Title: Introduction to Effective Field Theories - Lecture 11

Date: Feb 14, 2014 09:00 AM

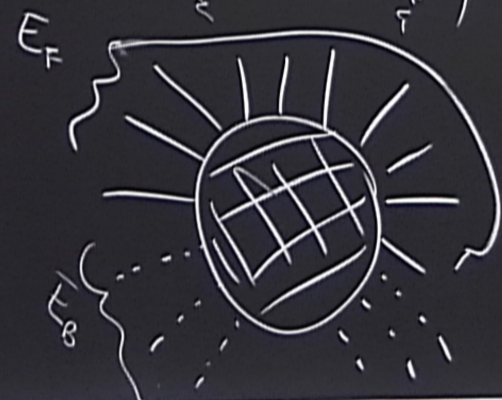
URL: http://pirsa.org/14020089

Abstract:

POWER-COUNTING WITH FERMIONS $\pm$ BOSONS

$$\mathcal{L} = \sum_i c_i \, \mathcal{O}\left(\frac{\phi}{v_B}, \frac{\psi}{v_F}, \frac{\partial\phi}{M v_B}, \frac{\partial\psi}{M v_F}\right)$$

$$\boxed{A_{E_F E_B}(q)}\,(2\pi)^4\delta^4(q)$$



$$\mathcal{L}_{0B} \propto \partial\phi\,\partial\phi + m_B^2\phi^2 \qquad G_B(p) \sim \frac{1}{p^2}$$

$$\mathcal{L}_{0F} \propto \bar{\psi}(\partial\!\!\!/ + m_F)\psi \qquad G_F(p) \sim \frac{1}{p}$$

$\mathcal{L}_{UF} \propto \bar{\psi}(\partial\!\!\!/ + m_F)\psi \qquad G_F(p) \sim \frac{1}{p}$

Identities: $L - (I_B + I_F) + \sum_{k_B k_F d} V_{k_B k_F d} = 1$

Cons of Ends: $E_F + 2I_F = \sum k_F V_{k_F k_B d}$

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POWER-COUNTING WITH FERMIONS + BOSONS

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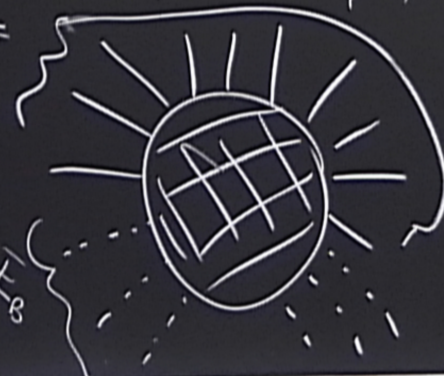

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$$A_{E_B E_F}(q) \sim f^4 \left(\frac{1}{v_B}\right)^{E_B} \left(\frac{1}{v_F}\right)^{3E_F/2} \left(\frac{Mq}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{R}$$

$$R = 2 - \frac{1}{2}E_F + \sum \left(d + \frac{k_F}{2} - 2\right) V_{k_B k_F d}$$

$$\sum \frac{k_F}{2} V_{k_F k_B d} = -\frac{1}{2}E_F + I_F \geq -\frac{1}{2}E_F .$$

$$\mathcal{L} = \sqrt{-g}\Big[\underset{\substack{\uparrow \\ \text{0-deriv}}}{c_0 M^4} + \underset{\substack{\uparrow \\ \text{2-deriv}}}{\frac{M_p^2}{2} R} + \underbrace{c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} + c_4 \Box R}_{\text{4-derivs}} + \underbrace{\frac{c_5 R^3}{M^2}}_{\text{6 derivs.}} + \dots$$

$$R = g^{\mu\nu} R_{\mu\nu} \qquad g^{\mu\nu} g_{\nu\lambda} = \delta^\mu_\lambda$$

$$R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$$

$$R^\mu{}_{\nu\lambda\rho} = \partial_\rho \Gamma^\mu_{\nu\lambda} - \partial_\lambda \Gamma^\mu_{\nu\rho} + \left(\Gamma\Gamma - \Gamma\Gamma\right)$$

$$\Gamma^\mu_{\nu\lambda} = \frac{1}{2} g^{\mu\sigma}\left(\partial_\nu g_{\sigma\lambda} + \partial_\lambda g_{\sigma\nu} - \partial_\sigma g_{\lambda\nu}\right)$$

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1) if $m \sim M_p$ then can use our lagrangian (*) with $f = v = M = M_p$.

