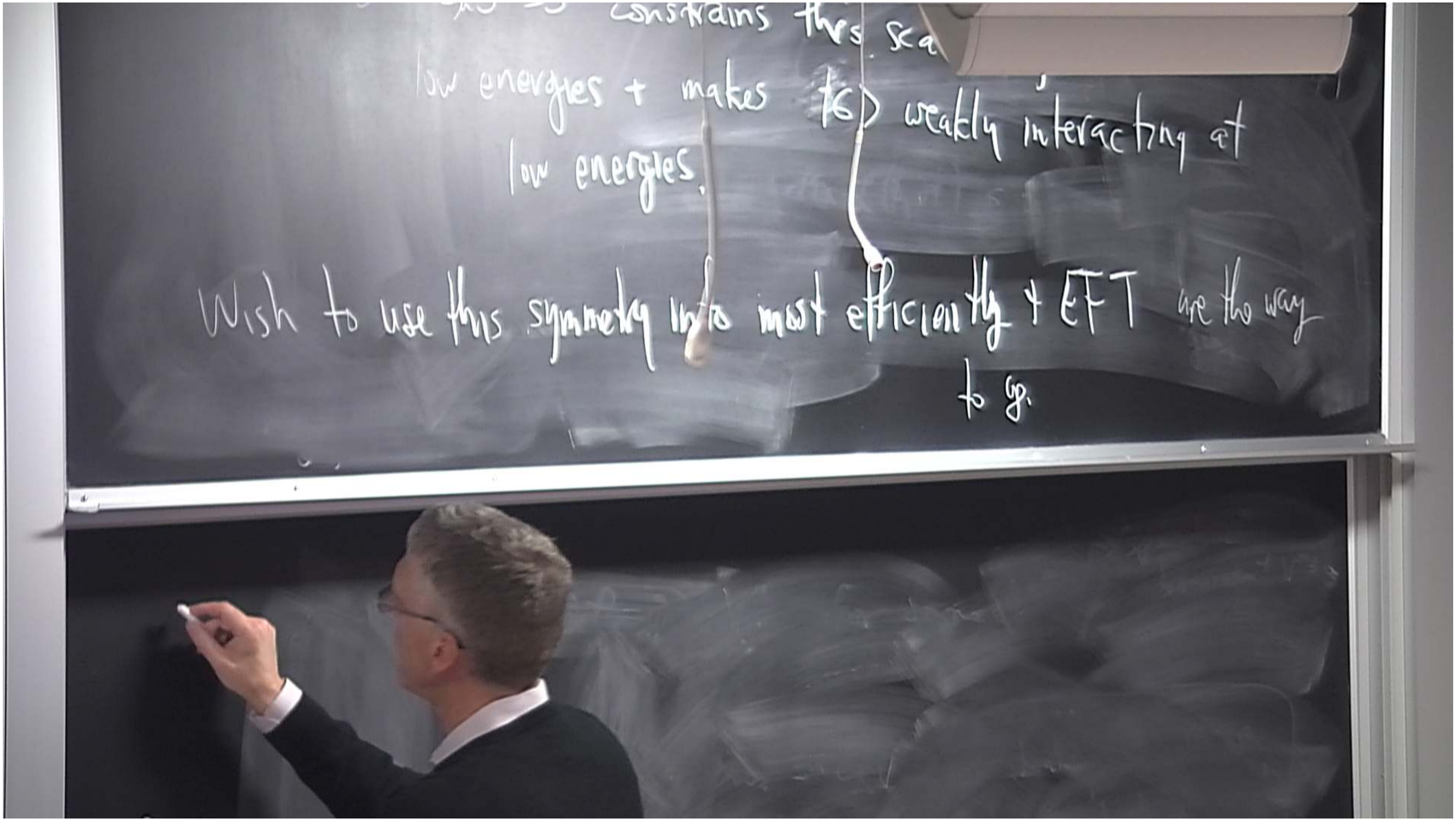
$$1)\ 0 \neq \langle \Omega | \phi | \Omega \rangle = \langle \Omega | \delta\psi | \Omega \rangle = i\omega^a \langle \Omega | Q_a \psi - \psi Q_a | \Omega \rangle$$

