

Title: Introduction to Effective Field Theories - Lecture 14

Date: Feb 28, 2014 02:30 PM

URL: <http://pirsa.org/14020094>

Abstract:

$$\mathcal{L} = -\frac{1}{4} Z_{\text{ph}} F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$$

$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e_{\text{phys}} J^\mu A_\mu$$

$e$

$\alpha_0$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$$

$$+ e_{\text{phys}} J^\mu A_\mu$$

$$e_{\text{ph}} = \frac{e_0}{\sqrt{\epsilon_{\text{ph}}}}$$

$$\alpha_{\text{ph}} = \frac{e_{\text{ph}}^2}{4\pi} = \frac{\alpha_0}{\epsilon_{\text{ph}}}$$

$$\ln\left(\frac{m_0^2}{m^2}\right)$$

$$\mathcal{L} = -\frac{1}{4} Z_{ph} F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$$

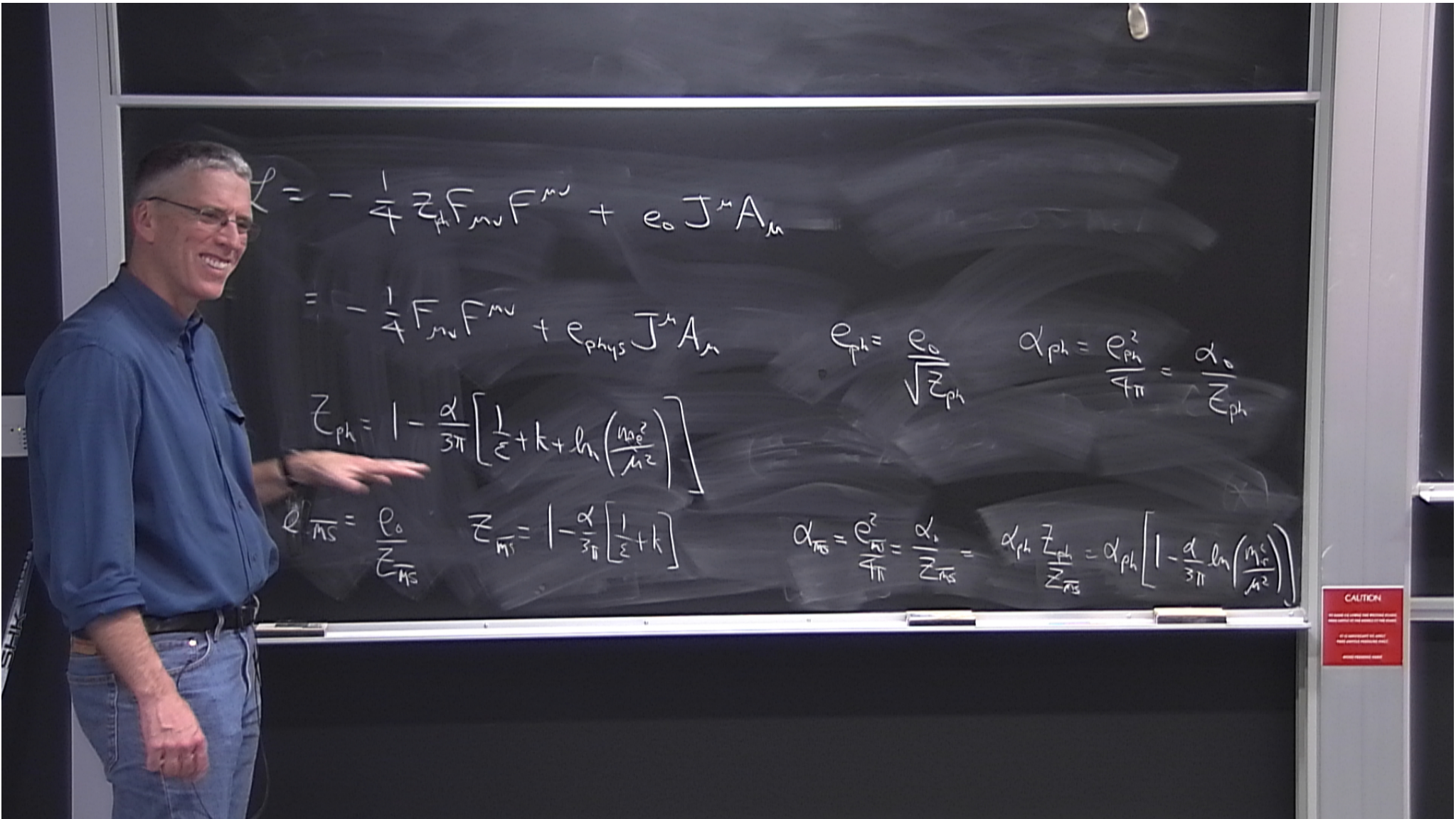
$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e_{phys} J^\mu A_\mu$$

$$Z_{ph} = 1 - \frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon} + k + \ln\left(\frac{m_0^2}{\mu^2}\right) \right]$$

$$Z_{MS} = \frac{\rho_0}{Z_{MS}} \quad Z_{MS} = 1 - \frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon} + k \right]$$

$$e_{ph} = \frac{\rho_0}{\sqrt{Z}}$$

$$\alpha_{MS} = \frac{\rho_0^2}{4\pi^2 \epsilon}$$



$$\mathcal{L} = -\frac{1}{4} Z_{ph} F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$$

$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e_{phys} J^\mu A_\mu$$

$$e_{ph} = \frac{e_0}{\sqrt{Z_{ph}}}$$

$$\alpha_{ph} = \frac{e_{ph}^2}{4\pi} = \frac{\alpha_0}{Z_{ph}}$$

$$Z_{ph} = 1 - \frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon} + k + \ln\left(\frac{m_e^2}{\mu^2}\right) \right]$$

$$Z_{MS} = \frac{e_0}{e_{MS}}$$

$$Z_{MS} = 1 - \frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon} + k \right]$$

$$\alpha_{MS} = \frac{e_{MS}^2}{4\pi} = \frac{\alpha_0}{Z_{MS}} = \alpha_{ph} \frac{Z_{ph}}{Z_{MS}} = \alpha_{ph} \left[ 1 - \frac{\alpha}{3\pi} \ln\left(\frac{m_e^2}{\mu^2}\right) \right]$$

CAUTION  
The board is heavy and should be moved with care.  
All equipment is used with safety in mind.  
Please do not touch the board.

$$-\frac{1}{4} Z_{ph} F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$$

$$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e_{phys} J^\mu A_\mu$$

$$e_{ph} = \frac{e_0}{\sqrt{Z_{ph}}}$$

$$\alpha_{ph} = \frac{e_{ph}^2}{4\pi} = \frac{\alpha_0}{Z_{ph}}$$

$$Z_{ph} = 1 - \frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon} + k + \ln\left(\frac{m_e^2}{\mu^2}\right) \right]$$

$$\frac{e_0}{Z_{MS}}$$

$$Z_{MS} = 1 - \frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon} + k \right]$$

$$\alpha_{MS} = \frac{e_0^2}{4\pi Z_{MS}} = \frac{\alpha_0}{Z_{MS}} = \alpha_{ph} \frac{Z_{ph}}{Z_{MS}} = \alpha_{ph} \left[ 1 - \frac{\alpha}{3\pi} \ln\left(\frac{m_e^2}{\mu^2}\right) \right]$$

CAUTION  
 The board is heavy and should never be used as a support for the board.  
 All components of the board and energy storage device are protected under patent.

$$\mathcal{L} = -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$$

$$= -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + e_{\text{phys}} J^\mu A_\mu$$

$$e_{\text{ph}} = \frac{e}{\sqrt{\epsilon_{\text{ph}}}}$$

$$\alpha_{\text{ph}} = \frac{e_{\text{ph}}^2}{4\pi} = \frac{\alpha_0}{\epsilon_{\text{ph}}}$$

$$= \frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon + k} + \ln\left(\frac{m_0^2}{\mu^2}\right) \right]$$

$$\frac{d\alpha}{d\ln\mu} = -\frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon + k} \right]$$

$$\alpha_{\text{ph}} = \frac{e_{\text{ph}}^2}{4\pi} = \frac{\alpha}{\epsilon_{\text{ph}}} = \alpha_{\text{ph}} \frac{\epsilon_{\text{ph}}}{\epsilon_{\text{ph}}} = \alpha_{\text{ph}} \left[ 1 - \frac{d}{3\pi} \ln\left(\frac{m_0^2}{\mu^2}\right) \right]$$

CAUTION  
 Do not touch the board when the board is hot.  
 Do not touch the board when the board is hot.  
 Do not touch the board when the board is hot.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$$

$$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e_{\text{phys}} J^\mu A_\mu$$

$$e_{\text{ph}} = \frac{e}{\sqrt{Z_{\text{ph}}}}$$

$$\alpha_{\text{ph}} = \frac{e_{\text{ph}}^2}{4\pi} = \frac{\alpha_0}{Z_{\text{ph}}}$$

$$1 - \frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon} + k + \ln\left(\frac{m_0^2}{\mu^2}\right) \right]$$

$$Z_{\overline{\text{MS}}} = 1 - \frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon} + k \right]$$

$$\alpha_{\overline{\text{MS}}} = \frac{e^2}{4\pi} = \frac{\alpha}{Z_{\overline{\text{MS}}}^2} = \alpha_{\text{ph}} Z_{\overline{\text{MS}}} = \alpha_{\text{ph}} \left[ 1 - \frac{\alpha}{3\pi} \ln\left(\frac{m_0^2}{\mu^2}\right) \right]$$



$$\mathcal{L} = \frac{1}{2} Z_{ph} F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$$

$$\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + e_{phys} J^\mu A_\mu$$

$$e_{ph} = \frac{e}{\sqrt{Z_{ph}}}$$

$$\alpha_{ph} = \frac{e_{ph}^2}{4\pi} = \frac{\alpha_0}{Z_{ph}}$$

$$= \frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon} + k + \ln\left(\frac{m_0^2}{\mu^2}\right) \right]$$

$$Z_{\overline{MS}} = 1 - \frac{\alpha}{3\pi} \left[ \frac{1}{\epsilon} + k \right]$$

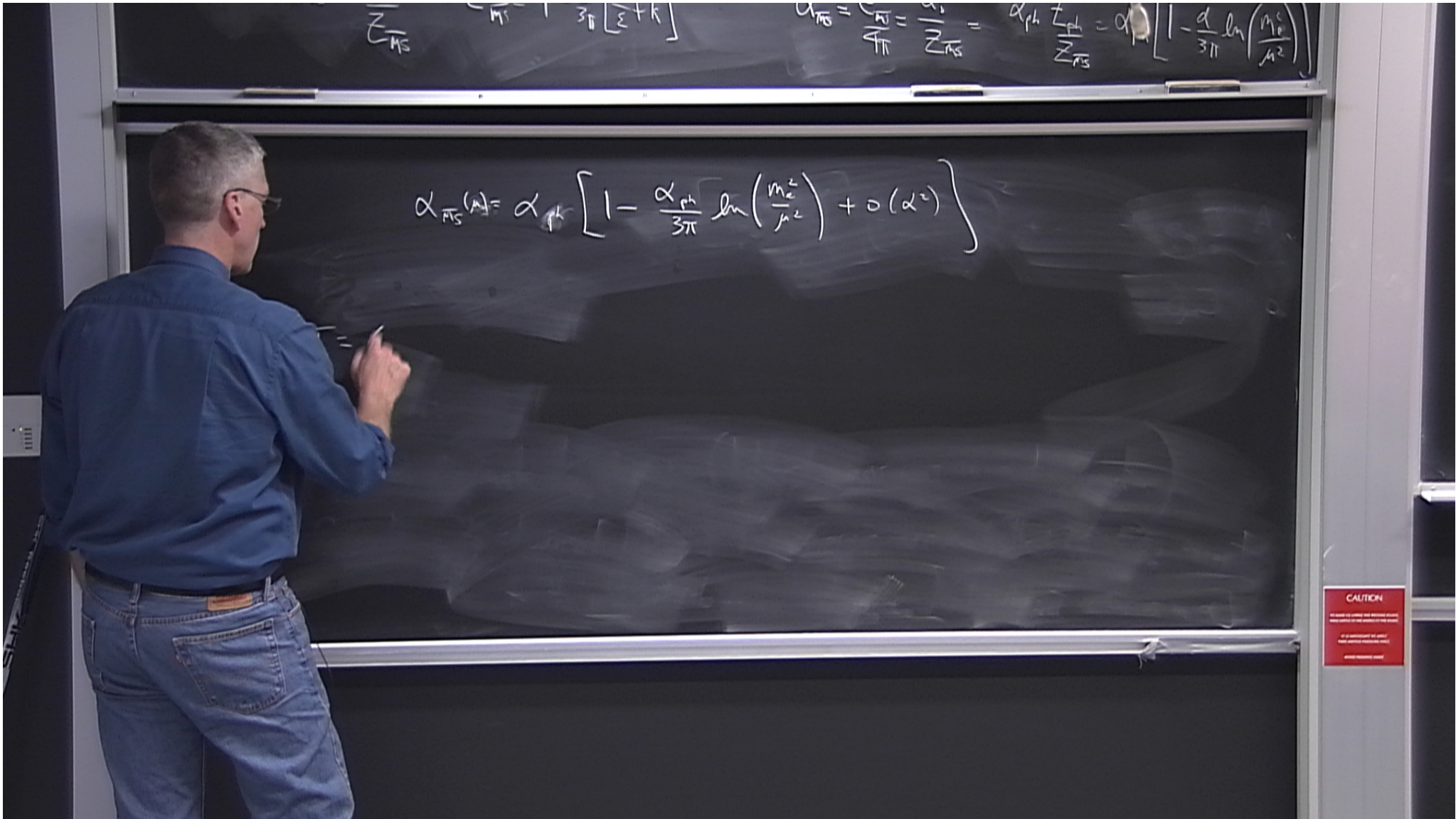
$$\alpha_{\overline{MS}} = \frac{e^2}{4\pi} = \frac{\alpha}{Z_{\overline{MS}}} = \alpha_{ph} Z_{\overline{MS}} = \alpha_{ph} \left[ 1 - \frac{\alpha}{3\pi} \ln\left(\frac{m_0^2}{\mu^2}\right) \right]$$

CAUTION  
 Do not touch the board when it is hot.  
 All components are protected by a fuse.  
 Please do not touch the board when it is hot.

$$\frac{Z_{MS}}{Z_{MS}} = \frac{m_s}{3\pi[\epsilon+k]} \quad \alpha_{MS} = \frac{\alpha_{ph}}{4\pi} = \frac{\alpha_s}{Z_{MS}} = \alpha_{ph} \frac{Z_{ph}}{Z_{MS}} = \alpha_{ph} \left[ 1 - \frac{\alpha}{3\pi} \ln\left(\frac{m_s}{\mu^2}\right) \right]$$

$$\alpha_{MS} = \alpha_{ph} \left[ 1 - \frac{\alpha_{ph}}{3\pi} \ln\left(\frac{m_s^2}{\mu^2}\right) + o(\alpha^2) \right]$$

**CAUTION**  
 Do not touch the board when it is hot.  
 All components are hot and may be damaged.  
 Please handle with care.



