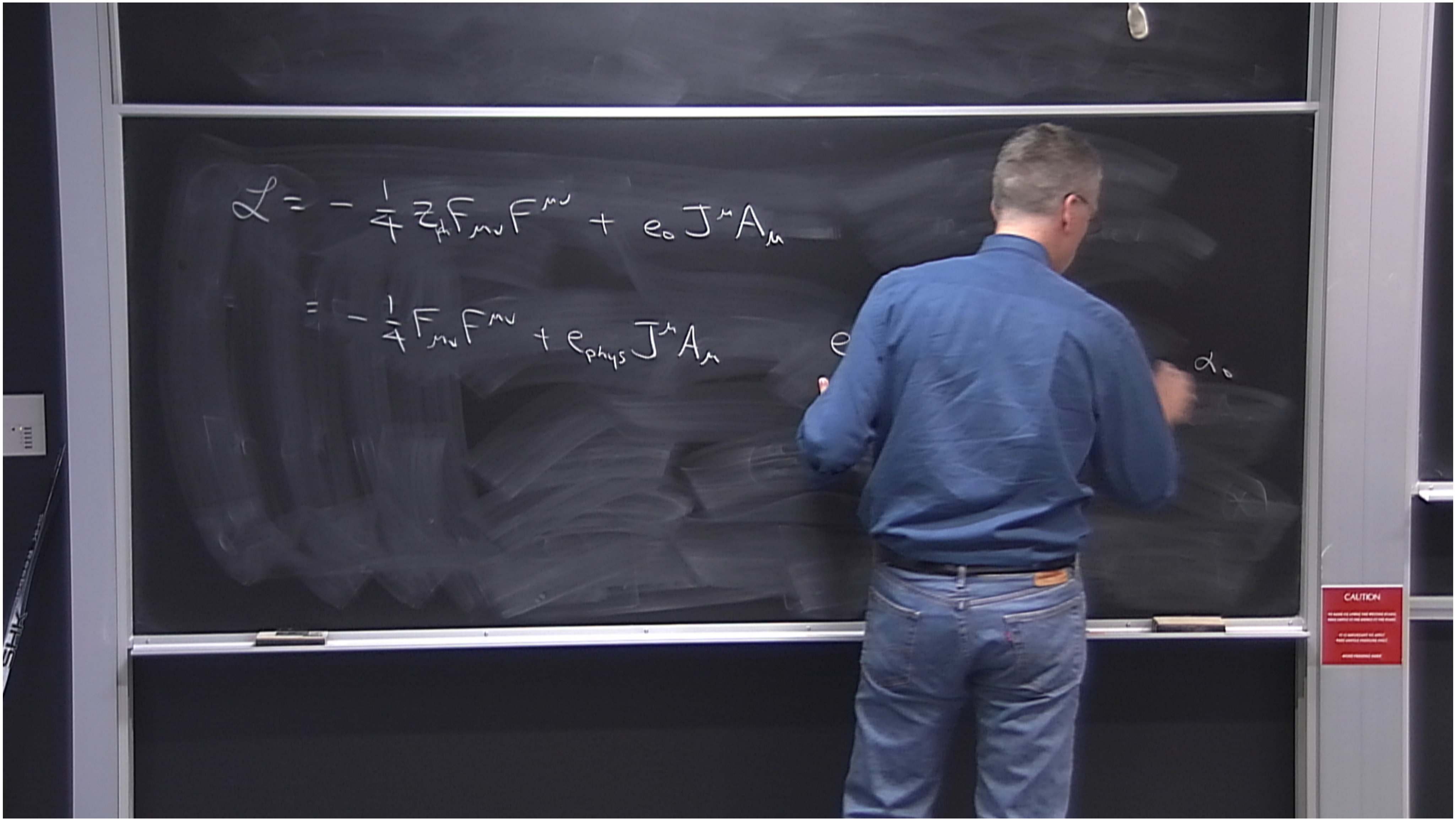
Title: Introduction to Effective Field Theories - Lecture 14

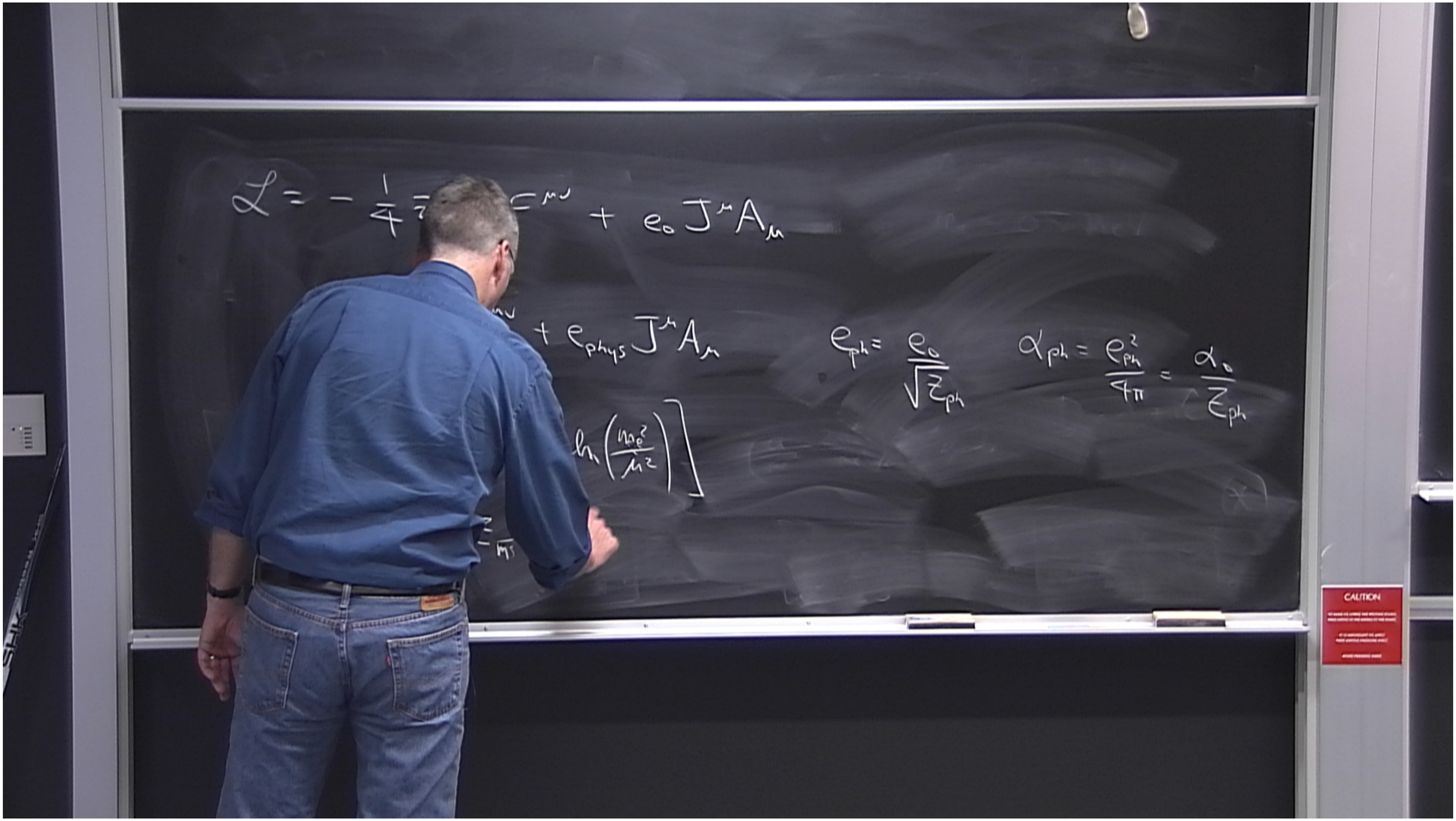
Date: Feb 28, 2014 02:30 PM

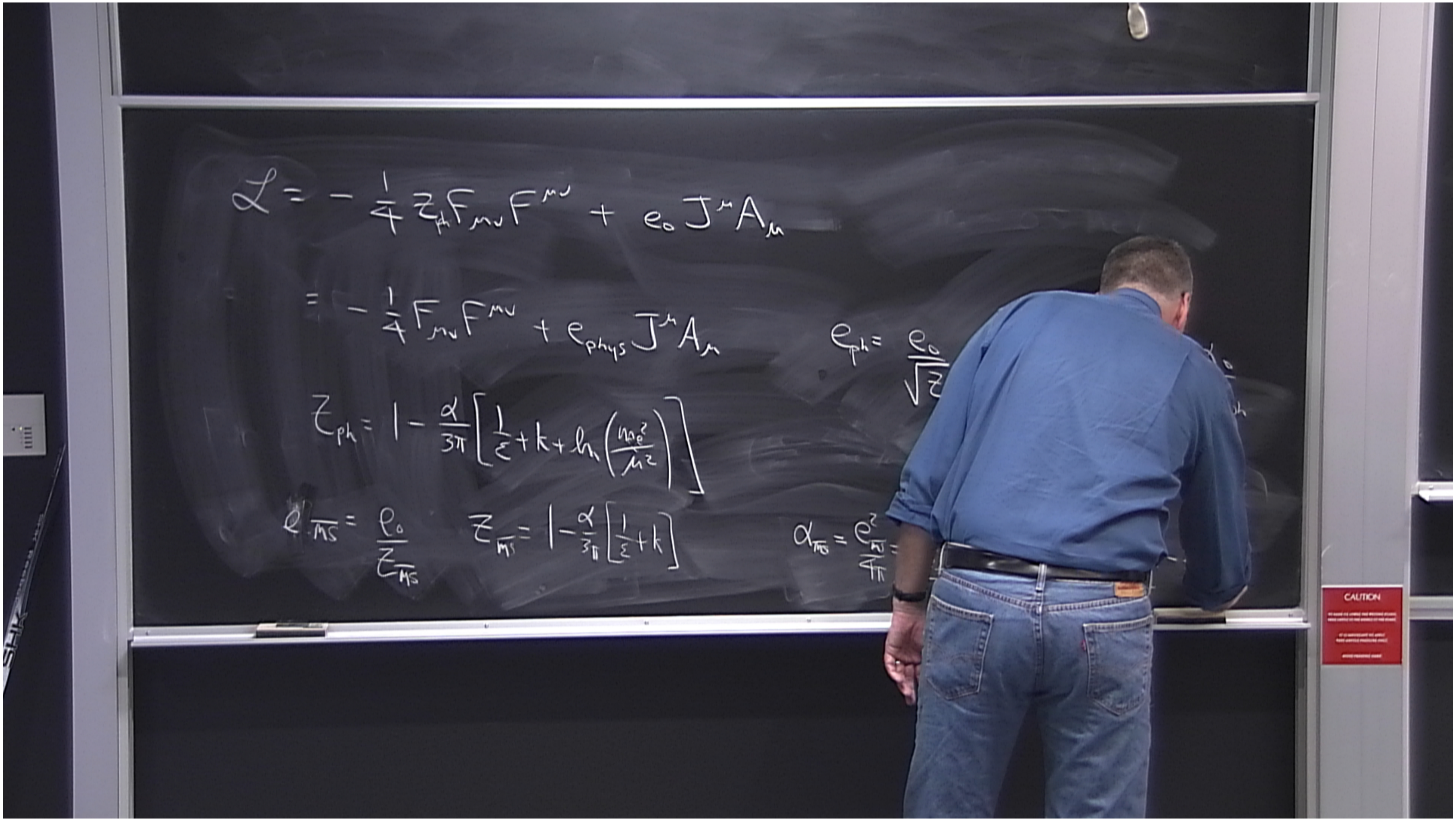
URL: http://pirsa.org/14020094

Abstract:

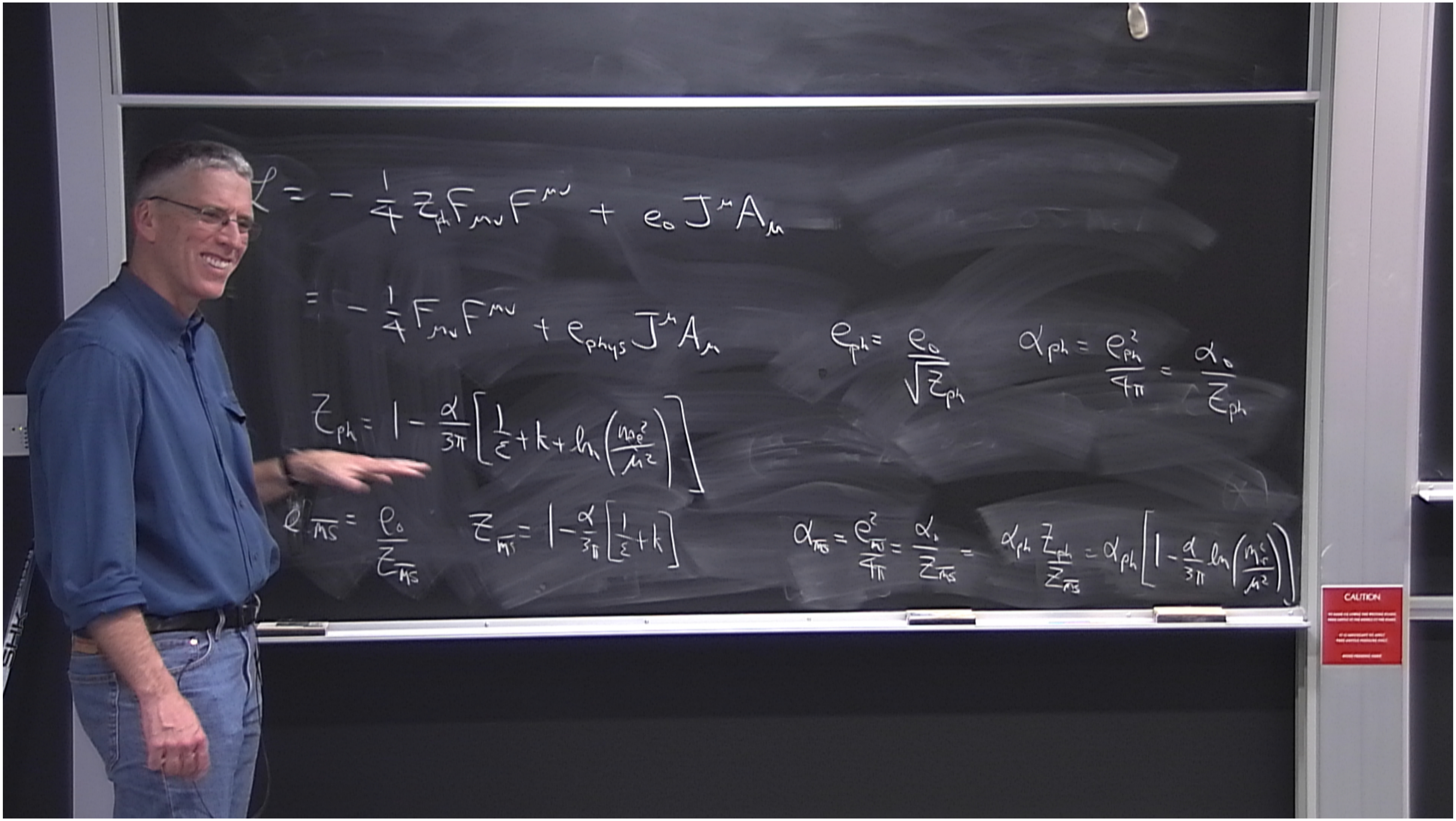


$\mathcal{L} = -\frac{1}{4} Z_A F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$
$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e_{phys} J^\mu A_\mu$
α_0
CAUTION


$e_{ph} = \frac{e_0}{\sqrt{Z_{ph}}}$
$\alpha_{ph} = \frac{e_{ph}^2}{4\pi} = \frac{\alpha_0}{Z_{ph}}$
CAUTION



$\mathcal{L} = -\frac{1}{4} Z_{ph} F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$
$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e_{phys} J^\mu A_\mu$
$e_{ph} = \frac{e_0}{\sqrt{Z}}$
$Z_{ph} = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k + \ln\left(\frac{m_e^2}{\mu^2}\right)\right]$
$e_{\overline{MS}} = \frac{e_0}{Z_{\overline{MS}}}$
$Z_{\overline{MS}} = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k\right]$
$\alpha_{\overline{MS}} = \frac{e^2_{\overline{MS}}}{4\pi}$
CAUTION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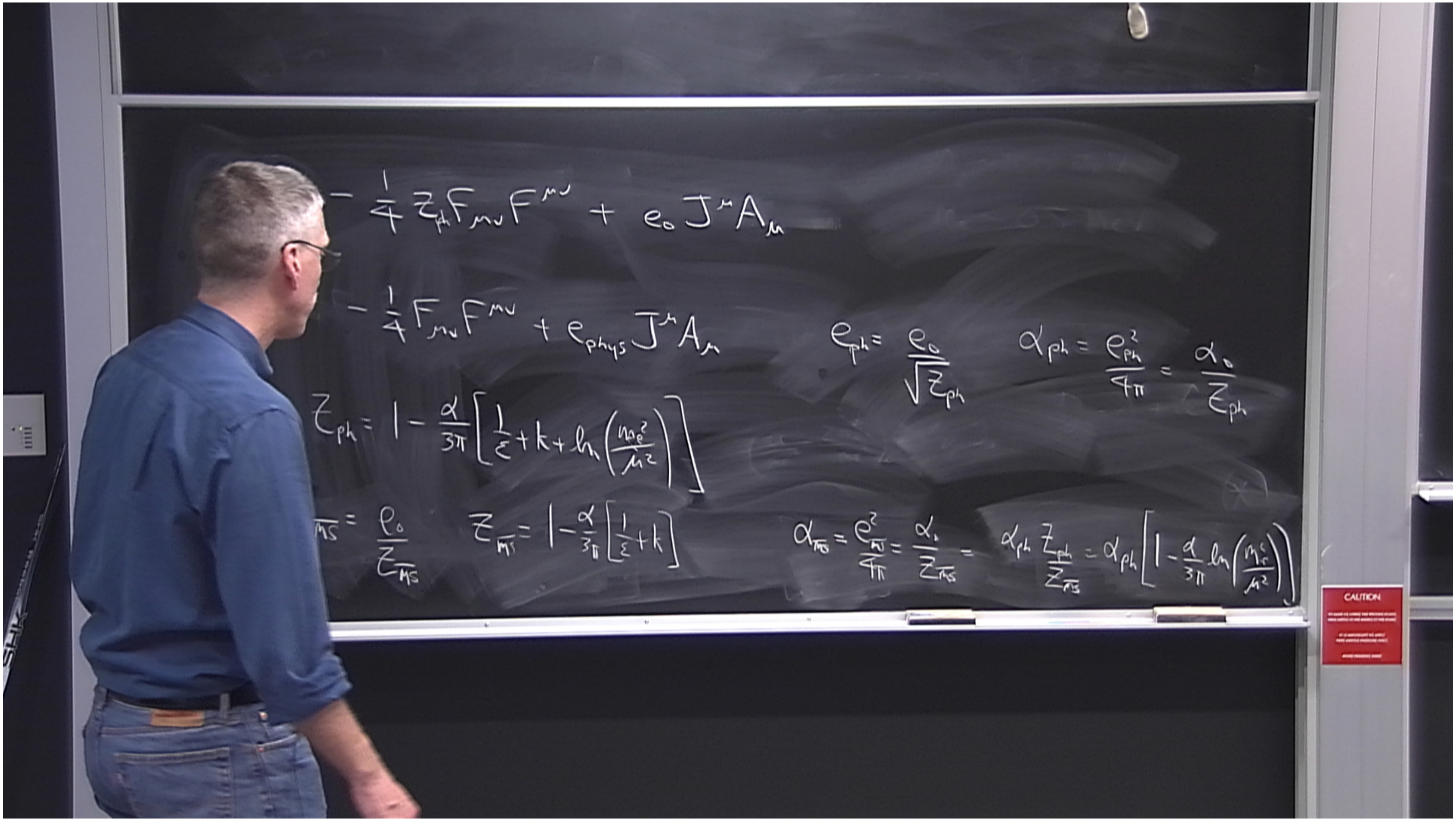
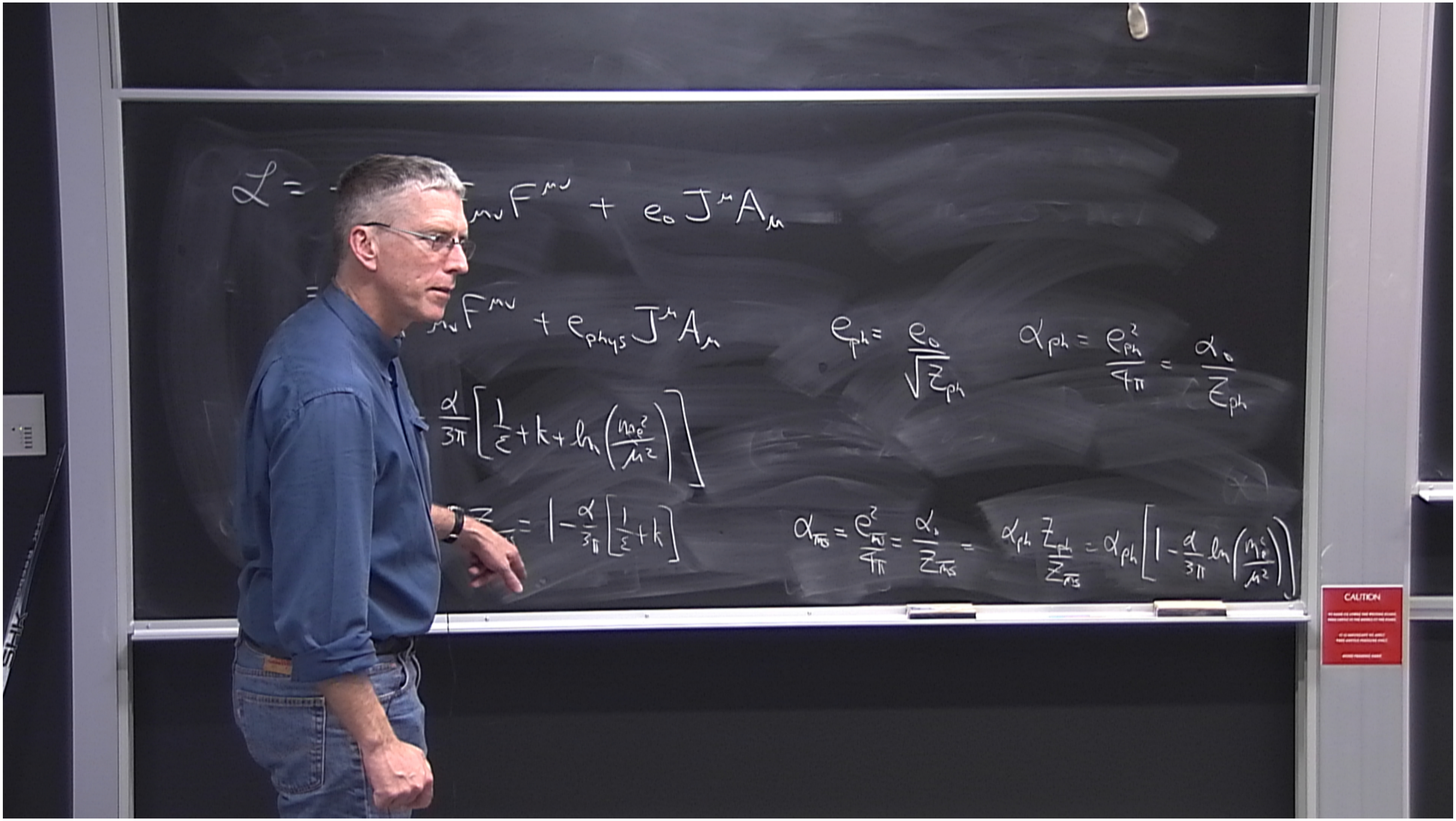
$\alpha_{\overline{MS}} = \frac{e_{\overline{MS}}^2}{4\pi} = \frac{\alpha_0}{Z_{\overline{MS}}} = \alpha_{ph}\frac{Z_{ph}}{Z_{\overline{MS}}} = \alpha_{ph}\left[1 - \frac{\alpha}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right)\right]$
CAUTION


$-\frac{1}{4} Z_{ph} F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$
$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e_{phys} J^\mu A_\mu$
$e_{ph} = \frac{e_0}{\sqrt{Z_{ph}}}$
$\alpha_{ph} = \frac{e_{ph}^2}{4\pi} = \frac{\alpha_0}{Z_{ph}}$
$Z_{ph} = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k + \ln\left(\frac{m_e^2}{\mu^2}\right)\right]$
$\overline{MS} = \frac{e_0}{Z_{\overline{MS}}}$
$Z_{\overline{MS}} = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k\right]$
$\alpha_{\overline{MS}} = \frac{e_{\overline{MS}}^2}{4\pi} = \frac{\alpha_0}{Z_{\overline{MS}}} = \alpha_{ph}\frac{Z_{ph}}{Z_{\overline{MS}}} = \alpha_{ph}\left[1 - \frac{\alpha}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right)\right]$
CAUTION


$F^{\mu\nu} + e_0 J^\mu A_\mu$
$F^{\mu\nu} + e_{phys} J^\mu A_\mu$
$\frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k + \ln\left(\frac{m_e^2}{\mu^2}\right)\right]$
$= 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k\right]$
$e_{ph} = \frac{e_0}{\sqrt{Z_{ph}}}$
$\alpha_{ph} = \frac{e_{ph}^2}{4\pi} = \frac{\alpha_0}{Z_{ph}}$
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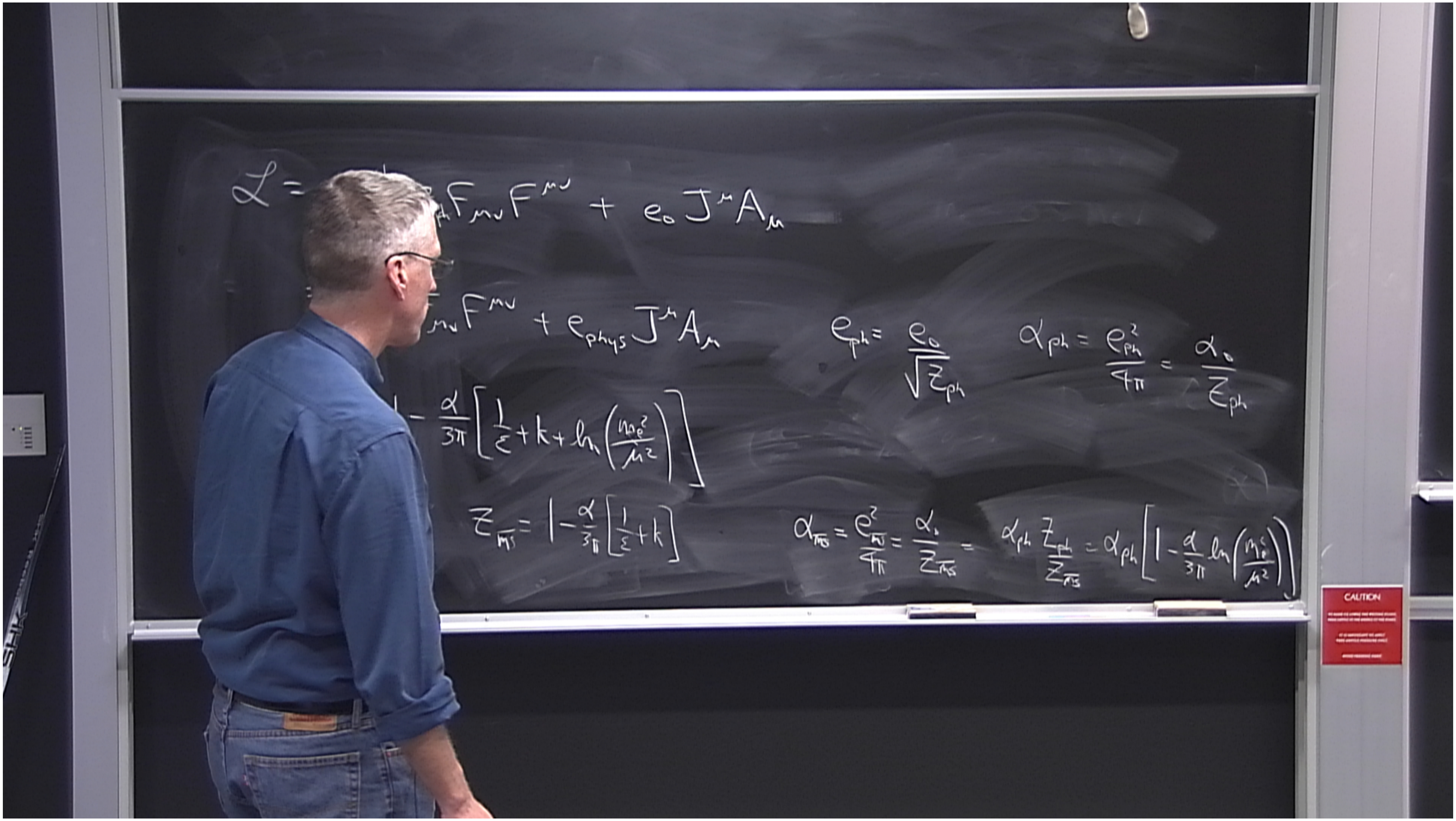
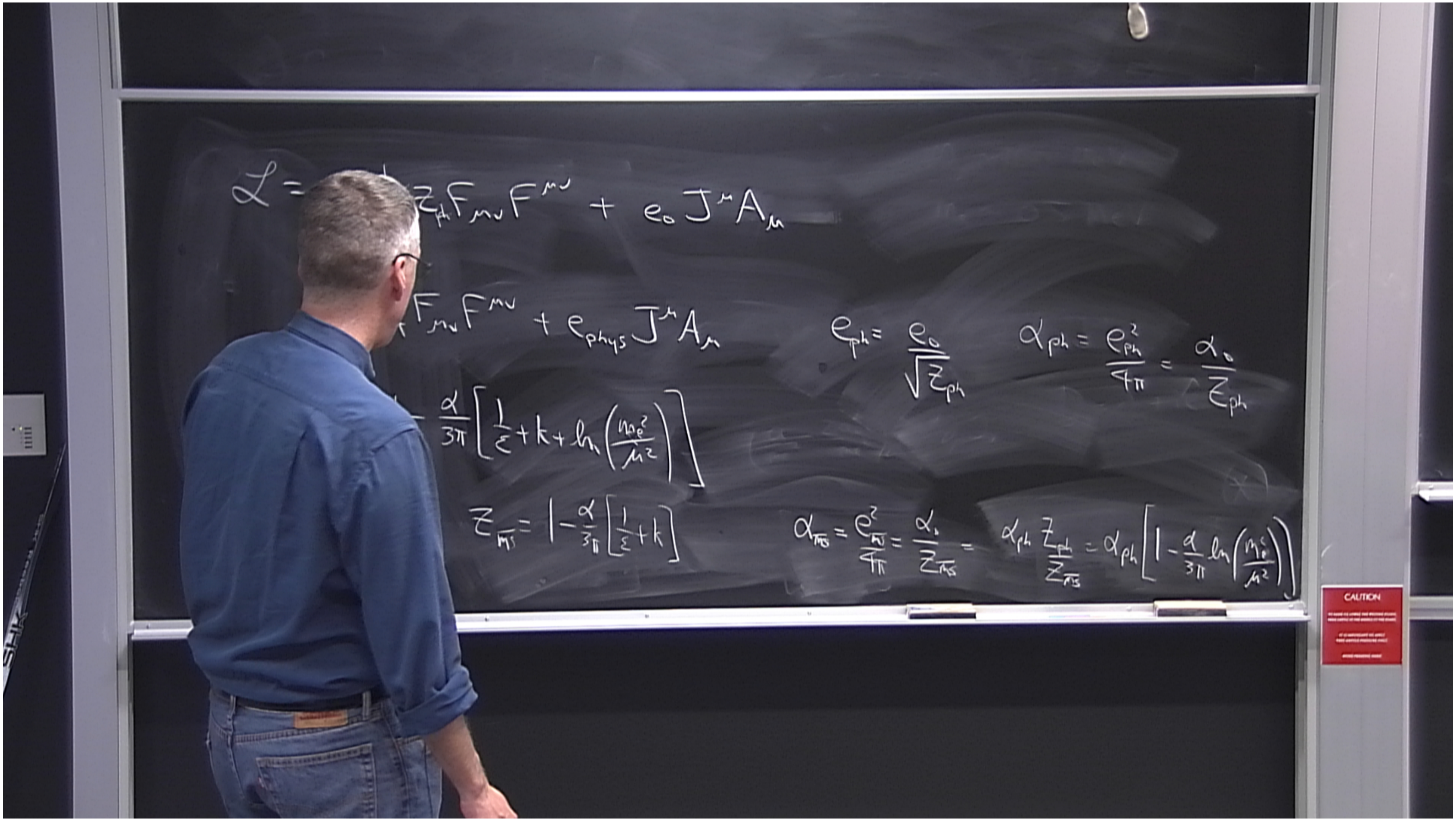
CAUTION


