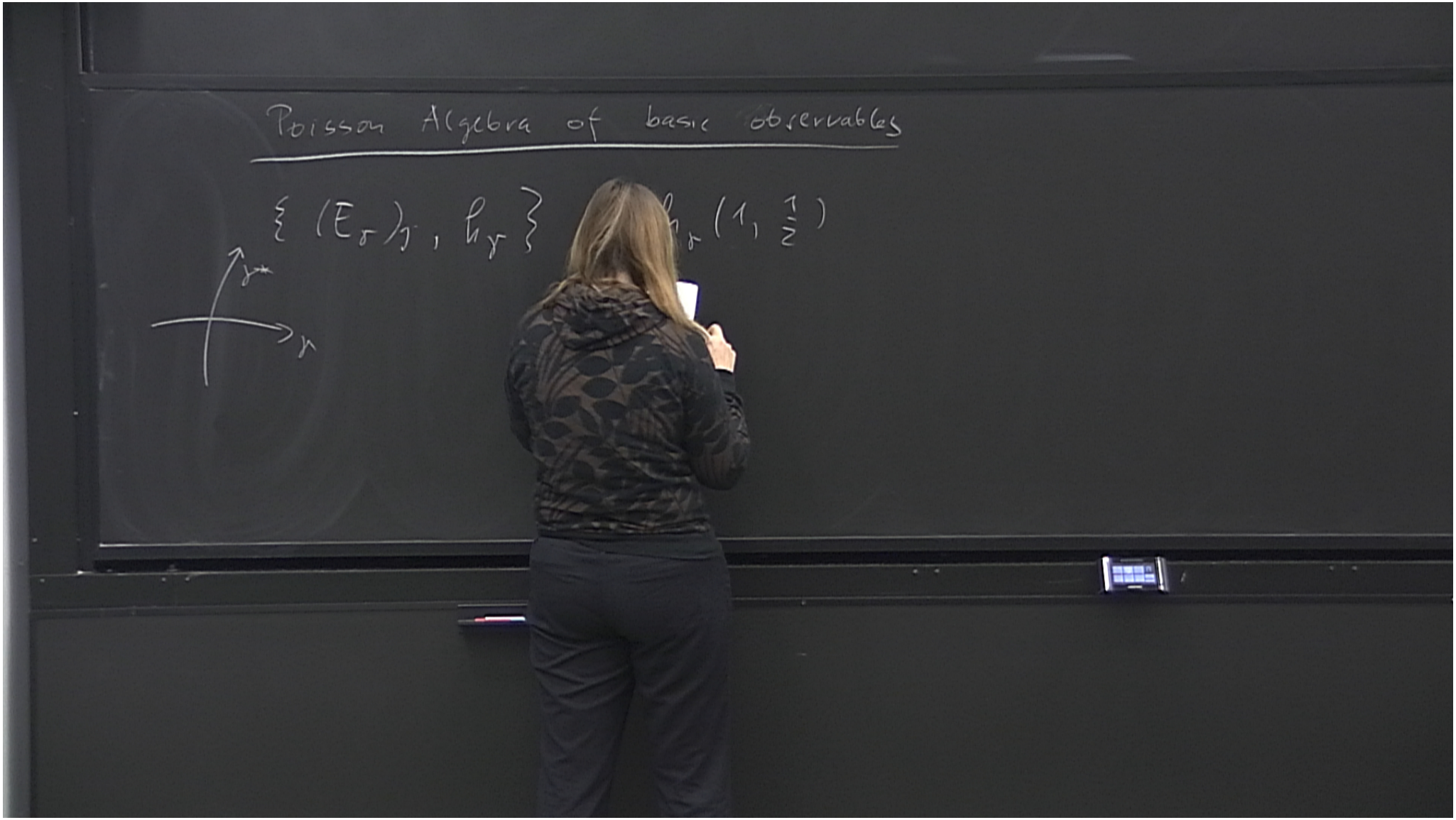
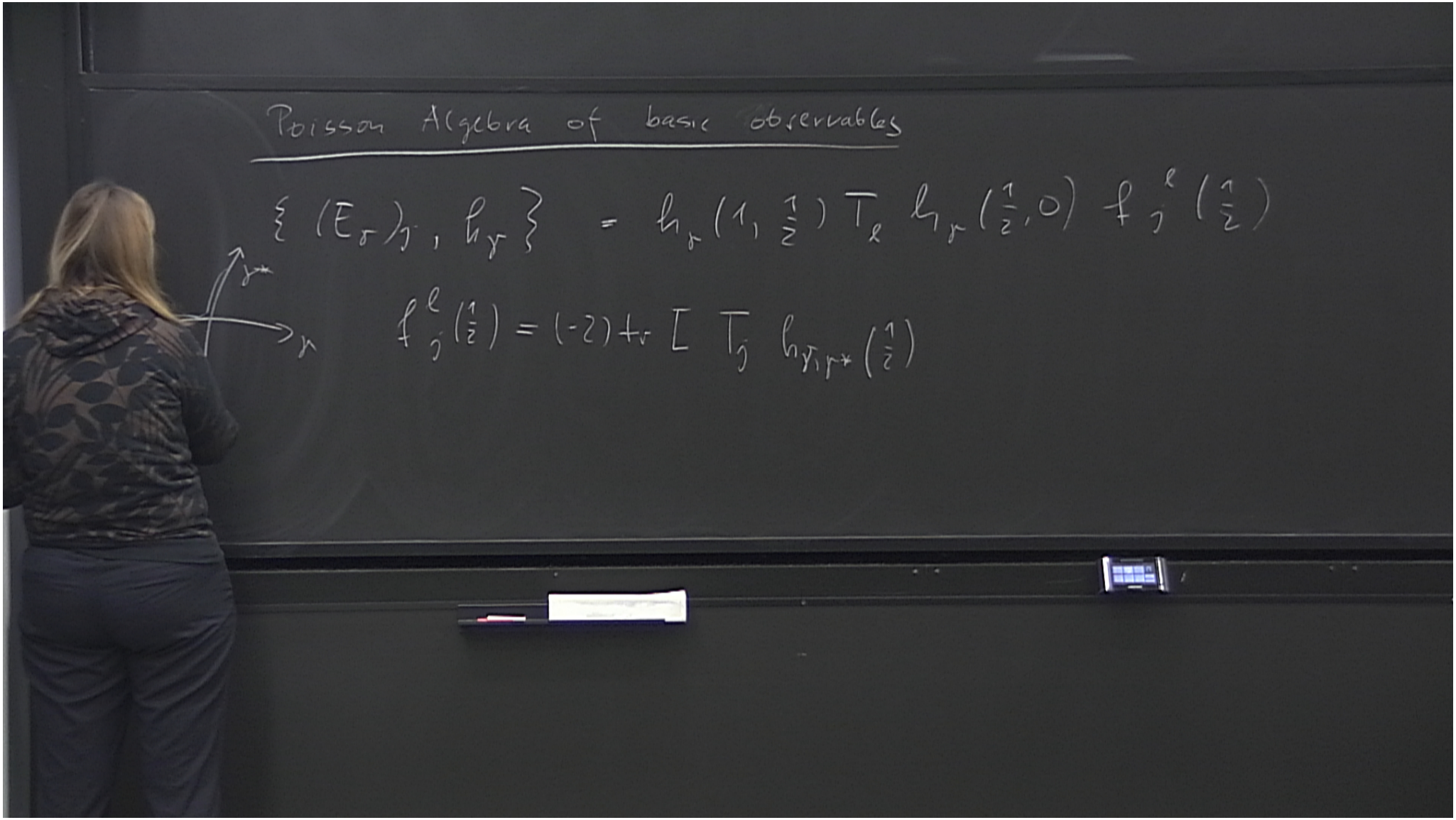
Title: 13/14 PSI - Quantum Gravity Review - Lecture 9

