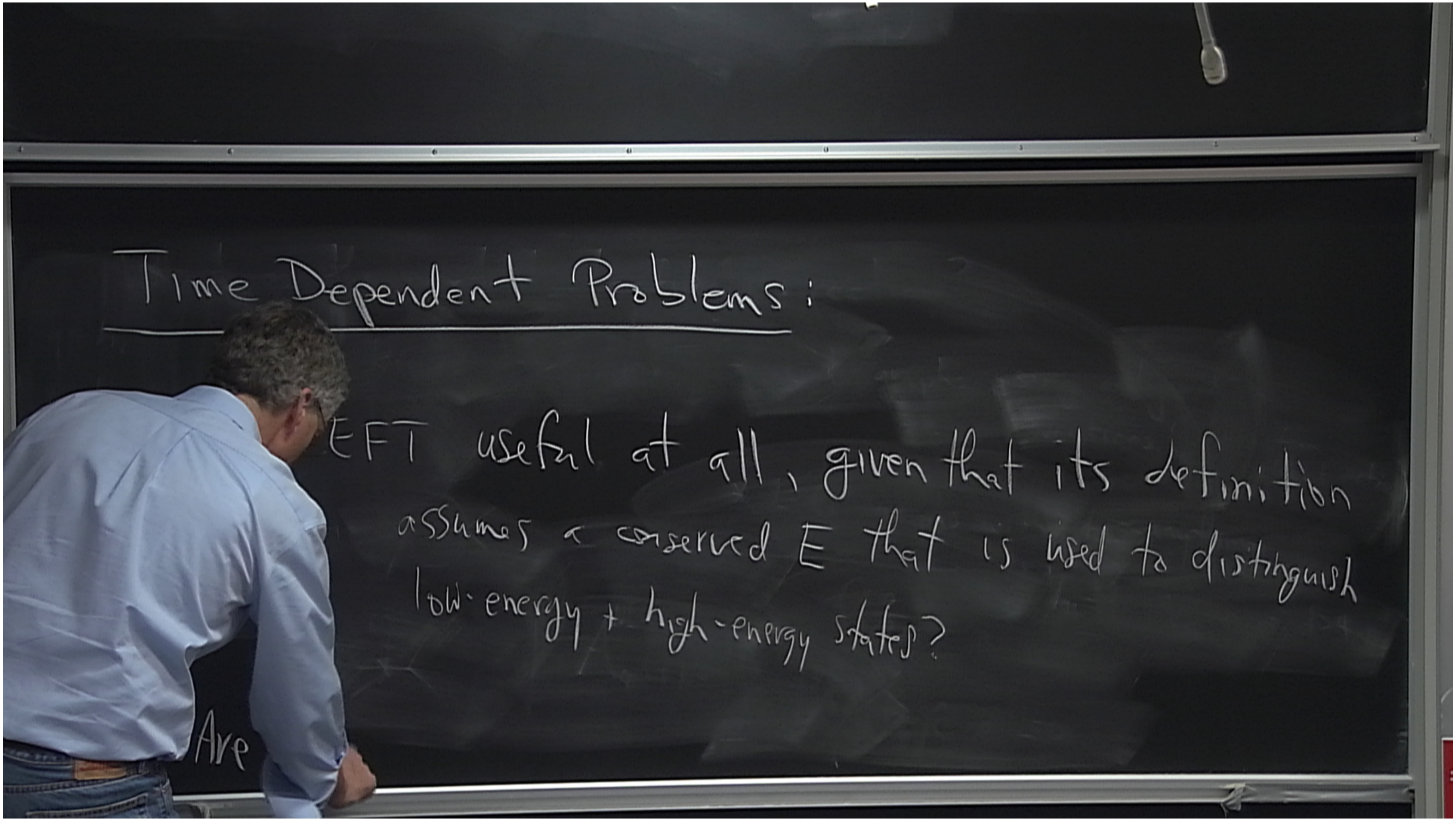
Title: Introduction to Effective Field Theories - Lecture 12

Date: Feb 14, 2014 02:30 PM

URL: http://pirsa.org/14020093

Abstract:


Time Dependent Problems:
EFT useful at all, given that its definition
assumes a conserved E that is used to distinguish
low-energy + high-energy states?
Are

Q2: When do solutions of the EFT capture behaviour of sol'ns of full theory?

...iabatic evolution: $\varphi(t)$ $\omega := \frac{1}{\varphi}\frac{d\varphi}{dt}$ $\omega \ll M$, which defines the heavy particles

...$E(t)\,dt'$.

$H(t)\psi_n = E_n(t)\psi_n$.

Use $E_n(t)$ to differentiate Low from high energy

$g(t) \ll \Lambda \ll m(t)$

Q2: When do solutions of the EFT capture behaviour of sol'ns of full theory?

Adiabatic evolution: $\varphi(t)$ $\quad \omega := \frac{1}{\varphi}\frac{d\varphi}{dt}$ $\quad \omega \ll M$, which defines the heavy particles

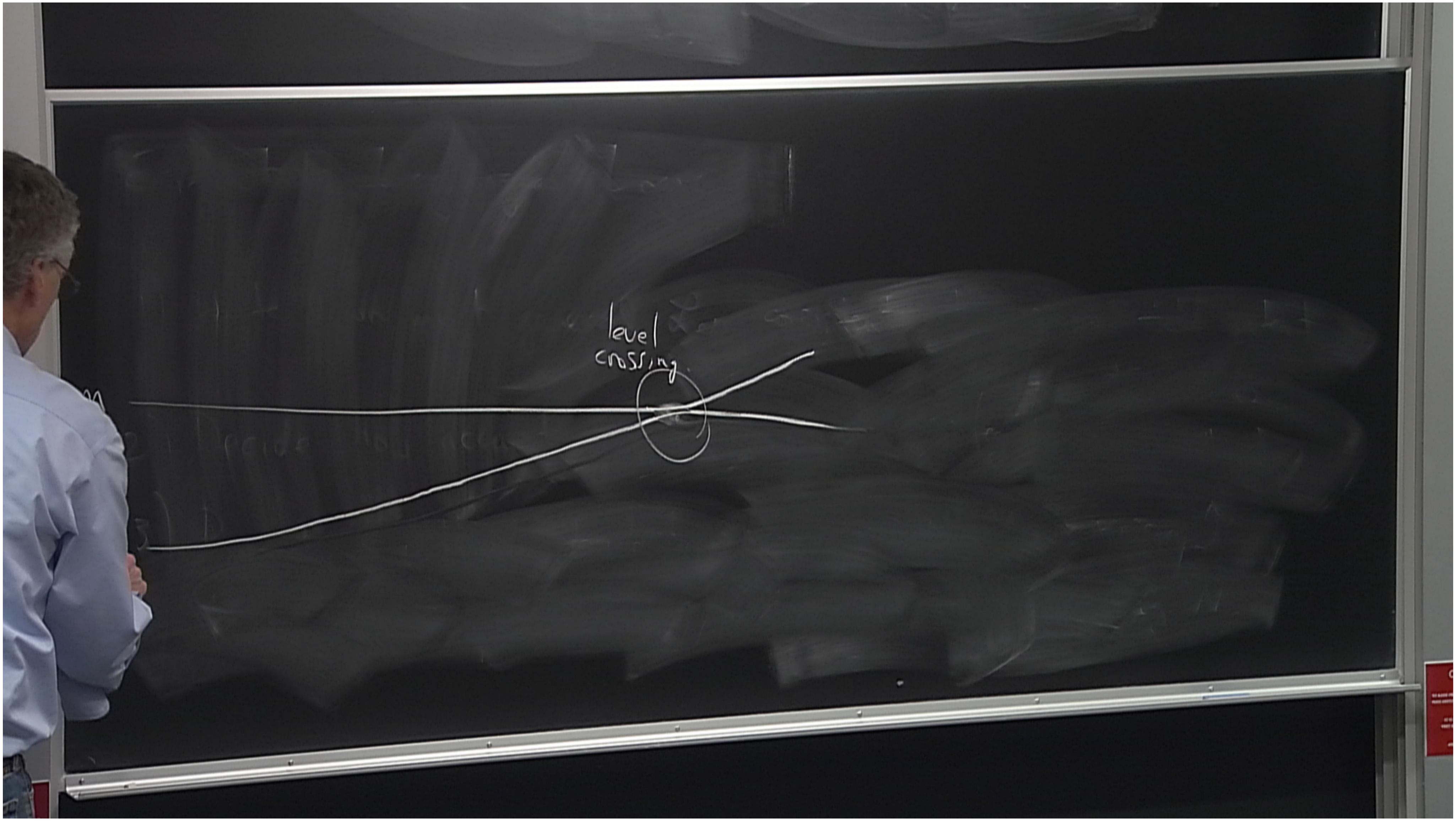
$\psi \sim e^{-i\int E(t)\,dt'}$.

$H(t)\psi_n = E_n(t)\psi_n$.