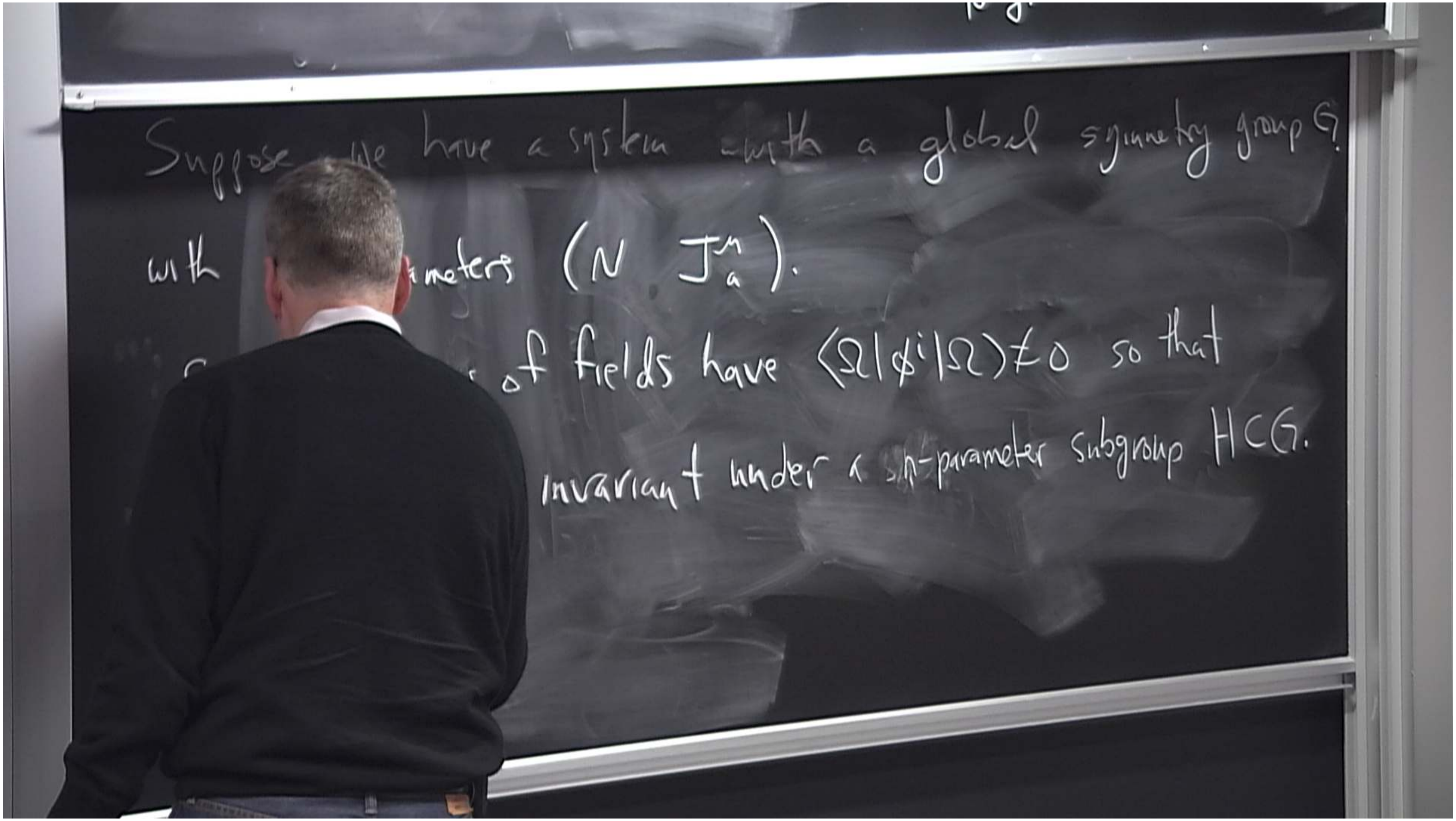
$$= i\omega^a \int d^3x \, \langle \Omega | J^0_a \psi - \psi J^0_a | \Omega \rangle$$