$F_{\mu\nu}F^{\mu\nu} + e_0 J^\mu A_\mu$
$F^{\mu\nu} + e_{phys} J^\mu A_\mu$
$e_{ph} = \frac{e_0}{\sqrt{Z_{ph}}}$
$\alpha_{ph} = \frac{e_{ph}^2}{4\pi} = \frac{\alpha_0}{Z_{ph}}$
$- \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k + \ln\left(\frac{m_e^2}{\mu^2}\right)\right]$
$Z_{\overline{MS}} = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k\right]$
$\alpha_{\overline{MS}} = \frac{e_{\overline{MS}}^2}{4\pi} = \frac{\alpha_0}{Z_{\overline{MS}}} = \alpha_{ph}\frac{Z_{ph}}{Z_{\overline{MS}}} = \alpha_{ph}\left[1 - \frac{\alpha}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right)\right]$


$\mathcal{L} = \ldots Z_{ph} F_{\mu\nu} F^{\mu\nu} + e_0 J^\mu A_\mu$
$\ldots F_{\mu\nu} F^{\mu\nu} + e_{phys} J^\mu A_\mu$
$e_{ph} = \frac{e_0}{\sqrt{Z_{ph}}}$
$\alpha_{ph} = \frac{e_{ph}^2}{4\pi} = \frac{\alpha_0}{Z_{ph}}$
$\ldots \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k + \ln\left(\frac{m_e^2}{\mu^2}\right)\right]$
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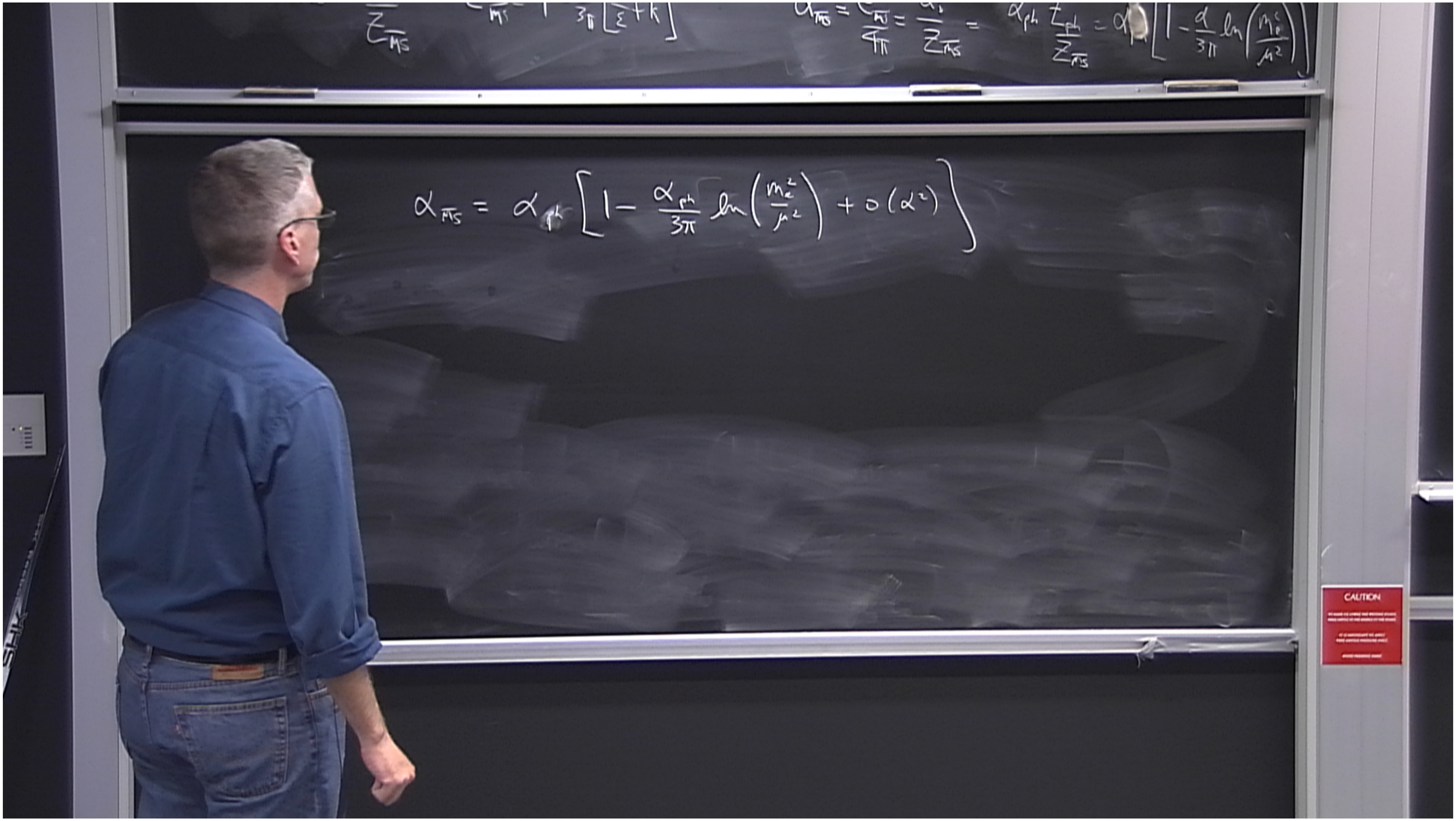
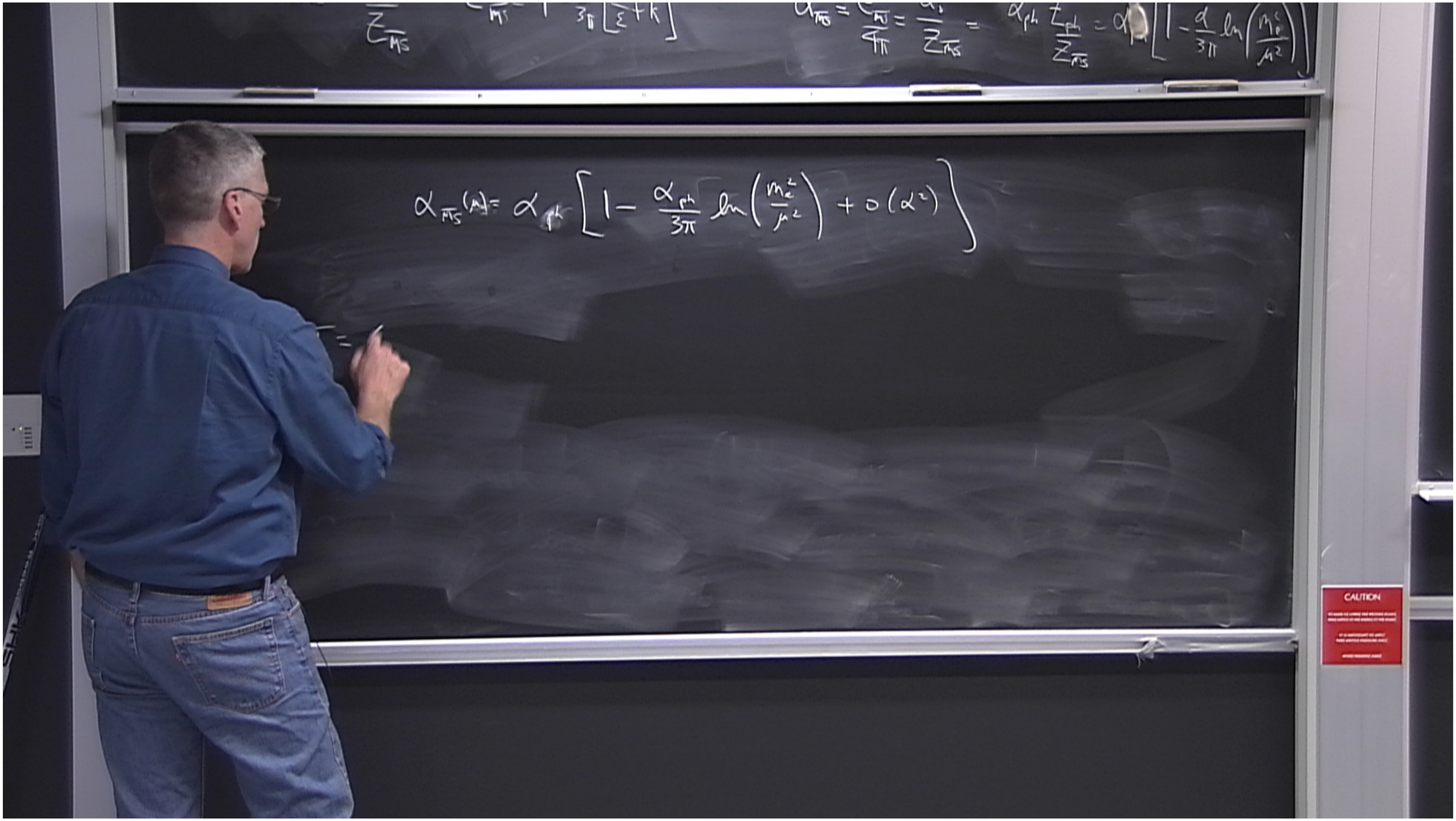
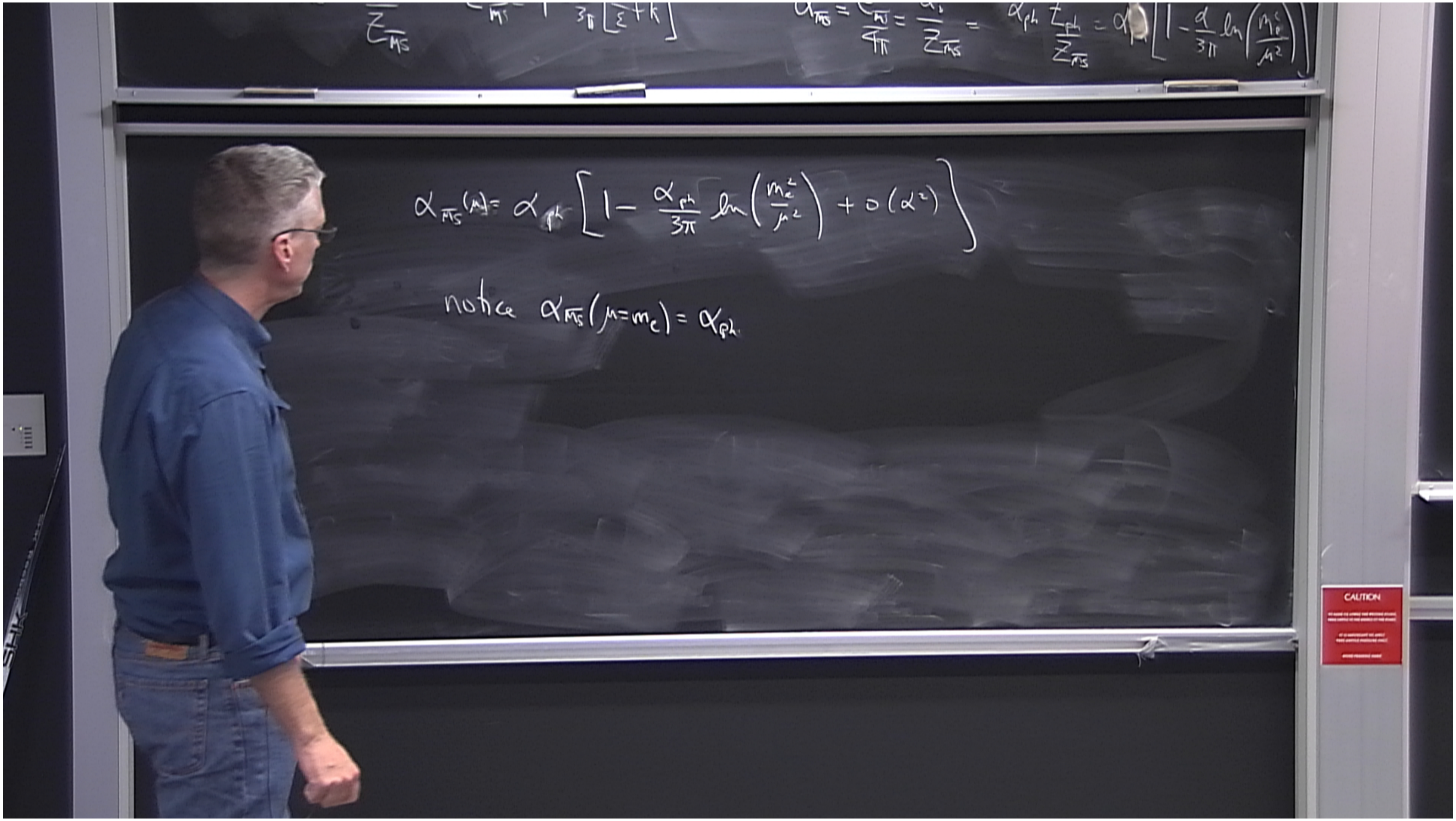
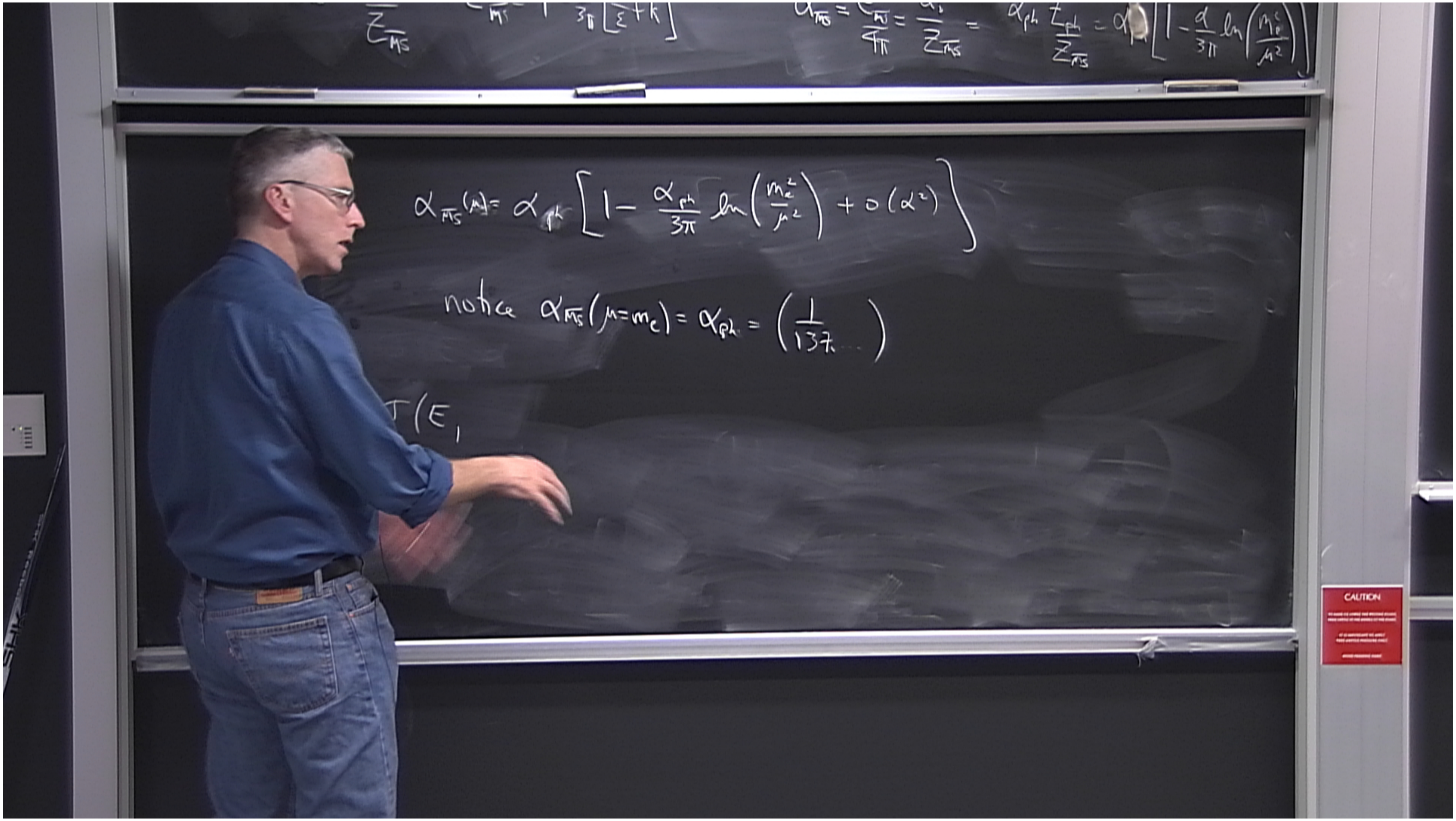
CAUTION

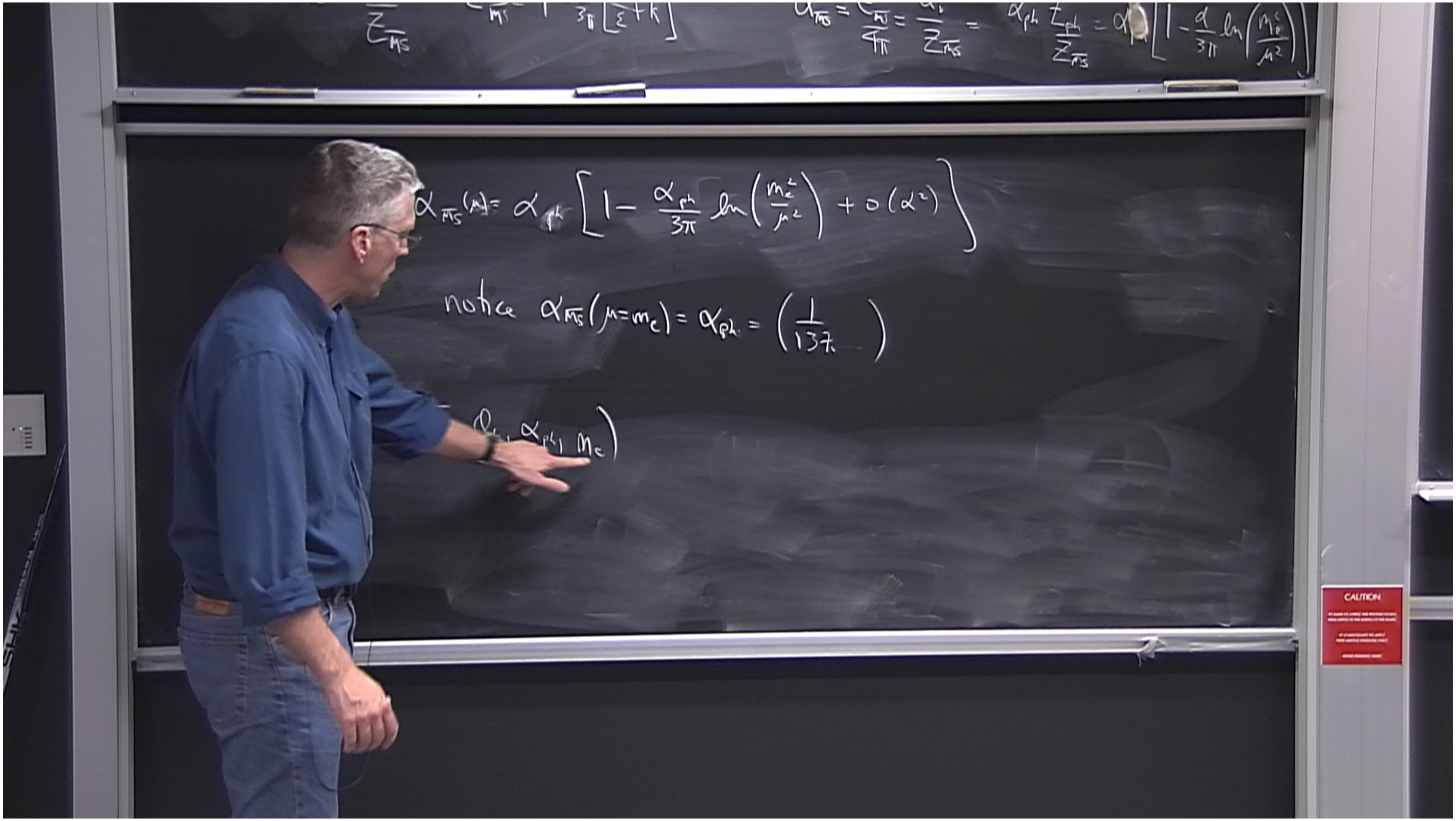

$\alpha_{\overline{MS}} = \alpha_{ph}\left[1 - \frac{\alpha_{ph}}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right) + O(\alpha^2)\right]$


$\alpha_{\overline{MS}}(\mu) = \alpha_{ph}\left[1 - \frac{\alpha_{ph}}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right) + O(\alpha^2)\right]$

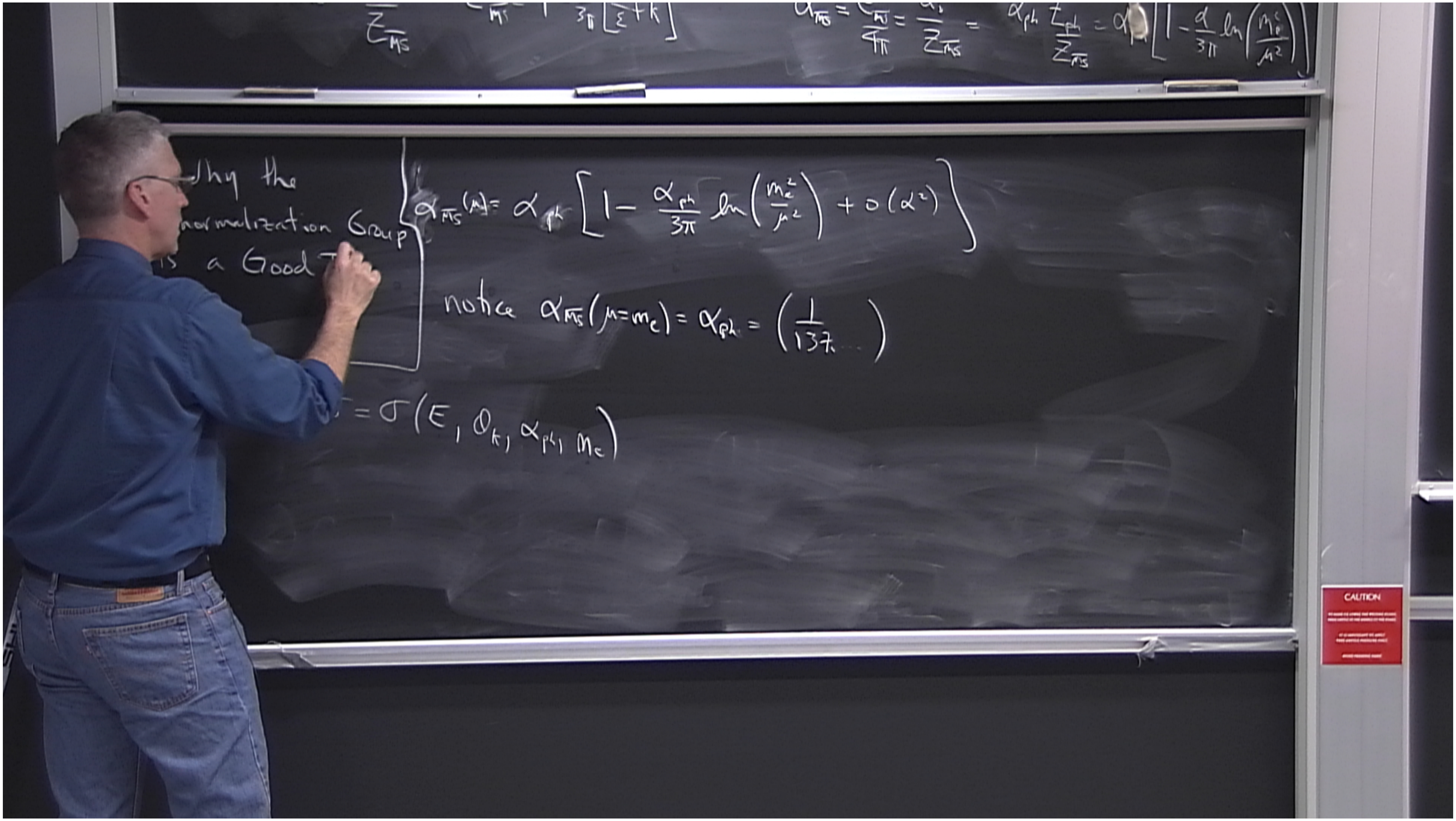


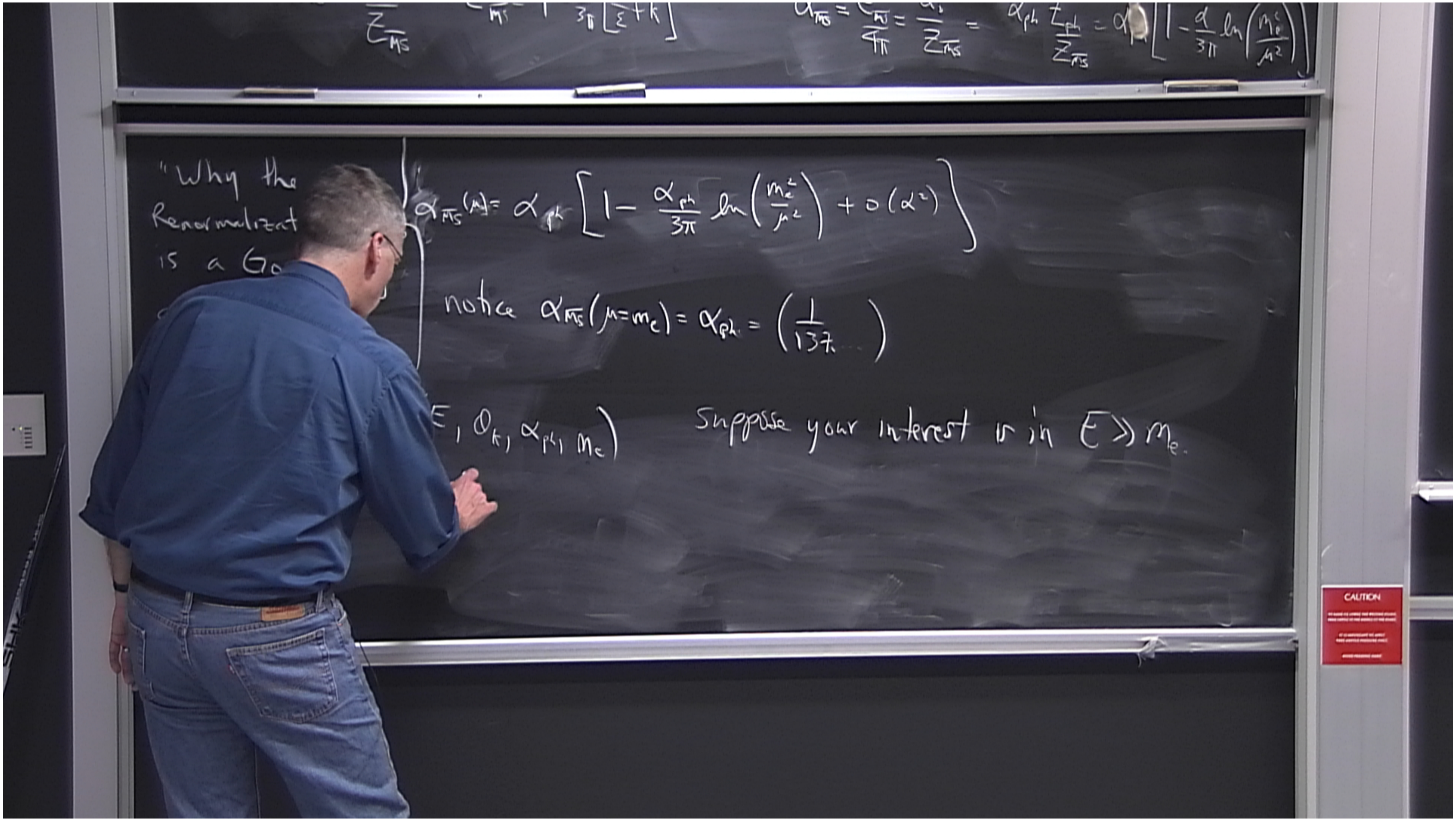
$\alpha_{\overline{MS}}(\mu) = \alpha_{ph}\left[1 - \frac{\alpha_{ph}}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right) + O(\alpha^2)\right]$
notice $\alpha_{\overline{MS}}(\mu = m_e) = \alpha_{ph}$

