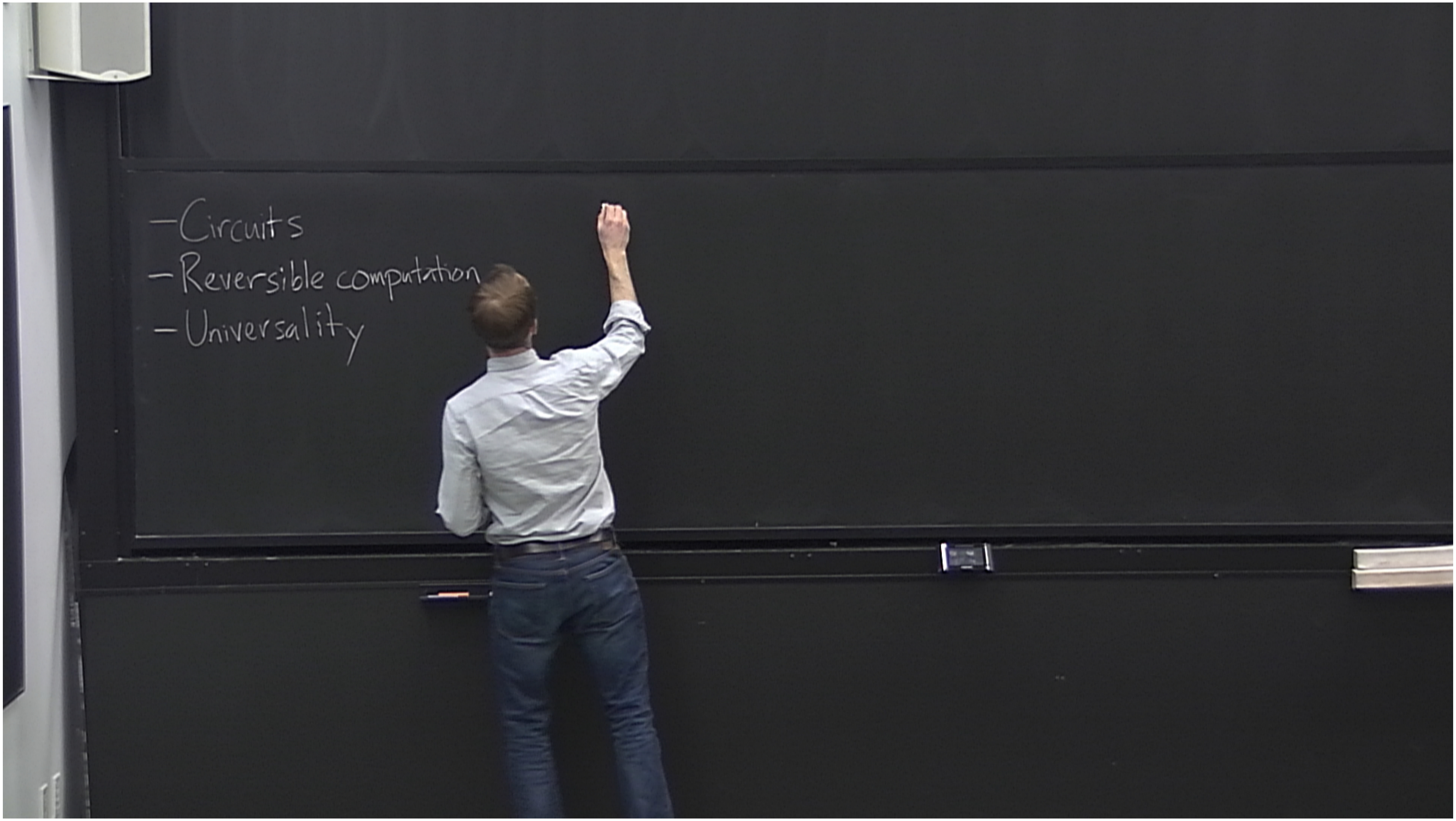
Title: 13/14 PSI - Quantum Information Review - Lecture 2

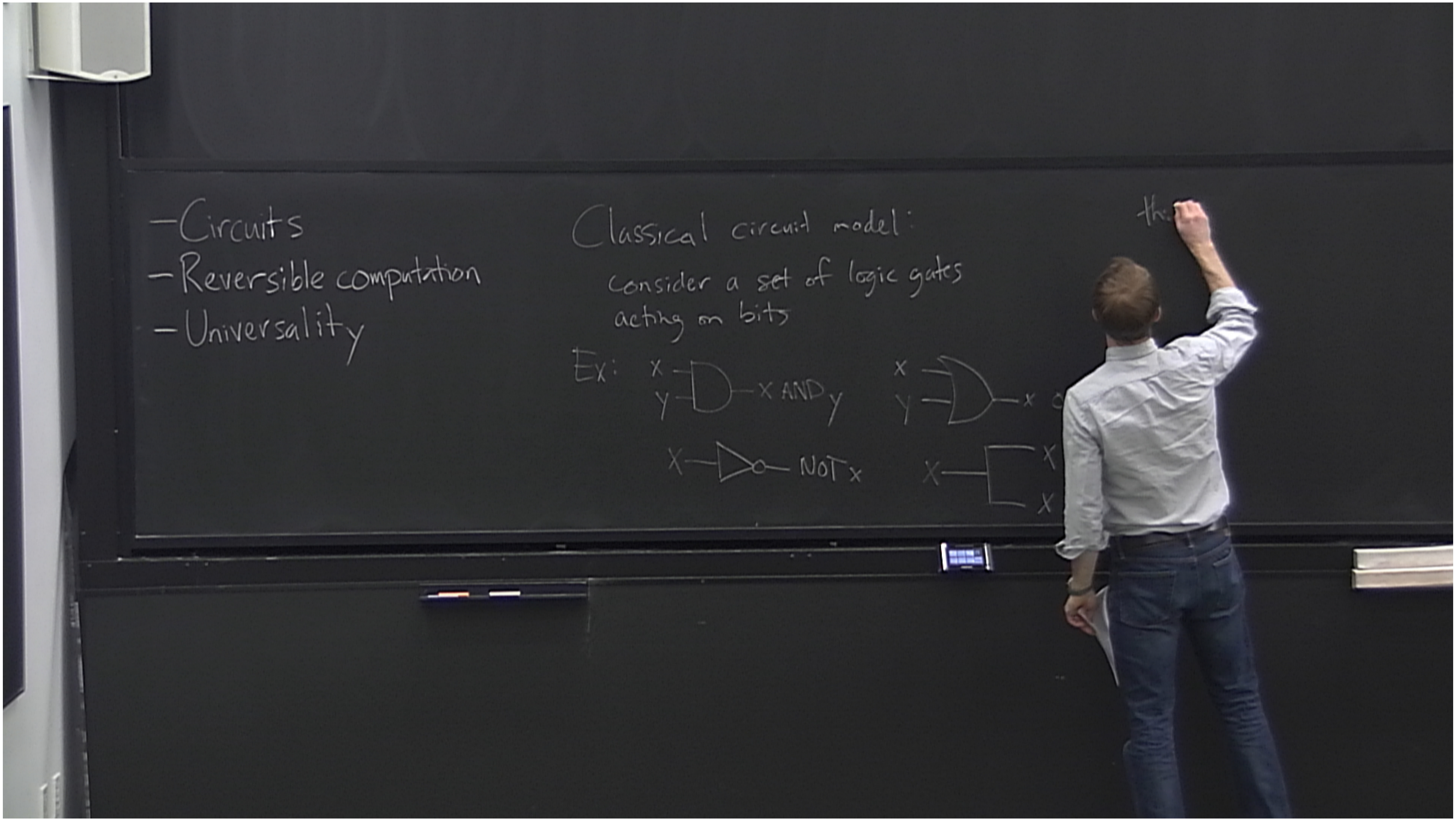
Date: Feb 19, 2014 11:30 AM

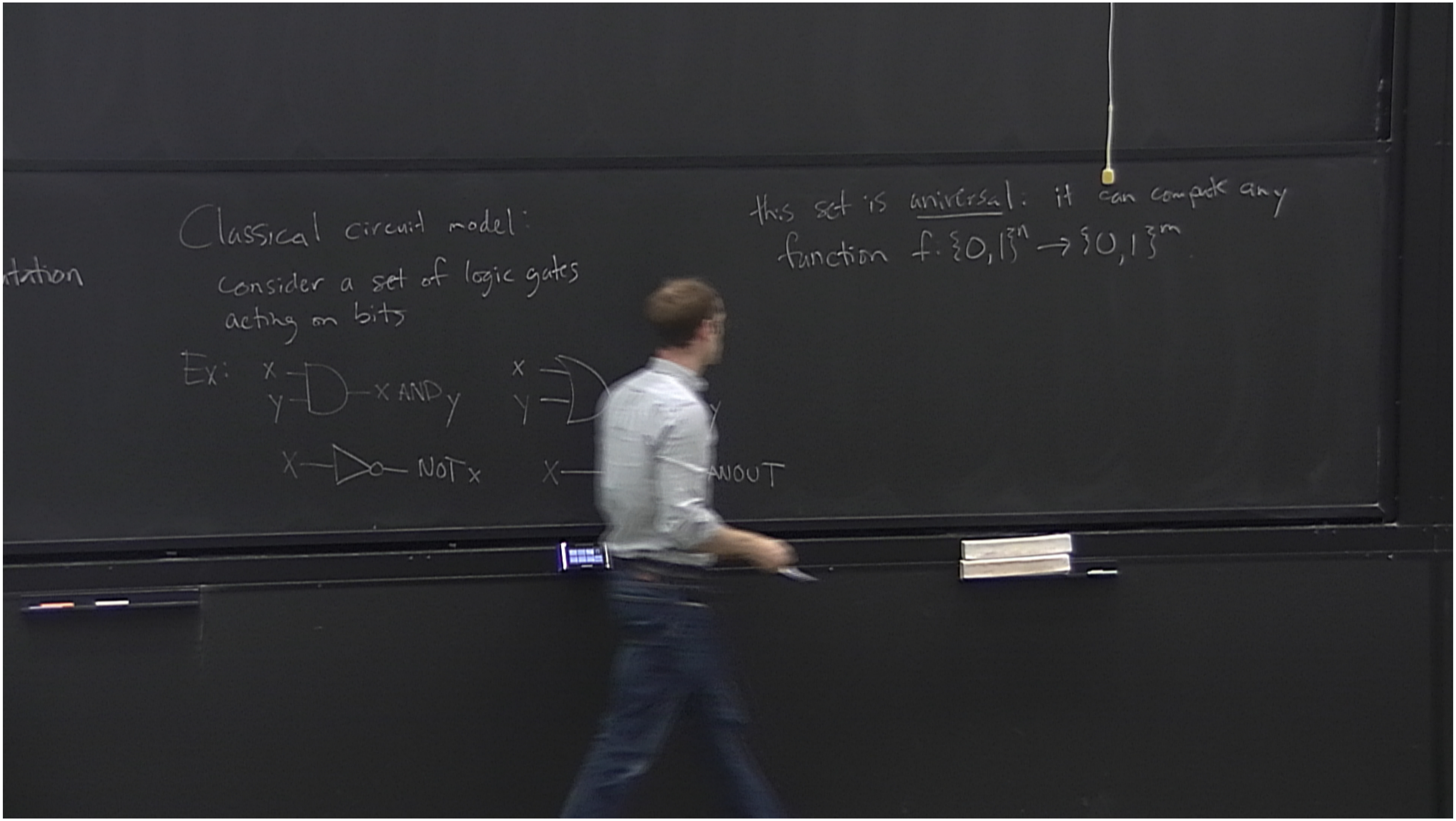
URL: http://pirsa.org/14020073

Abstract:



- Circuits
- Reversible computation
- Universality


-Circuits
-Reversible computation
-Universality
Classical circuit model:
consider a set of logic gates
acting on bits
Ex:
x AND y
NOT x


Classical circuit model:
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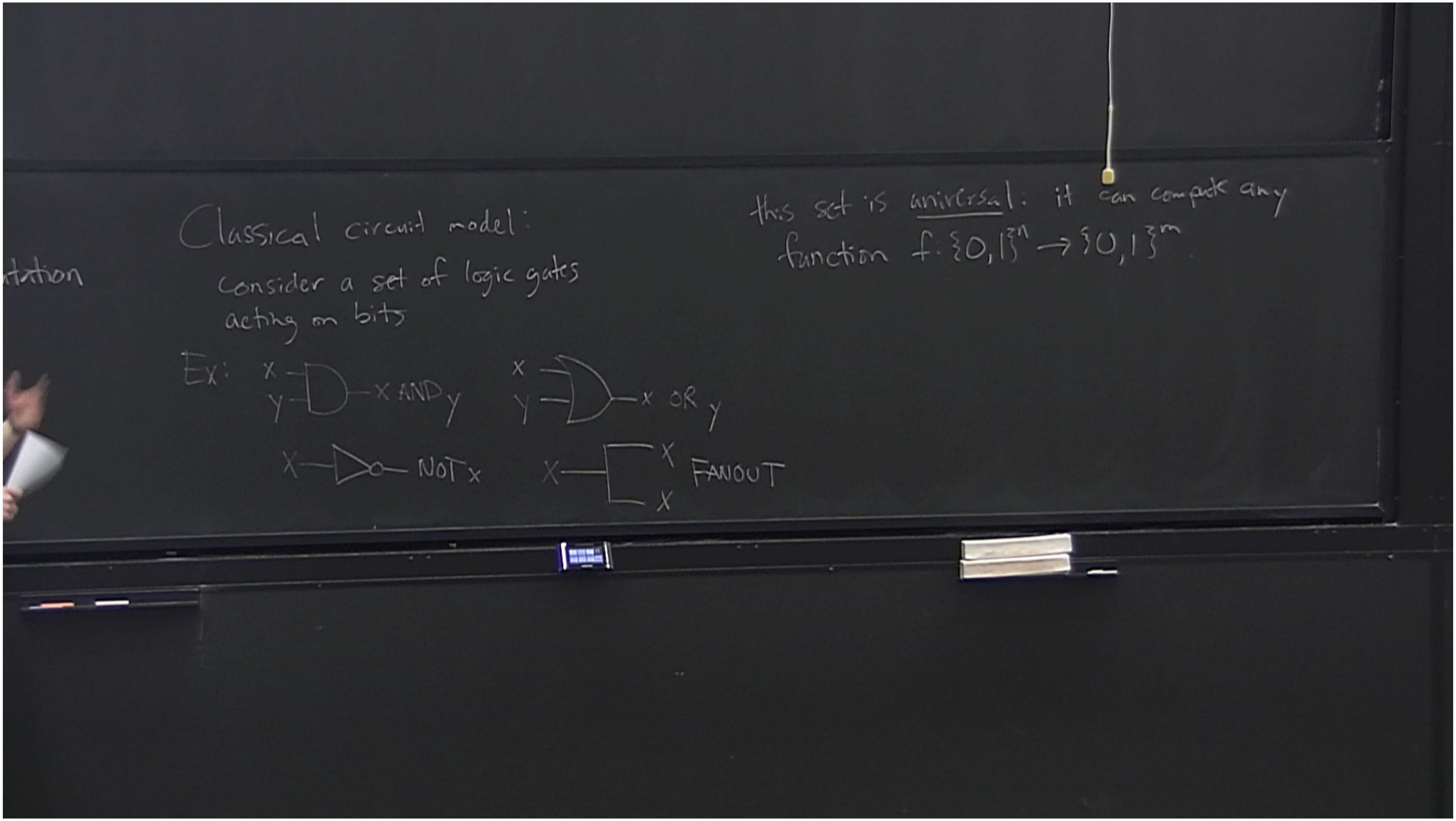
Ex:
x AND y
NOT x
this set is universal: it can compute any
function f: {0,1}^n → {0,1}^m

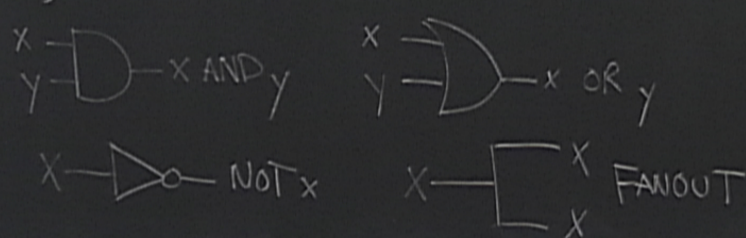

Classical circuit model:
consider a set of logic gates
acting on bits
Ex:
x
y
x AND y
x
y
x OR y
x
NOT x
x
x
x
FANOUT
this set is universal: it can compute any
function f: {0,1}^n → {0,1}^m

