Title: Introduction to Effective Field Theories - Lecture 13

Date: Feb 28, 2014 09:00 AM

URL: http://pirsa.org/14020090

Abstract:

For $E \ll m_e$

$$+\cancel{F_{\mu\nu}\tilde{F}^{\mu\nu}}$$

$$\mathcal{L}_W = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} + \overbrace{\left[\frac{a_e}{m_e^2} + \frac{a_\mu}{m_\mu^2} + \dots\right] F_{\mu\nu}\Box F^{\mu\nu} + \left[\frac{a'_e}{m_e^2} + \frac{a'_\mu}{m_\mu^2}\right]\partial_\lambda F^{\mu\nu}\partial^\lambda F_{\lambda\nu}}^{\left(\frac{a}{m_e^2} - 2\frac{a'_e}{m_e^2}\right)\partial^\mu J^\nu \partial_\mu J_\nu} + \left[\frac{b_e}{m_e^4} + \frac{b_\mu}{m_\mu^4} + \dots\right]\left(F_{\mu\nu}F^{\mu\nu}\right)^2$$

$$+ \left[\frac{c_e}{m_e^4} + \frac{c_\mu}{m_\mu^4}\right]\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2 + \dots + e_0 J^\mu A_\mu$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$$

$$\ldots \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left[\frac{a_e}{m_e^2} + \frac{a_\mu}{m_\mu^2} + \ldots\right] F_{\mu\nu} \Box F^{\mu\nu} + \left[\frac{a'_e}{m_e^2} + \frac{a'_\mu}{m_\mu^2}\right] \partial_\lambda F^{\mu\nu} \partial^\lambda F_{\lambda\nu} + \left[\frac{b_e}{m_e^4} + \frac{b_\mu}{m_\mu^4} + \ldots\right] (F_{\mu\nu} F^{\mu\nu})^2$$

$$+ \left[\frac{c_e}{m_e^4} + \frac{c_\mu}{m_\mu^4}\right] (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \ldots + \underline{e_0 J^\mu A_\mu}$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} F^{\lambda\rho}$$

$$\partial_\mu J^\mu = 0$$

Points to make:

1) $1/m_e$ terms dominate $1/m_\mu$ terms: (lightest mass wins in a denominator)

$$\left[\frac{\ }{m_e^2}+\frac{\ }{m_\mu^2}+\dots\right]F_{\mu\nu}\Box F^{\mu\nu} + \left[\frac{a_e}{m_e^2}+\frac{a_\mu}{m_\mu^2}\right]\partial_\lambda F^{\mu\nu}\partial^\lambda F_{\lambda\nu} + \left[\frac{b_e}{m_e^4}+\frac{b_\mu}{m_\mu^4}+\dots\right](F_{\mu\nu}F^{\mu\nu})^2$$

$$+\left[\frac{c_e}{m_e^4}+\frac{c_\mu}{m_\mu^4}\right](F_{\mu\nu}\tilde{F}^{\mu\nu})^2+\dots\dots + \underline{e_0 J^\mu A_\mu}$$

$$\tilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$$

$$\partial_\mu J^\mu = 0$$

Points to make:

1) $\frac{1}{m_e}$ terms dominate $1/m_\mu$ terms: (lightest mass wins in a denominator)

2) Corrections to classical EM at low energies are controlled by E^2/m_e^2 <u>except</u> for the <u>value</u> of the electric charge, e_0.

$$\left[\frac{\ldots}{m_e^2}+\frac{\ldots}{m_\mu^2}+\ldots\right]F_{\mu\nu}F^{\mu\nu} + \left[\frac{a_e}{m_e^2}+\frac{a_\mu}{m_\mu^2}\right]\partial_\lambda F^{\mu\nu}\partial^\lambda F_{\mu\nu} + \left[\frac{b_e}{m_e^4}+\frac{b_\mu}{m_\mu^4}+\ldots\right]\left(F_{\mu\nu}F^{\mu\nu}\right)^2$$

$$+\left[\frac{c_e}{m_e^4}+\frac{c_\mu}{m_\mu^4}\right]\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2 + \cdots\cdots + \underbrace{e_0 J^\mu A_\mu}$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$$

$$\partial_\mu J^\mu = 0$$

Points to make:

1) $\frac{1}{m_e}$ terms dominate $1/m_\mu$ terms: (lightest mass wins in a denominator)

2) Corrections to classical EM at low energies are controlled by E^2/m_e^2 <u>except</u> for the <u>value</u> of the electric charge, e_0. [and it is NOT a series in α]

$$\left[\frac{\ }{m_e^2}+\frac{\ }{m_\mu^2}+\dots\right]F_{\mu\nu}\Box F^{\mu\nu}+\left[\frac{a_e}{m_e^2}+\frac{a_\mu}{m_\mu^2}\right]\partial_\lambda F^{\mu\nu}\partial^\lambda F_{\lambda\nu}+\left[\frac{b_e}{m_e^4}+\frac{b_\mu}{m_\mu^4}\right](F_{\mu\nu}F^{\mu\nu})^2$$

$$+\left[\frac{c_e}{m_e^4}+\frac{c_\mu}{m_\mu^4}\right](F_{\mu\nu}\tilde{F}^{\mu\nu})^2+\dots+\underline{e_0 J^\mu A_\mu}$$

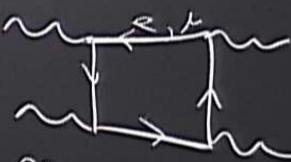
$$\tilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$$

$$\partial_\mu J^\mu=0$$

Points to make:

1) $1/m_e$ terms dominate $1/m_\mu$ terms: (lightest mass wins in a denominator)

2) Corrections to classical EM at low energies are controlled by E^2/m_e^2 <u>except</u> for the <u>value</u> of the electric charge, e_0. [and it is NOT just a series in α]

3) Scattering of light by light [in QED  $+ \ldots$]

at energies $\ll m_e$ is captured by the coefficients b, c.

Q: which interactions in $\mathcal{L}_W$ ~~in~~ which graphs contribute to $\sigma(\gamma\gamma \to \gamma\gamma)$?

$$A_E(q) \simeq f^2 q^2 \left(\frac{1}{f}\right)^E \left(\frac{M q}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{\sum (d-2) V_{dk}}$$

$$\mathcal{L} = f^4 \sum_i \mathcal{O}\left(\frac{\phi}{f}, \frac{\partial \phi}{M f}\right)$$

$(*)$ has this form with $f \simeq v = M = m_e$.

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ | $\phi \to A_\mu$.

$$A_E(q) \sim m_e^2 q^2 \left(\frac{1}{m_e}\right)^E \left(\frac{q}{4\pi m_e}\right)^{2L} \left(\frac{q}{m_e}\right)^{\sum (d-2) V_{dk}}$$

also because A_μ only enters thru $F_{\mu\nu}$ $d \geq k$ for all V_{dk}.

consequently $d = 2 \Rightarrow k \leq 2$ but $k \geq 2$ so only $d = 2$ term has $k = 2$ + is the free lagrangian $\mathcal{L}_0 = -\frac{Z}{4} F_{\mu\nu} F^{\mu\nu}$ so don't appear in vertices.

$\Rightarrow$ for all vertices $d \geq 4$.

$, V_{dk}=0 \;\forall\, d>4 \;;\; \text{or } L=0 \quad \sum (d-2) N_{dk} = 4$

$$A_4 \overset{\text{leading}}{\simeq} \frac{g^4}{m_e^4} \rightarrow \sigma \sim \left(\frac{g}{m_e}\right)^8$$

Real calculation: QED box: $b = \frac{4}{7} c = \frac{\alpha^2}{90} \overset{+O(\alpha^3)}{\uparrow} \left[\frac{e^4}{(4\pi)^2} \text{ where } \alpha = \frac{e^2}{4\pi} \right]$

$$\frac{d\sigma}{d\Omega}(\gamma\gamma \rightarrow \gamma\gamma) = \frac{139}{4\pi^2} \left(\frac{\alpha^2}{90}\right)^2 \left(\frac{E_{cm}^6}{m_e^8}\right) \left(3 + \cos^2\theta\right)^2 \left[1 + O\left(\frac{E_{cm}^2}{m_e^2}\right)\right]$$

4) At zeroth order in $1/m_e^2$ the sole influence of virtual e^+e^- $(+\mu^+\mu^-)$ is in Z.