2)

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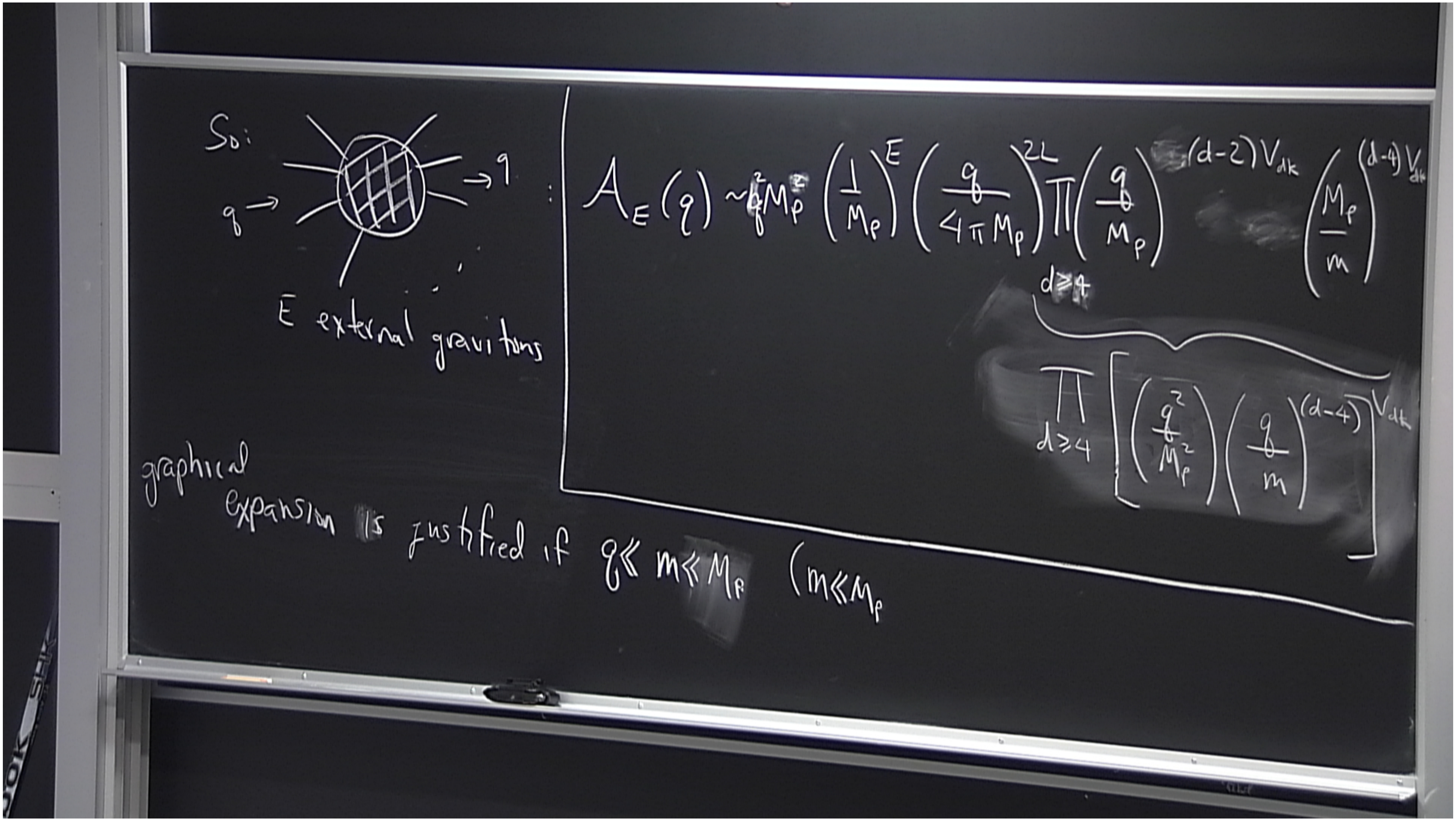
2) For $d \geq 6$ interactions $c_i = \hat{c}_i \left(\frac{M_p}{m}\right)^{d-4}$

$$\frac{g}{4\pi M_p}\Bigg)^{2L} \prod \left(\frac{g}{M_p}\right)^{(d-2)V_{dk}} \left(\frac{M_p}{m}\right)^{(d-4)V_{dk}}$$