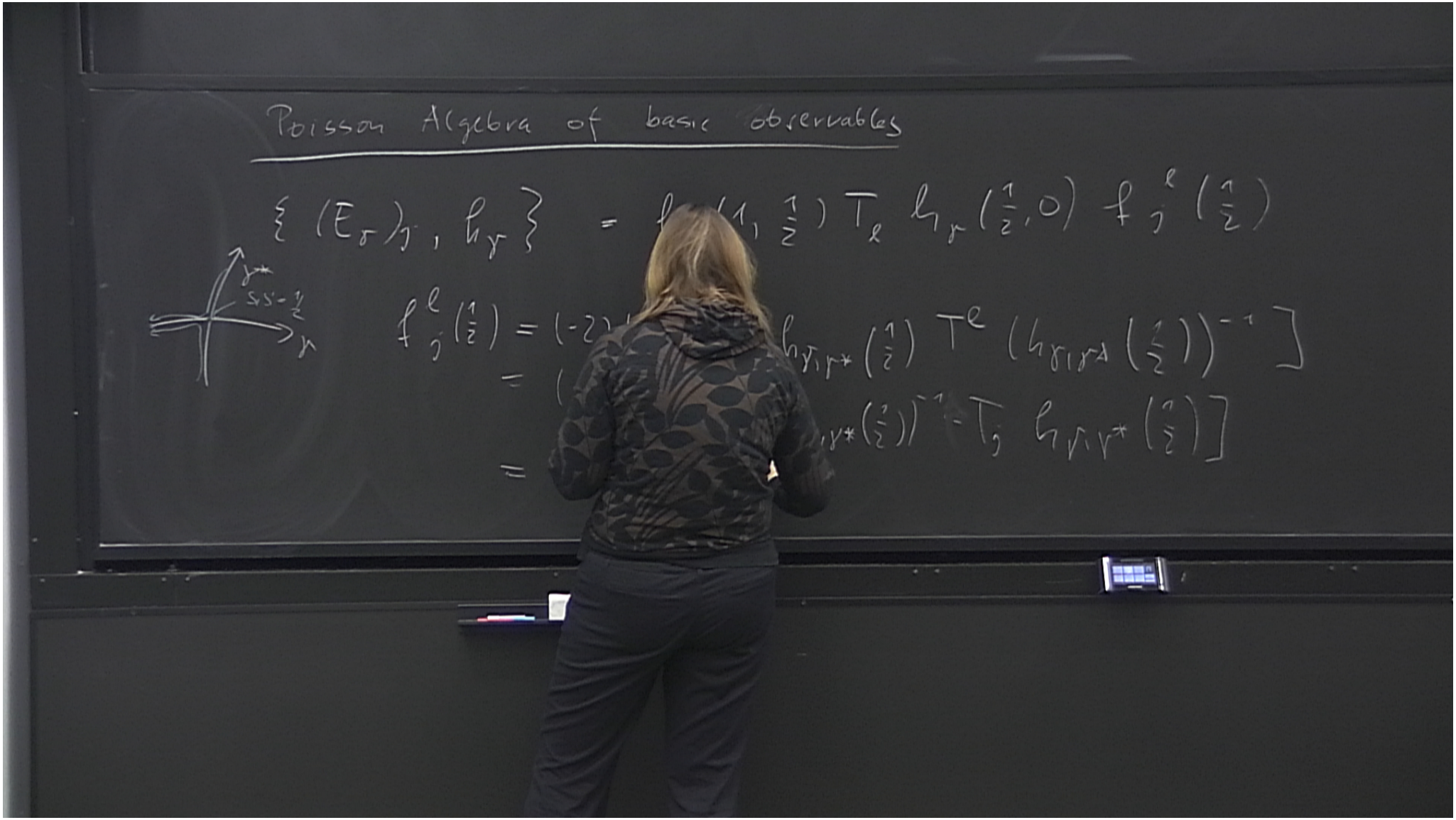
Date: Feb 28, 2014 10:15 AM

URL: http://pirsa.org/14020071

Abstract:

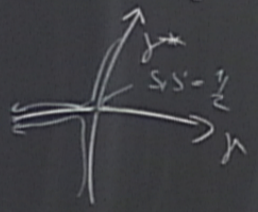

Poisson Algebra of basic observables


Poisson Algebra of basic observables
$\{ (E_\sigma)_j , h_\gamma \} = h_\gamma(1, \frac{1}{2}) T_\ell h_\gamma(\frac{1}{2}, 0) f_j^\ell(\frac{1}{2})$
$f_j^\ell(\frac{1}{2}) = (-2) \mathrm{tr}[T_j h_{\gamma_1 \gamma^*}(\frac{1}{2})$


Poisson Algebra of basic observables

Poisson Algebra of basic observables

$$\{ (E_\gamma)_j , h_\gamma \} = h_\gamma(1, \tau) T_\ell \, h_\gamma(\tfrac{1}{2}, 0) \, f_j^{\ell}(\tfrac{1}{2})$$



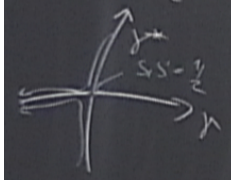
$$f_j^{\ell}(\tfrac{1}{2}) = (-2) \, \mathrm{tr} \, T^{\ell} \, [\, h_{\gamma,\gamma*}(\tfrac{1}{2}) \, T^{\ell} \, (h_{\gamma,\gamma*}(\tfrac{1}{2}))^{-1} \,]$$

$$= (-2) \, \mathrm{tr} \, T^{\ell} \, [\, h_{\gamma,\gamma*}(\tfrac{1}{2}) \, T_j \, h_{\gamma,\gamma*}(\tfrac{1}{2}) \,]$$

$$= (-2) \, \mathrm{tr} \, T^{\ell} \, [\, h_\gamma(\tfrac{1}{2}, 0) \, T_j \, h_\gamma(\tfrac{1}{2}, 0)^{-1} \,]$$

Poisson Algebra of basic observables

$$\{(E_\sigma)_j, h_\gamma\} = h_\gamma(1, \tfrac{1}{2}) T_\ell h_\gamma(\tfrac{1}{2}, 0) (-2) \mathrm{tr}[\tau^\ell h_\gamma(\tfrac{1}{2}, 0) T_j h_\gamma(\tfrac{1}{2}, 0)^{-1} \tau_\cdot]$$