$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} + e_0 A_\mu J^\mu$$

divergence

where gives Z to be: $$Z = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k + \ln\left(\frac{m_e^2}{\mu^2}\right)\right]$$

where $n = 4-2\varepsilon$, $k =$

$$\mathcal{L}_W = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} + \left[\frac{a_e}{m_e^2} + \cancel{\frac{a_\mu}{m_\mu^2}} + \dots\right] F_{\mu\nu}\Box F^{\mu\nu} + \left[\frac{a'_e}{m_e^2} + \cancel{\frac{b'_\mu}{m_\mu^2}}\right]\partial_\lambda F^{\mu\nu}\partial^\lambda F_{\lambda\nu} + \left[\frac{b_e}{m_e^4} + \cancel{\frac{b_\mu}{m_\mu^4}} + \dots\right]\left(F_{\mu\nu}F^{\mu\nu}\right)^2$$

4) At zeroth order in $1/m_e^2$ the sole influence of virtual e^+e^- $(+\mu^+\mu^-)$ is in Z.

$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} + \hat{e}_0 A_\mu J^\mu$$

where  gives Z to be: $Z = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k + \ln\left(\frac{m_e^2}{\mu^2}\right)\right]$ ← divergence

where $n = 4 - 2\varepsilon$, k = Euler-Mascheroni $+ \ln(4\pi)$ $= \gamma - \ln 4\pi$, $\hat{e}_0 = e_0 \mu^\varepsilon$

$$\mathcal{L}_W = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} + \left[\frac{a_e}{m_e^2} + \cancel{\frac{a_\mu}{m_\mu^2}} + \dots\right] F_{\mu\nu}\Box F^{\mu\nu} + \left[\frac{a'_e}{m_e^2} + \cancel{\frac{a'_\mu}{m_\mu^2}}\right]\partial_\lambda F^{\mu\nu}\partial^\lambda F_{\mu\nu} + \left[\frac{b_e}{m_e^4} + \cancel{\frac{b_\mu}{m_\mu^4}} + \dots\right]\left(F_{\mu\nu}F^{\mu\nu}\right)^2$$

4) At zeroeth order in $1/m_e^2$ the sole influence of virtual e^+e^- $(+\mu^+\mu^-)$ is in Z.

$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} + \hat{e}_0 A_\mu J^\mu$$

where  gives Z to be: $Z = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k + \ln\left(\frac{m_e^2}{\mu^2}\right)\right]$ ← divergence

where $n = 4 - 2\varepsilon$, k = Euler-Mascheroni + $\ln(4\pi)$ $= \gamma - \ln 4\pi$, $\hat{e}_0 = e_0\mu^\varepsilon$

$$\ldots \frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} + \left[\frac{a_e}{m_e^2} + \cancel{\frac{a_\mu}{m_\mu^2}} + \ldots\right] F_{\mu\nu}\Box F^{\mu\nu} + \left[\frac{a'_e}{m_e^2} + \cancel{\frac{b'_\mu}{m_\mu^2}}\right]\partial_\lambda F^{\mu\nu}\partial^\lambda F_{\lambda\nu} + \left[\frac{b_e}{m_e^4} + \cancel{\frac{b_\mu}{m_\mu^4}} + \ldots\right]\left(F_{\mu\nu}F^{\mu\nu}\right)^2$$

4) At zeroth order in $1/m_e^2$ the sole influence of virtual e^+e^- $(+\mu^+\mu^-)$ is in Z.

$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} + \hat{e}_0 A_\mu J^\mu$$

where $\overset{\varepsilon}{\sim\!\bigcirc\!\sim}$ gives Z to be: $Z = 1 - \frac{\alpha}{3\pi}\left[\underbrace{\frac{1}{\varepsilon}}_{\text{divergence}} + k + \ln\left(\frac{m_e^2}{\mu^2}\right)\right]$

where $n = 4 - 2\varepsilon$, k = Euler-Mascheroni + $\ln(4\pi)$ $= \gamma - \ln 4\pi$, $\hat{e}_0 = e_0\mu^\varepsilon$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Absorb Z into A_μ by rescaling $A_\mu \to \frac{A_\mu^{(R)}}{\sqrt{Z}}$