...mputation

Classical circuit model:

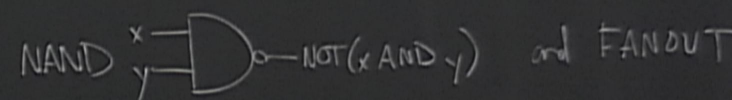
consider a set of logic gates acting on bits

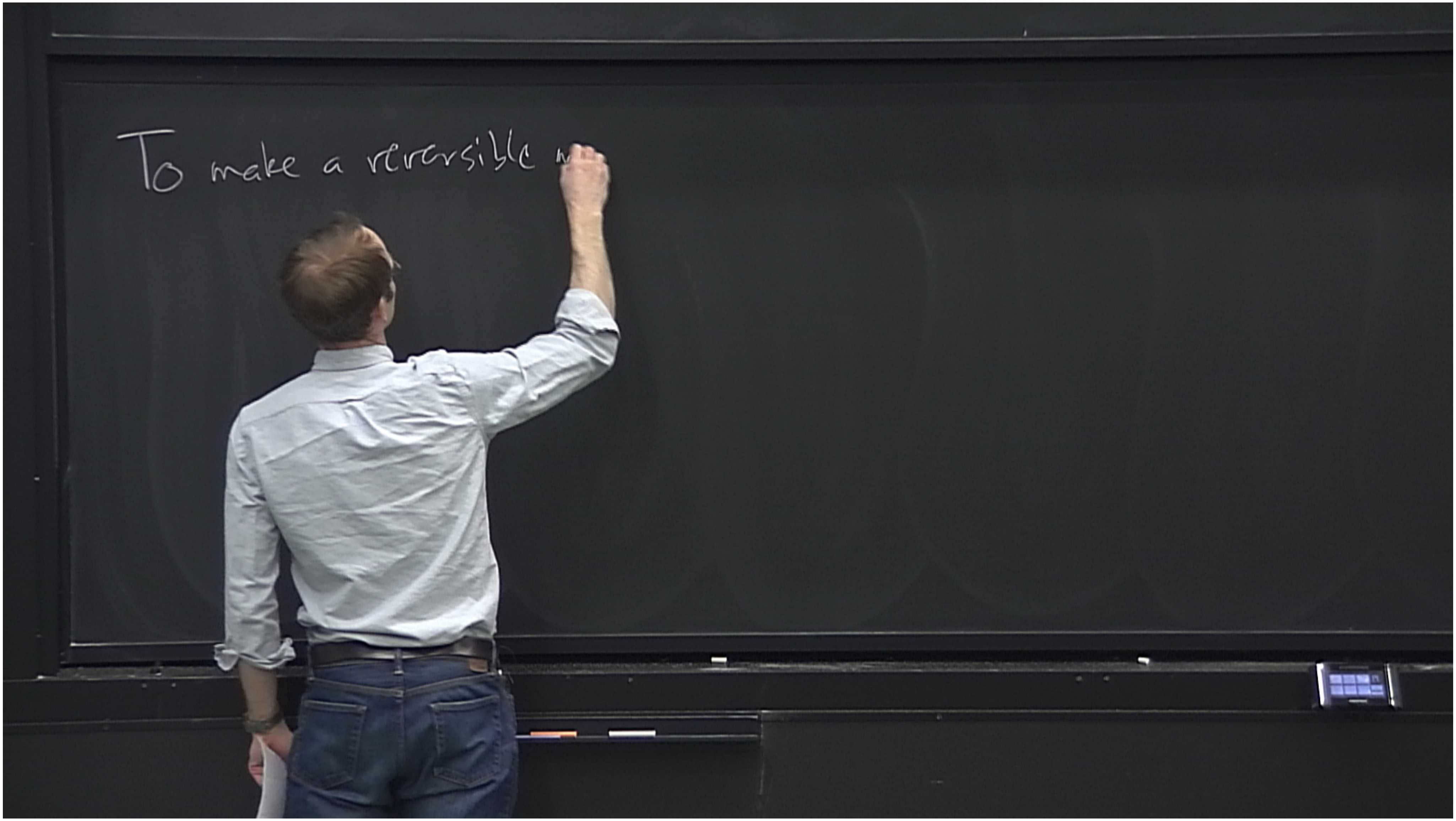
Ex:

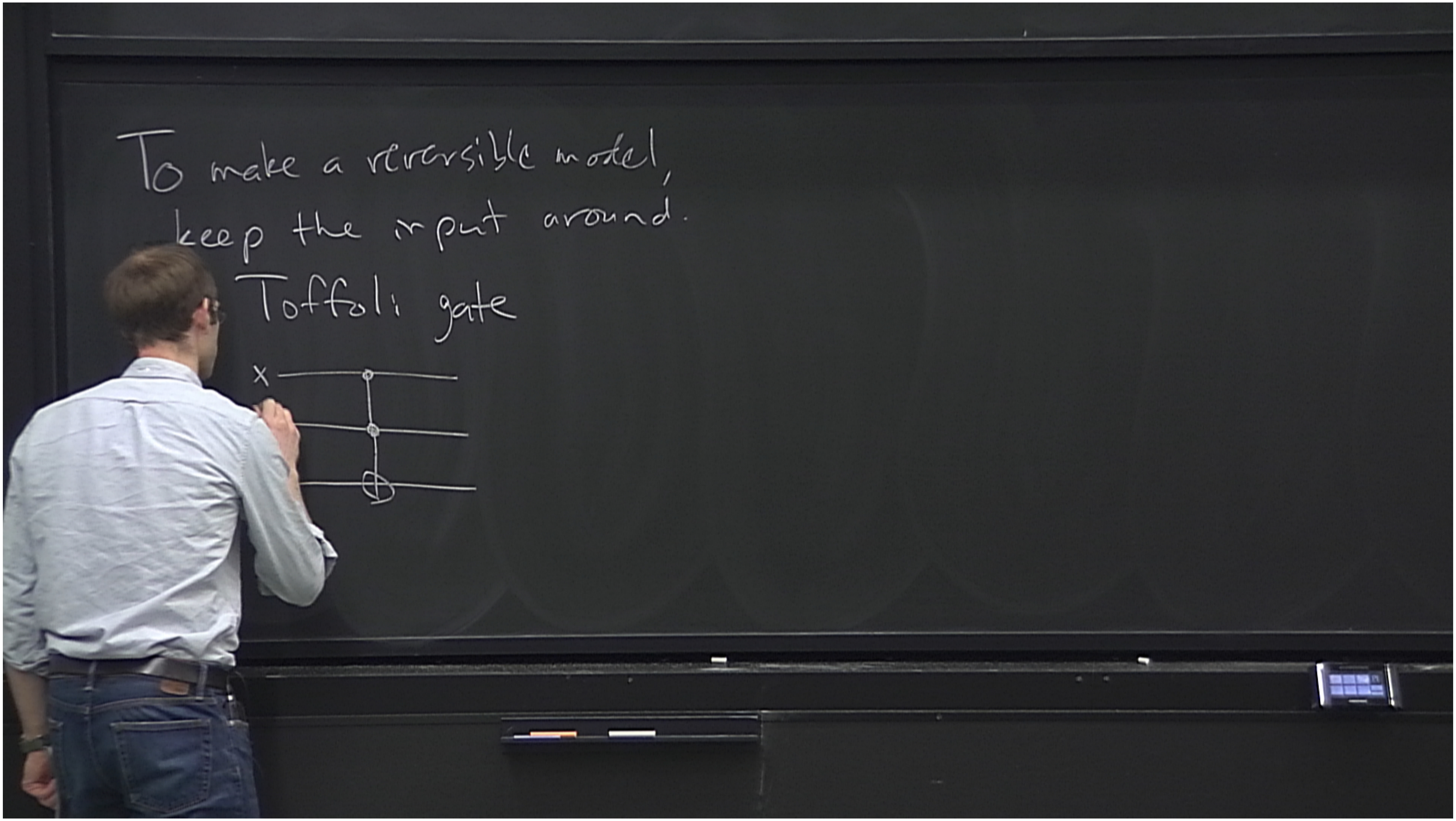


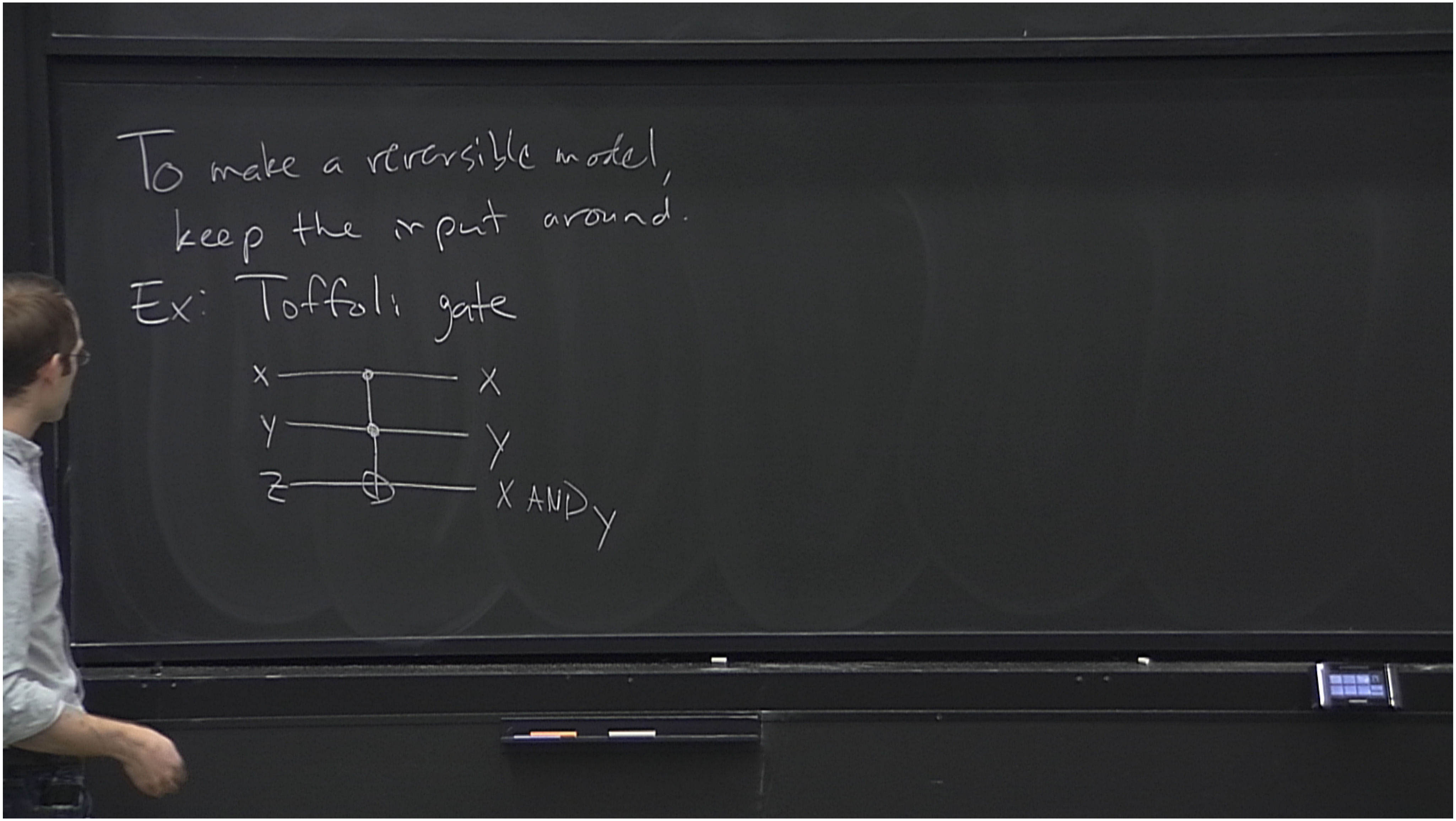
this set is universal: it can compute any function $f: \{0,1\}^n \to \{0,1\}^m$

Ex:



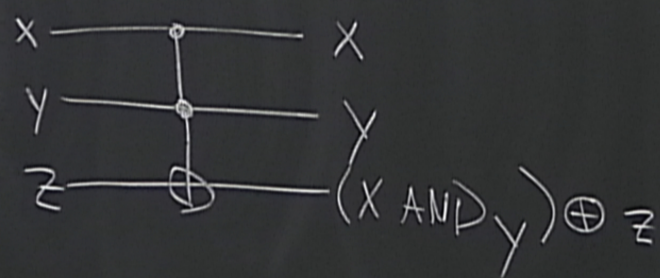

To make a reversible m

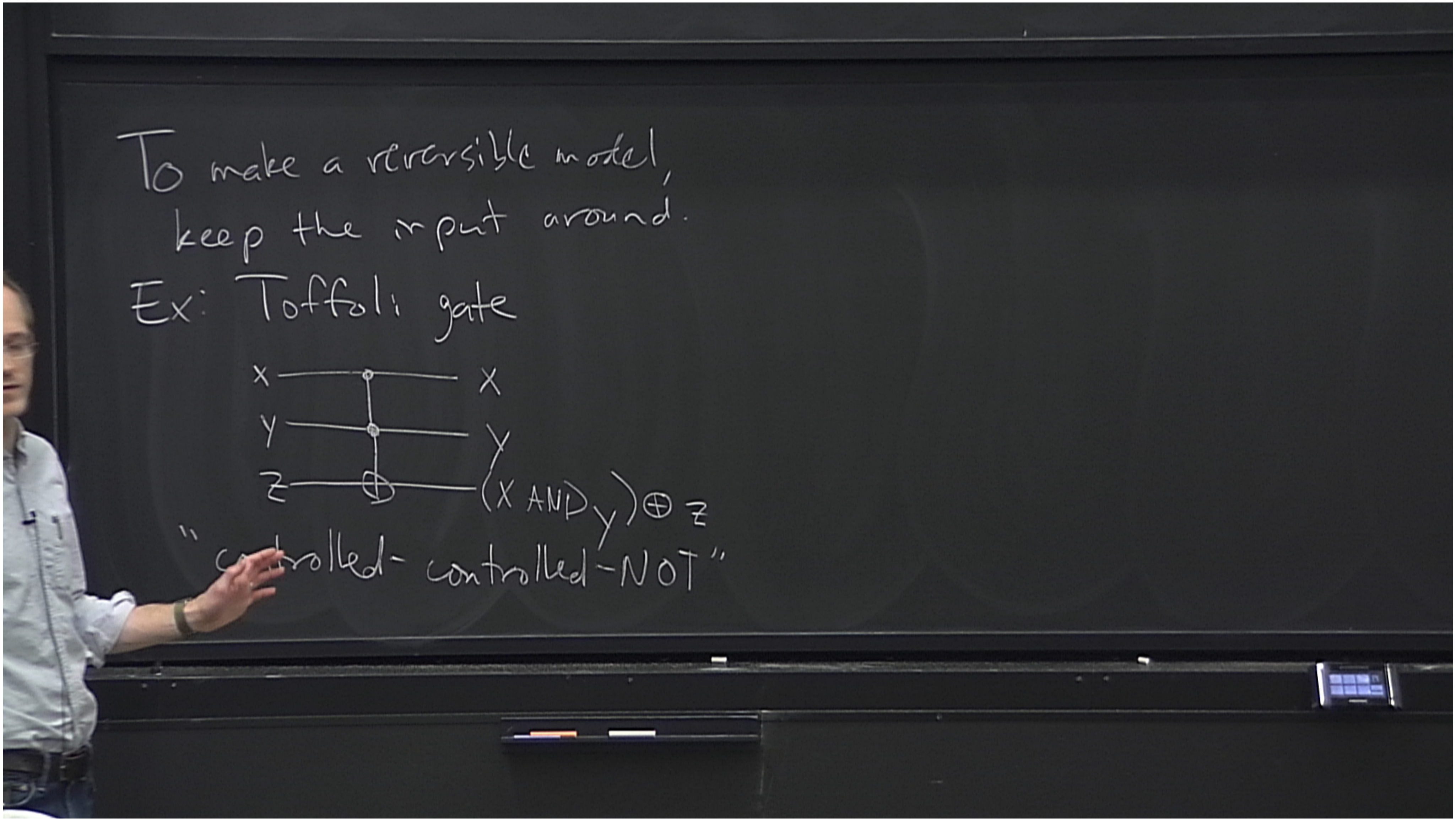

To make a reversible model,
keep the input around.
Toffoli gate
x

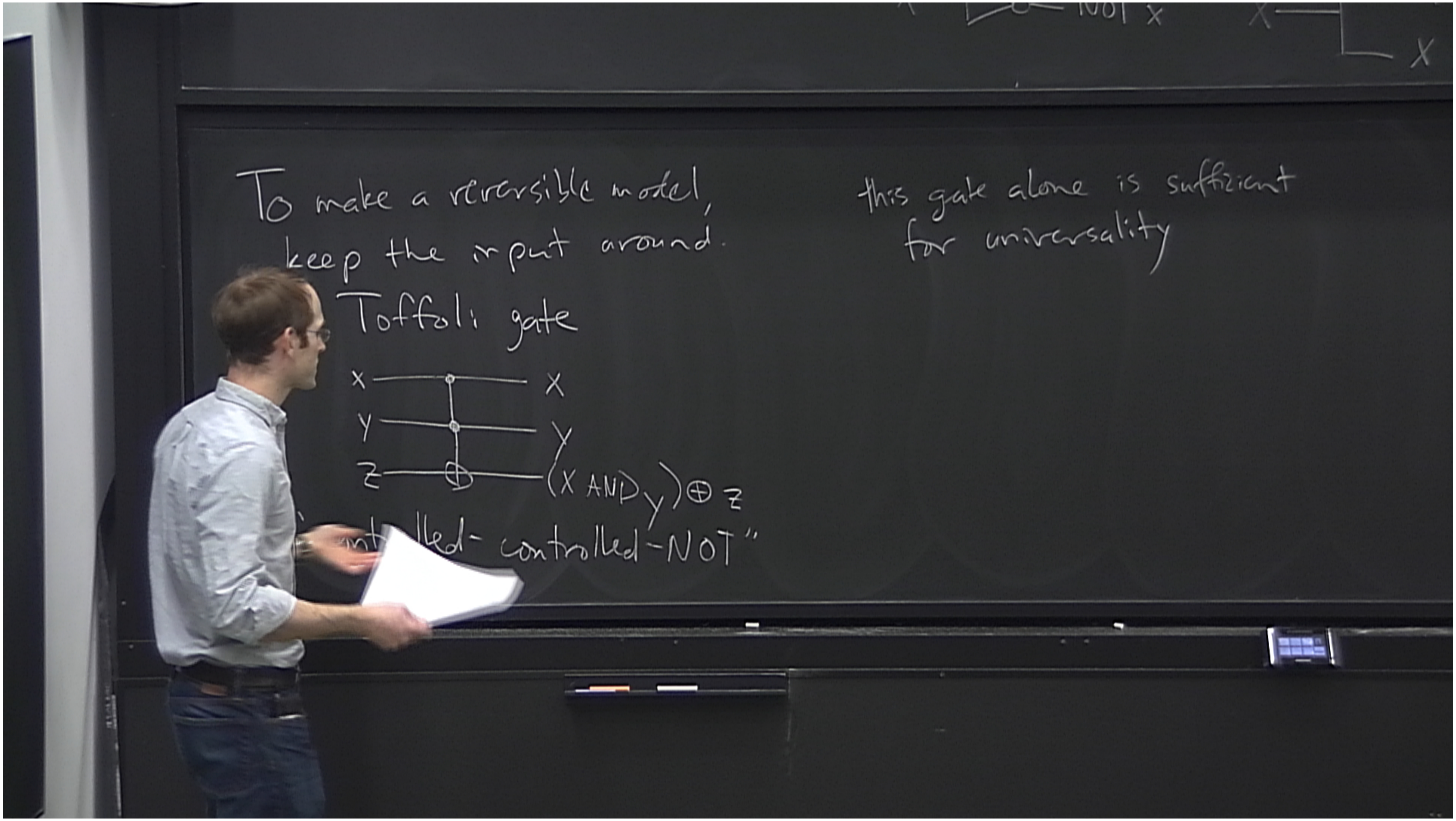

To make a reversible model,
keep the input around.
Ex: Toffoli gate
x
y
z
x
y
x AND y

To make a reversible model,
keep the input around.

Ex: Toffoli gate




To make a reversible model,
keep the input around.
Ex: Toffoli gate
x
y
z
x
y
(x AND y) ⊕ z
"controlled-controlled-NOT"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y
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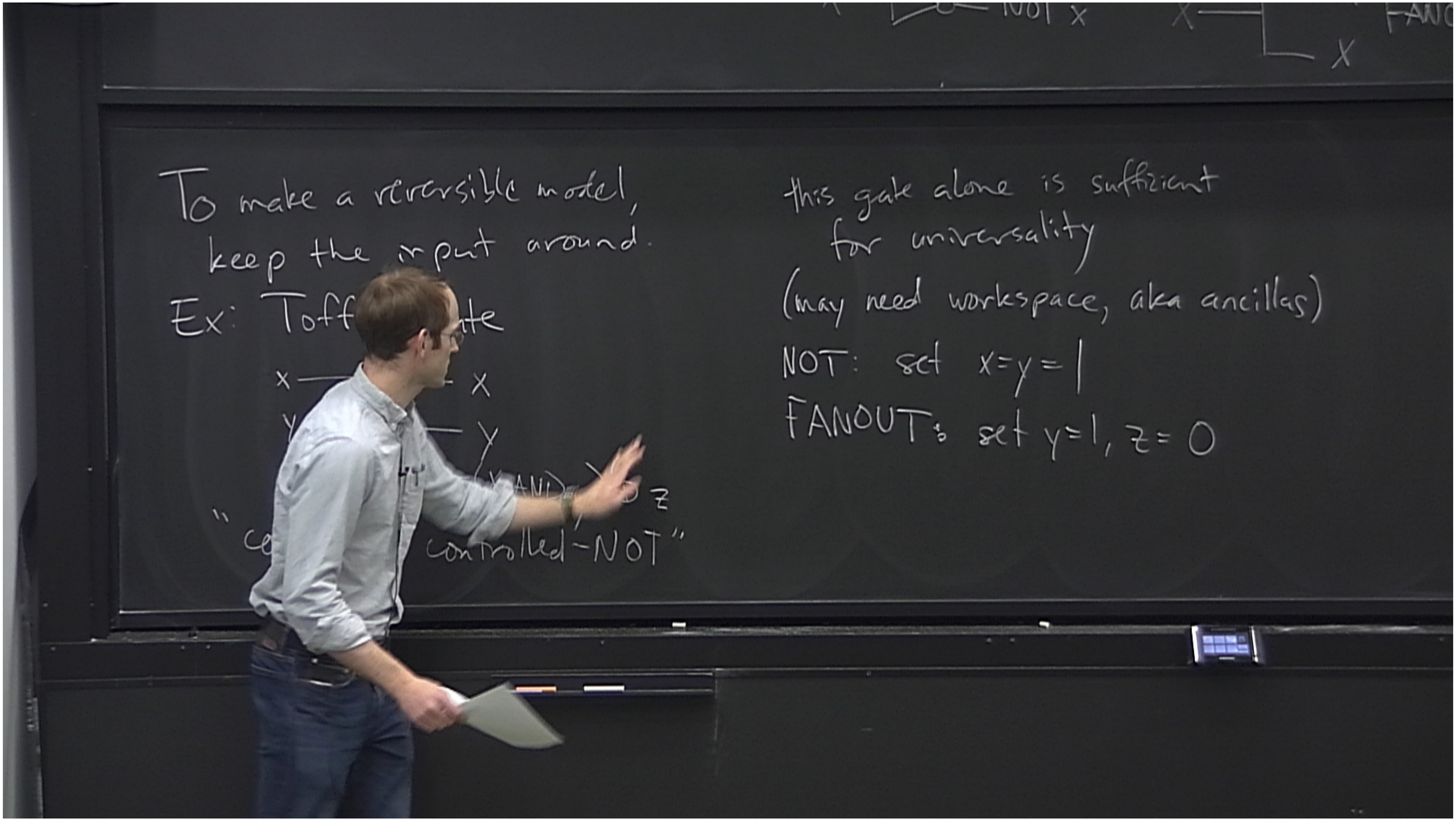
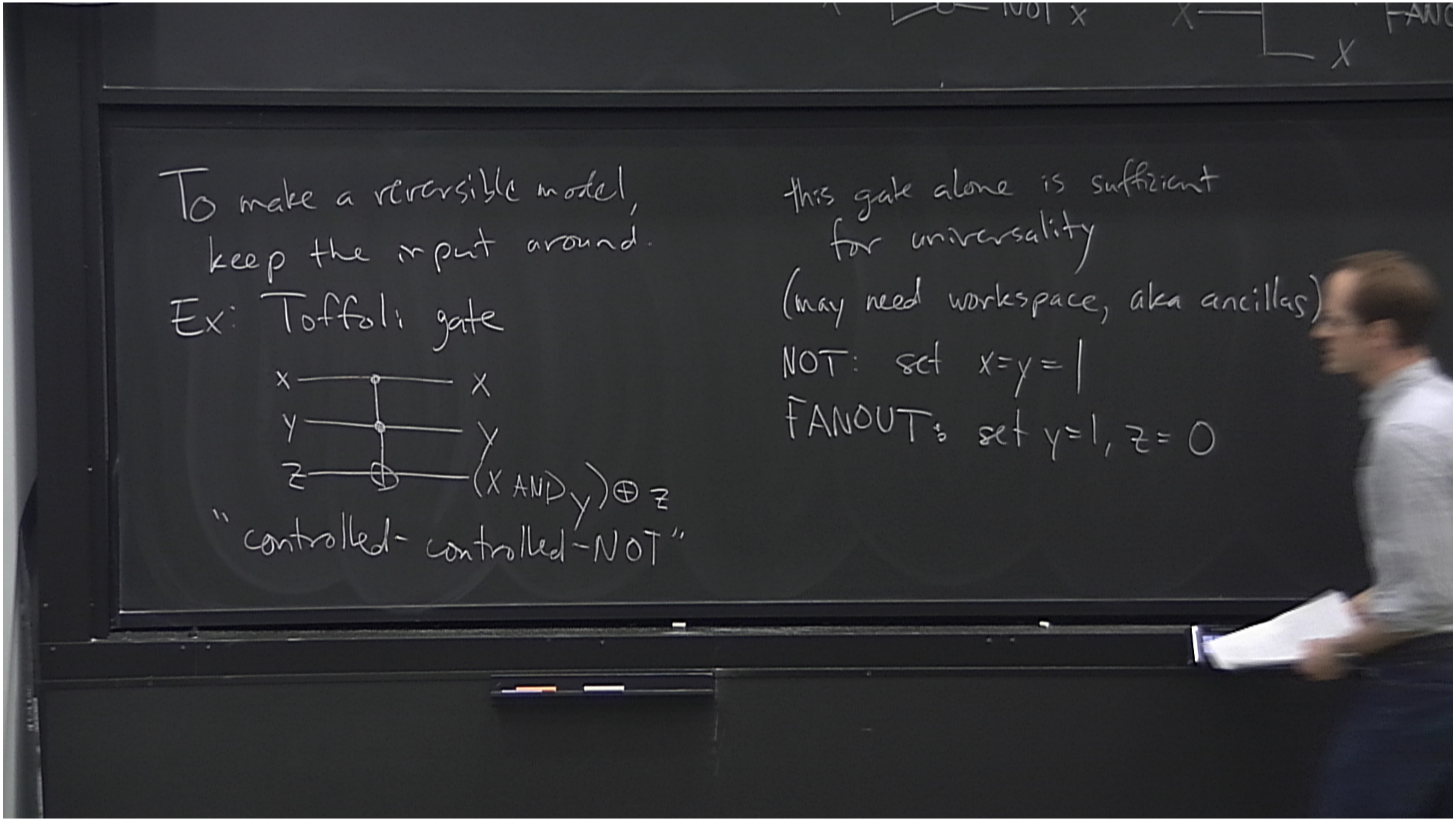
this gate alone is sufficient
for universality


To make a reversible model,
keep the input around.
this gate alone is sufficient
for universality
(may need workspace, aka ancillas)
NOT: set x=y=1
FANOUT: set y=1, z=0
controlled-NOT"