$d \geq 4$

$$\left(\frac{1}{M_p}\right)^{E}\left(\frac{g}{4\pi M_p}\right)^{2L}\underbrace{\prod_{d\geq 4}\left(\frac{g}{M_p}\right)^{(d-2)V_{dk}}\left(\frac{M_p}{m}\right)^{(d-4)V_{dk}}}_{\displaystyle \prod_{d\geq 4}\left[\left(\frac{g^2}{M_p^2}\right)\left(\frac{g}{m}\right)^{(d-4)}\right]^{V_{dk}}}$$


So:
q →
→ q
E external gravitons
$A_E(q) \sim q^2 M_P^2 \left(\frac{1}{M_P}\right)^E \left(\frac{q}{4\pi M_P}\right)^{2L} \prod \left(\frac{q}{M_P}\right)^{(d-2)V_{dk}} \left(\frac{M_P}{m}\right)^{(d-4)V_{dk}}$
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graphical expansion is justified if $q \ll m \ll M_P$ $(m \ll M_P$

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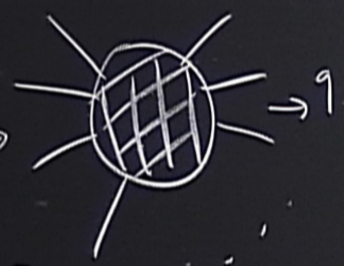

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5a) If you know the high-energy theory (UV completion) then compute the relevant c_i's from it.

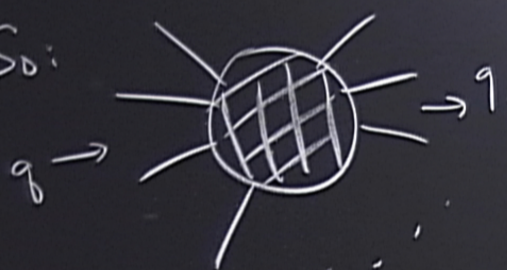
... which) coefficients $\{c_i\}$ are needed; say N.

5a) If you know the high-energy theory (UV completion) then compute the relevant c_i's from it.

5b) If you do not know the UV completion (or you do but cannot calculate with it) then take c_i's from experiment.

Predictive if you have more than N observables

So:



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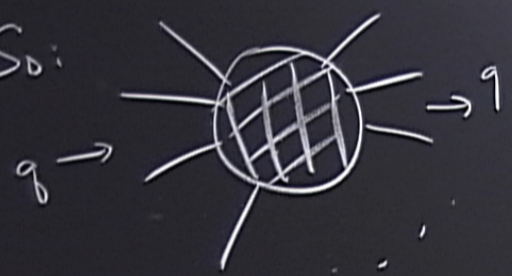
0-deriv 2-deriv 4-derivs 6 derivs. $M_4^2 = M_6^2 r$

For gravity: which graphs and interactions dominate?

eg graviton-graviton scattering: $E=4$ 

at energy q.

So:



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$$\mathcal{L} = \sqrt{-g}\Big[\cancel{\hat{c}_0 M^4} + \frac{M_P^2}{2} R + \underbrace{\hat{c}_1 R^2 + \hat{c}_2 R_{\mu\nu}R^{\mu\nu} + \hat{c}_3 R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} + \hat{c}_4 \Box R}_{\text{4-derivs}} + \underbrace{\frac{\hat{c}_5}{m^2} R^3 + \dots}_{\text{6 derivs.}}$$

$m \ll M_P$

$\hat{c}_0 M^4$: 0-deriv ?

$\frac{M_P^2}{2} R$: 2-deriv

$M_4^2 = M_6^2 r$

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* Leading terms have: $L=0$, $V_{dk}=0$ except V_{2k} = anything.

ie: classical graphs using only Einstein-Hilbert action.

aphs: * $L=1$ graphs using only $d=2$ vertices (ie Einstein Hilbert action

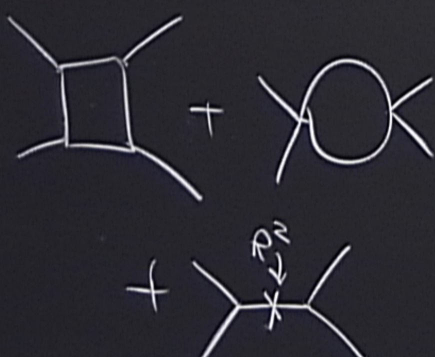
* $L=0$ graphs with V_{2k} = anything and $V_{4k}=1$, $V_{dk}=0$ if $d>4$.

… is justified if $q \ll m \ll M_P$

What is the next-to-leading correction?