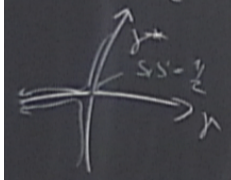
$$f^\ell_j(\tfrac{1}{2}) = (-2)\mathrm{tr}[\tau^\ell \quad (\tfrac{1}{2}) T^e (h_{\gamma\gamma'}(\tfrac{1}{2}))^{-1}]$$

$$= (-2) \quad h_{\gamma\gamma'*}(\tfrac{1}{2}))^{-1} - T_j \, h_{\gamma\gamma'*}(\tfrac{1}{2})]$$

$$= (-2 \quad h_\gamma(\tfrac{1}{2}, 0) \, T_j \, h_\gamma(\tfrac{1}{2}, 0)^{-1}]$$

Poisson Algebra of basic observables

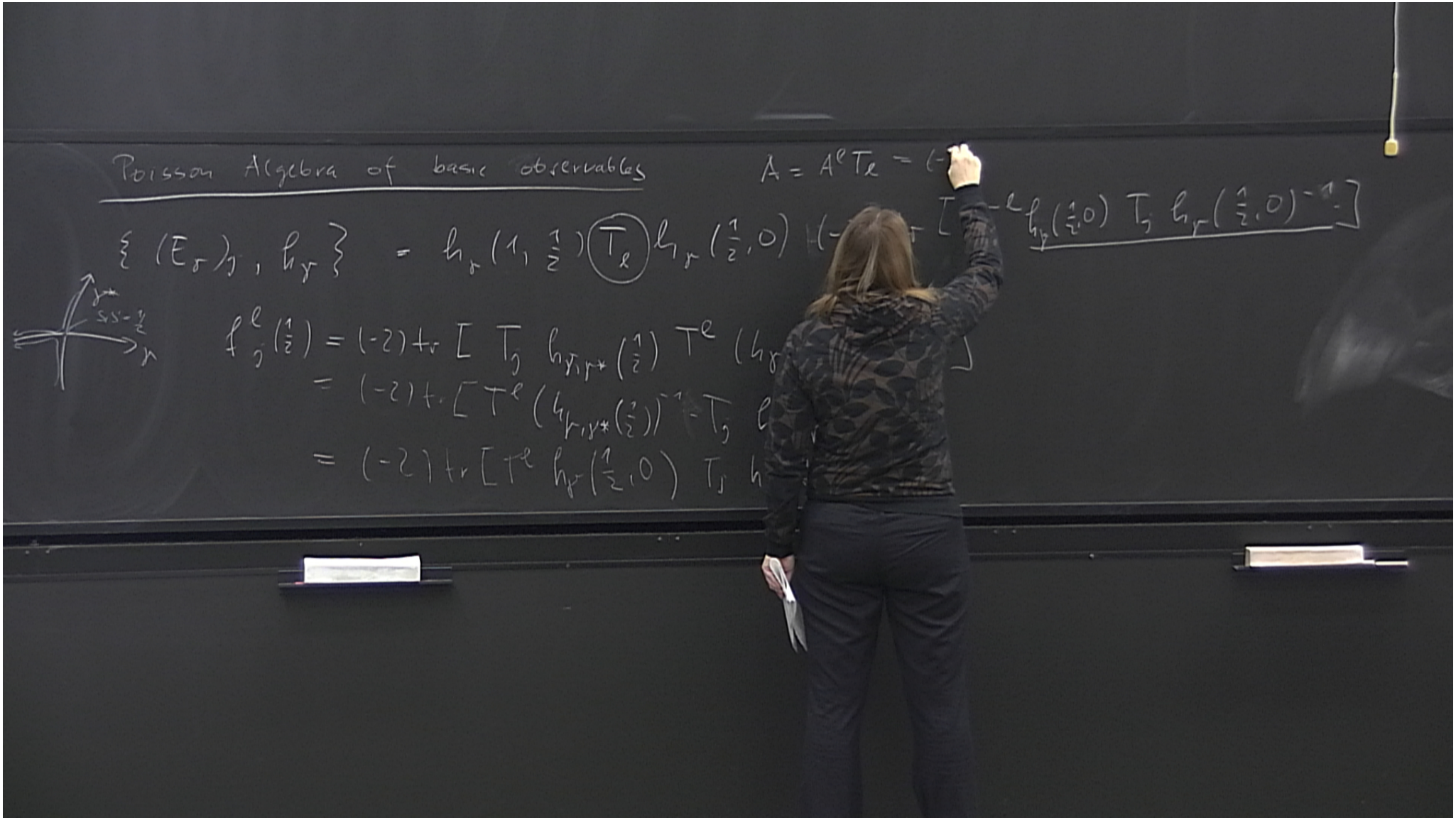
$$\{(E_r)_j, h_\gamma\} = h_\gamma(1, \tfrac{1}{2})\, T_\ell\, h_\gamma(\cdot, 0)\, (-2)\, \mathrm{tr}\left[T^\ell h_\gamma(\tfrac{1}{2}, 0)\, T_j\, h_\gamma(\tfrac{1}{2}, 0)^{-1}\right]$$