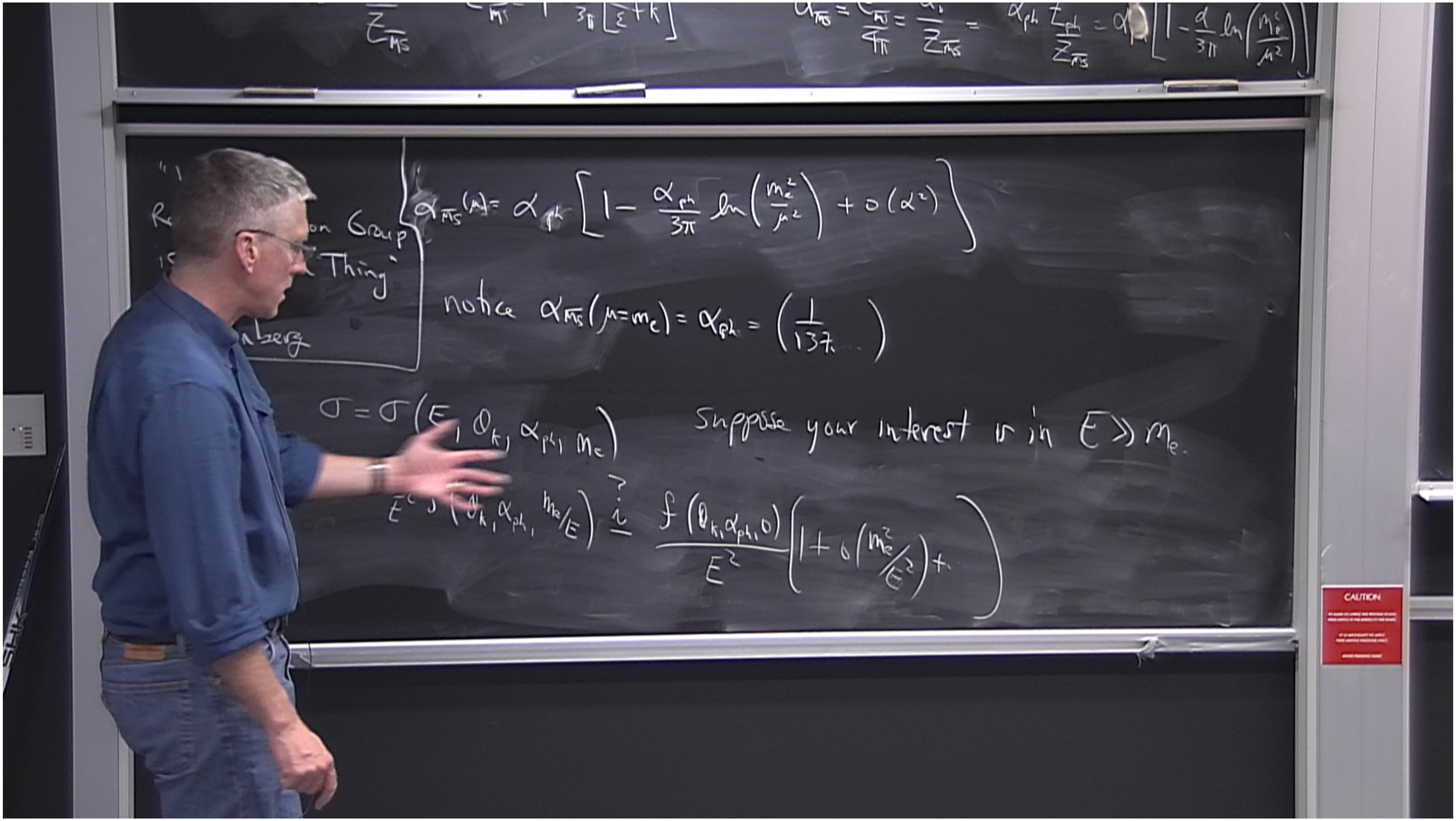

$\alpha_{\overline{MS}}(\mu) = \alpha_{ph}\left[1 - \frac{\alpha_{ph}}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right) + O(\alpha^2)\right]$
notice $\alpha_{\overline{MS}}(\mu = m_e) = \alpha_{ph} = \left(\frac{1}{137\ldots}\right)$

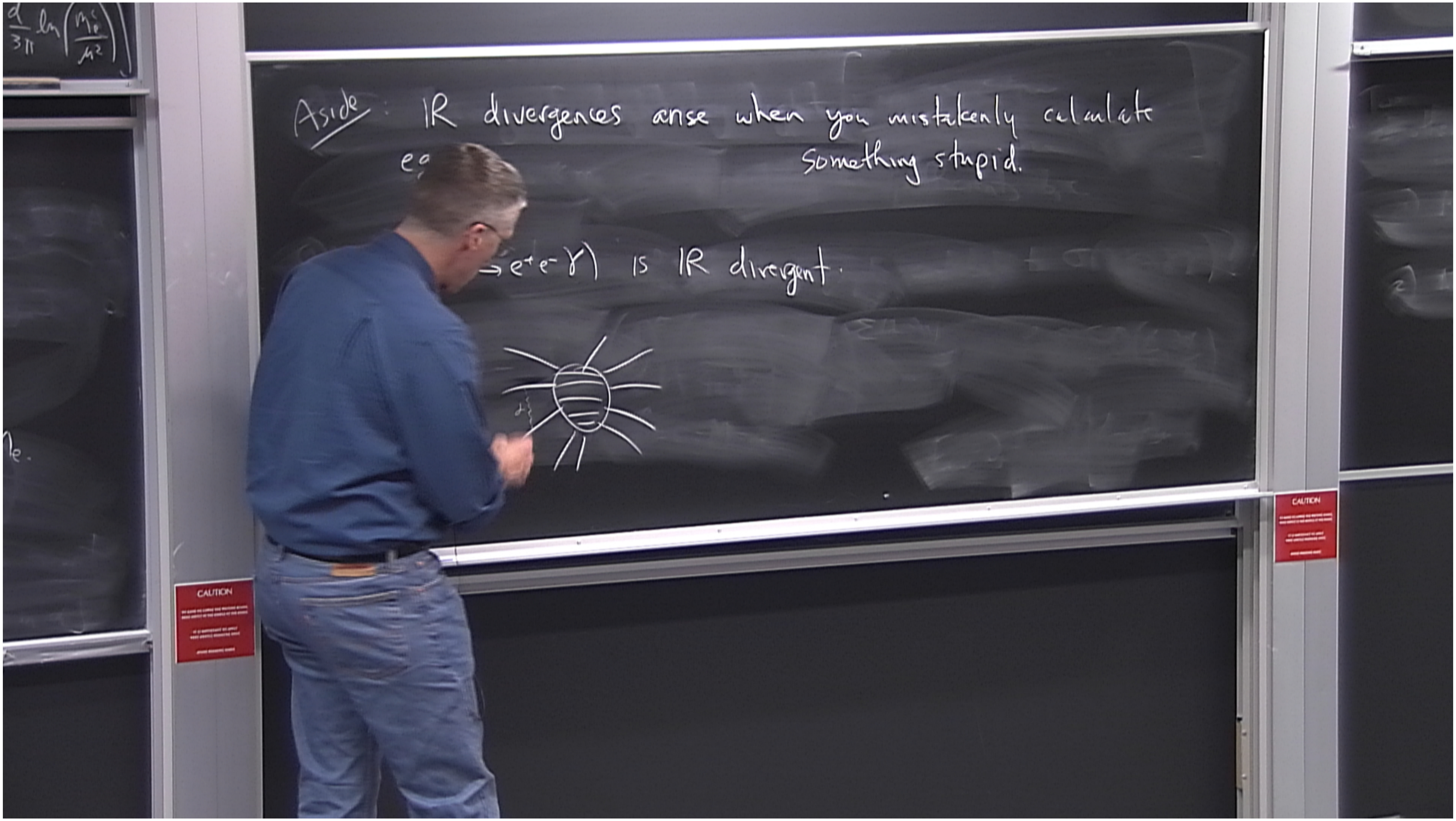


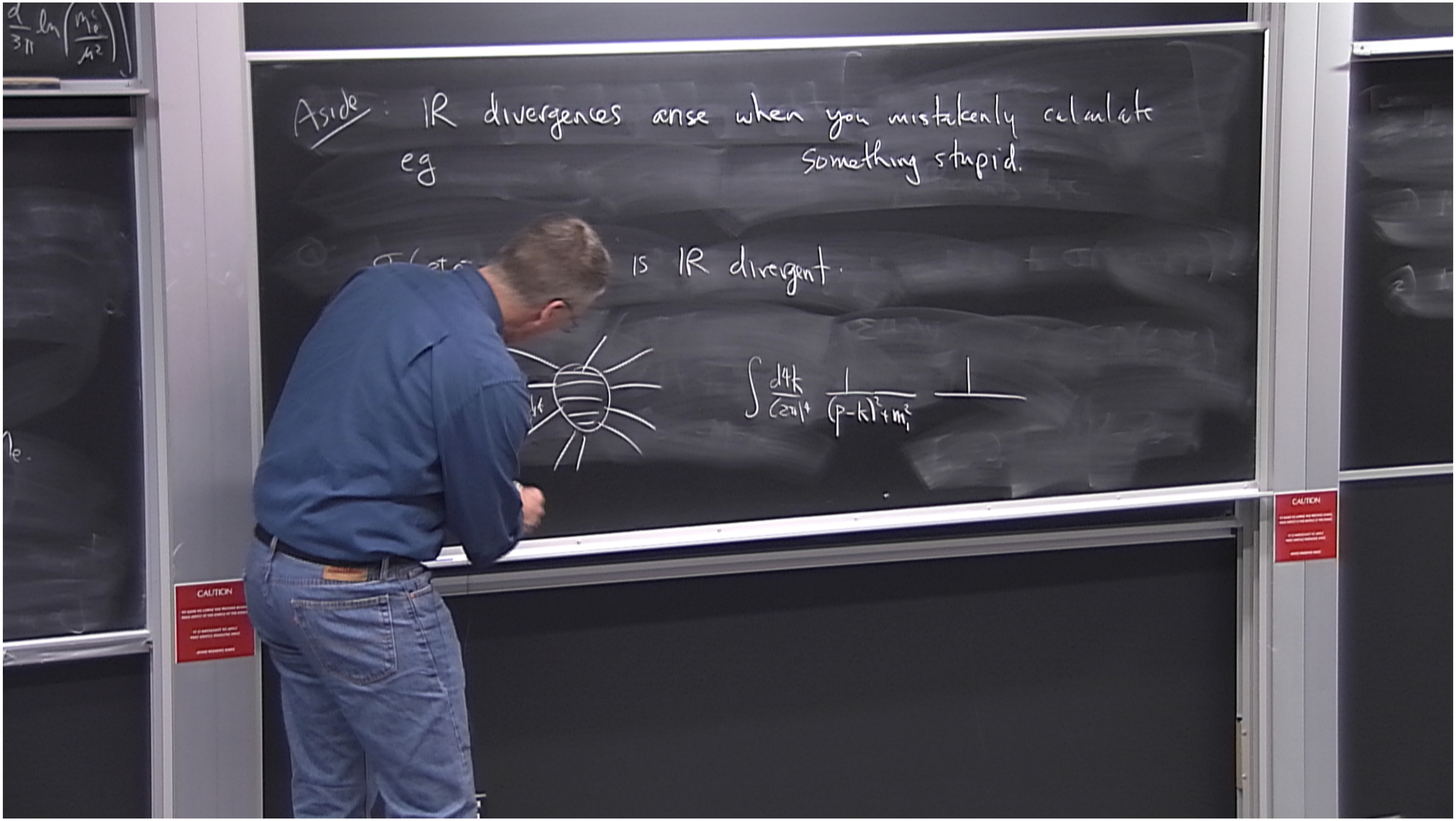
α_MS(μ) = α_ph [1 − α_ph/3π ln(m_e²/μ²) + O(α²)]
notice α_MS(μ=m_e) = α_ph = (1/137...)
e, α_ph, m_e)

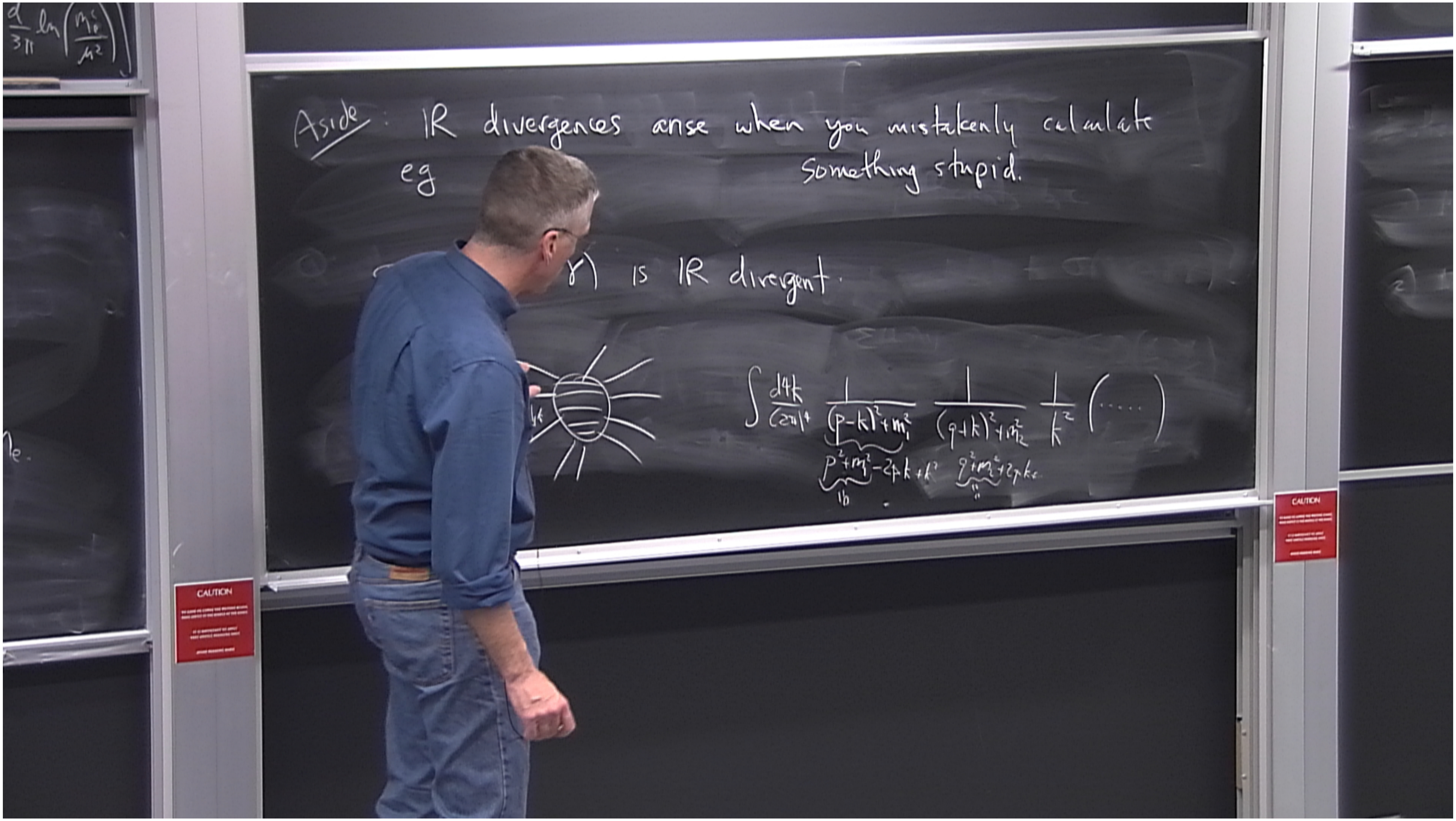

hy the
normalization Group
is a Good
$\alpha_{\overline{MS}}(\mu) = \alpha_{ph}\left[1 - \frac{\alpha_{ph}}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right) + O(\alpha^2)\right]$
notice $\alpha_{\overline{MS}}(\mu = m_e) = \alpha_{ph} = \left(\frac{1}{137.\ldots}\right)$
$= \sigma(E, \theta_k, \alpha_{ph}, m_e)$
CAUTION


"Why the
Renormalizat
is a Go
α_MS(μ) = α_ph [1 − (α_ph/3π) ln(m_e²/μ²) + O(α²)]
notice α_MS(μ=m_e) = α_ph = (1/137...)
E, θ_k, α_ph, m_e)
Suppose your interest is in E >> m_e.


Renormalization Group
Thing
$\alpha_{\overline{MS}}(\mu) = \alpha_{ph}\left[1 - \frac{\alpha_{ph}}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right) + O(\alpha^2)\right]$
notice $\alpha_{\overline{MS}}(\mu = m_e) = \alpha_{ph} = \left(\frac{1}{137.\ldots}\right)$
$\sigma = \sigma(E, \theta_k, \alpha_{ph}, m_e)$
Suppose your interest is in $E \gg m_e$.
$\overset{?}{\simeq} \frac{f(\theta_k, \alpha_{ph}, 0)}{E^2}\left(1 + O\left(\frac{m_e^2}{E^2}\right) + \ldots\right)$
CAUTION


Aside: IR divergences arise when you mistakenly calculate
something stupid.
is IR divergent.