2 kinds of graphs:

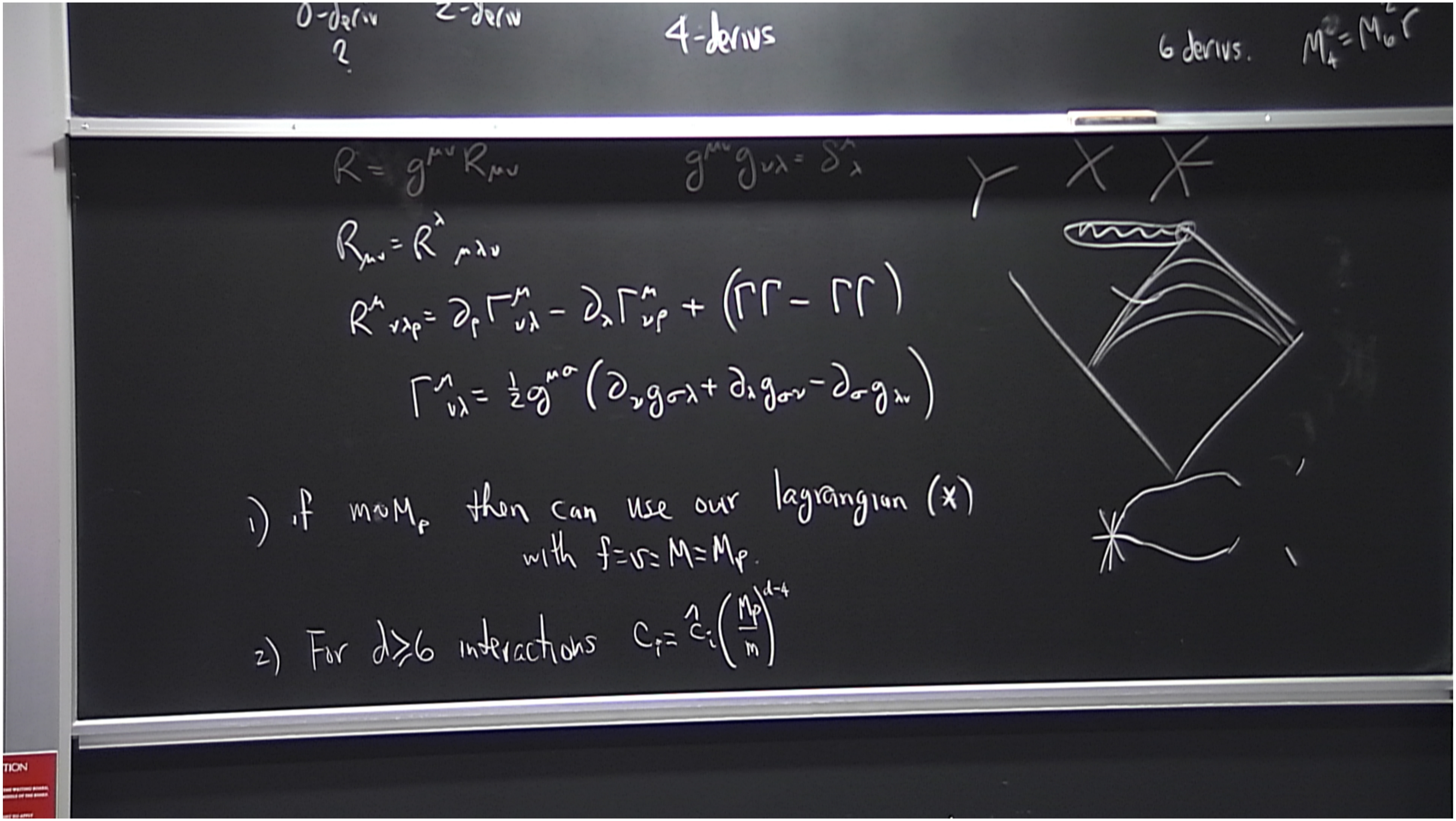
* $L=1$ graphs using only $d=2$ vertices (ie Einstein Hilbert action)
* $L=0$ graphs with $V_{2k} =$ anything and $V_{4k}=1$, $V_{dk}=0$ if $d>4$.



with V_{2k} = any

$$A_4^{NLO}(q) \simeq \left(\frac{g^2}{M_p^2}\right) * \begin{cases} \left(\frac{g}{4\pi M_p}\right)^2 & \text{if } L=1 \\ \left(\frac{g^2}{M_p^2}\right)\hat{c}_1 & \text{if } L=0 \end{cases}$$

and $V_{4k}=1$, V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