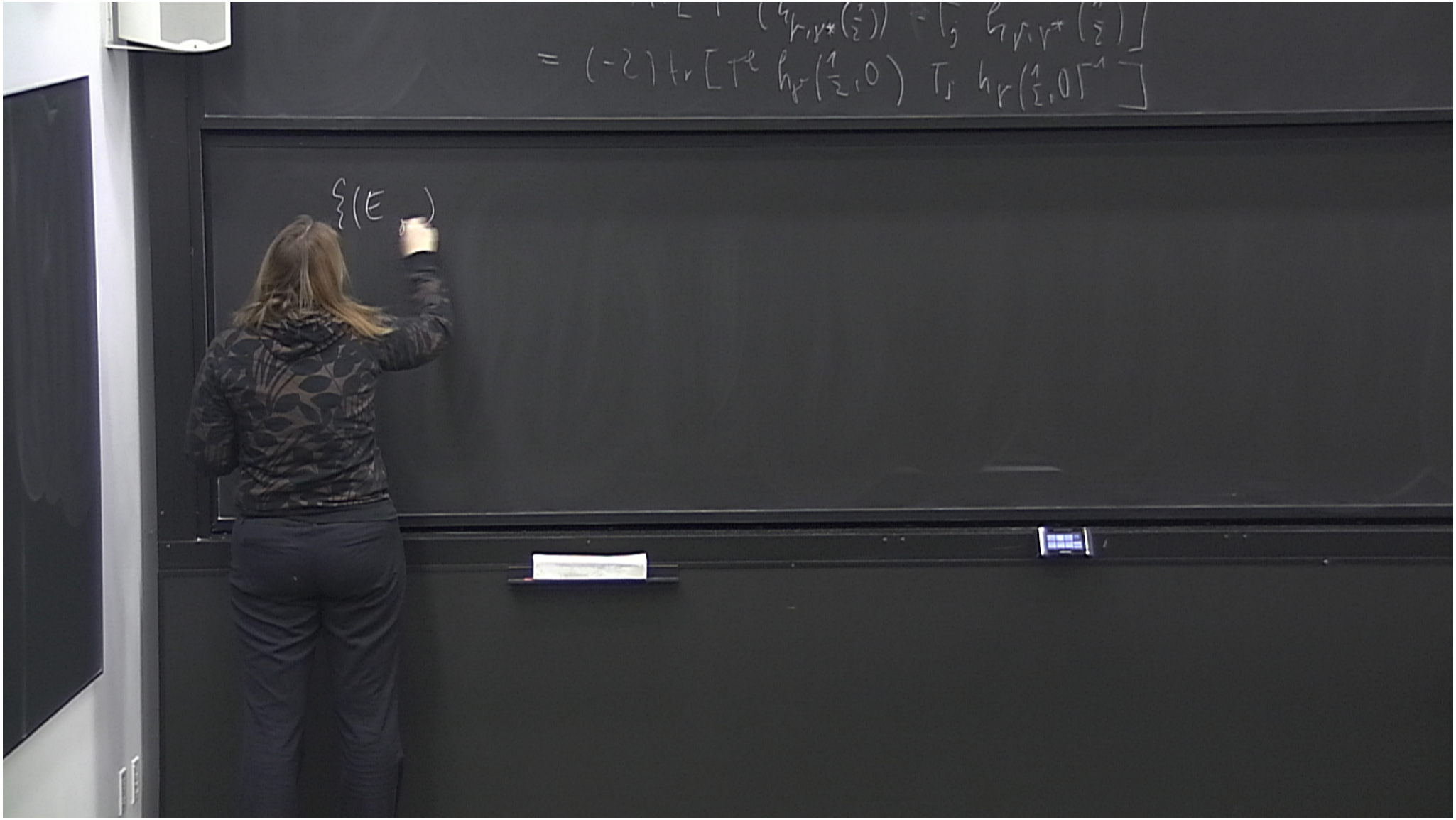


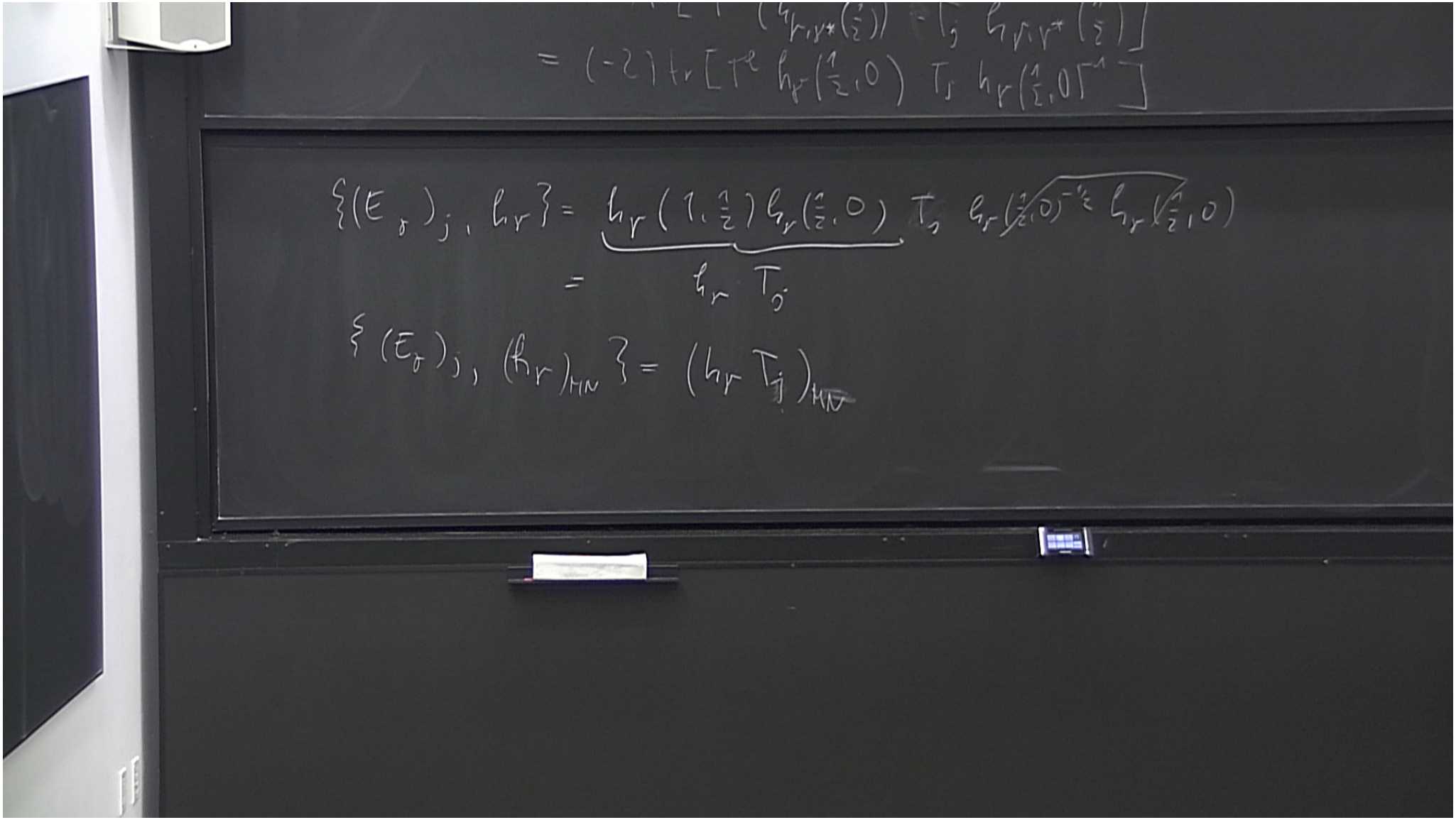
$$f^\ell_j(\tfrac{1}{2}) = (-2)\,\mathrm{tr}\left[T_j\, h_{\gamma_1,\gamma^*}(\cdot)\, h_{\gamma_1,\gamma^*}(\tfrac{1}{2})^{-1}\right]$$