Aside: IR divergences arise when you mistakenly calculate
eg something stupid.
is IR divergent.
∫ d4k/(2π)4 1/((p-k)2+m12) 1/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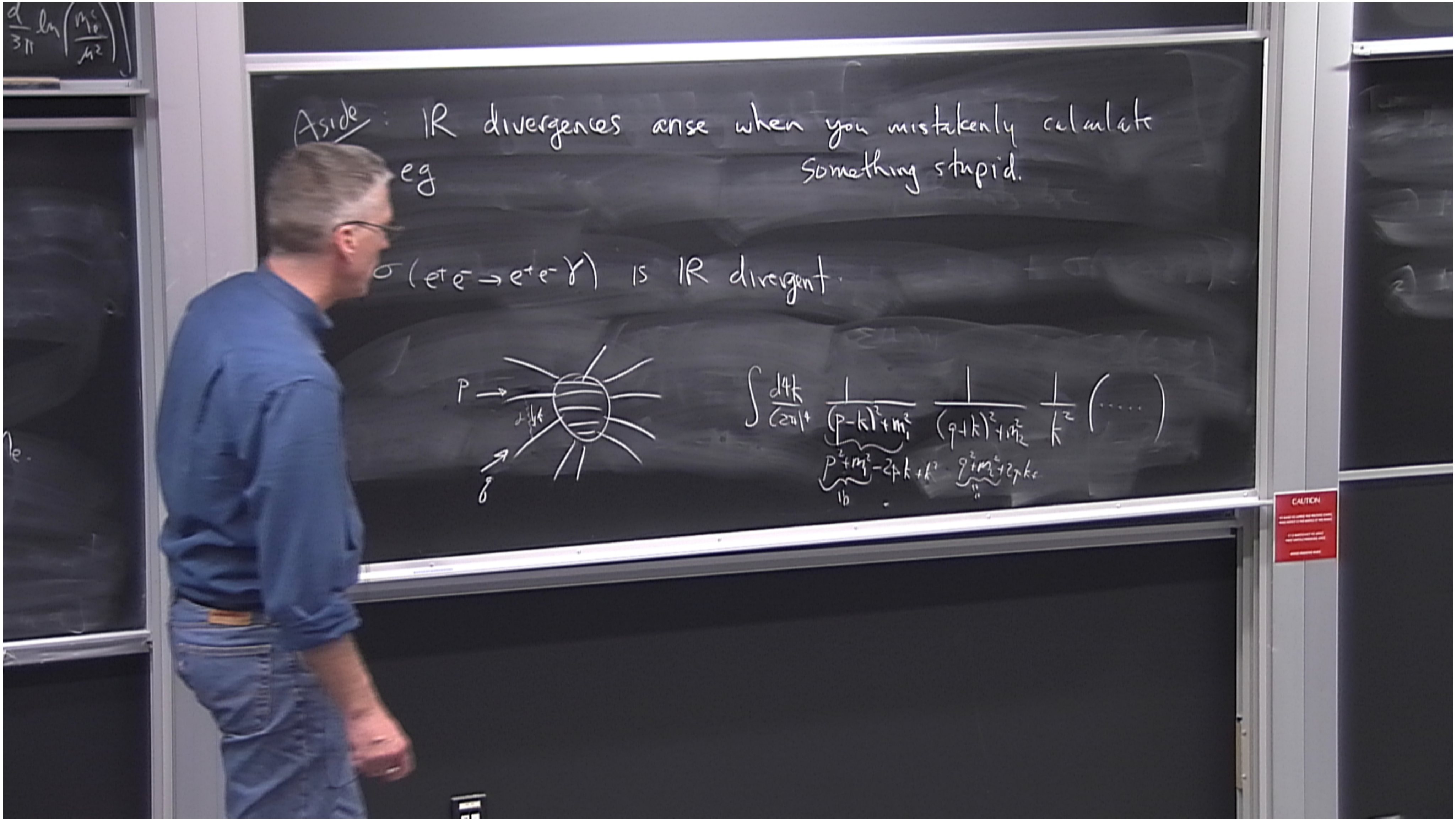
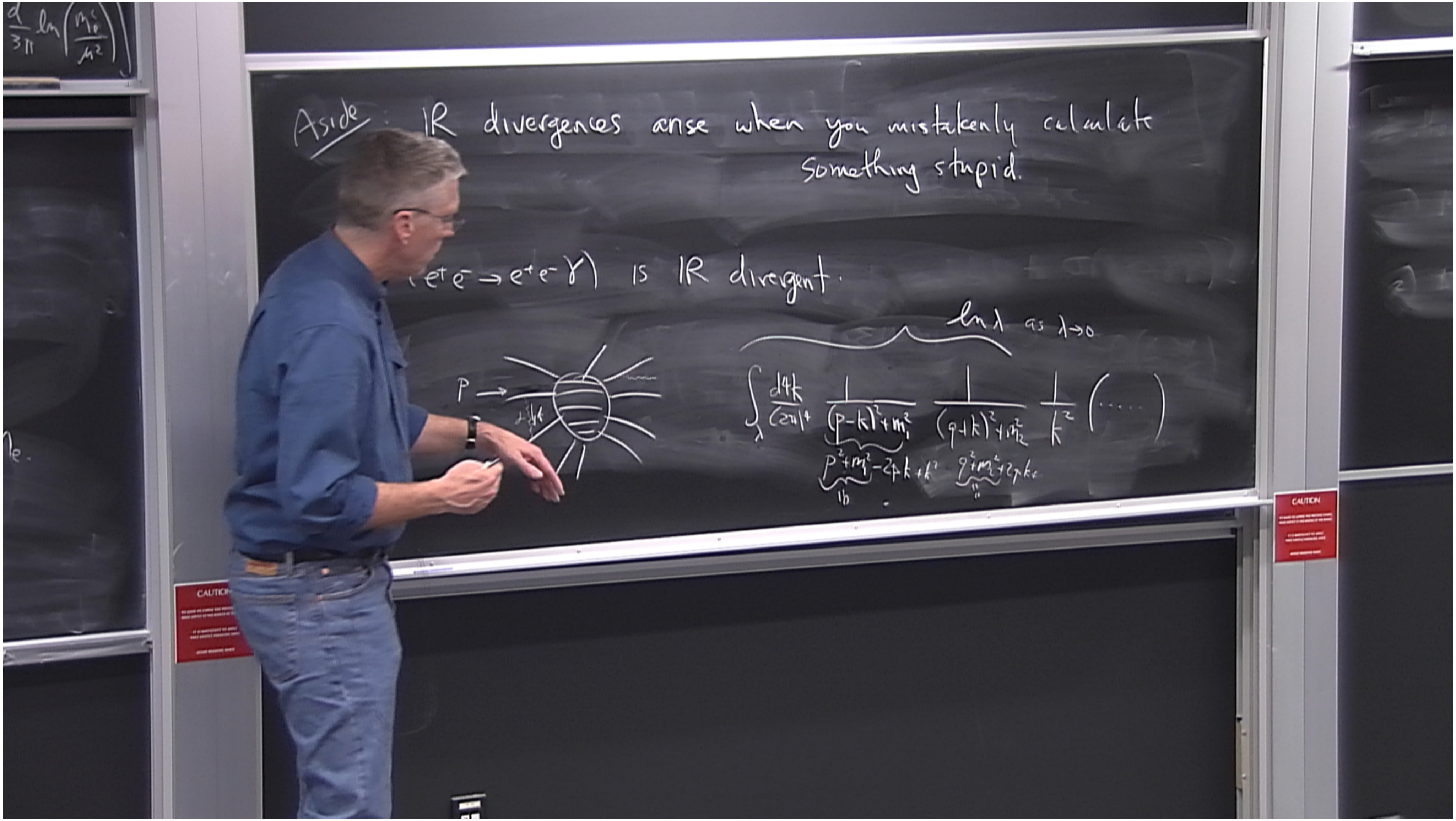
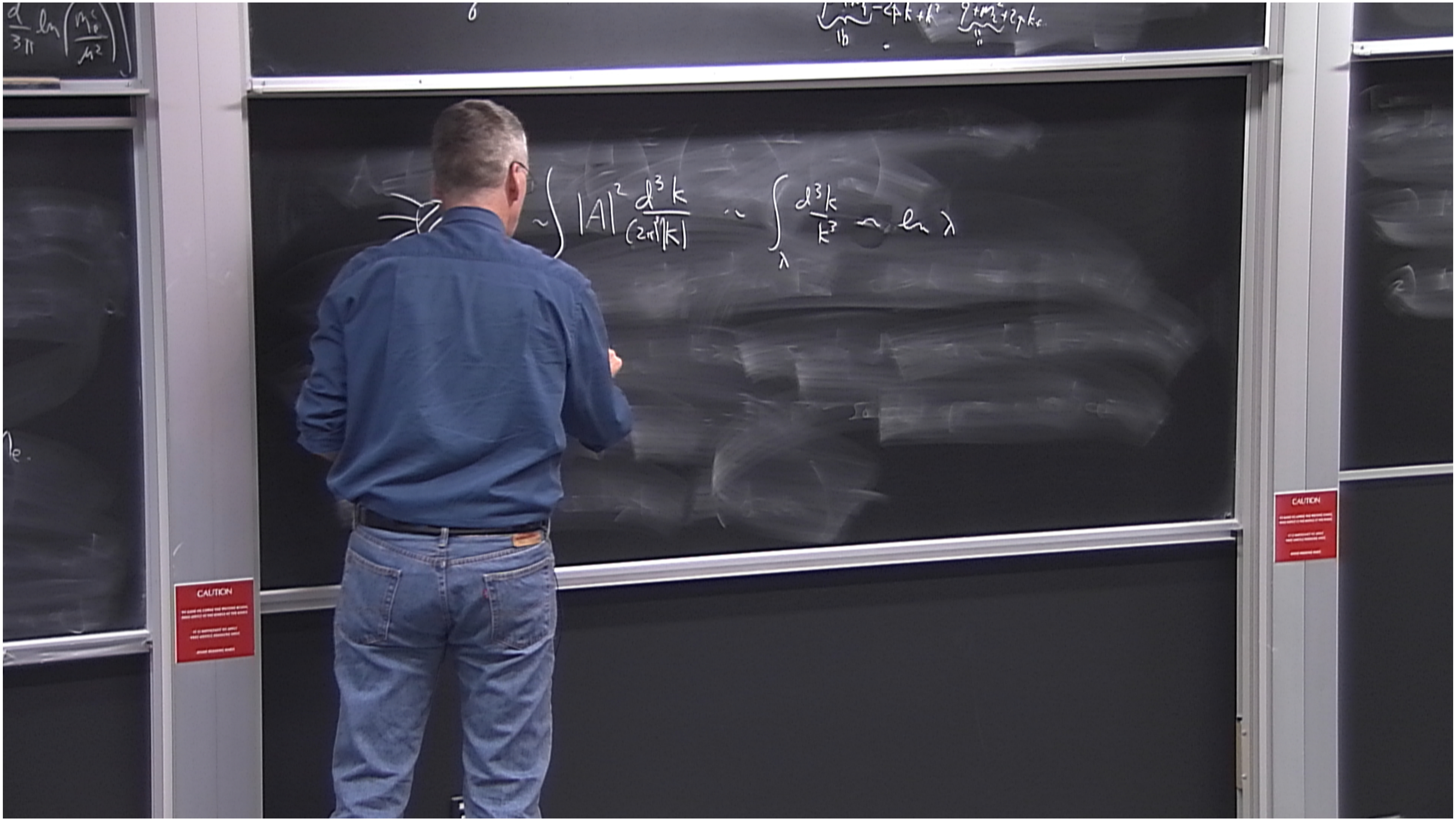
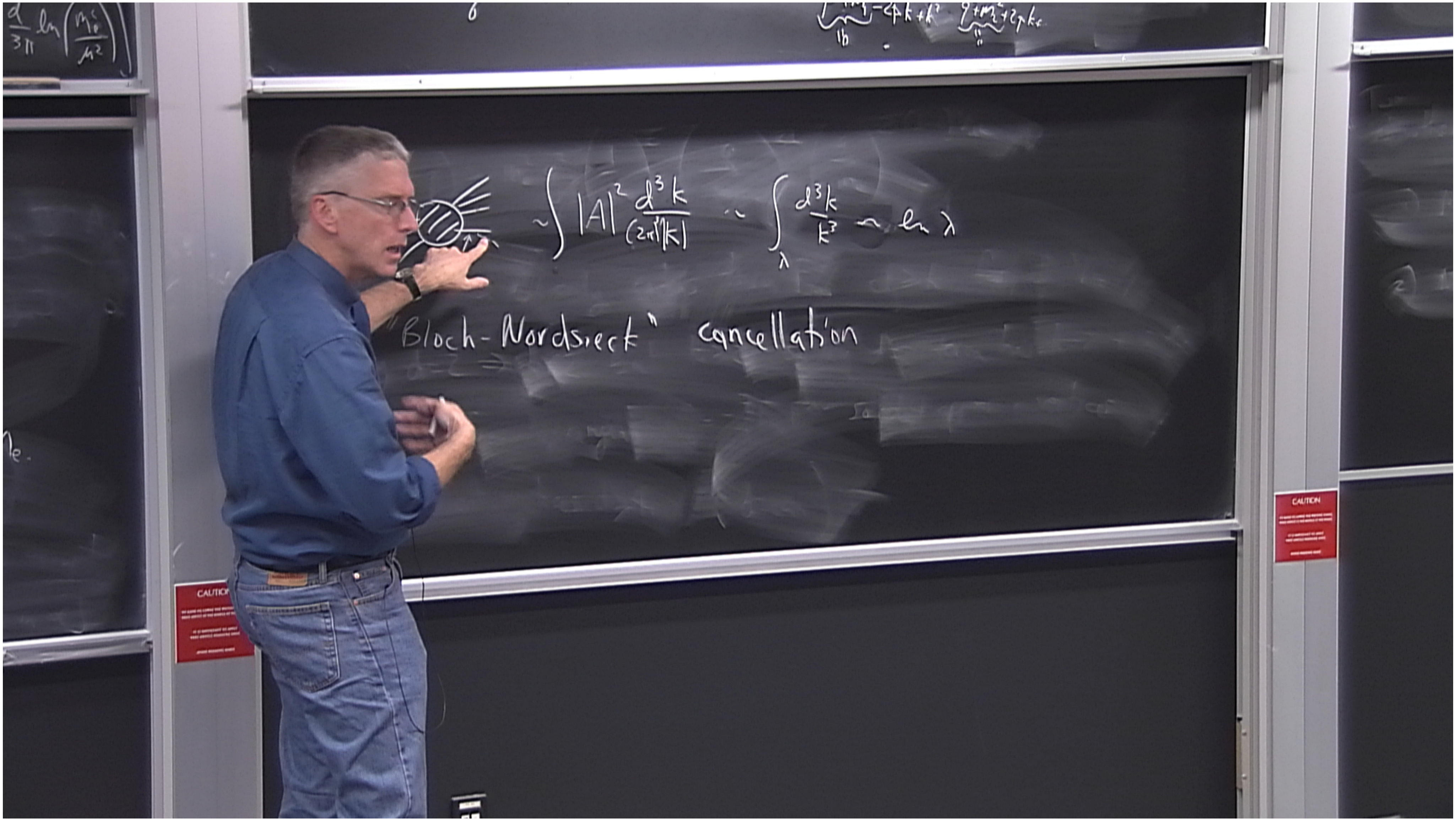
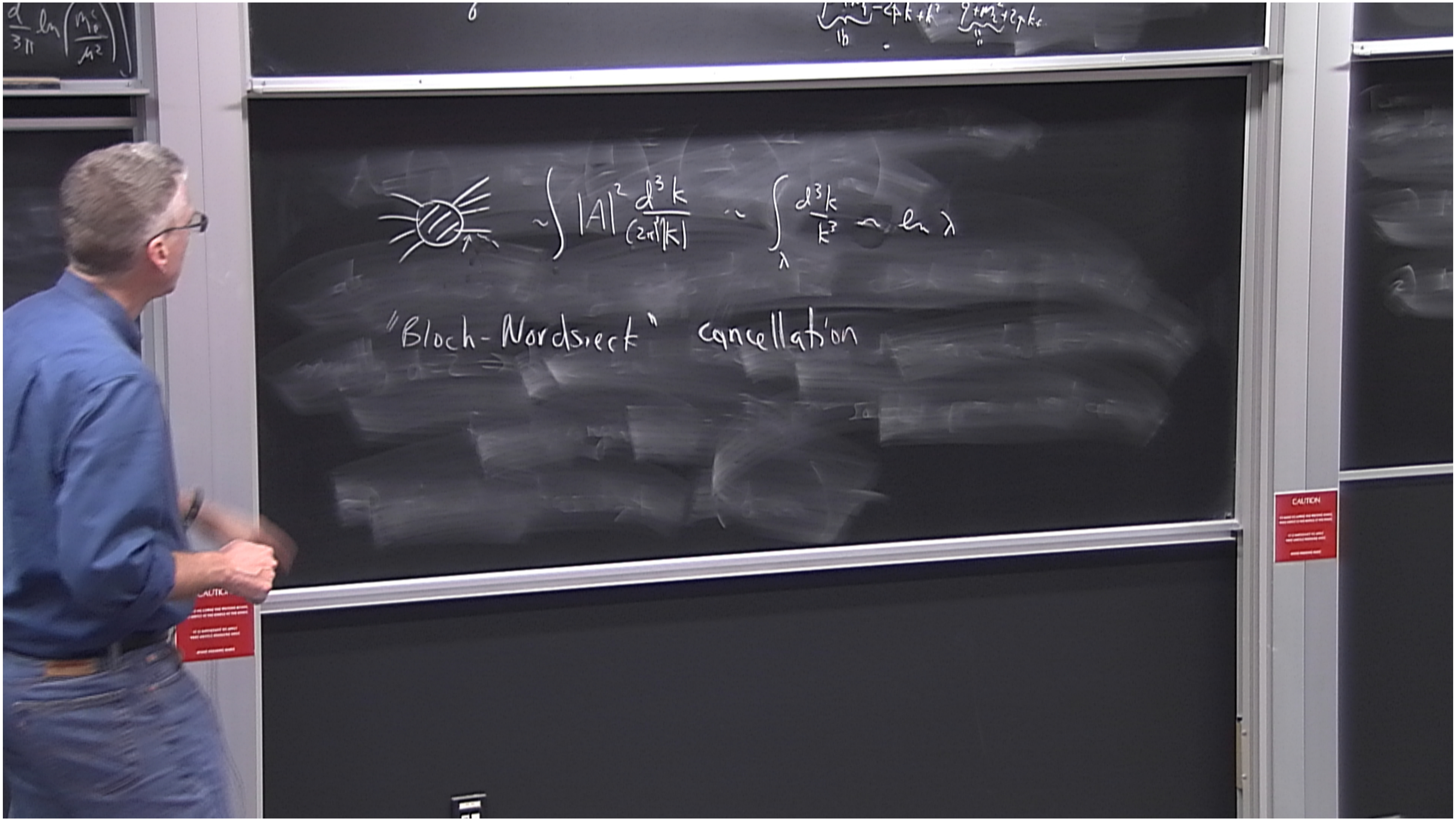
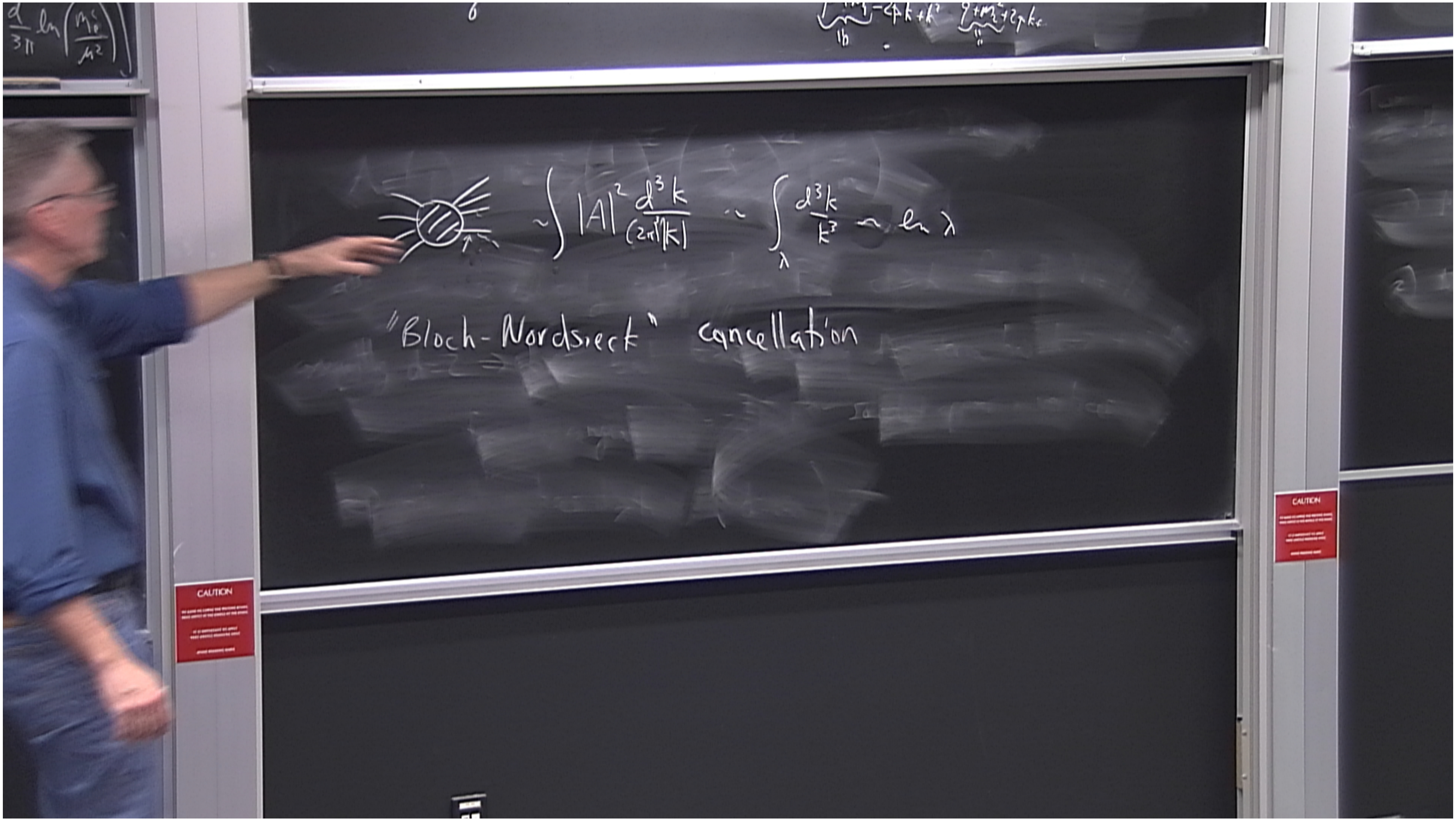

Aside: IR divergences arise when you mistakenly calculate something stupid.
eg
σ(e⁺e⁻ → e⁺e⁻γ) is IR divergent.
p
q
$\int \frac{d^4k}{(2\pi)^4} \frac{1}{\underbrace{(p-k)^2+m_1^2}_{p^2+m_1^2-2pk+k^2}} \frac{1}{\underbrace{(q+k)^2+m_2^2}_{q^2+m_2^2+2qk+k^2}} \frac{1}{k^2} (\ldots\ldots)$


Aside: IR divergences arise when you mistakenly calculate something stupid.
$(e^+e^- \to e^+e^-\gamma)$ is IR divergent.
$\ln\lambda$ as $\lambda \to 0$
$\int_\lambda \frac{d^4k}{(2\pi)^4} \frac{1}{(p-k)^2+m_1^2} \frac{1}{(q+k)^2+m_2^2} \frac{1}{k^2} (\cdots\cdots)$
$p^2+m_1^2-2pk+k^2$
CAUTION




~ ∫ |A|² d³k / (2π)³|k| ~ ∫λ d³k / k³ ~ ln λ
"Bloch-Nordsieck" cancellation


$\sim \int |A|^2 \frac{d^3k}{(2\pi)^3|k|} \sim \int_\lambda \frac{d^3k}{k^3} \sim \ln \lambda$
"Bloch-Nordsieck" cancellation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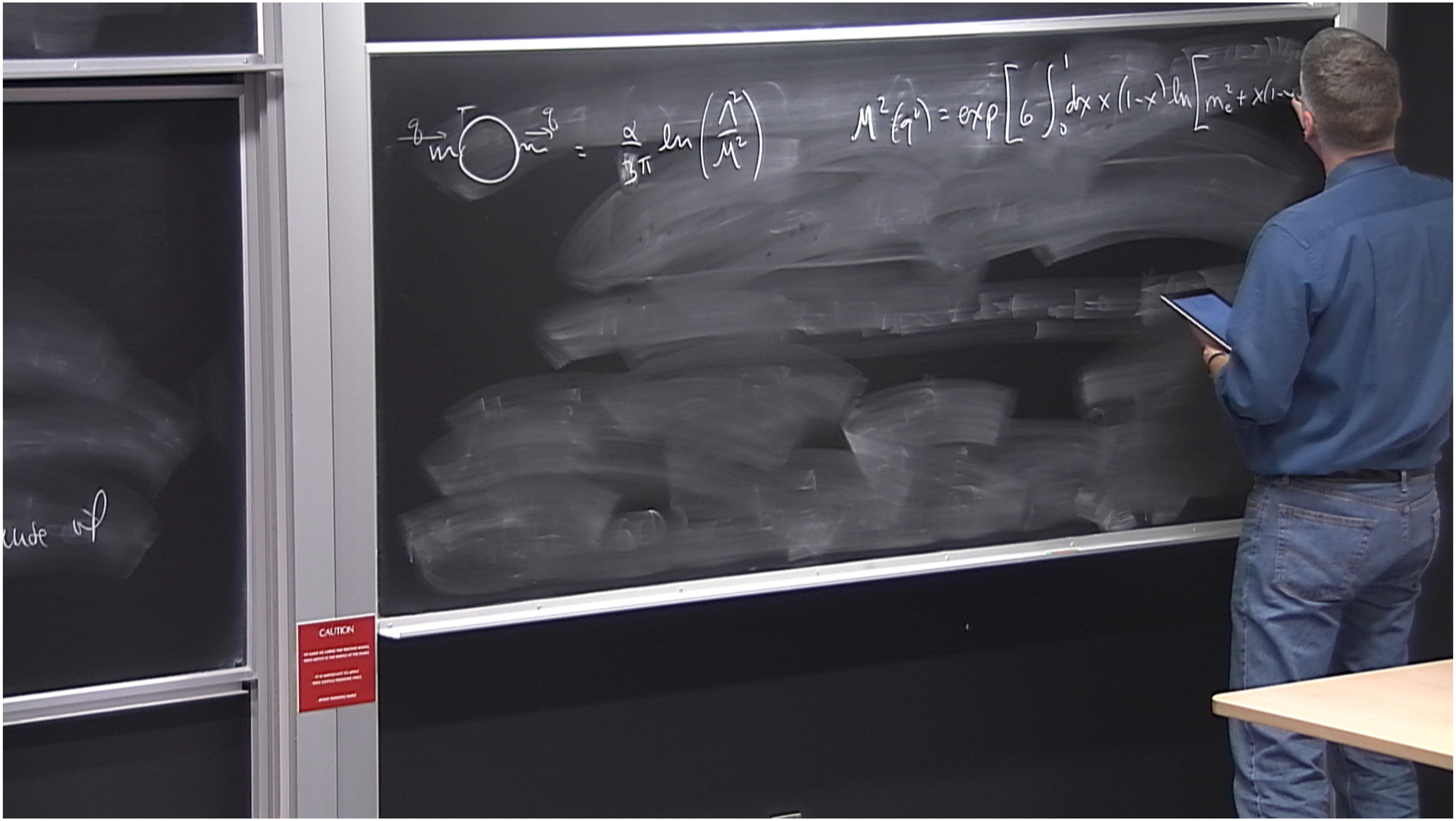
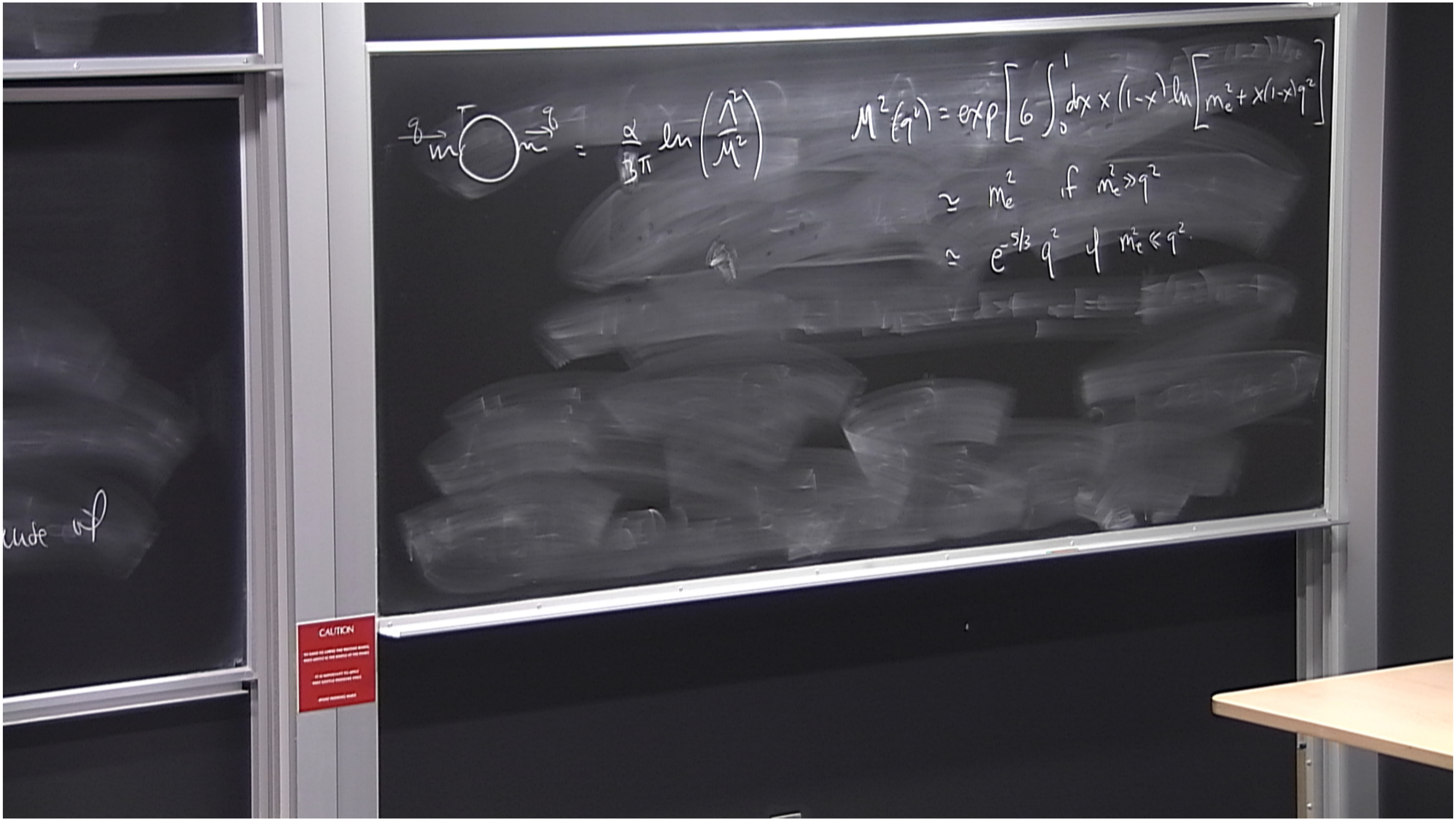
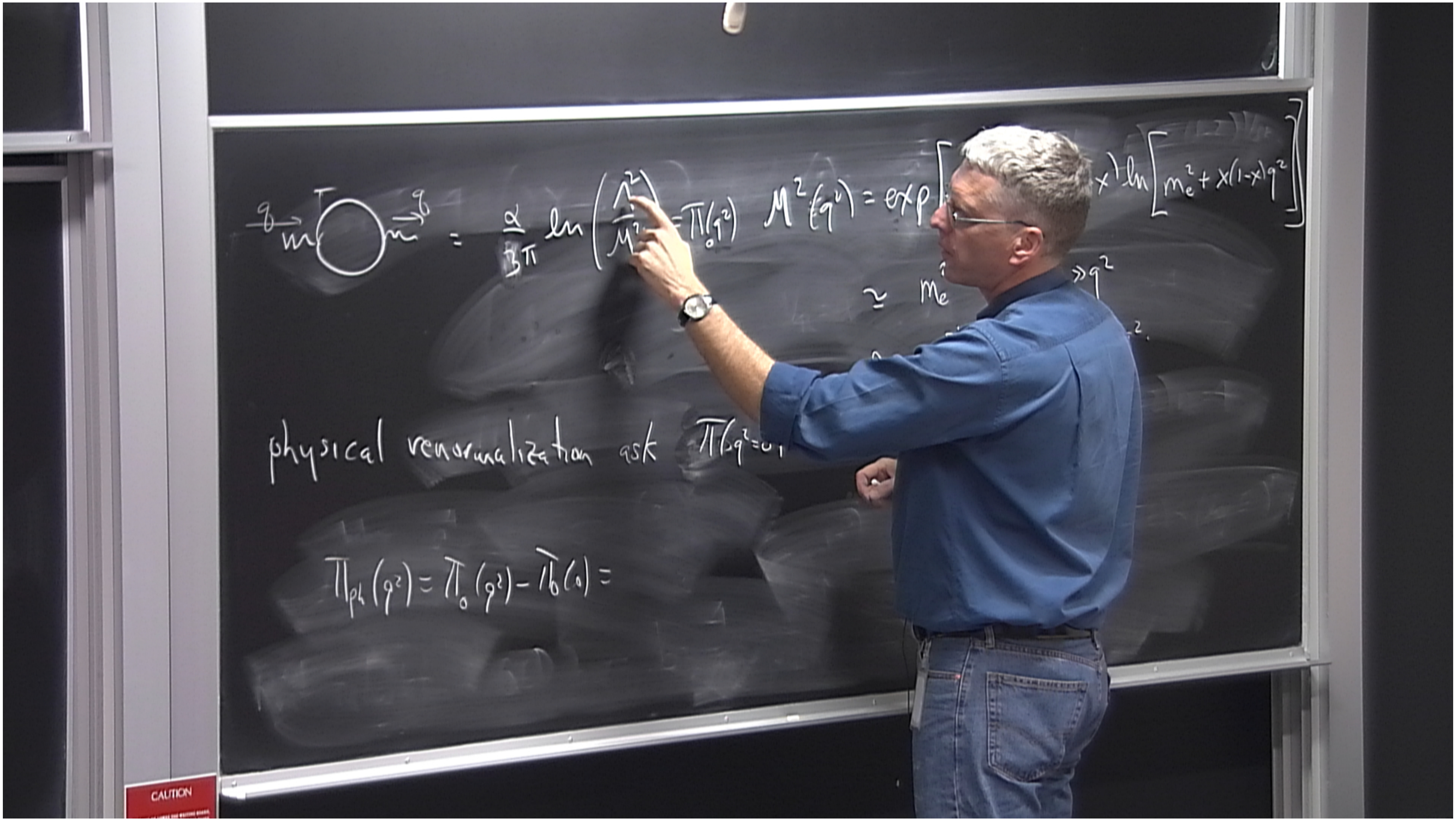
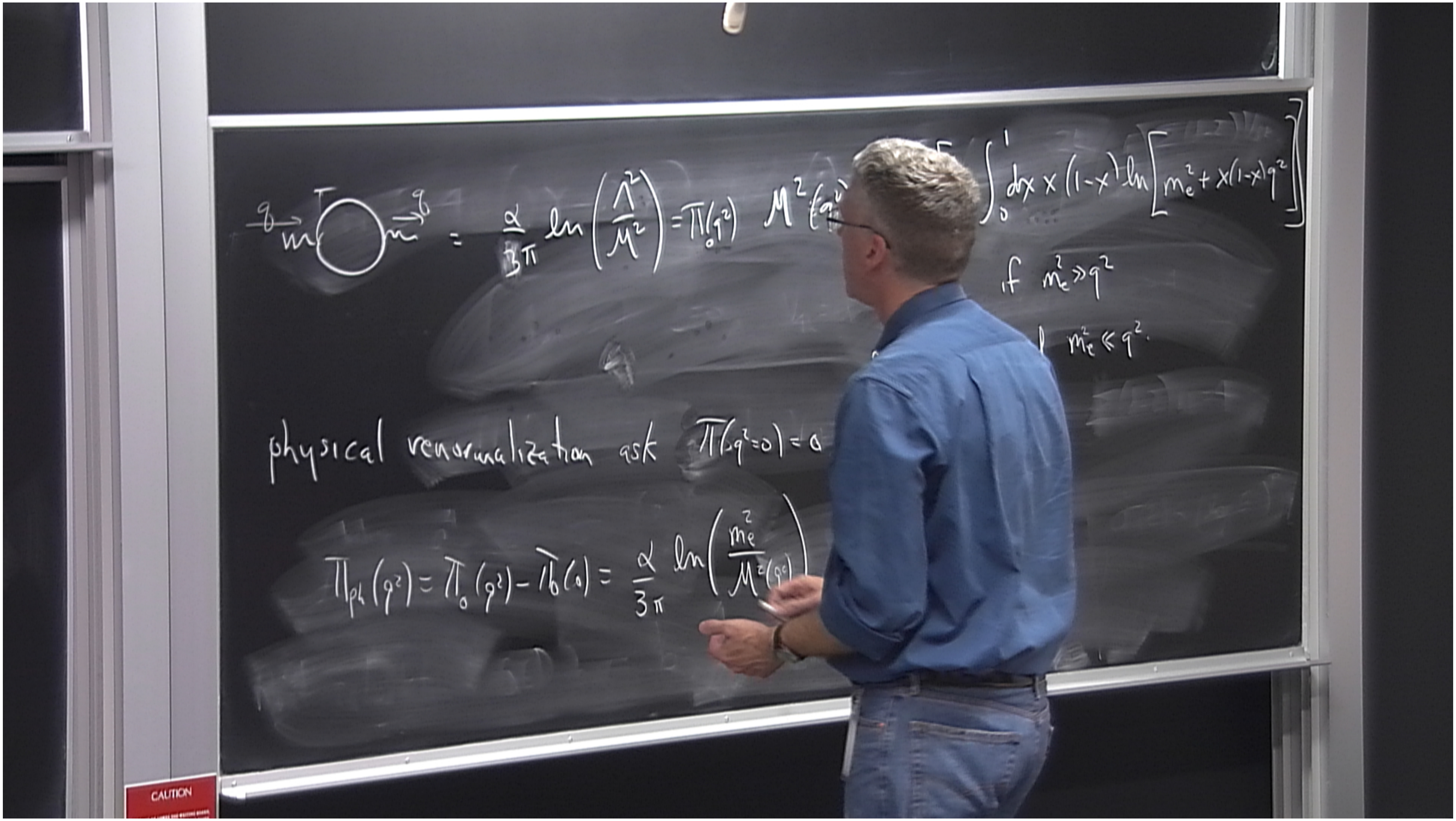
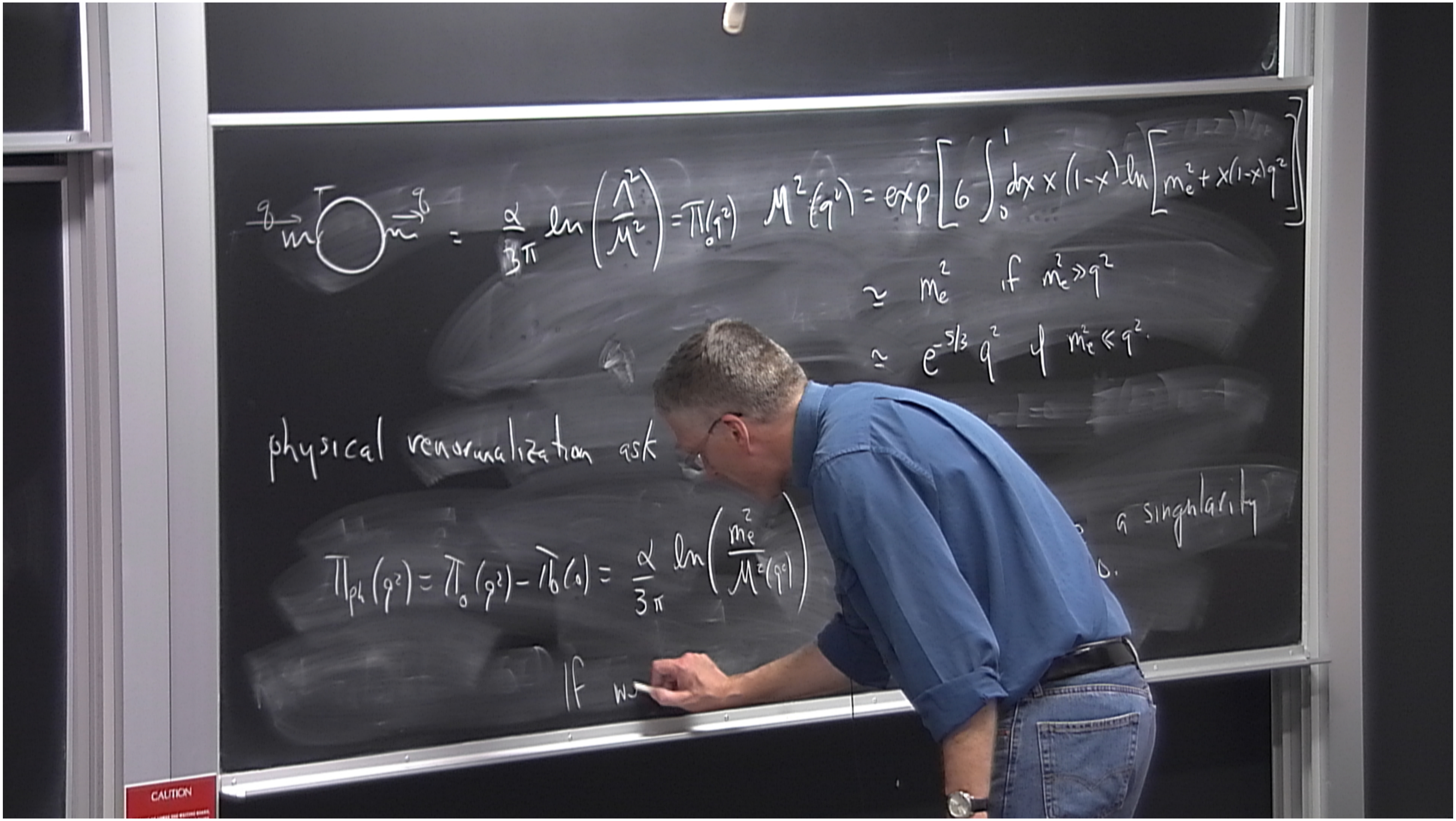

