

Title: Introduction to Effective Field Theories - Lecture 10

Date: Feb 07, 2014 02:30 PM

URL: <http://pirsa.org/14020092>

Abstract:

$$\mathcal{L}_w = -f^4 \sum c_i \mathcal{O}_i \left(\frac{\partial \phi}{\partial x}, \frac{\phi}{x} \right)$$

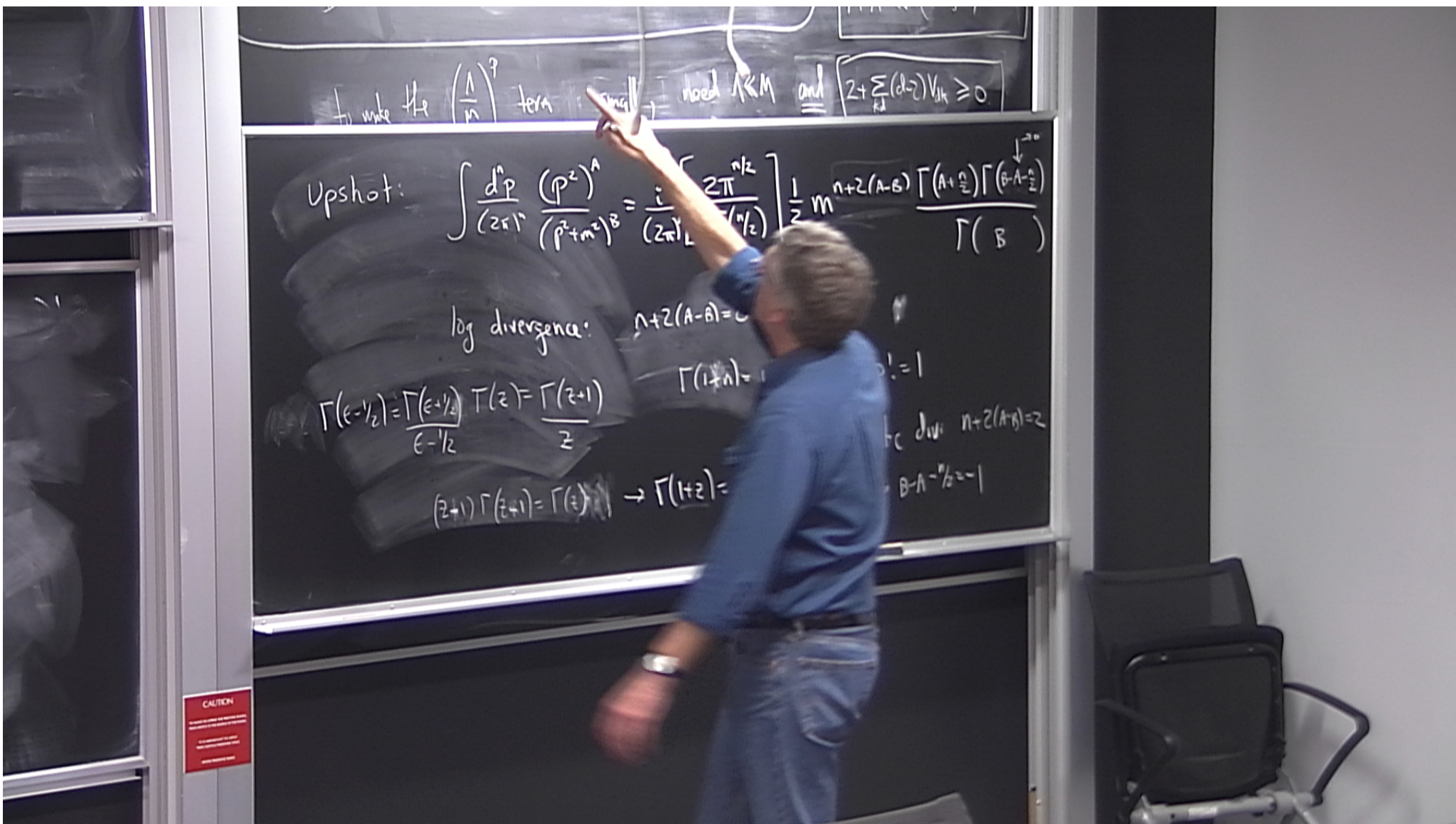
$$A_E(q)$$

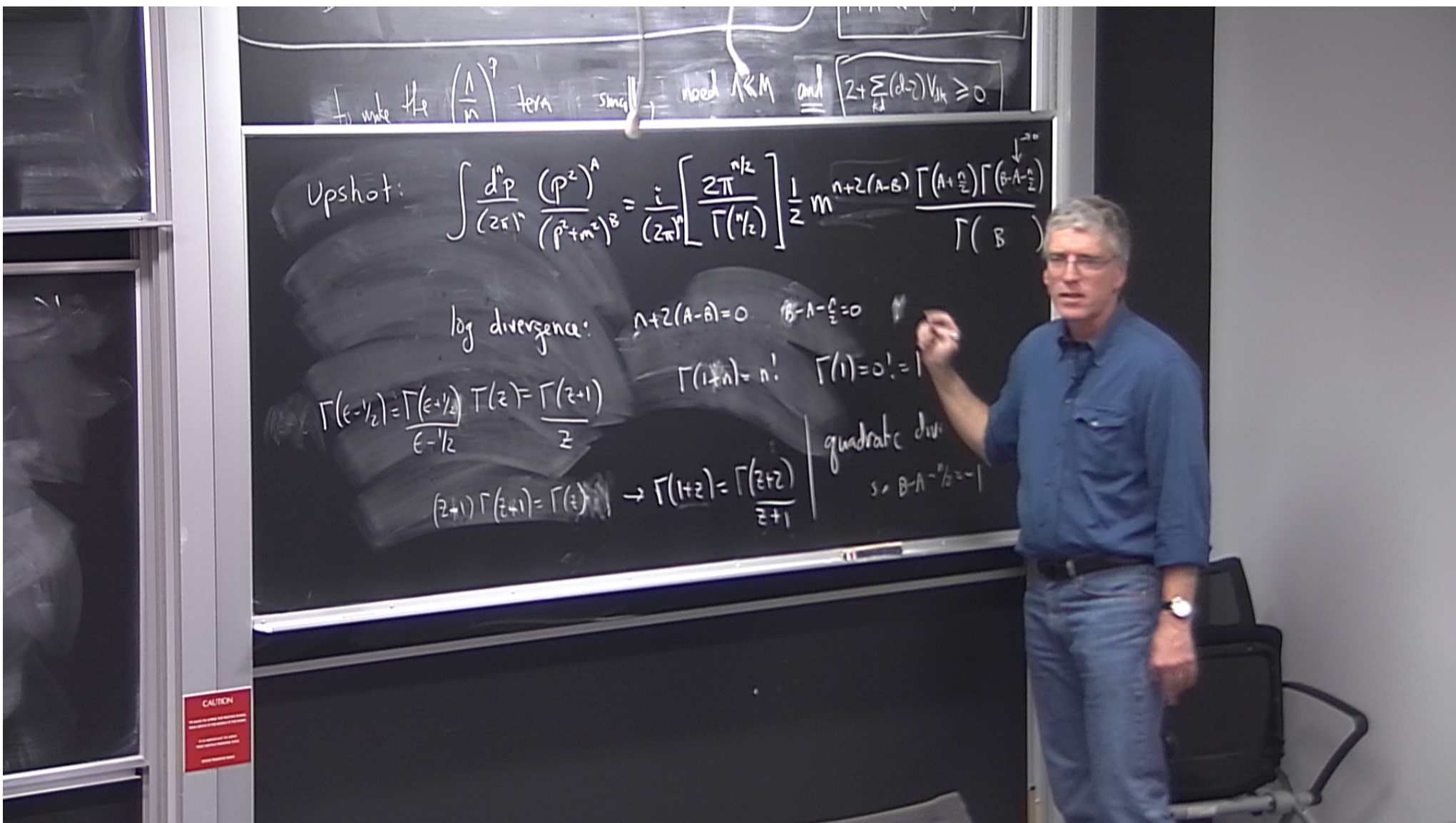


+ lines
+ "