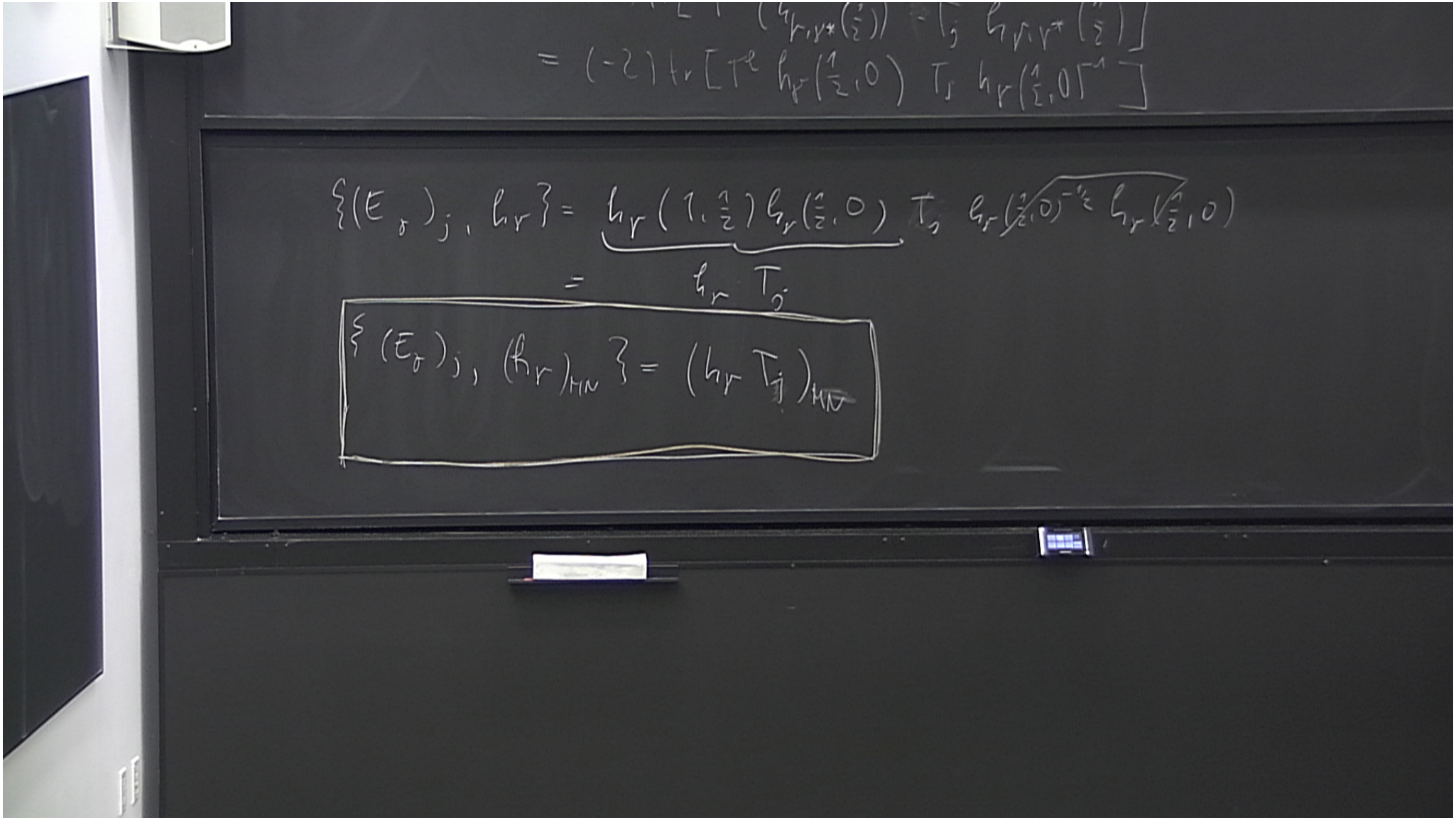
$$= (-2)\,\mathrm{tr}\left[T^\ell (h_{\gamma_1,\gamma^*}\cdots h_{\gamma_1,\gamma^*}(\tfrac{1}{2})\right]$$

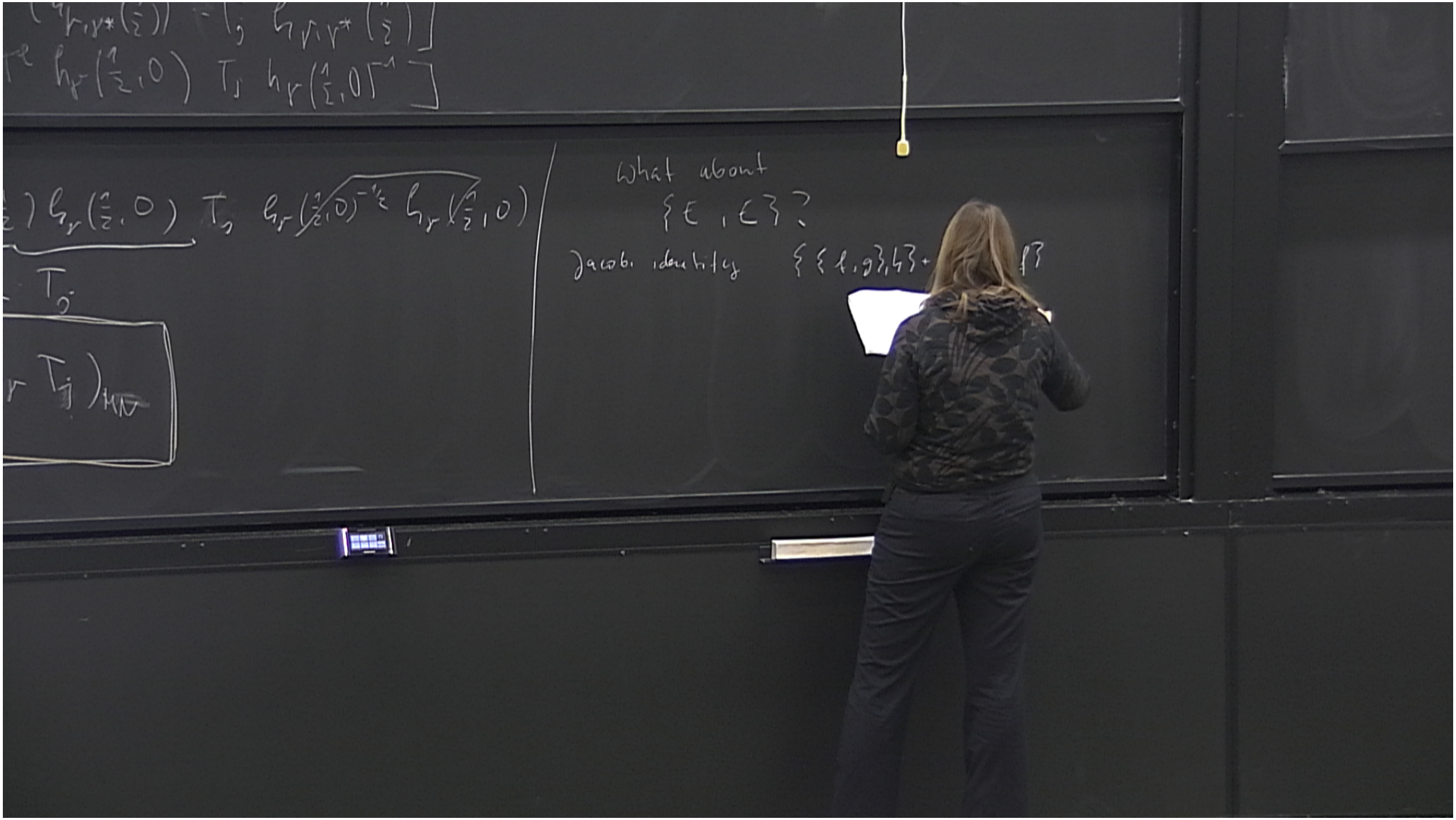
$$= (-2)\,\mathrm{tr}\left[T^\ell\, h_{\gamma'}(\tfrac{1}{2},\cdots,0)^{-1}\right]$$