Use $E_n(t)$ to differentiate Low from high energy

Must check: $q(t) \ll m(t)$ for all t of interest.

$q(t) \ll \Lambda \ll m(t)$

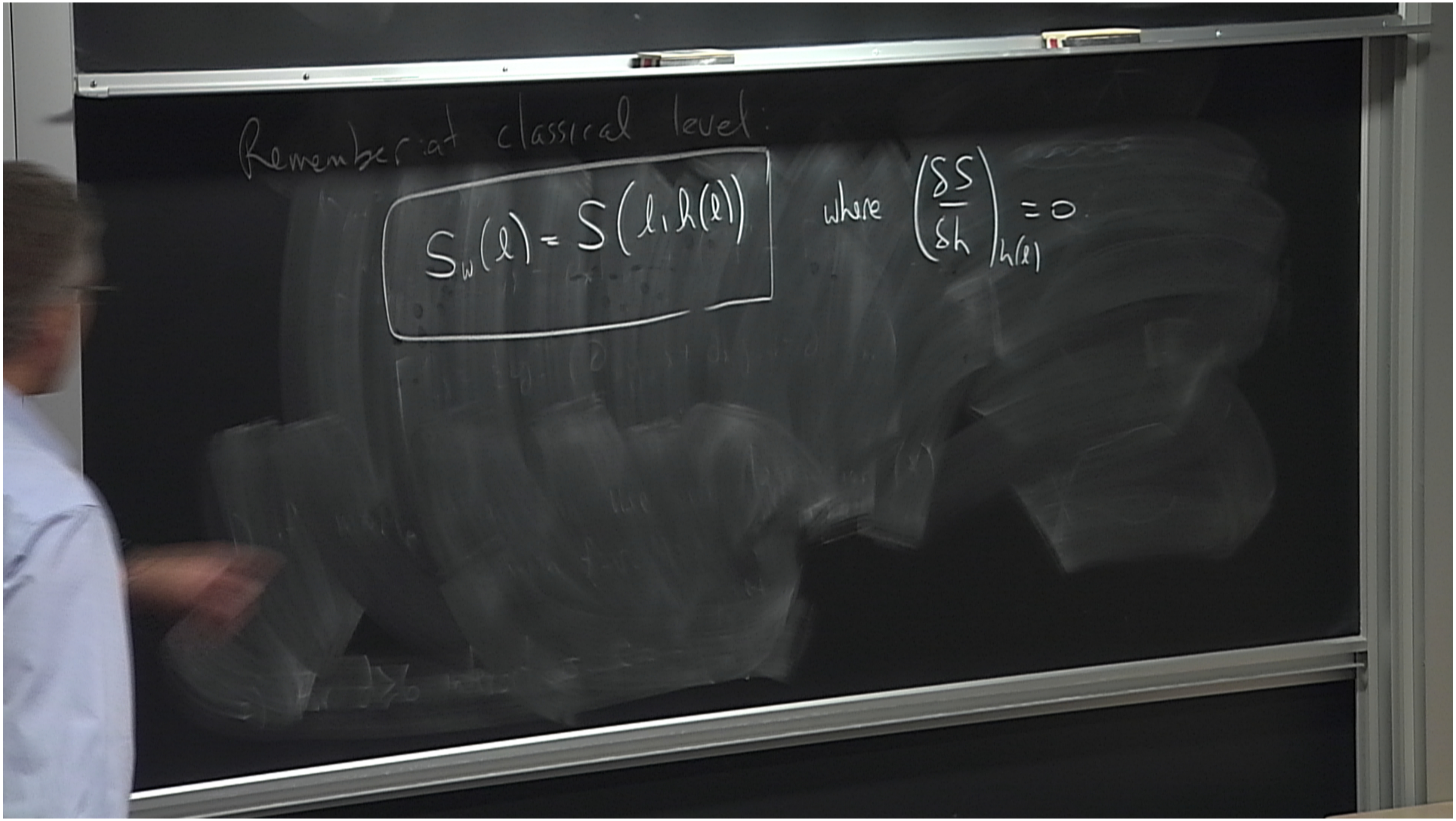

level
crossing

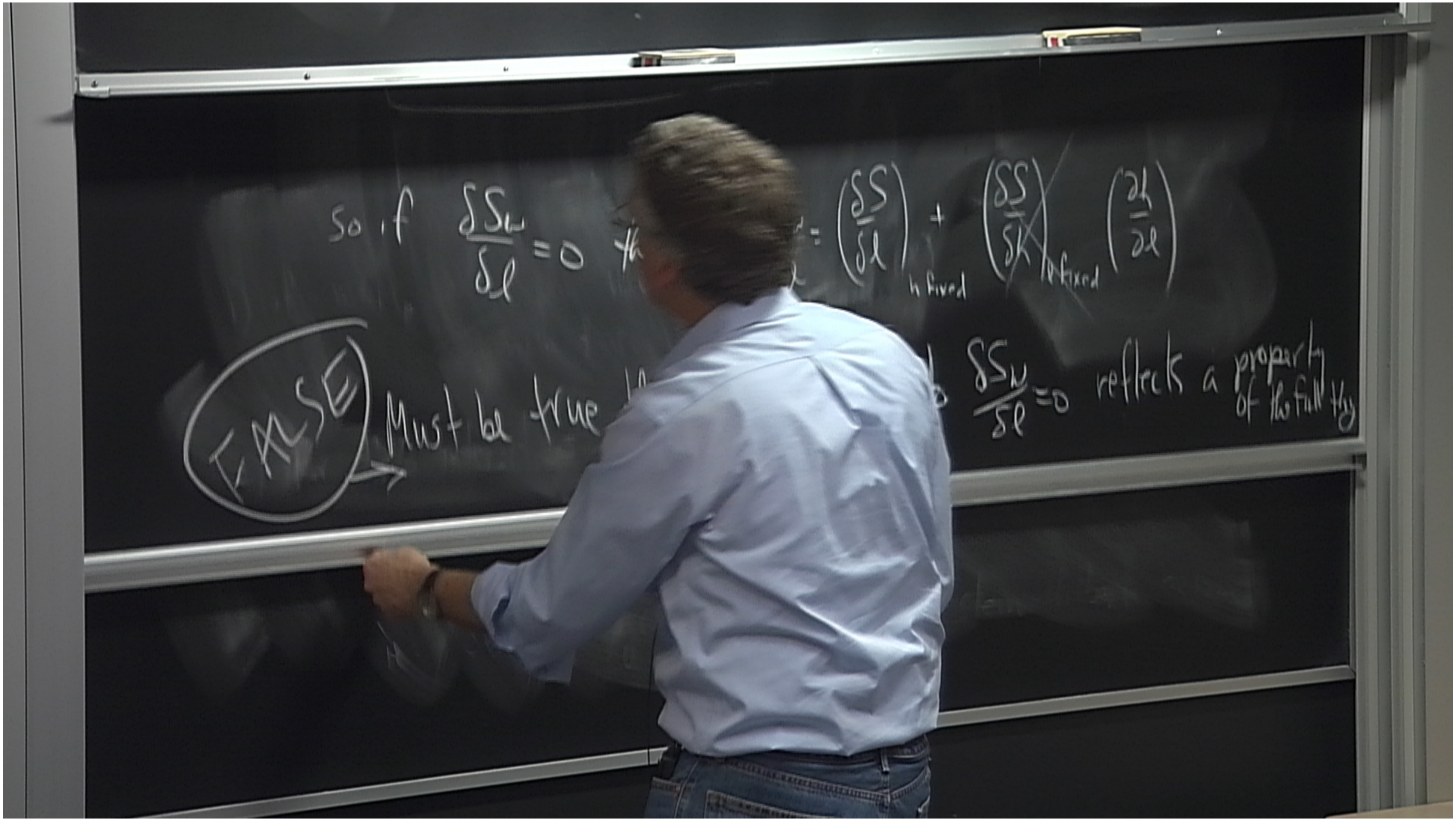
Q2: When do the solutions to the field eqⁿ/s for