CAUTION  
The board is heavy and should be used with care to avoid injury.  
All glass should be used with care to avoid injury.  
Always use proper technique.

$$\frac{Z}{\mu^2} \quad \frac{1}{\mu^2} \quad \frac{1}{3\pi} [\Sigma + k] \quad \alpha_{\overline{MS}} = \frac{\alpha_{\overline{MS}}}{4\pi} = \frac{\alpha_s}{Z_{\overline{MS}}} = \alpha_{\overline{MS}} \left[ 1 - \frac{\alpha_{\overline{MS}}}{3\pi} \ln \left( \frac{\mu_0}{\mu^2} \right) \right]$$

$$\alpha_{\overline{MS}}(\mu) = \alpha_{\overline{MS}} \left[ 1 - \frac{\alpha_{\overline{MS}}}{3\pi} \ln \left( \frac{\mu_c^2}{\mu^2} \right) + O(\alpha^2) \right]$$

notice  $\alpha_{\overline{MS}}(\mu = \mu_c) = \alpha_{\overline{MS}}$

**CAUTION**  
 Do not touch the board when it is hot.  
 All glass is under pressure and may shatter if hit.  
 Please do not touch the board when it is hot.

$$\frac{Z}{m_s} \quad \frac{1}{3\pi} [\epsilon + k] \quad \alpha_{\pi\pi} = \frac{1}{4\pi} = \frac{\alpha_s}{Z_{\pi\pi}} = \alpha_{ph} \frac{Z_{ph}}{Z_{\pi\pi}} = \alpha_{ph} \left[ 1 - \frac{\alpha}{3\pi} \ln \left( \frac{m_c}{\mu^2} \right) \right]$$

$$\alpha_{\pi\pi}(M) = \alpha_{ph} \left[ 1 - \frac{\alpha_{ph}}{3\pi} \ln \left( \frac{m_c^2}{\mu^2} \right) + O(\alpha^2) \right]$$

notice  $\alpha_{\pi\pi}(m=m_c) = \alpha_{ph} = \left( \frac{1}{137} \right)$

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CAUTION  
 Do not touch the board when it is hot.  
 All glassware is used and safety instructions apply.  
 Always wear your seat belt.

$$\frac{Z}{M_S} \quad \frac{1}{M_S} \quad \frac{1}{3\pi} [\Sigma + k] \quad \alpha_{MS} = \frac{\alpha_{ph}}{4\pi} = \frac{\alpha_s}{Z_{MS}} = \alpha_{ph} \frac{Z_{ph}}{Z_{MS}} = \alpha_{ph} \left[ 1 - \frac{d}{3\pi} \ln \left( \frac{m_c}{\mu^2} \right) \right]$$

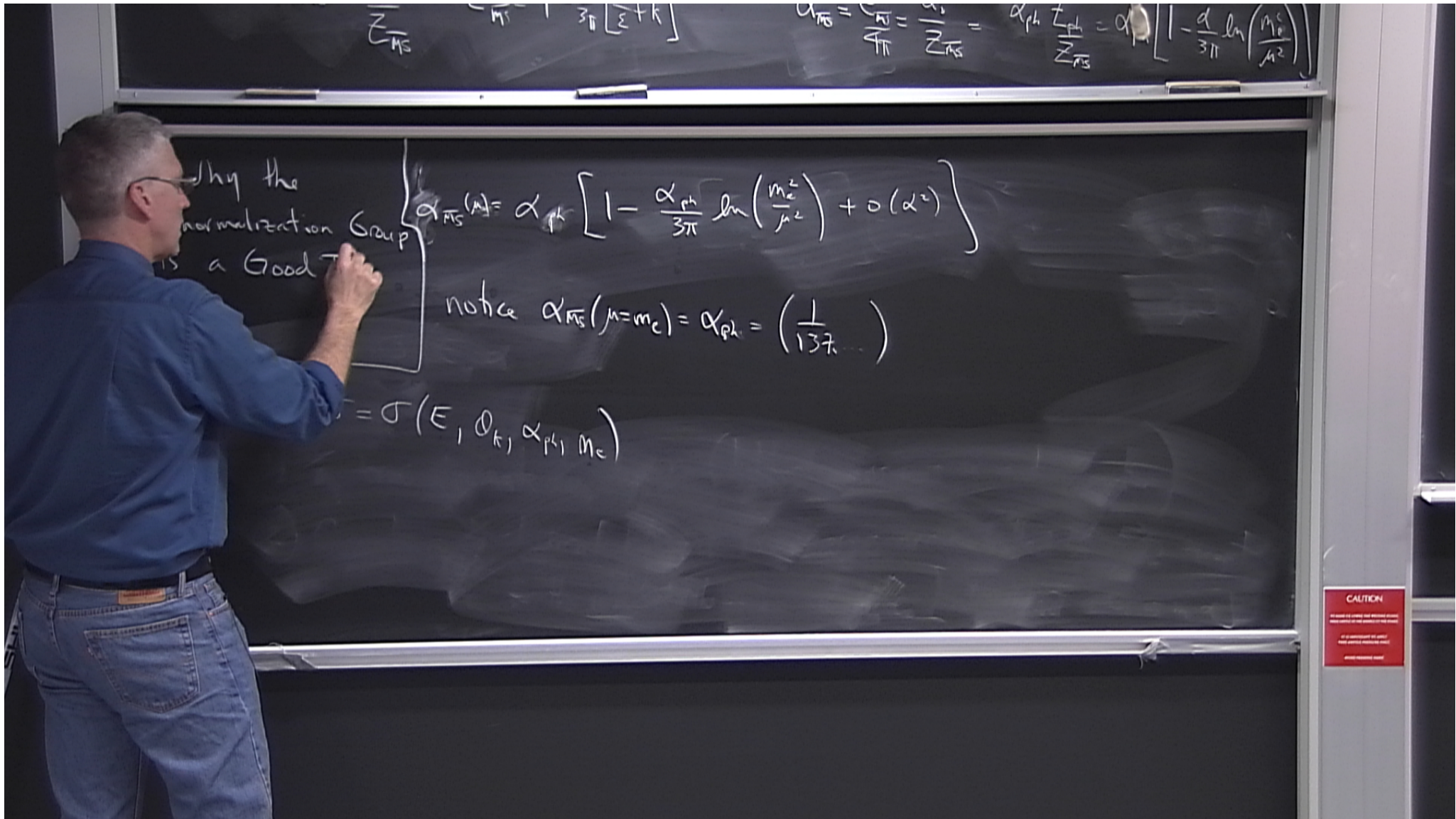
$$\alpha_{MS}(\mu) = \alpha_{ph} \left[ 1 - \frac{\alpha_{ph}}{3\pi} \ln \left( \frac{m_c}{\mu^2} \right) + O(\alpha^2) \right]$$

notice  $\alpha_{MS}(\mu = m_c) = \alpha_{ph} = \left( \frac{1}{137} \right)$

$(\alpha_{ph} = \alpha_{ph}(\mu = m_c))$



CAUTION  
 Do not touch the board when it is hot.  
 All glassware is used.  
 and safety instructions.



Why the normalization Group is a Good

$$\alpha_{MS}(\mu) = \alpha_{ph} \left[ 1 - \frac{\alpha_{ph}}{3\pi} \ln \left( \frac{m_c^2}{\mu^2} \right) + o(\alpha^2) \right]$$

notice  $\alpha_{MS}(\mu=m_c) = \alpha_{ph} = \left( \frac{1}{137} \right)$

$$\sigma = \sigma(E, Q_k, \alpha_{ph}, m_c)$$

CAUTION  
The board is heavy and should never  
be used as a shelf or for other  
purposes.  
All glassware on board  
and nearby should be used  
with extreme care.

$$\frac{Z}{\mu^2} \quad \frac{1}{\mu^2} \quad \frac{1}{3\pi} [\epsilon + k]$$

$$\alpha_{\overline{MS}} = \frac{\alpha_s}{4\pi} = \frac{\alpha_s}{Z_{\overline{MS}}} = \alpha_{ph} \frac{Z_{ph}}{Z_{\overline{MS}}} = \alpha_{ph} \left[ 1 - \frac{d}{3\pi} \ln \left( \frac{\mu_c}{\mu^2} \right) \right]$$

"Why the Renormalization is a Go

$$\alpha_{\overline{MS}}(\mu) = \alpha_{ph} \left[ 1 - \frac{\alpha_{ph}}{3\pi} \ln \left( \frac{\mu_c^2}{\mu^2} \right) + O(\alpha^2) \right]$$

notice  $\alpha_{\overline{MS}}(\mu = \mu_c) = \alpha_{ph} = \left( \frac{1}{137} \right)$

$(E, O_k, \alpha_{ph}, \mu_c)$  Suppose your interest is in  $E \gg \mu_c$ .





$$\frac{Z}{\mu^2} \quad \frac{1}{\mu^2} \quad \frac{1}{3\pi} [\epsilon + k] \quad \alpha_{\text{MS}} = \frac{\alpha_{\text{ph}}}{4\pi} = \frac{\alpha_0}{Z_{\text{MS}}} = \alpha_{\text{ph}} \frac{Z_{\text{ph}}}{Z_{\text{MS}}} = \alpha_{\text{ph}} \left[ 1 - \frac{\alpha}{3\pi} \ln \left( \frac{\mu_0}{\mu^2} \right) \right]$$

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Thing  
berg

$$\alpha_{\text{MS}}(\mu) = \alpha_{\text{ph}} \left[ 1 - \frac{\alpha_{\text{ph}}}{3\pi} \ln \left( \frac{\mu_0^2}{\mu^2} \right) + o(\alpha^2) \right]$$

notice  $\alpha_{\text{MS}}(\mu = m_e) = \alpha_{\text{ph}} = \left( \frac{1}{137} \right)$

$$\sigma = \sigma(E, \theta_k, \alpha_{\text{ph}}, m_e)$$

Suppose your interest is in  $E \gg m_e$ .

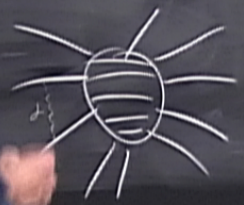
$$E \gg m_e \Rightarrow \sigma(E, \theta_k, \alpha_{\text{ph}}, m_e/E) \approx \frac{f(\theta_k, \alpha_{\text{ph}}, 0)}{E^2} \left( 1 + o\left(\frac{m_e^2}{E^2}\right) \right)$$

CAUTION  
All work on energy and electrical circuits  
shall comply with the standards of the Bureau  
of Standards and the National Fire Protection Association  
and the National Electrical Code.

$$\frac{d}{3\pi} \ln \left( \frac{m_1}{m_2} \right)$$

Aside: IR divergences arise when you mistakenly calculate something stupid.

$\rightarrow e^{e-\gamma}$  is IR divergent.



CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD ITSELF  
OR THE BOARD ITSELF

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$$\frac{d}{3\pi} \ln \left( \frac{m_0}{M^2} \right)$$

Aside: IR divergences arise when you mistakenly calculate something stupid.

$\phi(k)$  is IR divergent.



$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(p-k)^2 + m_i^2} \frac{1}{k^2}$$

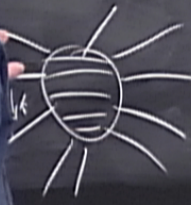
CAUTION  
DO NOT TOUCH THE BOARD WHEN IT IS HOT  
IT IS HEATED BY THE BOARD  
PLEASE BE CAREFUL

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PLEASE BE CAREFUL

$$\frac{d}{3\pi} \ln \left( \frac{m_1}{m_2} \right)$$

Aside: IR divergences arise when you mistakenly calculate something stupid.  
eg

$\gamma)$  is IR divergent.



