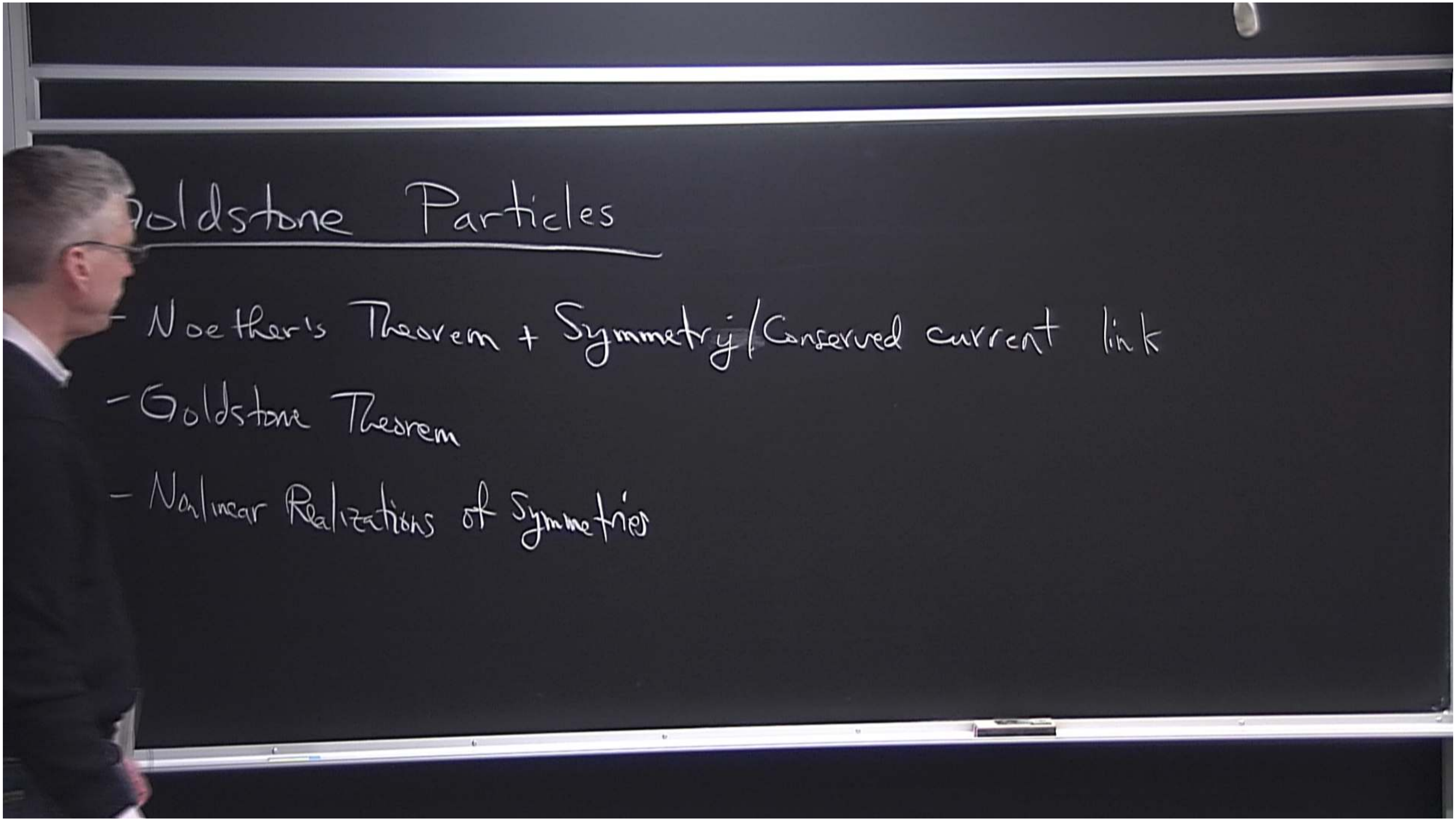
Title: Introduction to Effective Field Theories - Lecture 15

