

Title: 13/14 PSI - Quantum Gravity Review - Lecture 8

Date: Feb 27, 2014 10:15 AM

URL: <http://pirsa.org/14020070>

Abstract:

Holonomies

parallel transport vectors along a curve

$$V(s) = L(s) V(0) L(s)^{-1}$$

$$\frac{d}{ds} V(s) = \dot{\gamma} \underline{D_a V} = 0$$

$$D_a V^j = \partial_a V^j + \epsilon^j_{kl} A^k_a V^l$$

$$[D_a V]^j = \partial_a V^j$$

Holonomies

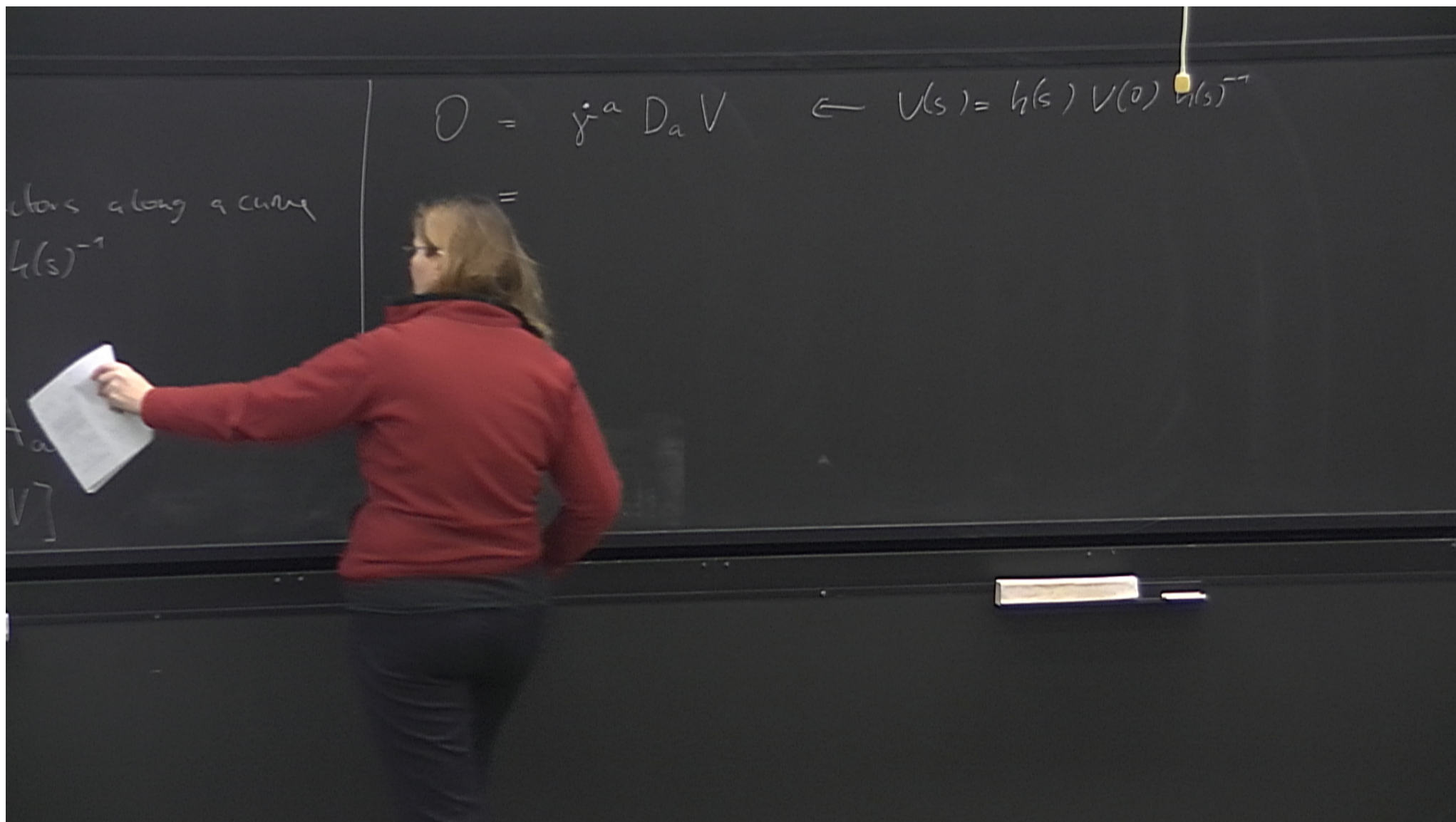
• parallel transport vectors along a curve