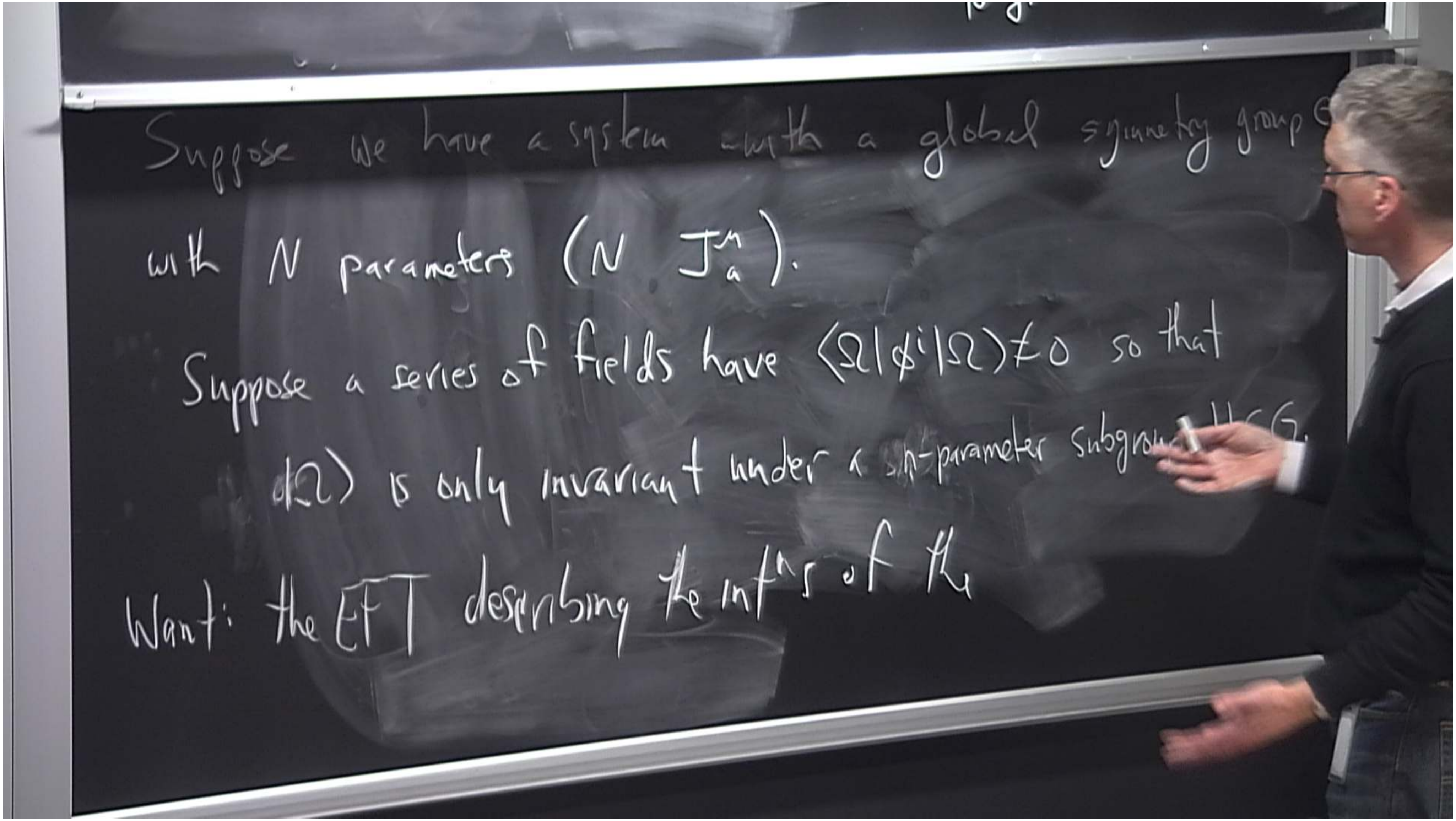
$$1 = \sum |\chi\rangle\langle\chi|$$

$|\chi(t)\rangle$

Scattering of $|G\rangle$ states can be obtained from correlation functions of $J^{\mu}(x)$ [LSZ reduction],


constrains this
low energies + makes (G) weakly interacting at
low energies.
Wish to use this symmetry info most efficiently + EFT are the way
to go.


Suppose we have a system with a global symmetry group G?
with
meters (N J^μ_a).
of fields have ⟨Ω|φ^i|Ω⟩≠0 so that
invariant under a
-parameter subgroup H⊂G.


Suppose we have a system with a global symmetry group G
with N parameters (N J^μ_a).
Suppose a series of fields have $\langle\Omega|\phi^i|\Omega\rangle \neq 0$ so that
$|\Omega\rangle$ is only invariant under a sub n-parameter subgroup $H \subset G$.
Want: the EFT describing the int^ns of the

$$\phi(x) = \rho(x) e^{i\theta(x)} \qquad \phi \to e^{i\alpha}\phi \longrightarrow \begin{array}{l}\rho \to \rho \\ \boxed{\theta \to \theta + \alpha}\end{array}$$

$$E = \sqrt{\vec{q}^{\,2} + m^2}$$

$$\langle\Omega| J^{\mu}_{a}(x) |\Omega\rangle \neq 0.$$

$$) \quad 0 \neq \langle\Omega|\phi|\Omega\rangle = \langle\Omega|\delta\psi|\Omega\rangle = i\omega^a \langle\Omega| Q_a\psi - \psi Q_a |\Omega\rangle$$