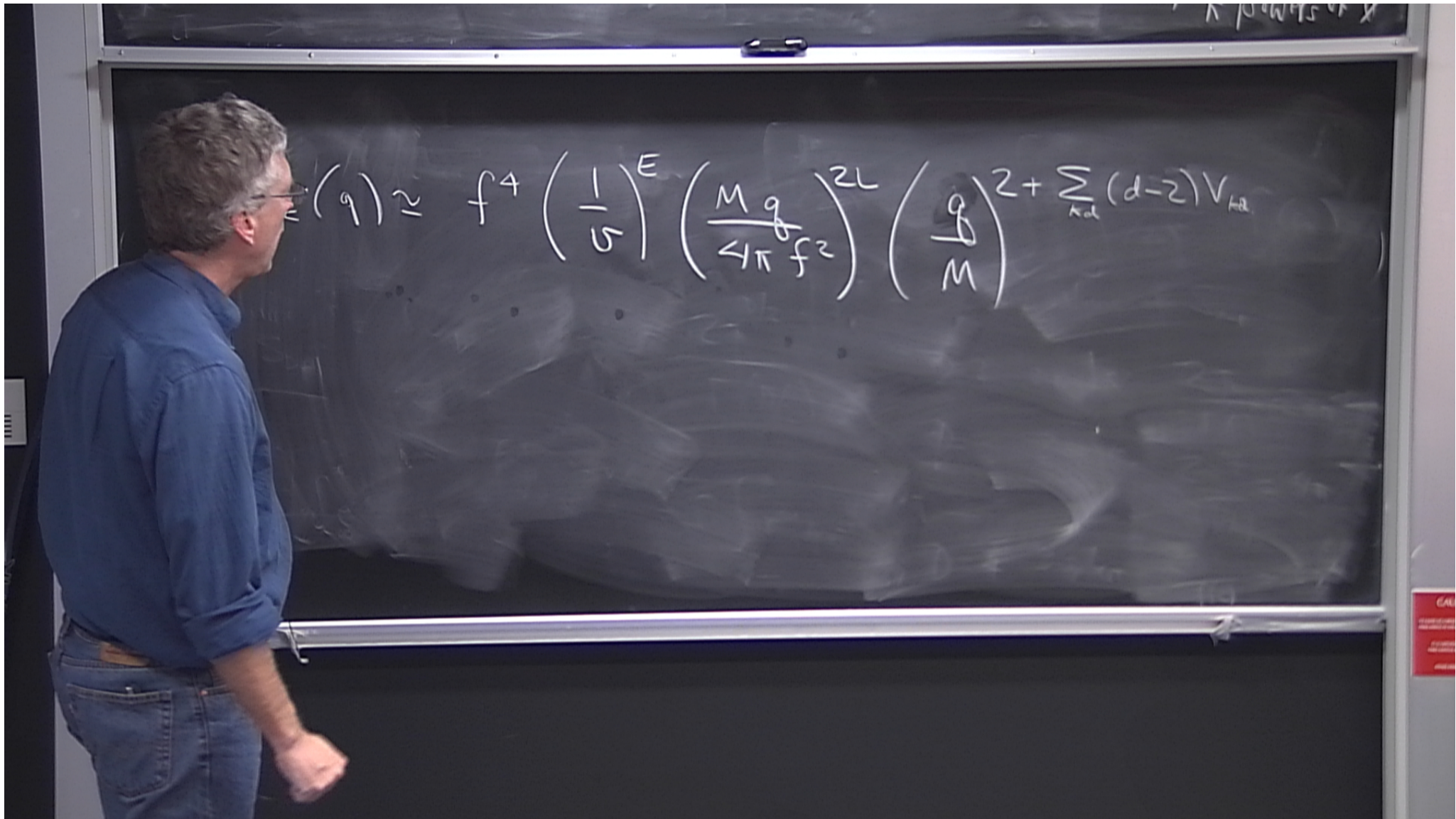
Vdf

W





$$A_E(q) \approx f^4 \left(\frac{1}{v} \right)^E \left(\frac{M q}{4\pi f^2} \right)^{2L} \left(\frac{q}{M} \right)^{2 + \sum_{\text{rel}} (d_i - 1)}$$