To make a reversible model,
keep the input around.
Ex: Toffoli gate
x
y
z
x
y
(x AND y) ⊕ z
"controlled-controlled-NOT"
this gate alone is sufficient
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NOT: set x=y=1
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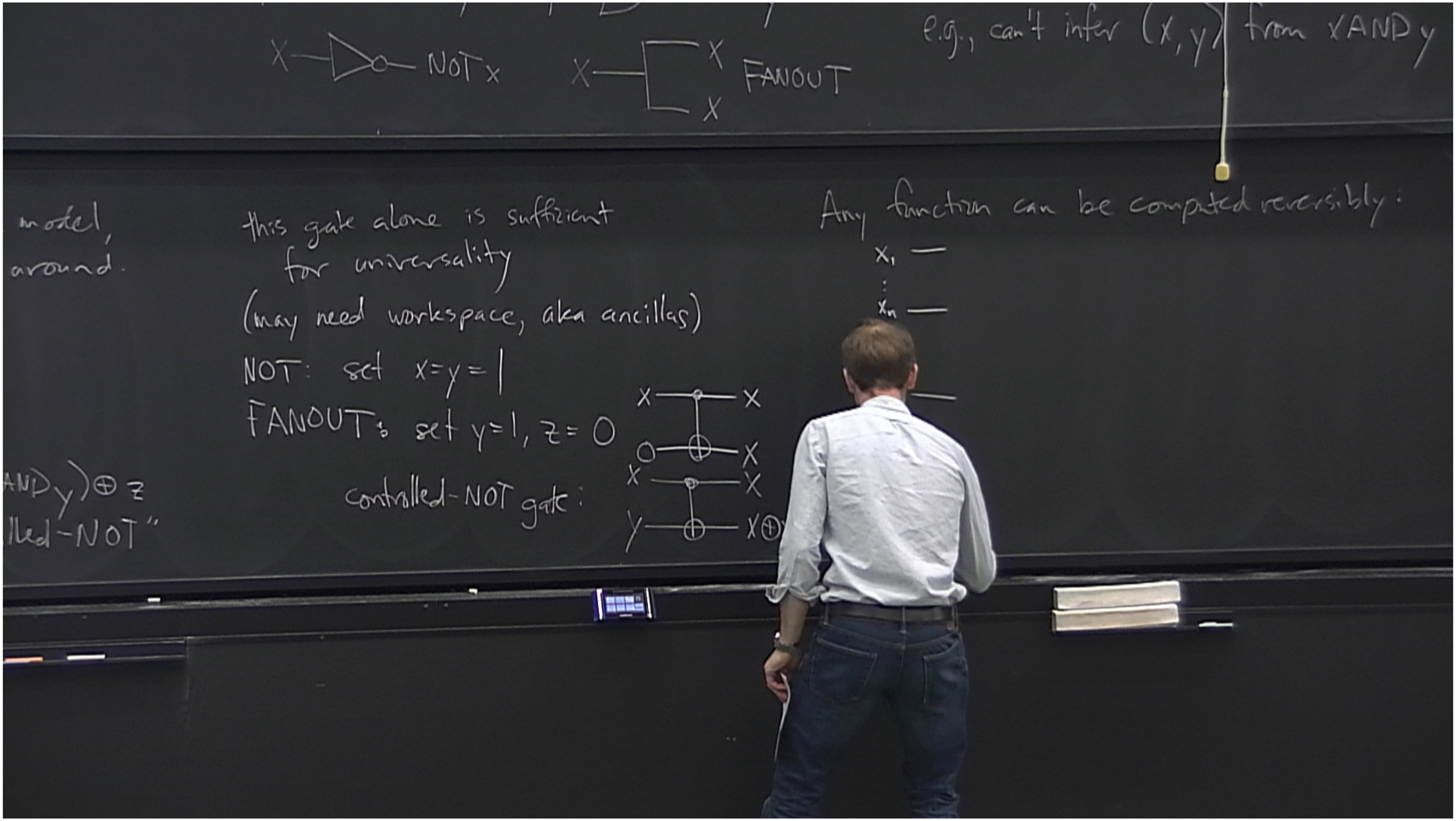
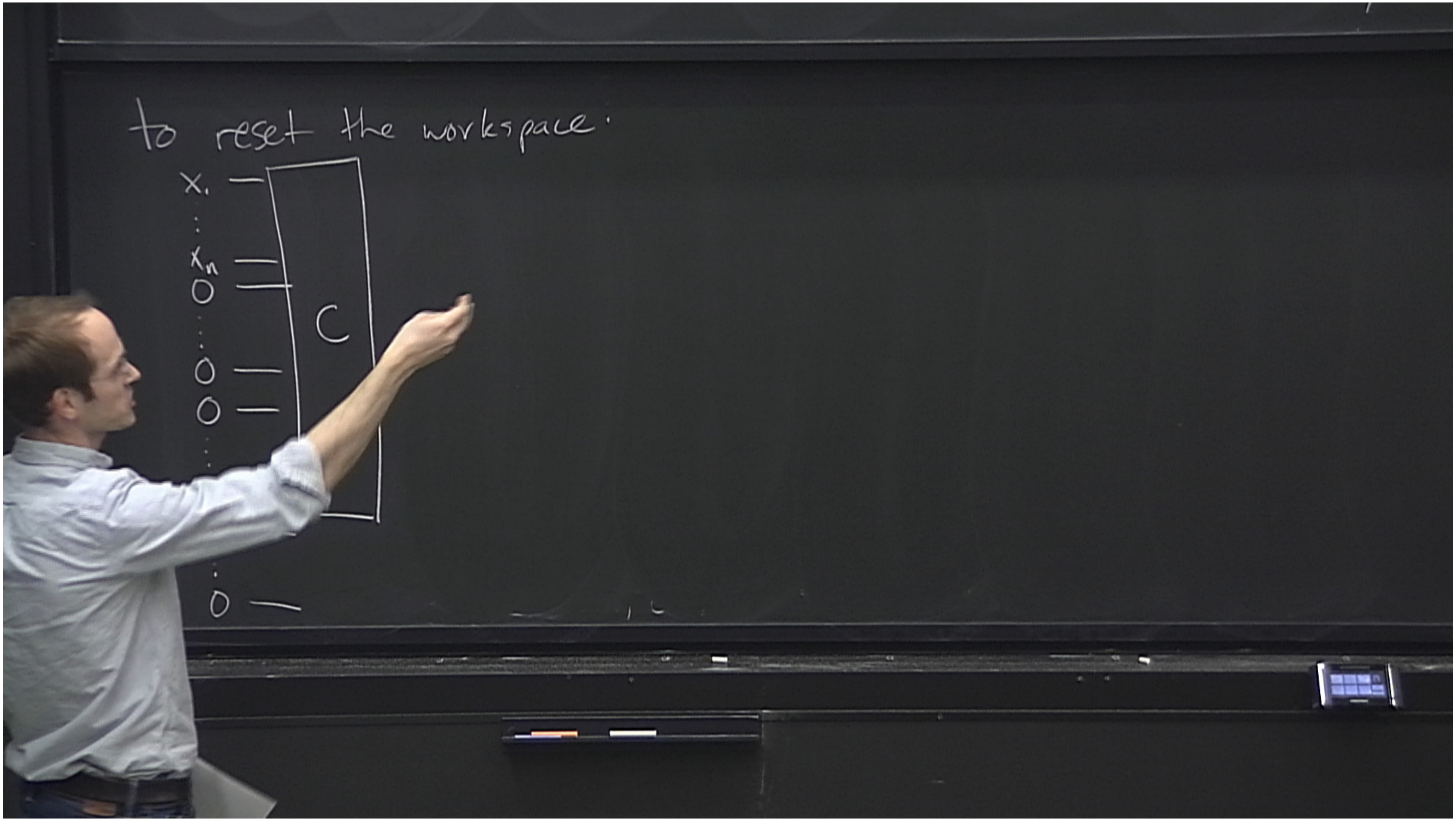
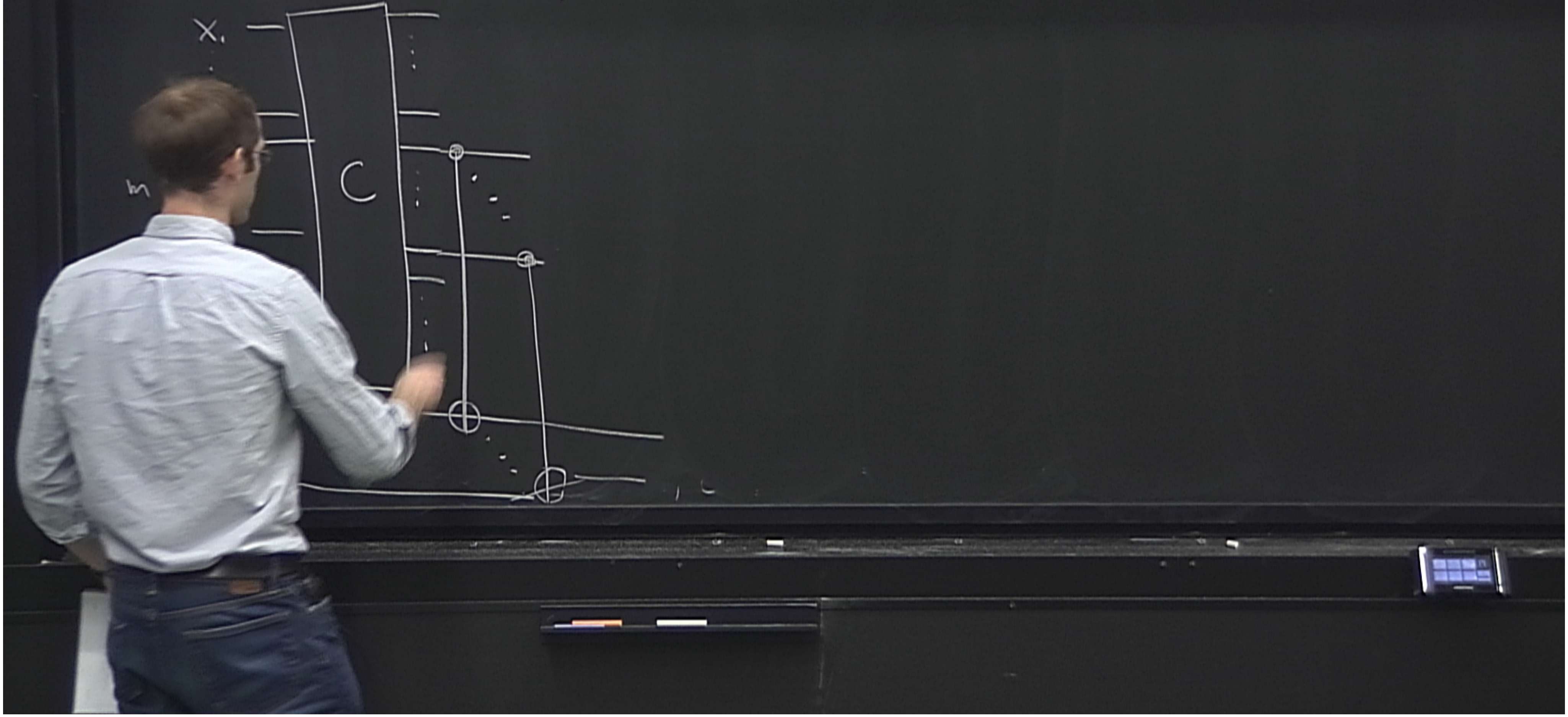
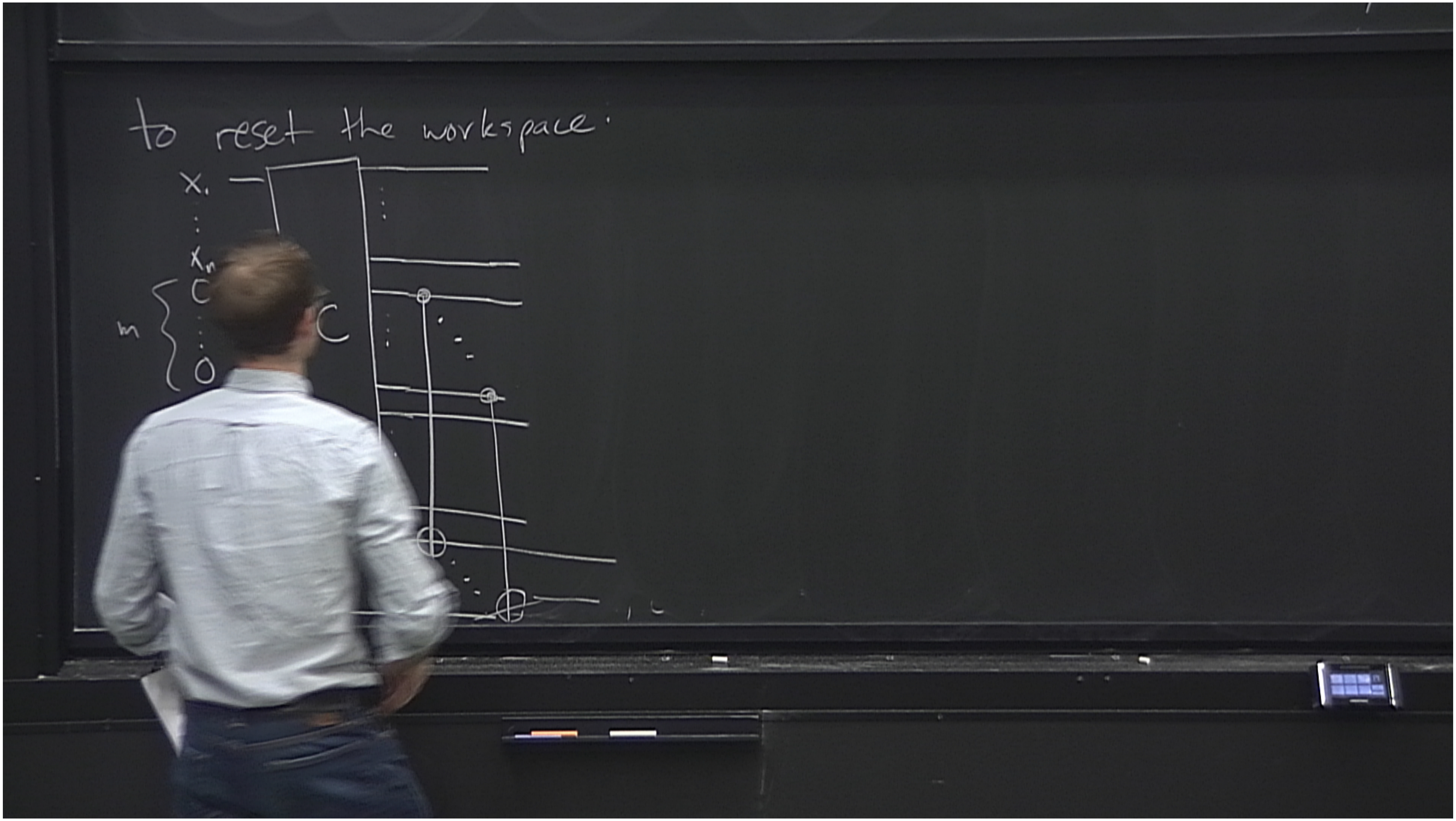
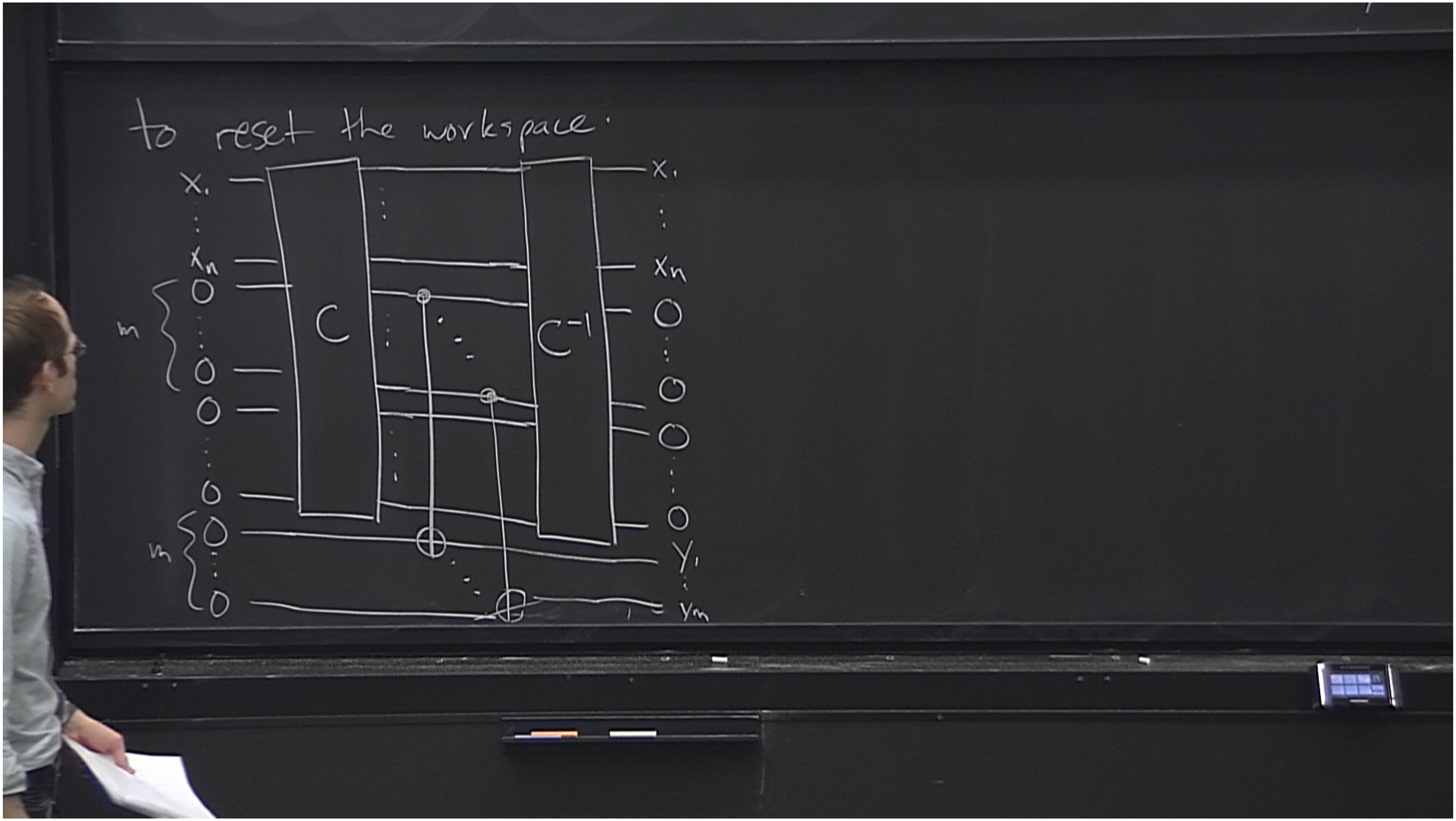

e.g., can't infer (x, y) from x AND y
NOT x
FANOUT
model,
around.
this gate alone is sufficient for universality
(may need workspace, aka ancillas)
NOT: set x = y = 1
FANOUT: set y = 1, z = 0
controlled-NOT gate:
AND y) ⊕ z
lled-NOT"
Any function can be computed reversibly:

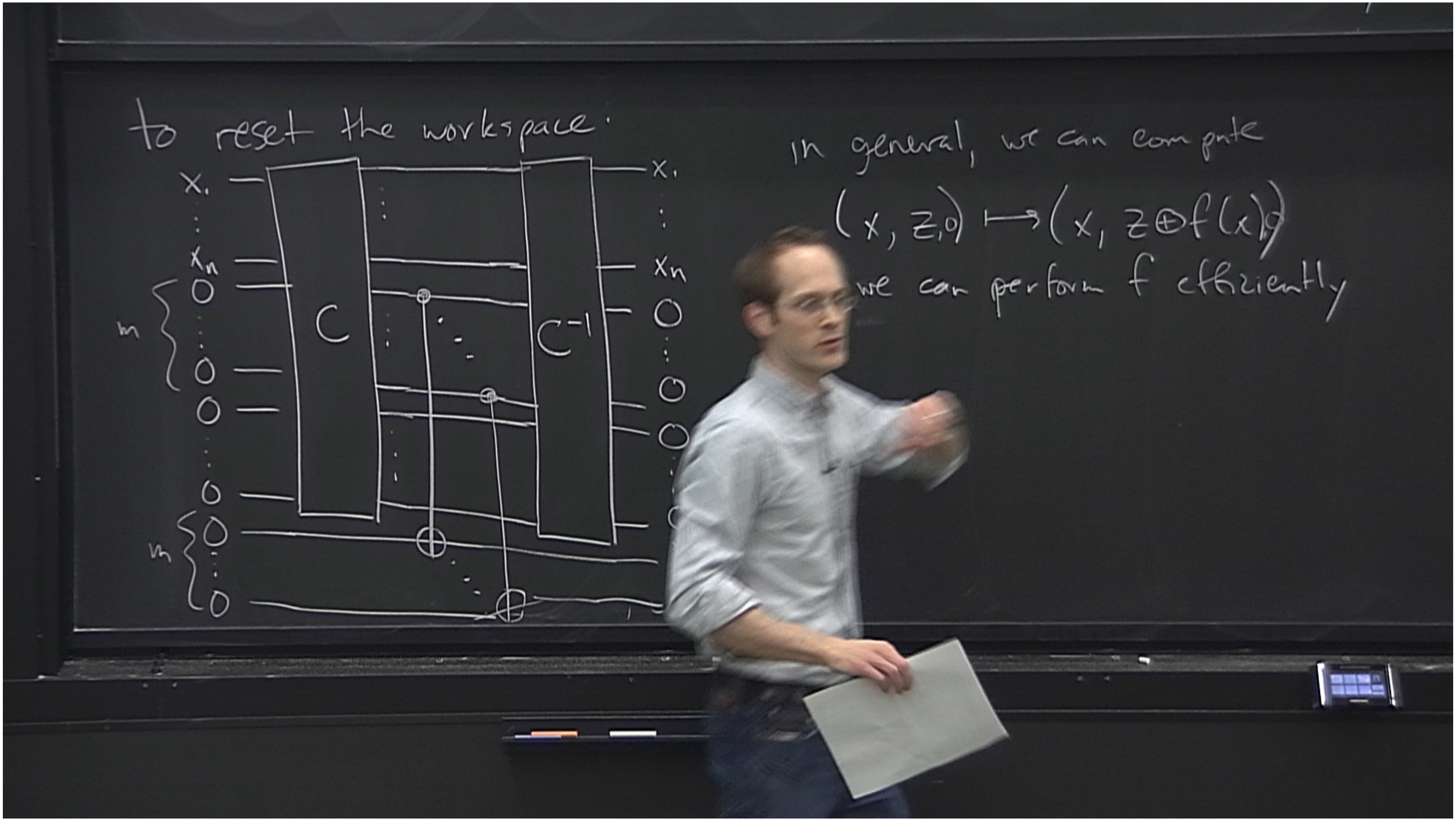

to reset the workspace:
x1
xn
0
0
0
0
C

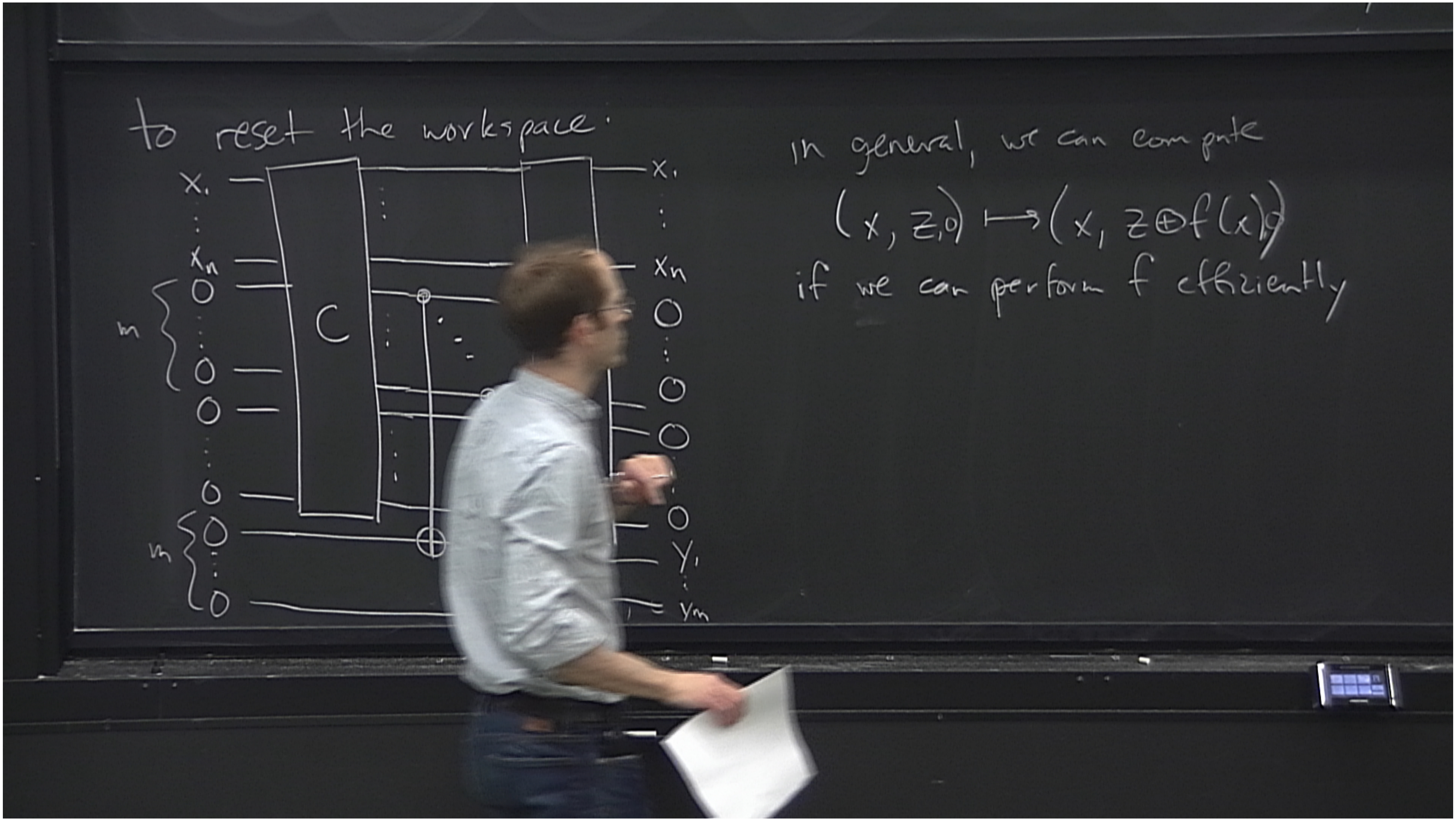
to reset the workspace:

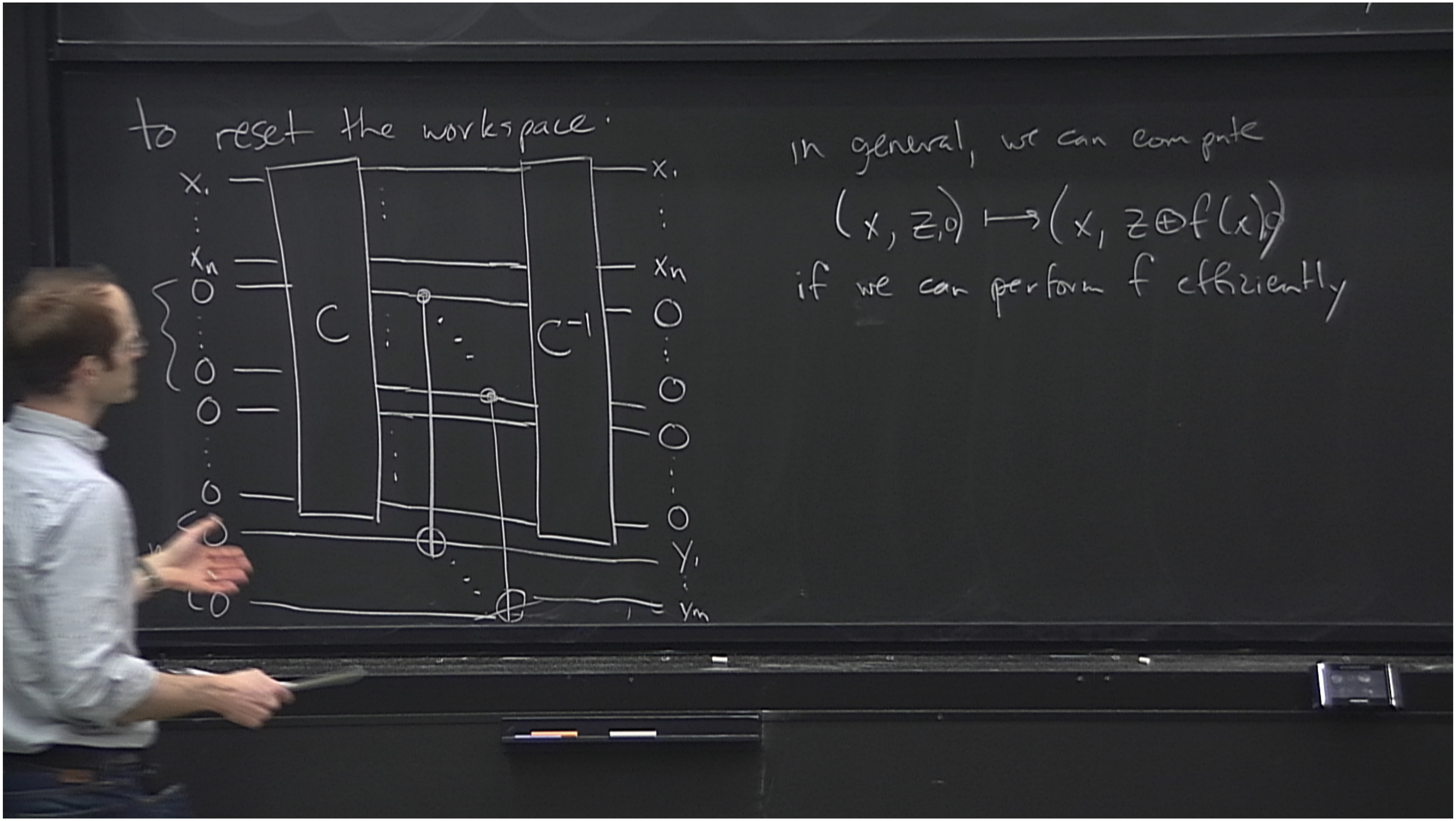


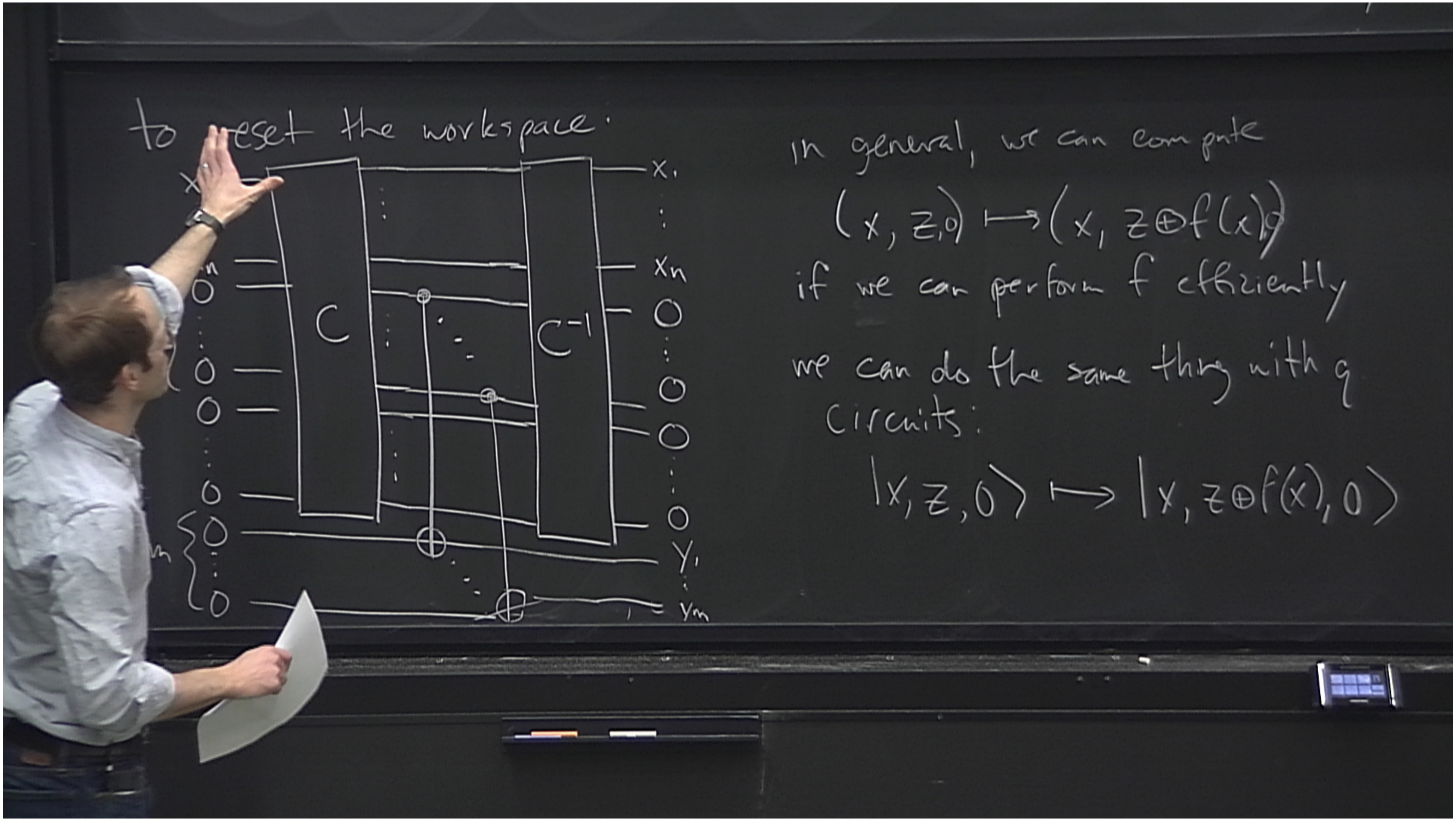

to reset the workspace:
C
m


to reset the workspace.
x_1
x_n
m
0
C
C^{-1}
y_1
y_m


to reset the workspace:
C
C⁻¹
in general, we can compute
(x, z, 0) ⟼ (x, z ⊕ f(x), 0)
we can perform f efficiently


to reset the workspace:
x1
xn
m
0
C
m
y1
ym
in general, we can compute
(x, z, 0) ↦ (x, z ⊕ f(x), 0)
if we can perform f efficiently


to reset the workspace:
x1
xn
C
C^-1
y1
ym
in general, we can compute
(x, z, 0) ↦ (x, z ⊕ f(x), 0)
if we can perform f efficiently


to reset the workspace:
C
C⁻¹
x1
xn
y1
ym
in general, we can compute
(x, z, 0) ↦ (x, z ⊕ f(x), 0)
if we can perform f efficiently
we can do the same thing with q circuits:
|x, z, 0⟩ ↦ |x, z ⊕ f(x), 0⟩