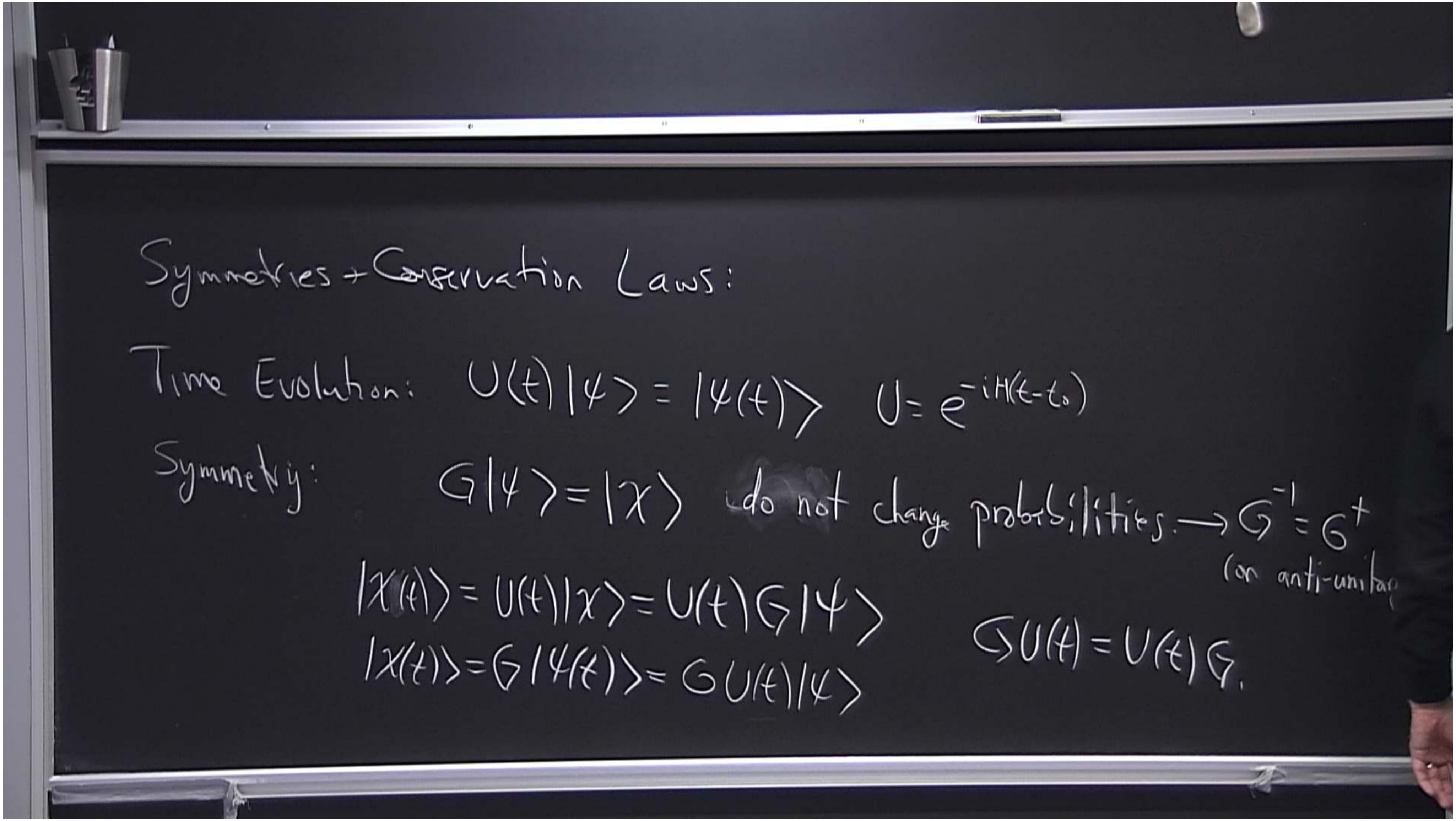
Date: Mar 07, 2014 09:00 AM

URL: http://pirsa.org/14020091

Abstract:



oldstone Particles
- Noether's Theorem + Symmetry/Conserved current link
- Goldstone Theorem
- Nonlinear Realizations of Symmetries


Symmetries + Conservation Laws:
Time Evolution: $U(t)|\psi\rangle = |\psi(t)\rangle$ $U = e^{-iH(t-t_0)}$
Symmetry: $G|\psi\rangle = |\chi\rangle$ do not change probabilities $\to G^{-1} = G^{\dagger}$
(or anti-unitar
$|\chi(t)\rangle = U(t)|\chi\rangle = U(t)G|\psi\rangle$
$|\chi(t)\rangle = G|\psi(t)\rangle = GU(t)|\psi\rangle$
$GU(t) = U(t)G$.

G_1, G_2 both symmetries $\longrightarrow$ $G_1 G_2 = G_3$ is also; $\{G\}$ form a group. if $G = G(\theta^\alpha)$ is labelled by a continuous parameter set (Lie Group)

Define $Q_\alpha = G^{-1} \frac{\partial G}{\partial \theta^\alpha}$ so $G = e^{i\theta^\alpha Q_\alpha}$ $Q = Q^\dagger$ because $G^{-1} = G^\dagger$.

→ conserved quantum #s.

Operators transform under the symmetry as $\mathcal{O} \to G \mathcal{O} G^{\dagger}$

→ for infinitesimal transformations

$$\mathcal{O} \to (1 + i\theta Q + \dots)\,\mathcal{O}\,(1 - i\theta Q + \dots)$$

$$= \mathcal{O} + i\theta\,[Q, \mathcal{O}] + O(\theta^2)$$

Noether's theorem: for local field theories symmetries don't just give conserved charges, they give conserved currents:

a symmetry in field theory is a

$Q^\dagger = Q$ because $G^\dagger = G$.

$GU = UG \to HQ = QH$ so eigenvalues of Q can label energy eigenstates:

$$Q|\psi\rangle = q|\psi\rangle \qquad Q|\psi(t)\rangle = QU(t)|\psi\rangle = U(t)Q|\psi\rangle = qU(t)|\psi\rangle = q|\psi(t)\rangle$$

$\longrightarrow$ conserved quantum #s. ; if $|\chi\rangle = iQ|\psi\rangle$ then $[Q,H]=0 \Rightarrow E_\chi = E_\psi$.