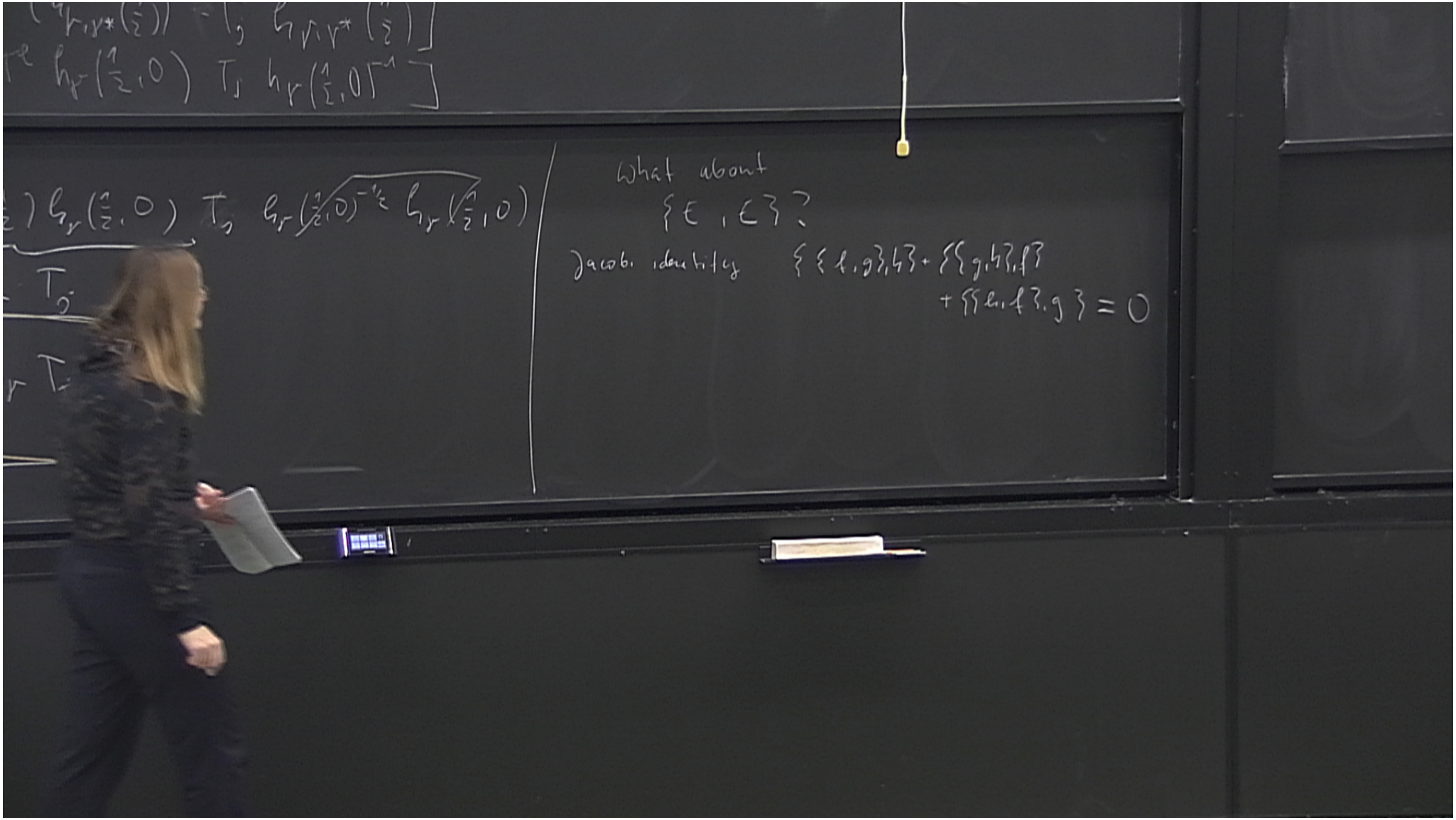

Poisson Algebra of basic observables
A = A^e T_e = (-
{ (E_r)_j , h_γ } = h_γ(1, 1/2) T_e h_γ(1/2, 0)
f^e_j(1/2) = (-2) tr [T_j h_{γ,γ*}(1/2) T^e (h_γ
= (-2) tr [T^e (h_{γ,γ*}(1/2))^{-1} T_j e
= (-2) tr [T^e h_γ(1/2, 0) T_j h
s(s-1/2)