$$V(s) = L(s) V(0) L(s)^{-1}$$

$$= \dot{\gamma} \cdot \underline{D_a V} = 0$$

$$= \partial_a V^i + \epsilon^j_{kl} A^k_a V^l$$

$$\partial_a V^i = 0 \implies V + [A_a, V] \quad \rightsquigarrow [A^k_a T^k, V^l T^l] = \epsilon^{jk}_l T^j A^k_a V^l$$



ctors along a curve
 $(s)^{-1}$

$$\begin{aligned} 0 &= \dot{y}^a D_a V \quad \leftarrow V(s) = h(s) V(0) h(s)^{-1} \\ &= \left(\frac{d}{ds} h(s) \right) V h(s)^{-1} + h(s) V \left(\frac{d}{ds} h(s)^{-1} \right) \\ &\quad + \dot{y}^a A_a h(s) V h(s)^{-1} - h(s) V h(s)^{-1} \dot{y}^a A_a \end{aligned}$$

along a curve
 $h(s)^{-1}$

$A_a^k V^k$
 $V]$

$$\begin{aligned}
 0 &= \dot{y}^a D_a V \quad \leftarrow V(s) = h(s) V(0) h(s)^{-1} \\
 &= \left(\frac{d}{ds} h(s) \right) V h(s)^{-1} + h(s) V \left(\frac{d}{ds} h(s)^{-1} \right) \\
 &\quad + \dot{y}^a A_a h(s) V h(s)^{-1} - h(s) V h(s)^{-1} \dot{y}^a A_a
 \end{aligned}$$

$$0 = \dot{\gamma}^a D_a V \quad \leftarrow V(s) = h(s) V(0) h(s)^{-1}$$

$$= \left(\frac{d}{ds} h(s) \right) V h(s)^{-1} + h(s) V \left(\frac{d}{ds} h(s)^{-1} \right)$$