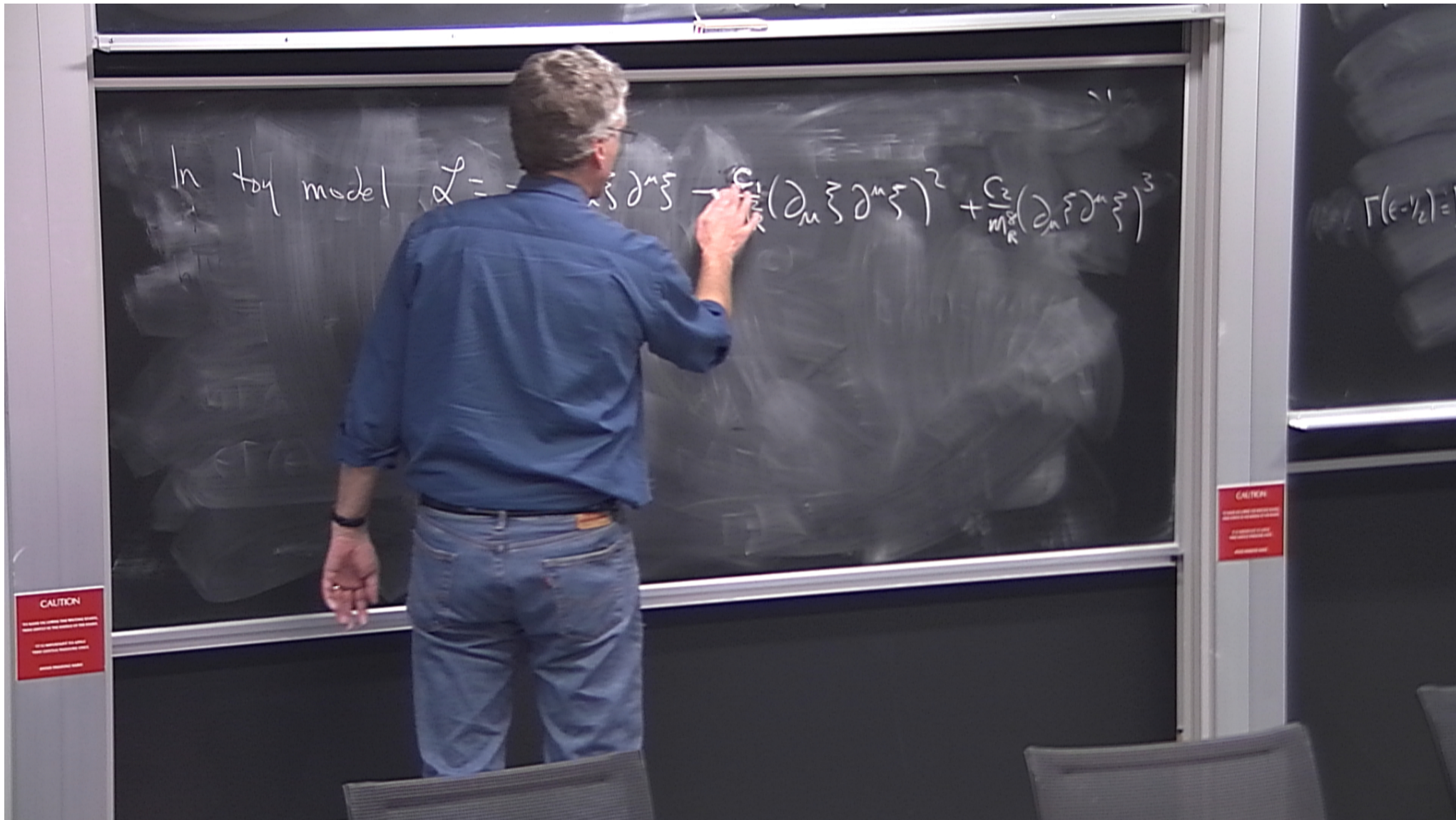
$$\epsilon(q) \approx f^4 \left(\frac{1}{U} \right)^E \left(\frac{M g}{4\pi f^2} \right)^{2L} \left(\frac{g}{M} \right)^{2 + \sum_{n=1}^{\infty} (d-2) V_n}$$