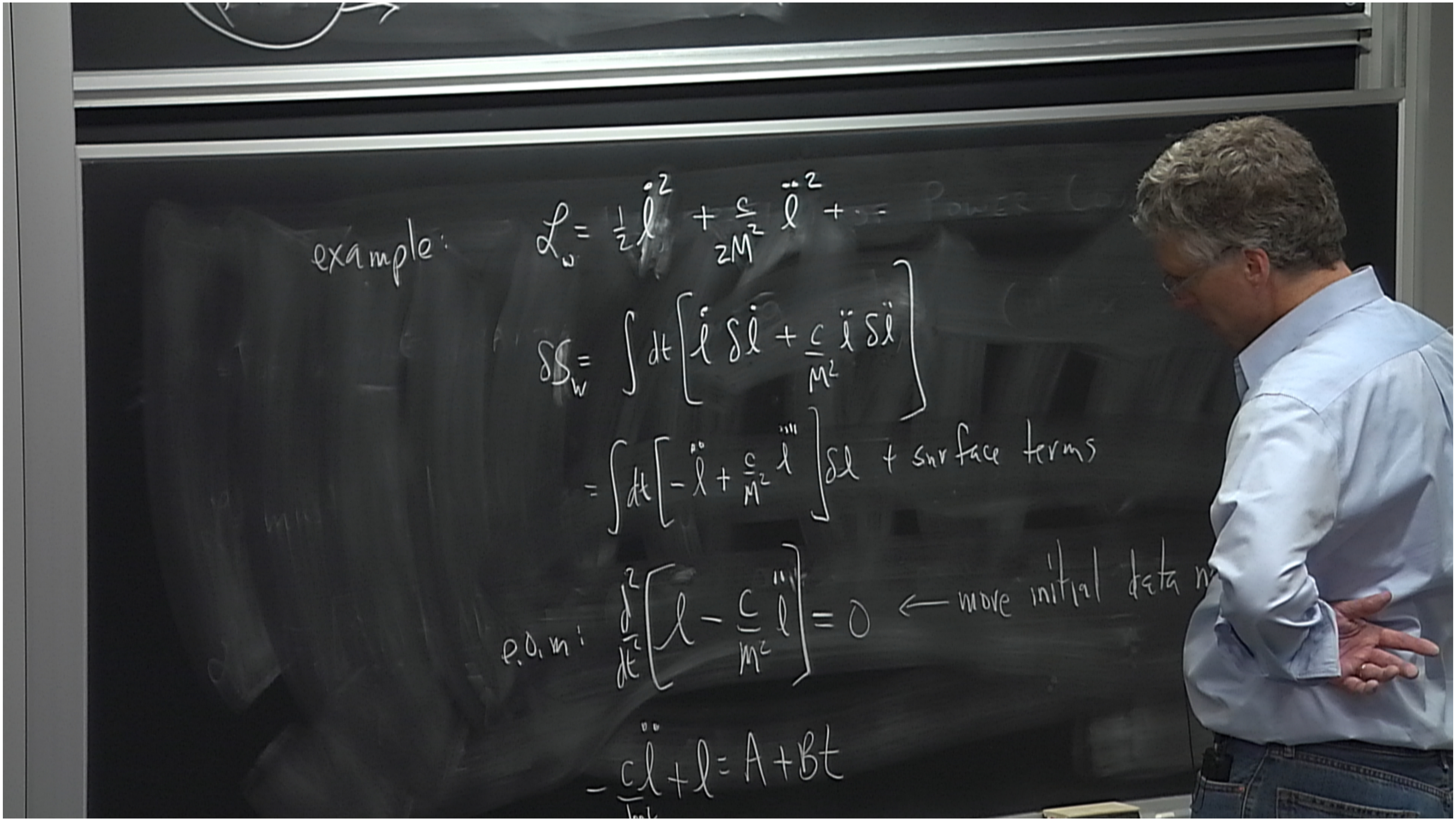
$$\mathcal{L}_{eff} \simeq -\frac{1}{2}(\partial_\mu \xi \partial^\mu \xi) - \frac{a}{m_R^4}(\partial_\mu \xi \partial^\mu \xi)^2 + \cdots$$

capture the solutions to

$$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi) \quad ?$$


Remember at classical level:
$S_w(\ell) = S(\ell, h(\ell))$
where $\left(\frac{\delta S}{\delta h}\right)_{h(\ell)} = 0$