$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{\underbrace{(p-k)^2 + m_1^2}_{p^2 + m_1^2 - 2pk + k^2}} \frac{1}{\underbrace{(q-k)^2 + m_2^2}_{q^2 + m_2^2 - 2qk + k^2}} \frac{1}{k^2} (\dots)$$

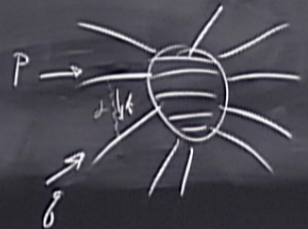
CAUTION  
DO NOT TOUCH THE BOARD WHEN THE BOARD IS BEING USED BY THE LECTURER.  
IF IT IS NECESSARY TO TOUCH THE BOARD, PLEASE ASK THE LECTURER.

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$$\frac{d}{3\pi} \ln \left( \frac{m_0}{\Lambda^2} \right)$$

Aside: IR divergences arise when you mistakenly calculate something stupid.

eg  $\sigma(e^+e^- \rightarrow e^+e^- \gamma)$  is IR divergent.



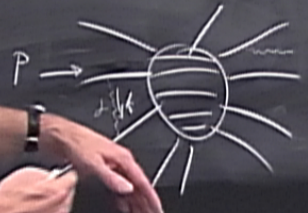
$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{\underbrace{(p-k)^2 + m_e^2}_{p^2 + m_e^2 - 2pk + k^2}} \frac{1}{\underbrace{(q-k)^2 + m_e^2}_{q^2 + m_e^2 - 2qk + k^2}} \frac{1}{k^2} (\dots)$$

CAUTION

$$\frac{d}{3\pi} \ln \left( \frac{m_1}{m_2} \right)$$

Aside: IR divergences arise when you mistakenly calculate something stupid.

$(e^+e^- \rightarrow e^+e^- \gamma)$  is IR divergent.



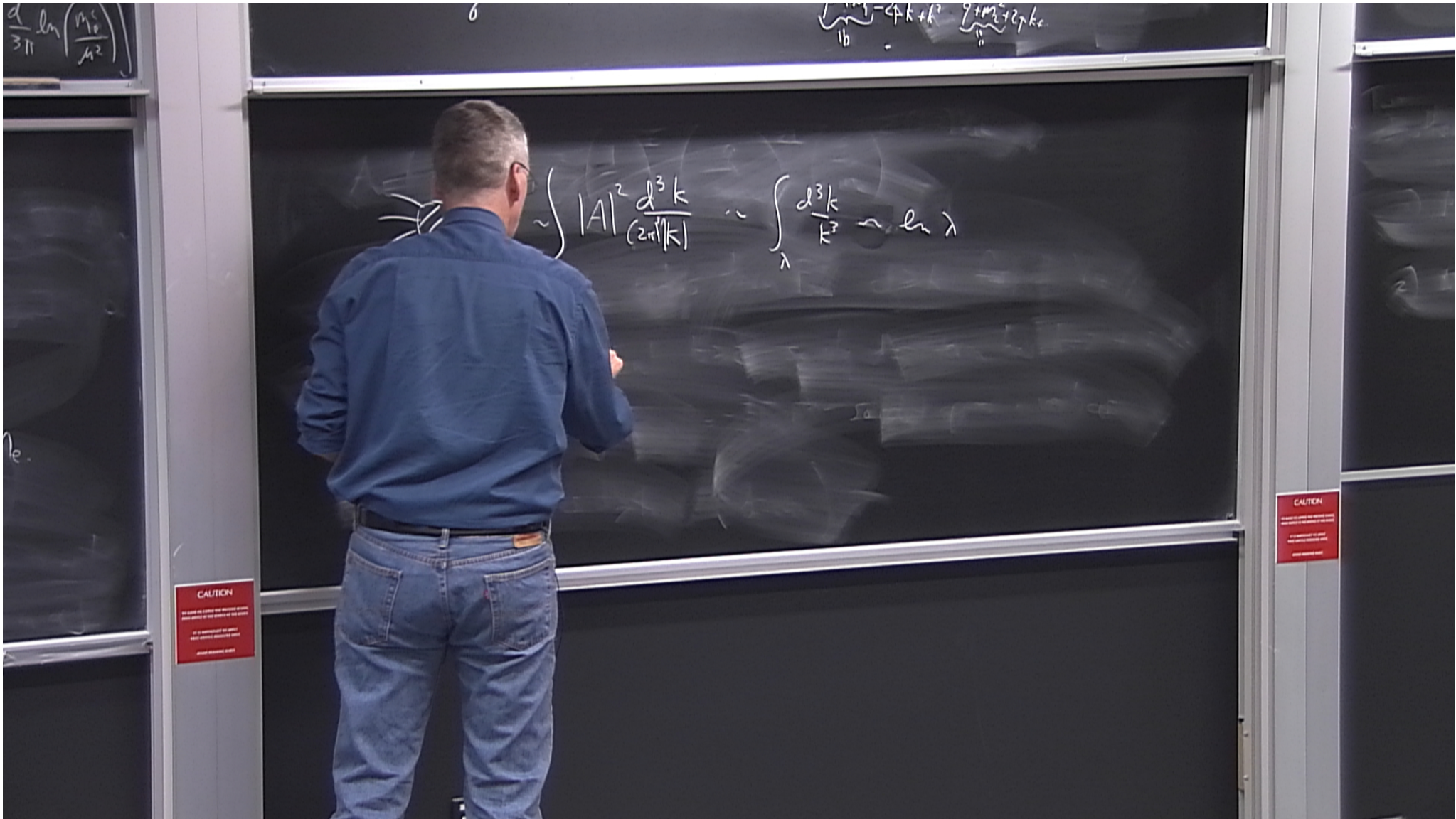
ln  $\lambda$  as  $\lambda \rightarrow 0$

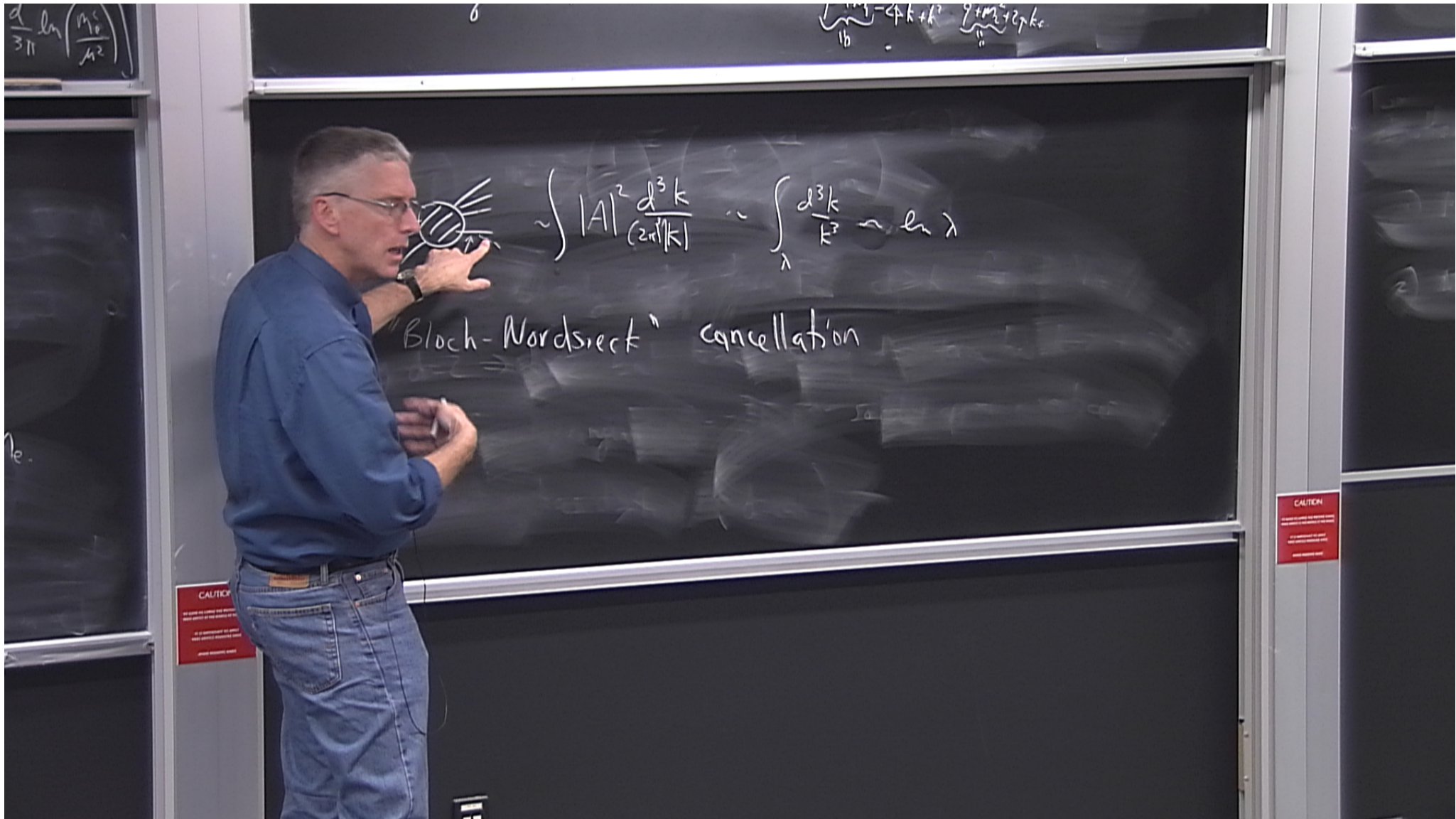
$$\int \frac{dk}{2\pi} \frac{1}{(p-k)^2 + m_1^2} \frac{1}{(q+k)^2 + m_2^2} \frac{1}{k^2} (\dots)$$

$\underbrace{p^2 + m_1^2 - 2pk}_{\text{b}}$       $\underbrace{q^2 + m_2^2 + 2pk}_{\text{c}}$

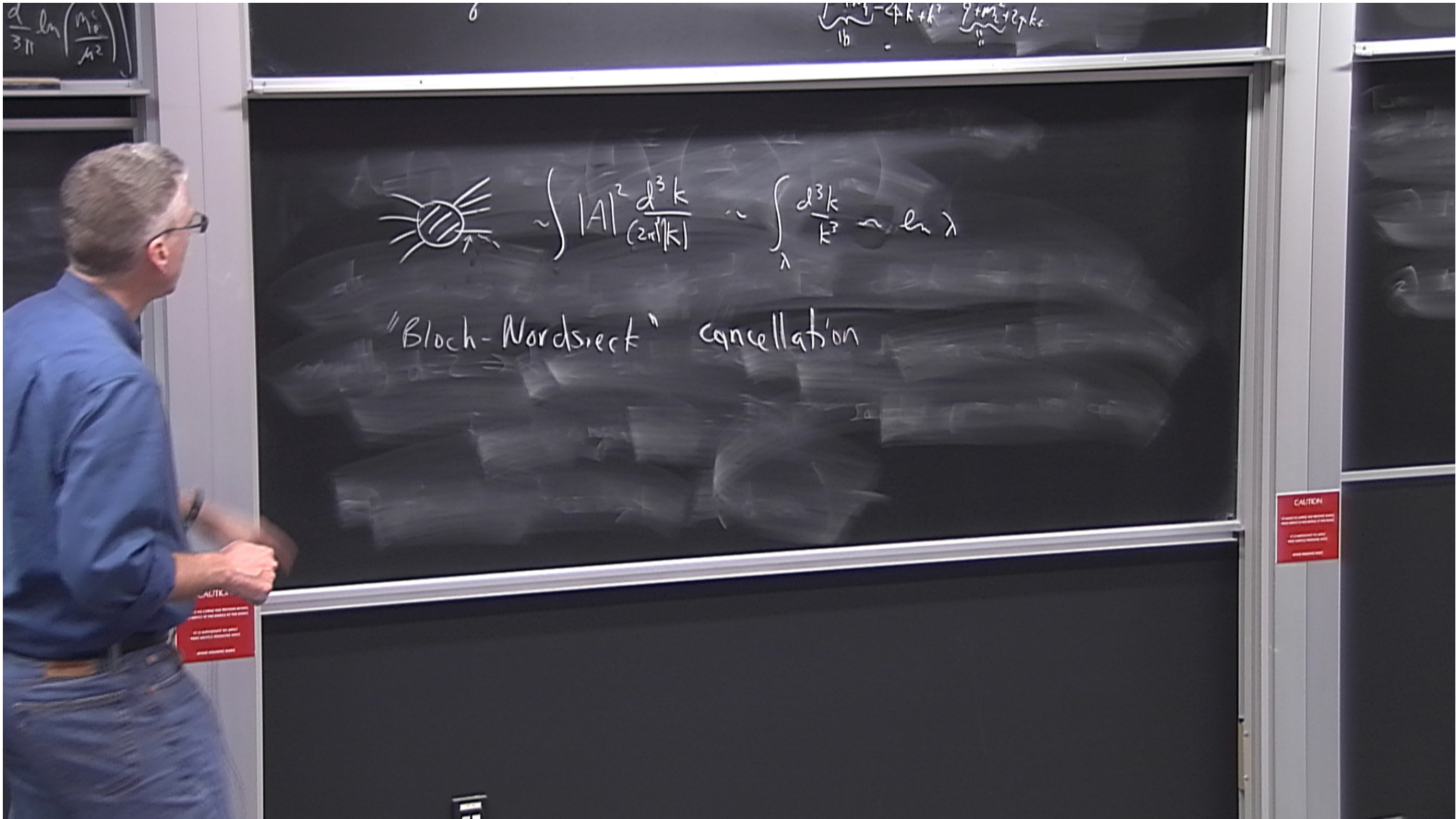
CAUTION  
DO NOT TOUCH THE BOARD  
IF IT IS DAMAGED OR  
IF IT IS NOT WORKING  
PLEASE REPORT TO THE  
TECHNICAL STAFF