So $-\frac{1}{4} Z F_{\mu\nu} F^{\mu\nu} \to \underbrace{-\frac{1}{4} F^{(R)}_{\mu\nu} F^{(R)\mu\nu}}_{\text{canonical form}}$.

$$\mathcal{L} = -\frac{1}{4} F^{(R)}_{\mu\nu} F^{(R)\mu\nu} + \boxed{\frac{e_0}{\sqrt{Z}}} A^{(R)}_\mu J^\mu$$

$e_{phys} = \frac{e_0}{\sqrt{Z}}$ "physical renormalization scheme"

$$\alpha_{phys} = \frac{e_{phys}^2}{4\pi} = \frac{1}{137\ldots}$$

Real calculation: QED box: $b = \frac{4}{7} c = \frac{\alpha^2}{90} \left[\frac{e^2}{(4\pi)^2} \text{ where } \alpha = \frac{e^2}{4\pi} \right]$

Two other renormalization scheme:

1) Minimal Subtraction: $Z_{MS} = 1 - \frac{\alpha}{3\pi\varepsilon}$, $e_{MS} = \frac{e_0}{\sqrt{Z_{MS}}}$

2) Modified Minimal subtraction: $Z_{\overline{MS}} = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k\right]$, $e_{\overline{MS}} = \frac{e_0}{\sqrt{Z_{\overline{MS}}}}$

Real calculation: QED box: $b = \frac{4}{7} c = \frac{\alpha^2}{90} \left[\frac{e^4}{(4\pi)^2} \text{ where } \alpha = \frac{e^2}{4\pi} \right]$

Two other renormalization scheme:

1) Minimal Subtraction: $Z_{MS} = 1 - \frac{\alpha}{3\pi\varepsilon}$, $e_{MS} = \frac{e_0}{\sqrt{Z_{MS}}}$

2) Modified Minimal subtraction: $Z_{\overline{MS}} = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k\right]$, $e_{\overline{MS}} = \frac{e_0}{\sqrt{Z_{\overline{MS}}}}$

$$e_{\overline{MS}} = \frac{e_0}{\sqrt{Z_{\overline{MS}}}} = e_{phys}\sqrt{\frac{Z_{phys}}{Z_{\overline{MS}}}} = 1 - \frac{\alpha}{6\pi}\left[\cancel{\frac{1}{\varepsilon}} + k + \ln\frac{m^2}{\mu^2}\right] + \frac{\alpha}{6\pi}\left[\cancel{\frac{1}{\varepsilon}} + k\right]$$

$$= 1 - \frac{\alpha}{6\pi}\ln\left(\frac{m_e^2}{\mu^2}\right)$$

Real calculation: QED box: $b = \frac{4}{7} c = \frac{\alpha^2}{90} \left[\frac{e^2}{(4\pi)^2} \text{ where } \alpha = \frac{e^2}{4\pi} \right]$

Two other <u>renormalization scheme:</u>

1) Minimal Subtraction: $Z_{MS} = 1 - \frac{\alpha}{3\pi\varepsilon}$, $e_{MS} = \frac{e_0}{\sqrt{Z_{MS}}}$

2) Modified Minimal subtraction: $Z_{\overline{MS}} = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + k\right]$, $e_{\overline{MS}} = \frac{e_0}{\sqrt{Z_{\overline{MS}}}}$

$$e_{\overline{MS}} = \frac{e_0}{\sqrt{Z_{\overline{MS}}}} = e_{phys}\sqrt{\frac{Z_{phys}}{Z_{\overline{MS}}}} = e_{phys}\left[1 - \frac{\alpha}{6\pi}\left[\cancel{\frac{1}{\varepsilon}} + k - \ln\left(\frac{m_e^2}{\mu^2}\right)\right] + \frac{\alpha}{6\pi}\left[\cancel{\frac{1}{\varepsilon}} + k\right]\right]$$

$$= \left[1 - \frac{\alpha}{6\pi}\ln\left(\frac{m_e^2}{\mu^2}\right)\right] e_{phys} \Longrightarrow e_{\overline{MS}}(\mu)$$