$$\frac{d}{ds} (h^{-1} h) = 0$$

$$+ \dot{\gamma}^a A_a h(s)^{-1} - h(s) V h(s)^{-1} \dot{\gamma}^a A_a$$

$$\frac{d}{ds} h(s)$$

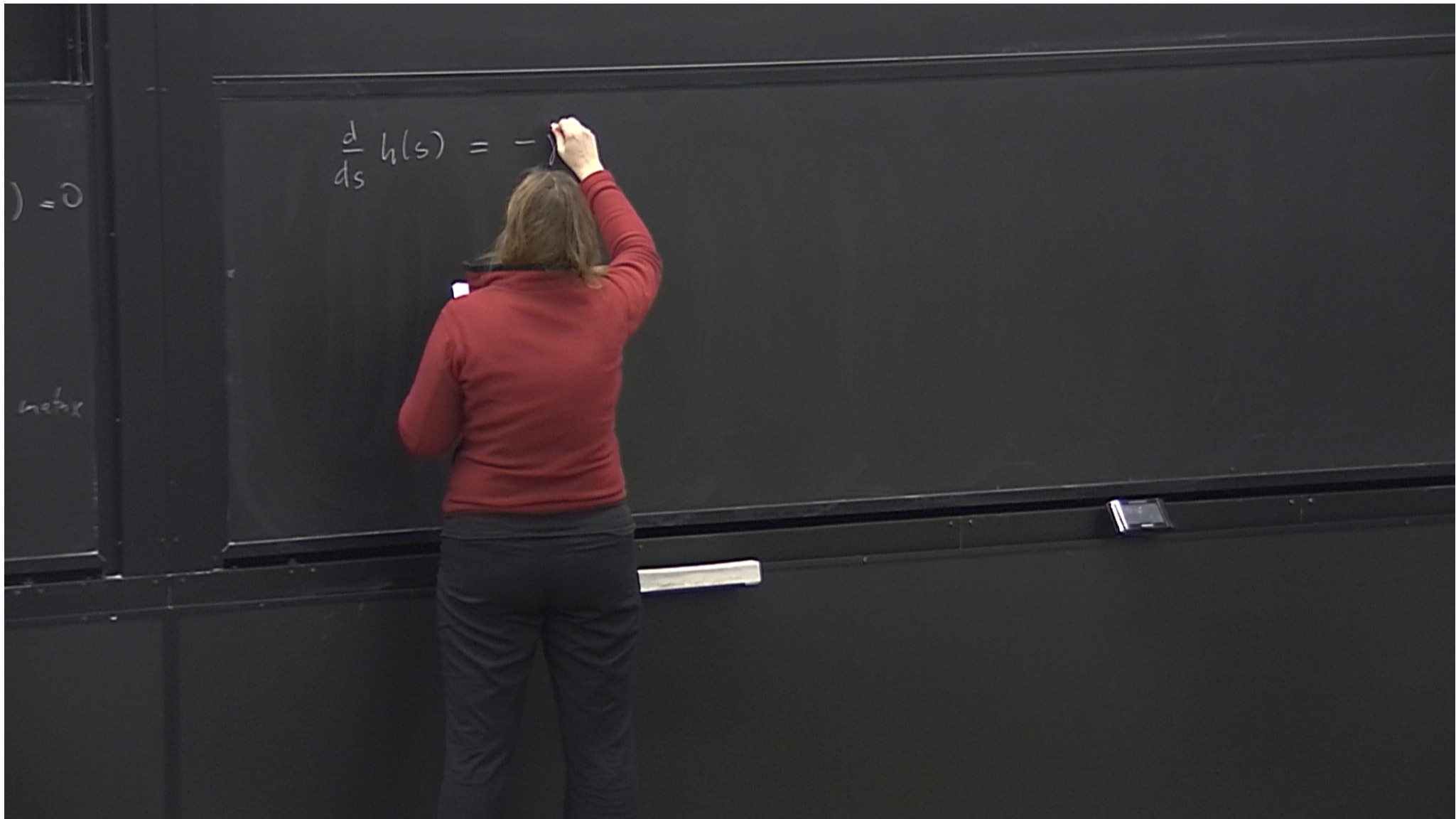
$$0 = \dot{\gamma}^a D_a V \quad \leftarrow V(s) = h(s) V(0) h(s)^{-1}$$

$$= \left(\frac{d}{ds} h(s) \right) V h(s)^{-1} + h(s) V \left(\frac{d}{ds} h(s)^{-1} \right)$$