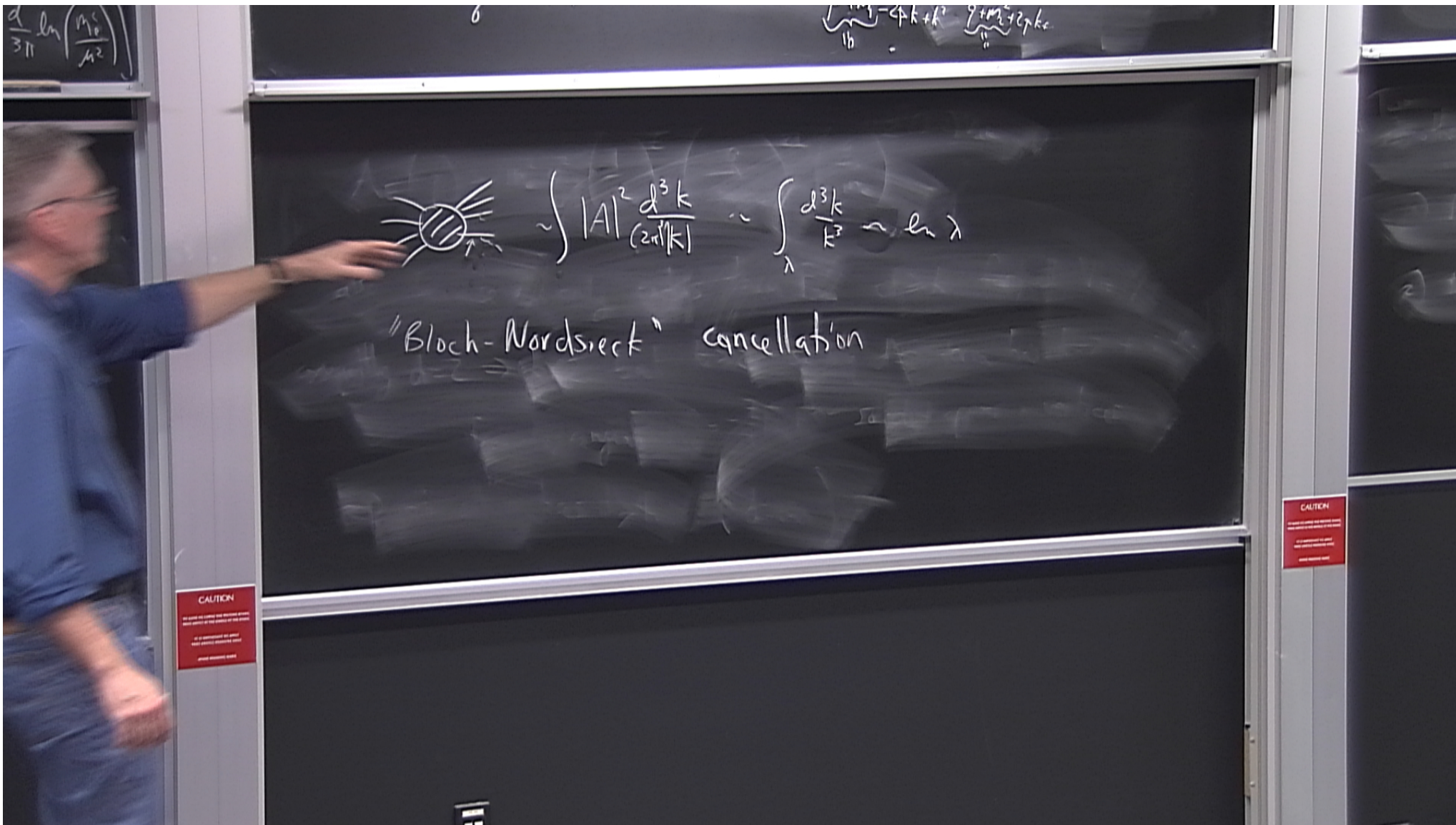
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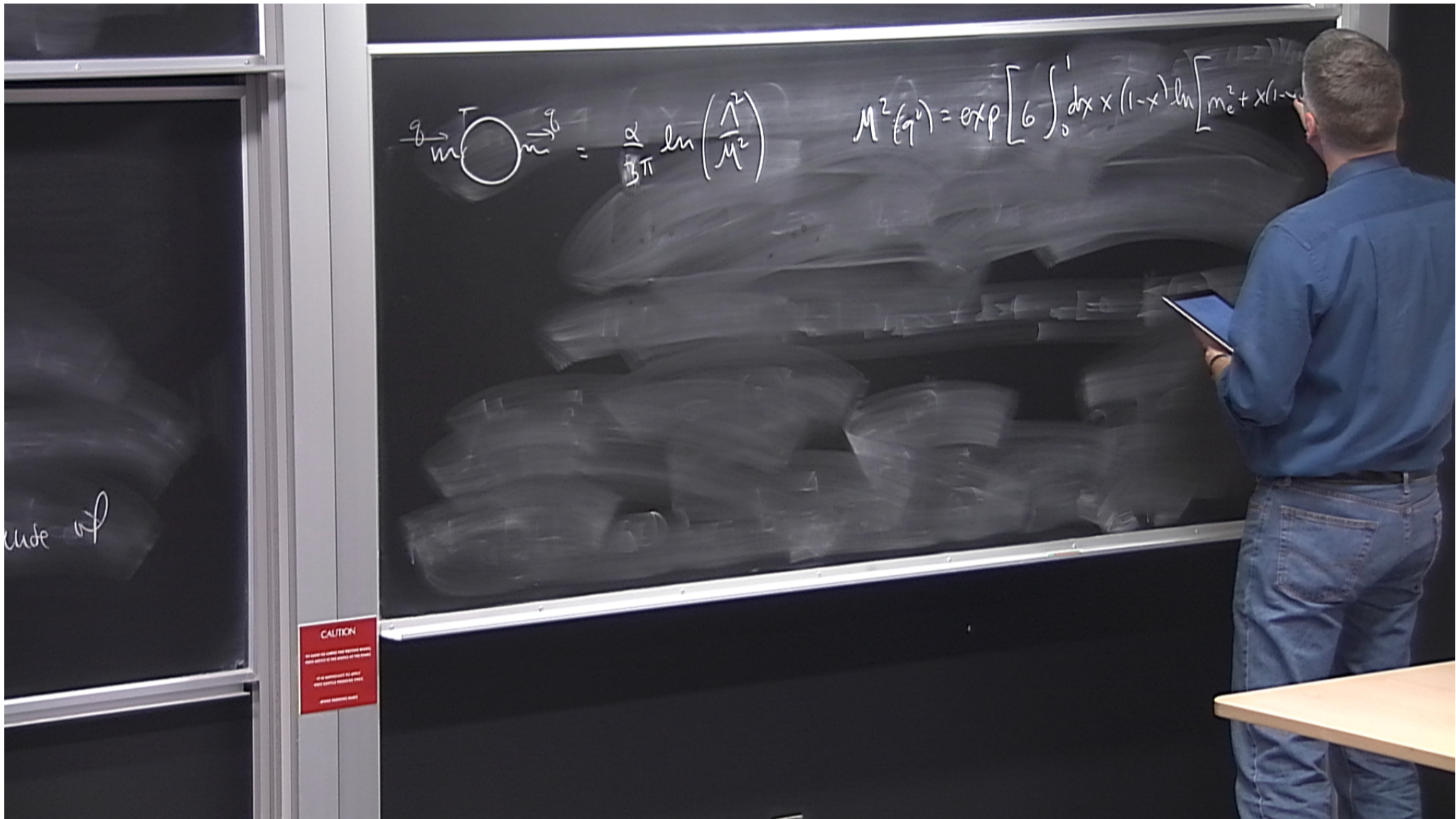












$$\frac{g}{m} = \frac{\alpha}{3\pi} \ln\left(\frac{\Lambda^2}{M^2}\right)$$

$$M^2(g^2) = \exp\left[6 \int_0^1 dx x(1-x) \ln[m_c^2 + x(1-x)M^2]\right]$$

CAUTION  
DO NOT TOUCH THE BOARD WHEN  
IT IS BEING USED BY THE INSTRUCTOR  
OR STUDENTS

$$g \rightarrow m \circlearrowleft \overset{T}{=} \overset{q}{m} = \frac{\alpha}{3\pi} \ln\left(\frac{\Lambda^2}{M^2}\right)$$

$$M^2(q^2) = \exp\left[6 \int_0^1 dx x(1-x) \ln\left[m_c^2 + x(1-x)q^2\right]\right]$$

$$\approx m_c^2 \quad \text{if } m_c^2 \gg q^2$$

$$\approx e^{-5/3} q^2 \quad \text{if } m_c^2 \ll q^2$$

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CAUTION  
 Do not lean on the chalkboard.  
 Do not use the chalkboard as a desk.  
 Do not use the chalkboard as a shelf.  
 Do not use the chalkboard as a table.



$$g \rightarrow m \overset{\text{T}}{\circ} \vec{m} \rightarrow g = \frac{\alpha}{3\pi} \ln\left(\frac{\Lambda^2}{M^2}\right) = \Pi_0(q^2) M^2(q^2)$$

$$\int_0^1 dx x(1-x) \ln[m_c^2 + x(1-x)q^2]$$

$$\text{if } m_c^2 \gg q^2$$

$$m_c^2 \ll q^2$$

physical renormalization ask  $\Pi(q^2=0) = 0$

$$\Pi_{ph}(q^2) = \Pi_0(q^2) - \Pi_0(0) = \frac{\alpha}{3\pi} \ln\left(\frac{m_c^2}{M^2(q^2)}\right)$$

CAUTION

$$\begin{aligned}
 \vec{q} \rightarrow m \text{---} \text{---} \text{---} \vec{q} &= \frac{\alpha}{3\pi} \ln\left(\frac{\Lambda^2}{M^2}\right) = \Pi_0(q^2) \quad M^2(q^2) = \exp\left[6 \int_0^1 dx x(1-x) \ln\left[m_c^2 + x(1-x)q^2\right]\right] \\
 &\approx m_c^2 \quad \text{if } m_c^2 \gg q^2 \\
 &\approx e^{-5/3} q^2 \quad \text{if } m_c^2 \ll q^2.
 \end{aligned}$$

physical renormalization ask

$$\Pi_{ph}(q^2) = \Pi_0(q^2) - \Pi_0(0) = \frac{\alpha}{3\pi} \ln\left(\frac{m_c^2}{M^2(q^2)}\right)$$

singularity

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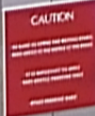
CAUTION

$$\begin{aligned}
 \frac{q \rightarrow T}{m} \bigcirc \frac{q \rightarrow b}{m} &= \frac{\alpha}{3\pi} \ln\left(\frac{\Lambda^2}{M^2}\right) = \overline{\Pi}(q^2) \quad M^2(q^2) = \exp\left[6 \int_0^1 dx x(1-x) \ln\right] \\
 &\approx m_e^2 \quad \text{if } m_e^2 \gg q^2 \\
 &\approx e^{-5/3} q^2 \quad \text{if } m_e^2 \ll q^2.
 \end{aligned}$$