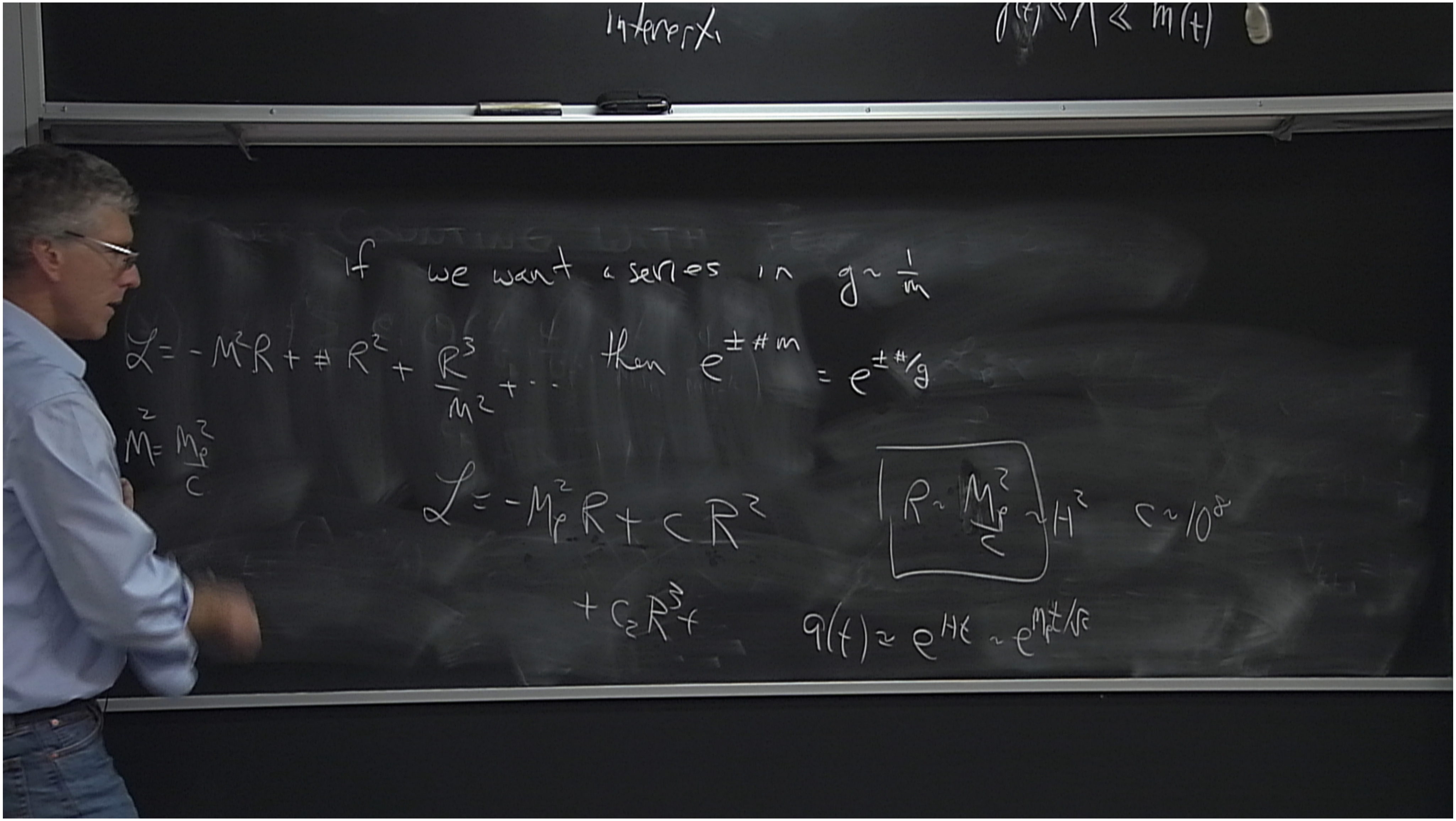

So if
FALSE
Must be true
reflects a property
of the full thy
h fixed
l fixed

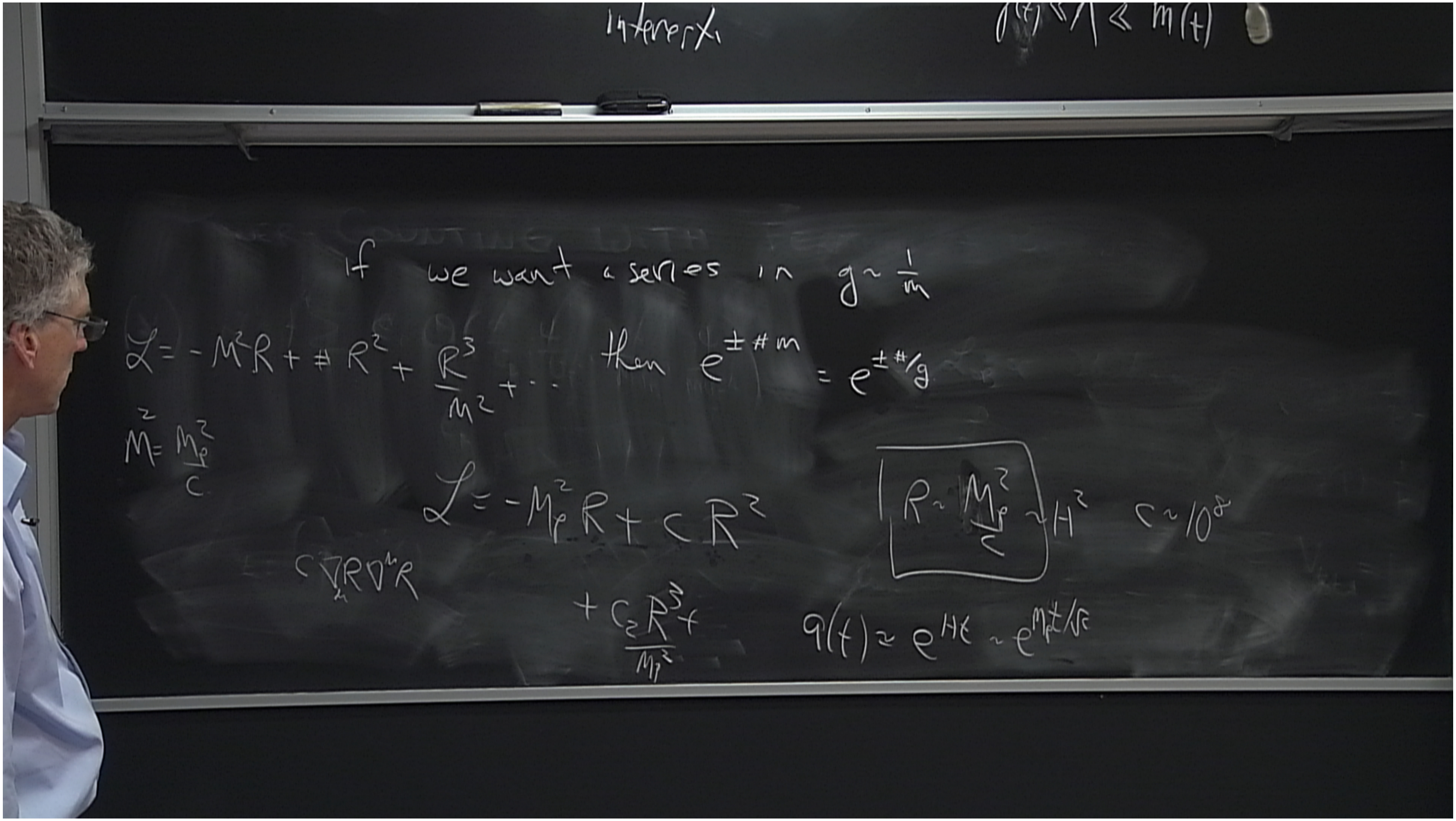

example: $\mathcal{L}_w = \frac{1}{2}\dot{l}^2 + \frac{c}{2M^2}\ddot{l}^2 + \dots$
$\delta S_w = \int dt \left[\dot{l}\,\delta\dot{l} + \frac{c}{M^2}\ddot{l}\,\delta\ddot{l}\right]$
$= \int dt \left[-\ddot{l} + \frac{c}{M^2}\ddddot{l}\right]\delta l$ + surface terms
e.o.m: $\frac{d^2}{dt^2}\left[l - \frac{c}{M^2}\ddot{l}\right] = 0$ ← more initial data
$-\frac{c}{M^2}\ddot{l} + l = A + Bt$

ℓ_h solves $\ddot{\ell}_h - \frac{m^2}{c}\ell_h = 0$

$$\ell_h = C_+ e^{mt/\sqrt{c}} + C_- e^{-mt/\sqrt{c}}$$

$$\ell(t) = \ell_h(t) +$$


interact.
if we want a series in g ~ 1/m
L = -M²R + # R² + R³/M² + ...
then e^{±#m} = e^{±#/g}
M² = Mp²/c
L = -Mp² R + c R²
+ c₂R³ +
R ~ Mp²/c ~ H²
c ~ 10⁸
a(t) ≈ e^{Ht} ~ e^{Mp t/√c}


interact.
if we want a series in $g \sim \frac{1}{m}$
$\mathcal{L} = -M^2 R + \# R^2 + \frac{R^3}{M^2} + \cdots$ then $e^{\pm \# m} = e^{\pm \#/g}$
$M^2 = \frac{M_p^2}{c}$
$\mathcal{L} = -M_p^2 R + c R^2$
$c \nabla_\mu R \nabla^\mu R$
$+ \frac{c_2 R^3}{M_p^2} +$
$\boxed{R \sim \frac{M_p^2}{c}} \sim H^2$
$c \sim 10^8$
$a(t) \simeq e^{Ht} \sim e^{M_p t/\sqrt{c}}$

interaction $\quad \rho(t) \ll \Lambda \ll m(t)$

if we want a series in $g \sim \frac{1}{m}$

$$\mathcal{L} = -M^2 R + \# R^2 + \frac{R^3}{M^2} + \cdots \quad \text{then } e^{\pm \# m} = e^{\pm \#/g}$$

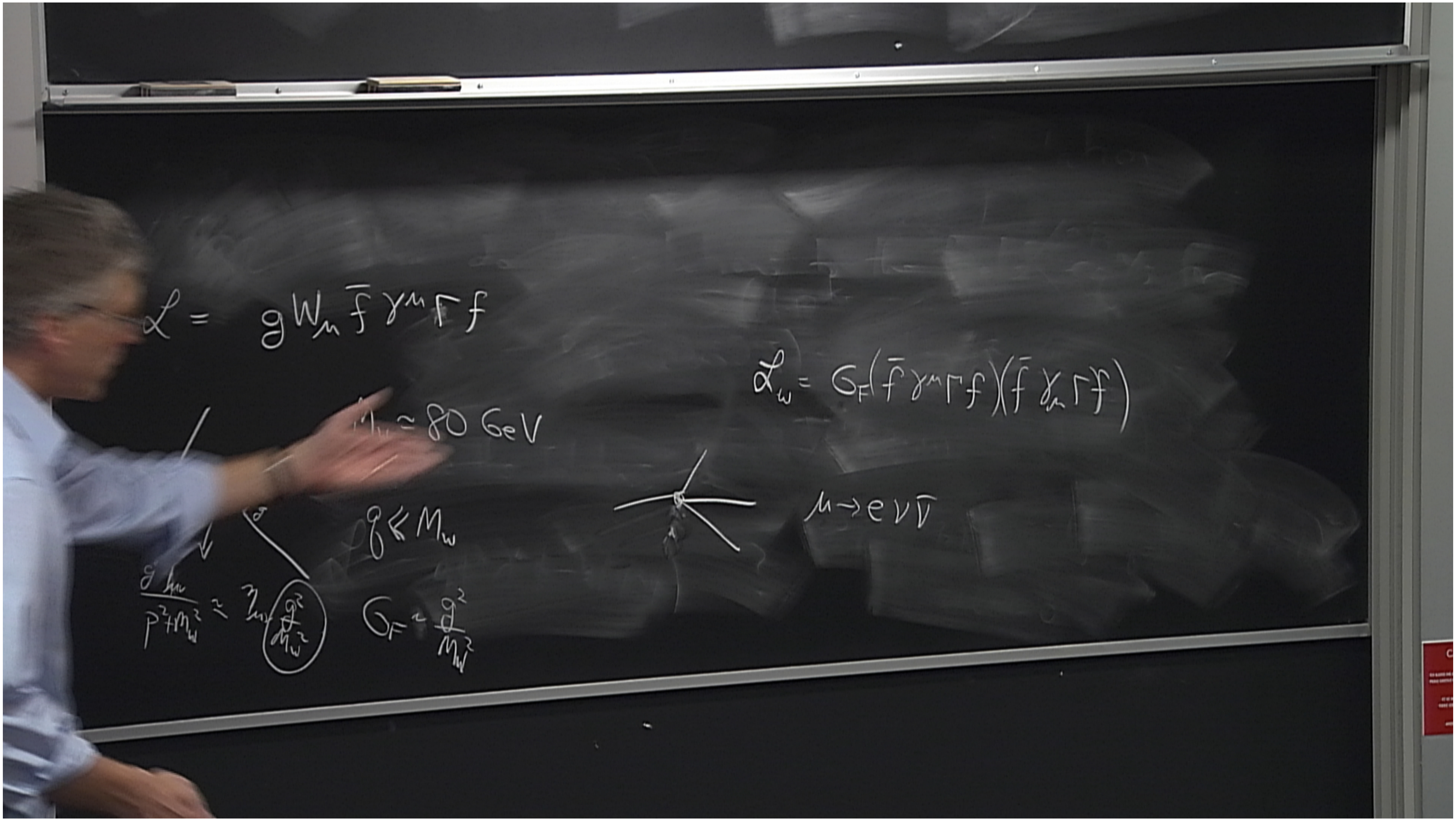
$$M^2 = \frac{M_p^2}{c}$$

$$\mathcal{L} = -M_p^2 R + c R^2 \qquad \boxed{R \sim \frac{M_p^2}{c}} \sim H^2 \qquad c \sim 10^8$$