$$= i\omega^a \int d^3x \, \langle\Omega| J^0_a \psi - \psi J^0_a |\Omega\rangle$$

$$1 = \sum |x\rangle\langle x|$$

$E = \sqrt{\vec{q}^{\,2} + m^2}$

Need: how symmetries act on the GB fields

Aside: consider an example where $\phi^i(x) = \begin{pmatrix} \phi^1 \\ \vdots \\ \phi^5 \end{pmatrix} = \phi^{ix}$

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \phi^T \partial^\mu \phi - V(\phi^T\phi)$$

Symmetry $\phi \to g\phi$ $\quad g = 5\times5$ orthogonal matrix $G = \{g\} = O(5)$

$E = \sqrt{\vec{q}^2 + m^2}$

Need: how symmetries act on the GB fields

Aside: consider an example where $\phi^i(x) = \begin{pmatrix} \phi^1 \\ \vdots \\ \phi^5 \end{pmatrix} = \phi^{i*}$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi^T \partial^\mu \phi - V(\phi^T \phi)$$

symmetry $\phi \to g\phi$ $g = 5\times5$ orthogonal matrix $G = \{g\} = O(5)$

Define: $\phi = g(\theta)\underline{\chi}$

where if $g(\omega) = e^{i\omega^a T_a} \in G$ then $g(\theta) = e^{i\theta^a(x) T_a}$

demand χ be $\perp$ Goldstone boson directions

Suppose $v = \begin{pmatrix} v_1 \\ \vdots \\ v_J \end{pmatrix}$ is the vacuum value of ϕ. $\delta v = i\omega^a T_a v \leftarrow$ GB direction

Demand $v^T T_a \chi = 0$ (ie χ is $\perp$ to GB direction)