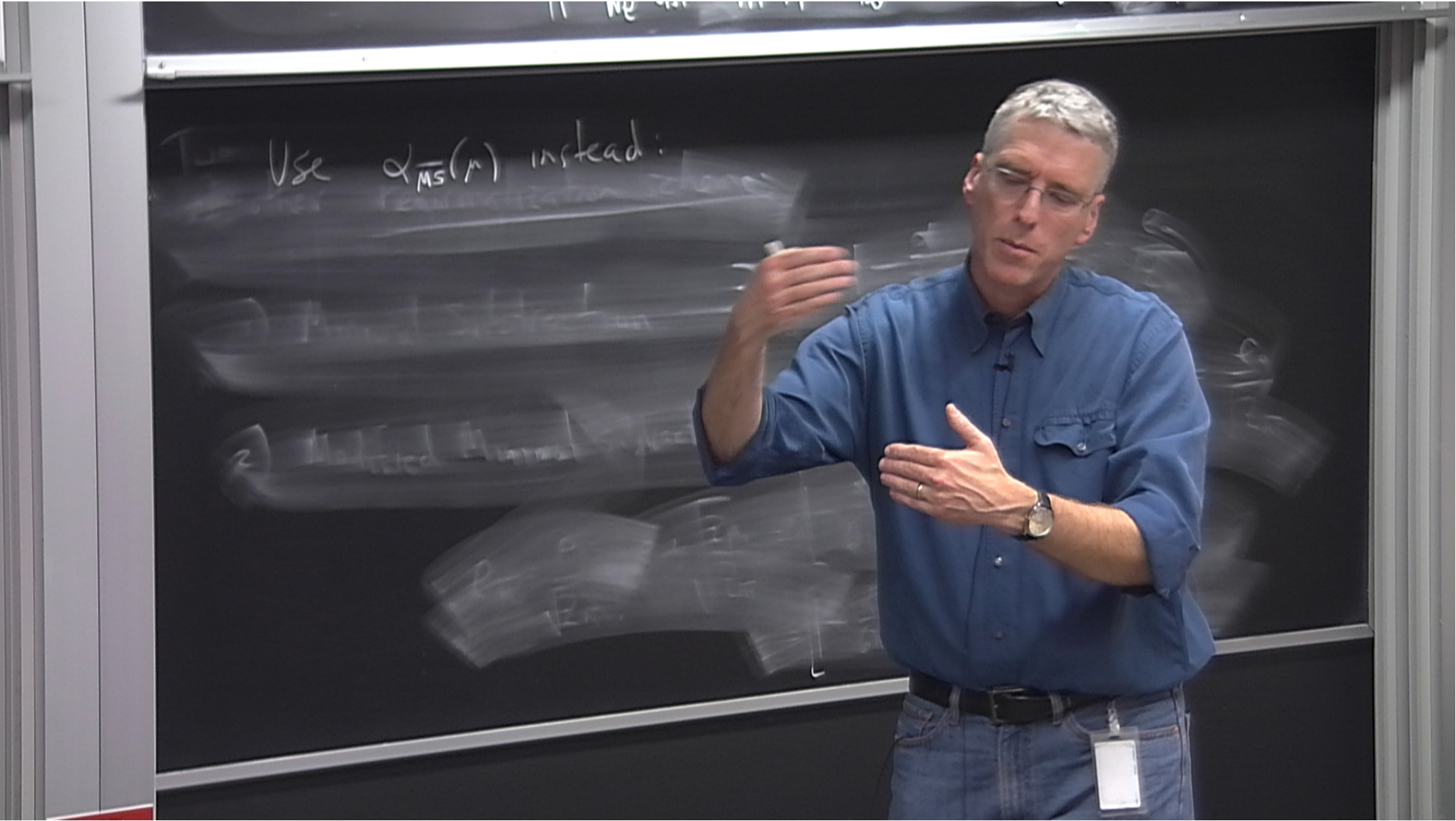
physical renormalization ask  $\overline{\Pi}(q^2=0) = 0$

$$\overline{\Pi}_{ph}(q^2) = \overline{\Pi}_0(q^2) - \overline{\Pi}_0(0) = \frac{\alpha}{3\pi} \ln\left(\frac{m_e^2}{M^2(q^2)}\right) \leftarrow \text{Now we have as } m_0 \rightarrow 0.$$

If we ask  $\overline{\Pi}(0) \neq 0$  the answer is again smooth







Use  $\alpha_{MS}(\mu)$  instead in the dimensional argument, since  
then  $f$  won't generically have a singularity as  $m_e \rightarrow 0$ .

$$\sigma = \sigma(E, Q_k, \alpha_{MS}(\mu), \mu, m_e) = \frac{1}{E^2} f\left(Q_k, \alpha_{MS}(\mu), \frac{\mu}{E}, \frac{m_e}{E}\right)$$

Use  $\alpha_{MS}(\mu)$  instead in the dimensional argument, since  
 then  $f$  won't generically have a singularity as  $m_e \rightarrow 0$ .

$$\sigma = \sigma(E, Q_k, \alpha_{MS}(\mu), \mu, m_e) = \frac{1}{E^2} f\left(Q_k, \alpha_{MS}(\mu), \frac{m_e}{E}\right)$$

- 3) Facts:
- 1)  $\sigma$  is independent of  $\mu$
  - 2) if  $\mu = m_e$ , then  $\alpha_{MS}(m_e) = \alpha_{ph} + f$  has a  $\log m_e$  in it.
  - 3) if  $\mu \neq m_e$  then there are no  $\log m_e$ 's in  $f$ .

$\alpha_{ph} \left[ 1 - \frac{\alpha_{ph}}{3\pi} \ln\left(\frac{m_e^2}{\mu^2}\right) + o(\alpha^2) \right]$

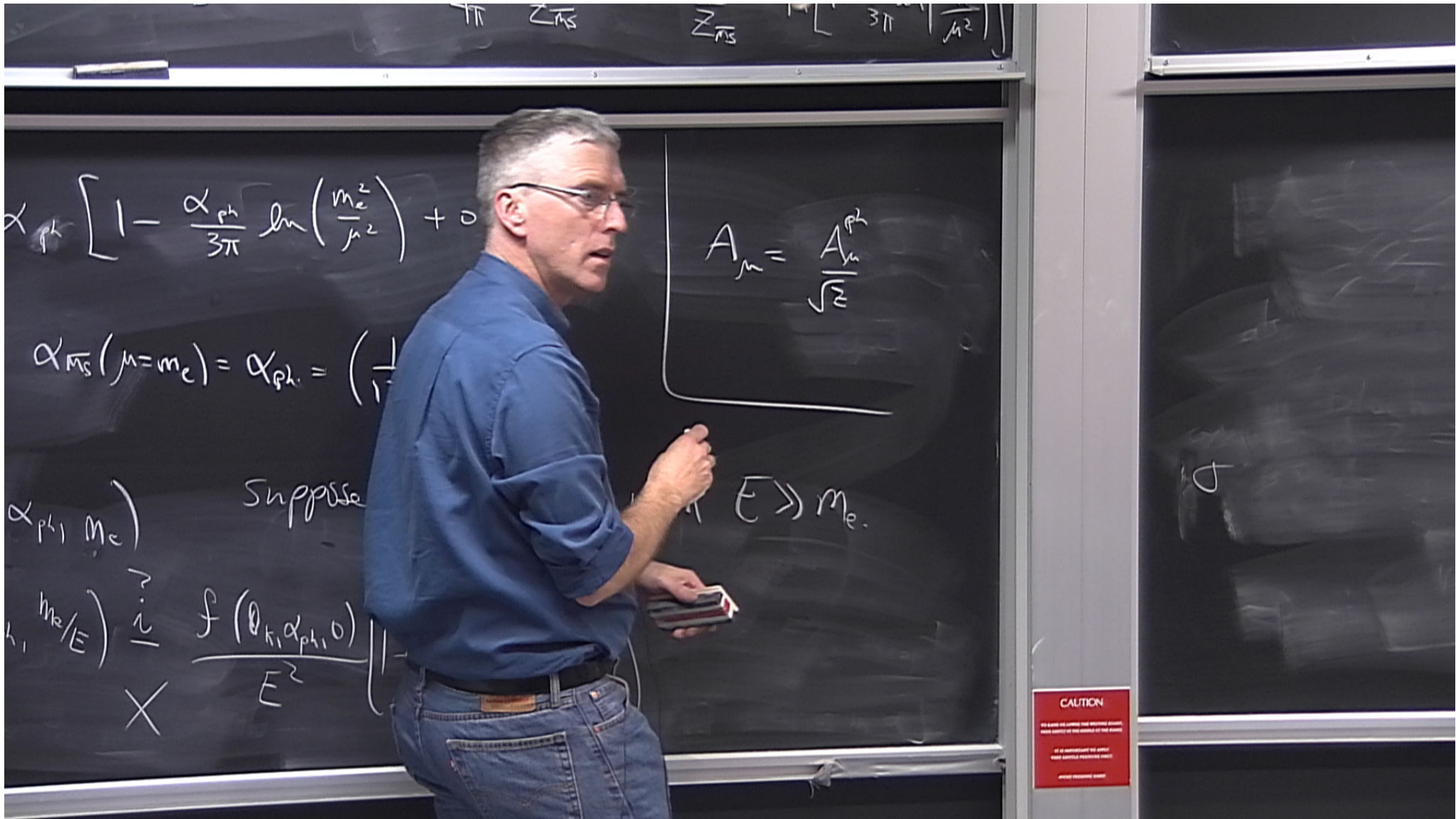
$\alpha_{MS}(\mu = m_e) = \alpha_{ph} = \left(\frac{1}{137}\right)$

Suppose  $y_0 \gg r_{te}$

$\frac{f(\theta_{k_i}, \alpha_{ph}, 0)}{E^2} \left( 1 + o(\dots) \right)$

$\alpha_{ph} m_e$   
 $m_e/E$

**CAUTION**  
 DO NOT STAND IN FRONT OF THE BEAMLINE  
 WHEN ENERGY IS BEING DELIVERED  
 ALWAYS WEAR YOUR SAFETY GOGGLES



$$\alpha_{ph} \left[ 1 - \frac{\alpha_{ph}}{3\pi} \ln \left( \frac{m_e^2}{\mu^2} \right) \right] + 0$$

$$A_m = \frac{A_{ph}}{\sqrt{2}}$$

$$\alpha_{MS}(\mu = m_e) = \alpha_{ph} = \left( \frac{1}{1} \right)$$

Suppose

$$E \gg m_e$$

$$\alpha_{ph}(m_e) \sim \frac{f(\theta_k, \alpha_{ph}, 0)}{E^2}$$

CAUTION  
DO NOT TOUCH THE BOARD  
IF IT IS NEARLY FULL  
PLEASE REMOVE YOUR HANDS  
IMMEDIATELY

$$\alpha_{ph} \left[ 1 - \frac{\alpha_{ph}}{3\pi} \ln\left(\frac{m_e^2}{\mu^2}\right) + o(\alpha^2) \right]$$

$$\alpha_{MS}(\mu = m_e) = \alpha_{ph} = \left( \frac{1}{137} \right)$$

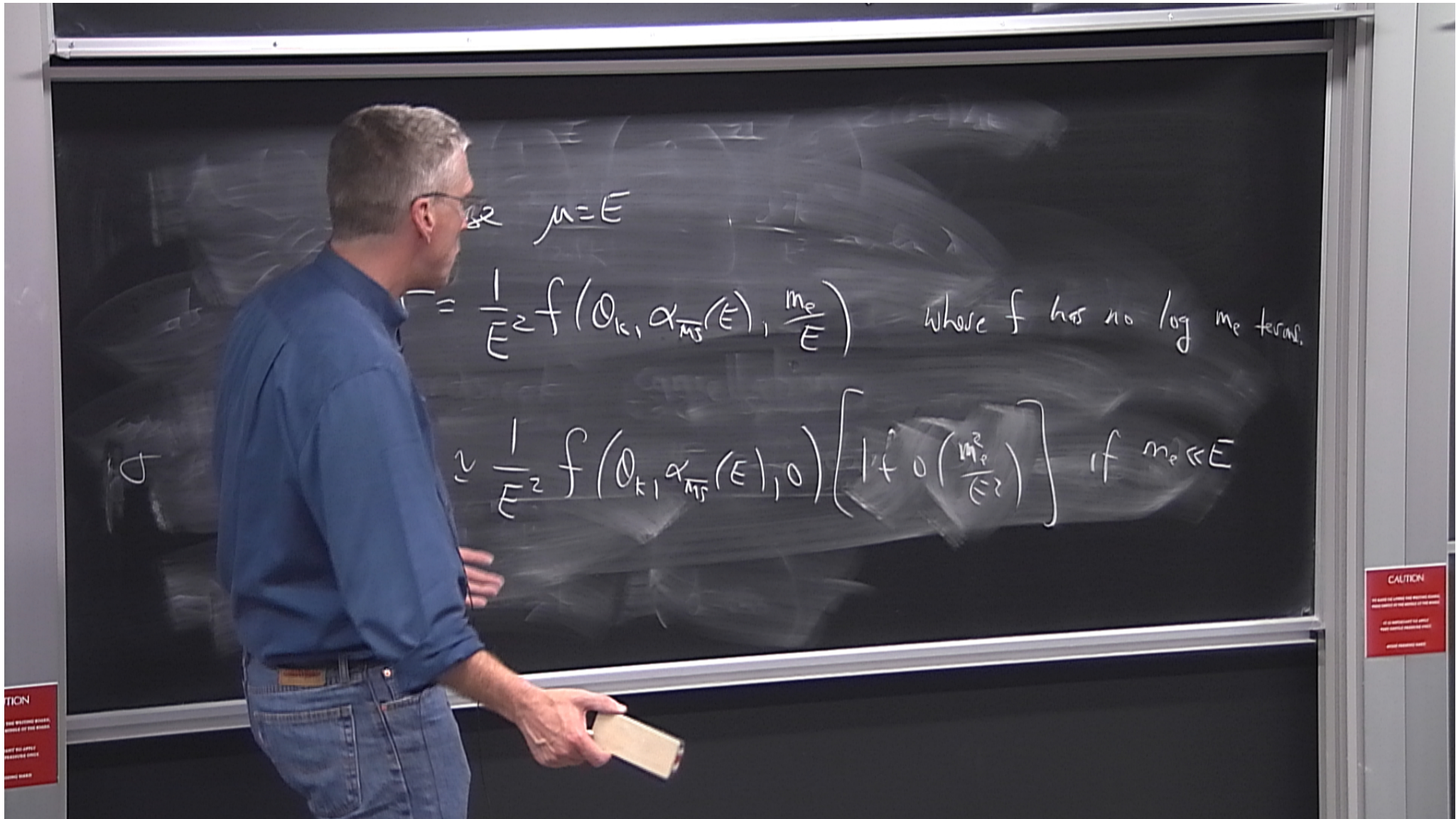
$$\alpha_{ph}(m_e) \sim \frac{f(\theta_k, \alpha_{ph}, 0)}{E^2}$$

Suppose

$$= \frac{A_{ph}}{\sqrt{2}}$$

$$A_{ph} + \frac{\alpha}{2\pi} \left( \frac{1}{\epsilon^+} \right) A_{ph}$$

CAUTION  
DO NOT TOUCH THE SURFACE OF THE BOARD.  
IF IT MOVES IT WILL BE DAMAGED.  
PLEASE REPORT TO THE STAFF.