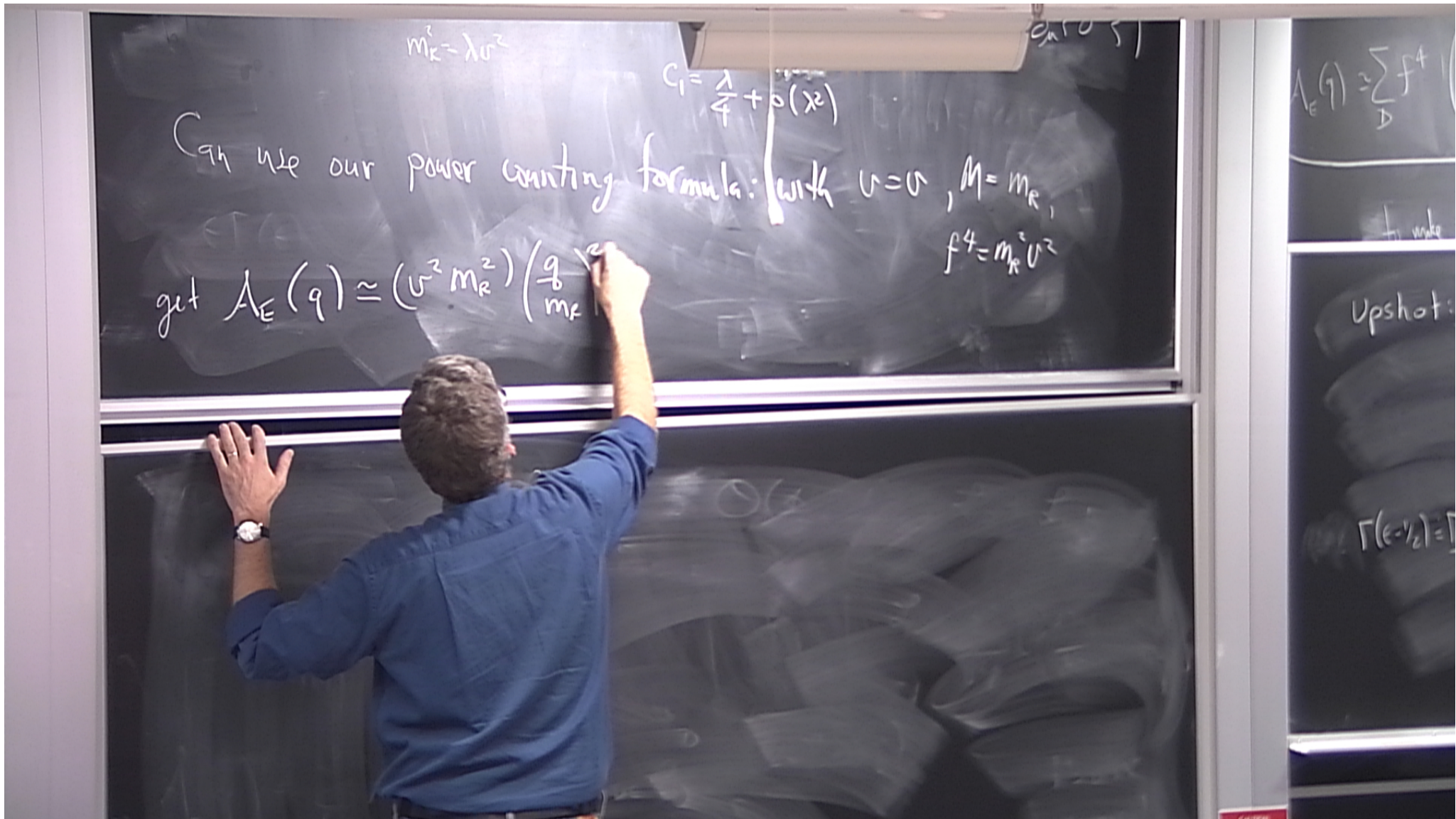
$$A_E(q) \approx f^4 \left(\frac{1}{v} \right)^E \left(\frac{Mg}{4\pi f^2} \right)^{2L} \left(\frac{q}{M} \right)^{2 + \sum_{k \neq 0} (d-2) V_{k,0}}$$

ask $Mg \ll 4\pi f^2$, $q \ll M$ + no $d=0$ interactions
(or sufficiently suppressed)

$$A_E(q) \approx f^4 \left(\frac{1}{v} \right)^E \left(\frac{M q}{4\pi f^2} \right)^{2L} \left(\frac{q}{M} \right)^{2 + \sum_{\alpha d} (d-2) V_{\alpha d}}$$

ask $Mq \ll 4\pi f^2$, $q \ll M$ + no $d=0$ interactions
(or sufficiently suppressed)





get $\Lambda_E(q) \simeq (q^2 v^2) \left(\frac{1}{v} \right) \left(\frac{1}{4\pi v} \right) \left(\frac{1}{m_R} \right)$

$\xi + \bar{\xi} \rightarrow \xi + \bar{\xi}$

(E

All interactions have $d \geq k$
and $k \geq 4$.

What are dominant L s?

