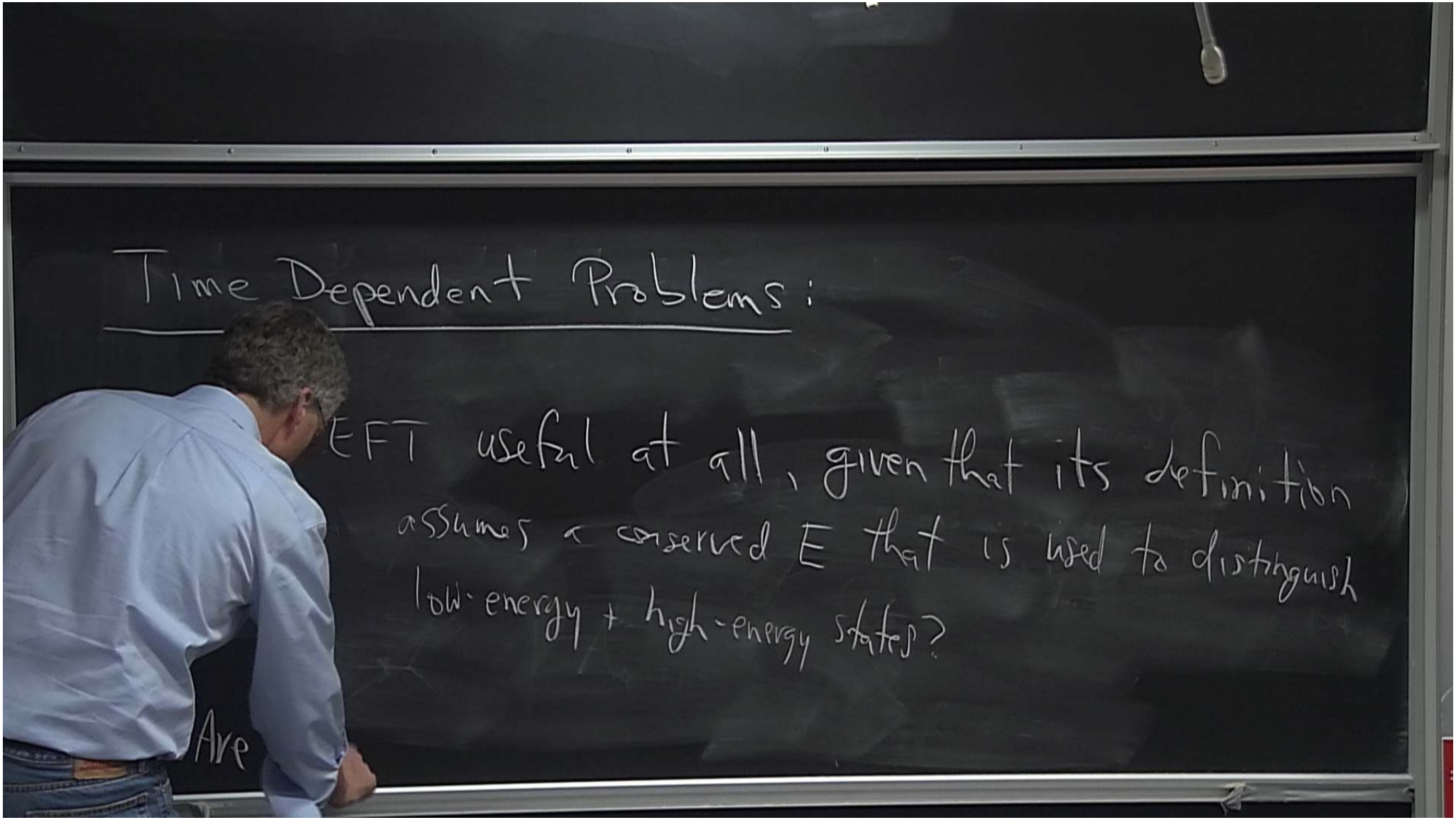
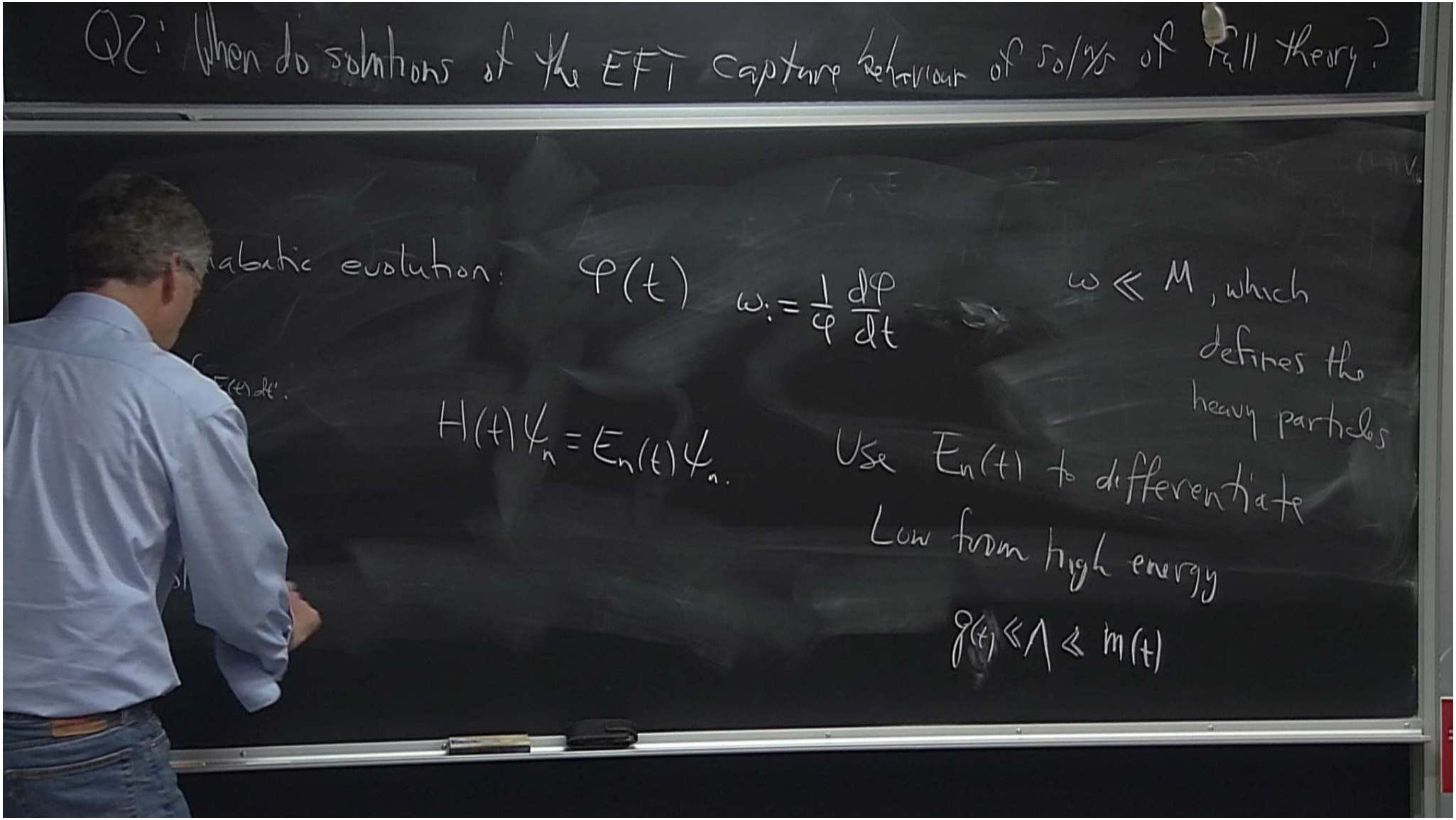
Title: Introduction to Effective Field Theories - Lecture 12

Date: Feb 14, 2014 02:30 PM

URL: http://pirsa.org/14020093

Abstract:


Time Dependent Problems:
EFT useful at all, given that its definition
assumes a conserved E that is used to distinguish
low-energy + high-energy states?
Are


Q2: When do solutions of the EFT capture behaviour of sol'n of full theory?
abatic evolution: $\varphi(t)$ $\omega := \frac{1}{\varphi}\frac{d\varphi}{dt}$ $\omega \ll M$, which defines the heavy particles
$H(t)\psi_n = E_n(t)\psi_n$.
Use $E_n(t)$ to differentiate
Low from high energy
$g(t) \ll \Lambda \ll m(t)$

Q2: When do solutions of the EFT capture behaviour of sol'ns of full theory?

Adiabatic evolution: $\varphi(t)$ $\omega := \frac{1}{\varphi}\frac{d\varphi}{dt}$ $\omega \ll M$, which defines the heavy particles

$\psi \sim e^{-i\int E(t)dt}$.

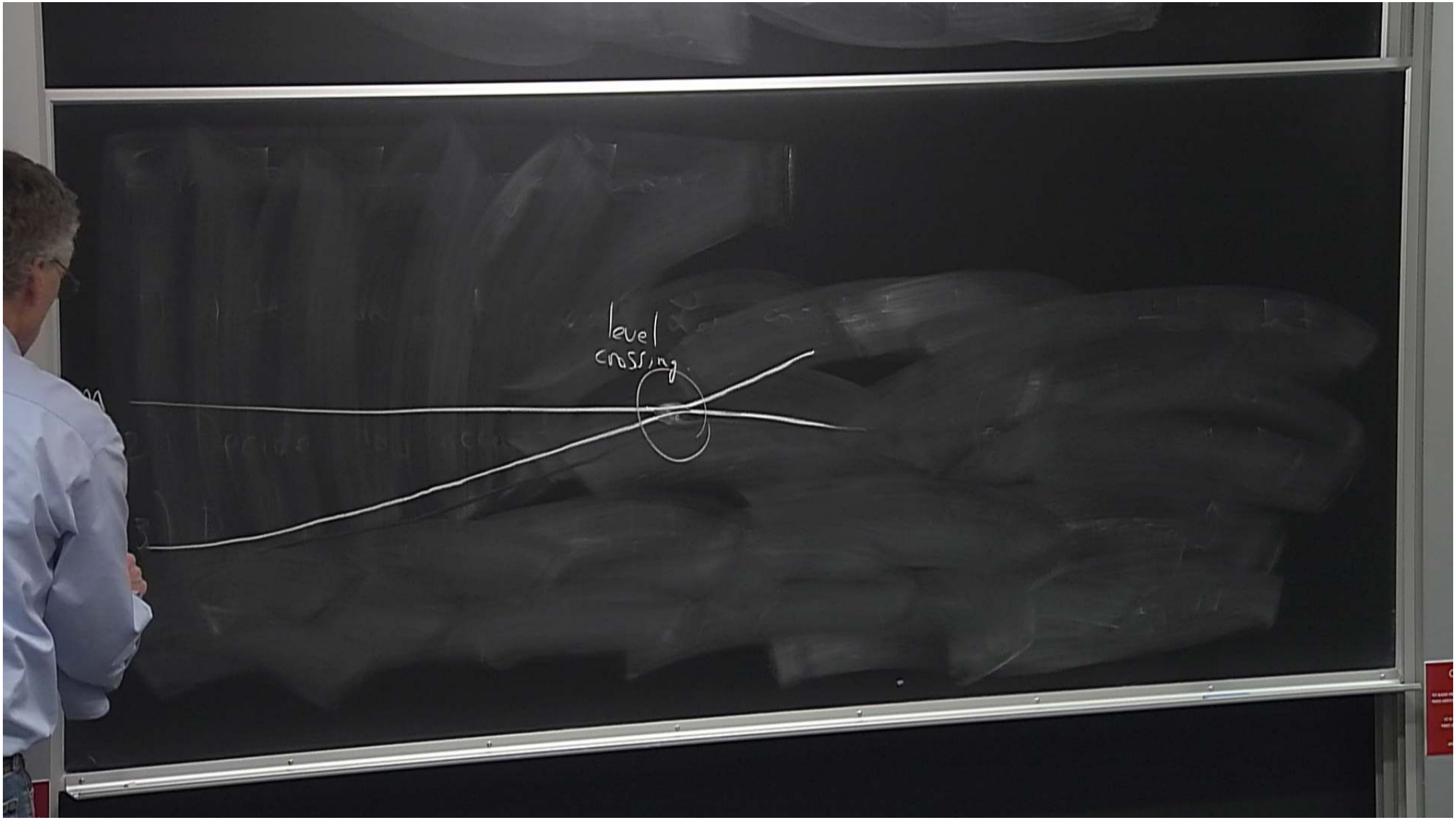
$H(t)\psi_n = E_n(t)\psi_n$.