Operators transform under the symmetry as $\mathcal{O} \to G\mathcal{O}G^\dagger$

$\to$ for infinitesimal transformations

$$\mathcal{O} \to (1+i\theta Q + \cdots)\,\mathcal{O}\,(1-i\theta Q+\cdots)$$
$$= \mathcal{O} + i\theta\left[Q,\mathcal{O}\right] + O(\theta^2)$$

Noether's theorem: for local field theories symmetries don't just give conserved charges, they give conserved currents:

a symmetry in field theory is a transformation of the fields $\delta\phi^i = \omega^a \xi_a^i(\phi)$

$\uparrow$ par

$$\delta S = 0 \longrightarrow \delta\mathcal{L} = \omega^a \partial_\mu V^\mu_a \quad \text{for some } V^\mu \begin{cases} \rightarrow \text{spacetime transform}^{n}\ V^\mu \neq 0 \\ \text{"internal" } V^\mu = 0 \end{cases}$$

$$\frac{\partial\mathcal{L}}{\partial\phi^i} \underbrace{\omega^a \xi^i_a}_{\delta\phi^i} + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi^i)}\,\omega^a \partial_\mu \xi^i_a \equiv \omega^a \partial_\mu V^\mu_a .$$

identity for all ω^a, and all ϕ^i

$$\underbrace{\frac{\partial\mathcal{L}}{\partial\phi^i}\,\xi^i_a - \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi^i)}\,\xi^i_a\right]}_{=0 \text{ is the e.o.m.}} = \partial_\mu\left[V^\mu - \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi^i)}\,\xi^i_a\right] \quad \text{for all } \phi^i$$

So symmetry + e.o.m. $\Longrightarrow$ $\exists\ J^\mu_a$ such that

$$\partial_\mu J^\mu_a = 0.$$

where: $J^\mu_a = V^\mu_a - \dfrac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} \xi^i_a$

So symmetry + e.o.m. $\Longrightarrow$ $\exists\ J^\mu_a$ such that

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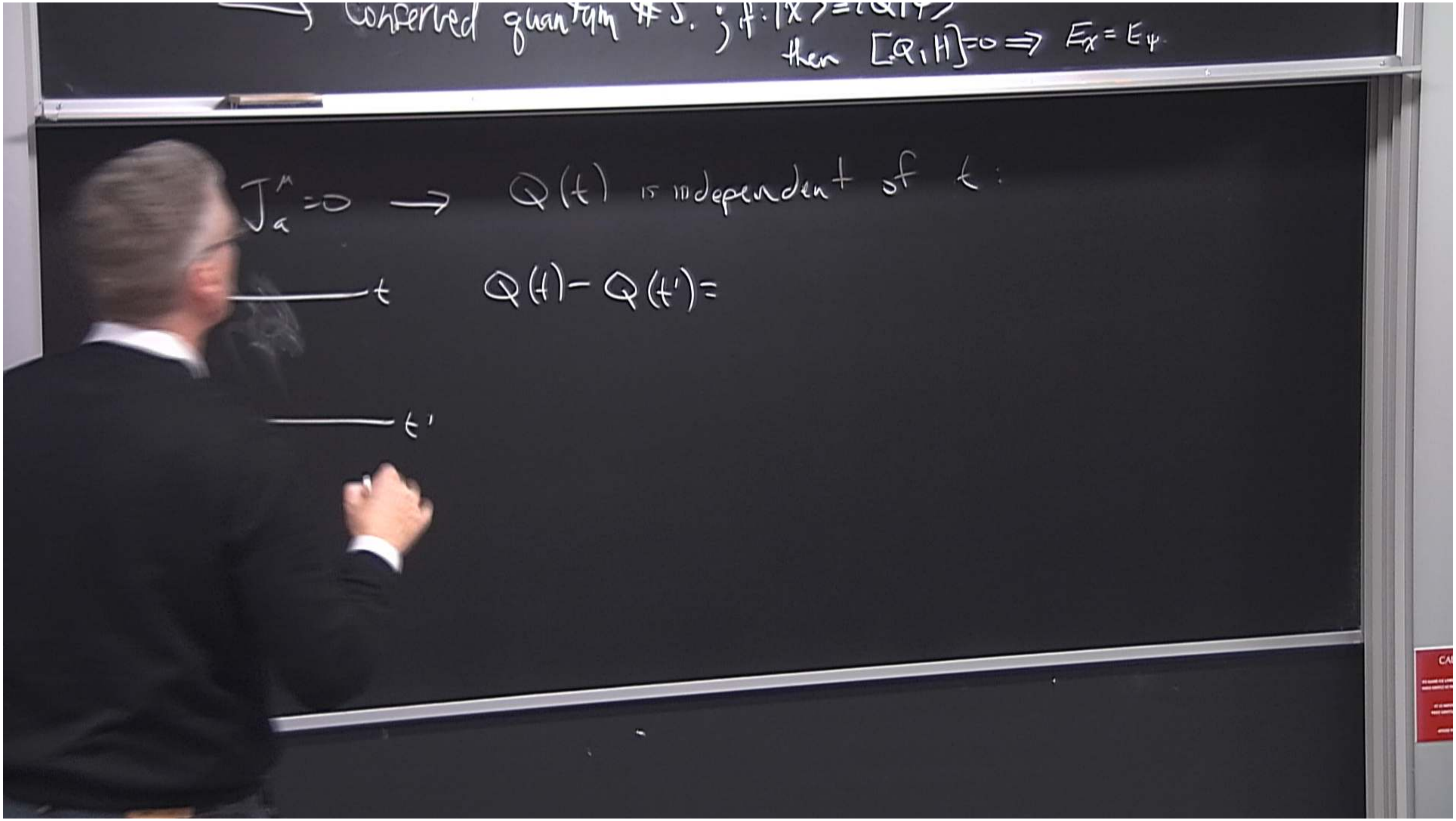
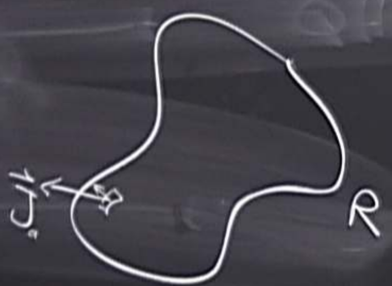
if we define $Q_a(t) = \displaystyle\int_{\text{const } t} d^3x\, J^0_a$ $\left[Q = \displaystyle\int_\Sigma d\Sigma_\mu\, J^\mu_a \right]$


