

Title: 13/14 PSI - Quantum Information Review - Lecture 8

Date: Feb 26, 2014 11:30 AM

URL: <http://pirsa.org/14020080>

Abstract:

Unstructured search

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black box: U_x

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$$U_x | \psi \rangle$$

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Can work in span $\{ | \psi \rangle \}$

$$\begin{aligned} U_x | \gamma \rangle &= (I - 2|w\rangle\langle w|) | \gamma \rangle \\ &= | \gamma \rangle - 2\langle w | \gamma \rangle | w \rangle \end{aligned}$$

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Quantum: Grover's algorithm

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 U_x |\gamma\rangle &= (I - 2|\omega\rangle\langle\omega|) |\gamma\rangle \\
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 V|w\rangle &= (2|\gamma\rangle\langle\gamma| - I)|w\rangle \\
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$$V|\gamma\rangle = |\gamma\rangle$$

$$U_x |\omega\rangle = -|\omega\rangle$$

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$$|y\rangle = \frac{1}{\sqrt{n}} |w\rangle + \sqrt{1 - \frac{1}{n}} |w^\perp\rangle$$

$$= \sin(\theta) |w\rangle + \cos(\theta) |w^\perp\rangle$$

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$$= \sin\theta |w\rangle + \cos\theta |w^\perp\rangle$$

$$\sin\theta = \frac{1}{\sqrt{n}}$$

in $\{|\omega\rangle, |\omega^\perp\rangle\}$ basis:

$$U_x = I - 2|\omega\rangle\langle\omega| = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

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a rotation by 2θ .

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$$(2k+1)\theta \approx \frac{\pi}{2} \Rightarrow k \approx \frac{\pi}{4\theta} \approx \frac{\pi}{4} \sqrt{n}.$$

$$\begin{pmatrix} \sin 2k\theta \\ \cos 2k\theta \end{pmatrix} \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}$$

So $O(\sqrt{n})$ queries allow us to find w .