$E = \sqrt{\vec{q}^{\,2} + m^2}$

Need: how symmetries act on the GB fields

Aside: consider an example where $\phi^i(x) = \begin{pmatrix} \phi^1 \\ \vdots \\ \phi^5 \end{pmatrix} = \phi^{i*}$

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Suppose v break G down to H. $hv = v$ if $h \in H$

$g = e^{i\omega^a T_a}$ $\qquad \underbrace{\{T_a\}}_{N} = \{\underbrace{t_i}_{n}, \underbrace{X_\alpha}_{N-n}\}$ $\qquad t_i v = 0$

$X_\alpha v \neq 0$

$$\boxed{v^T X_\alpha \chi = 0}$$

$E = \sqrt{\vec{q}^{\,2} + m^2}$

How does G act on θ^α and χ?

$$\left.\begin{array}{l} \theta \to \tilde{\theta} \\ \chi \to \tilde{\chi} \end{array}\right\} \to \phi \to \tilde{\phi}$$

$i\theta^\alpha$

$$\phi = U(\theta)\chi \qquad \phi \to \tilde{\phi} = g\phi$$

$$\tilde{\phi} = U(\tilde{\theta})\tilde{\chi}$$

$E = \sqrt{\vec{q}^{\,2} + m^2}$

How does G act on θ^α and χ?

$$\left.\begin{array}{l} \theta \to \tilde{\theta} \\ \chi \to \tilde{\chi} \end{array}\right\} \to \phi \to \tilde{\phi}$$