= α/3π ln(Λ²/M²)
M²(q²) = exp[6 ∫₀¹ dx x(1-x) ln[mₑ² + x(1-x)
CAUTION


$= \frac{\alpha}{3\pi} \ln\left(\frac{\Lambda^2}{M^2}\right)$
$M^2(q^2) = \exp\left[6\int_0^1 dx\, x(1-x) \ln\left[m_e^2 + x(1-x)q^2\right]\right]$
$\simeq m_e^2$ if $m_e^2 \gg q^2$
$\simeq e^{-5/3} q^2$ if $m_e^2 \ll q^2$.
ude of

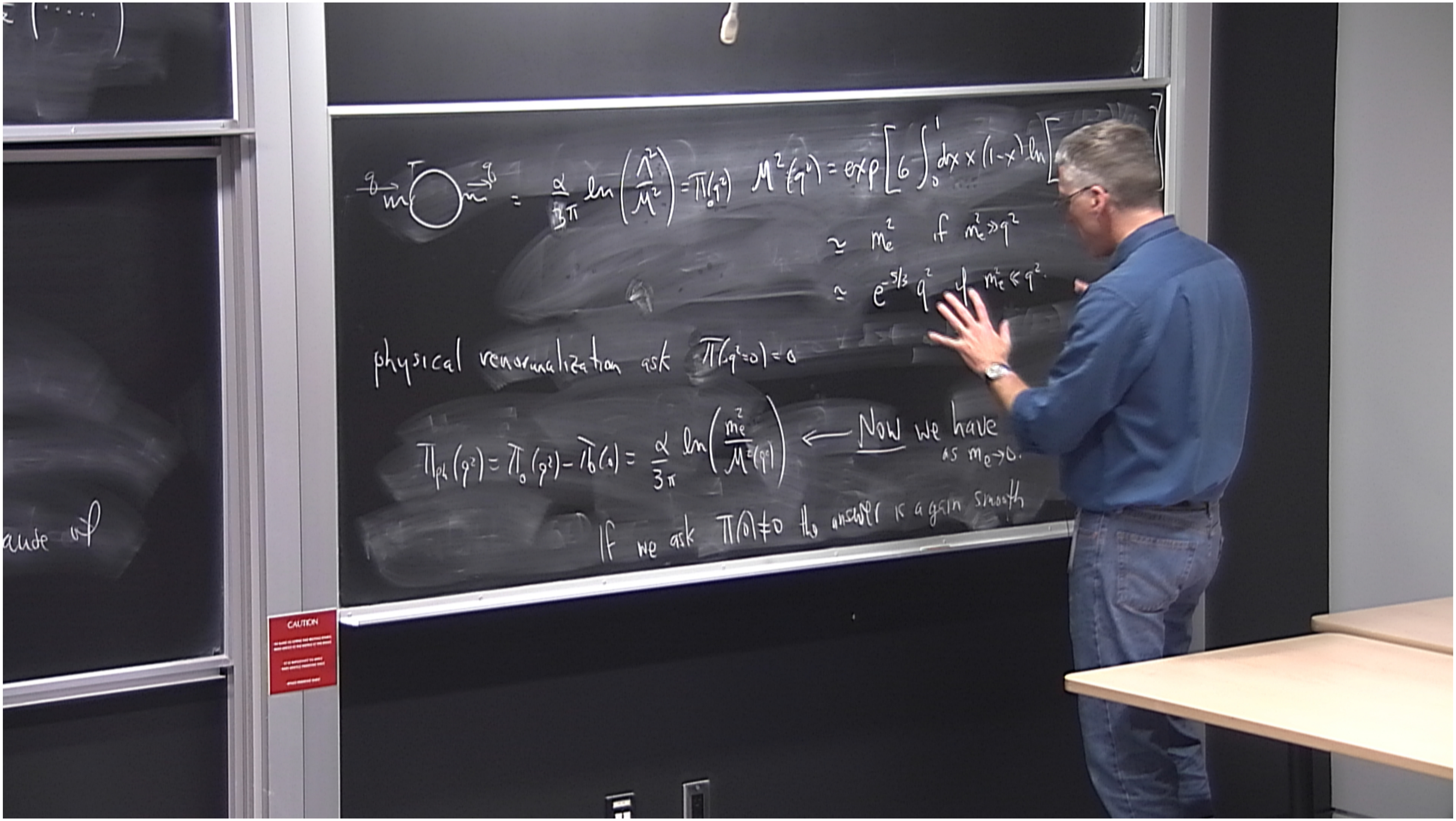


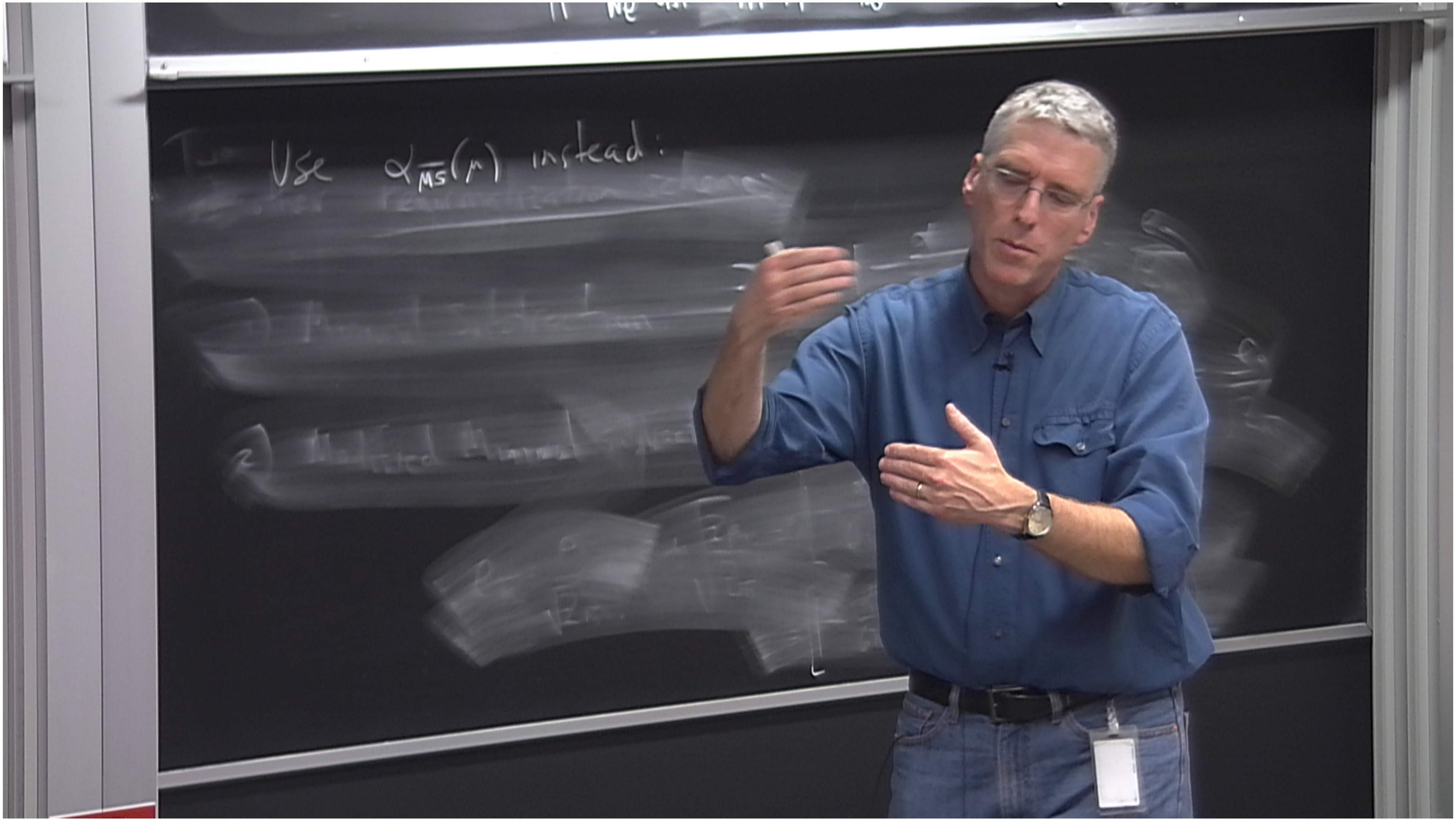
$= \frac{\alpha}{3\pi} \ln\left(\frac{\Lambda^2}{M^2}\right) = \Pi_0(q^2)$
$M^2(q^2) = \exp$
$\ln\left[m_e^2 + x(1-x)q^2\right]$
physical renormalization ask
$\Pi_{ph}(q^2) = \Pi_0(q^2) - \Pi_0(0) =$
CAUTION


physical renormalization ask
if $m_e^2 \gg q^2$
$m_e^2 \ll q^2$.
CAUTION

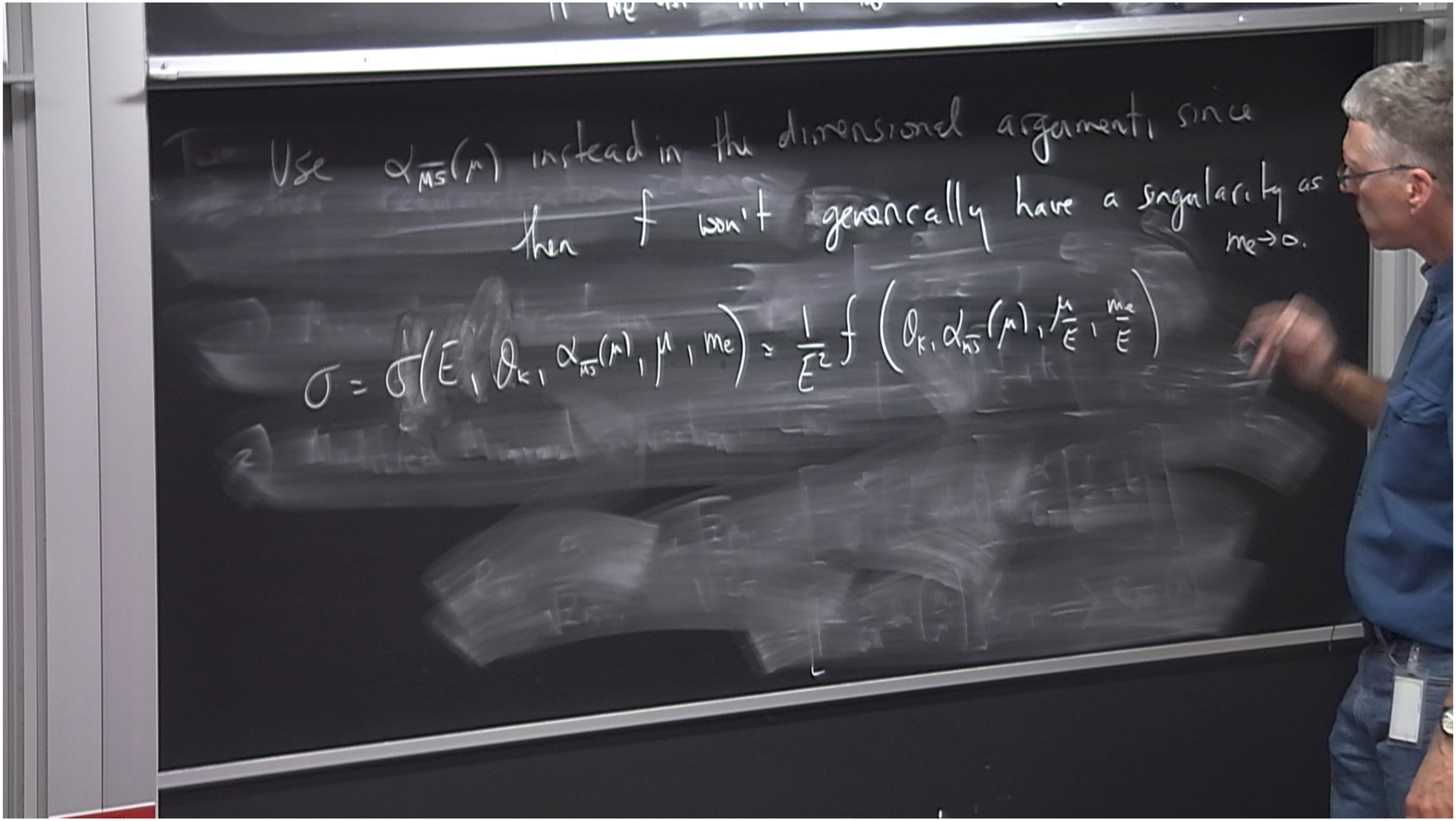


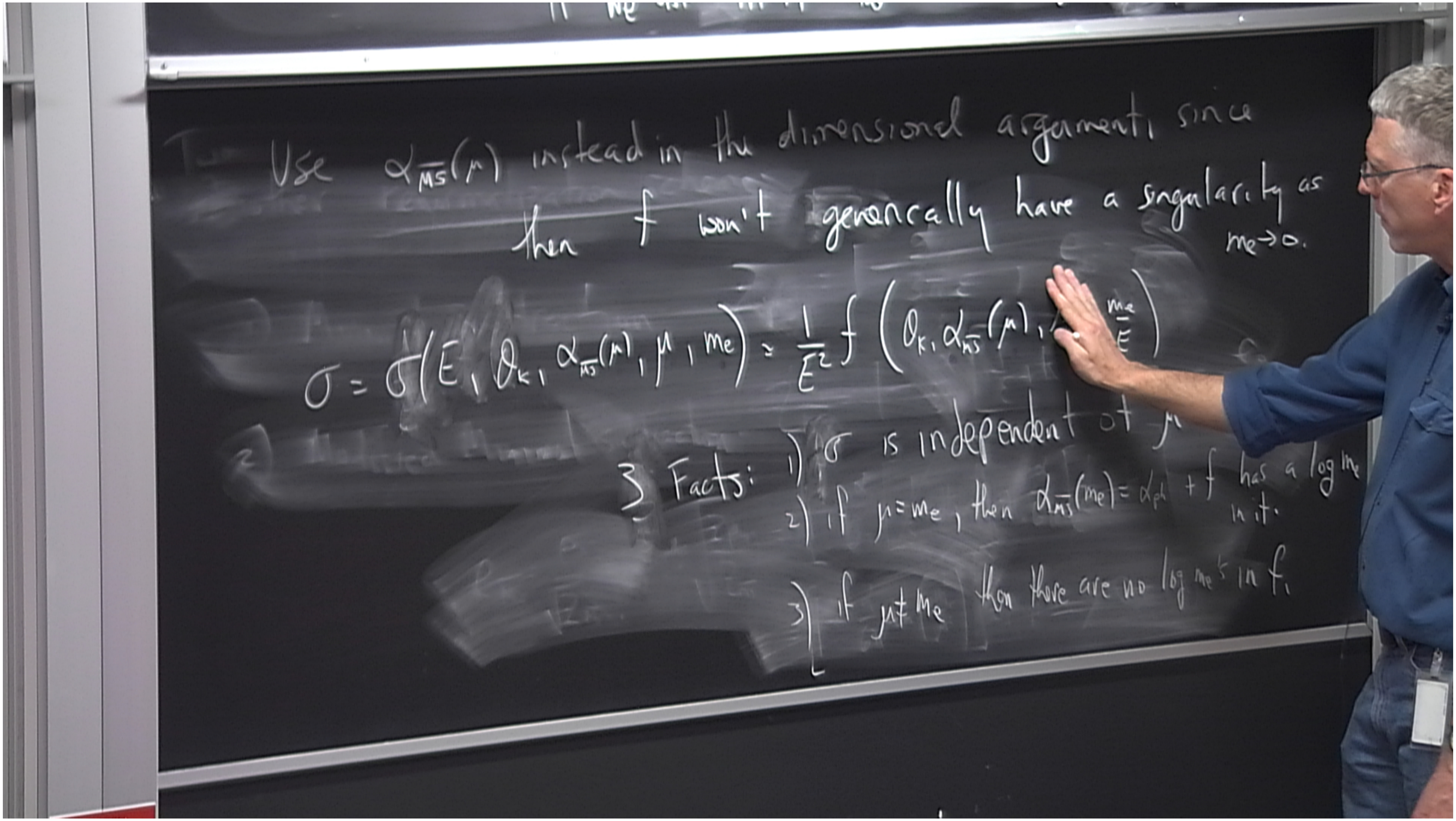
= α/3π ln(Λ²/M²) = Π₀(q²)
M²(q²) = exp[6 ∫₀¹ dx x(1-x) ln[mₑ² + x(1-x)q²]]
≃ mₑ² if mₑ² ≫ q²
≃ e^{-5/3} q² if mₑ² ≪ q².
physical renormalization ask
a singularity
Π_ph(q²) = Π₀(q²) − Π₀(0) = α/3π ln(mₑ²/M²(q²))
If w
CAUTION


physical renormalization ask $\Pi(q^2=0)=0$
$\Pi_{ph}(q^2) = \Pi_0(q^2) - \Pi_0(0) = \frac{\alpha}{3\pi} \ln\left(\frac{m_e^2}{M^2(q^2)}\right)$ ← Now we have
as $m_e \to 0$.
If we ask $\Pi(0) \neq 0$ the answer is again smooth
$\simeq m_e^2$ if $m_e^2 \gg q^2$
$\simeq e^{-5/3} q^2$ if $m_e^2 \ll q^2$



Use $\alpha_{\overline{MS}}(\mu)$ instead:


Use $\alpha_{\overline{MS}}(\mu)$ instead in the dimensional argument, since
then f won't generically have a singularity as
$m_e \to 0$.
$\sigma = \sigma(E, \theta_k, \alpha_{\overline{MS}}(\mu), \mu, m_e) = \frac{1}{E^2} f\left(\theta_k, \alpha_{\overline{MS}}(\mu), \frac{\mu}{E}, \frac{m_e}{E}\right)$