conserved quantum #s. ; if: $|\chi\rangle = Q|\psi\rangle$
then $[Q, H] = 0 \Rightarrow E_\chi = E_\psi$.
$J^\mu_a = 0 \rightarrow Q(t)$ is independent of t:
t
$Q(t) - Q(t') =$
t'

$\rightarrow$ conserved quantum #s. ; if: $|\chi\rangle = Q|\psi\rangle$
then $[Q, H] = 0 \Rightarrow E_\chi = E_\psi$.

$$\partial_\mu J^\mu_a = 0 \longrightarrow Q(t) \text{ is independent of } t : \ \partial_t J^0_a + \nabla \cdot \vec{J}_a = 0$$



$$Q_a(t,R) = \int_R d^3x \, J^0_a$$

$$\boxed{\frac{\partial Q_a}{\partial t}(t,R) = \int_R d^3x \, \frac{\partial J^0_a}{\partial t} = -\int_R d^3x \, \nabla \cdot \vec{J}_a = -\int_{\partial R} d^2S \, \vec{n} \cdot \vec{J}_a}$$

$\Rightarrow$ conserved quantum #s. ; if $|\chi\rangle = Q|\psi\rangle$
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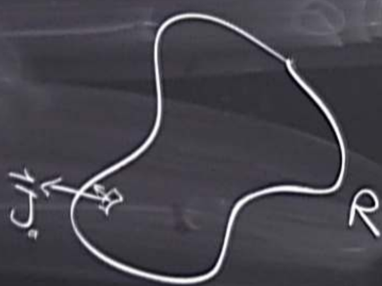
$\partial_\mu J^\mu_a = 0 \rightarrow Q(t)$ is independent of t : $\partial_t J^0_a + \nabla\cdot\vec{J}_a = 0$



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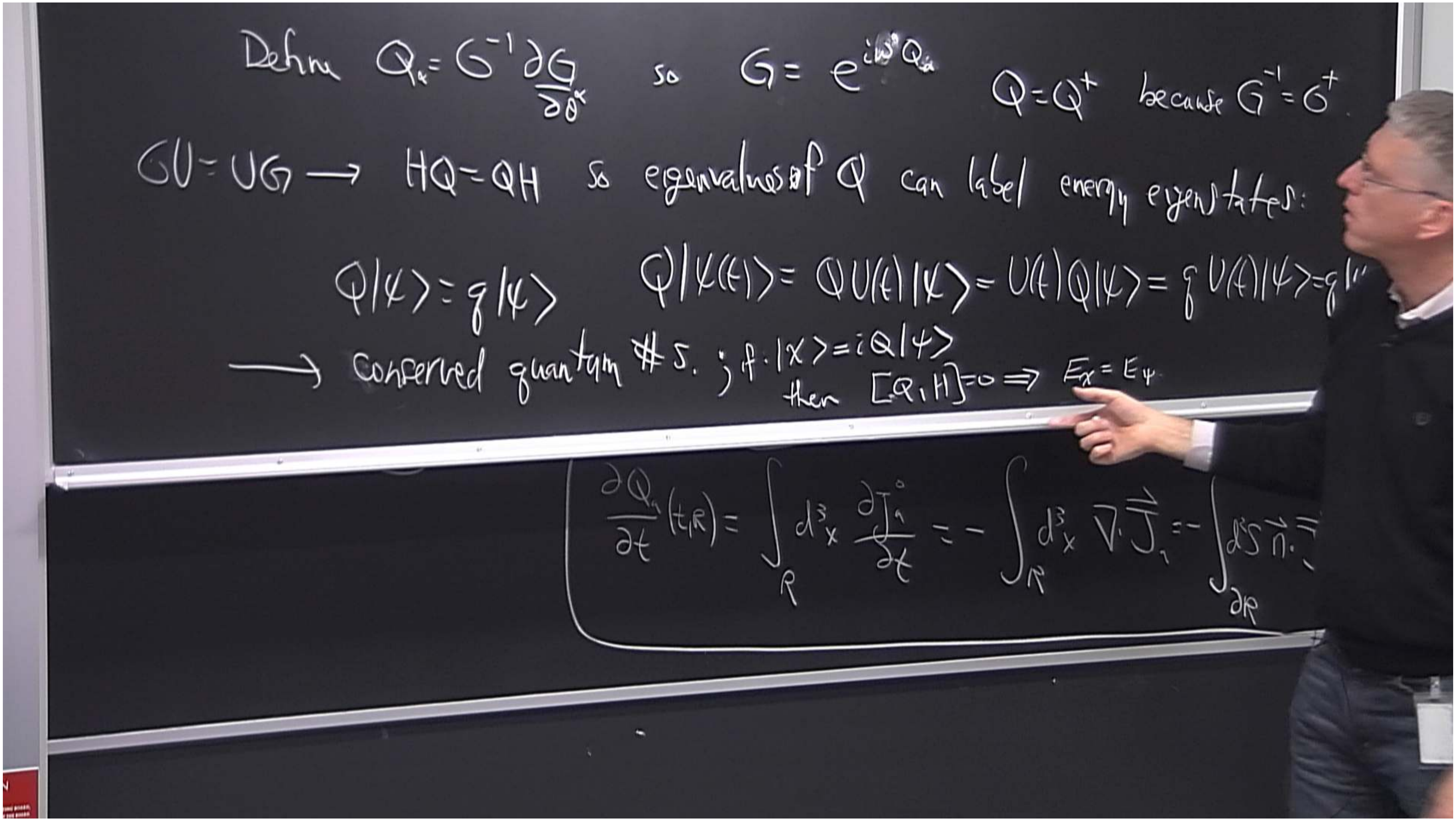
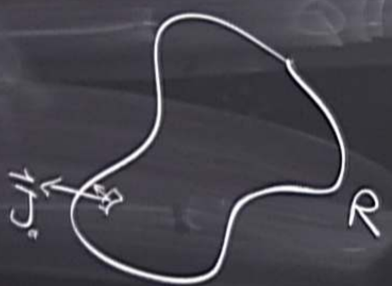