compute

$z \oplus f(x)\rangle$

f efficiently

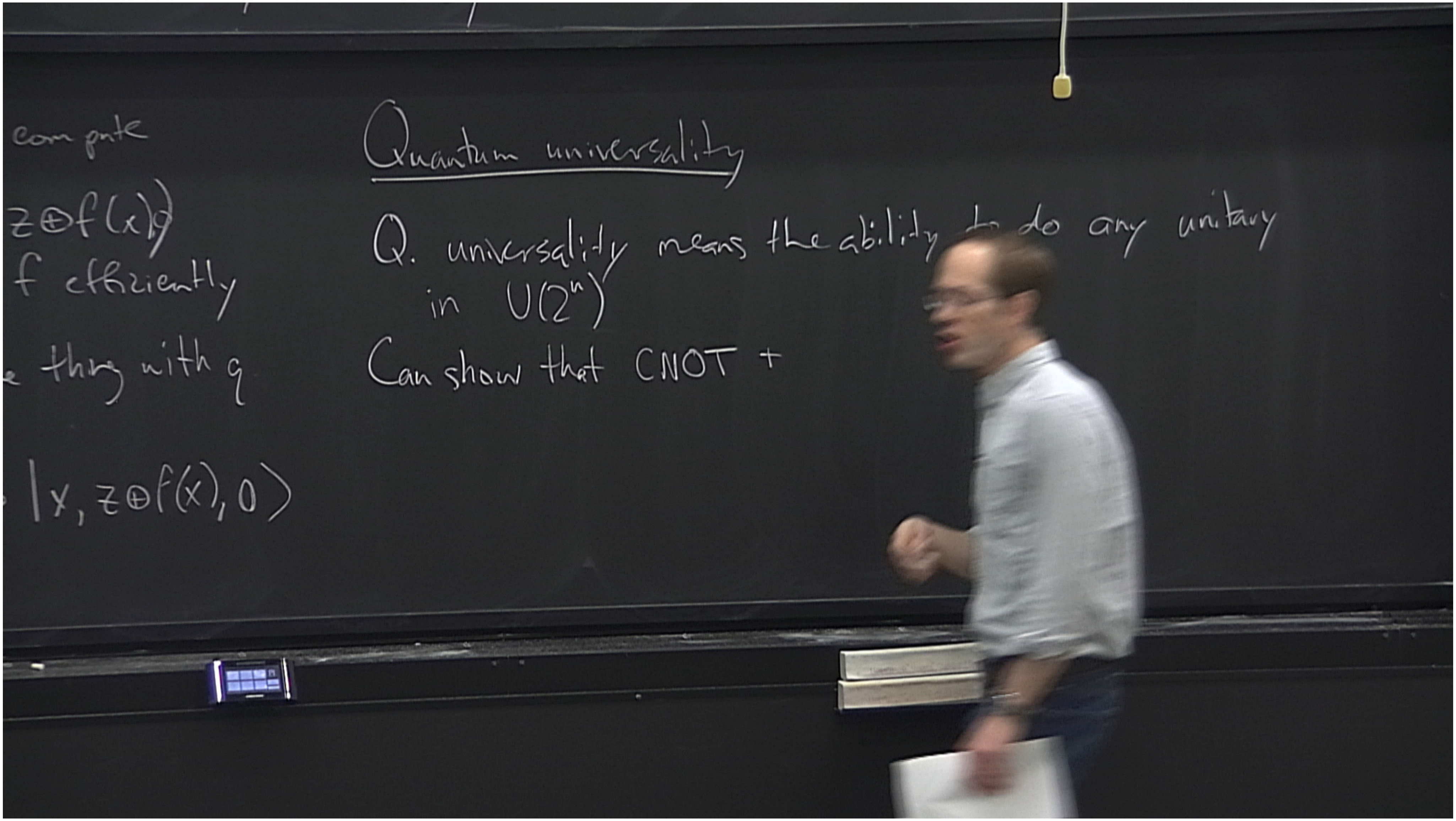
thing with q

$|x, z \oplus f(x), 0\rangle$

Quantum universality

Q. universality means the ability to do any unitary in $U(2^n)$

Can show


compute
z⊕f(x)⟩
f efficiently
thing with q
|x, z⊕f(x), 0⟩
Quantum universality
Q. universality means the ability to do any unitary
in U(2^n)
Can show that CNOT +

compute

$z \oplus f(x)\rangle$

f efficiently

thing with q

$|x, z \oplus f(x), 0\rangle$

Quantum universality

Q. universality means the ability to do any unitary in $U(2^n)$

Can show that CNOT + all single-qubit gates is a universal set

compute

$z \oplus f(x) \rangle$

f efficiently

thing with q

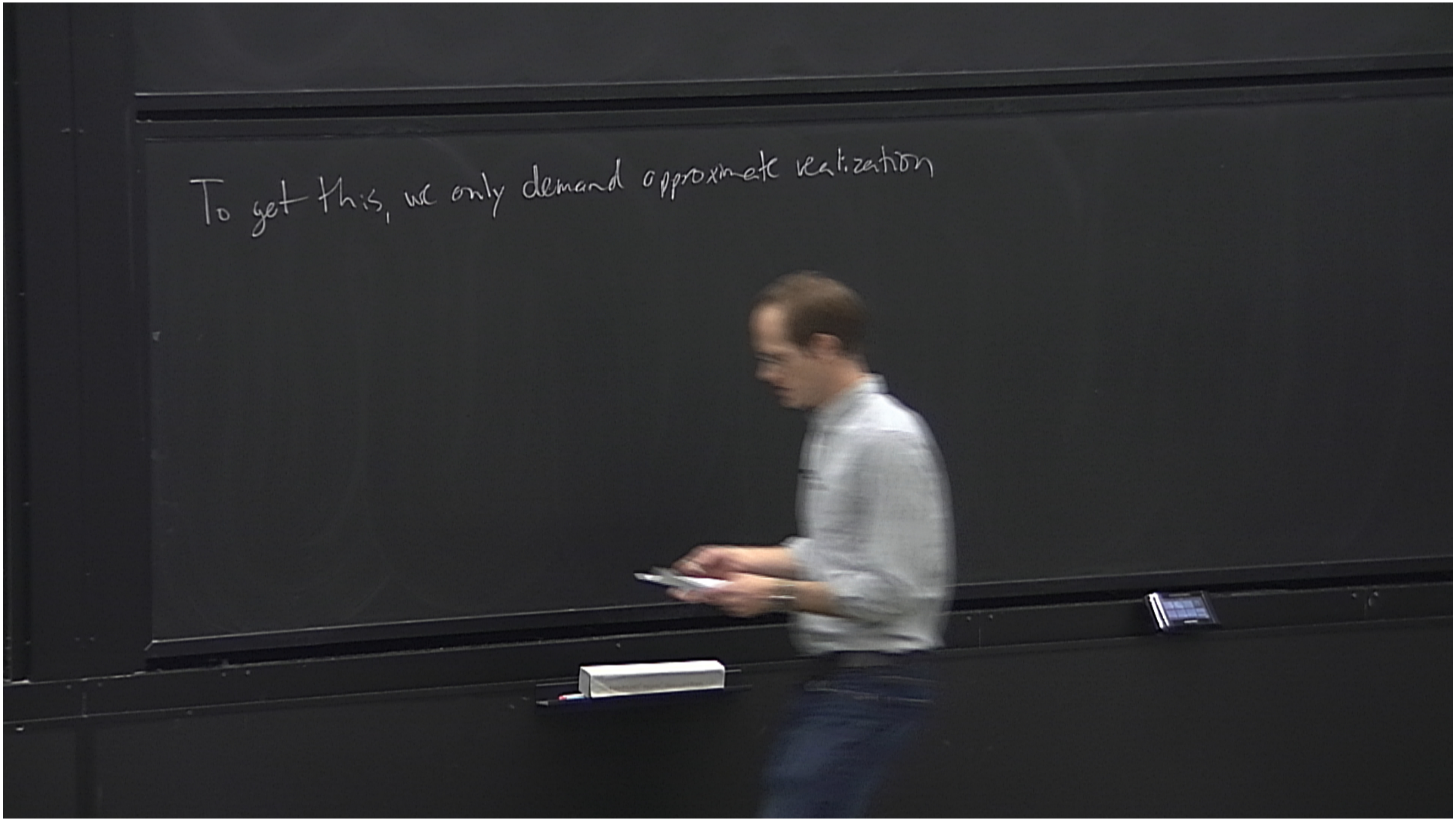
$|x, z \oplus f(x), 0\rangle$

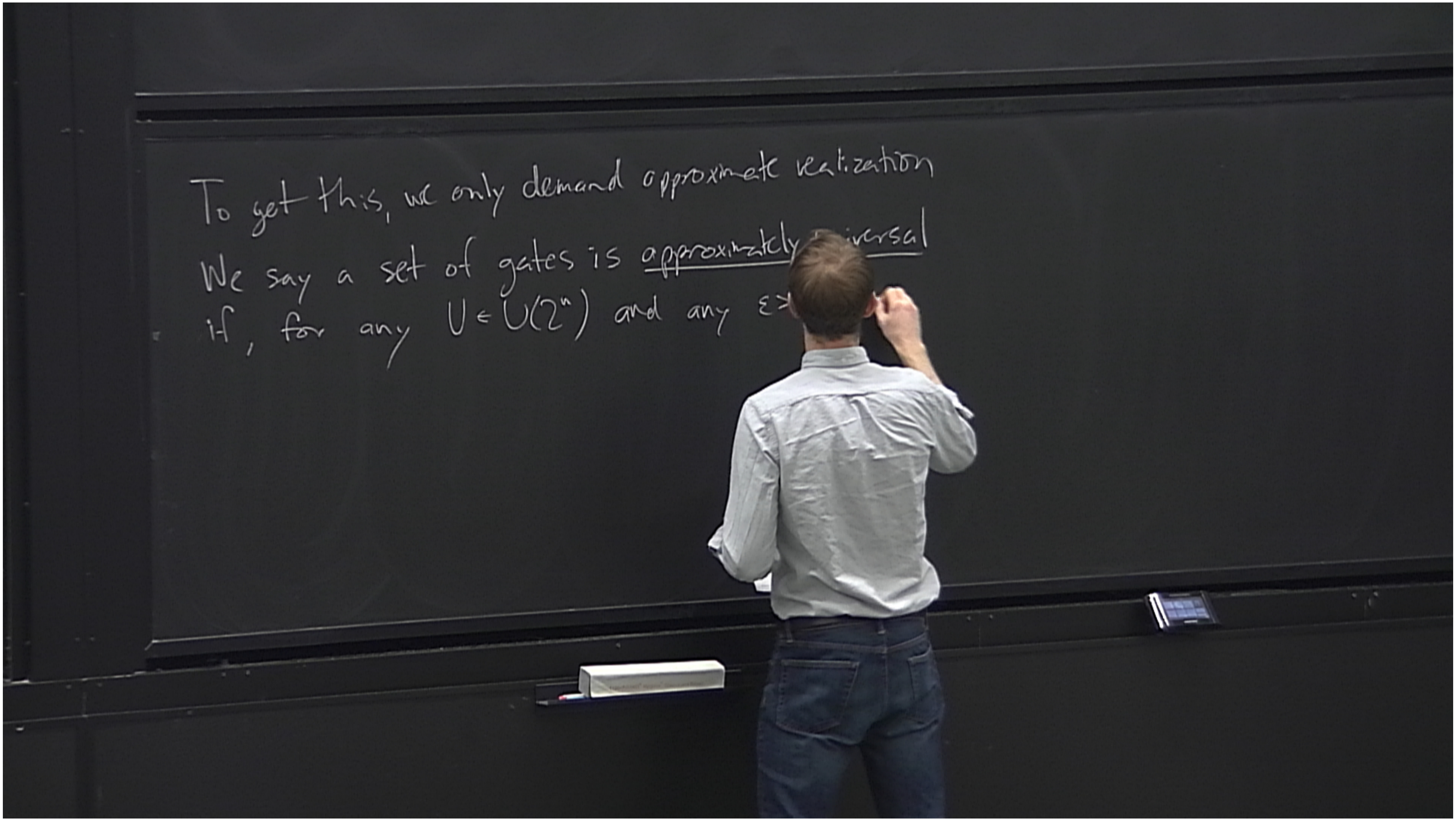
Quantum universality

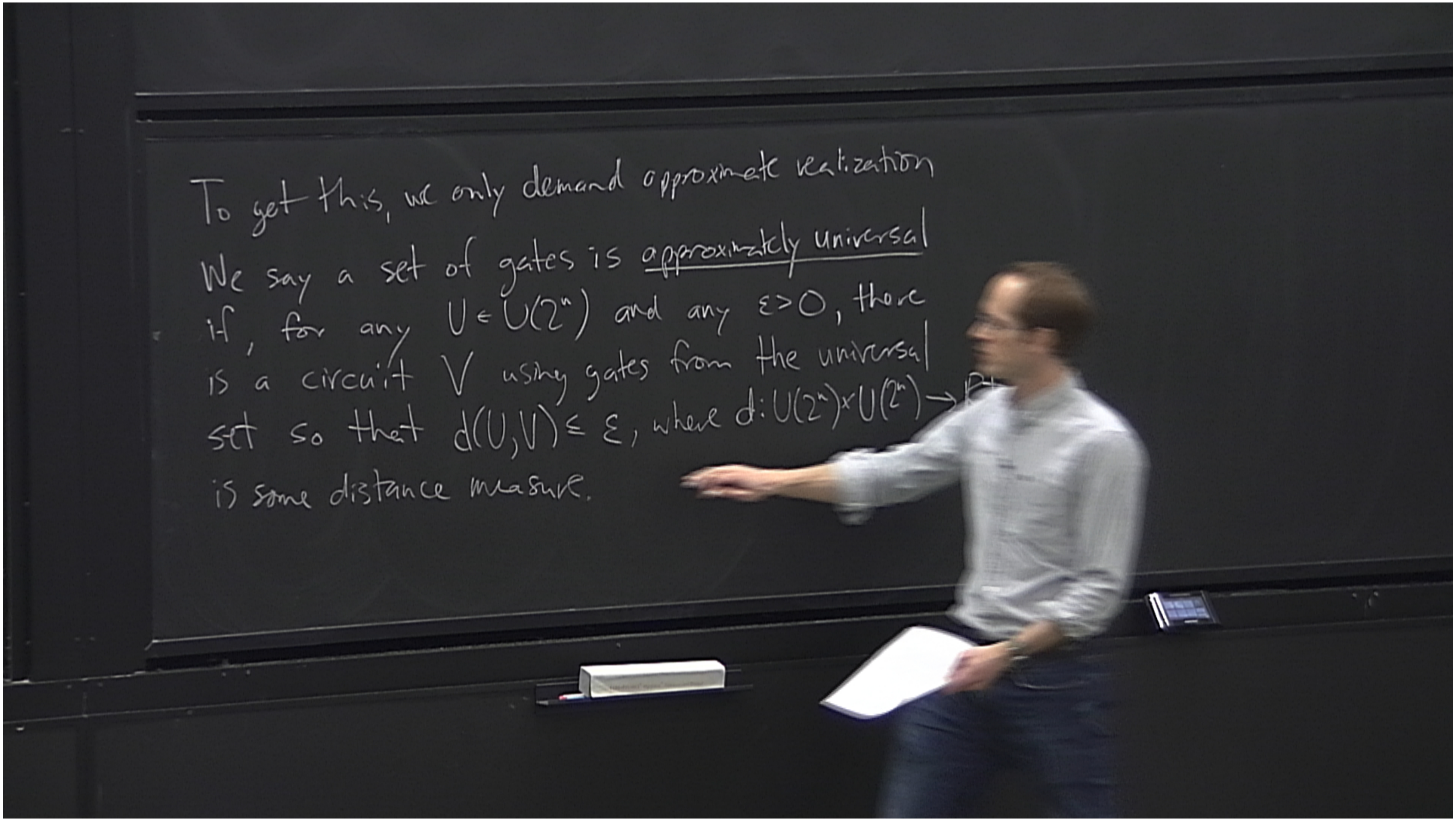
Q. universality means the ability to do any unitary in $U(2^n)$