Use $\alpha_{\overline{MS}}(\mu)$ instead in the dimensional argument, since
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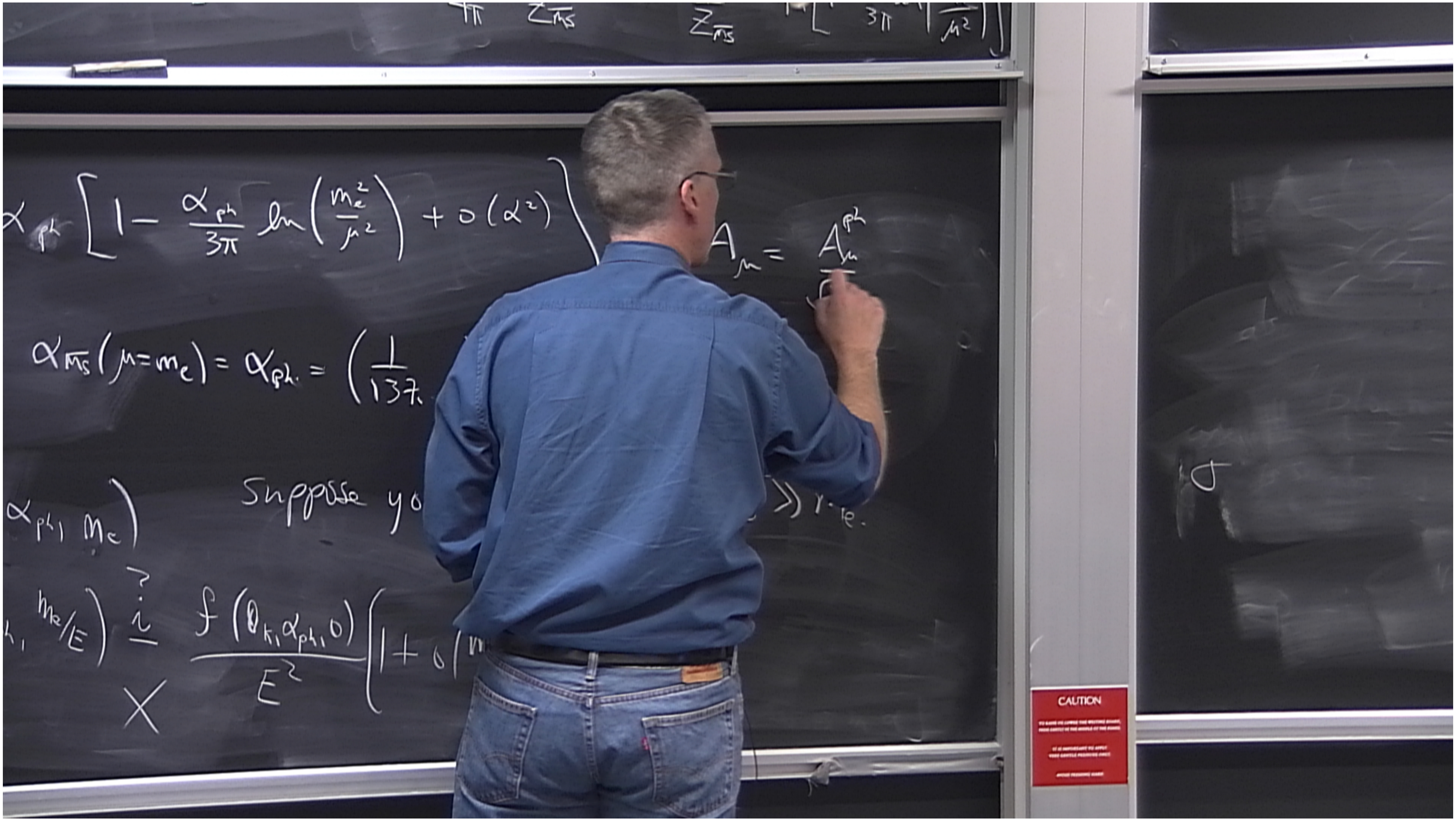
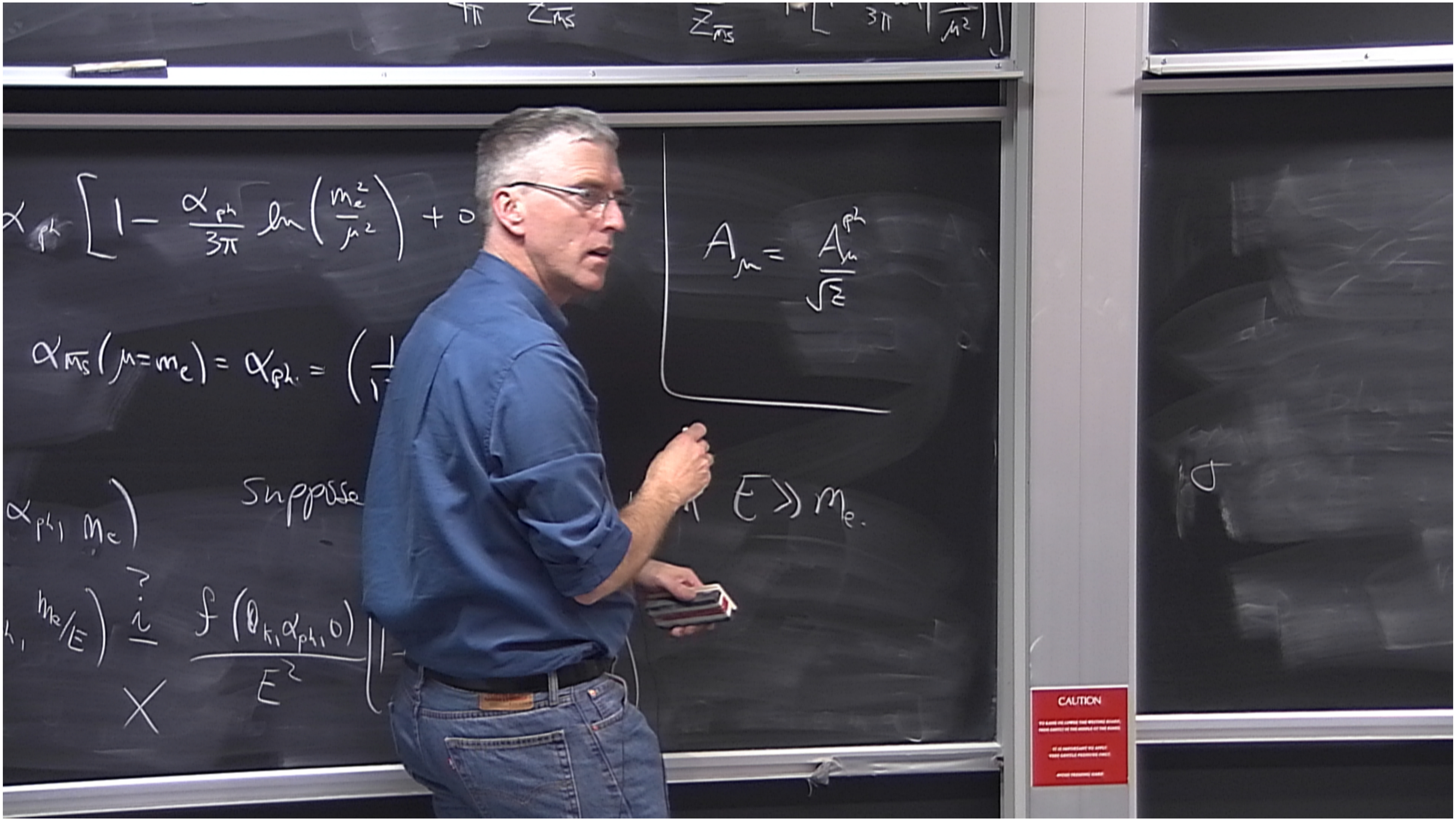
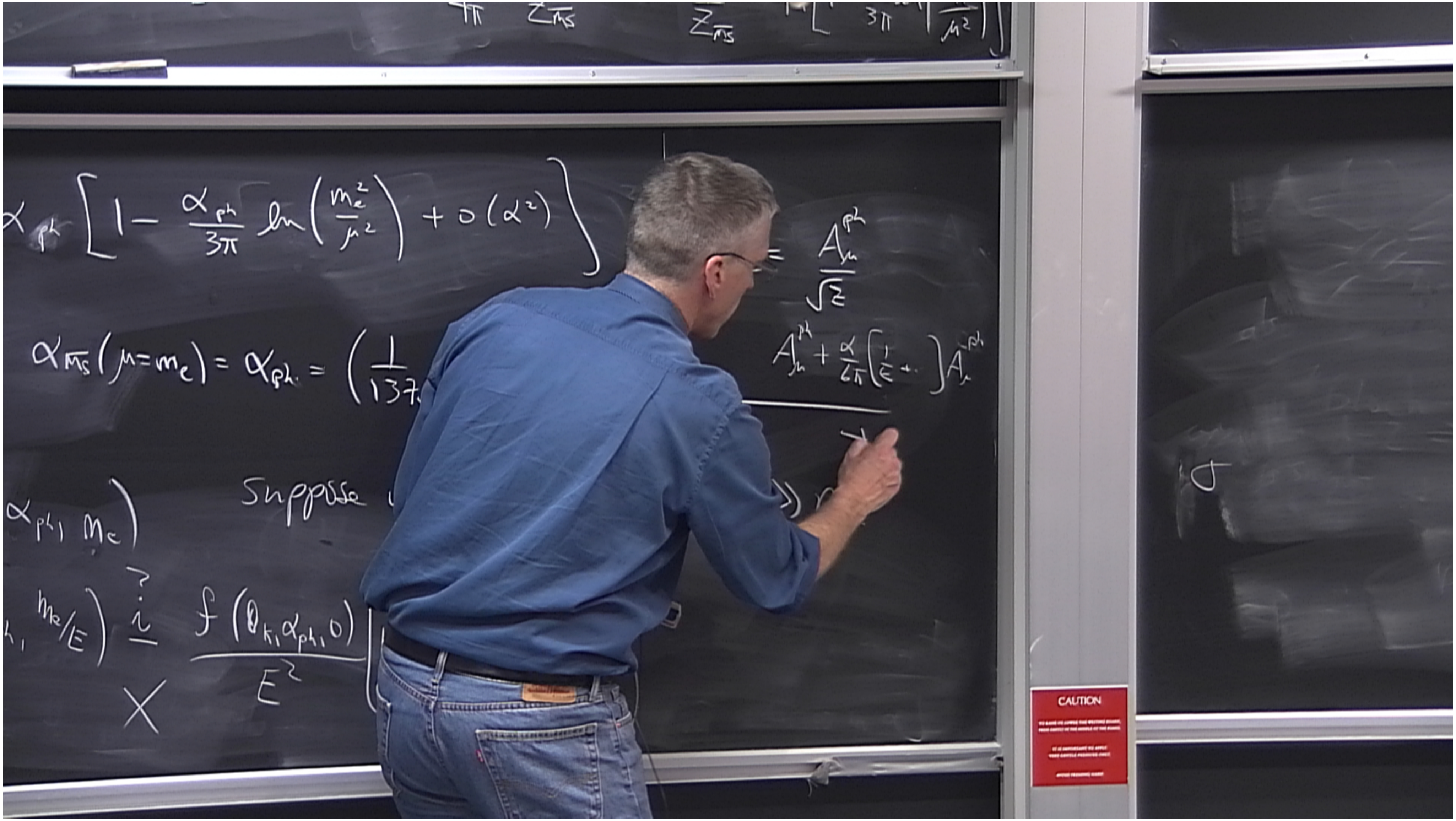
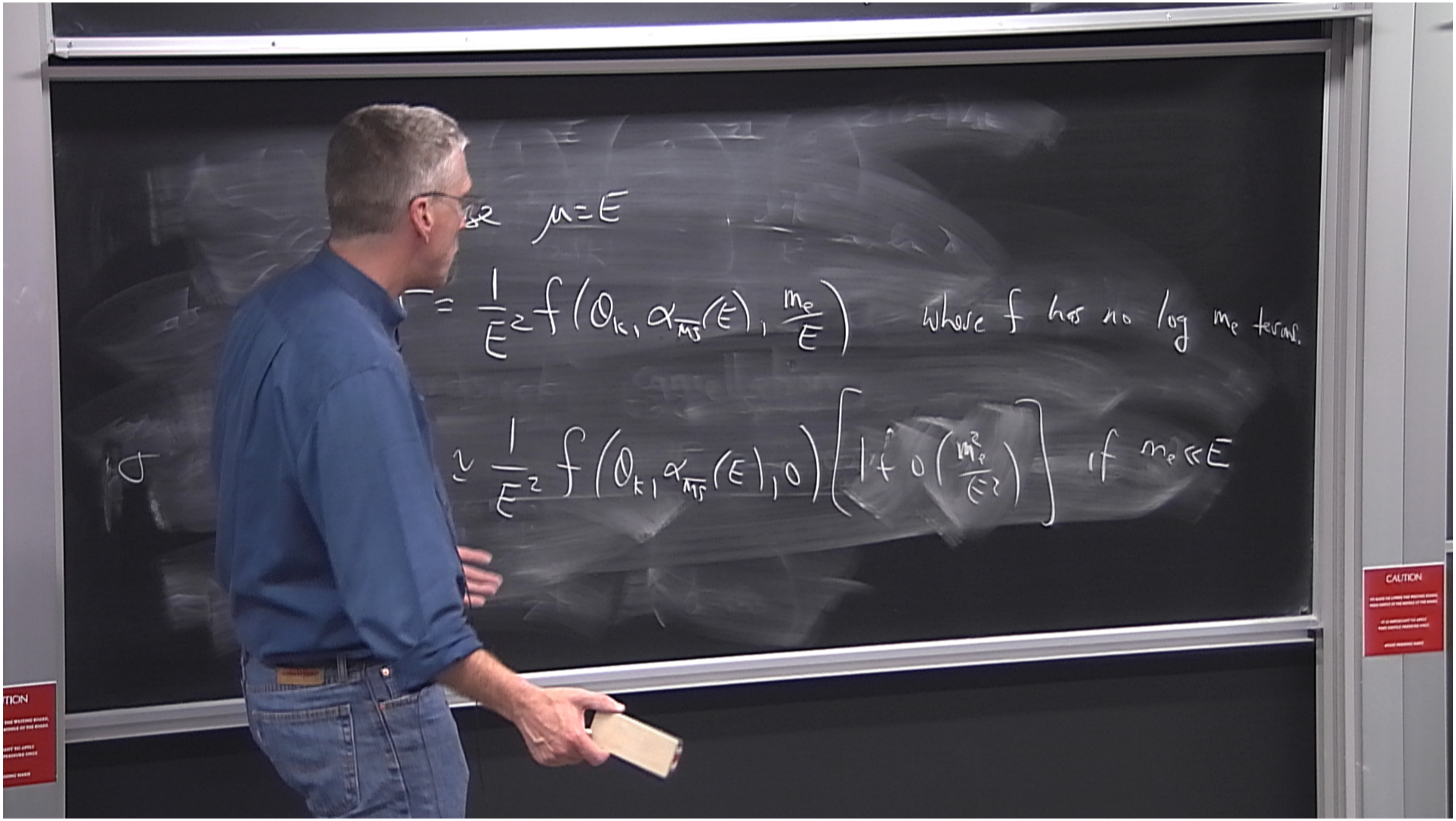
$\sigma = \sigma(E, \theta_k, \alpha_{\overline{MS}}(\mu), \mu, m_e) = \frac{1}{E^2} f\left(\theta_k, \alpha_{\overline{MS}}(\mu), \frac{m_e}{E}\right)$
3 Facts:
1) σ is independent of μ
2) if $\mu = m_e$, then $\alpha_{\overline{MS}}(m_e) = \alpha_{ph}$ + f has a log m_e in it.
3) if $\mu \neq m_e$ then there are no log m_e's in f.

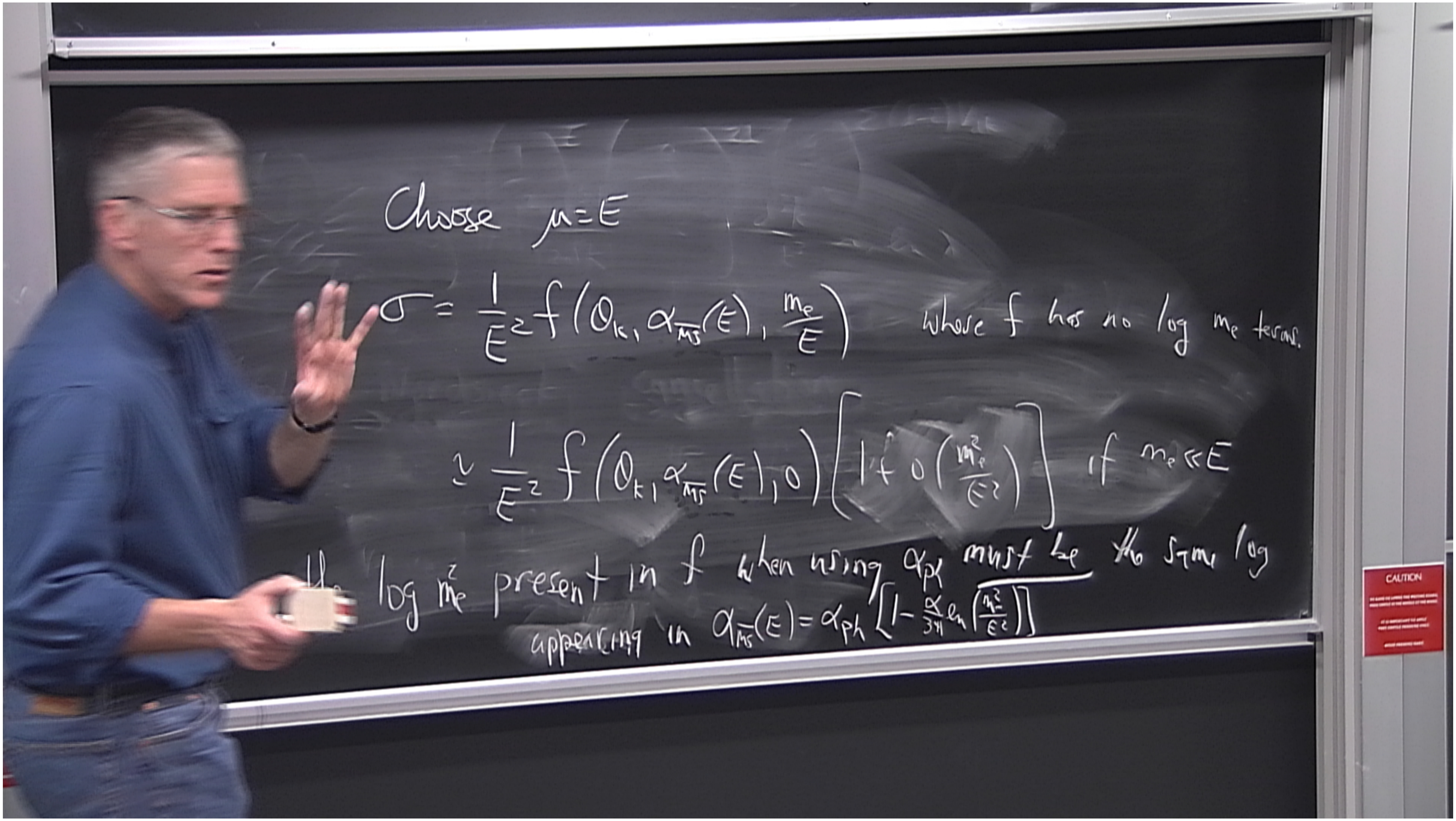

$\alpha_{\overline{MS}}(\mu = m_e) = \alpha_{ph.} = \left(\frac{1}{137.}\right.$
Suppose yo
$A_\mu = \frac{A_\mu^{ph}}{}$
$\frac{f(\theta_k, \alpha_{ph}, 0)}{E^2}\left(1 + o\right.$
CAUTION

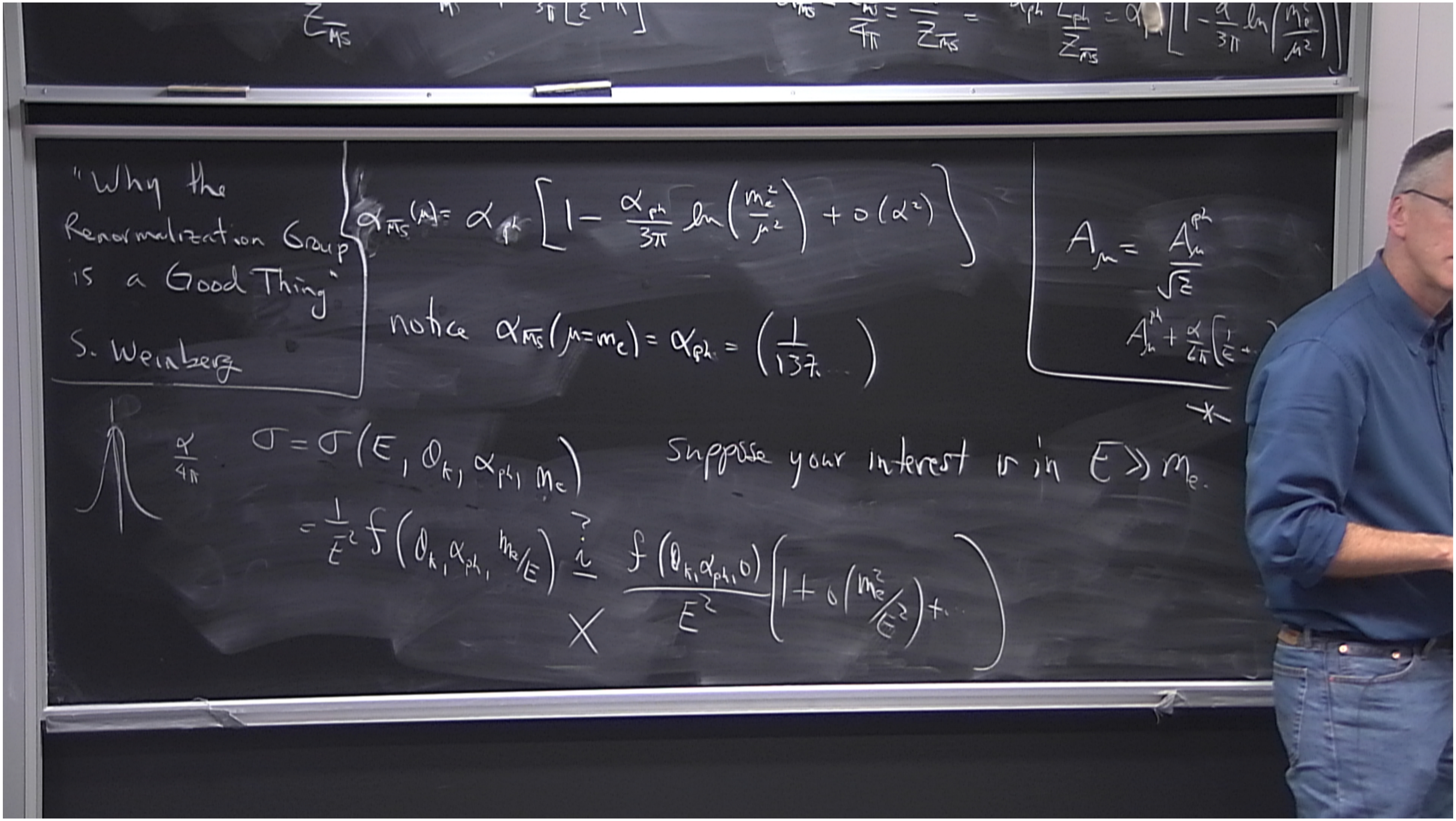

suppose
CAUTION

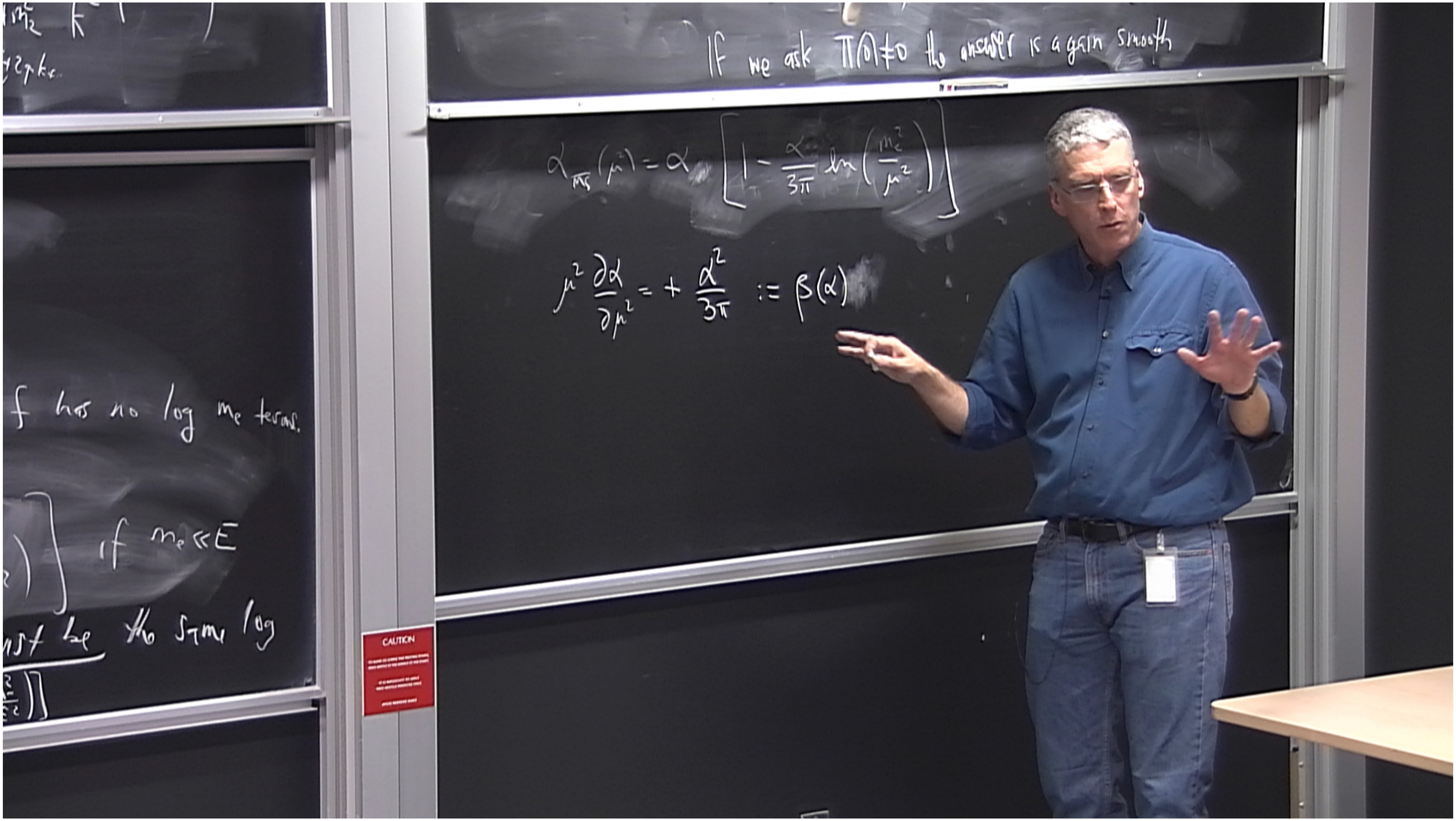

suppose
CAUTION

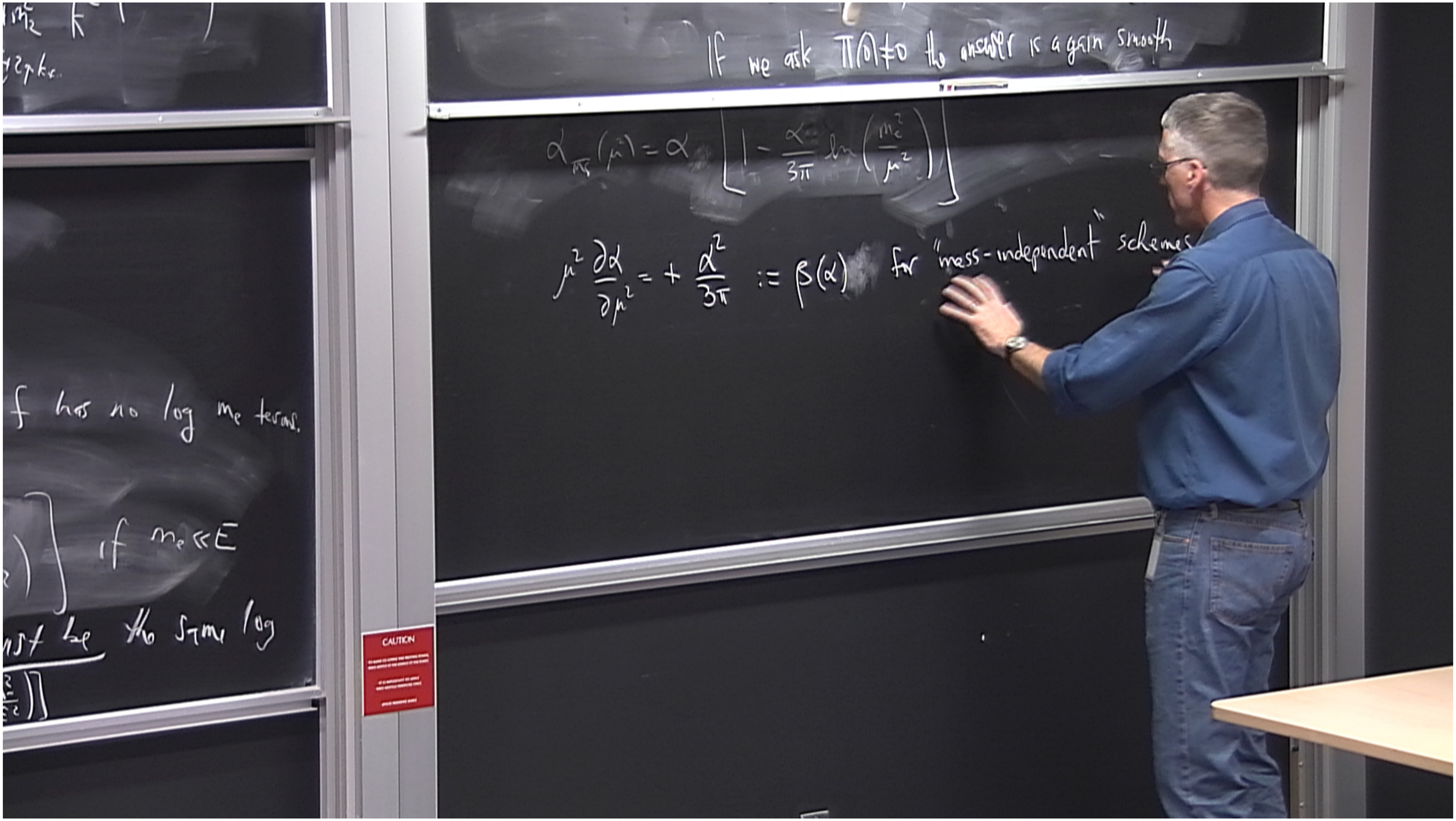


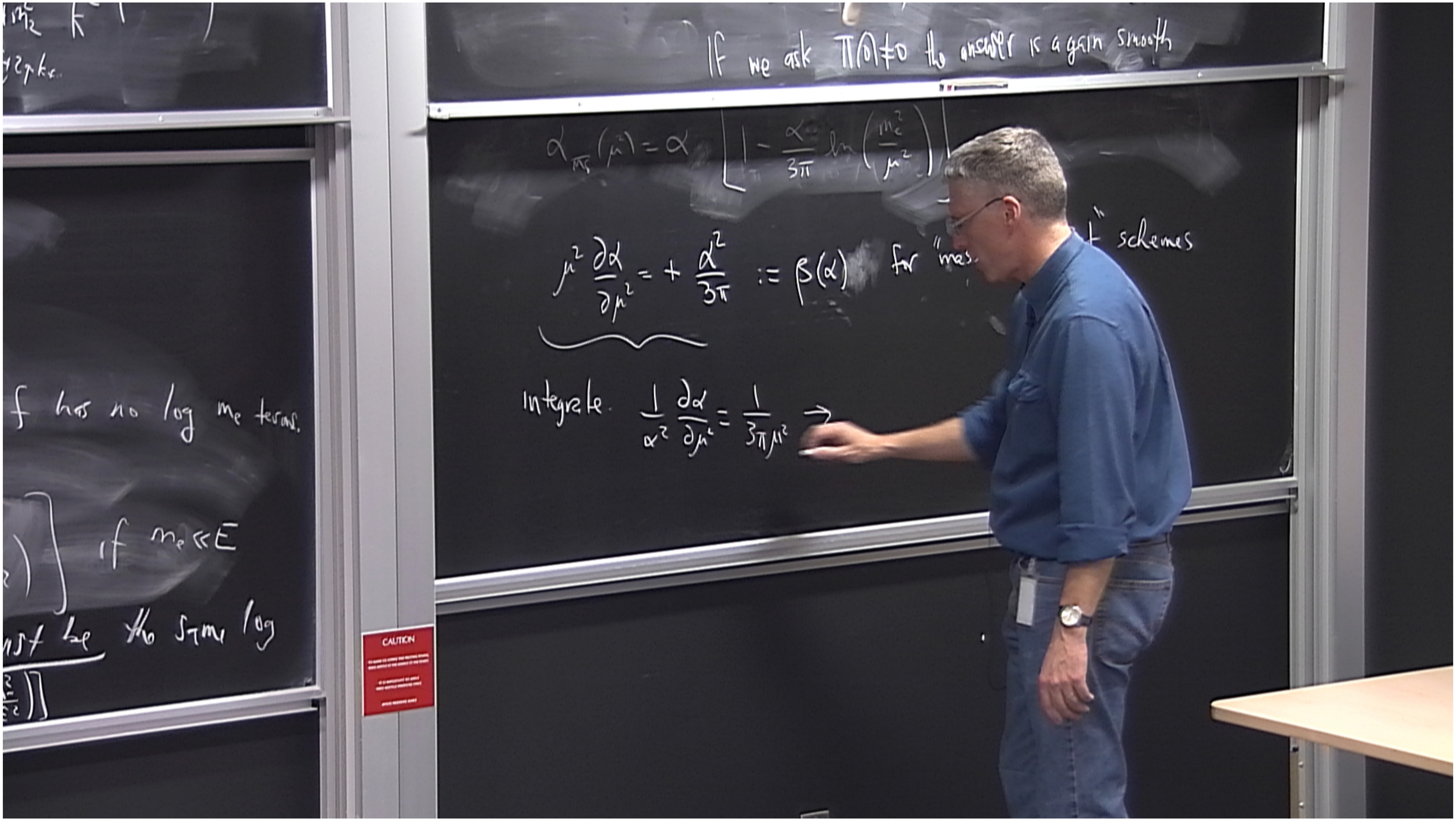
$\mu = E$
$\Gamma = \frac{1}{E^2} f\left(\theta_k, \alpha_{\overline{MS}}(E), \frac{m_e}{E}\right)$ where f has no log m_e terms.
$\simeq \frac{1}{E^2} f\left(\theta_k, \alpha_{\overline{MS}}(E), 0\right)\left[1 + O\left(\frac{m_e^2}{E^2}\right)\right]$ if $m_e \ll E$
CAUTION