Define Q_α = G^{-1} ∂G/∂θ^α so G = e^{iθ^α Q_α} Q = Q^† because G^{-1} = G^†.
GU = UG → HQ = QH so eigenvalues of Q can label energy eigenstates:
Q|ψ> = q|ψ> Q|ψ(t)> = QU(t)|ψ> = U(t)Q|ψ> = q U(t)|ψ> = q|ψ
→ conserved quantum #s. ; if |χ> = iQ|ψ> then [Q,H]=0 ⇒ E_χ = E_ψ
∂Q_a/∂t (t,R) = ∫_R d^3x ∂J^0_a/∂t = -∫_R d^3x ∇·J_a = -∫_∂R d^2S n·J

$\Rightarrow$ conserved quantum #s. ; if $Q|\chi\rangle = |\psi\rangle$
then $[Q,H]=0 \Rightarrow E_\chi = E_\psi$.

$\partial_\mu J^\mu_a = 0 \longrightarrow Q(t)$ is independent of t : $\partial_t J^0_a + \nabla\cdot\vec{J}_a = 0$



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$\Rightarrow$ conserved quantum #s. ; if: $|x\rangle = \ldots$
then $[Q,H]=0 \Rightarrow E_x = E_\psi$.

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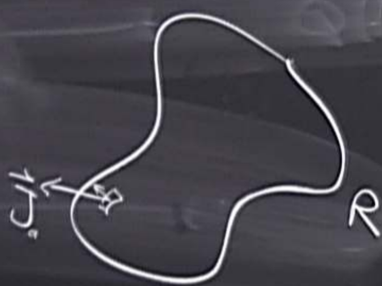

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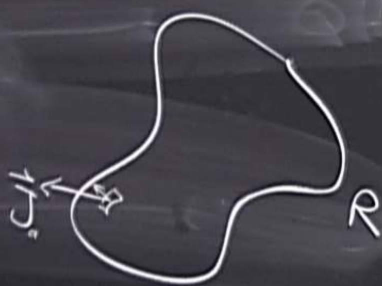

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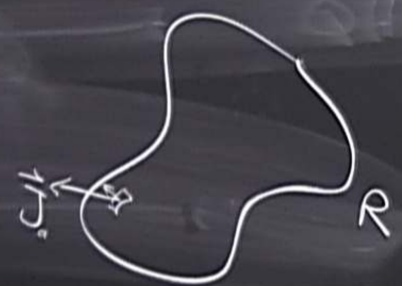

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operators transform under the symmetry as $\mathcal{O} \to G \mathcal{O} G^\dagger$

$\rightarrow$ for infinitesimal transformations

$$\mathcal{O} \to (1 + i\theta Q + \ldots)\,\mathcal{O}\,(1 - i\theta Q + \ldots)$$
$$= \mathcal{O} + i\theta [Q, \mathcal{O}] + O(\theta^2)$$

$$\delta\phi^i = \omega^a \xi^i_a(\phi) = i\omega^a [Q, \phi^i]$$

→ conserved quantum #s. ; if $Q|\chi\rangle = \dots$ then $[Q, H] = 0 \Rightarrow E_\chi = E_\psi$.

operators transform under the symmetry as $O \to G O G^\dagger$

→ for infinitesimal transformations

$$O \to (1 + i\theta Q + \dots)\, O\, (1 - i\theta Q + \dots)$$
$$= O + i\theta\,[Q, O] + O(\theta^2)$$

$$\delta\phi^i = \omega^a\, \xi^i_a(\phi) = i\omega^a\,[Q, \phi^i] = i\omega \int d^3x\, [J^0, \phi^i]$$