Use $E_n(t)$ to differentiate

Must check: $g(t) \ll m(t)$ for all t of interest

Low from high energy

$g(t) \ll \Lambda \ll m(t)$

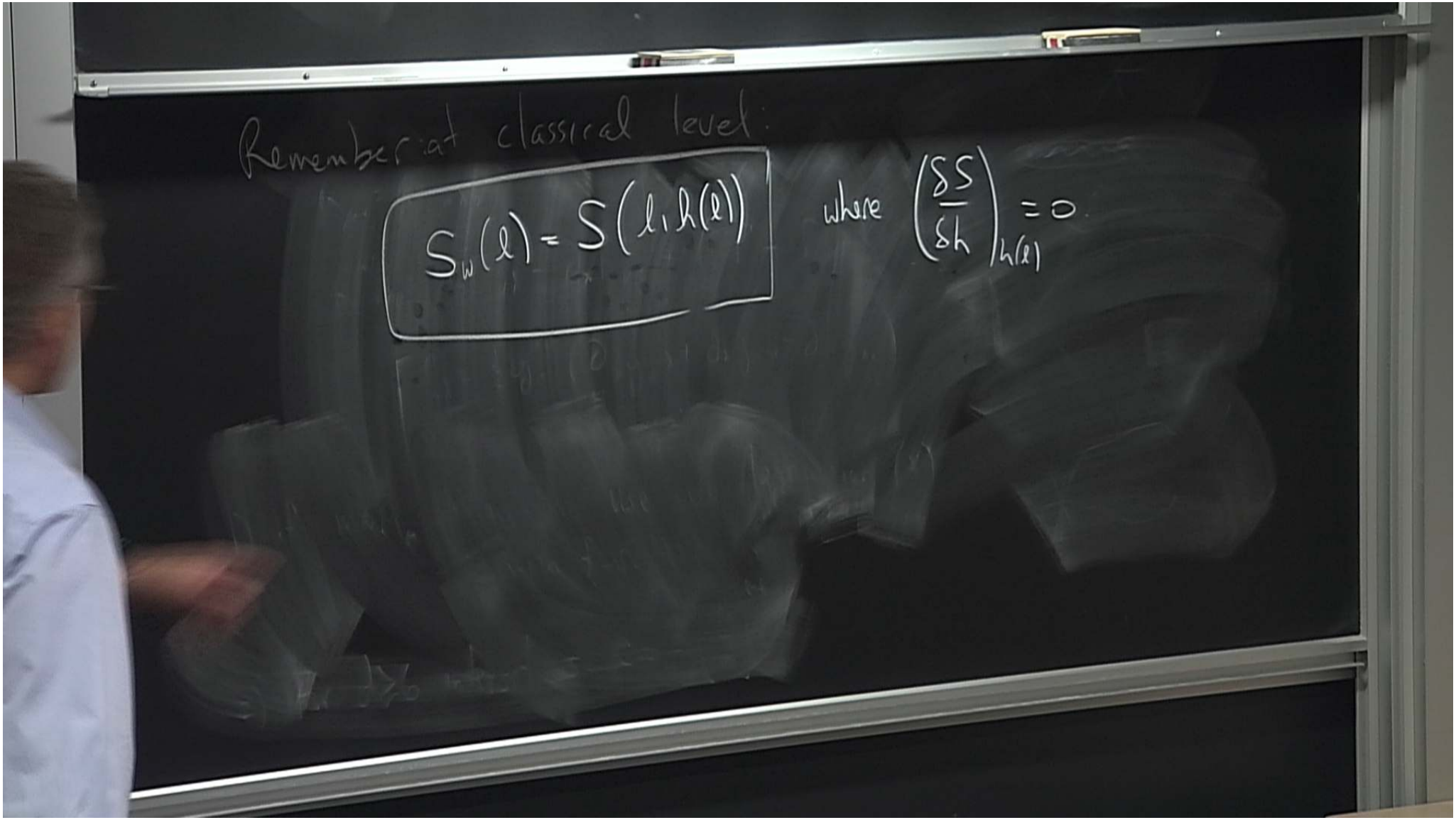

level
crossing

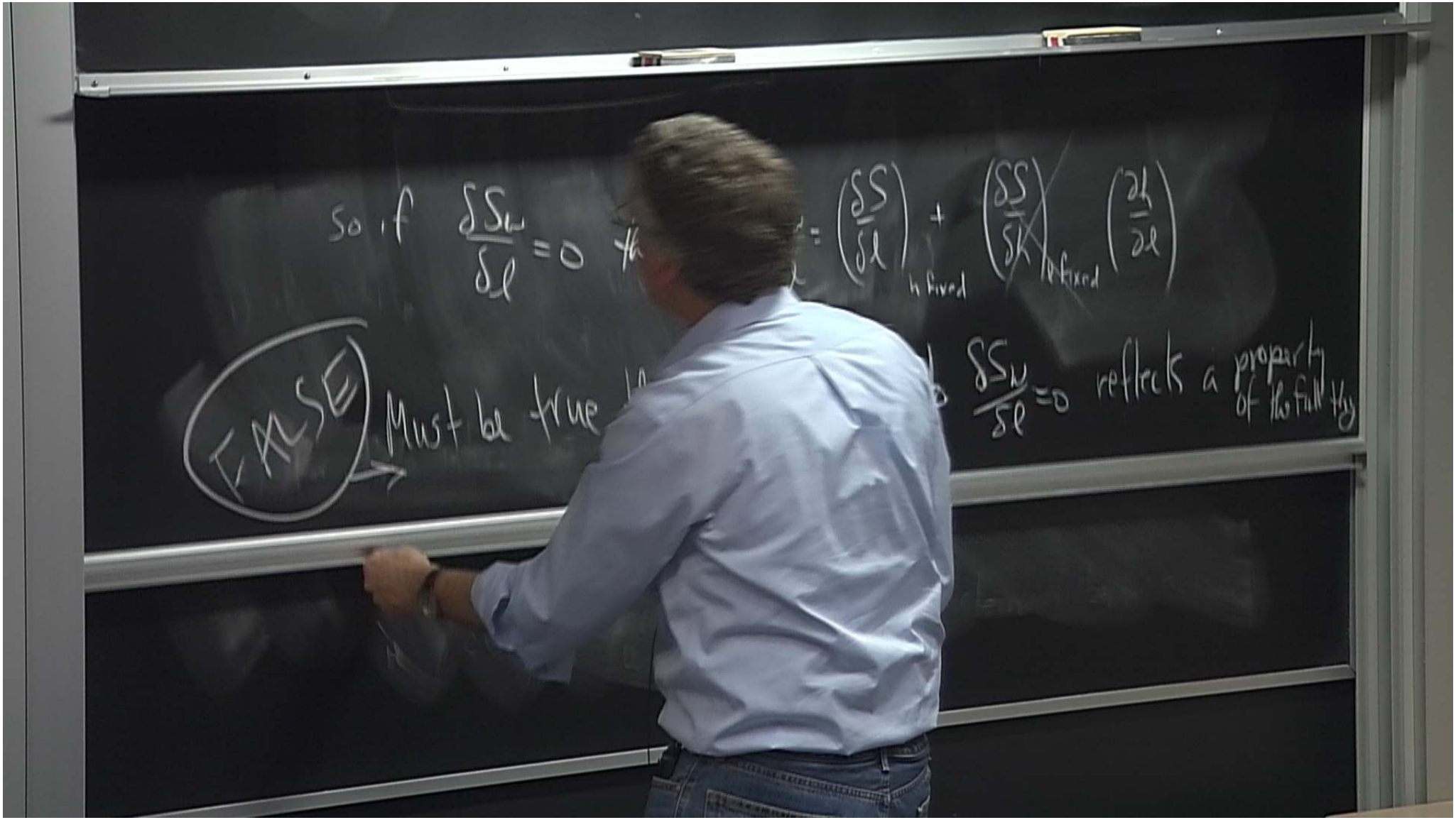
Q2: When do the solutions to the field eq^{n}/s for

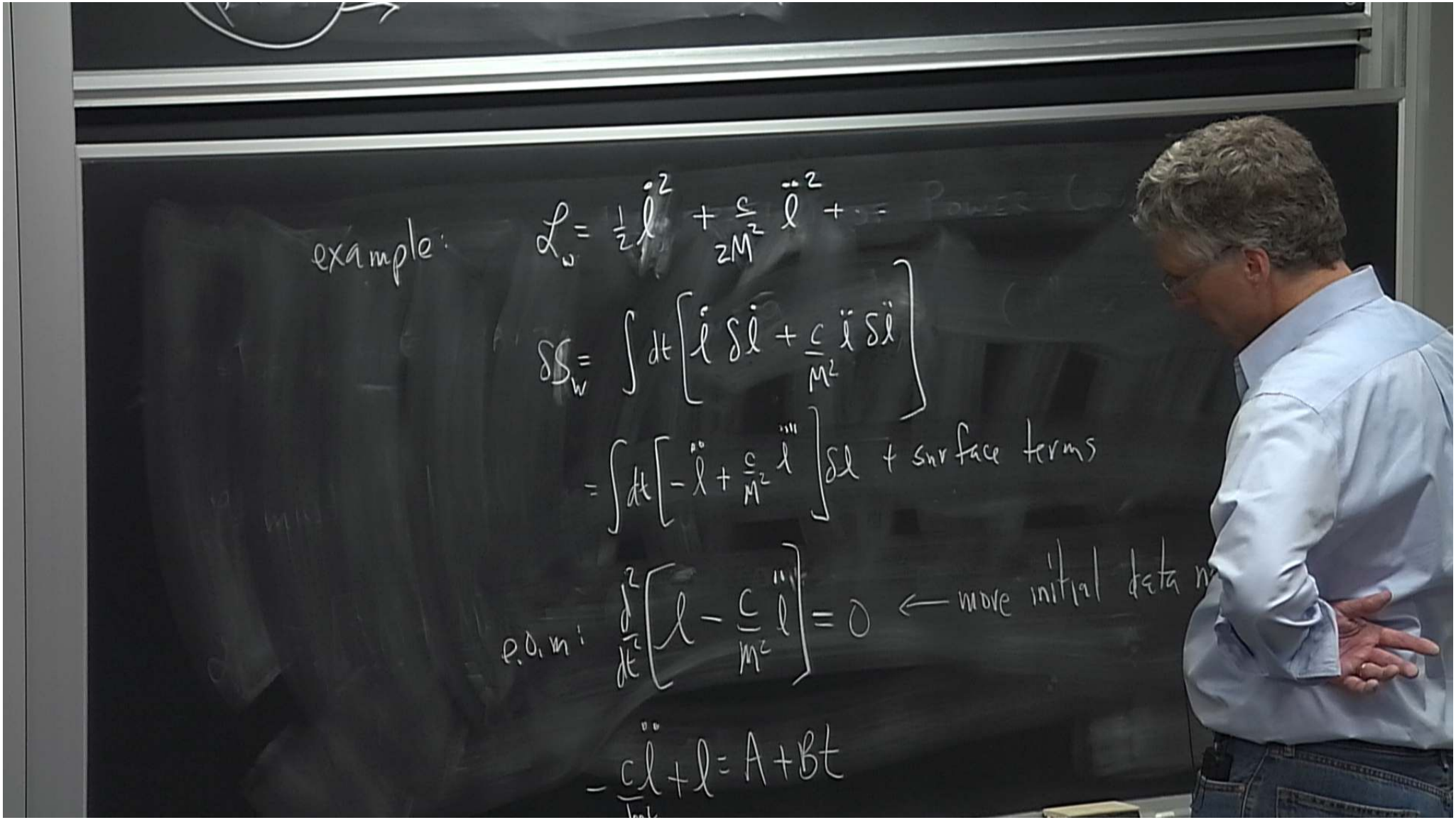
$$\mathcal{L}_{eff} \simeq -\frac{1}{2}(\partial_\mu \xi \partial^\mu \xi) - \frac{a}{m_R^4}(\partial_\mu \xi \partial^\mu \xi)^2 + \ldots$$