Can show that CNOT + all single-qubit gates is a universal set

But we'd like to have a finite gate set

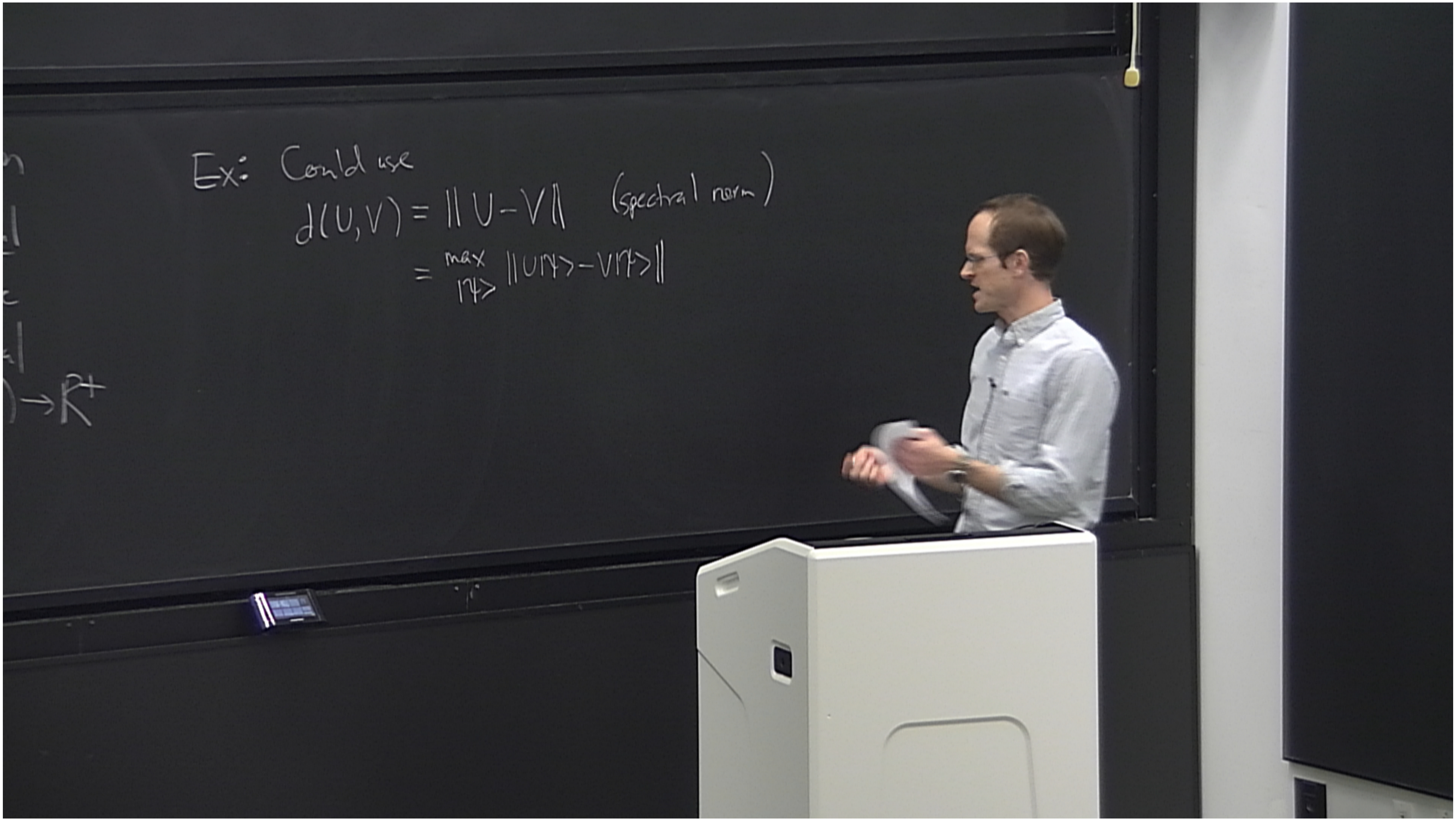

To get this, we only demand approximate realization

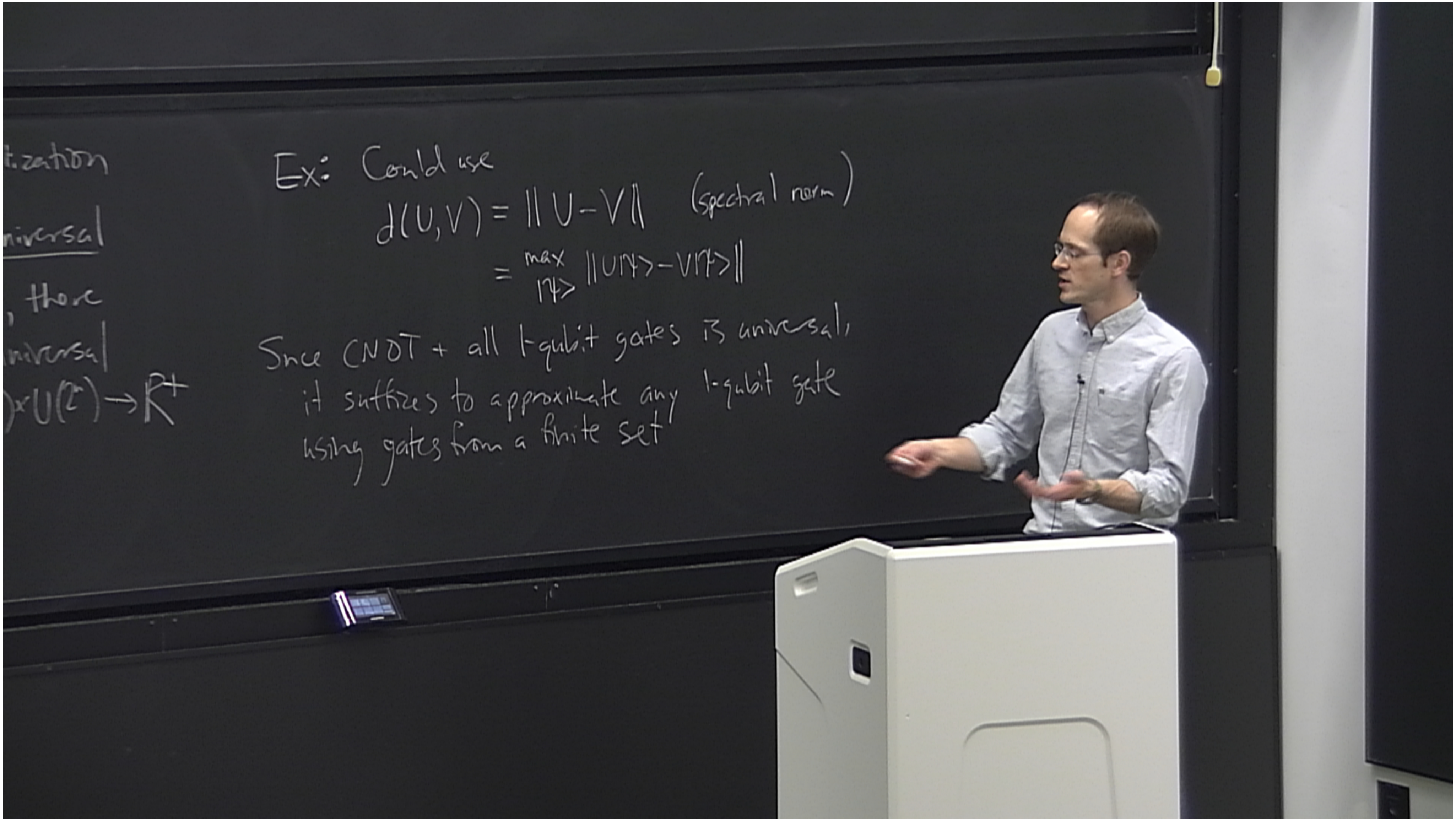

To get this, we only demand approximate realization
We say a set of gates is approximately iversal
if, for any $U \in U(2^n)$ and any $\varepsilon >$

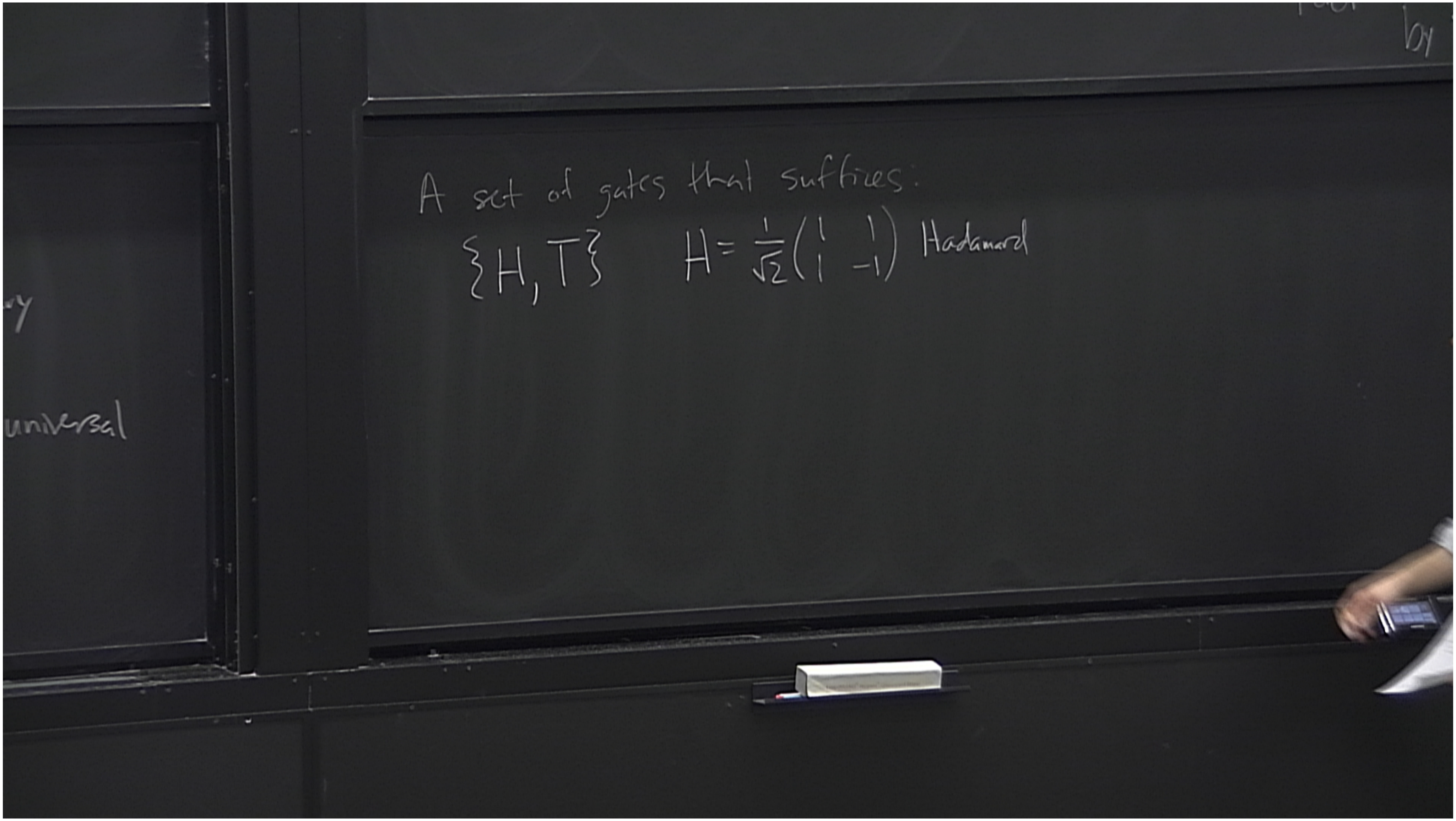

To get this, we only demand approximate realization
We say a set of gates is approximately universal
if, for any U ∈ U(2^n) and any ε>0, there
is a circuit V using gates from the universal
set so that d(U,V) ≤ ε, where d: U(2^n)×U(2^n) →
is some distance measure.

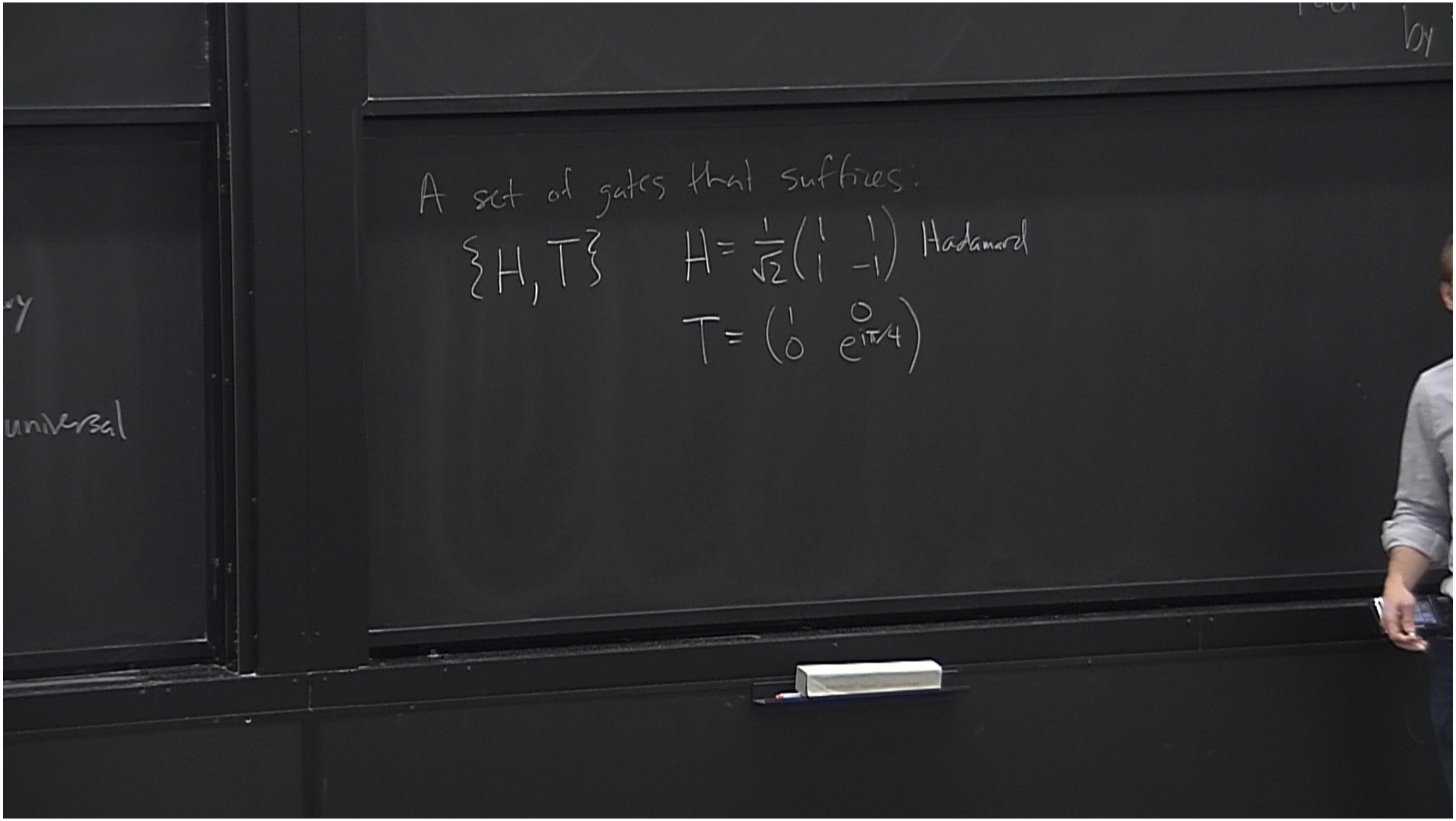
To get this, we only demand approximate realization

We say a set of gates is <u>approximately universal</u> if, for any $U \in U(2^n)$ and any $\varepsilon > 0$, there is a circuit V using gates from the universal set so that $d(U,V) \leq \varepsilon$, where $d: U(2^n) \times U(2^n) \to \mathbb{R}^+$ is some distance measure.