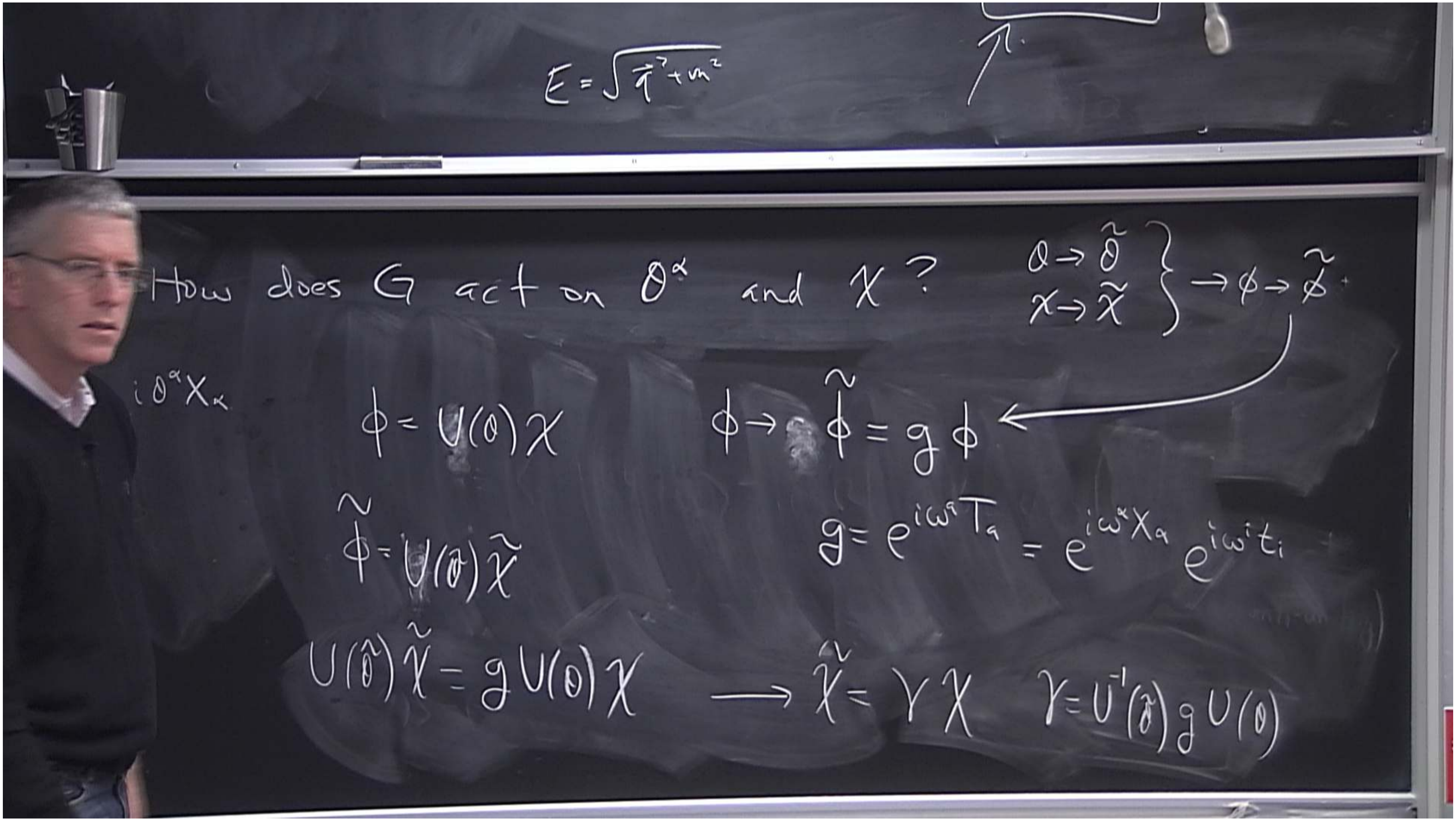
$U = e^{i\theta^\alpha X_\alpha}$

$\phi = U(\theta)\chi$

$\tilde{\phi} = U(\tilde{\theta})\tilde{\chi}$

$\phi \to \tilde{\phi} = g\phi$

$g = e^{i\omega^a T_a}$



$E = \sqrt{\vec{q}^2 + m^2}$
How does G act on θ^α and χ?
$\theta \to \tilde{\theta}$, $\chi \to \tilde{\chi}$ $\} \to \phi \to \tilde{\phi}$
$i\theta^\alpha X_\alpha$
$\phi = U(\theta)\chi$
$\phi \to \tilde{\phi} = g\phi$
$\tilde{\phi} = U(\tilde{\theta})\tilde{\chi}$
$g = e^{i\omega^a T_a} = e^{i\omega^\alpha X_\alpha} e^{i\omega^i t_i}$
$U(\tilde{\theta})\tilde{\chi} = g U(\theta)\chi \longrightarrow \tilde{\chi} = \gamma\chi$
$\gamma = U^{-1}(\tilde{\theta})\, g\, U(\theta)$

$E = \sqrt{\vec{q}^{\,2} + m^2}$

How does G act on θ^α and χ?

$\left.\begin{array}{l}\theta \to \tilde{\theta} \\ \chi \to \tilde{\chi}\end{array}\right\} \to \phi \to \tilde{\phi}$

$e^{i\theta^\alpha X_\alpha}$

$\phi = U(\theta)\chi \qquad \phi \to \tilde{\phi} = g\phi$

$\tilde{\phi} = U(\tilde{\theta})\tilde{\chi} \qquad g = e^{i\omega^a T_a} = e^{i\omega^\alpha X_\alpha} e^{i\omega^i t_i}$

$U(\tilde{\theta})\tilde{\chi} = g\,U(\theta)\chi \longrightarrow \tilde{\chi} = \gamma\chi \qquad \gamma = U^{-1}(\tilde{\theta})\,g\,U(\theta)$

$E = \sqrt{\vec{q}^{\,2} + m^2}$

How does G act on θ^α and χ?

$$\left.\begin{array}{l} \theta \to \tilde{\theta} \\ \chi \to \tilde{\chi} \end{array}\right\} \to \phi \to \tilde{\phi}$$