$$\mu = E$$

$$\sigma = \frac{1}{E^2} f(O_k, \alpha_{MS}(E), \frac{m_e}{E}) \quad \text{where } f \text{ has no } \log m_e \text{ terms.}$$

$$\sigma \approx \frac{1}{E^2} f(O_k, \alpha_{MS}(E), 0) \left[ 1 + O\left(\frac{m_e^2}{E^2}\right) \right] \quad \text{if } m_e \ll E$$

Choose  $\mu = E$

$$\sigma = \frac{1}{E^2} f(Q_k, \alpha_{MS}(E), \frac{m_e}{E}) \quad \text{where } f \text{ has no } \log m_e \text{ terms.}$$

$$\approx \frac{1}{E^2} f(Q_k, \alpha_{MS}(E), 0) \left[ 1 + O\left(\frac{m_e^2}{E^2}\right) \right] \quad \text{if } m_e \ll E$$

The  $\log m_e$  present in  $f$  when using  $\alpha_{ph}$  must be the same  $\log$  appearing in  $\alpha_{MS}(E) = \alpha_{ph} \left[ 1 - \frac{\alpha}{3\pi} \ln \left( \frac{m_e^2}{E^2} \right) \right]$



"Why the Renormalization Group is a Good Thing"

S. Weinberg

$$\alpha_{\overline{MS}}(\mu) = \alpha_{ph} \left[ 1 - \frac{\alpha_{ph}}{3\pi} \ln\left(\frac{m_e^2}{\mu^2}\right) + o(\alpha^2) \right]$$

notice  $\alpha_{\overline{MS}}(\mu=m_e) = \alpha_{ph} = \left(\frac{1}{137}\right)$

$$A_\mu = \frac{A_{ph}}{\sqrt{Z}}$$

$$A_\mu^R + \frac{\alpha}{2\pi} \left( \frac{1}{\epsilon} + \dots \right)$$



$\frac{\alpha}{4\pi}$

$$\sigma = \sigma(E, \theta_k, \alpha_{ph}, m_e)$$

Suppose your interest is in  $E \gg m_e$ .

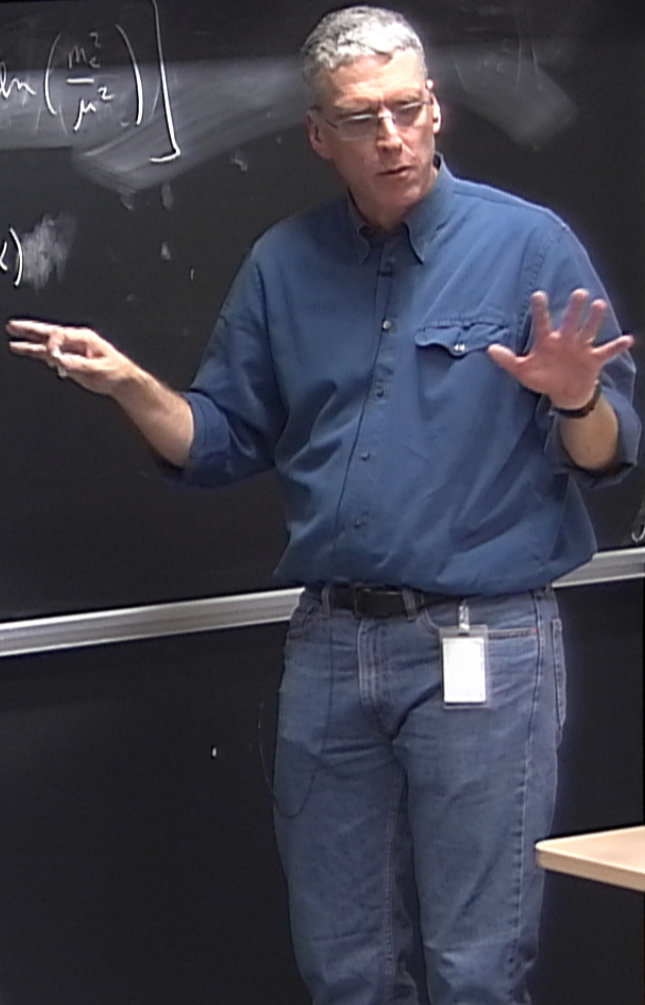
$$= \frac{1}{E^2} f(\theta_k, \alpha_{ph}, m_e/E) \approx \frac{f(\theta_k, \alpha_{ph}, 0)}{E^2} \left( 1 + o\left(\frac{m_e^2}{E^2}\right) + \dots \right)$$

$m_e^2$   $k$   
 $\mu^2$   $k$   
 $f$  has no  $\log m_e$  terms.  
 $\left. \begin{array}{l} \\ \end{array} \right\}$  if  $m_e \ll E$   
 must be the same log  
 $\left. \begin{array}{l} \\ \end{array} \right\}$

If we ask  $\Pi(0) \neq 0$  the answer is a gain smooth

$$\alpha_{MS}(\mu^2) = \alpha \left[ 1 - \frac{\alpha_s}{3\pi} \ln\left(\frac{m_e^2}{\mu^2}\right) \right]$$

$$\mu^2 \frac{\partial \alpha}{\partial \mu^2} = + \frac{\alpha^2}{3\pi} := \beta(\alpha)$$



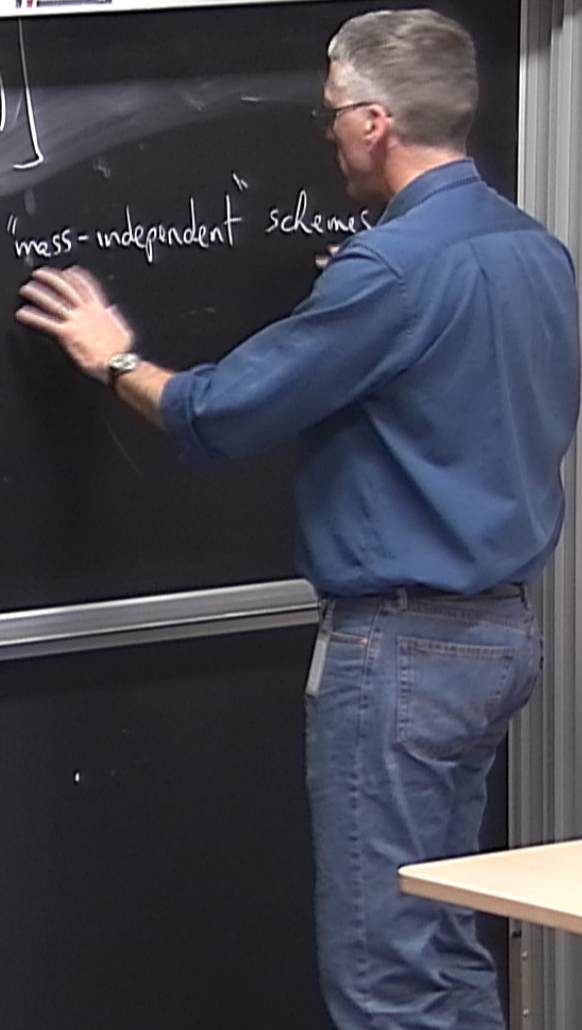
CAUTION  
 DO NOT TOUCH THE BOARD WHEN THE BOARD IS HOT  
 DO NOT TOUCH THE BOARD WHEN THE BOARD IS HOT  
 DO NOT TOUCH THE BOARD

$m_c^2$   $f$   
 $f$  has no  $\log m_c$  terms.  
 $\left. \begin{array}{l} \\ \end{array} \right\}$  if  $m_c \ll E$   
 must be the same log  
 $\left. \begin{array}{l} \\ \end{array} \right\}$

If we ask  $\Pi(0) \neq 0$  the answer is a gain smooth  

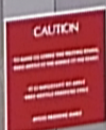
$$\alpha_{\overline{MS}}(\mu^2) = \alpha \left[ 1 - \frac{\alpha}{3\pi} \ln \left( \frac{m_c^2}{\mu^2} \right) \right]$$

$$\mu^2 \frac{\partial \alpha}{\partial \mu^2} = + \frac{\alpha^2}{3\pi} := \beta(\alpha) \text{ for "mass-independent" schemes}$$



$\mu^2$   $f$   
 $\mu^2$   $f$   
 $f$  has no  $\log \mu_e$  terms.  
 $\left. \begin{array}{l} \text{if } m_e \ll E \\ \text{not be the same log} \end{array} \right\}$   
 $\left. \begin{array}{l} \text{if } m_e \ll E \\ \text{not be the same log} \end{array} \right\}$

If we ask  $\Pi(0) \neq 0$  the answer is a gain smooth  
 $\alpha_{\overline{MS}}(\mu^2) = \alpha \left[ 1 - \frac{\alpha}{3\pi} \ln\left(\frac{\mu^2}{\mu^2}\right) \right]$   
 $\underbrace{\mu^2 \frac{\partial \alpha}{\partial \mu^2} = + \frac{\alpha^2}{3\pi}}_{\text{for "mas."}} := \beta(\alpha)$  for "mas."  $f$  schemes  
 Integrate.  $\frac{1}{\alpha^2} \frac{\partial \alpha}{\partial \mu^2} = \frac{1}{3\pi \mu^2} \rightarrow$



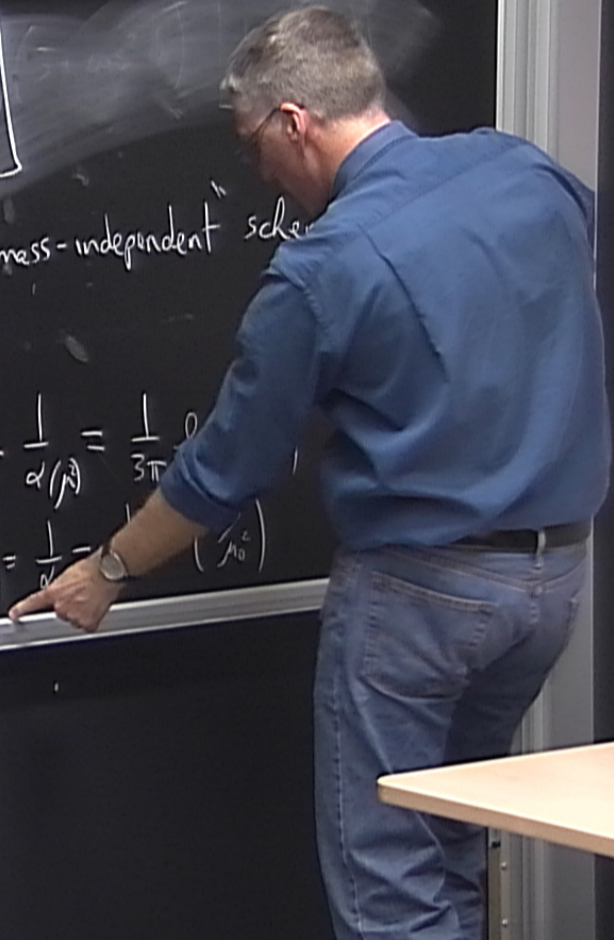
$m_e^2 k$   
 $\mu^2 k$   
 $f$  has no  $\log m_e$  terms.  
 if  $m_e \ll E$   
 not be the same log  
 $\left(\frac{m_e^2}{\mu^2}\right)$

If we ask  $\Pi(0) \neq 0$  the answer is a gain smooth

$$\alpha_{MS}(\mu^2) = \alpha \left[ 1 - \frac{\alpha}{3\pi} \ln \left( \frac{m_e^2}{\mu^2} \right) \right]$$

$$\mu^2 \frac{\partial \alpha}{\partial \mu^2} = + \frac{\alpha^2}{3\pi} := \beta(\alpha) \text{ for "mass-independent" scheme}$$

Integrate  $\int_{\mu_0}^{\mu} \frac{1}{\alpha^2} d\alpha = \int_{\mu_0}^{\mu} \frac{d\mu^2}{3\pi \mu^2} \rightarrow \frac{1}{\alpha_0} - \frac{1}{\alpha(\mu)} = \frac{1}{3\pi} \ln \left( \frac{\mu^2}{\mu_0^2} \right)$



CAUTION  
 DO NOT TOUCH THE BOARD OR THE CHALK  
 IF YOU NEED TO USE THE BOARD PLEASE ASK THE LECTURER

$m_e^2$   $k$   
 $\mu^2$   $k$   
 $f$  has no  $\log m_e$  terms.  
 if  $m_e \ll E$   
 not be the same log  
 $\left(\frac{m_e^2}{\mu^2}\right)$