Best: $L=0$

$V_{44} = 1$, all others
 $V_{dk} = 0$

$m_R^2 = \lambda v^2$

$\frac{q^4}{m_R^2 v^2} \simeq \frac{\lambda q^4}{m_R^4}$

$$m_K = \lambda v^2$$

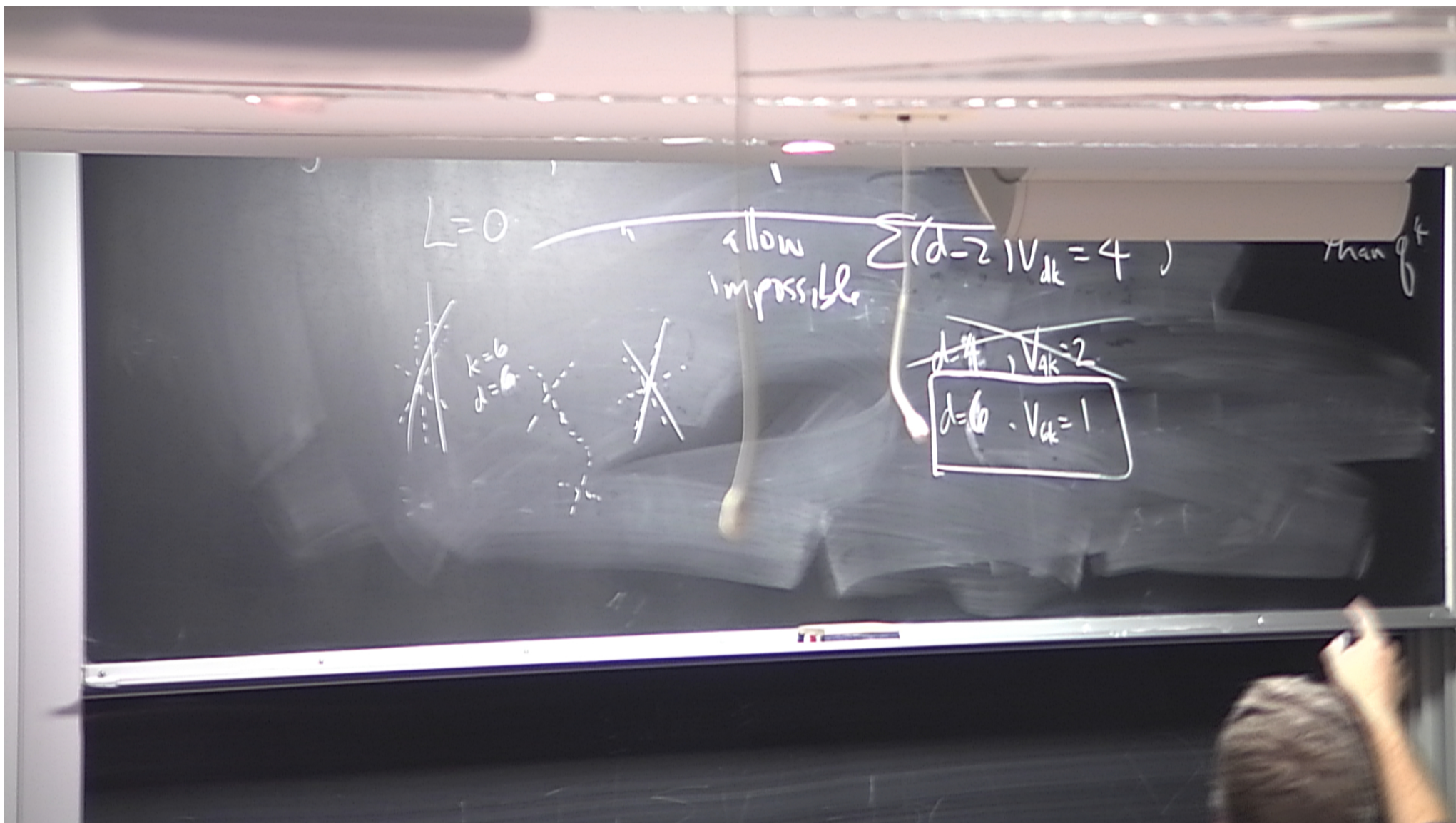
$$C_1 = \frac{\lambda}{4} + o(\lambda^2)$$

Can use our power counting formula: with $v = v$, $M = m_K$,

$$\text{get } \Lambda_E(q) \approx (q^2 v^2) \left(\frac{1}{v}\right)^E \left(\frac{q}{4\pi v}\right)^{2L} \left(\frac{q}{m_K}\right)^{\sum_{k \geq 2} (d-2) V_{dk}} \quad \left\{ \begin{array}{l} 4 = m_K^2 v^2 \\ q \ll 4\pi v \\ q \ll m_K \end{array} \right.$$

$\xi + \bar{\xi} \rightarrow \xi + \bar{\xi} \quad (E=4) \quad || \quad \text{All interactions have } d \geq k$
and $k \geq 4$.

What are dominant graphs? Best: $L=0$ $V_{44} = 1$, all others $V_{dk} = 0$.



Try $L=2$ $\sum (d-2)V_{dk} = 2$

$L=1$ $\sum (d-2)V_{dk} = 4$ ✖

$L=0$ $\sum (d-2)V_{dk} = 6$

$(\partial_\mu \xi \partial^\mu \xi) \square^2 (\partial^\mu \xi \partial_\mu \xi)$

Try $L=2$ $\sum (d-2)V_{dk} = 2$

$L=1$ $\sum (d-2)V_{dk} = 4$ ✖

$L=0$ $\sum (d-2)V_{dk} = 6$

$(\partial_\mu \xi \partial^\mu \xi) \square^2 (\partial^\mu \xi \partial_\mu \xi)$

Try

$$L=2 \quad \Sigma(d-2)V_{dk} = 2 \times$$

$$L=1 \quad \Sigma(d-2)V_{dk} = 4 \checkmark$$

$$L=0 \quad \Sigma(d-2)V_{dk} = 6 ?$$

$d=6 \quad k=6$

$$(\partial_\mu \xi \partial^\mu \xi) (\partial_\nu \xi \partial^\nu \xi)$$

$$(\partial_\mu \xi \partial^\mu \xi)$$

$$(\partial_\mu \xi \partial^\mu \xi) (\partial_\nu \xi \partial^\nu \xi)$$

is there a non redundant
 $d=8, k=4$ sp?