$U = e^{i\theta^\alpha X_\alpha}$

$$\phi = U(\theta)\chi \qquad \phi \to \tilde{\phi} = g\,\phi$$

$$\tilde{\phi} = U(\tilde{\theta})\tilde{\chi} \qquad g = e^{i\omega^a T_a} = e^{i\omega^\alpha X_\alpha}\, e^{i\omega^i t_i}$$

$$U(\tilde{\theta})\tilde{\chi} = g\,U(\theta)\chi \longrightarrow \tilde{\chi} = \gamma\chi \qquad \gamma = U^{-1}(\tilde{\theta})\, g\, U(\theta)$$

Rule: $U(\theta) = e^{i\theta^\alpha X_\alpha}$.

decompose $g\,U(\theta) = U(\hat{\theta})\,\gamma$. where $\gamma = e^{iu^i t_i} \in H$

$U(\hat{\theta}) = e^{i\hat{\theta}^\alpha X_\alpha}$ for some $\hat{\theta}^\alpha$

notice $\gamma = \gamma(\theta, \hat{\theta}, g)$

$u^i = u^i(\theta, \hat{\theta}, g)$

Coleman – Callan – Wess – Zumino

nonlinear realization

Rule: $\theta \to \tilde{\theta}$: $U(\theta) = e^{i\theta^\alpha X_\alpha}$.

decompose $\quad g\,U(\theta) = U(\tilde{\theta})\,\gamma \quad$ where $\gamma = e^{i u^i t_i} \in H$