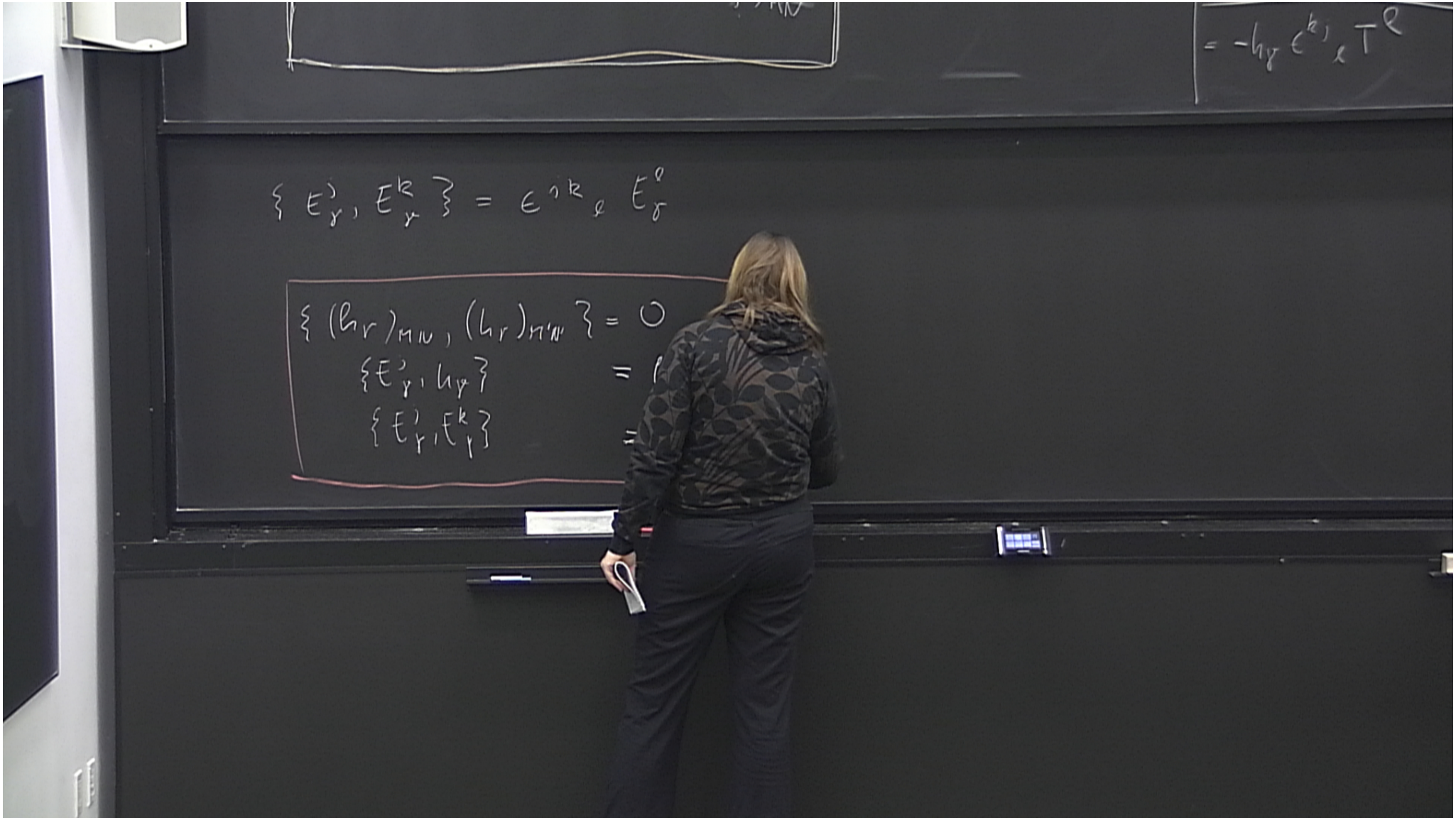


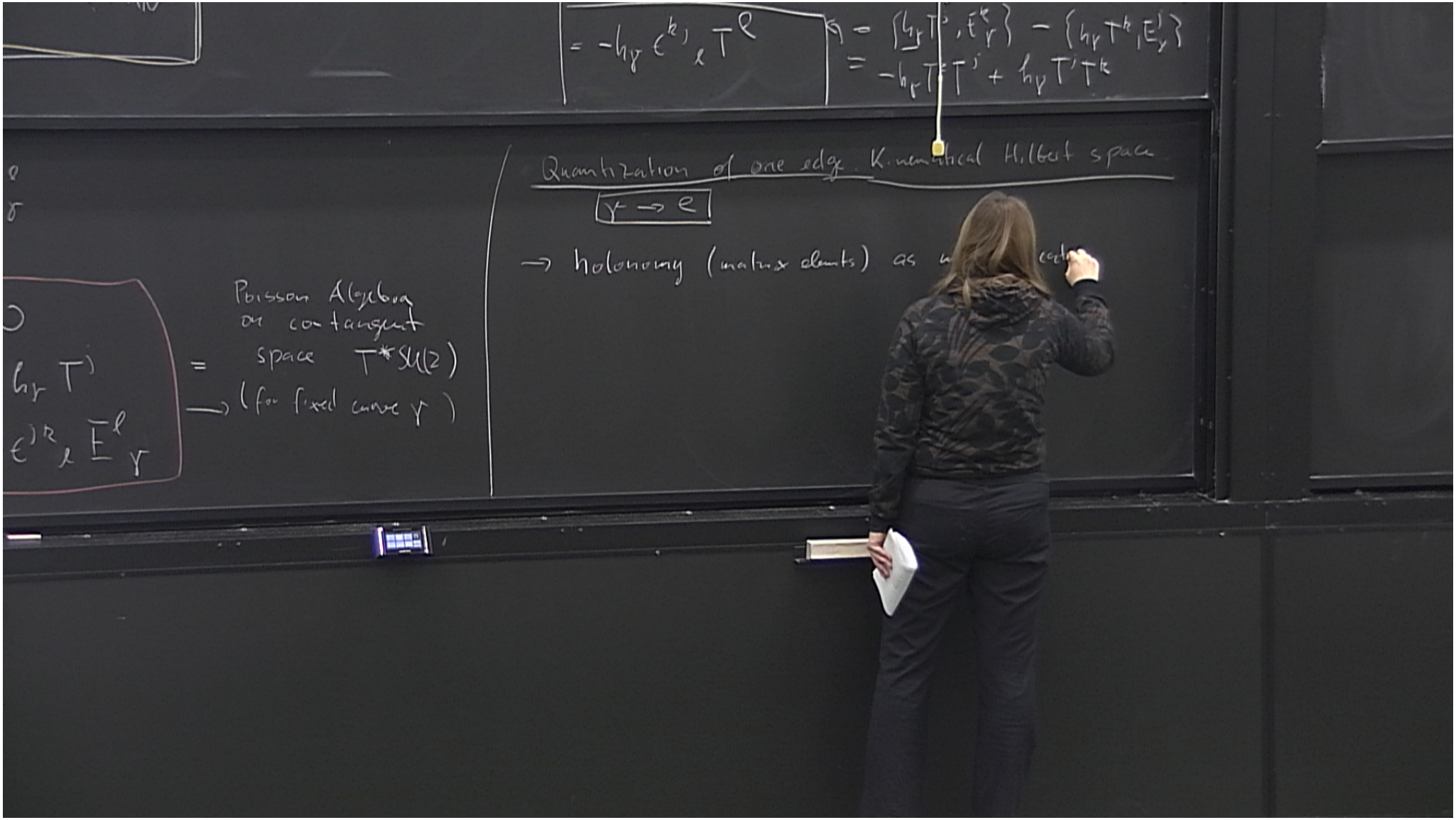

$= (-2)\,\mathrm{tr}\left[T^a\, h_r\left(\tfrac{1}{2},0\right) T_j\, h_r\left(\tfrac{1}{2},0\right)^{-1}\right]$
$\{(E_r)_j, h_r\} = h_r\left(1,\tfrac{1}{2}\right) h_r\left(\tfrac{1}{2},0\right) T_j\, h_r\left(\tfrac{1}{2},0\right)^{-1} h_r\left(\tfrac{1}{2},0\right)$
$= h_r\, T_j$
$\{(E_r)_j, (h_r)_{MN}\} = (h_r\, T_j)_{MN}$