$\dot\phi^*\dot\phi -$

$$\mathcal{L} = -\partial_\mu\phi^*\partial^\mu\phi - V(\phi^*\phi)$$

$\delta\phi = i\omega\phi \quad \delta\phi^* = -i\omega\phi^*$

$\delta\mathcal{L} = 0$

$J^\mu = -\partial^\mu\phi\,\phi^* + i\partial^\mu\phi^*\,\phi$

$J^0 = +i\dot\phi\,\phi^* - i\dot\phi^*\,\phi$

$\pi = \frac{\delta S}{\delta\dot\phi} = \dot\phi^* \quad \pi^* = \dot\phi$

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$\dot\phi^*\dot\phi -$

$\mathcal{L} = -\partial_\mu\phi^*\partial^\mu\phi - V(\phi^*\phi)$

$\delta\phi = i\omega\phi \quad \delta\phi^* = i\omega\phi^*$

$\delta\mathcal{L} = 0$

$J^\mu = -\partial^\mu\phi\,\phi^* + i\,\partial^\mu\phi^*\,\phi$

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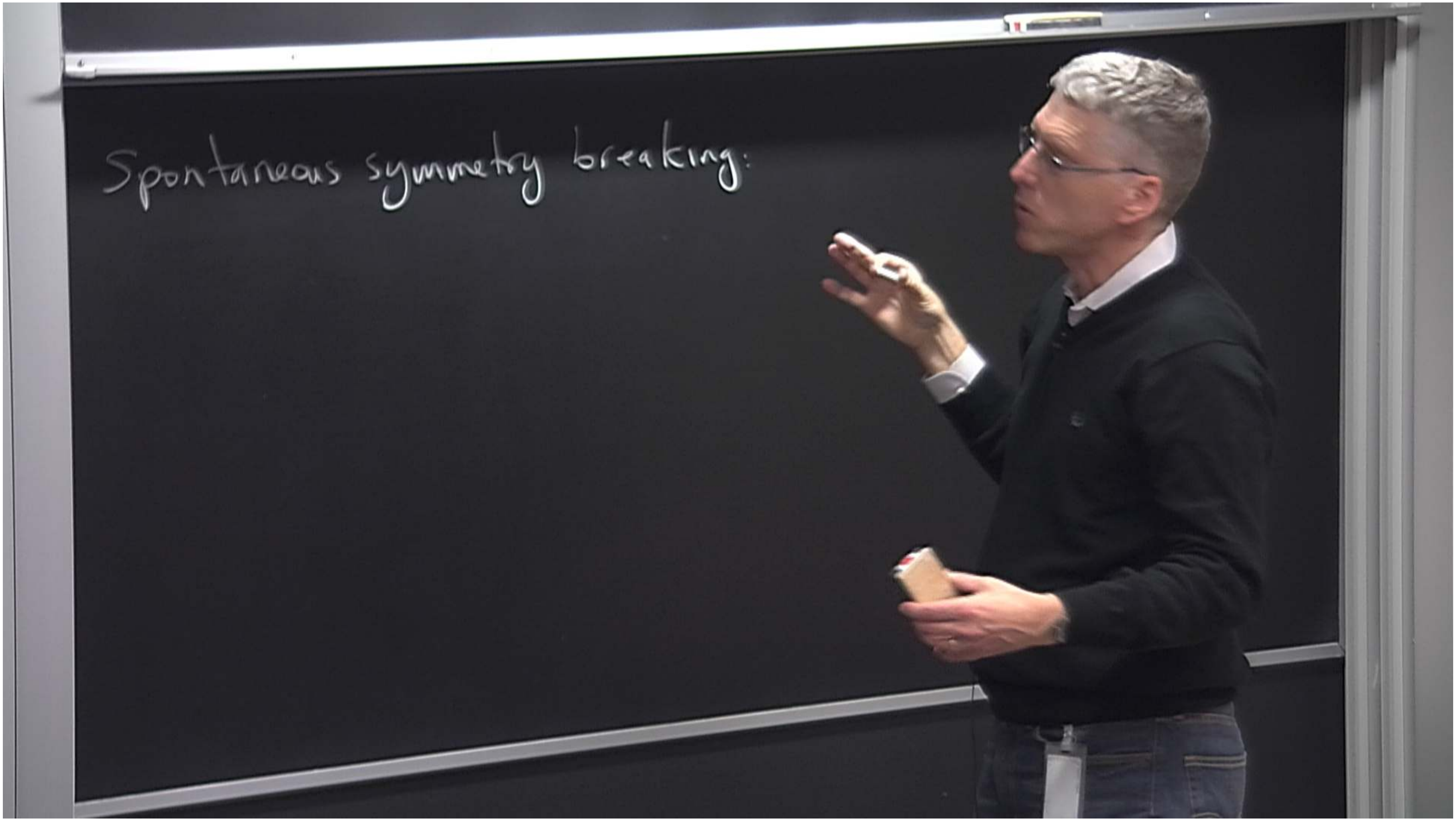
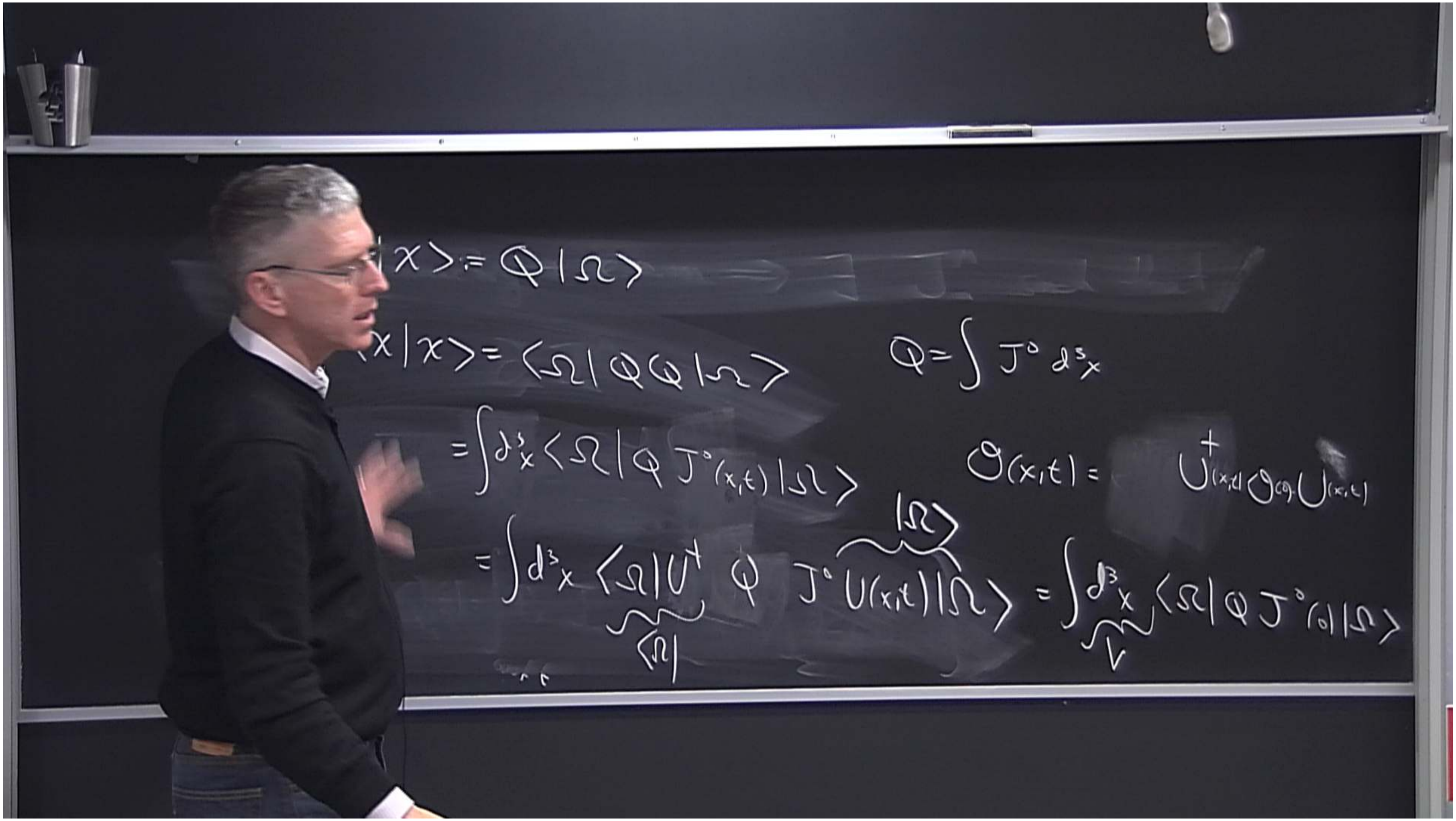
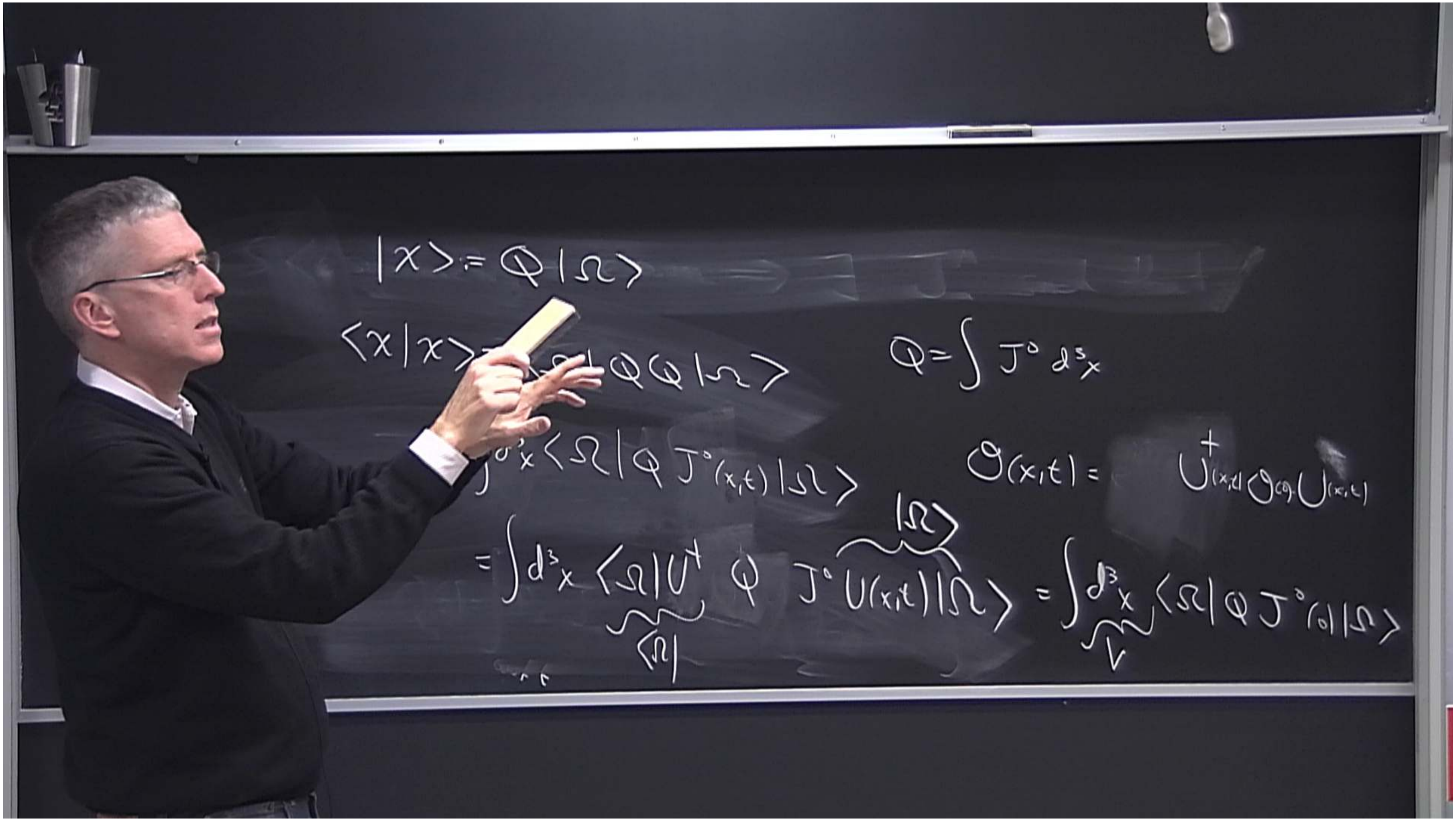