$$U(\tilde{\theta}) = e^{i\tilde{\theta}^\alpha X_\alpha}$$

for some $\tilde{\theta}^\alpha$

notice $\gamma = \gamma(\theta, \tilde{\theta}, g)$

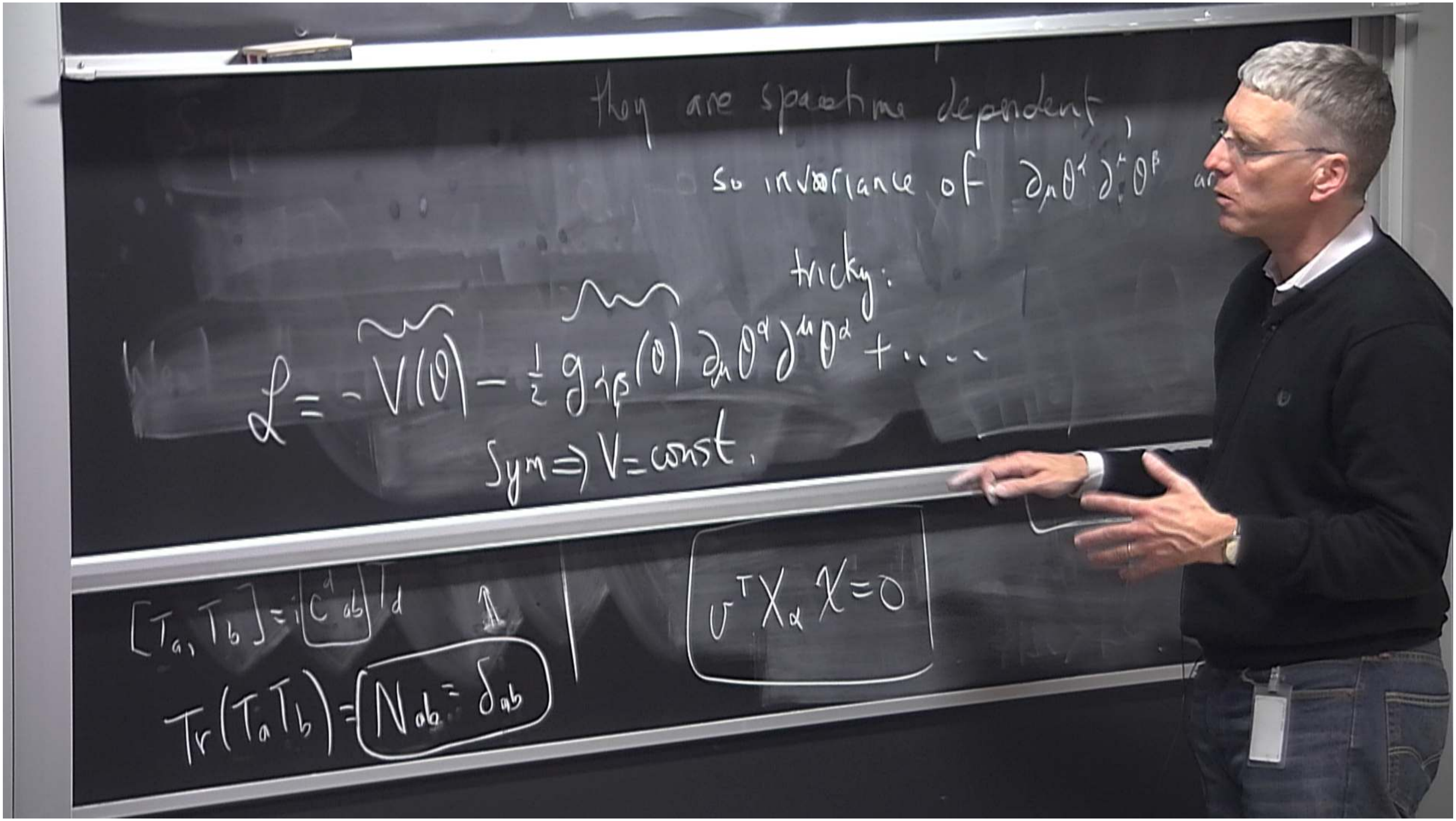
$u^i = u^i(\theta, \tilde{\theta}, g)$

rule for $\chi \to \tilde{\chi}$: $\quad \tilde{\chi} = \gamma\,\chi$

$$\delta\theta^\alpha = \tilde{\theta}^\alpha - \theta^\alpha = \omega^\alpha \underbrace{- C^\alpha{}_{\beta\gamma}}_{\text{"adjoint" representation}} \omega^\beta \theta^\gamma + O(\theta^2)$$

$$(T_\beta)^\alpha{}_\gamma = i C^\alpha{}_{\beta\gamma}$$

$$\gamma = e^{i u^i t_i} \qquad u^i = -C^i{}_{\alpha\beta}\,\omega^\alpha \theta^\beta + O(\theta^2)$$


they are spacetime dependent,
so invariance of $\partial_\mu \theta^\alpha \partial^\mu \theta^\beta$
tricky:
$\mathcal{L} = -V(\theta) - \frac{1}{2} g_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\alpha + \dots$
Sym $\Rightarrow V = \text{const.}$
$[T_a, T_b] = i\, C^d{}_{ab}\, T_d$
$\mathrm{Tr}(T_a T_b) = N_{ab} = \delta_{ab}$
$v^T X_\alpha \chi = 0$

Coleman – Callan – Wess – Zumino

nonlinear realization

Rule: $\theta \to \tilde{\theta}$: $U(\theta) = e^{i\theta^\alpha X_\alpha}$.

decompose $g\,U(\theta) = U(\tilde{\theta})\,\gamma$. where $\gamma = e^{iu^i t_i} \in H$

$U(\tilde{\theta}) = e^{i\tilde{\theta}^\alpha X_\alpha}$ for some $\tilde{\theta}^\alpha$

notice $\gamma = \gamma(\theta, \tilde{\theta}, g)$

$u^i = u^i(\theta, \tilde{\theta}, g)$

rule for $\chi \to \tilde{\chi}$: $\tilde{\chi} = \gamma\,\chi$

$A_\mu(\theta)$

$D_\mu e_\alpha{}^\beta(\theta) = \partial_\mu e_\alpha{}^\beta + A$ term.