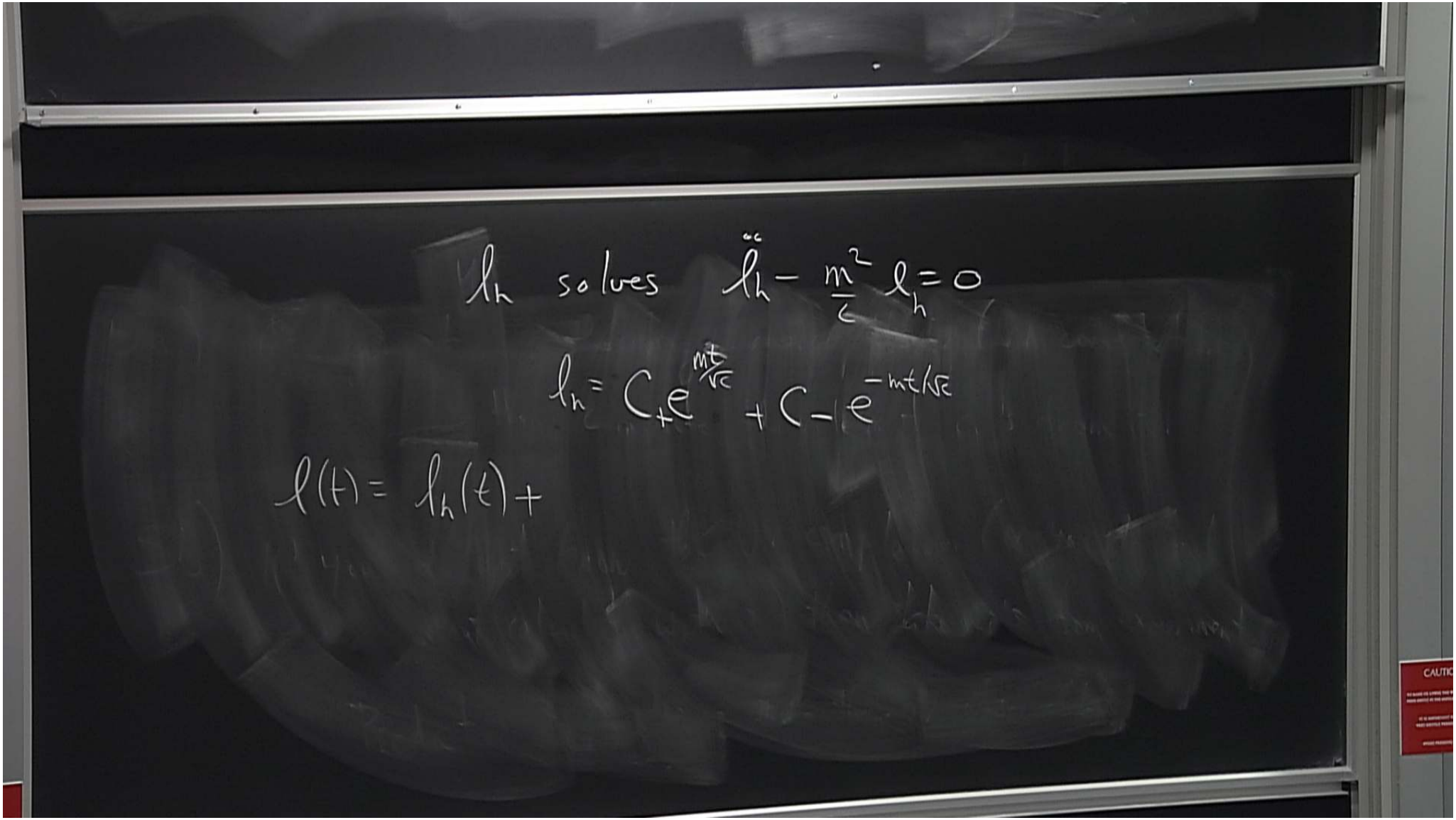
capture the solutions to

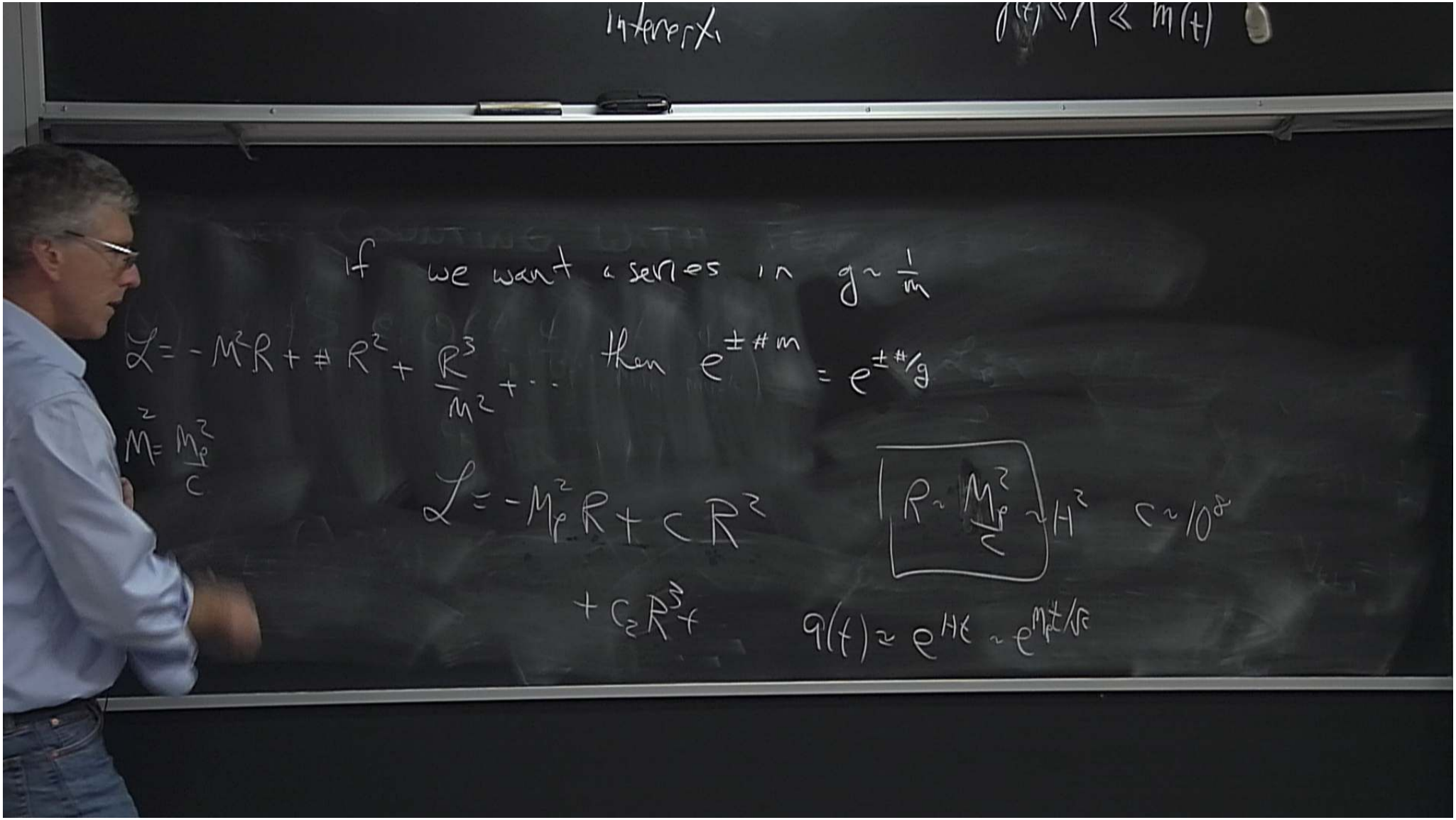
$$\mathcal{L} \simeq -\partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi) \quad ?$$

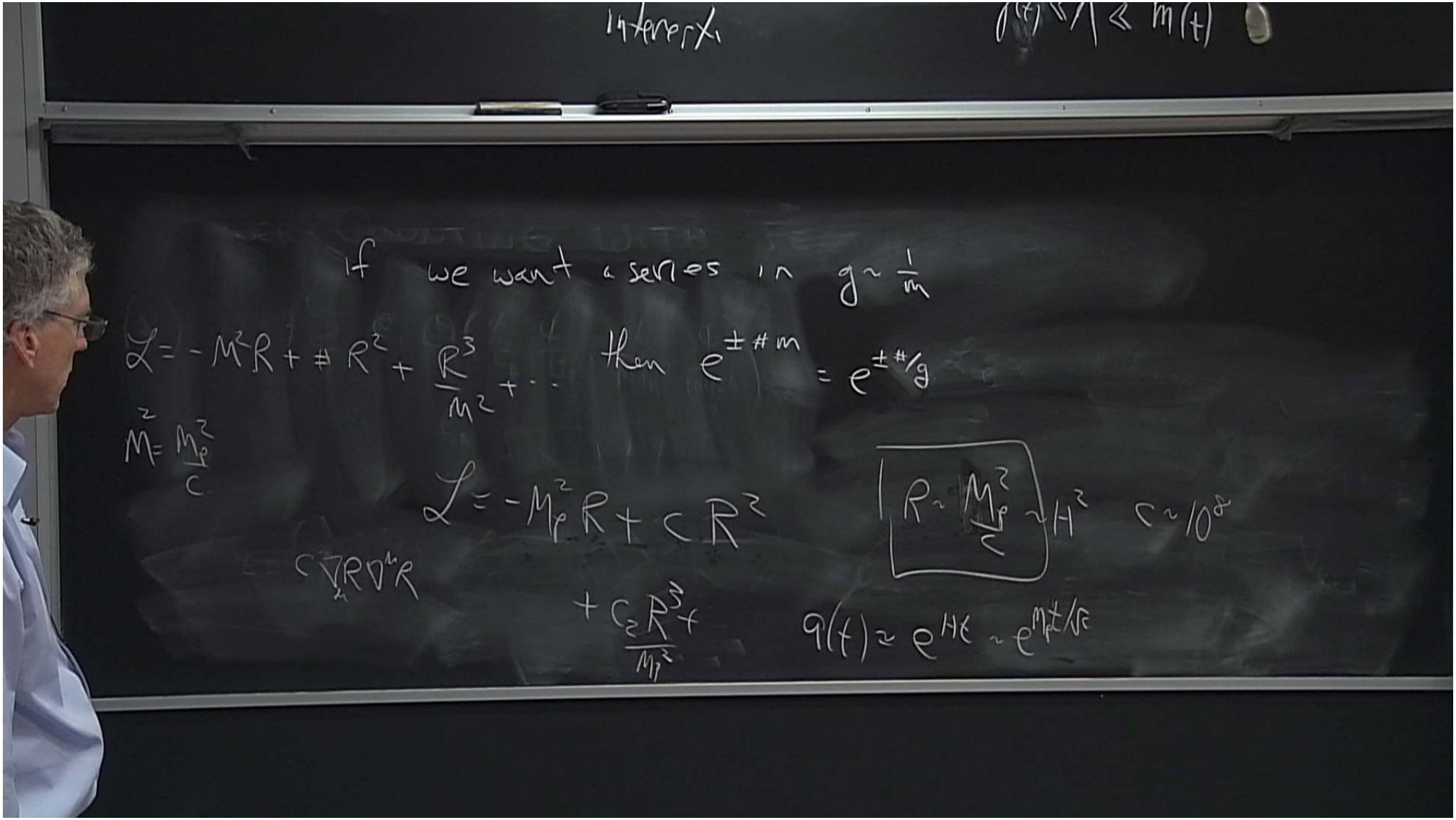

Remember at classical level:
$S_W(\ell) = S(\ell, h(\ell))$
where $\left(\frac{\delta S}{\delta h}\right)_{h(\ell)} = 0$


FALSE
reflects a property of the full thy


example:
$\mathcal{L}_\pi = \frac{1}{2}\dot{\ell}^2 + \frac{c}{2M^2}\ddot{\ell}^2 + \cdots$
$\delta S_\pi = \int dt \left[\dot{\ell}\,\delta\dot{\ell} + \frac{c}{M^2}\ddot{\ell}\,\delta\ddot{\ell}\right]$
$= \int dt \left[-\ddot{\ell} + \frac{c}{M^2}\ell''''\right]\delta\ell$ + surface terms
e.o.m: $\frac{d^2}{dt^2}\left[\ell - \frac{c}{M^2}\ell''''\right] = 0$ ← more initial data
$-\frac{c}{M^2}\ddot{\ell} + \ell = A + Bt$