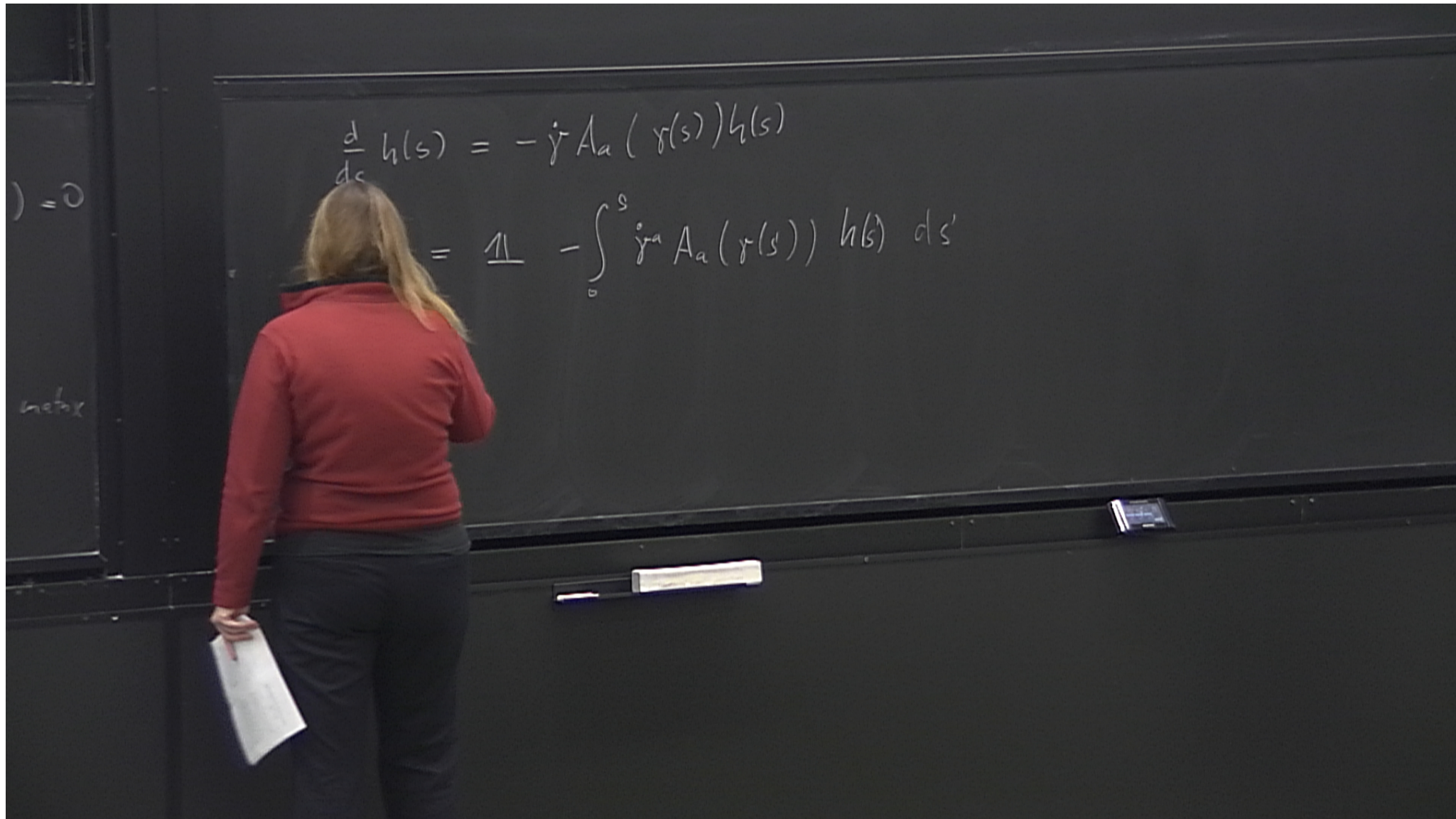
$$\frac{d}{ds} (h^{-1} h) = 0$$

$$+ \dot{\gamma}^a A_a h(s) V h(s)^{-1} - h(s) V h(s)^{-1} \dot{\gamma}^a A_a$$

$$\frac{d}{ds} h(s) = - \dot{\gamma}^a A_a(\gamma(s)) \cdot h(s) \quad , \quad h(0) = \underline{1} \leftarrow \text{identity matrix}$$