$c \nabla_\mu R \nabla^\mu R$

$$+ \frac{c_2 R^3}{M_p^2} +$$

$$a(t) \simeq e^{Ht} \sim e^{M_p t/\sqrt{c}}$$


$\mathcal{L} = g W_\mu \bar{f} \gamma^\mu \Gamma f$
$M_W \simeq 80$ GeV
$\frac{g\,\eta_{\mu\nu}}{p^2+M_W^2} \simeq \eta_{\mu\nu} \frac{g^2}{M_W^2}$
$q \ll M_W$
$G_F \simeq \frac{g^2}{M_W^2}$
$\mathcal{L}_W = G_F (\bar{f} \gamma^\mu \Gamma f)(\bar{f} \gamma_\mu \Gamma f)$
$\mu \to e \nu \bar{\nu}$

Example Quantum Electrodynamics:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

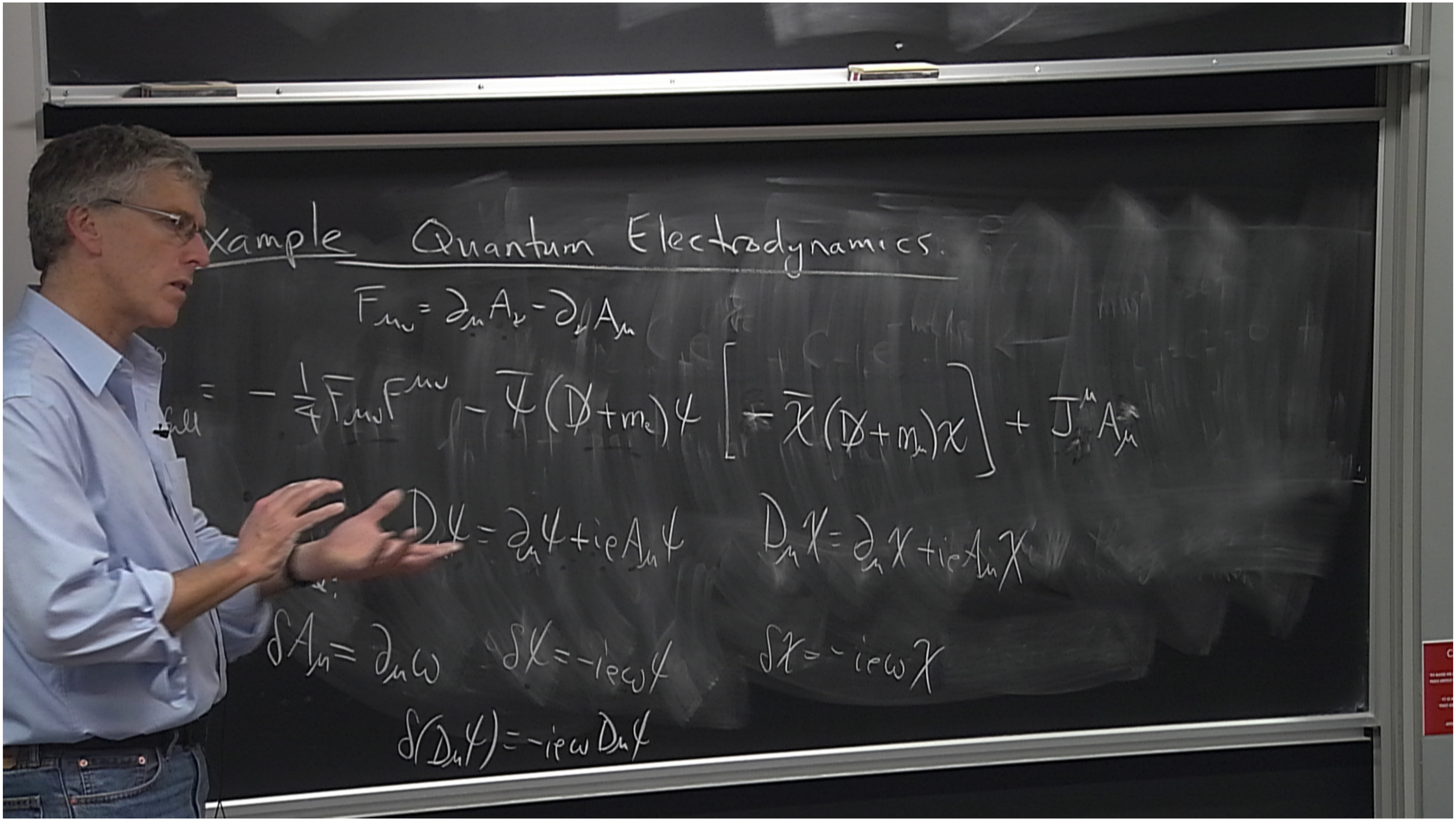
$$\mathcal{L}_{full} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{D} + m_e)\psi \left[- \bar{\chi}(\not{D} + m_\mu)\chi \right] + J^\mu A_\mu$$

$$D_\mu \psi = \partial_\mu \psi + ieA_\mu \psi \qquad D_\mu \chi = \partial_\mu \chi + ieA_\mu \chi$$

gauge invariance:

$$\delta A_\mu = \partial_\mu \omega \qquad \delta\psi = -ie\omega\psi \qquad \delta\chi = -ie\omega\chi$$

$$\delta(D_\mu \psi) = -ie\omega D_\mu \psi$$


xample Quantum Electrodynamics.
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
full $= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{D} + m_e)\psi \left[- \bar{\chi}(\not{D} + m_\mu)\chi \right] + J^\mu A_\mu$
$D\psi = \partial_\mu\psi + ieA_\mu\psi$
$D_\mu\chi = \partial_\mu\chi + ieA_\mu\chi$
$\delta A_\mu = \partial_\mu\omega$
$\delta\psi = -ie\omega\psi$
$\delta\chi = -ie\omega\chi$
$\delta(D_\mu\psi) = -ie\omega D_\mu\psi$

Example Quantum Electrodynamics:

$$J^\mu = \delta^\mu_0 \sum_i Q_i\, \delta^3(\vec{r}-\vec{r}_i)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L}_{full} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{D}+m_e)\psi \left[-\bar{\chi}(\not{D}+m_\mu)\chi\right] + J^\mu A_\mu$$

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Example: Quantum Electrodynamics:

$$J^\mu = \delta^\mu_0 \sum_i Q\, \delta^3(\vec{r}-\vec{r}_i)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L}_{full} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{D}+m_e)\psi \cancel{\left[-\bar{\chi}(\not{D}+m_\mu)\chi\right]} + J^\mu A_\mu \qquad \left[-\bar{\nu}(\not{\partial}+m_\nu)\nu\right]$$

$$D_\mu\psi = \partial_\mu\psi + ieA_\mu\psi \qquad D_\mu\chi = \partial_\mu\chi + ieA_\mu\chi$$

$\nu \to e^{i\omega\gamma_5}\nu$

"chiral"

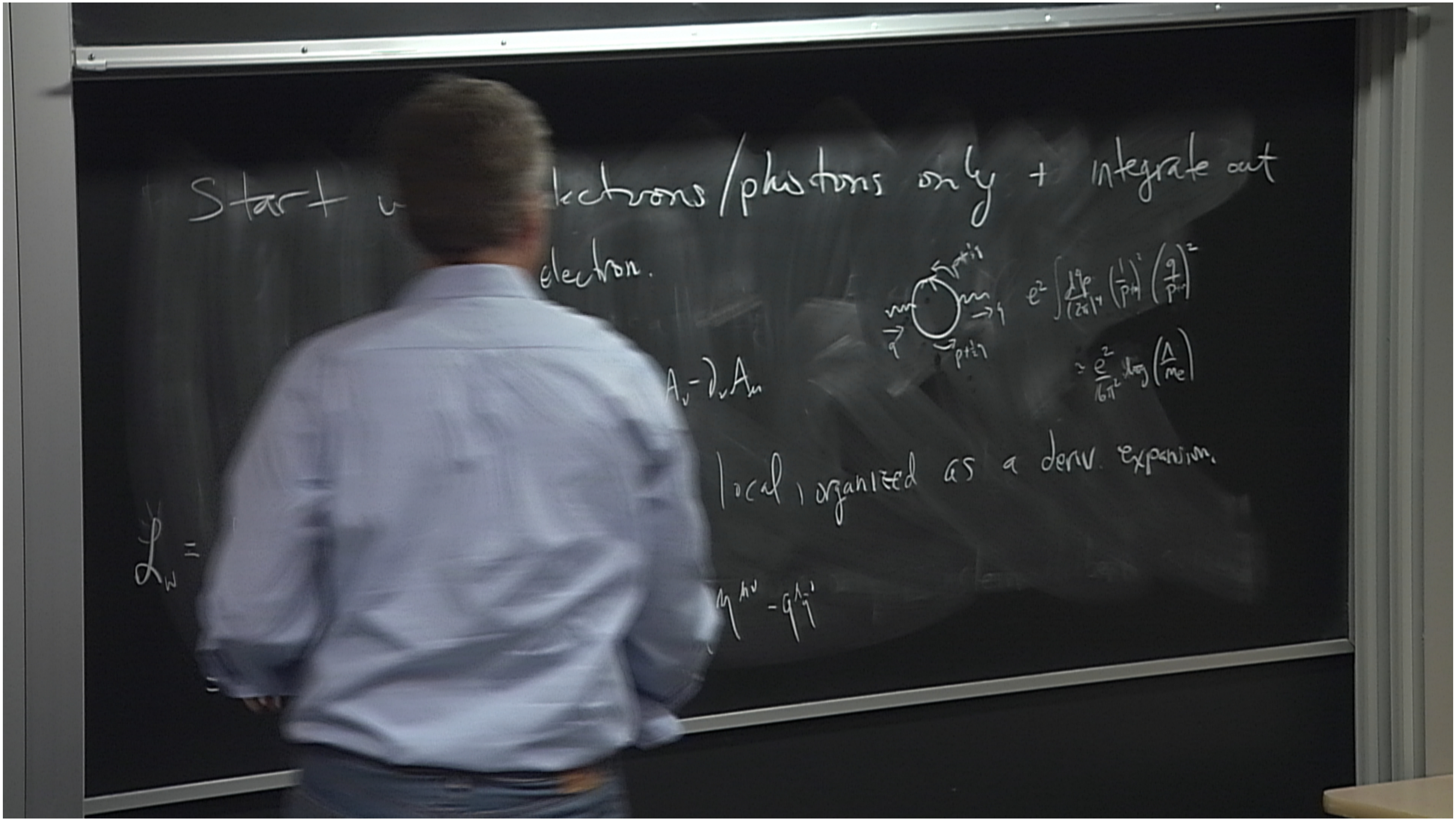
gauge invariance:

$$\delta A_\mu = \partial_\mu\omega \qquad \delta\psi = -ie\omega\psi \qquad \delta\chi = -ie\omega\chi$$

$$\delta(D_\mu\psi) = -ie\omega D_\mu\psi$$

Start with electrons/photons only + integrate out electron.

$q \ll m_e$


Start w lectrons/photons only + integrate out
electron.
local, organized as a deriv. expansion.

Start with electrons/photons only + integrate out

electron.

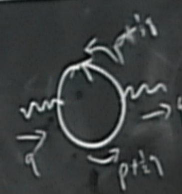
$q \ll m_e$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$\delta A_\mu = \partial_\mu \omega$, local

$\mathcal{L}_W$ = function of A_μ ,

$= -\frac{Z}{4} F_{\mu\nu} F^{\mu\nu} +$



$e^2 \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{p+q}\right)^2 \left(\frac{q}{p+q}\right)^2$

$\approx \frac{e^2}{16\pi^2} \log\left(\frac{\Lambda}{m_e}\right)$

$Z = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k + \log\left(\frac{m_e^2}{\mu^2}\right)\right]$

$n = 4 - 2\varepsilon$ $\quad k = \gamma - \log 4\pi$ Euler Mascheroni

$\gamma = 0.577215\ldots$

Aside on dimensional analysis in dim reg:

$$4-n=2\epsilon$$

$$n=4-2\epsilon$$

$$e=(\text{mass})^{\epsilon}=e_0\mu^{\epsilon}$$

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\bar{\psi}(\not{\partial}+ie\not{A})\psi+\ldots$$

$$\not{A}=\gamma^{\mu}A_{\mu}\qquad \not{\partial}=\gamma^{\mu}\partial_{\mu}\qquad \bar{\psi}=i\psi^{\dagger}\gamma_0$$

$$S=\int d^n x\,\mathcal{L}\qquad [\mathcal{L}]=n\qquad [\partial]=1\qquad [A_{\mu}]=\frac{n-2}{2}\qquad [\psi]=\frac{n-1}{2}\qquad [e]=n-\left(\frac{n-2}{2}\right)-(n-1)=\frac{4-n}{2}$$

n dim reg:

$$f(n)\frac{1}{\epsilon} = \frac{f(4)}{\epsilon} + f'(4) + O(\epsilon)$$

$$4-n = 2\epsilon$$

$$n = 4-2\epsilon$$

$+ \ldots$

$$e = (\text{mass})^{\epsilon} = e_0\mu^{\epsilon}$$

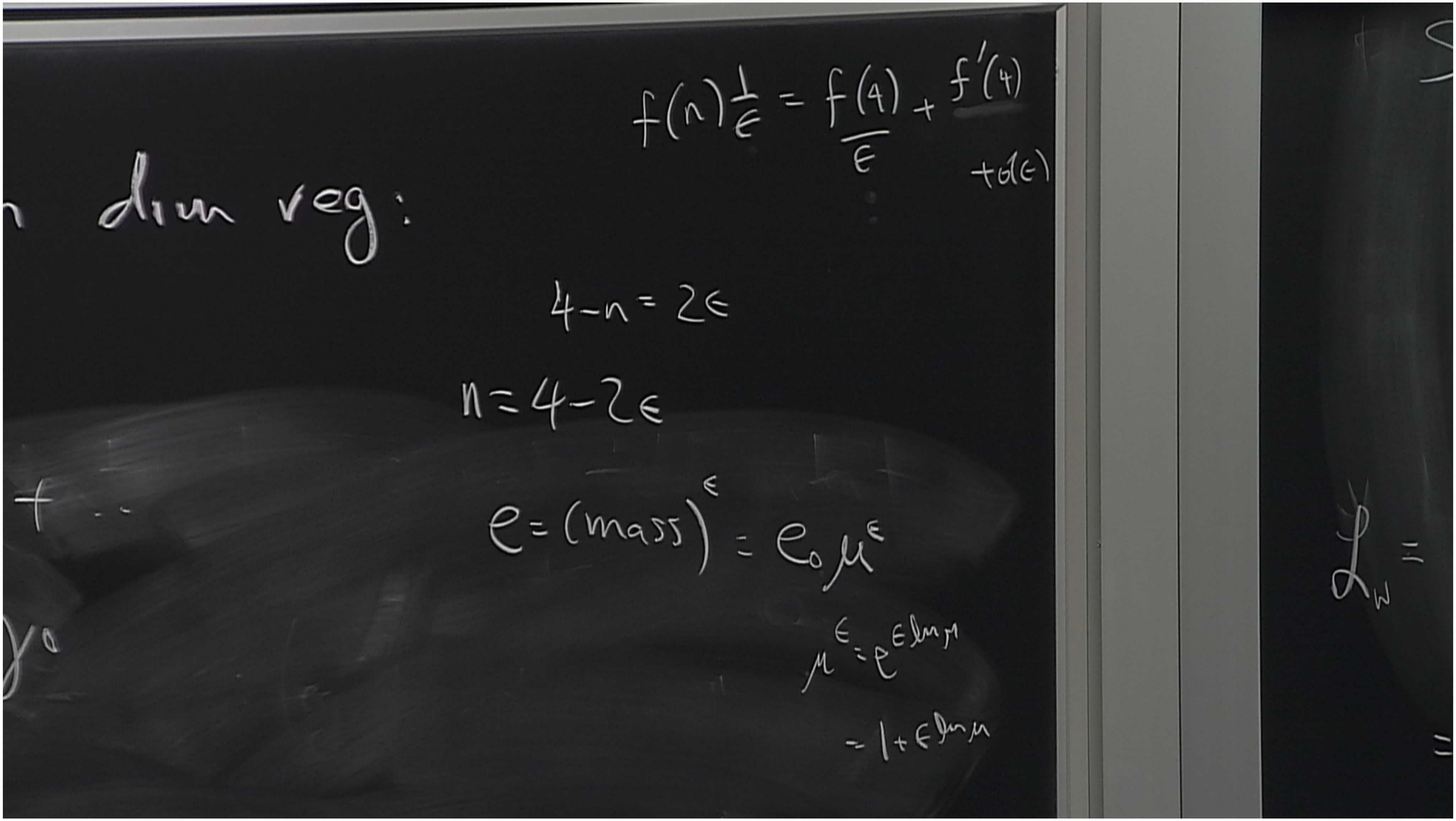
g_0

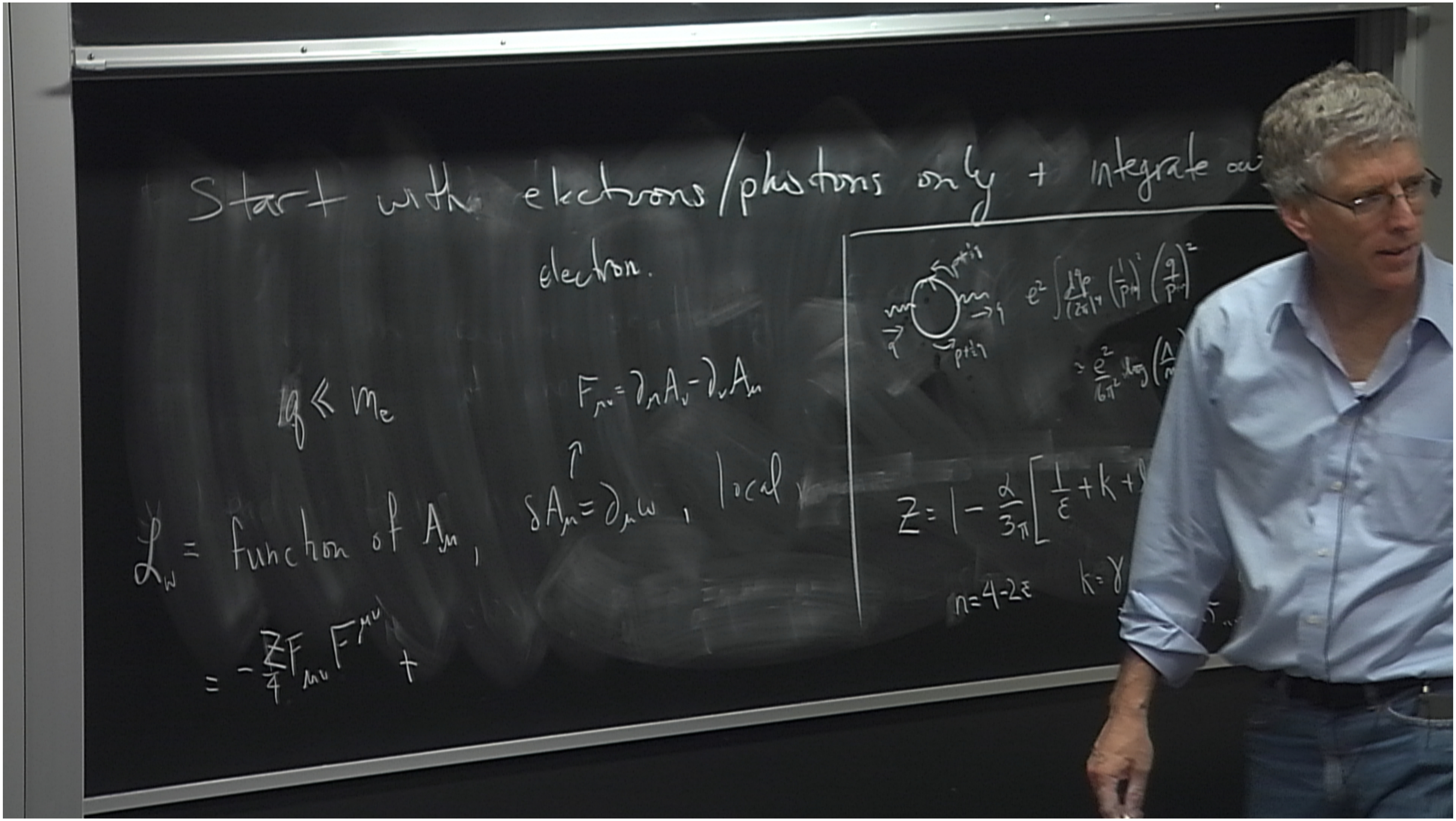
$$\mu^{\epsilon} = e^{\epsilon \ln \mu}$$

$$= 1 + \epsilon \ln \mu$$

$\mathcal{L}_W =$

$=$

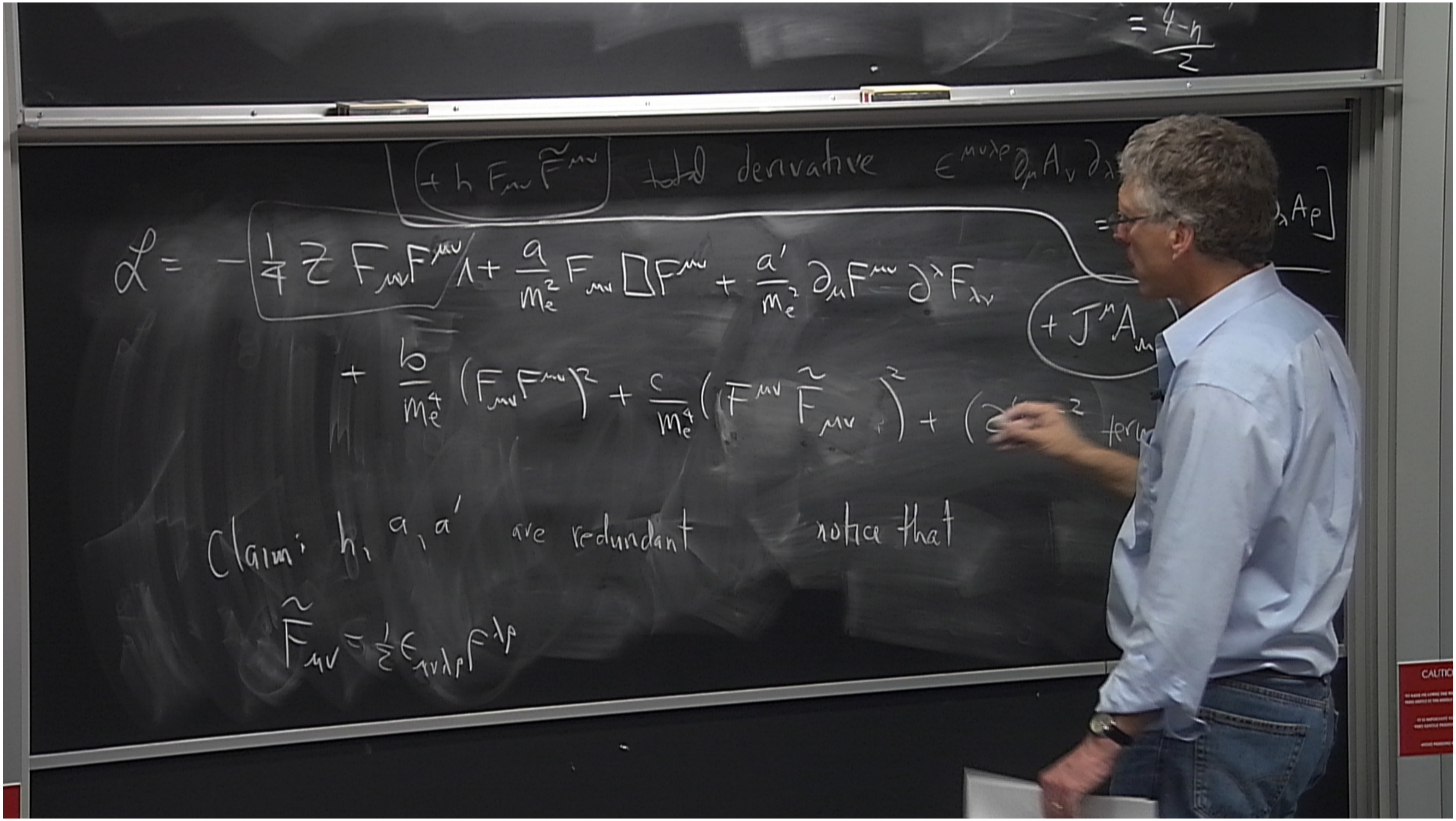

dim reg:
f(n) 1/ε = f(4)/ε + f'(4) + O(ε)
4 − n = 2ε
n = 4 − 2ε
e = (mass)^ε = e₀ μ^ε
μ^ε = e^(ε ln μ)
= 1 + ε ln μ
L_W =


Start with electrons/photons only + integrate out
electron.
q ≪ m_e
F_μν = ∂_μA_ν − ∂_νA_μ
L_w = function of A_μ , δA_μ = ∂_μω , local
= − Z/4 F_μν F^μν +
Z = 1 − α/3π [1/ε + k +
n = 4 − 2ε
k = γ

$$= \frac{4-n}{2}$$

$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} + \frac{a}{m_e^2} F_{\mu\nu}\Box F^{\mu\nu} + \frac{a'}{m_e^2}\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu}$$

$$+ \frac{b}{m_e^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{c}{m_e^4}\left(\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\right)^2$$


= (4-n)/2
+ h F_{μν} F̃^{μν}   total derivative   ε^{μνλρ} ∂_μ A_ν ∂_λ
∂_λ A_ρ]
ℒ = − 1/4 Z F_{μν} F^{μν} + a/m_e^2 F_{μν} □ F^{μν} + a′/m_e^2 ∂_μ F^{μν} ∂^λ F_{λν}
+ J^μ A_μ
+ b/m_e^4 (F_{μν} F^{μν})^2 + c/m_e^4 (F^{μν} F̃_{μν})^2 + (∂ ...)^2 terms
Claim: h, a, a′ are redundant   notice that
F̃_{μν} = 1/2 ε_{μνλρ} F^{λρ}

$$= \frac{4-n}{2}$$

$+ h F_{\mu\nu}\tilde{F}^{\mu\nu}$ total derivative $\epsilon^{\mu\nu\lambda\rho}\partial_\mu A_\nu \partial_\lambda A_\rho$

$$= \partial_\mu\left[\epsilon^{\mu\nu\lambda\rho} A_\nu \partial_\lambda A_\rho\right]$$

$$\mathcal{L} = \boxed{-\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu}} + \frac{a}{m_e^2} F_{\mu\nu}\Box F^{\mu\nu} + \frac{a'}{m_e^2}\underbrace{\partial_\mu F^{\mu\nu}\,\partial^\lambda F_{\lambda\nu}}_{\frac{a'}{Z m_e^2} J^\nu J_\nu} + J^\mu A_\mu$$

$$+ \frac{b}{m_e^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{c}{m_e^4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2 + \left(\partial^4 F^2 \text{ terms} + \dots\right)$$