ℓ_h solves $\ddot{\ell}_h - \frac{m^2}{c}\ell_h = 0$
$\ell_h = C_+ e^{mt/\sqrt{c}} + C_- e^{-mt/\sqrt{c}}$
$\ell(t) = \ell_h(t) +$


interest
if we want a series in $g \sim \frac{1}{m}$
$\mathcal{L} = -M^2 R + \# R^2 + \frac{R^3}{M^2} + \cdots$
then $e^{\pm \# m} = e^{\pm \#/g}$
$M^2 = \frac{M_p^2}{c}$
$\mathcal{L} = -M_p^2 R + c R^2$
$+ c_2 R^3 +$
$R \sim \frac{M_p^2}{c} \sim H^2$
$c \sim 10^8$
$a(t) \sim e^{Ht} \sim e^{M_p t/\sqrt{c}}$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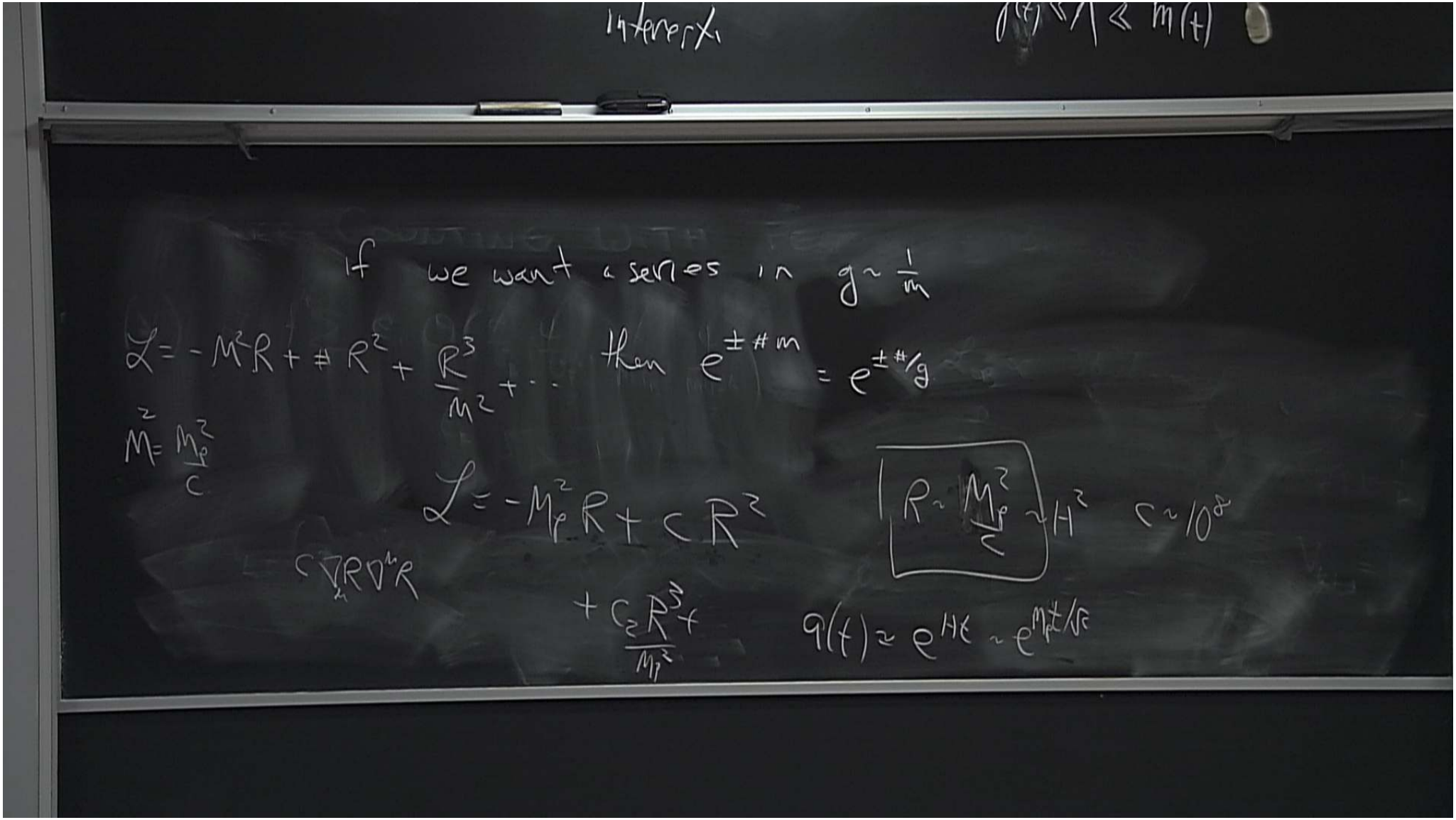
if we want a series in $g \sim \frac{1}{m}$
$\mathcal{L} = -M^2 R + \# R^2 + \frac{R^3}{M^2} + \cdots$ then $e^{\pm \# m} = e^{\pm \# / g}$
$M^2 = \frac{M_P^2}{c}$
$\mathcal{L} = -M_P^2 R + c R^2$
$R \sim \frac{M_P^2}{c} \sim H^2$ $c \sim 10^{\alpha}$
$c \nabla_\mu R \nabla^\mu R$
$+ c_2 \frac{R^3}{M_P^2} +$
$a(t) \approx e^{Ht} \sim e^{M_P t / \sqrt{c}}$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if we want a series in $g \sim \frac{1}{m}$
$\mathcal{L} = -M^2 R + \# R^2 + \frac{R^3}{M^2} + \cdots$ then $e^{\pm \# m} = e^{\pm \# / g}$
$M^2 = \frac{M_P^2}{c}$
$\mathcal{L} = -M_P^2 R + c R^2$
$c \nabla R \nabla^n R$
$+ c_2 \frac{R^3}{M_P^2} +$
$R \sim \frac{M_P^2}{c} \sim H^2$
$c \sim 10^{\alpha}$
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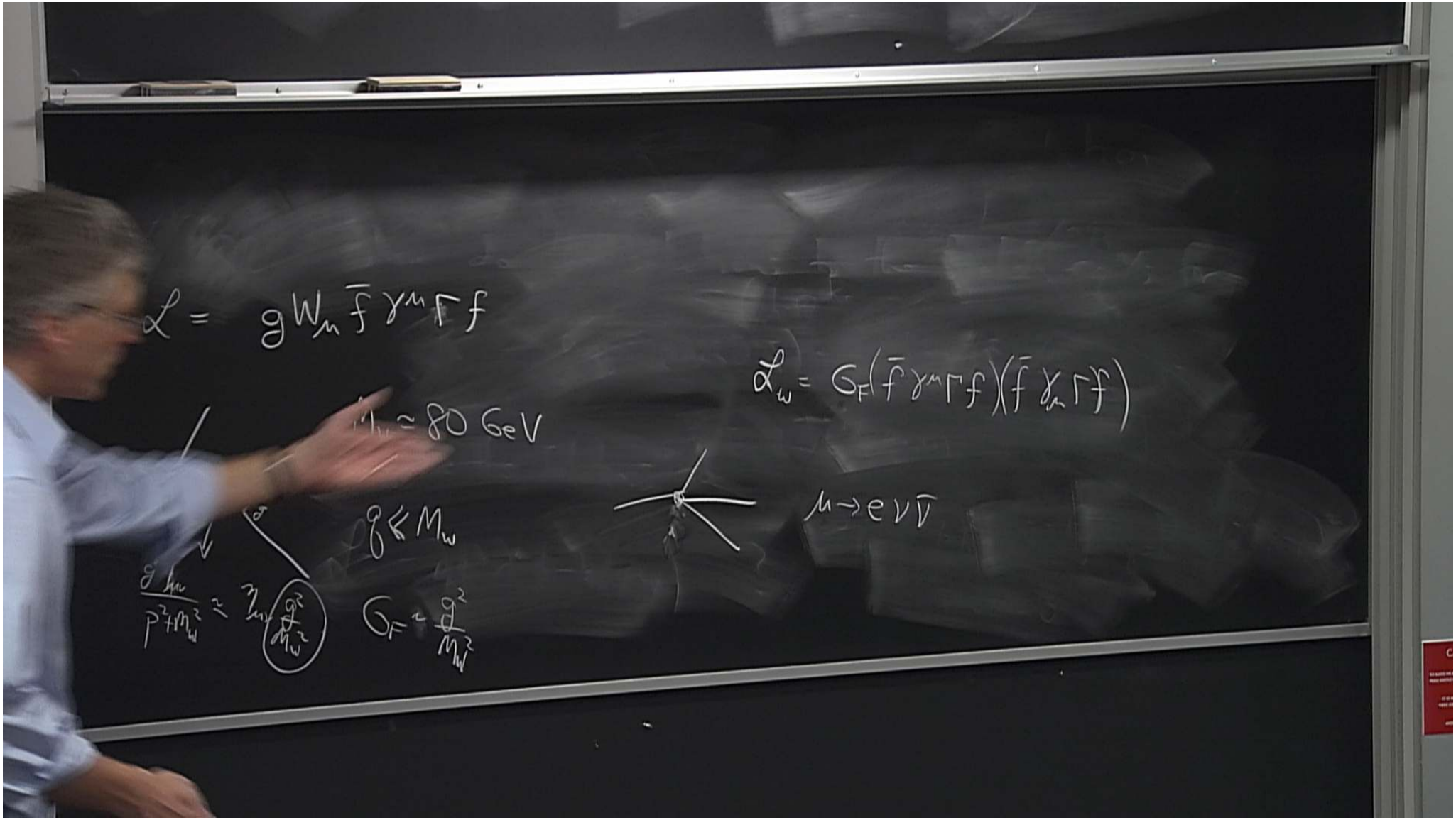

$\mathcal{L} = g W_\mu \bar{f} \gamma^\mu \Gamma f$
$M_W \simeq 80\ \mathrm{GeV}$
$q \ll M_W$
$\frac{g}{p^2 + M_W^2} \simeq \frac{g}{M_W^2}$
$G_F \sim \frac{g^2}{M_W^2}$
$\mathcal{L}_W = G_F (\bar{f} \gamma^\mu \Gamma f)(\bar{f} \gamma_\mu \Gamma f)$
$\mu \to e \nu \bar{\nu}$

Example Quantum Electrodynamics.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

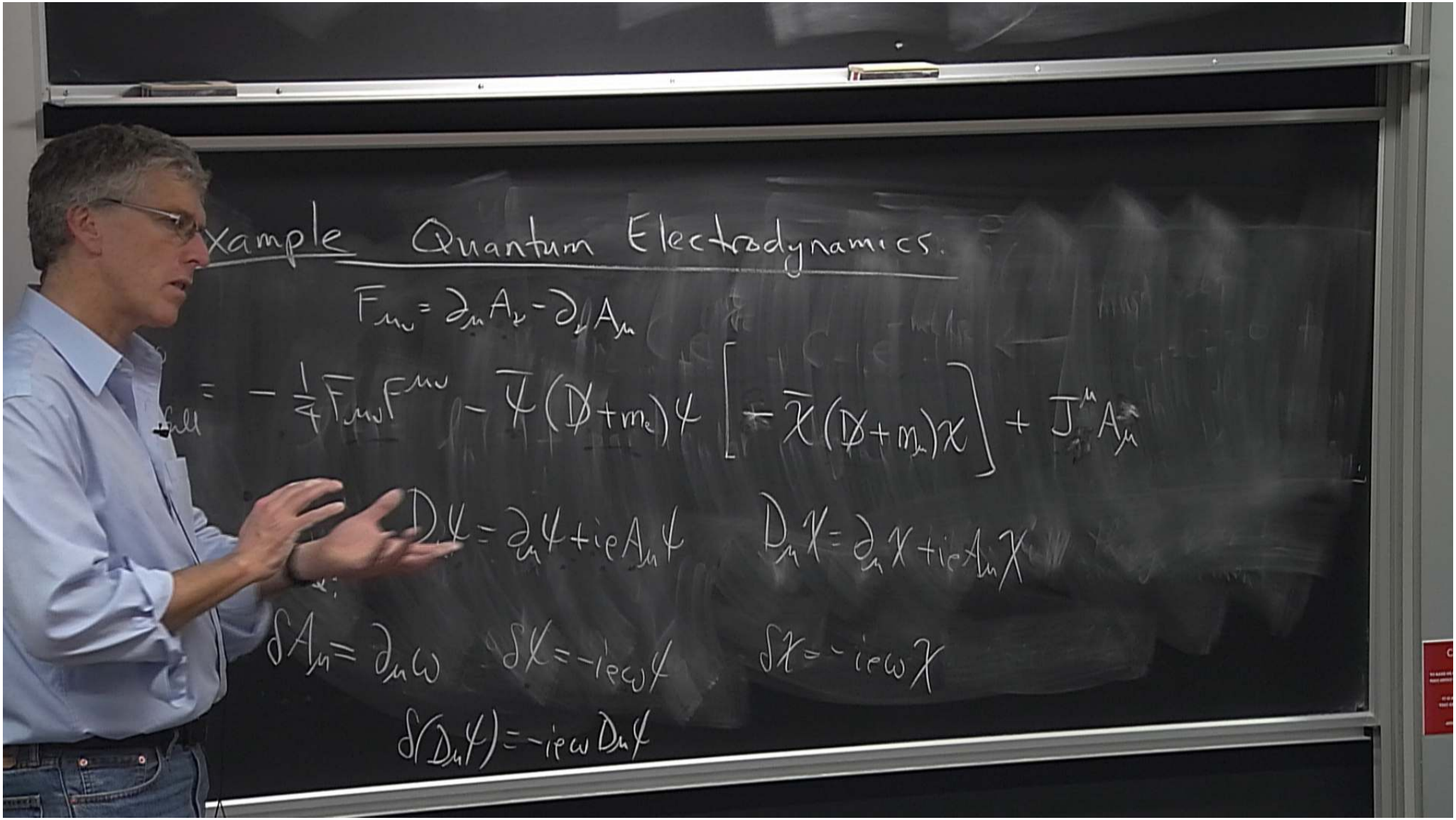
$$\mathcal{L}_{full} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{D} + m_e)\psi \left[- \bar{\chi}(\not{D} + m_\mu)\chi \right] + J^\mu A_\mu$$