$$\begin{pmatrix} \sin 2(L+1)\theta \\ \cos 2(L+1)\theta \end{pmatrix}$$

$$\sin^2(2(L+1)\theta)$$

$\times k$ to make

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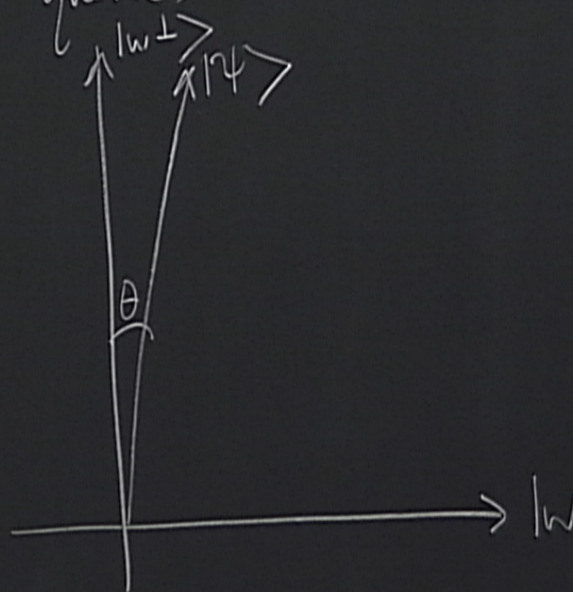
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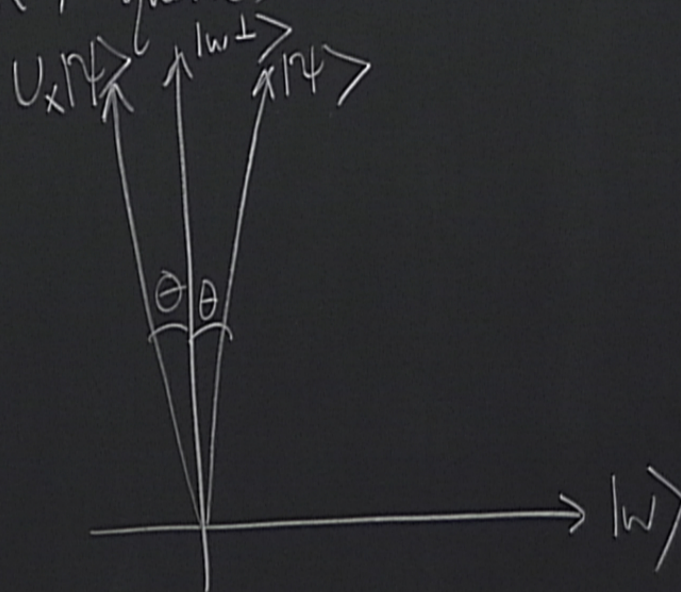
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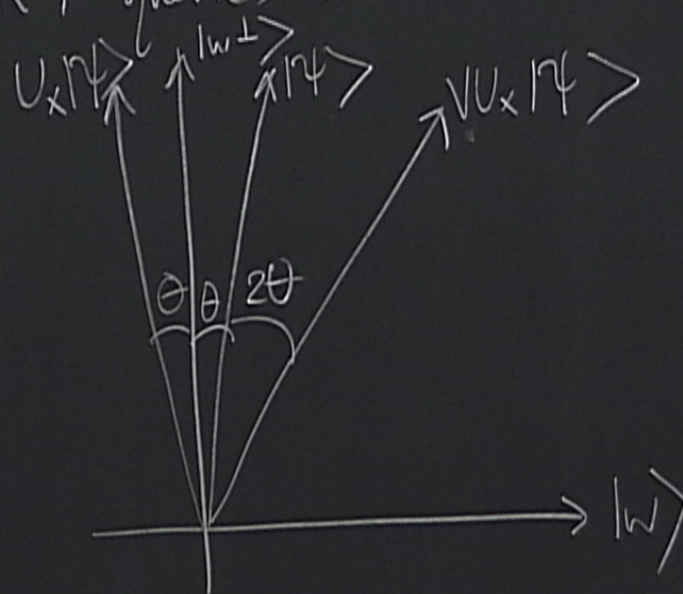
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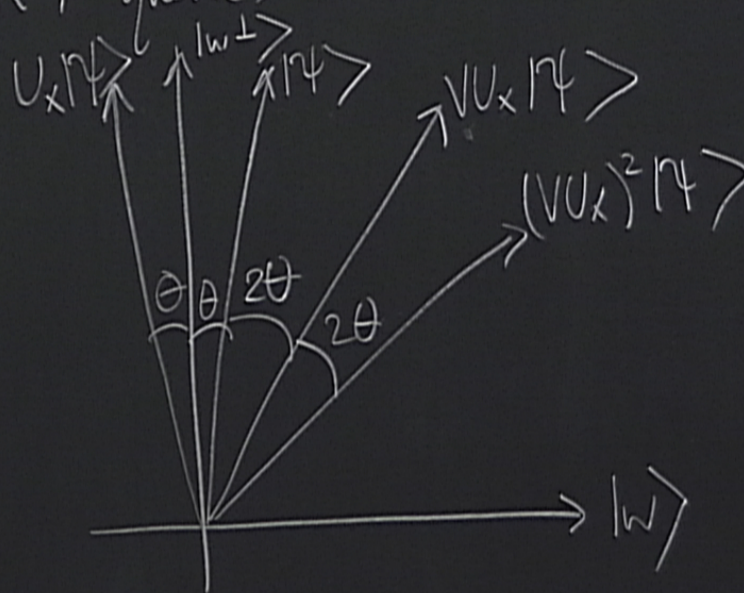


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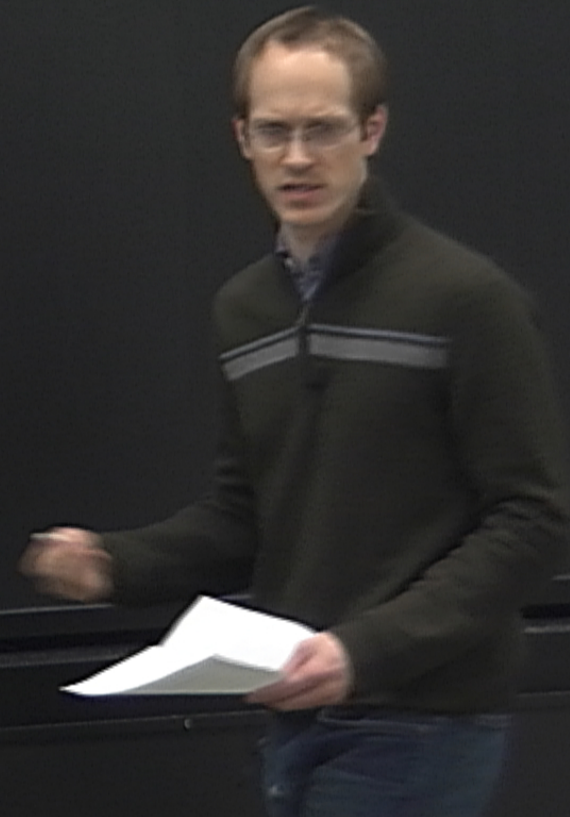
$\Omega(\sqrt{n})$ queries are necessary