If we ask  $\Pi(0) \neq 0$  the answer is a gain smooth

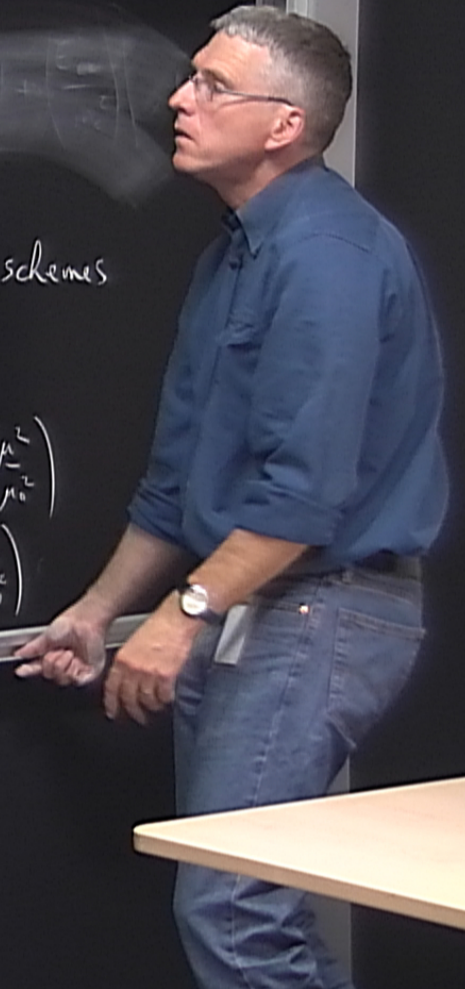
$$\alpha_{\overline{MS}}(\mu^2) = \alpha \left[ 1 - \frac{\alpha}{3\pi} \ln\left(\frac{m_e^2}{\mu^2}\right) \right]$$

$$\mu^2 \frac{\partial \alpha}{\partial \mu^2} = + \frac{\alpha^2}{3\pi} := \beta(\alpha) \quad \text{for "mass-independent" schemes}$$

Integrate  $\int_{\mu_0}^{\mu} \frac{d\alpha}{\alpha^2} = \int_{\mu_0}^{\mu} \frac{d\mu^2}{3\pi \mu^2} \rightarrow$

$$\frac{1}{\alpha_0} - \frac{1}{\alpha(\mu)} = \frac{1}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha_0} - \frac{1}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$



CAUTION  
 DO NOT TOUCH THE BOARD  
 OR THE CHALK  
 OR THE ERASER  
 OR THE MARKERS  
 OR THE WIPER  
 OR THE DUST  
 OR THE BOARD

$$\begin{aligned}
 & \frac{1}{p^2 + m_1^2 - 2pk + k^2} = \frac{1}{(p+k)^2 + m_2^2} - \frac{1}{(p-k)^2 + m_2^2} \\
 & \frac{1}{p^2 + m_1^2 - 2pk + k^2} = \frac{1}{(p+k)^2 + m_2^2} - \frac{1}{(p-k)^2 + m_2^2}
 \end{aligned}$$

$$\frac{1}{\alpha(\mu^2)} = \frac{1}{\alpha_0} - \frac{1}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

$$\alpha_0 = \alpha - \frac{\alpha\alpha_0}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right) \quad \alpha(\mu^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)} \approx \alpha_0 \left[ 1 + \frac{\alpha}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right) \right]$$

CAUTION  
DO NOT TOUCH THE WALLS OR THE BOARD.  
IT IS IMPORTANT TO KEEP THE BOARD CLEAN.

CAUTION  
DO NOT TOUCH THE WALLS OR THE BOARD.  
IT IS IMPORTANT TO KEEP THE BOARD CLEAN.

$$\frac{1}{\alpha(\mu^2)}$$

$$= \frac{1}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

$$\alpha(\mu^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)} \approx \alpha_0 \left[ 1 + \frac{\alpha}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right) + \dots \right]$$

free to all orders in  $\alpha \ln\left(\frac{\mu^2}{\mu_0^2}\right)$

but does not include terms of order  $\alpha^2 \ln\left(\frac{\mu^2}{\mu_0^2}\right)$



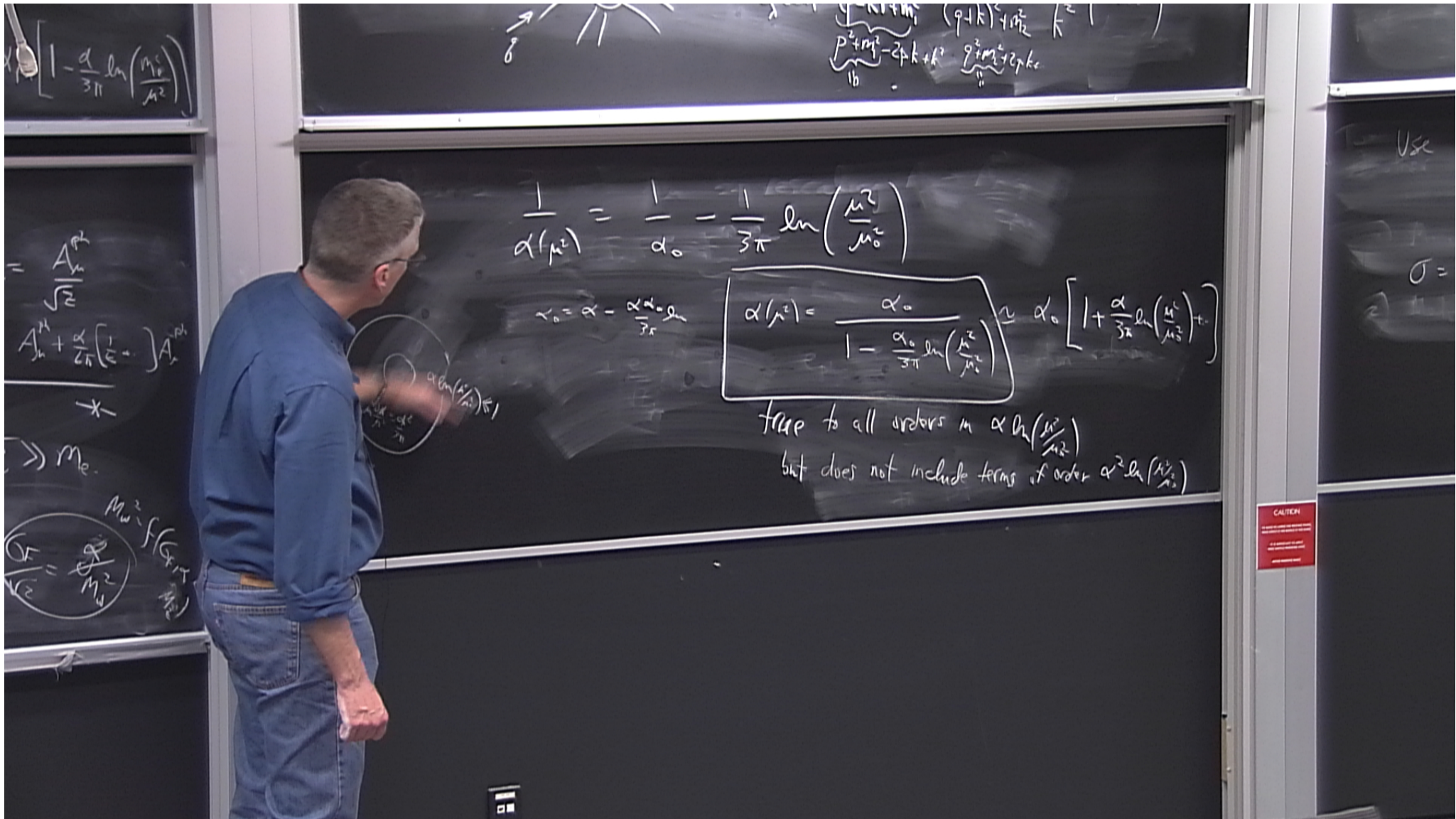
$$\begin{aligned}
 & \frac{1}{p^2 + m^2} = \frac{1}{(p-k)^2 + m^2} + \frac{1}{p^2 + m^2} \\
 & \frac{1}{p^2 + m^2} - \frac{1}{(p-k)^2 + m^2} = \frac{2pk + k^2}{(p^2 + m^2)((p-k)^2 + m^2)}
 \end{aligned}$$

$$= \frac{1}{\alpha_0} - \frac{1}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

$$\alpha_0 = \alpha - \frac{\alpha \alpha_0}{3\pi} \ln$$

$$\alpha(\mu^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)} \approx \alpha_0 \left[ 1 + \frac{\alpha}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right) + \dots \right]$$

free to all orders in  $\alpha \ln(\frac{\mu^2}{\mu_0^2})$   
 but does not include terms of order  $\alpha^2 \ln(\frac{\mu^2}{\mu_0^2})$



$$\frac{1}{\alpha(\mu^2)} = \frac{1}{\alpha_0} - \frac{1}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

$$\alpha_0 = \alpha - \frac{\alpha^2}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

$$\alpha(\mu^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)} \approx \alpha_0 \left[ 1 + \frac{\alpha_0}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right) + \dots \right]$$

free to all orders in  $\alpha \ln(\mu^2/\mu_0^2)$   
 but does not include terms of order  $\alpha^2 \ln(\mu^2/\mu_0^2)$

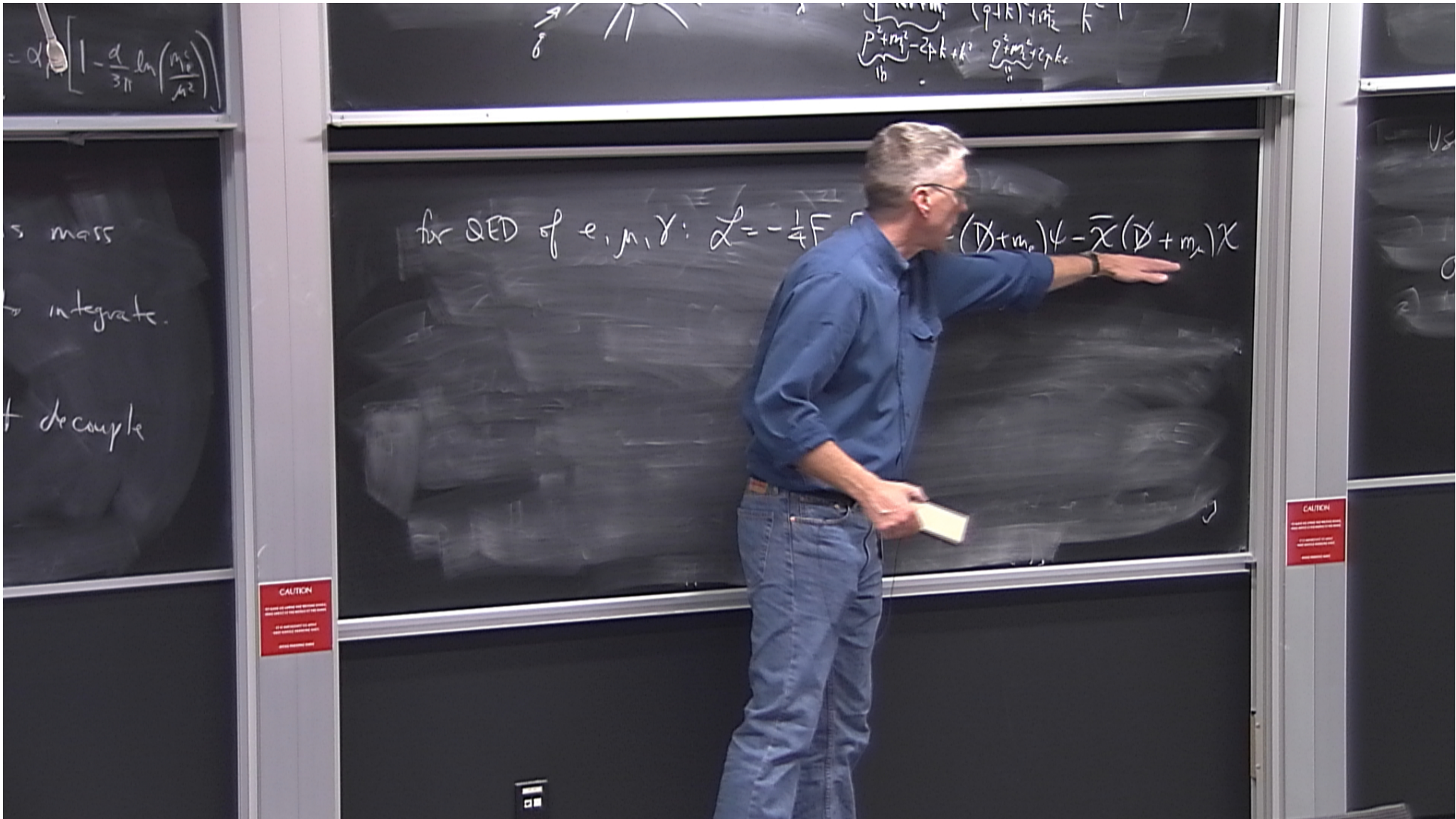
$m_e = 0.511 \text{ MeV}$   
 $m_\mu = 105.66 \text{ MeV}$   
 $m_\tau = 1.777 \text{ GeV}$