$$D_\mu \psi = \partial_\mu \psi + ie A_\mu \psi \qquad D_\mu \chi = \partial_\mu \chi + ie A_\mu \chi$$

gauge invariance:

$$\delta A_\mu = \partial_\mu \omega \qquad \delta \psi = -ie\omega\psi \qquad \delta \chi = -ie\omega\chi$$

$$\delta(D_\mu \psi) = -ie\omega D_\mu \psi$$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$D_\mu \psi = \partial_\mu \psi + ieA_\mu \psi$
$D_\mu \chi = \partial_\mu \chi + ieA_\mu \chi$
$\delta A_\mu = \partial_\mu \omega$
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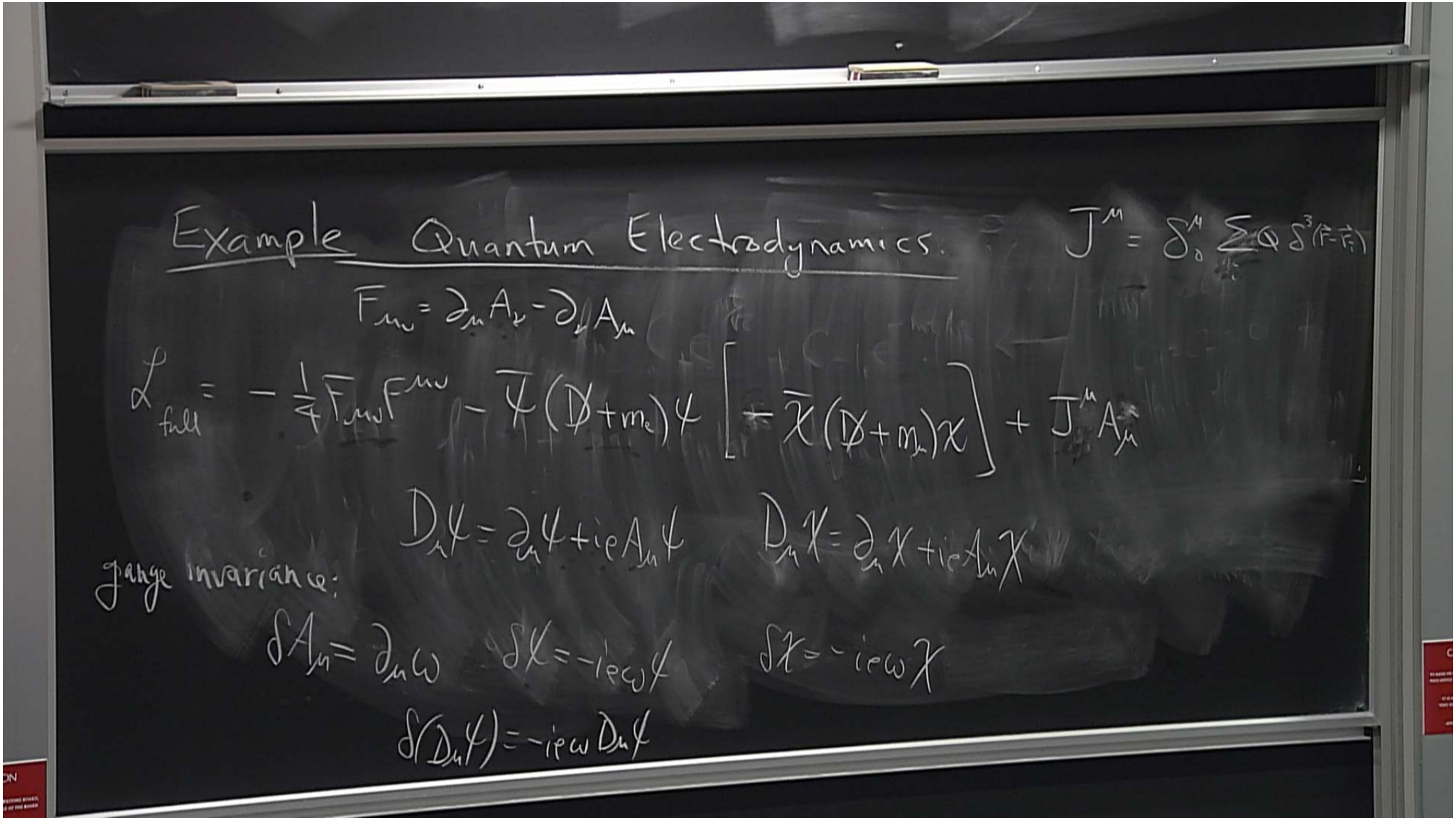
Example Quantum Electrodynamics.
$J^\mu = \delta^\mu_0 \sum_i Q \delta^3(\vec{r}-\vec{r}_i)$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\mathcal{L}_{full} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{D}+m_e)\psi \left[- \bar{\chi}(\not{D}+m_\mu)\chi \right] + J^\mu A_\mu$
$D_\mu \psi = \partial_\mu \psi + ieA_\mu \psi$
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gauge invariance:
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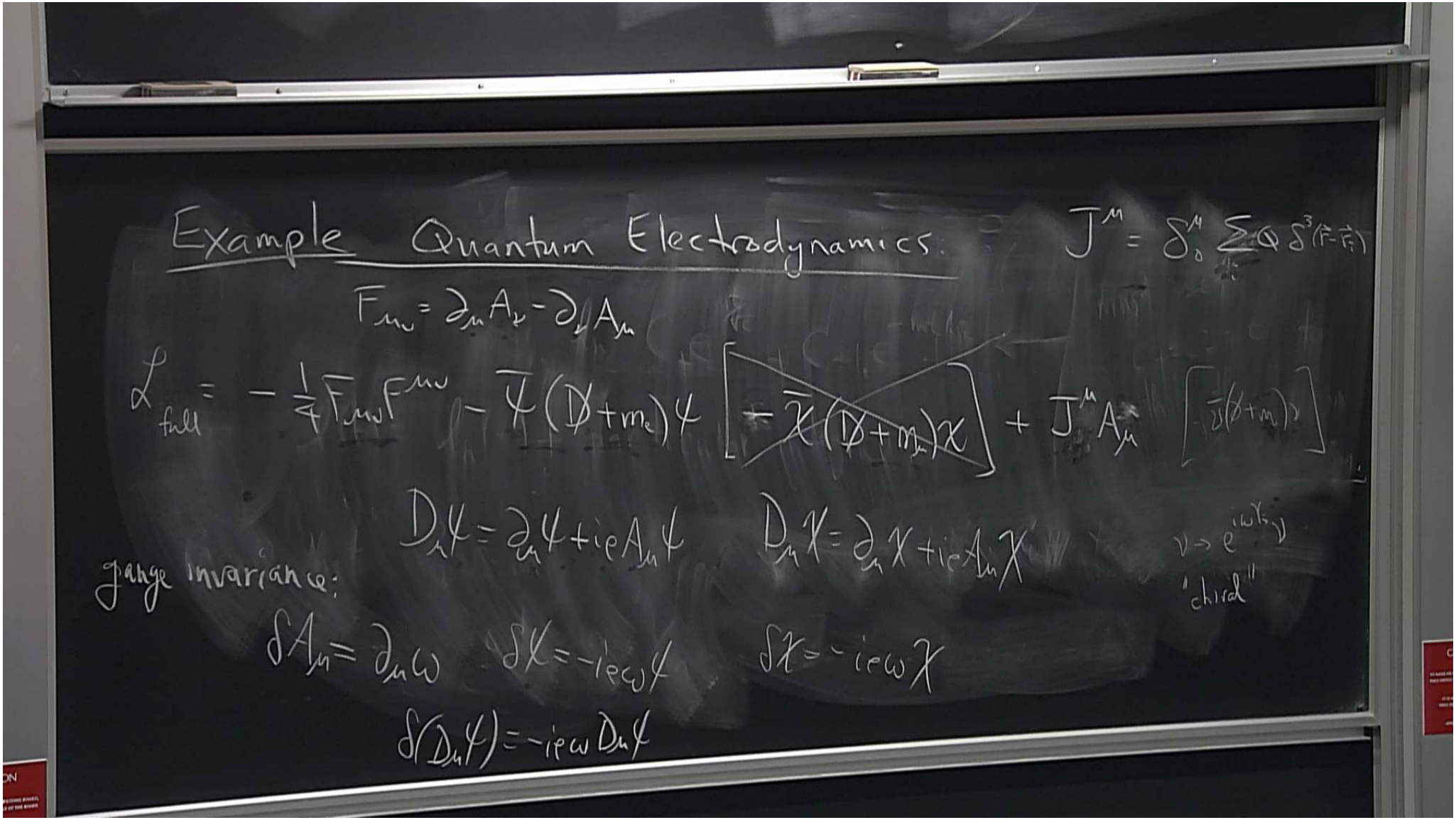
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$\mathcal{L}_{full} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{D}+m_e)\psi$ $-\bar{\chi}(\not{D}+m_e)\chi$ $+ J^\mu A_\mu$ $-\bar{\nu}(\not{D}+m_\nu)\nu$
$D_\mu\psi = \partial_\mu\psi + ieA_\mu\psi$ $D_\mu\chi = \partial_\mu\chi + ieA_\mu\chi$
$\nu \to e^{i\omega\gamma_5}\nu$
"chiral"
gauge invariance:
$\delta A_\mu = \partial_\mu\omega$ $\delta\psi = -ie\omega\psi$ $\delta\chi = -ie\omega\chi$
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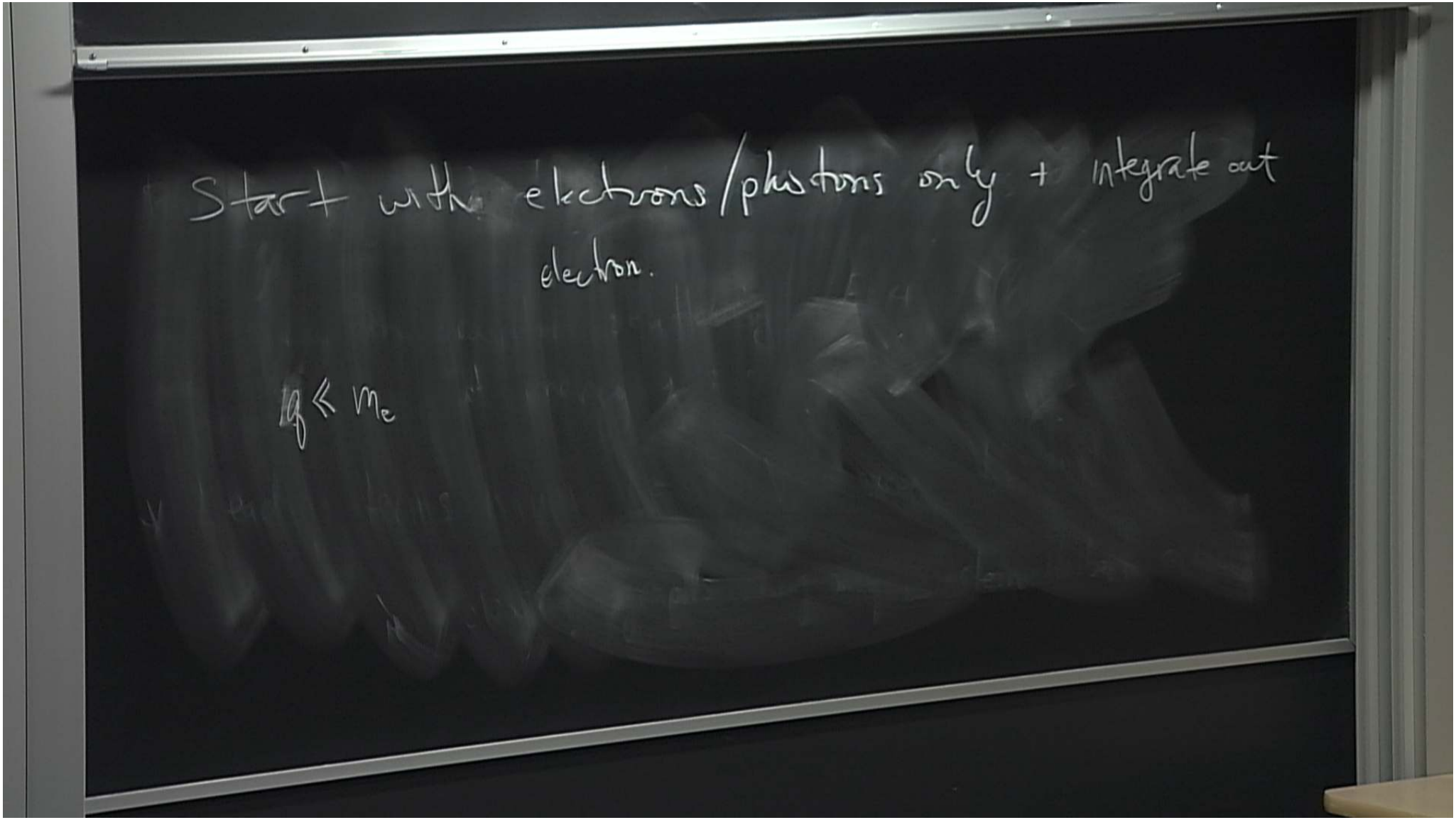
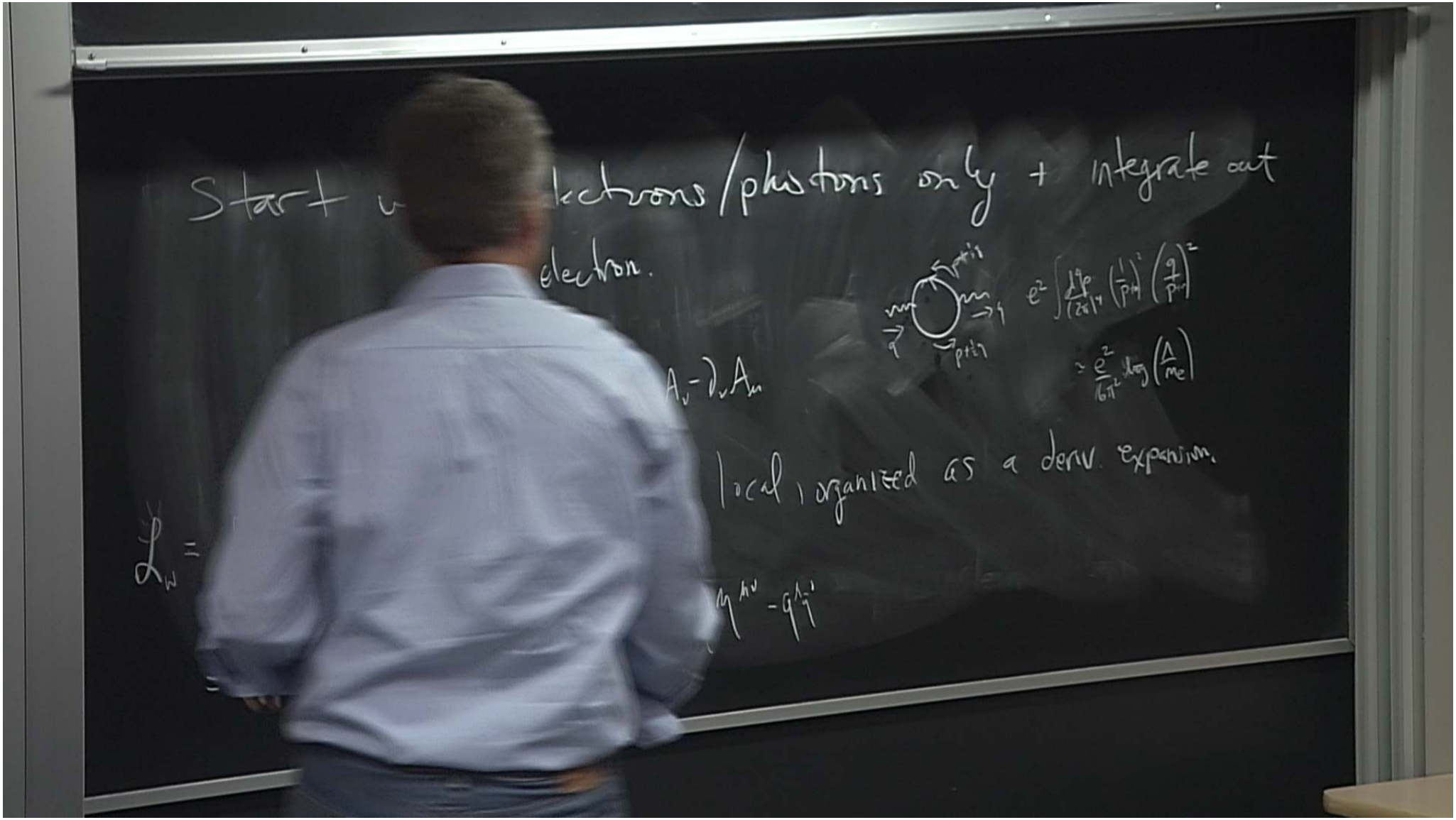