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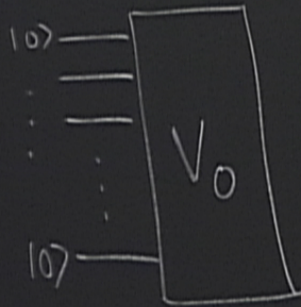
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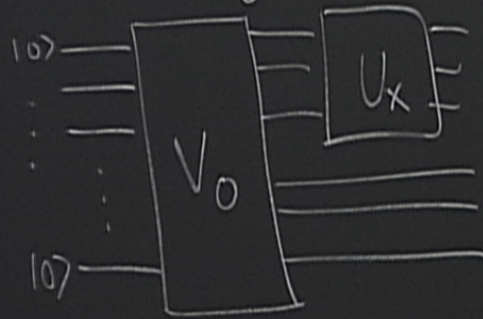
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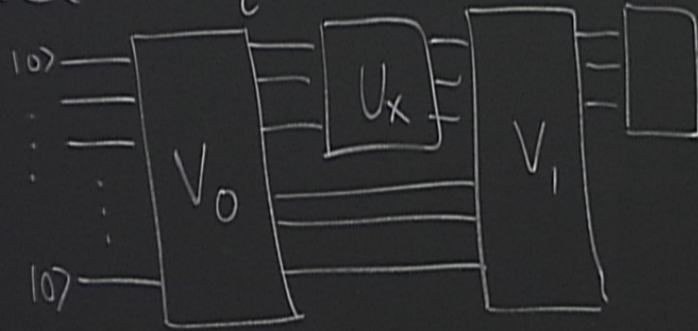
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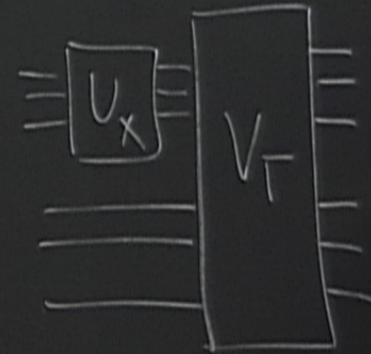
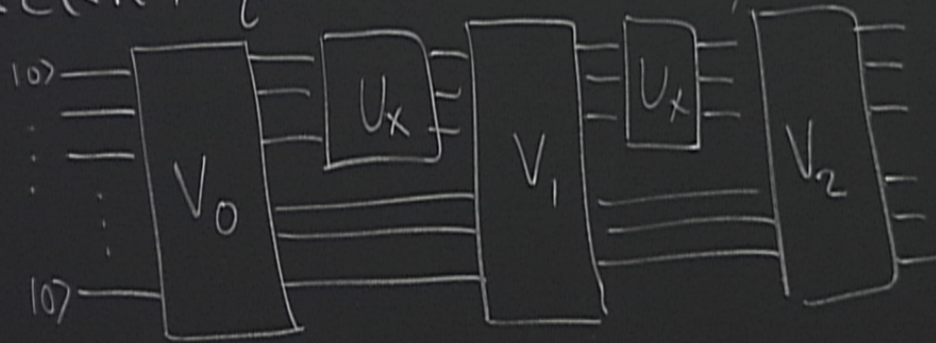
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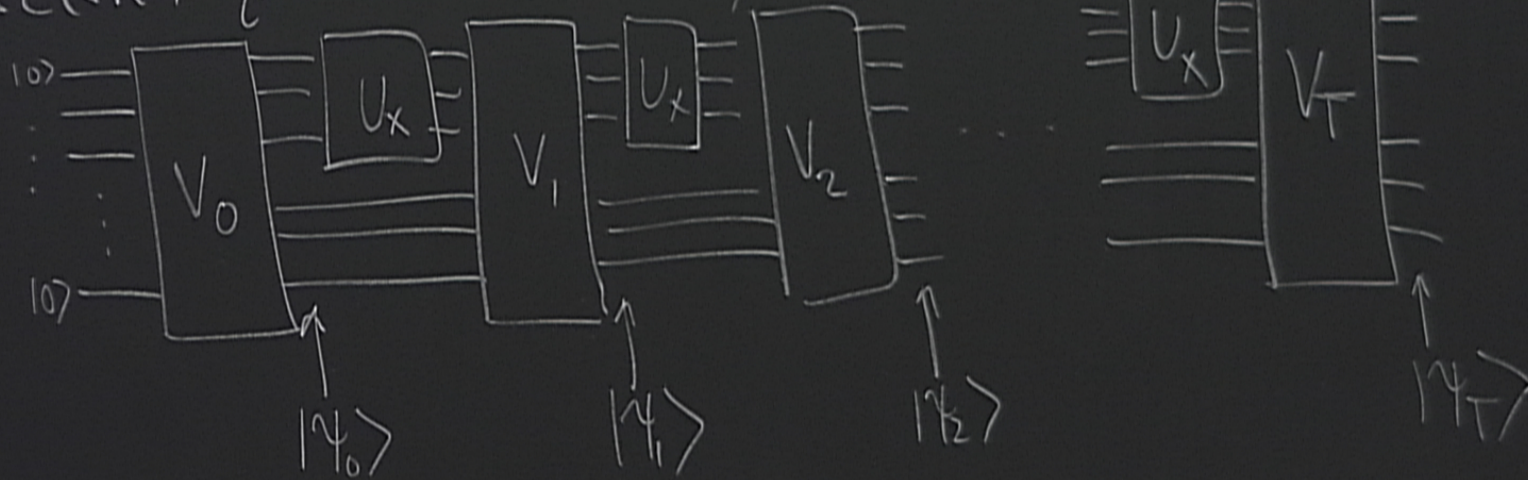
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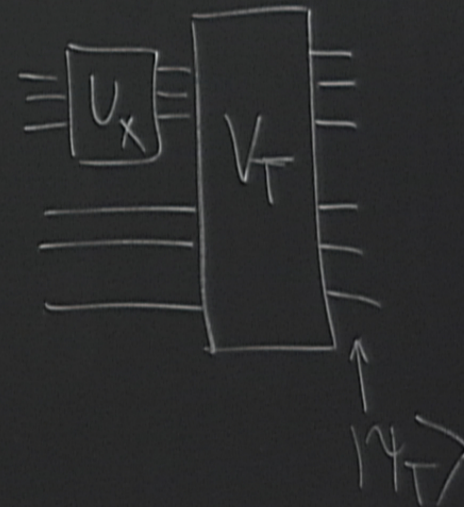