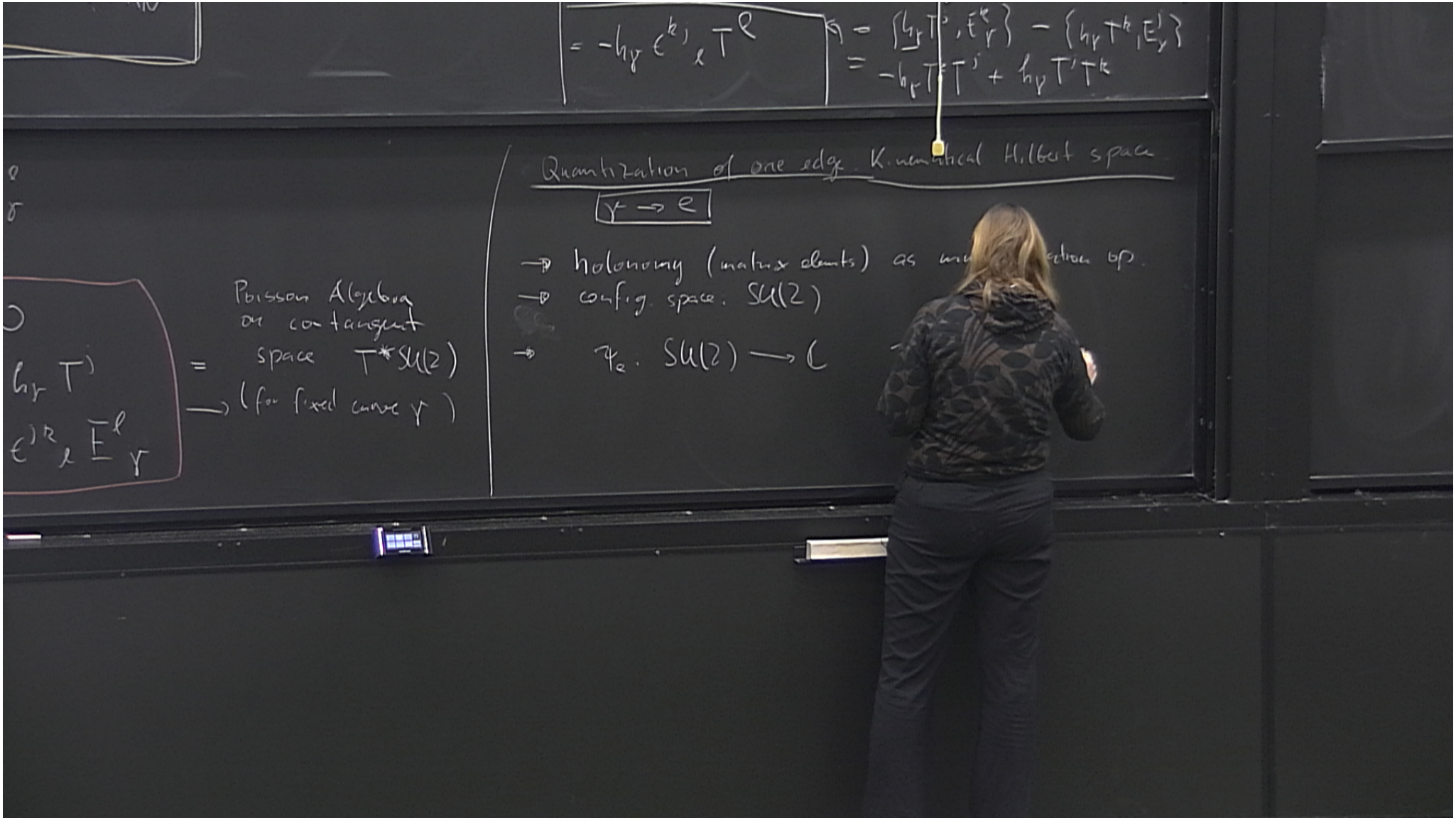

$= (-2)\,\mathrm{tr}\left[T^a\, h_r\left(\tfrac{1}{2},0\right)\, T_j\, h_r\left(\tfrac{1}{2},0\right)^{-1}\right]$
$\{(E_r)_j, h_r\} = h_r(1,\tfrac{1}{2})\, h_r(\tfrac{1}{2},0)\, T_j\, h_r(\tfrac{1}{2},0)^{-\frac{1}{2}}\, h_r(\tfrac{1}{2},0)$
$= h_r\, T_j$
$\{(E_r)_j, (h_r)_{MN}\} = (h_r\, T_j)_{MN}$


What about
{E, E}?
Jacobi identity


What about
{E, E}?
Jacobi identity {{f,g},h} + {{g,h},f}
+ {{h,f},g} = 0


$\{ E^j_\gamma, E^k_\rho \} = \epsilon^{jk}{}_\ell \, E^\ell_\gamma$
$\{ (h_r)_{mn}, (h_r)_{m'n'} \} = 0$
$\{ E^j_\gamma, h_\gamma \}$
$\{ E^j_\gamma, E^k_\gamma \}$
$= -h_\gamma \epsilon^{kj}{}_\ell T^\ell$


$= -h_\gamma \epsilon^{k}{}_{\ell} T^{\ell}$
$= \{h_\gamma T^j, \tilde{E}^k_\gamma\} - \{h_\gamma T^k, \tilde{E}^j_\gamma\}$
$= -h_\gamma T^k T^j + h_\gamma T^j T^k$
Quantization of one edge. Kinematical Hilbert space
$\gamma \to e$
→ holonomy (matrix elements) as
$h_\gamma T^j$
$\epsilon^{jk}{}_{\ell} \tilde{E}^{\ell}_\gamma$
=
Poisson Algebra on cotangent space $T^* SU(2)$
⟶ (for fixed curve γ)


$= -h_\gamma \epsilon^{k}{}_{\ell} T^{\ell}$
$= \{h_\gamma T^j, \tilde{E}^k_\gamma\} - \{h_\gamma T^k, \tilde{E}^j_\gamma\}$
$= -h_\gamma T^k T^j + h_\gamma T^j T^k$
Quantization of one edge. Kinematical Hilbert space
$\gamma \to e$
holonomy (matrix elements) as
config space. SU(2)
$\mathcal{H}_e : SU(2) \to \mathbb{C}$
Poisson Algebra on cotangent space $T^*SU(2)$
(for fixed curve γ)
$h_\gamma T^j$
$\epsilon^{jk}{}_{\ell} \tilde{E}^{\ell}_\gamma$

$f(g_e) \qquad (g_e)_{MN} = D_{MN}(g_e) \qquad D^j$

- looking at a constant fct.: $f \equiv 1 \Rightarrow |1\rangle$

$$i\hbar \overbrace{(L^j_e f_e)|1\rangle} = i\hbar \overbrace{\{E^j, f_e(h_e)\}}\ |1\rangle$$

$$= [\hat{E}^j, \hat{f}_e]\,|1\rangle$$

$$= \hat{E}^j|f_e\rangle - \hat{f}_e \underbrace{\hat{E}^j|1\rangle}_{\text{set} = 0}$$

$f(g_e) \quad (\gamma_e)_{MN} = D_{MN}(\gamma_e)$

- looking at a constant fct.: $f \equiv 1 \Rightarrow |1\rangle$

$$i\hbar \underbrace{(L^j_e f_e)|1\rangle} = i\hbar \overbrace{\{E^j, f_e(h_e)\}}\,|1\rangle$$

$$= [\hat{E}^j, \hat{f}_e]\,|1\rangle$$

$$= \hat{E}^j|f_e\rangle - \hat{f}_e \underbrace{\hat{E}^j|1\rangle}_{\text{set} = 0}$$

$i\hbar\,|L^j_e f_e\rangle$

$$\boxed{\hat{E}^j_e\,|\gamma_e\rangle = i\hbar\,|L^j_e \gamma_e\rangle}$$

$$(L^j_e \circ L^k_e - L^k_e \circ L^j_e)(h_e)_{MN}$$

$$= \frac{d}{ds}\Big|_{s=0} \frac{d}{dt}\Big|_{t=0} \left((h_e e^{sT^j} e^{tT^k})_{MN} - (h_e e^{sT^k} e^{tT^j})_{MN} \right)$$

$$= (h_e [T^j, T^k])_{MN} = (h_e \epsilon^{jk}{}_l T^l)_{MN} = \epsilon^{jk}{}_l \frac{d}{dt}\Big|_{t=0} (h_e e^{tT^l})_{MN}$$

$$= \epsilon^{jk}{}_l (L^l_e(h_e))_{MN}$$

$$\boxed{[\hat{E}^j_e, \hat{E}^k_e] = i\hbar \epsilon^{jk}{}_l \hat{E}^l_e}$$