Start with electrons/photons only + integrate out
electron.
$q \ll m_e$


Start u
lectrons/photons only + integrate out
electron.
$A_\nu - \partial_\nu A_m$
local, organized as a deriv. expansion.
$\mathcal{L}_W =$
$\eta^{\mu\nu} - q^\mu q^\nu$
$e^2 \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{p+q}\right)^2 \left(\frac{q}{p+q}\right)^2$
$\simeq \frac{e^2}{16\pi^2} \log\left(\frac{\Lambda}{m_e}\right)$

Start with electrons/photons only + integrate out electron.

$q \ll m_e$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$\mathcal{L}_w$ = function of A_μ, $\delta A_\mu = \partial_\mu \omega$, local

$= -\frac{Z}{4} F_{\mu\nu} F^{\mu\nu} + \ldots$

$e^2 \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{p^2}\right)^2 \left(\frac{q}{p}\right)^2$

$\approx \frac{e^2}{16\pi^2} \log\left(\frac{\Lambda}{m_e}\right)$

$Z = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\epsilon} + k + \log\left(\frac{m_e^2}{\mu^2}\right)\right]$

$n = 4 - 2\epsilon$ $k = \gamma - \log 4\pi$ Euler

$\gamma = 0.577215\ldots$ Mascheroni

Aside on dimensional analysis in dim reg:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{\partial} + ie\not{A})\psi + \dots$$

$$\not{A} = \gamma^\mu A_\mu \qquad \not{\partial} = \gamma^\mu \partial_\mu \qquad \bar{\psi} = i\psi^\dagger \gamma^0$$

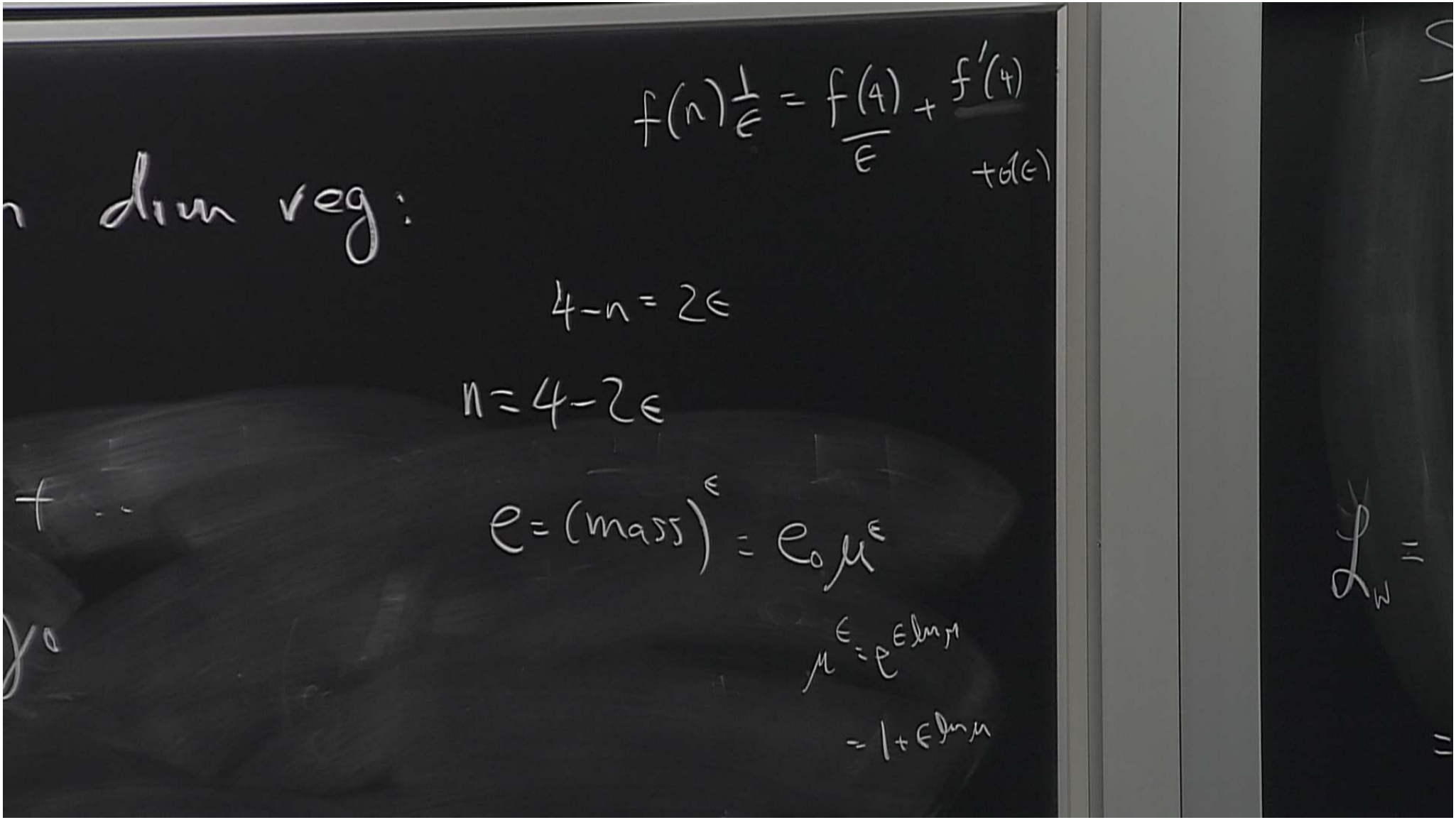
$$S = \int d^n x \, \mathcal{L} \qquad [\mathcal{L}] = n \qquad [\partial] = 1 \qquad [A_\mu] = \frac{n-2}{2} \qquad [\psi] = \frac{n-1}{2}$$