Ex: Could use
$d(U,V) = \|U - V\|$ (spectral norm)
$= \max_{|\psi\rangle} \| U|\psi\rangle - V|\psi\rangle \|$


Ex: Could use
d(U,V) = ||U−V|| (spectral norm)
= max |ψ> ||U|ψ>−V|ψ>||
Since CNOT + all 1-qubit gates is universal,
it suffices to approximate any 1-qubit gate
using gates from a finite set


A set of gates that suffices:
$\{H, T\}$ $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ Hadamard
universal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$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ Hadamard
$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$
universal
by

by irrational

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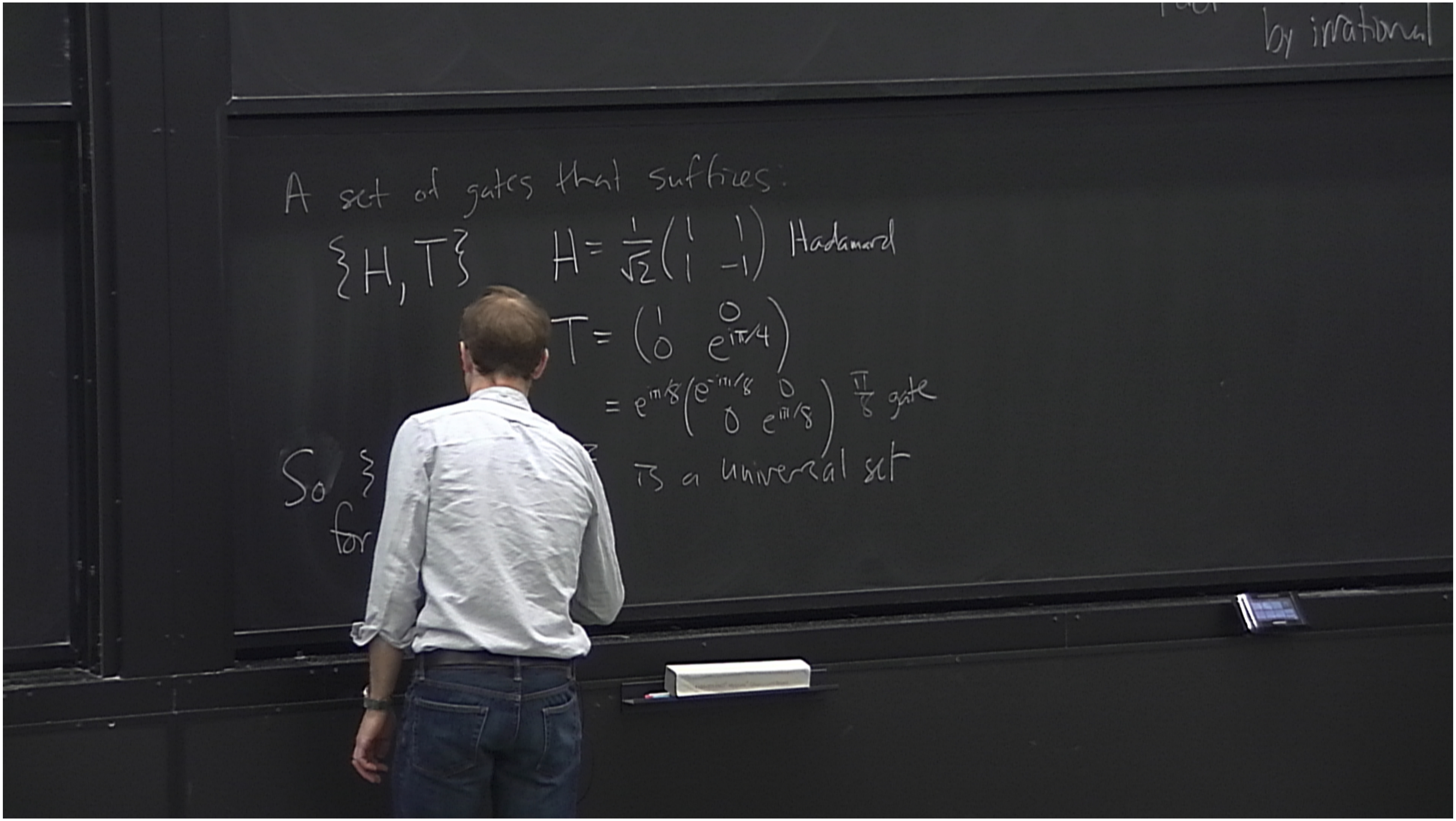
$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

$$= e^{i\pi/8}\begin{pmatrix} e^{-i\pi/8} & 0 \\ 0 & e^{i\pi/8} \end{pmatrix} \quad \frac{\pi}{8} \text{ gate}$$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$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$
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So $\{$ is a universal set
for

by irrational

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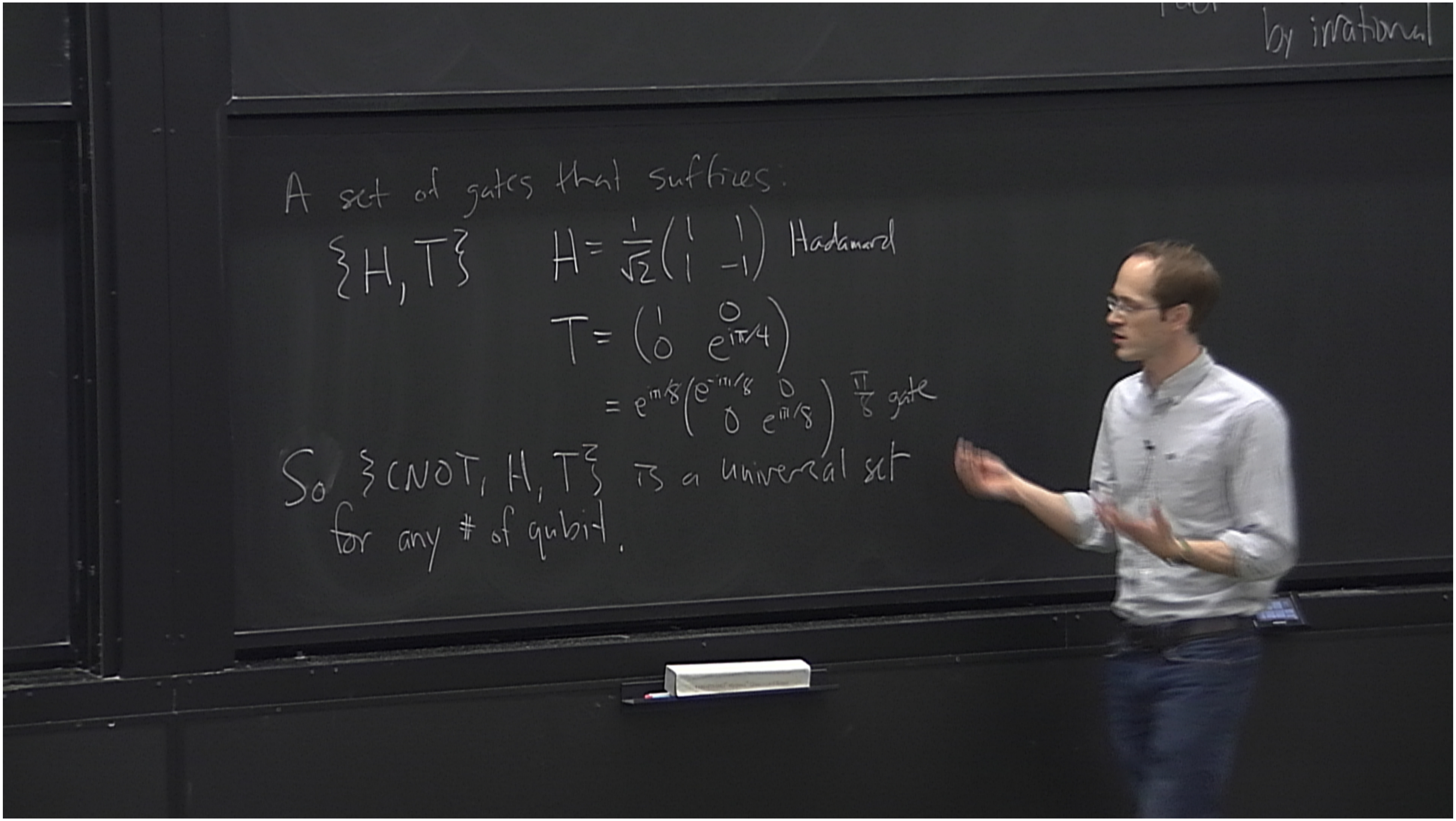
So $\{CNOT, H, T\}$ is a universal set for any # of qubit.



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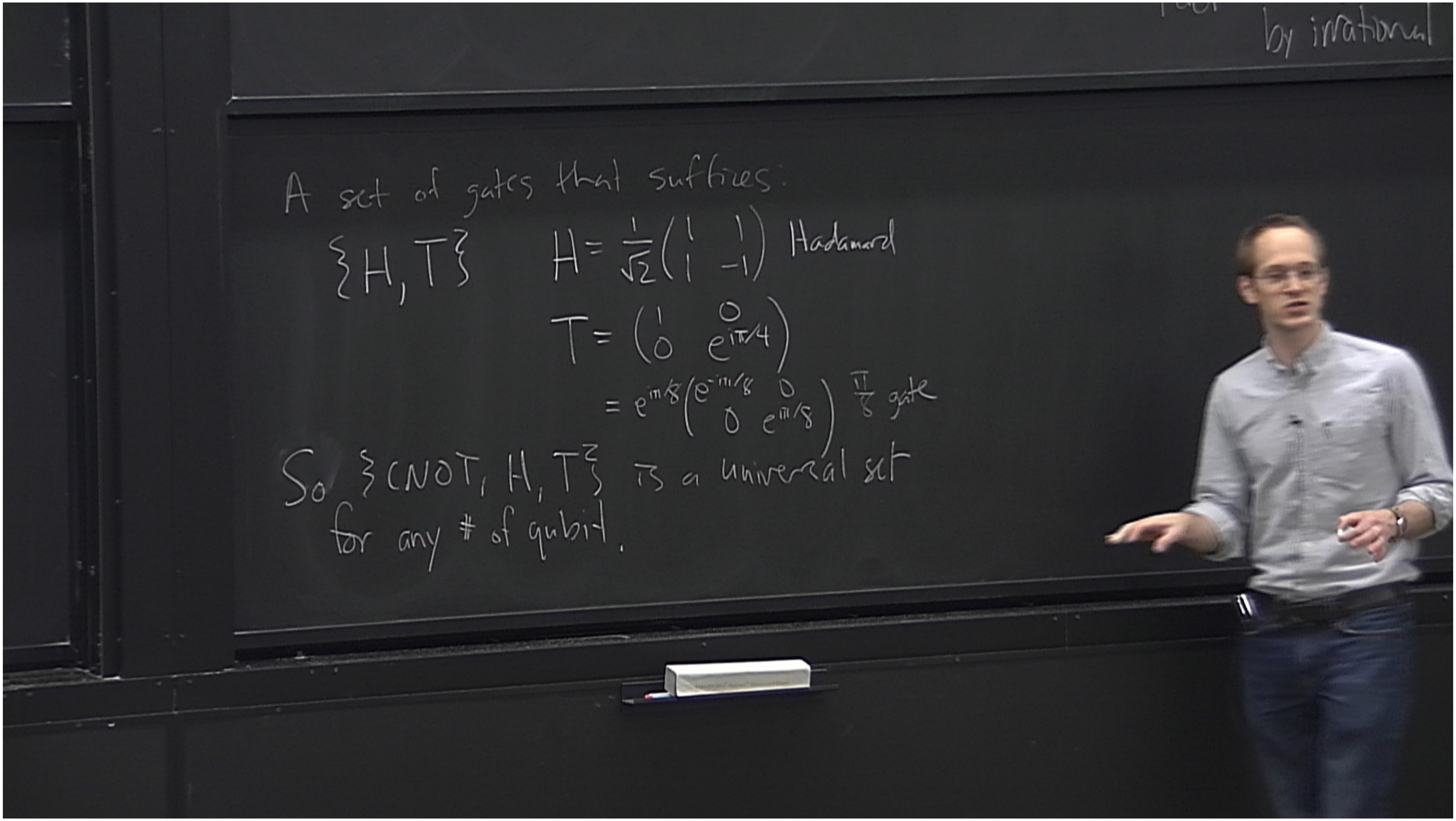
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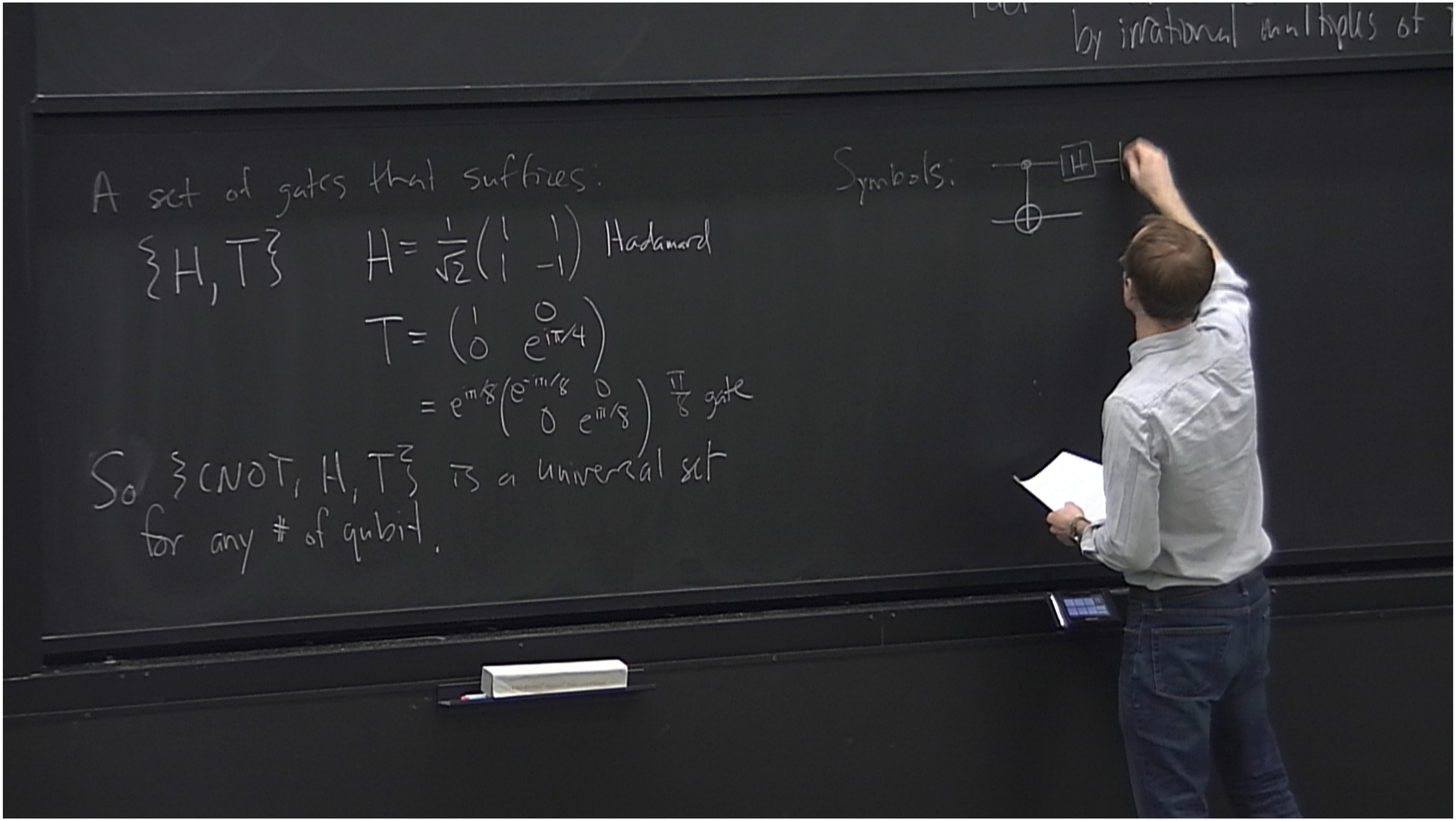
by irrational multiples of
A set of gates that suffices:
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Symbols:
H

by irrational multiples of

A set of gates that suffices:

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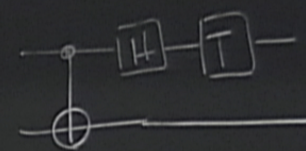
Symbols:



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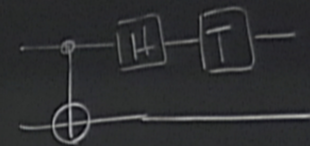
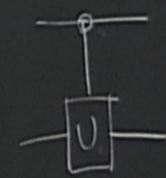
Symbols:



Another common notation:



$$\begin{pmatrix} I & 0 \\ 0 & U \end{pmatrix}$$