Try

$L=2$	$\Sigma(d-2)V_{dk} = 2 \times$
$L=1$	$\Sigma(d-2)V_{dk} = 4 \checkmark$
$L=0$	$\Sigma(d-2)V_{dk} = 6 ?$



$d=6 \quad k=6$

$(\partial_\mu \xi \partial^\mu \xi)(\partial_\nu \xi \partial^\nu \xi)$
 $(\partial_\lambda \xi \partial^\lambda \xi)$



$c_2 (\partial_\mu \xi \partial^\mu \xi) (\partial_\nu \xi \partial^\nu \xi)$
 $c_3 (\partial_\mu \xi \partial^\mu \xi)$

$d=8, k=4$
 $\lambda \frac{g^8}{(4\pi)^2 m_R^8}$

$A_4^{NLO}(q) \approx (q^2 v^2) \left(\frac{1}{v}\right)^4 \left(\frac{g}{4\pi v}\right)^2 \left(\frac{g}{m_R}\right)^4$
 $\sim \frac{g^8}{(4\pi)^2 v^4 m_R^4} \sim \frac{\lambda^2 g^8}{(4\pi)^2 m_R^8}$

or $c_2 q^2 v^2 \left(\frac{1}{v}\right)^4 \left(\frac{g}{m_R}\right)^6 \sim \frac{c_2 g^8}{v^2 m_R^6} = \frac{\lambda c_2 g^8}{m_R^8} \sim \frac{\lambda}{m_R^2}$

Try $L=2$ $\Sigma(d-2)V_{dk} = 2 \times$

$L=1$ $\Sigma(d-2)V_{dk} = 4 \checkmark$

$L=0$ $\Sigma(d-2)V_{dk} = 6 ?$



$d=6 \quad k=6$

$(\partial_\mu \xi \partial^\mu \xi)(\partial_\nu \xi \partial^\nu \xi)$
 $(\partial_\lambda \xi \partial^\lambda \xi)$

$c_2 (\partial_\mu \xi \partial^\mu \xi) (\partial_\nu \xi \partial^\nu \xi)$

is there a non redundant
 $d=8, k=4$ sp?



$A_4^{NLO}(q) \approx (q^2 v^2) \left(\frac{1}{v}\right)^4 \left(\frac{g}{4\pi v}\right)^2 \left(\frac{g}{m_R}\right)^4$
 $\sim \frac{g^8}{(4\pi)^2 v^4 m_R^4} \sim \frac{\lambda^2 g^8}{(4\pi)^2 m_R^8}$

or $c_2 q^2 v^2 \left(\frac{1}{v}\right)^4 \left(\frac{g}{m_R}\right)^6 \sim \frac{c_2 g^8}{v^2 m_R^6} = \frac{\lambda c_2 g^8}{m_R^8} \sim \frac{\lambda}{(4\pi)^2}$

$$\alpha = f \sum_{d \neq 0} c_i \mathcal{O}_i \left(\frac{\phi}{v}, \frac{\partial \phi}{\partial v} \right) + f^4 \left(\frac{f_v^4}{f^4} \right) \sum_{d=0} c_i \mathcal{O}_i \left(\frac{\phi}{v} \right)$$

How small must f_v be to salvage dangerous $\left(\frac{g}{m} \right)^{\#}$ terms?