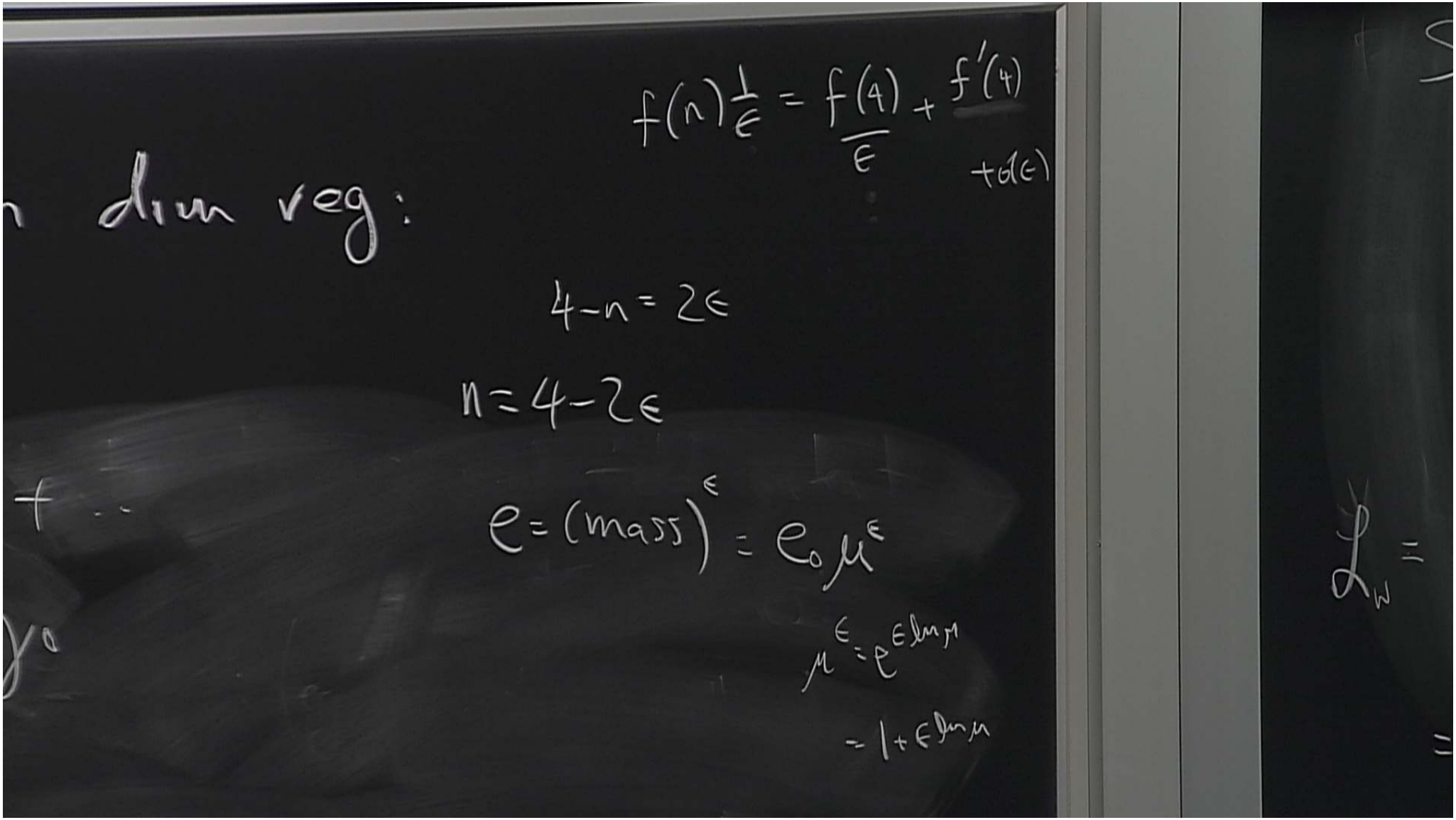
$$[e] = n - \left(\frac{n-2}{2}\right) - (n-1) = \frac{4-n}{2}$$

$$4 - n = 2\epsilon$$

$$n = 4 - 2\epsilon$$

$$e = (\text{mass})^\epsilon = e_0 \mu^\epsilon$$


$f(n)\frac{1}{\epsilon} = \frac{f(4)}{\epsilon} + f'(4) + O(\epsilon)$
dim reg:
$4-n = 2\epsilon$
$n = 4-2\epsilon$
$e = (\text{mass})^{\epsilon} = e_0 \mu^{\epsilon}$
$\mu^{\epsilon} = e^{\epsilon \ln \mu}$
$= 1 + \epsilon \ln \mu$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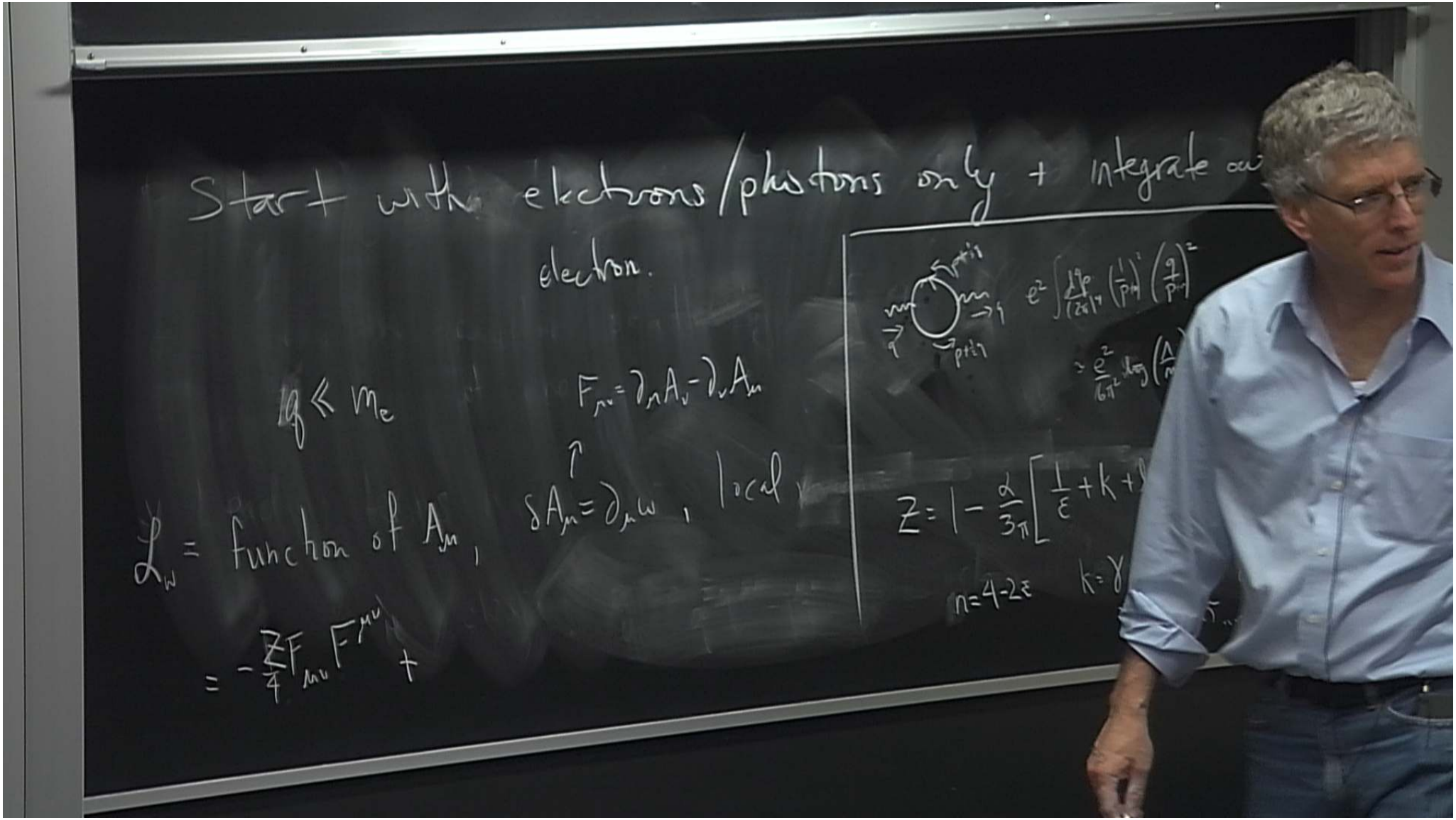
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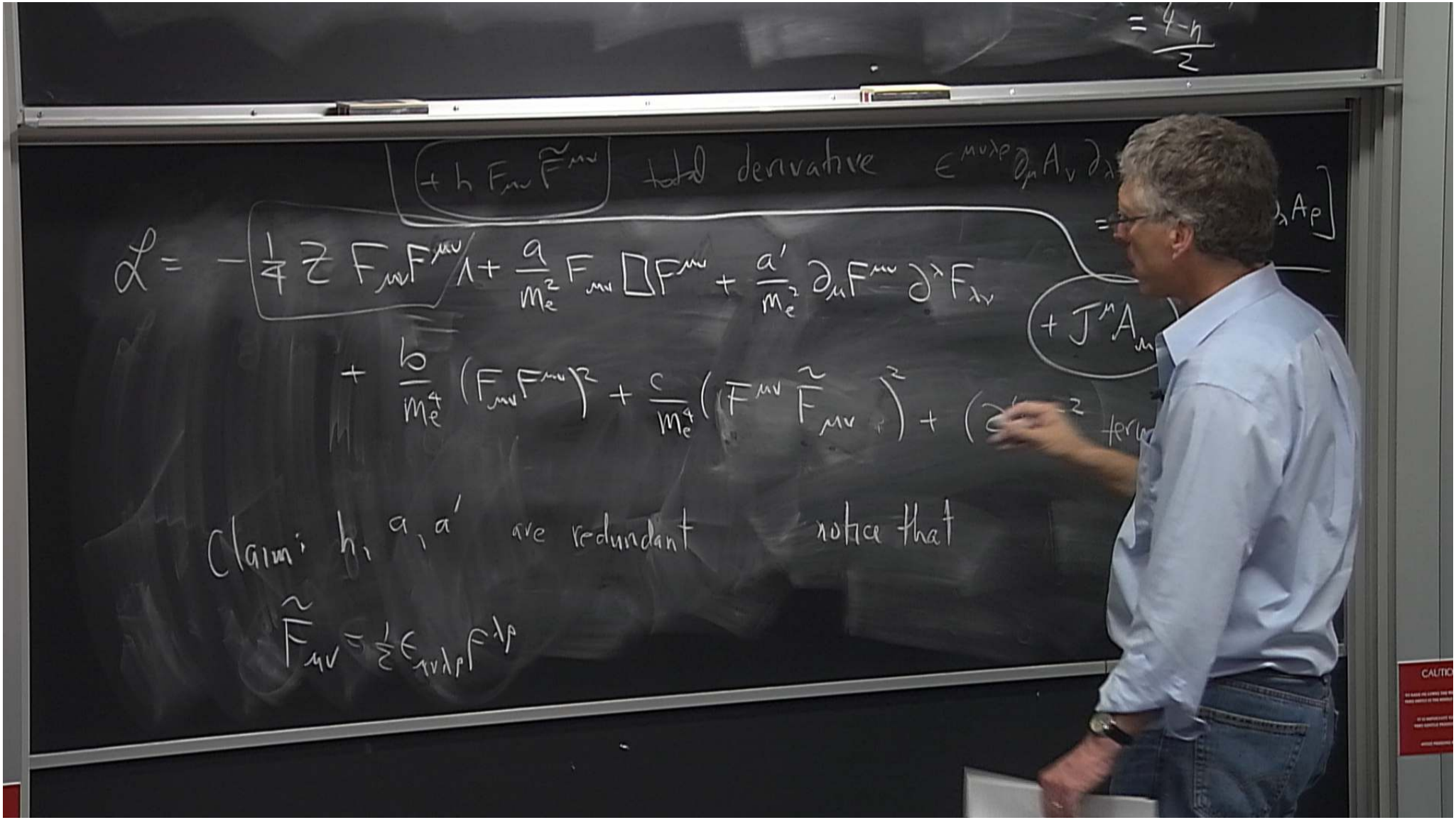
$\mathcal{L}_w$ = function of A_μ, $\delta A_\mu = \partial_\mu \omega$, local
$= -\frac{Z}{4} F_{\mu\nu} F^{\mu\nu} +$
$Z = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\epsilon} + k + \right.$
$n = 4 - 2\epsilon$
$k = \gamma$

$$= \frac{4-\eta}{2}$$

$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu} F^{\mu\nu} + \frac{a}{m_e^2} F_{\mu\nu} \Box F^{\mu\nu} + \frac{a'}{m_e^2} \partial_\mu F^{\mu\nu} \partial^\lambda F_{\lambda\nu}$$

$$+ \frac{b}{m_e^4} \left(F_{\mu\nu} F^{\mu\nu} \right)^2 + \frac{c}{m_e^4} \left(\epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} \right)^2$$


$= \frac{4-\eta}{2}$
$+ h F_{\mu\nu}\tilde{F}^{\mu\nu}$ total derivative $\epsilon^{\mu\nu\rho}\partial_\mu A_\nu \partial_\lambda$
$= \ldots \partial_\lambda A_\rho]$
$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu}\Lambda + \frac{a}{m_e^2} F_{\mu\nu}\Box F^{\mu\nu} + \frac{a'}{m_e^2}\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu}$
$+ J^\mu A_\mu$
$+ \frac{b}{m_e^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{c}{m_e^4}(F^{\mu\nu}\tilde{F}_{\mu\nu})^2 + (\ldots)^2$ terms
Claim: h, a, a' are redundant notice that
$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$

$= \frac{4-\eta}{2}$

$+ h F_{\mu\nu}\tilde{F}^{\mu\nu}$ total derivative $\epsilon^{\mu\nu\lambda\rho}\partial_\mu A_\nu \partial_\lambda A_\rho = \partial_\mu\left[\epsilon^{\mu\nu\lambda\rho} A_\nu \partial_\lambda A_\rho\right]$