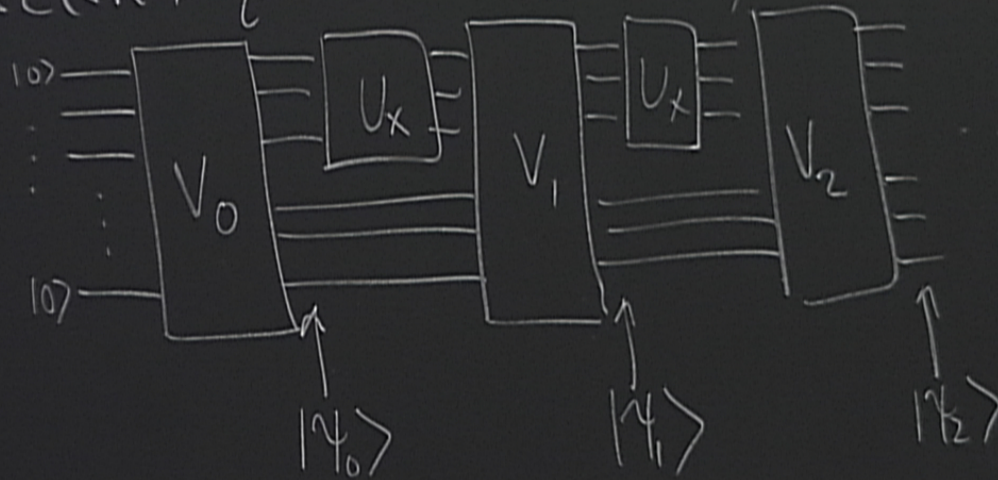
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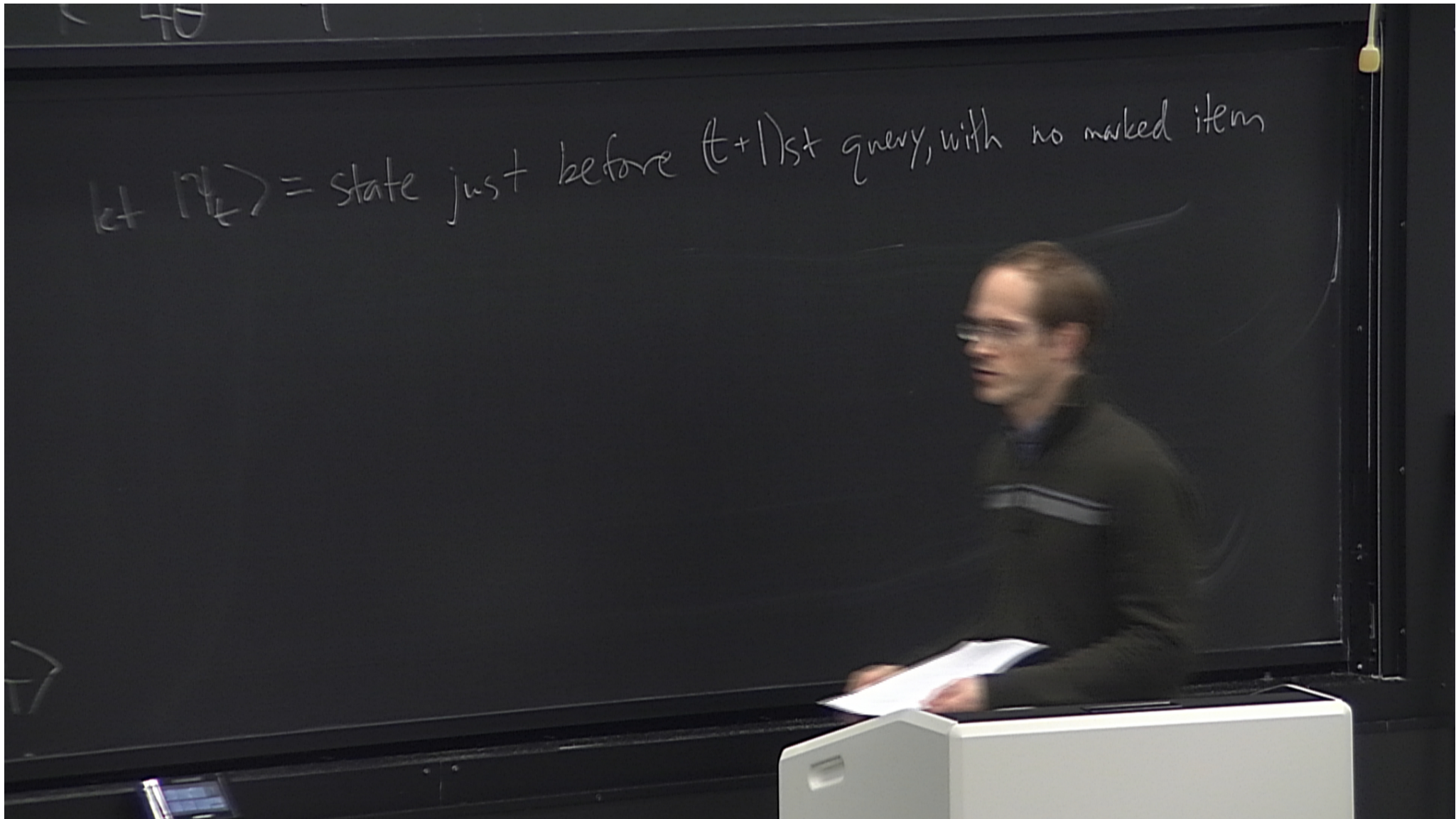


Optimality

$\Omega(\sqrt{n})$ queries are necessary



let $|y_t\rangle =$



$|v_t\rangle = \text{state just before } (t+1)\text{st query, with no marked item}$
 $|v_t^i\rangle = \text{ " " " " " " , with } x_i = 1 \text{ (marked item } i)$
 $\phantom{|v_t^i\rangle = \text{ " " " " " " , with } } x_j = 0 \quad j \neq i$

let $|\psi_t\rangle$ = state just before $(t+1)$ st query, with no marked item
 $|\psi_t^i\rangle$ = " " " " " " , with $x_i = 1$ (marked item i)
 $x_j = 0$ $j \neq i$

to distinguish "no marked item" from "marked item = i "
 must have $\|\psi_t\rangle - \psi_t^i\rangle\| \geq \epsilon$ for some $\epsilon > 0$

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this should hold $\forall i \Rightarrow \sum_i \|\psi_t\rangle - \psi_t^i\rangle\| \geq N\epsilon$

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$$\sum_i \| |\psi_0\rangle - |\psi_0^i\rangle \| = 0$$

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$$\sum_i \| |\psi_0\rangle - |\psi_0^i\rangle \| = 0$$