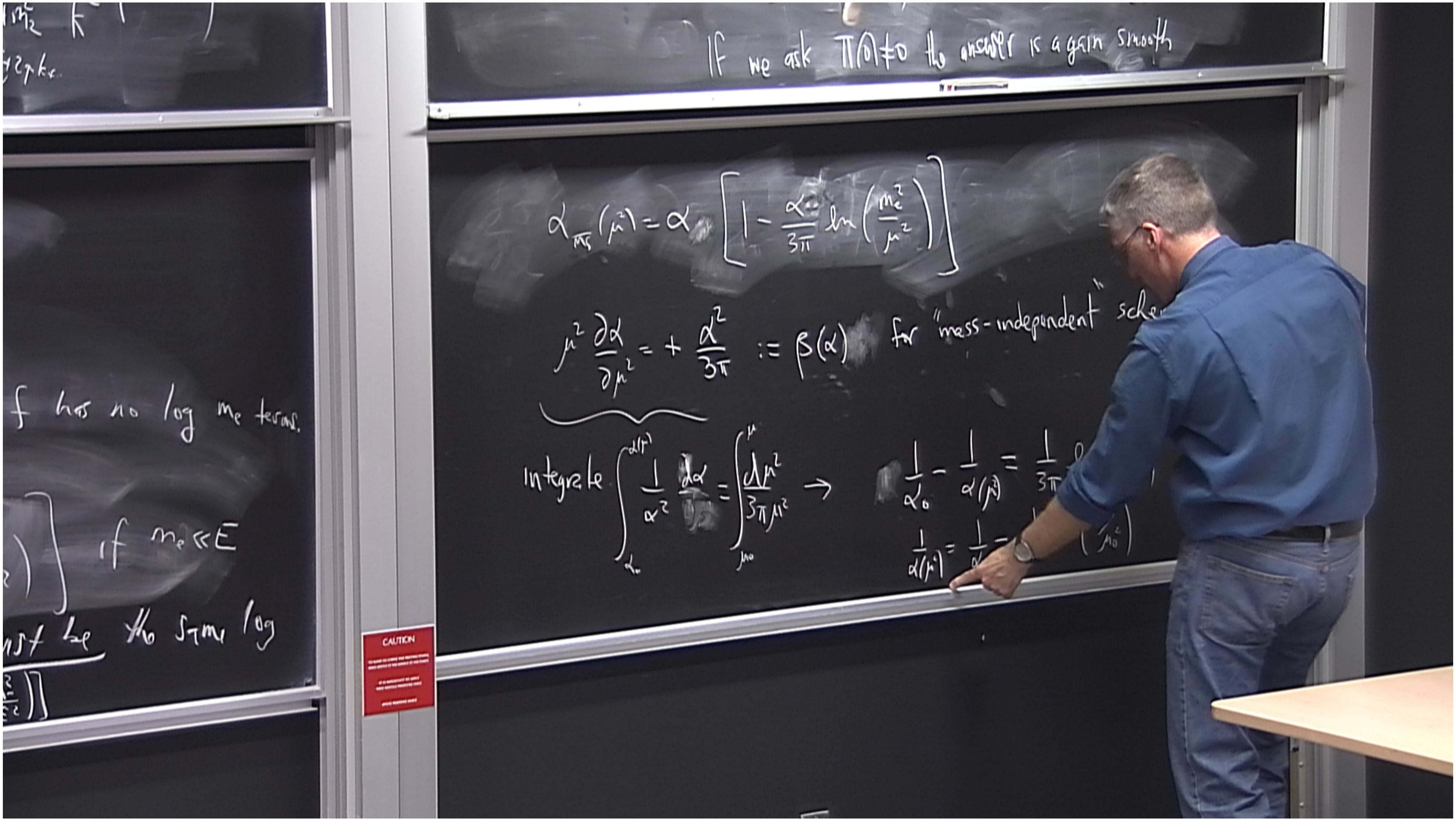

Choose $\mu = E$
$\sigma = \frac{1}{E^2} f\left(\theta_k, \alpha_{\overline{MS}}(E), \frac{m_e}{E}\right)$ where f has no log m_e terms.
$\simeq \frac{1}{E^2} f\left(\theta_k, \alpha_{\overline{MS}}(E), 0\right)\left[1 + O\left(\frac{m_e^2}{E^2}\right)\right]$ if $m_e \ll E$
the log m_e^2 present in f when using α_{ph} must be the same log
appearing in $\alpha_{\overline{MS}}(E) = \alpha_{ph}\left[1 - \frac{\alpha}{3\pi} \ln\left(\frac{m_e^2}{E^2}\right)\right]$
CAUTION

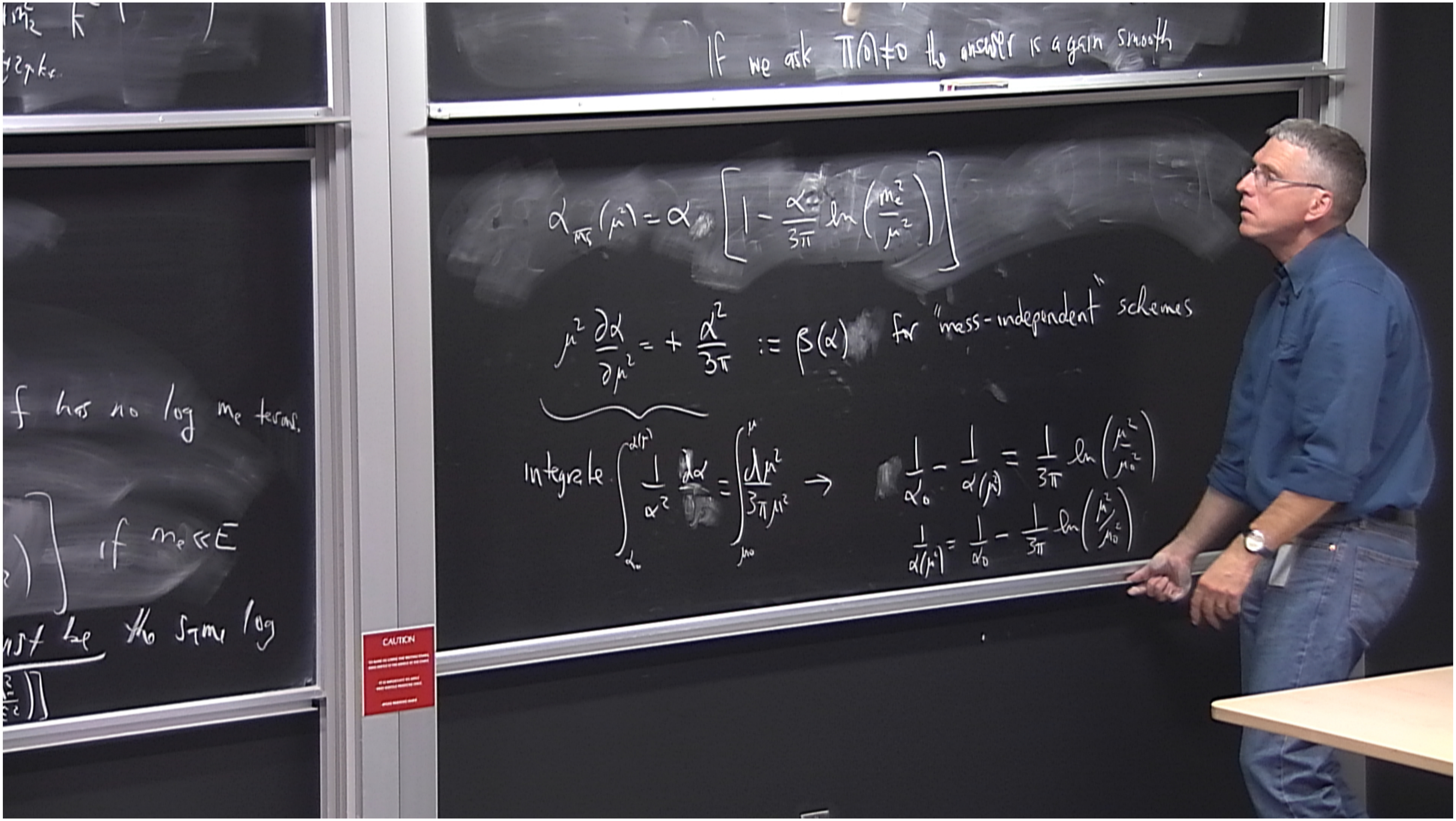

"Why the Renormalization Group is a Good Thing"
S. Weinberg
$\alpha_{\overline{MS}}(\mu) = \alpha_{ph}\left[1 - \frac{\alpha_{ph}}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right) + O(\alpha^2)\right]$
notice $\alpha_{\overline{MS}}(\mu = m_e) = \alpha_{ph} = \left(\frac{1}{137.\ldots}\right)$
$A_\mu = \frac{A_\mu^{ph}}{\sqrt{Z}}$
$\frac{\alpha}{4\pi}$
$\sigma = \sigma(E, \theta_k, \alpha_{ph}, m_e)$
suppose your interest is in $E \gg m_e$.
$= \frac{1}{E^2} f(\theta_k, \alpha_{ph}, m_e/E) \overset{?}{=} \frac{f(\theta_k, \alpha_{ph}, 0)}{E^2}\left(1 + O\left(\frac{m_e^2}{E^2}\right) + \ldots\right)$


If we ask $\Pi(0) \neq 0$ the answer is again smooth
$\alpha_{\overline{MS}}(\mu^2) = \alpha\left[1 - \frac{\alpha}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right)\right]$
$\mu^2 \frac{\partial\alpha}{\partial\mu^2} = + \frac{\alpha^2}{3\pi} := \beta(\alpha)$
f has no log m_e terms.
if $m_e \ll E$
be the same log
CAUTION


If we ask Π(0) ≠ 0 the answer is again smooth
α_MS(μ²) = α [1 − α/3π ln(m_e²/μ²)]
μ² ∂α/∂μ² = + α²/3π := β(α) for "mass-independent" schemes
f has no log m_e terms.
if m_e ≪ E
must be the same log
CAUTION


If we ask Π(0)≠0 the answer is again smooth
$\alpha_{\overline{MS}}(\mu^2) = \alpha \left[1 - \frac{\alpha}{3\pi} \ln\left(\frac{m_e^2}{\mu^2}\right) \right]$
$\mu^2 \frac{\partial \alpha}{\partial \mu^2} = + \frac{\alpha^2}{3\pi} := \beta(\alpha)$ for "mass…" schemes
integrate. $\frac{1}{\alpha^2}\frac{\partial \alpha}{\partial \mu^2} = \frac{1}{3\pi\mu^2} \rightarrow$
f has no log m_e terms.
if $m_e \ll E$
must be the same log
CAUTION

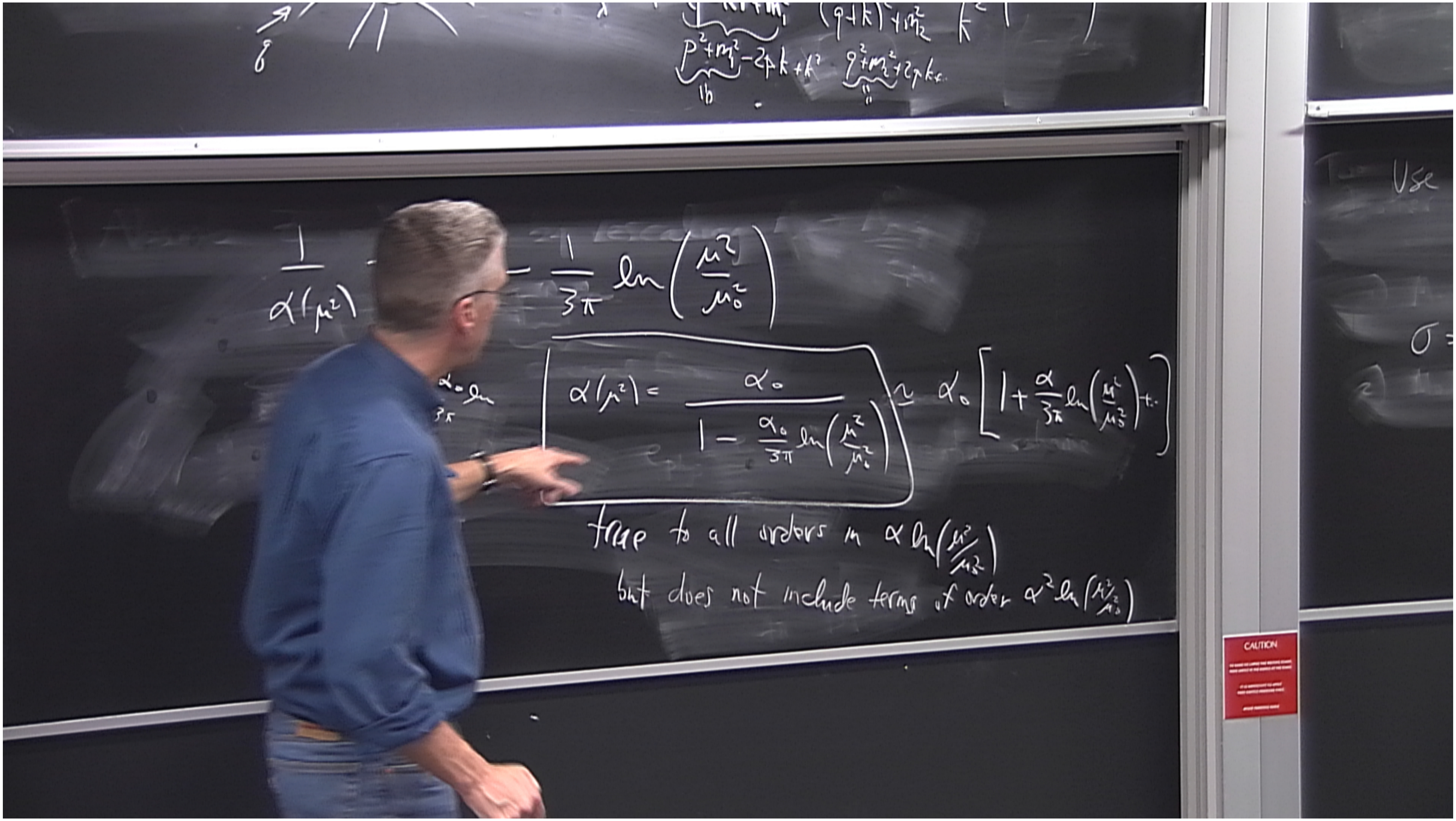

If we ask Π(0) ≠ 0 the answer is again smooth
f has no log me terms.
if me ≪ E
must be the same log
integrate
for "mass-independent" sche


If we ask $\Pi(0) \neq 0$ the answer is again smooth
f has no log m_e terms.
if $m_e \ll E$
must be the same log
$\alpha_{\overline{MS}}(\mu^2) = \alpha \left[1 - \frac{\alpha}{3\pi} \ln\left(\frac{m_e^2}{\mu^2}\right) \right]$
$\mu^2 \frac{\partial \alpha}{\partial \mu^2} = + \frac{\alpha^2}{3\pi} := \beta(\alpha)$ for "mass-independent" schemes
integrate $\int_{\alpha_0}^{\alpha(\mu^2)} \frac{1}{\alpha^2} d\alpha = \int_{\mu_0}^{\mu} \frac{d\mu^2}{3\pi \mu^2} \rightarrow$
$\frac{1}{\alpha_0} - \frac{1}{\alpha(\mu^2)} = \frac{1}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)$
$\frac{1}{\alpha(\mu^2)} = \frac{1}{\alpha_0} - \frac{1}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)$
CAUTION

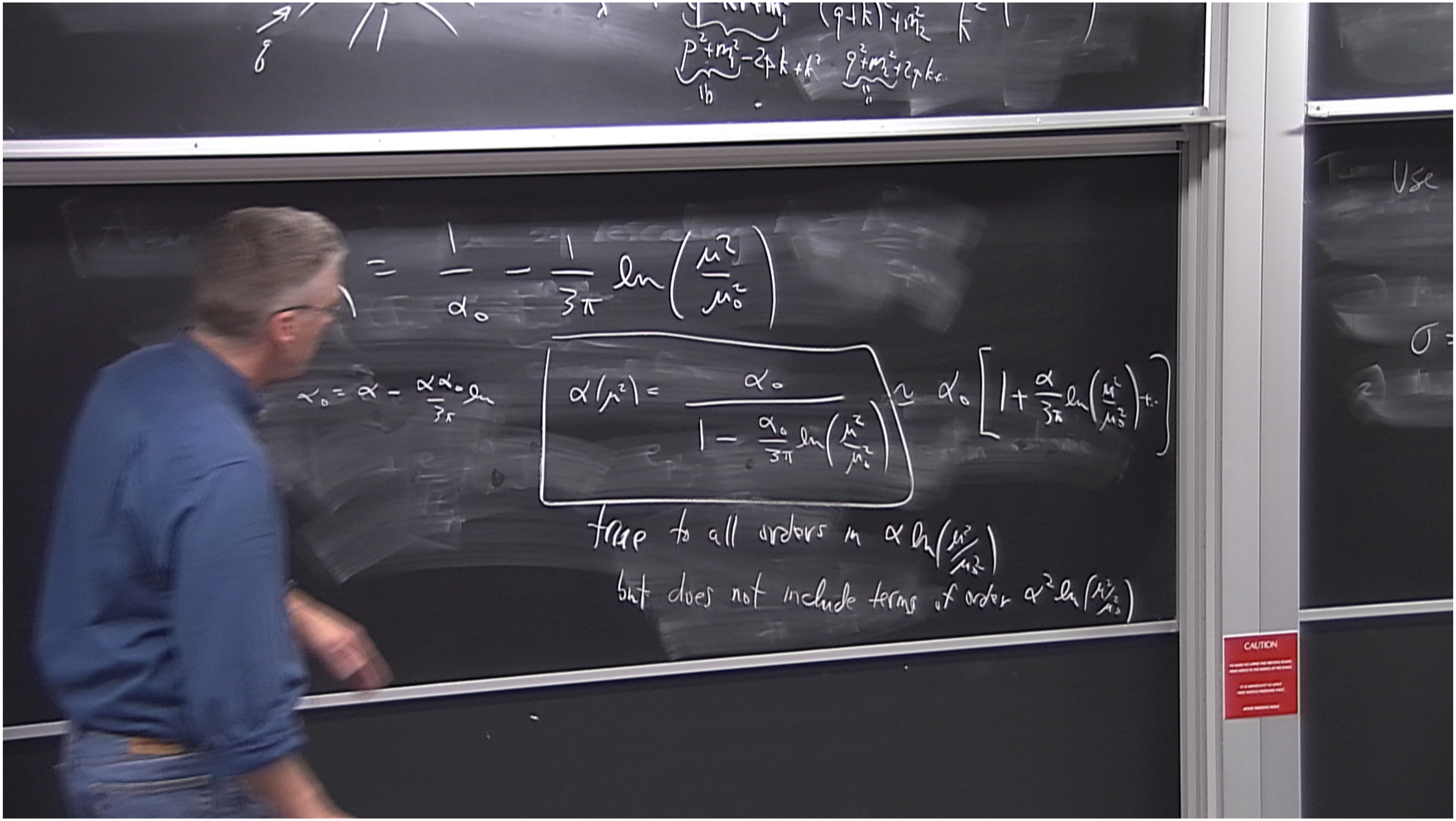
$$\underbrace{p^2+m_1^2}-2pk+k^2 \qquad \underbrace{q^2+m_2^2}+2qk$$

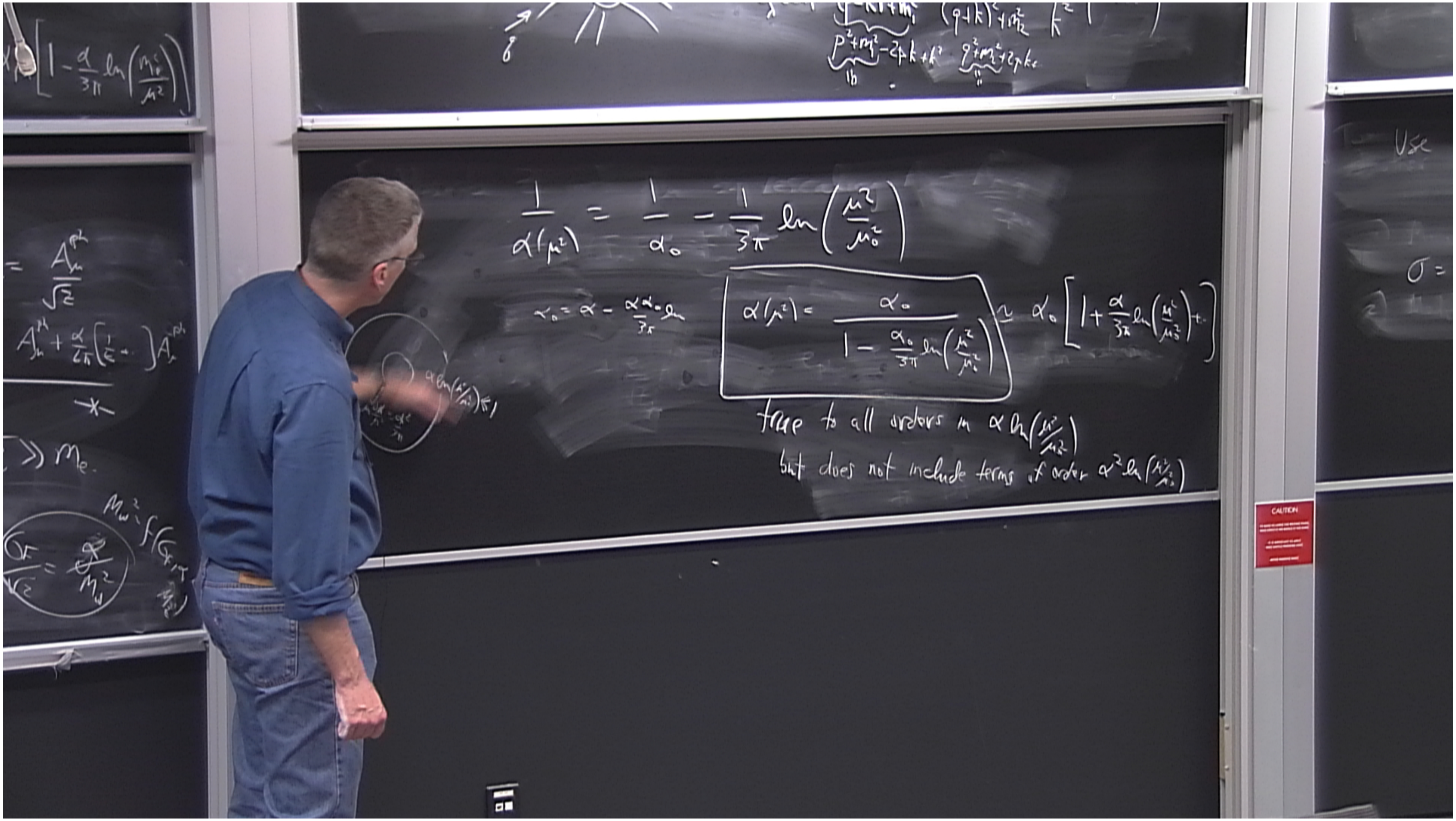
$$\frac{1}{\alpha(\mu^2)} = \frac{1}{\alpha_0} - \frac{1}{3\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

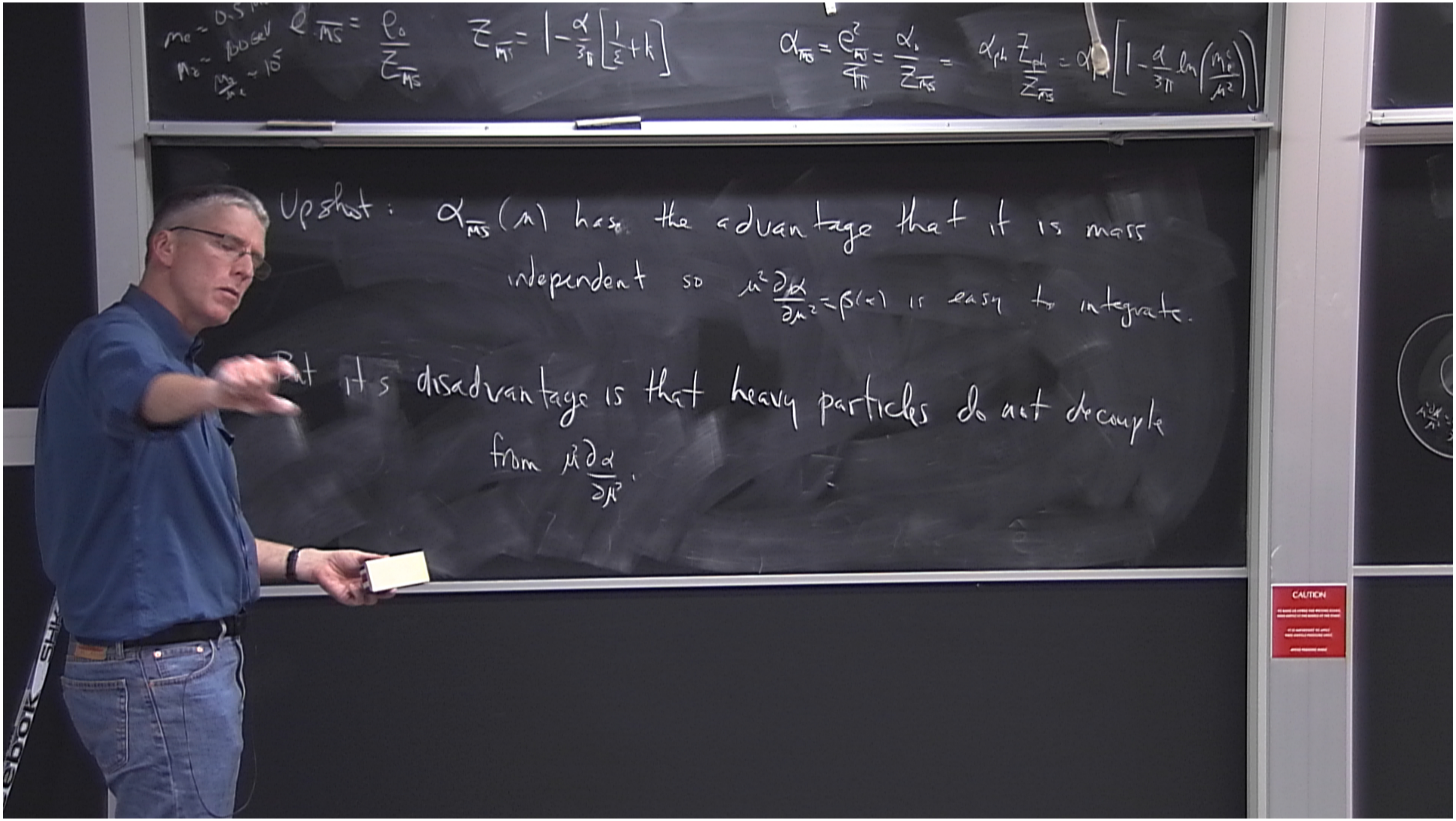
$$\alpha_0 = \alpha - \frac{\alpha\alpha_0}{3\pi}\ln \qquad \alpha(\mu^2) = \frac{\alpha_0}{1-\frac{\alpha_0}{3\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right)} \simeq \alpha_0\left[1+\frac{\alpha}{3\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right)+\dots\right]$$