$$\frac{d}{ds} h(s) = -$$



$$\frac{d}{ds} h(s) = -\dot{\gamma}^a A_a(\gamma(s)) h(s)$$

$$h(s) = \mathbb{1} - \int_0^s \dot{\gamma}^a A_a(\gamma(s')) h(s') ds'$$

iterating

$$\begin{aligned} h_p(s) &= \mathbb{P} \exp \left[- \int_{\gamma} A \right] := \\ &= \sum_{n=0}^{\infty} (-1)^n \int_0^s ds_{n-1} \cdots \int_0^{s_2} ds_1 A(s_n) A(s_{n-1}) \cdots A(s_1) A(s) \end{aligned}$$

$$\frac{d}{ds} h(s) = -\dot{\gamma}^a A_a(\gamma(s)) h(s)$$

$$h(s) = \mathbb{1} - \int_0^s \dot{\gamma}^a A_a(\gamma(s')) h(s') ds'$$

iterating

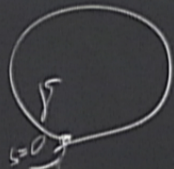
$$\begin{aligned} h_f(s) &= \mathbb{P} \exp \left[- \int_{\gamma} A \right] := \\ &= \sum_{n=0}^{\infty} (-1)^n \int_0^s ds_{n-1} \cdots \int_0^{s_2} ds_1 A(s_n) A(s_{n-1}) \cdots A(s_1) A(s) \end{aligned}$$

$$r(s) h(s)$$

$$\int A_a(r(s)) h(s) ds$$

$$A \int :=$$

$$\int_0^s ds_0 A(s_0) A(s_{0+1}) \dots A(s_r) A(s_1)$$



$$h(x) \rightarrow g(0) h(x) g(0)^{-1}$$

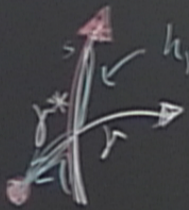
Wilson loops

$$D_a V = \partial_a V + [A_a, V]$$

$$h \approx \rho_{exp}$$

Fluxes $\rightarrow E^a_j$

$$[e_a] = e_a^j T_j = \tilde{E}_{ba} E^b_j T_j$$



$h_{y, y^*}(s)$: from $y^*(s)$ to y

$$E_y = \int_{y^*} h_y$$

$$D_a V = \partial_a V + [A_a, V]$$

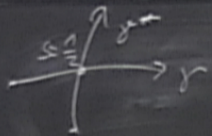
$$h = y \exp$$

Fluxes $\rightarrow E^a_s$

$$e = e_a^j T_j = \tilde{e}_{ba} E^b_j T_j$$

$h_{\gamma, \gamma^*}(s)$: from $\gamma^*(s)$ to $\gamma(0)$

$$\bar{E}_\gamma = \int_{\gamma^*} h_{\gamma, \gamma^*}(s) e_a(\gamma^*(s)) \dot{\gamma}^a(s) (h_{\gamma, \gamma^*}(s))^{-1} ds$$



$$h_Y = h_Y(1, \frac{1}{2} + \frac{\epsilon}{2}) \cdot h_Y(\frac{1}{2} + \frac{\epsilon}{2}, \frac{1}{2} - \frac{\epsilon}{2}) \cdot h_Y(\frac{1}{2} - \frac{\epsilon}{2}, 0)$$

$h_Y(s'', s')$ holonomy from s' to s'' along γ

$$h_Y(\frac{1}{2} + \frac{\epsilon}{2}, \frac{1}{2} - \frac{\epsilon}{2}) = \mathbb{1} - \int_{1/2 - \epsilon/2}^{1/2 + \epsilon/2} T_c \dot{\gamma}(s) A_c^e(\gamma(s)) ds + \mathcal{O}(\epsilon^2)$$



$$h(\gamma) \rightarrow g(0) h(\gamma) g(0)^{-1}$$

Wilson loops

$$\bar{E}_r = \int_{r^*} h_{r,r^*}(s) e_a(r^*(s)) (\dot{y}^a(s)) (h_{r,r^*}(s))^{-1} ds$$