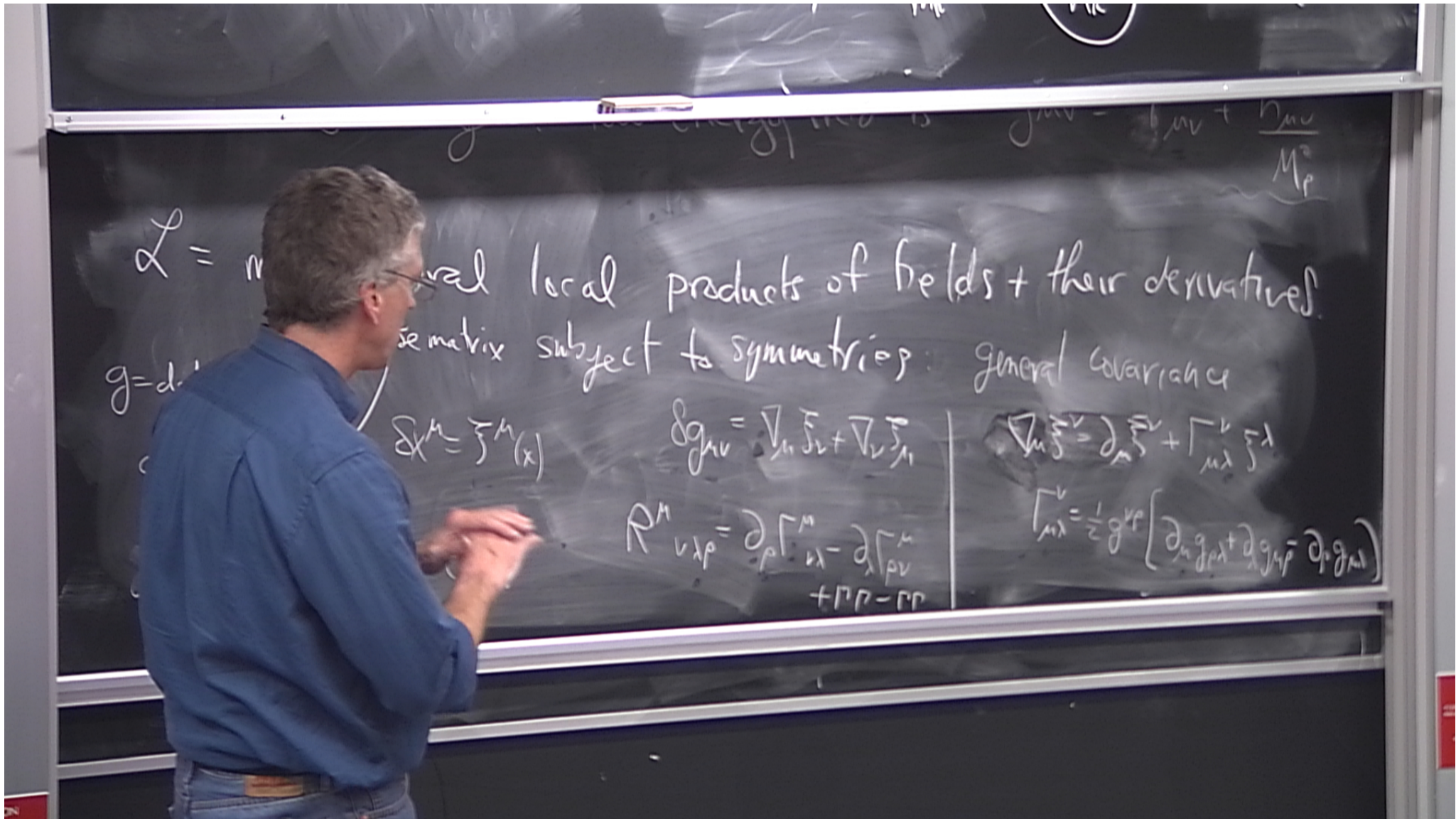
For gravity

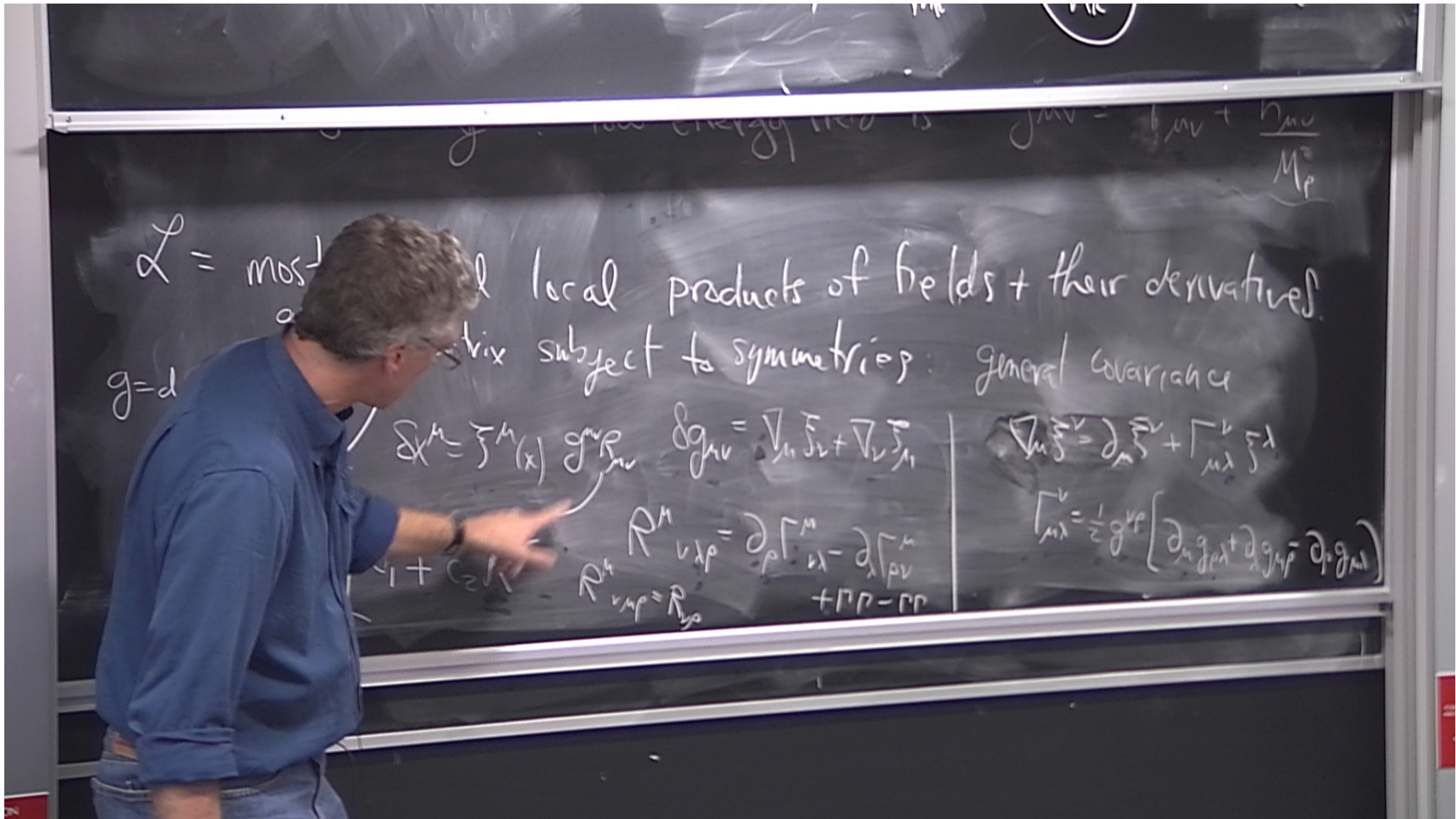
For gravity : low energy field is $g_{\mu\nu} \approx \bar{g}_{\mu\nu} + \frac{h_{\mu\nu}}{M_P^2}$

For gravity: low energy field is $g_{\mu\nu} \approx \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_p^2}$

general local products of fields + their derivatives
subject to symmetries: general covariance

$$\delta x^\mu = \xi^\mu(x) \quad \delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$