$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu}\Lambda + \frac{a}{m_e^2} F_{\mu\nu}\Box F^{\mu\nu} + \frac{a'}{m_e^2}\underbrace{\partial_\mu F^{\mu\nu}}_{\frac{a'}{Zm_e^2} J^\nu} \partial^\lambda F_{\lambda\nu} + J^\mu A_\mu$$

$$+ \frac{b}{m_e^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{c}{m_e^4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2 + \left(\partial^4 F^2 \text{ terms} + \ldots\right)$$

Claim: h, a, a' are redundant — notice that zeroth order e.o.m is

$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$

$-Z\partial_\mu F^{\mu\nu} + J^\nu = 0$

$= \frac{4-\eta}{2}$

$+ h F_{\mu\nu}\tilde{F}^{\mu\nu}$ total derivative $\epsilon^{\mu\nu\rho\lambda}\partial_\mu A_\nu \partial_\lambda A_\rho = \partial_\mu\left[\epsilon^{\mu\nu\rho\lambda} A_\nu \partial_\lambda A_\rho\right]$

$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu}\Lambda + \frac{a}{m_e^2} F_{\mu\nu}\Box F^{\mu\nu} + \frac{a'}{m_e^2}\underbrace{\partial_\mu F^{\mu\nu}}_{\frac{a'}{Zm_e^2} J^\nu} \partial^\lambda F_{\lambda\nu} + J^\mu A_\mu$$

$$+ \frac{b}{m_e^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{c}{m_e^4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2 + \left(\partial^4 F^2 \text{ terms} + \ldots\right)$$

Claim: h, a, a' are redundant

notice that zeroth order e.o.m is

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$$

$$-Z\partial_\mu F^{\mu\nu} + J^\nu = 0$$

$+h F_{\mu\nu}\tilde{F}^{\mu\nu}$ total derivative $\epsilon^{\mu\nu\lambda\rho}\partial_\mu A_\nu \partial_\lambda A_\rho = \partial_\mu\left[\epsilon^{\mu\nu\lambda\rho} A_\nu \partial_\lambda A_\rho\right]$

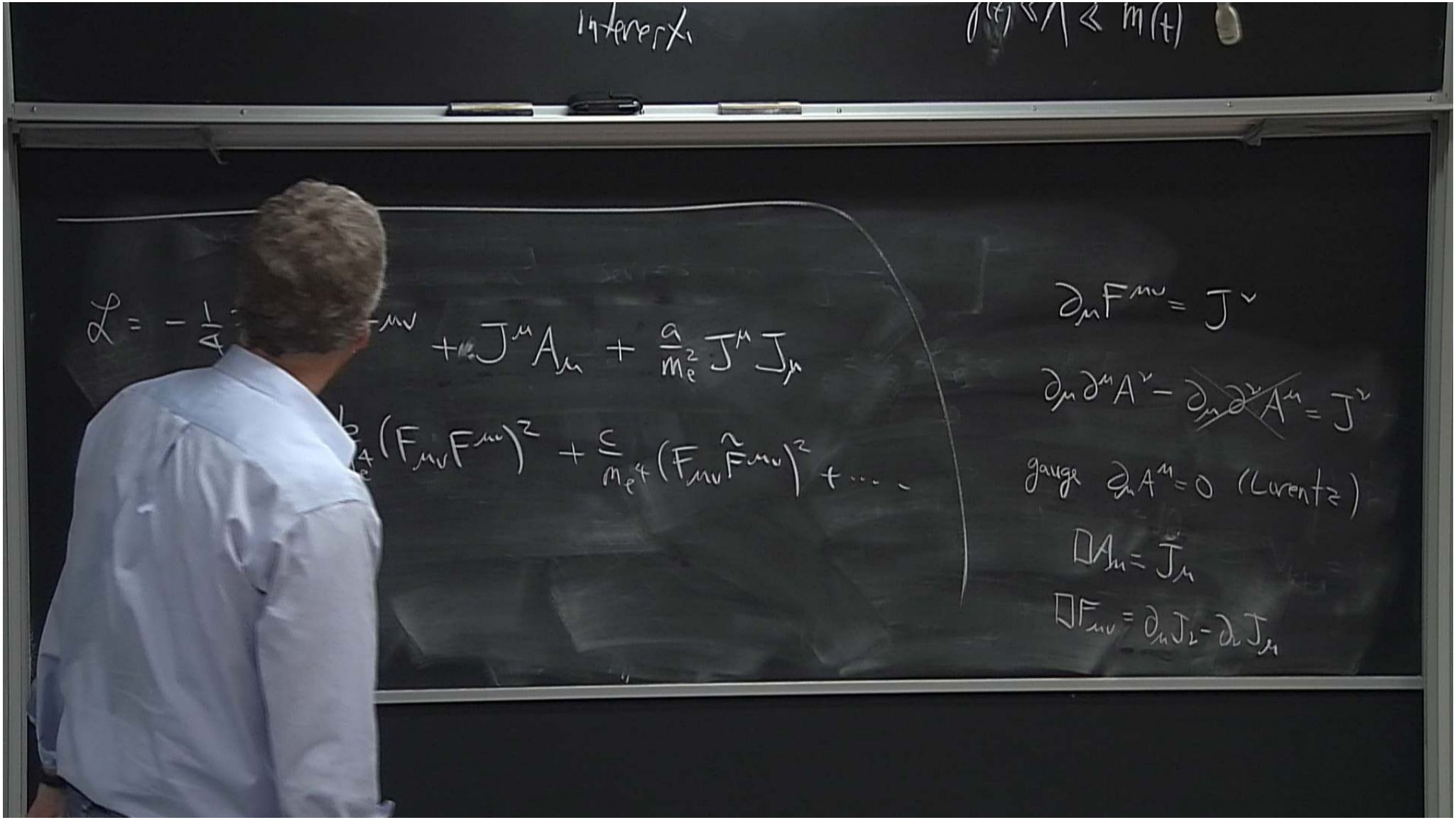
$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu}\Big(1 + \frac{a}{m_e^2} F_{\mu\nu}\underbrace{\Box F^{\mu\nu}}_{\partial J^\nu - \partial^\nu J^\mu} + \frac{a'}{m_e^2}\underbrace{\partial_\mu F^{\mu\nu}}_{\frac{a'}{2m_e^2} J^\nu} \partial^\lambda F_{\lambda\nu} + \frac{b}{m_e^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{c}{m_e^4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2 + \left(\partial^4 F^2 \text{ terms} + \dots\right)\Big) + J^\mu A_\mu$$

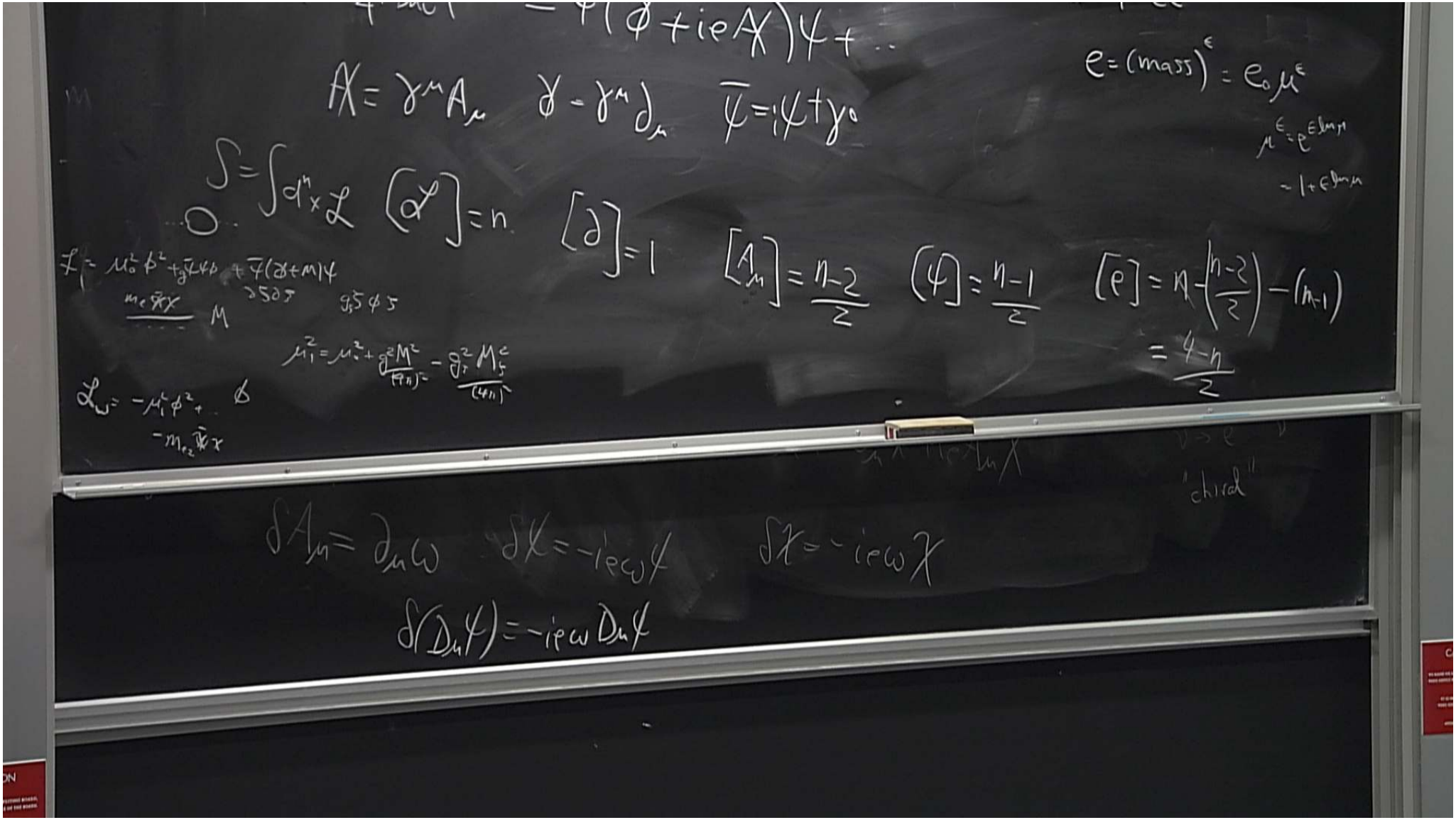
Claim: h, a, a' are redundant

notice that zeroeth order e.o.m is

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$$

$$-Z\partial_\mu F^{\mu\nu} + J^\nu = 0$$


interesting
$\mathcal{L} = -\frac{1}{4}$
$+ J^\mu A_\mu + \frac{e}{m_e^2} J^\mu J_\mu$
$(F_{\mu\nu}F^{\mu\nu})^2 + \frac{c}{m_e^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \dots$
$\partial_\mu F^{\mu\nu} = J^\nu$
$\partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu = J^\nu$
gauge $\partial_\mu A^\mu = 0$ (Lorentz)
$\Box A_\mu = J_\mu$
$\Box F_{\mu\nu} = \partial_\mu J_\nu - \partial_\nu J_\mu$


$= \bar{\psi}(\partial + ieA)\psi + \ldots$
$A = \gamma^\mu A_\mu \quad \partial = \gamma^\mu \partial_\mu \quad \bar{\psi} = i\psi^\dagger \gamma^0$
$e = (\text{mass})^\epsilon = e_0 \mu^\epsilon$
$\mu^\epsilon = e^{\epsilon \ln \mu} = 1 + \epsilon \ln \mu$
$S = \int d^n x \, \mathcal{L} \quad [x] = n \quad [\partial] = 1 \quad [A_\mu] = \frac{n-2}{2} \quad [\psi] = \frac{n-1}{2} \quad [e] = n - \left(\frac{n-2}{2}\right) - (n-1) = \frac{4-n}{2}$
$\mathcal{L} = \mu_0^2 \phi^2 + g \bar{\chi} \gamma^5 \phi \chi + \bar{\chi}(\partial + m)\chi$
$\mu_1^2 = \mu^2 + \frac{g^2 M^2}{(4\pi)^2} - \frac{g^2 M_\chi^2}{(4\pi)^2}$
$\mathcal{L}_\omega = -\mu_1^2 \phi^2 + \ldots$
$-m_{e_2} \bar{\chi} \chi$
"chiral"
$\delta A_\mu = \partial_\mu \omega \quad \delta \psi = -ie\omega\psi \quad \delta \chi = -ie\omega\chi$
$\delta(D_\mu \psi) = -ie\omega D_\mu \psi$