$$(E_r)_i = \int_{r^*} (-2) \text{tr} \left[\bar{E}_i h_{r,r^*}(s) T_k (h_{r,r^*}(s))^{-1} \right] \\ = \int_{r^*} f_{jk}(s) \bar{e}_{ba} \dot{y}^{*a}(s) E^{bk}(r(s))$$

$$\{ (E_r)_i, h_0 \} = -h_r \left(1, \frac{1}{2} + \frac{\varepsilon}{2} \right) T_e h_r \left(\frac{1}{2} - \frac{\varepsilon}{2}, 0 \right) \times$$

$$\int_{r^*} \int_{\frac{1}{2}-\varepsilon/2}^{\frac{1}{2}+\varepsilon/2} f_{jk}(s) \bar{e}_{ba} \dot{y}^{*a}(s) \dot{y}^c(s') \{ E^{bk}(r(s)), A_c^l(r(s')) \} ds ds' + O(\varepsilon^2)$$

$$\bar{E}_Y = \int_{Y^*} h_{Y,Y^*}(s) e_a(Y(s)) (\dot{Y}^a(s)) (h_{Y,Y^*}(s))^{-1} ds$$

$$(E_Y)_i = \int_{Y^*} (-2) T_i \left[\bar{E}_Y h_{Y,Y^*}(s) T_k h_{Y,Y^*}(s) \right] \\ = i \int_{Y^*} f_{jk}(s) \bar{E}_{ba} \dot{Y}^a(s) E^{bk}(s)$$

$$\{ (E_Y)_i, h_0 \} = -h_Y \left(1, \frac{1}{2} + \frac{\varepsilon}{2} \right) T_e h_Y \left(\frac{1}{2} - \frac{\varepsilon}{2}, 0 \right) \times$$

$$\int_{Y^*} \int_{\frac{1}{2}-\varepsilon/2}^{\frac{1}{2}+\varepsilon/2} f_{jk}(s) \underbrace{\bar{E}_{ba} \dot{Y}^a(s) \dot{Y}^c(s')}_{\geq 0} \underbrace{\left(-b_k(\dot{Y}^a(s), A_c^l(Y(s'))) \right)}_{\delta(\dot{Y}(s), Y(s')) \delta_c^b \delta^{kl}} ds ds' + O(\varepsilon^2)$$

$\delta^{kl} \delta(s, s')$

$$\bar{E}_Y = \int_{Y^*} h_{Y,Y^*}(s) e_a(Y^*(s)) (\dot{Y}^a(s)) (h_{Y,Y^*}(s))^{-1} ds$$

$$(E_Y)_i = \int_{Y^*} (-2) \text{tr} \left[\bar{E}_T^i h_{Y,Y^*}(s) T_k^i h_{Y,Y^*}(s) \right] \\ = \int_{Y^*} f_{jk}(s) \bar{E}_{ba} \dot{Y}^{*a}(s) E^{bk}(s)$$

$$\{ (E_Y)_i, h_0 \} = -h_Y(1, \frac{1}{2} + \frac{\varepsilon}{2}) T_a h_Y(\frac{1}{2} - \frac{\varepsilon}{2}, 0) \times$$

$$\int_{Y^*} \int_{\frac{1}{2}-\varepsilon/2}^{\frac{1}{2}+\varepsilon/2} f_{jk}(s) \underbrace{\bar{E}_{ba} \dot{Y}^{*a}(s) \dot{Y}^{*c}(s') \underbrace{E^{bk}(Y^*(s)), A_c^l(Y(s'))}_{\geq 0}}_{\delta^{ke} \delta(s-s')} ds ds' + O(\varepsilon^2)$$

$$\{ (E_Y)_i, h_0 \} = h_Y(1, \frac{1}{2}) T_a h_Y(\frac{1}{2}, 0) \dot{Y}^{*a}(\frac{1}{2})$$