Claim: h, a, a' are redundant

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$$

notice that zeroeth order e.o.m is

$$-Z\partial_\mu F^{\mu\nu} + J^\nu = 0$$

$$= \frac{4-n}{2}$$

$$+ h\, F_{\mu\nu}\tilde{F}^{\mu\nu} \quad \text{total derivative} \quad \epsilon^{\mu\nu\lambda\rho}\partial_\mu A_\nu \partial_\lambda A_\rho = \partial_\mu\left[\epsilon^{\mu\nu\lambda\rho} A_\nu \partial_\lambda A_\rho\right]$$

$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} \Lambda + \frac{a}{m_e^2} F_{\mu\nu}\Box F^{\mu\nu} + \frac{a'}{m_e^2} \underbrace{\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu}}_{\frac{a'}{Z m_e^2} J^\nu J_\nu} + J^\mu A_\mu$$

$$+ \frac{b}{m_e^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{c}{m_e^4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2 + \left(\partial^4 F^2 \text{ terms} + \ldots\right)$$

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notice that zeroeth order e.o.m is

$$-Z\partial_\mu F^{\mu\nu} + J^\nu = 0$$

$+ h\, F_{\mu\nu}\tilde{F}^{\mu\nu}$ total derivative $\epsilon^{\mu\nu\lambda\rho}\partial_\mu A_\nu \partial_\lambda A_\rho$

$= \partial_\mu\left[\epsilon^{\mu\nu\lambda\rho} A_\nu \partial_\lambda A_\rho\right]$

$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} + \frac{a}{m_e^2} F_{\mu\nu} \underbrace{\Box F^{\mu\nu}}_{\partial^\mu J^\nu - \partial^\nu J^\mu} + \frac{a'}{m_e^2} \underbrace{\partial_\mu F^{\mu\nu}\, \partial^\lambda F_{\lambda\nu}}_{\frac{a'}{2m_e^2} J^\nu J_\nu} + J^\mu A_\mu$$

$$+ \frac{b}{m_e^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{c}{m_e^4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2 + \left(\partial^4 F^2 \text{ terms} + \ldots\right)$$

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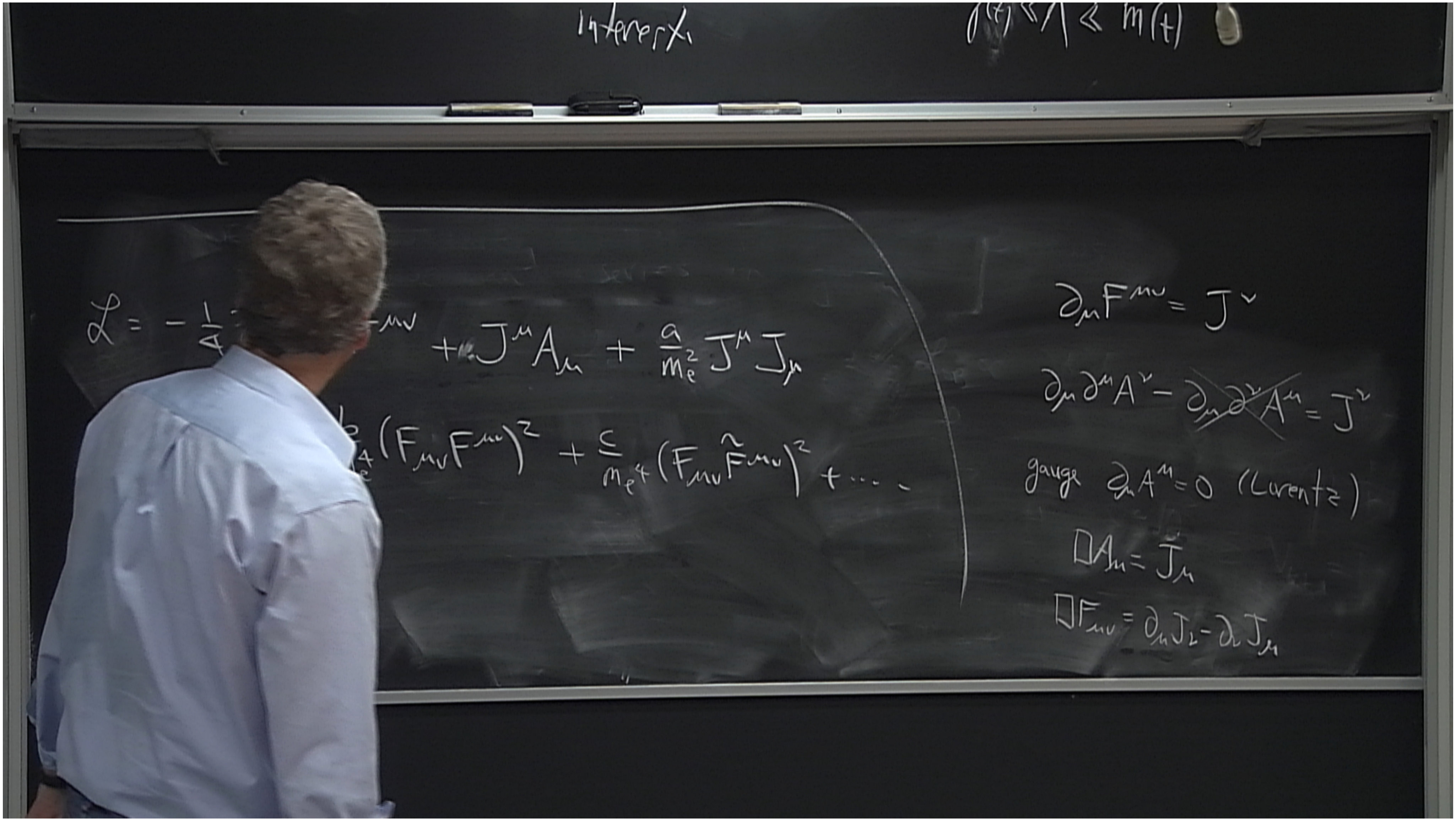
$+ J^\mu A_\mu + \frac{a}{m_e^2} J^\mu J_\mu$
$(F_{\mu\nu}F^{\mu\nu})^2 + \frac{c}{m_e^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \dots$
$\partial_\mu F^{\mu\nu} = J^\nu$
$\partial_\mu \partial^\mu A^\nu - \cancel{\partial_\mu \partial^\nu A^\mu} = J^\nu$
gauge $\partial_\mu A^\mu = 0$ (Lorentz)
$\Box A_\mu = J_\mu$
$\Box F_{\mu\nu} = \partial_\mu J_\nu - \partial_\nu J_\mu$

$$= \bar\psi(\partial\!\!\!/ + ieA\!\!\!/)\psi + \ldots$$

$$A\!\!\!/ = \gamma^\mu A_\mu \qquad \partial\!\!\!/ = \gamma^\mu \partial_\mu \qquad \bar\psi = i\psi^\dagger\gamma_0$$

$$e = (\text{mass})^\epsilon = e_0\mu^\epsilon$$

$$\mu^\epsilon = e^{\epsilon \ln\mu} \sim 1 + \epsilon\ln\mu$$

$$S = \int d^n x\, \mathcal{L} \qquad [\mathcal{L}] = n \qquad [\partial] = 1 \qquad [A_\mu] = \frac{n-2}{2} \qquad [\psi] = \frac{n-1}{2}$$

$$[e] = n - \left(\frac{n-2}{2}\right) - (n-1) = \frac{4-n}{2}$$

$$\mathcal{L}_1 = \mu_0^2\phi^2 + g\bar\psi\psi\phi + \bar\psi(\partial\!\!\!/ + m)\psi$$

$$m_e\bar\chi\chi \qquad M \qquad \bar\partial 5 \partial 5 \qquad g_5 \phi 5$$

$$\mu_1^2 = \mu_0^2 + \frac{g^2M^2}{(4\pi)^2} - \frac{g_5^2 M_5^2}{(4\pi)^2}$$

$$\mathcal{L}_\omega = -\mu_1^2\phi^2 + \ldots \qquad \phi$$

$$-m_{e_2}\bar\chi\chi$$

$$\ldots\chi\, ieA_\mu\chi$$

"chiral"

$$\delta A_\mu = \partial_\mu\omega \qquad \delta\psi = -ie\omega\psi \qquad \delta\chi = -ie\omega\chi$$

$$\delta(D_\mu\psi) = -ie\omega D_\mu\psi$$