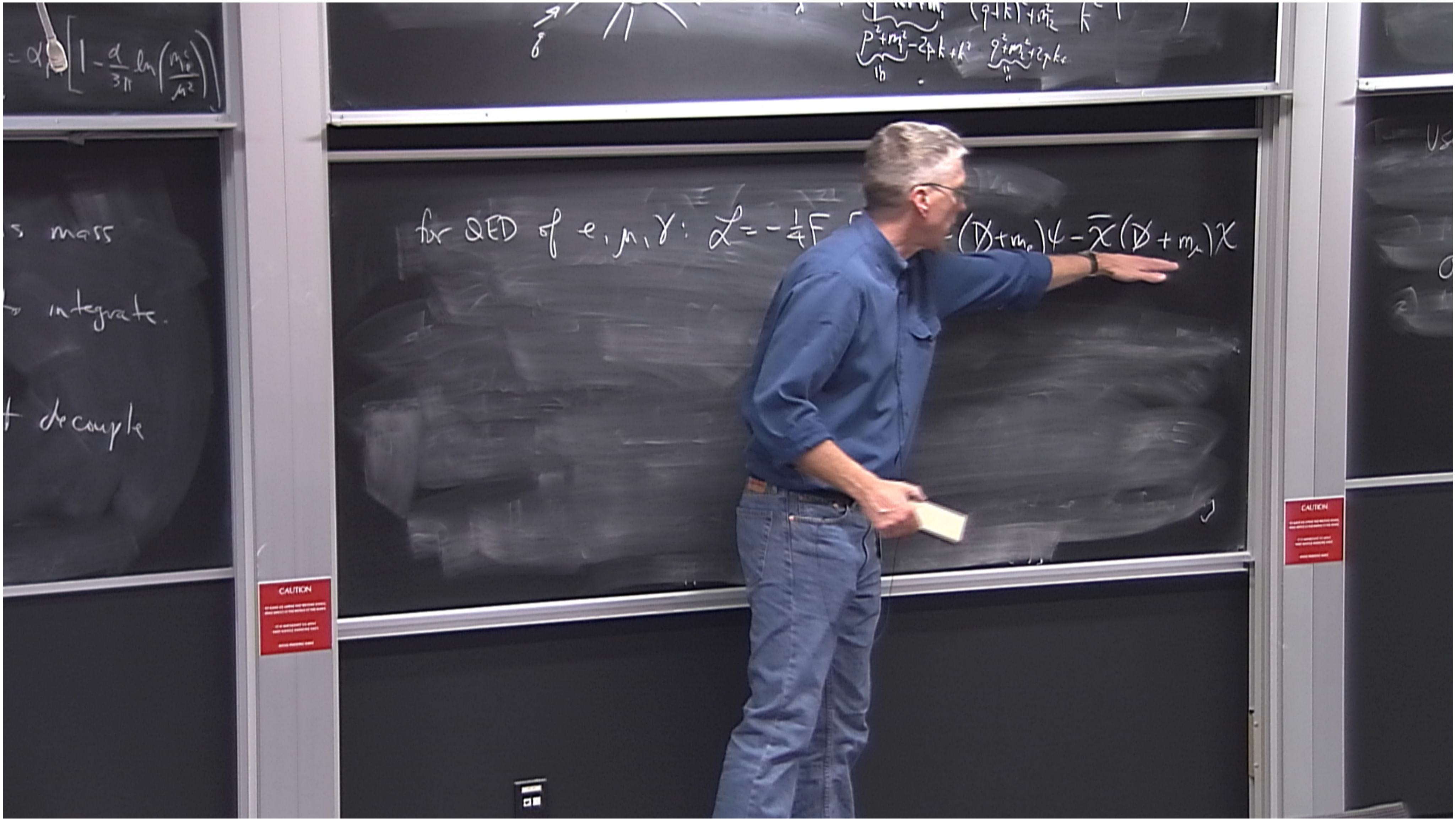


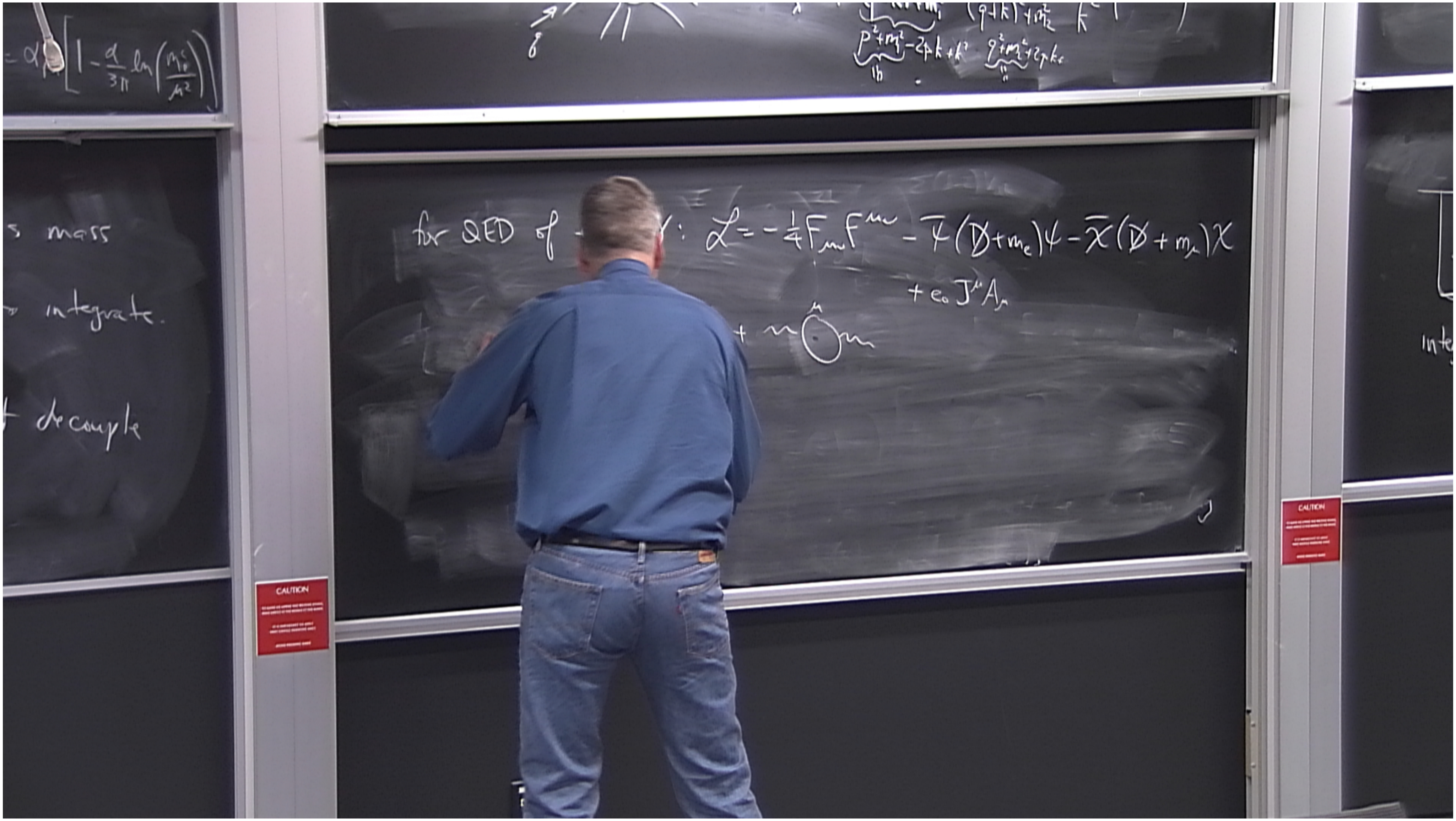
$\frac{1}{\alpha(\mu^2)}$ … $\frac{1}{3\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right)$
$\alpha(\mu^2) = \frac{\alpha_0}{1-\frac{\alpha_0}{3\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right)} \simeq \alpha_0\left[1+\frac{\alpha}{3\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right)+\dots\right]$
true to all orders in $\alpha\ln(\mu^2/\mu_0^2)$
but does not include terms of order $\alpha^2\ln(\mu^2/\mu_0^2)$
CAUTION

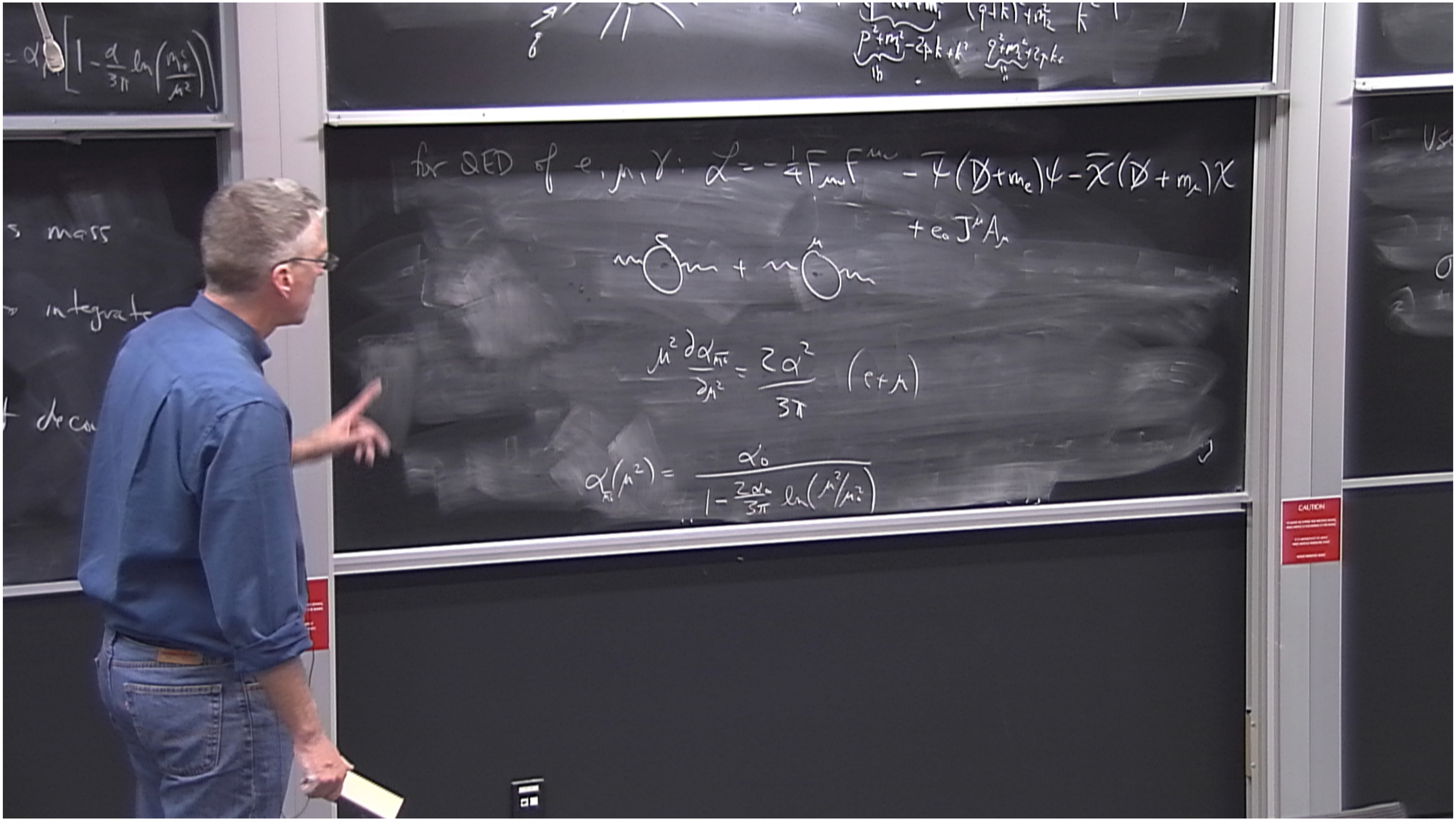

$= \frac{1}{\alpha_0} - \frac{1}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)$
$\alpha_0 = \alpha - \frac{\alpha\alpha_0}{3\pi} \ln$
$\alpha(\mu^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right)} \simeq \alpha_0\left[1 + \frac{\alpha}{3\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right) + \dots\right]$
true to all orders in $\alpha \ln(\mu^2/\mu_0^2)$
but does not include terms of order $\alpha^2 \ln(\mu^2/\mu_0^2)$
CAUTION

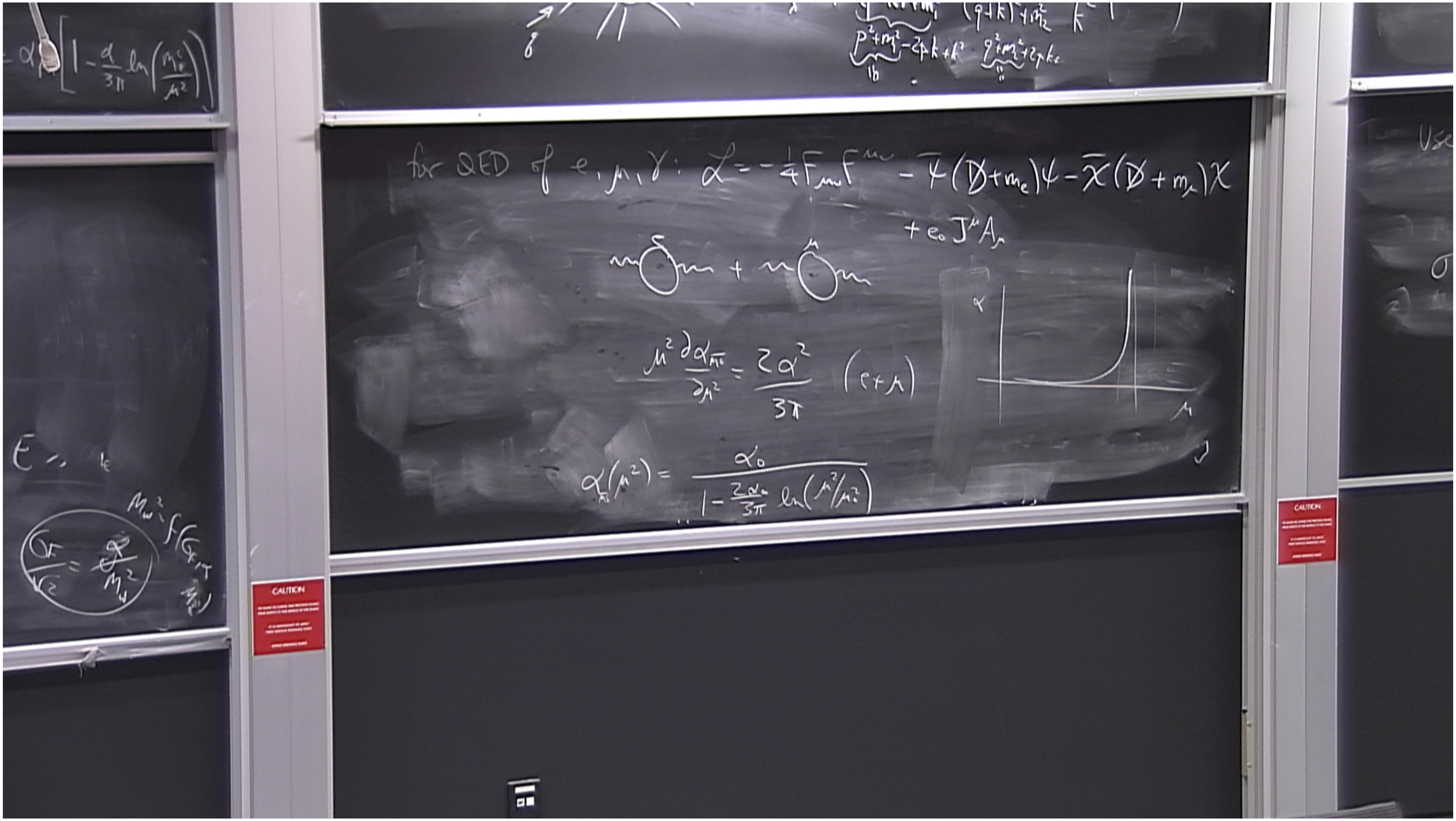

1/α(μ²) = 1/α₀ − 1/3π ln(μ²/μ₀²)
α₀ = α − αα₀/3π ln
α(μ²) = α₀ / (1 − α₀/3π ln(μ²/μ₀²)) ≃ α₀ [1 + α/3π ln(μ²/μ₀²) + …]
true to all orders in α ln(μ²/μ₀²)
but does not include terms of order α² ln(μ²/μ₀²)
α ln(μ²/μ₀²) ≪ 1
Use
CAUTION

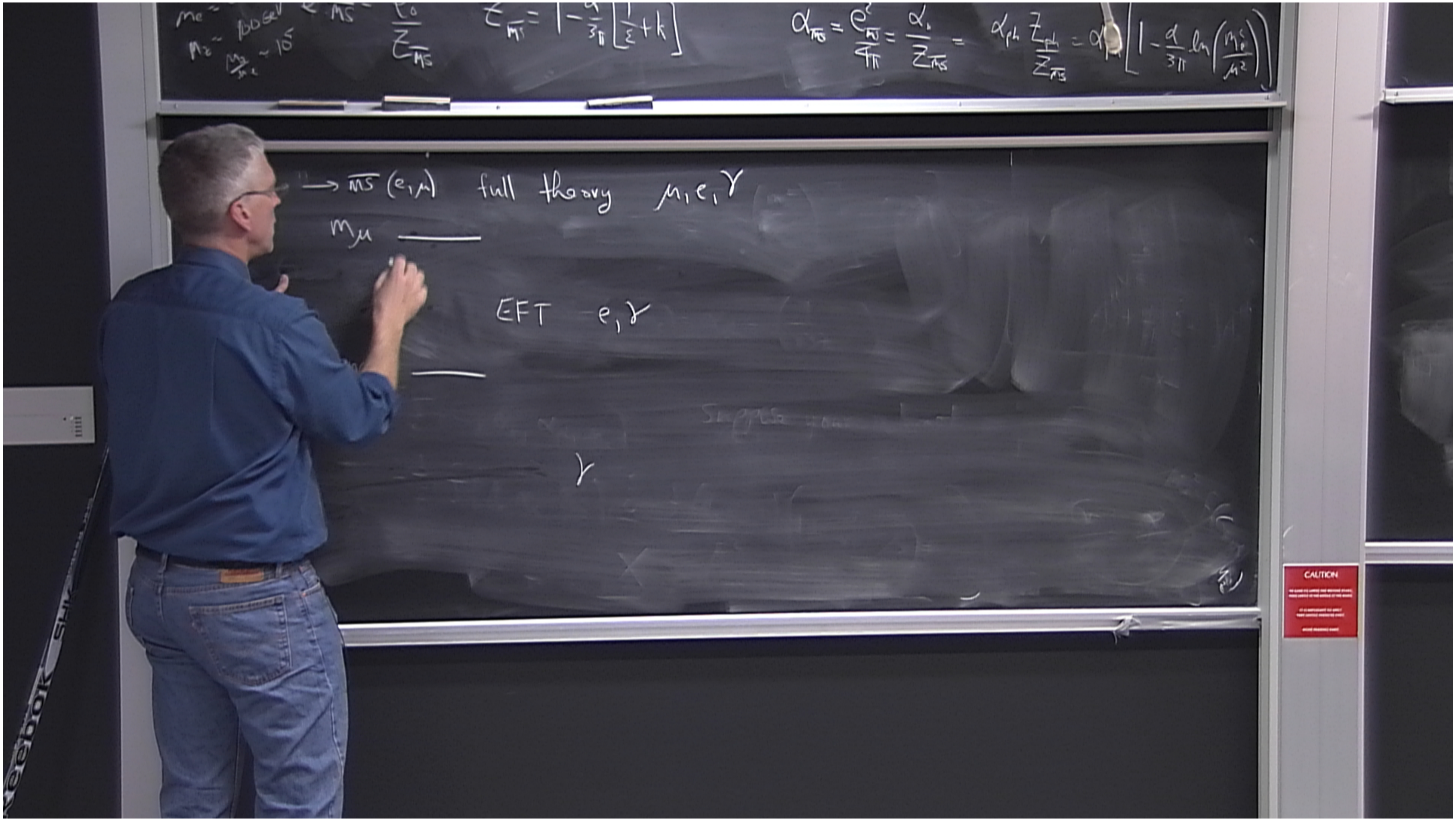

$m_e \sim 0.5$
$m_Z \sim 100$ GeV
$\frac{m_Z}{m_e} \sim 10^5$
$e_{\overline{MS}} = \frac{e_0}{Z_{\overline{MS}}}$
$Z_{\overline{MS}} = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\epsilon} + k\right]$
$\alpha_{\overline{MS}} = \frac{e^2_{\overline{MS}}}{4\pi} = \frac{\alpha_0}{Z_{\overline{MS}}} = \alpha_{ph}\frac{Z_{ph}}{Z_{\overline{MS}}} = \alpha_{ph}\left[1 - \frac{\alpha}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right)\right]$
Upshot: $\alpha_{\overline{MS}}(\mu)$ has the advantage that it is mass independent so $\mu^2\frac{\partial\alpha}{\partial\mu^2} = \beta(\alpha)$ is easy to integrate.
But its disadvantage is that heavy particles do not decouple from $\mu^2\frac{\partial\alpha}{\partial\mu^2}$.
CAUTION


mass
integrate.
decouple
for QED of e, μ, γ: ℒ = − ¼ F
(D̸ + m_e)ψ − χ̄ (D̸ + m_χ) χ
CAUTION

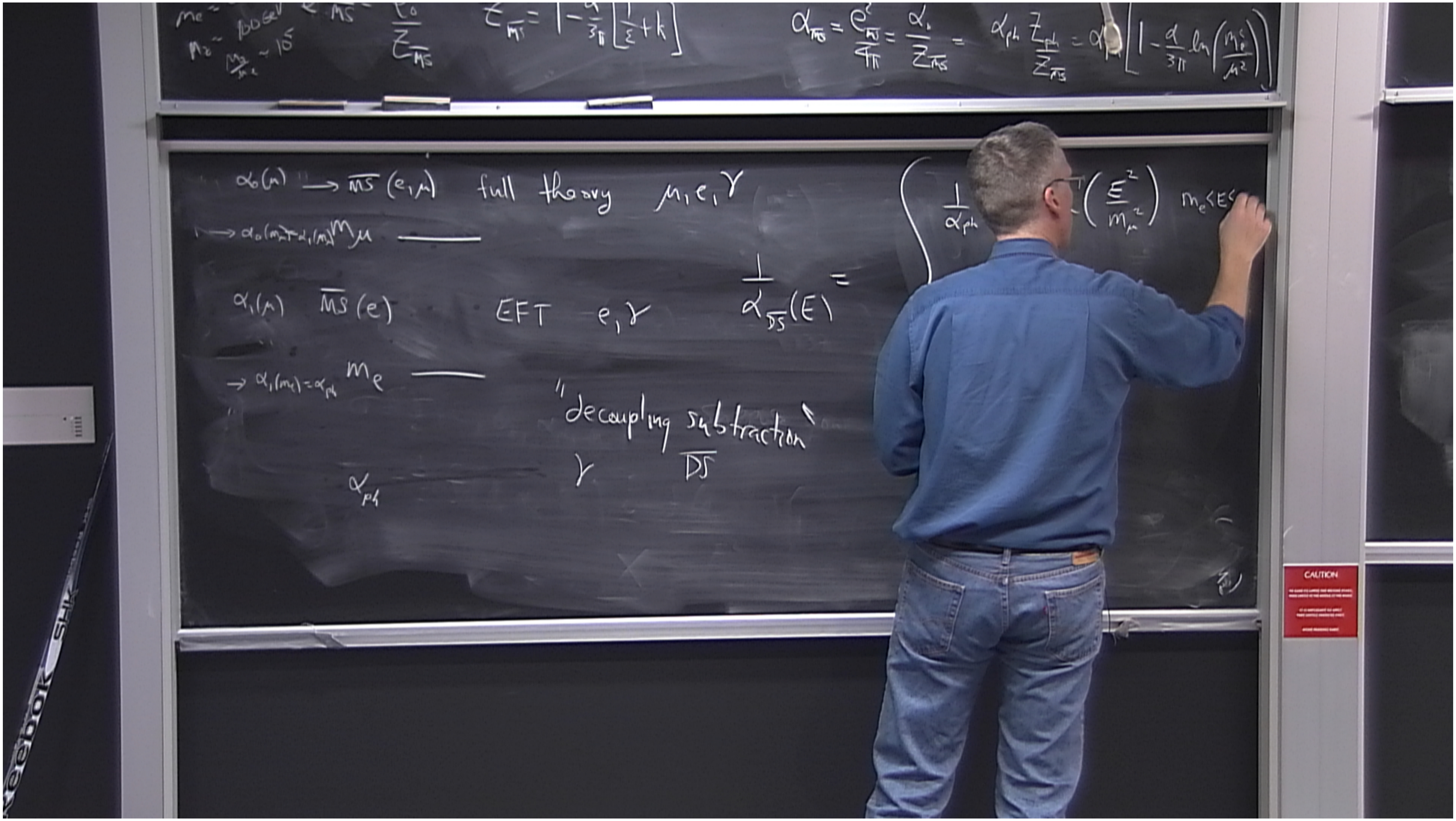

mass
integrate.
decouple
for QED of
$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{D}+m_e)\psi - \bar{\chi}(\not{D}+m_\mu)\chi$
$+ e_0 J^\mu A_\mu$

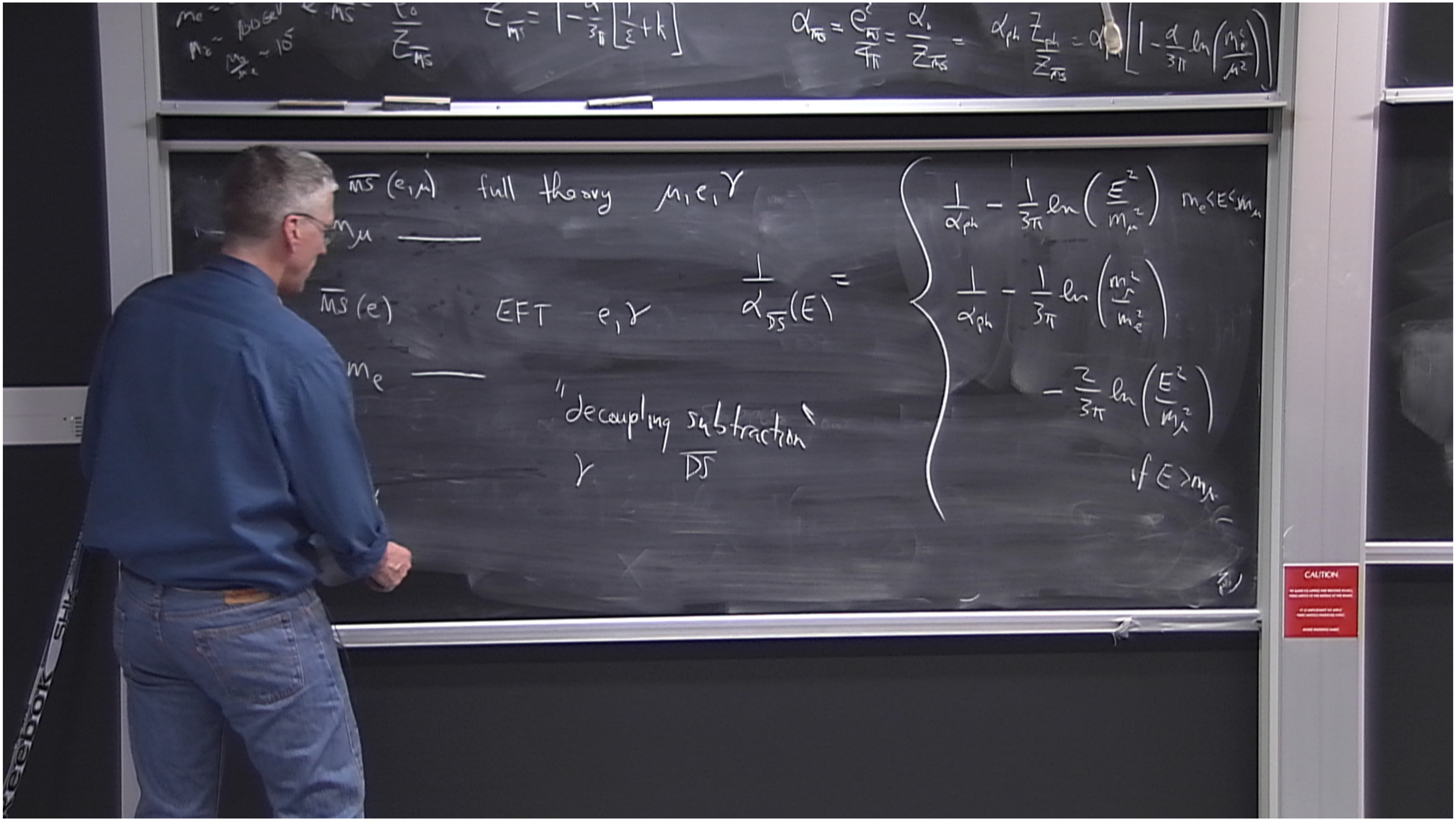

for QED of e, μ, γ: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{D}+m_e)\psi - \bar{\chi}(\not{D}+m_\mu)\chi$
$+ e_0 J^\mu A_\mu$
$\mu^2 \frac{\partial \alpha_{\mu e}}{\partial \mu^2} = \frac{2\alpha^2}{3\pi}$ (e+μ)
$\alpha_{\mu e}(\mu^2) = \frac{\alpha_0}{1 - \frac{2\alpha_0}{3\pi}\ln(\mu^2/\mu_0^2)}$
mass
integrate
CAUTION


for QED of e, μ, γ: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{D}+m_e)\psi - \bar{\chi}(\not{D}+m_\mu)\chi + e_0 J^\mu A_\mu$
$\mu^2 \frac{\partial \alpha_{\overline{MS}}}{\partial \mu^2} = \frac{2\alpha^2}{3\pi}$ (e + μ)
$\alpha_{\overline{MS}}(\mu^2) = \frac{\alpha_0}{1 - \frac{2\alpha_0}{3\pi}\ln(\mu^2/\mu_0^2)}$
$\alpha\left[1 - \frac{\alpha}{3\pi}\ln\left(\frac{m_e^2}{\mu^2}\right)\right]$
$\frac{G_F}{\sqrt{2}} = \frac{g^2}{M_W^2}$
$M_W^2 = f(G_F, \alpha, ...)$



$\alpha_{\overline{MS}} = \frac{e^2_{\overline{MS}}}{4\pi} = \frac{\alpha_0}{Z_{\overline{MS}}}$
$\to \overline{MS}\ (e, \mu)$ full theory μ, e, γ
m_μ
EFT e, γ
γ


full theory
EFT
"decoupling subtraction"
CAUTION


MS (e, μ) full theory μ, e, γ
m_μ
MS (e) EFT e, γ
m_e
"decoupling subtraction"
γ DS
1/α_DS(E) =
1/α_ph − 1/3π ln(E²/m_μ²) m_e < E < m_μ
1/α_ph − 1/3π ln(m_μ²/m_e²) − 2/3π ln(E²/m_μ²) if E > m_μ
α_MS = e²_MS/4π = α_0/Z_MS = α_ph Z_ph/Z_MS = α_ph [1 − α/3π ln(m_e²/μ²)]