$\mathcal{L}$ = most general local products of fields + their derivatives
 $g^{\mu\nu}$ = inverse matrix subject to symmetries: general covariance
 $g = \det(g_{\mu\nu})$
 $g_{\mu\nu} = \frac{\partial x^\alpha}{\partial x^\mu} \frac{\partial x^\beta}{\partial x^\nu} g_{\alpha\beta}$ (4x4)
 $= \sqrt{-g} [c_1 + c_2 R]$
 $\delta x^\mu = \xi^\mu(x)$
 $\delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$
 $R^\mu_{\nu\lambda\rho} = \partial_\lambda \Gamma^\mu_{\nu\rho} - \partial_\rho \Gamma^\mu_{\nu\lambda} + \Gamma^\mu_{\lambda\rho} \Gamma^\rho_{\nu\lambda} - \Gamma^\mu_{\nu\rho} \Gamma^\rho_{\lambda\lambda}$
 $R^\mu_{\nu\lambda\rho} = R^\mu_{\rho\lambda\nu}$
 $\nabla_\mu \xi^\nu = \partial_\mu \xi^\nu + \Gamma^\nu_{\mu\lambda} \xi^\lambda$
 $\Gamma^\nu_{\mu\lambda} = \frac{1}{2} g^{\nu\rho} (\partial_\mu g_{\rho\lambda} + \partial_\lambda g_{\mu\rho} - \partial_\rho g_{\mu\lambda})$

$$\mathcal{L} = \sqrt{-g}$$

$d=0$

$d=2$

$d=4$

$$1^2 R + c_3 R^2 + c_4 R_{\mu\nu} R^{\mu\nu} + c_5 R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}$$

$$+ c_6 \square R + \frac{1}{M^2} R^3$$

$$R_{\mu\nu\lambda\rho} = g_{\mu\sigma} R^{\sigma}_{\nu\lambda\rho}$$

constant: $(10^{-3} \text{ eV})^4$ $d=6$

$$M = M_P \quad M_P^2 = (8\pi G_N)^{-1}$$

$$\mathcal{L} = \sqrt{-g} \left[M^4 + M^2 R + \underbrace{c_3 R^2 + c_4 R_{\mu\nu} R^{\mu\nu} + c_5 R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}}_{d=4} + \underbrace{c_6 \square R + \frac{1}{M^2} R^3}_{d=6} + \dots \right]$$

Cosmological constant: $(10^{-3} \text{ eV})^4$

→ Einstein-Hilbert $M = M_P$ $M_P^2 = (8\pi G_N)^{-1}$

$$\mathcal{L} = \sqrt{-g} \left[M^4 + M^2 R + c_3 R^2 + c_4 R_{\mu\nu} R^{\mu\nu} + c_5 R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} \right]$$

$d=0$ $d=2$ $d=6$

$+ c_6 \square R + \frac{1}{M^2} R^3 + \dots$

Cosmological constant: $(10^{-3} \text{ eV})^4$

$\rightarrow$ Einstein-Hilbert $M = M_P$ $M_P^2 = (8\pi G_N)^{-1}$

also $\frac{1}{M_P^2}$

$$\mathcal{L} = \sqrt{-g} \left[M^4 + M^2 R + \underbrace{c_3 R^2 + c_4 R_{\mu\nu} R^{\mu\nu} + c_5 R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}}_{d=4} + \underbrace{c_6 \square R + \frac{1}{M^2} R^3}_{d=6} + \dots \right]$$

$$R_{\mu\nu\lambda\rho} = g_{\mu\sigma} R^{\sigma}_{\nu\lambda\rho}$$

Cosmological constant: $(10^{-3} \text{ eV})^4$

→ Einstein-Hilbert

$$M = M_P$$

$$M_P^2 = (8\pi G_N)^{-1}$$

$$\text{also } \frac{1}{m^2}$$

$$m \gg m_e?$$

$$G_N \sim \frac{1}{M_P^2}$$

$$\mathcal{L} = \sqrt{-g} \left[M^4 + M^2 R + \underbrace{c_3 R^2 + c_4 R_{\mu\nu} R^{\mu\nu}}_{d=2} + \underbrace{c_5 R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}}_{d=4} \right]$$

$d=0$ $d=2$ $d=4$

Cosmological constant: $(10^{-3} \text{ eV})^4$ $d=6$
 → Einstein-Hilbert $M = M_P$ $M_P^2 = (8\pi G_N)^{-1}$

$$\mathcal{L} = \sqrt{-g} \left[M^4 + M^2 R + \underbrace{c_3 R^2 + c_4 R_{\mu\nu} R^{\mu\nu}}_{d=2} + \underbrace{c_5 R}_{d=0} + \frac{1}{M^2} R^3 \right]$$

Cosmological constant: $(10^{-3} \text{ eV})^4$

→ Einstein-Hilbert $M = M_P$ $M_P^2 = \frac{1}{8\pi G}$

$$\mathcal{L} = \sqrt{-g} \left[M^4 + M^2 R + c_3 R^2 + c_4 R_{\mu\nu} R^{\mu\nu} + c_5 R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} \right]$$

$d=0$ $d=2$ $d=4$

$+ c_6 \left(\square R \right) + \left(\frac{1}{M^2} \right) R^3 - \left(\frac{M_p^2}{m} \right)^2$

$R_{\mu\nu\lambda\rho} = g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}$

Cosmological constant: $(10^{-3} \text{ eV})^4$ $d=6$

→ Einstein-Hilbert $M = M_p$ $M_p^2 = (8\pi G_N)^{-1}$

also
 $m \sim m_e?$