Spontaneous symmetry breaking:


$|X\rangle = Q|\Omega\rangle$
$\langle X|X\rangle = \langle\Omega|QQ|\Omega\rangle$
$Q = \int J^0 \, d^3x$
$= \int d^3x \, \langle\Omega|Q\, J^0(x,t)|\Omega\rangle$
$\mathcal{O}(x,t) = U^\dagger(x,t)\, \mathcal{O}(0)\, U(x,t)$
$= \int d^3x \, \underbrace{\langle\Omega|U^\dagger}_{\langle\Omega|} \, Q \, J^0 \, \overbrace{U(x,t)|\Omega\rangle}^{|\Omega\rangle} = \underbrace{\int d^3x}_{V} \langle\Omega|Q\, J^0(0)|\Omega\rangle$


$|x\rangle := Q|\Omega\rangle$
$\langle x|x\rangle = \langle\Omega|QQ|\Omega\rangle$
$Q = \int J^0 d^3x$
$\int d^3x \langle\Omega|Q J^0(x,t)|\Omega\rangle$
$O(x,t) = U^\dagger(x,t) O(0) U(x,t)$
$= \int d^3x \langle\Omega|U^\dagger Q J^0 U(x,t)|\Omega\rangle = \int d^3x \langle\Omega|Q J^0(0)|\Omega\rangle$
$|\Omega\rangle$
$\langle\Omega|$
V

Spontaneous symmetry breaking: when ground state (min. energy, $|\Omega\rangle$) is **NOT** invariant under a symmetry.

$$G|\Omega\rangle \neq |\Omega\rangle \rightarrow Q|\Omega\rangle \neq 0.$$

$$\langle v_1|\,\phi(x)\,\phi(y)\,|v_2\rangle - \langle\phi\rangle\langle\phi\rangle$$

For a field theory, we need a better criterion for S.S.B.

Assume: $\exists\ \phi, \psi$ such that $\delta\psi = \phi$ (order param.)
and $\langle\Omega|\phi|\Omega\rangle \neq 0$.

Nonlinear Realizations of Symmetries

$$Q|\psi\rangle = q|\psi\rangle$$

$$|\chi\rangle = Q|\psi\rangle$$

then $E_\chi = E_\psi$.

$$i[Q,\phi] = \delta\phi$$

$$\phi(x) = \sum_k u_k(x) a_k + \dots$$

$$i[Q, a_k^*] = b_k^*$$

$$|\chi\rangle = Q|\Omega\rangle$$

$$\langle\chi|\chi\rangle = \langle\Omega|QQ|\Omega\rangle \qquad Q = \int J^0 \, d^3x$$

$$= \int d^3x \langle\Omega|Q\, J^0(x,t)|\Omega\rangle \qquad \mathcal{O}(x,t) = U^\dagger(x,t)\,\mathcal{O}(0)\,U(x,t)$$

$$= \int d^3x \underbrace{\langle\Omega|U^\dagger}_{\langle\Omega|} Q\, J^0 \overbrace{U(x,t)|\Omega\rangle}^{|\Omega\rangle} = \underbrace{\int d^3x}_{V} \langle\Omega|Q\, J^0(0)|\Omega\rangle$$

So $Q|\Omega\rangle = 0 \Rightarrow \langle\Omega|Q = 0 \Rightarrow \langle\Omega|\delta\phi|\Omega\rangle = 0$

then

$\partial_\mu J_a = 0 \longrightarrow Q(t)$ is independent of t: $\partial_t J_a + \nabla\cdot \vec{J}_a = 0$

$$|\chi\rangle = b_k^* |\Omega\rangle \qquad |\psi\rangle = a_k^* |\Omega\rangle$$

$$|\chi\rangle = i\left[Q a_k^* - a_k^* Q\right]|\Omega\rangle$$

$$= iQ|\psi\rangle \boxed{- i a_k^* Q|\Omega\rangle}$$

Need $|\chi\rangle = iQ|\psi\rangle$ to conclude

$E_\chi = E_\psi$ if Q is a sym.

So SSB ($Q|\Omega\rangle \neq 0$) is of interest because it ruins the inference that ptcls come in deg mult.