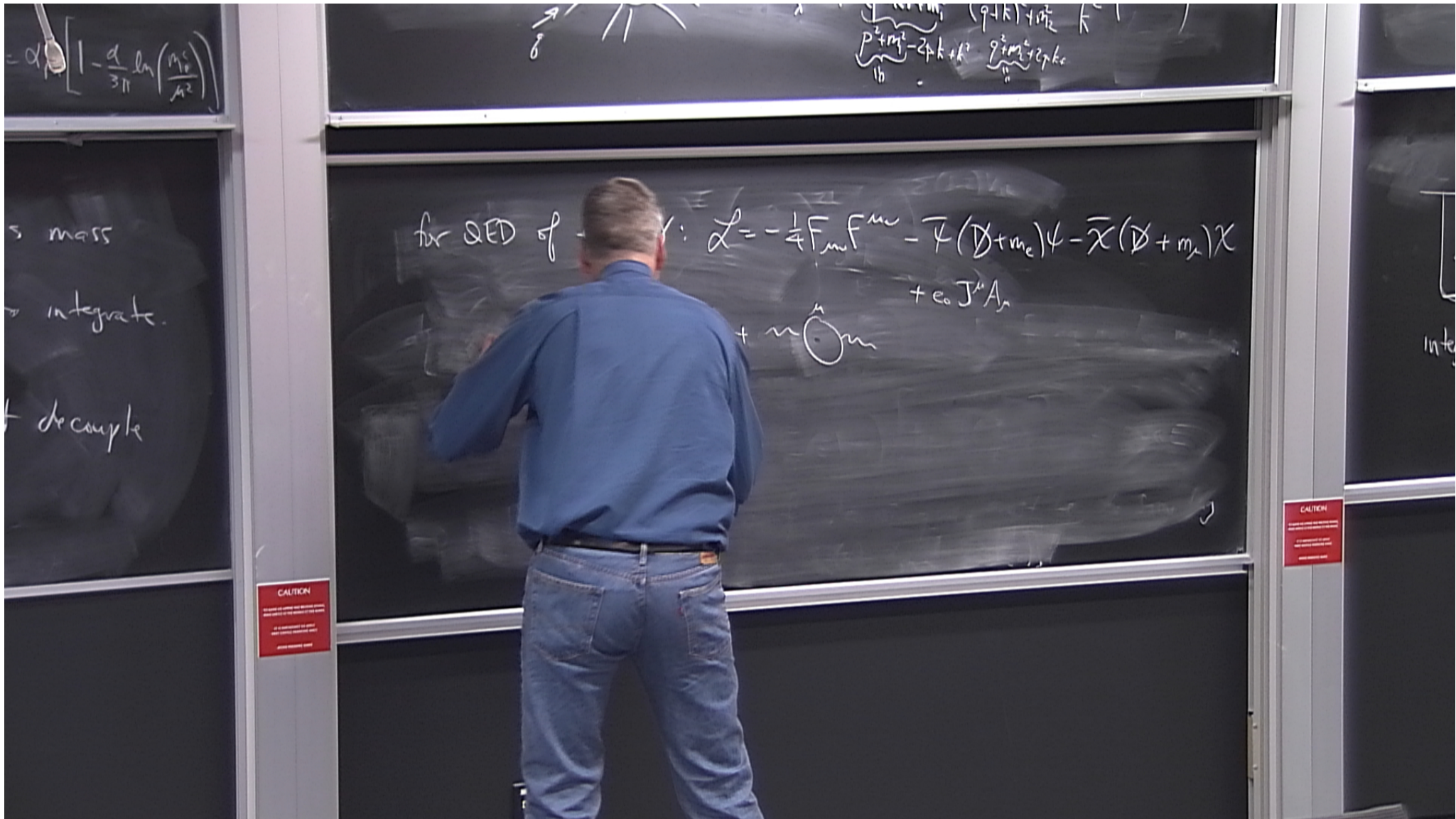
$$\alpha_{\overline{MS}} = \frac{e_0^2}{4\pi} \quad Z_{\overline{MS}} = \left[ 1 - \frac{\alpha}{3\pi} \left( \frac{1}{\epsilon} + k \right) \right]$$

$$\alpha_{\overline{MS}} = \frac{e^2}{4\pi} = \frac{\alpha_0}{Z_{\overline{MS}}} = \alpha_{\text{ph}} \frac{Z_{\text{ph}}}{Z_{\overline{MS}}} = \alpha_0 \left[ 1 - \frac{\alpha}{3\pi} \ln \left( \frac{m_0^2}{\mu^2} \right) \right]$$

Upshot:  $\alpha_{\overline{MS}}(\mu)$  has the advantage that it is mass independent so  $\mu^2 \frac{\partial \alpha}{\partial \mu^2} = \beta(\alpha)$  is easy to integrate.

But its disadvantage is that heavy particles do not decouple from  $\mu^2 \frac{\partial \alpha}{\partial \mu^2}$ .





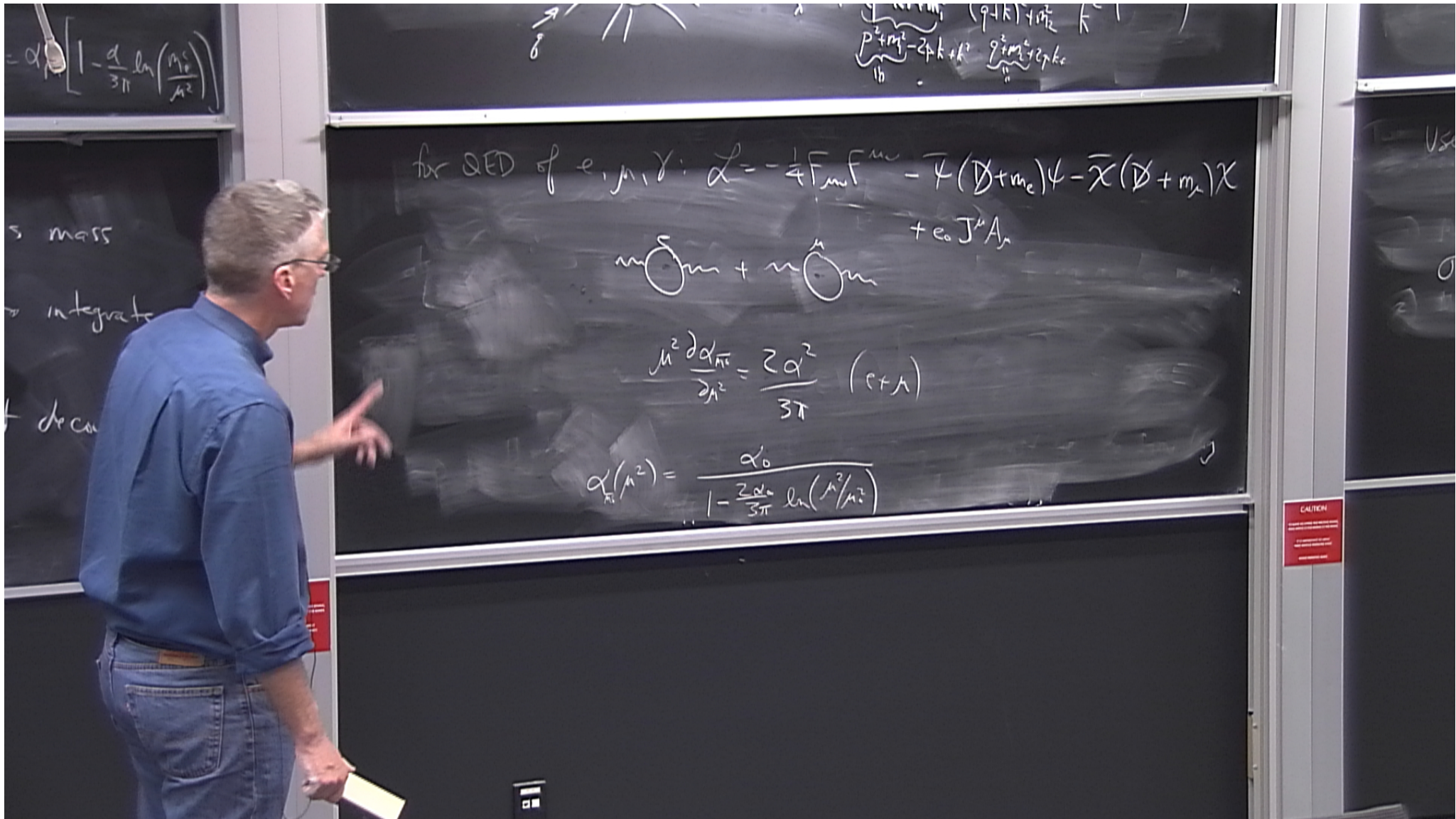
$$= \alpha \left[ 1 - \frac{\alpha}{3\pi} \ln \left( \frac{m_e}{m_\mu} \right) \right]$$

s mass  
→ integrate.  
+ decouple

for QED of  $\psi$ :  $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi} (\not{D} + m_e) \psi - \bar{\chi} (\not{D} + m_\chi) \chi + e_0 J^\mu A_\mu + m \bar{\psi} \psi$

CAUTION  
DO NOT TOUCH THE BOARD  
IF IT IS DAMAGED BY YOU  
YOU WILL BE FINED

CAUTION  
DO NOT TOUCH THE BOARD  
IF IT IS DAMAGED BY YOU  
YOU WILL BE FINED



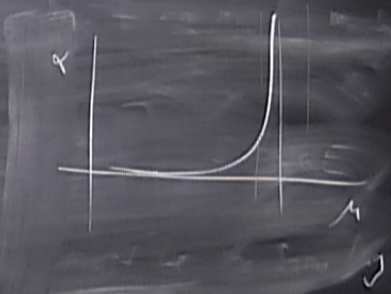
$$\alpha \left[ 1 - \frac{d}{3\pi} \ln \left( \frac{m_e}{\Lambda^2} \right) \right]$$

$$\frac{p^2 + m^2 - 2pk + k^2}{i} \frac{2im^2 + 2pk}{i}$$

for QED of  $e, \mu, \gamma$ :  $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi} (\not{D} + m_e) \psi - \bar{\chi} (\not{D} + m_\mu) \chi + e_0 J^\mu A_\mu$

$$m \int \bar{\psi} \psi + m \int \bar{\chi} \chi$$

$$\mu^2 \frac{d\alpha}{d\mu^2} = \frac{2\alpha^2}{3\pi} \quad (e+\mu)$$



$$\alpha(\mu^2) = \frac{\alpha_0}{1 - \frac{2\alpha_0}{3\pi} \ln(\mu^2/\mu_0^2)}$$

$E = \dots$

$$\frac{d\alpha}{d\ln \mu^2} = \frac{2\alpha^2}{3\pi}$$

$M_W^2 - f(G_F, M_W)$

CAUTION

CAUTION

$$m_e = \text{boson} \quad m_\mu = 10m_e \quad \frac{m_0}{m_s} \quad Z_{\overline{MS}} = 1 - \frac{d}{3\pi} \left[ \frac{1}{\epsilon} + k \right] \quad \alpha_{\overline{MS}} = \frac{e^2}{4\pi\epsilon_0} = \frac{\alpha_s}{Z_{\overline{MS}}} = \alpha_{ph} Z_{\overline{MS}} = \alpha_{ph} \left[ 1 - \frac{d}{3\pi} \ln \left( \frac{m_0}{\mu^2} \right) \right]$$

→  $\overline{MS}(e, \mu)$  full theory  $m, e, \gamma$

$m_\mu$  ———

EFT  $e, \gamma$

—————

$\gamma$

CAUTION  
 Do not touch the electrical parts  
 when the power is on.  
 Do not touch the power  
 supply when the power  
 is on.



$$m_e = 208 m_\mu, \quad m_e = 10^{-4} m_p, \quad \frac{m_e}{m_p} = 10^{-4}$$

$$Z_{\overline{MS}} = 1 - \frac{d}{3\pi} \left[ \frac{1}{\epsilon} + k \right]$$

$$\alpha_{\overline{MS}} = \frac{e^2}{4\pi\epsilon_0} = \alpha_{ph} \frac{Z_{ph}}{Z_{\overline{MS}}} = \alpha_{ph} Z_{ph} = \alpha_{ph} \left[ 1 - \frac{d}{3\pi} \ln \left( \frac{m_p}{m_e} \right) \right]$$

$\alpha_0(\mu) \rightarrow \overline{MS}(e, \mu)$  full theory  $m, e, \gamma$   
 $\rightarrow \alpha_0(m_e, m_e, m_e) \mu$

$\alpha_1(\mu) \overline{MS}(e)$  EFT  $e, \gamma$   $\frac{1}{\alpha_{DS}(E)} =$

$\rightarrow \alpha_1(m_e) = \alpha_{ph} \frac{m_e}{\mu}$   
 "decoupling subtraction"  
 $\gamma \quad \overline{DS}$   
 $\alpha_{ph}$

$$\frac{1}{\alpha_{ph}} \left( \frac{E^2}{m_e^2} \right) m_e \ll E$$

$$m_e = 206 m_e \quad m_e = 10^{-31} \text{ kg} \quad \frac{m_e}{m_p} = 10^{-4}$$

$$\frac{Z}{M} = \left[ -\frac{d}{3\pi} \left( \frac{1}{E+k} \right) \right]$$

$$\alpha_{\text{MS}} = \frac{e^2}{4\pi\epsilon_0} = \frac{\alpha_s}{Z_{\text{MS}}} = \alpha_{\text{ph}} \frac{Z_{\text{ph}}}{Z_{\text{MS}}} = \alpha_{\text{ph}} \left[ 1 - \frac{d}{3\pi} \ln \left( \frac{m_e}{M^2} \right) \right]$$

$\overline{MS}(e, \mu)$  full theory  $m, e, \gamma$   
 $m_\mu$  —————  
 $\overline{MS}(e)$  EFT  $e, \gamma$   
 $m_e$  —————

$$\frac{1}{\alpha_{\overline{MS}}(E)} =$$

"decoupling subtraction"  
 $\gamma$   $\overline{DS}$

$$\left\{ \begin{array}{l} \frac{1}{\alpha_{\text{ph}}} - \frac{1}{3\pi} \ln \left( \frac{E^2}{m_\mu^2} \right) \quad m_e \ll E \ll m_\mu \\ \frac{1}{\alpha_{\text{ph}}} - \frac{1}{3\pi} \ln \left( \frac{m_\mu^2}{m_e^2} \right) \\ - \frac{2}{3\pi} \ln \left( \frac{E^2}{m_\mu^2} \right) \quad \text{if } E > m_\mu \end{array} \right.$$

CAUTION

