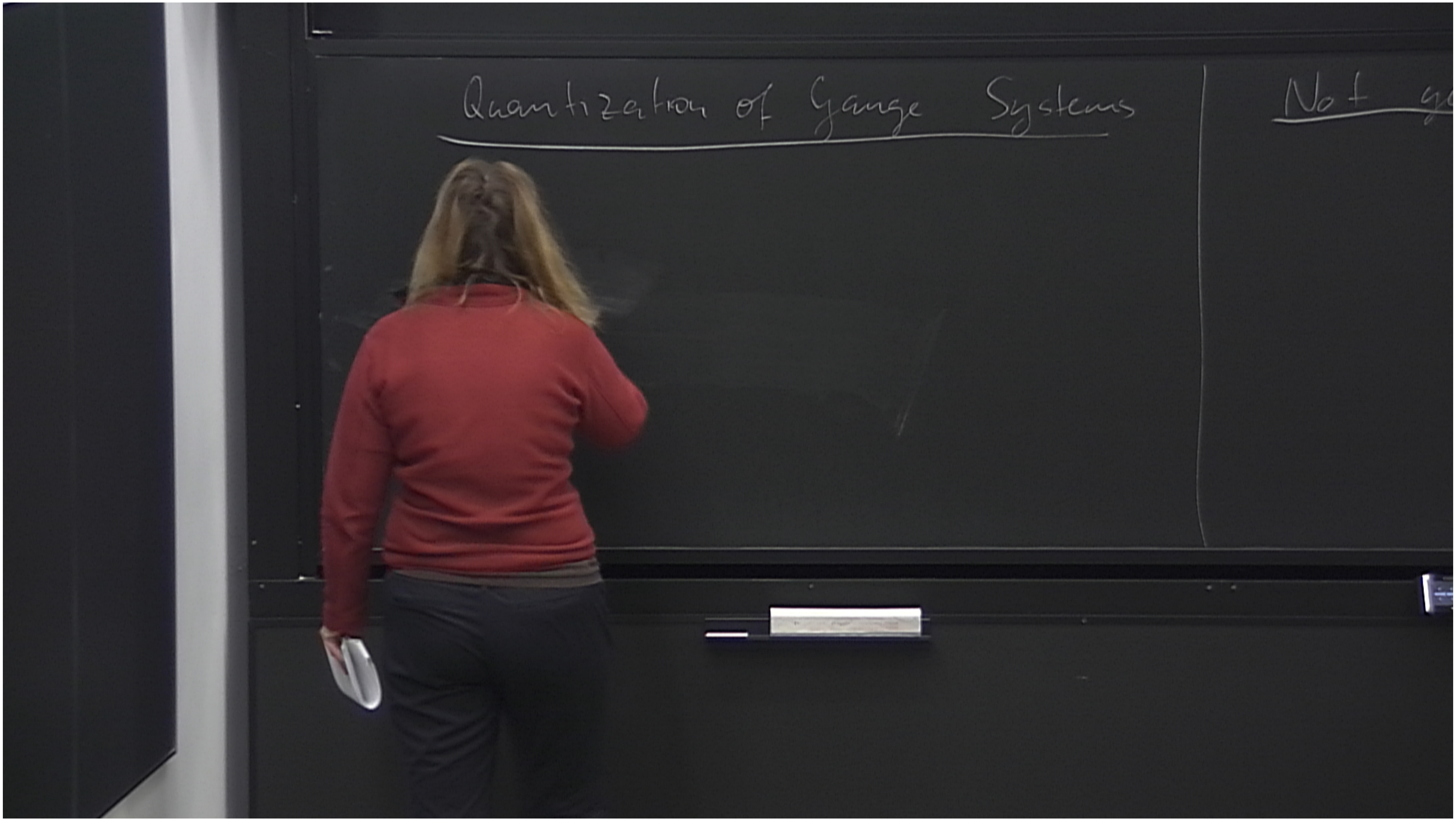
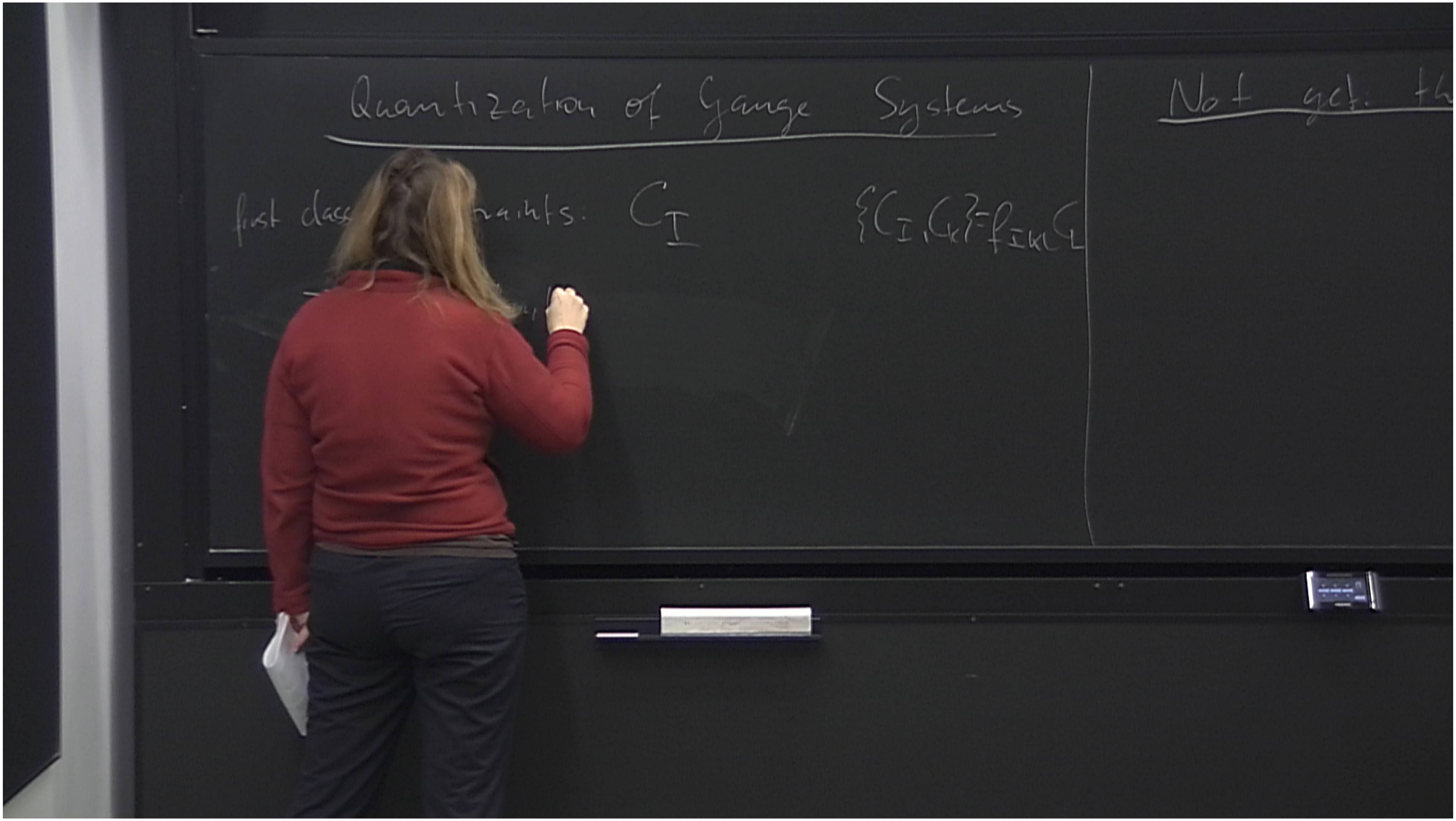
Title: 13/14 PSI - Quantum Gravity Review - Lecture 6

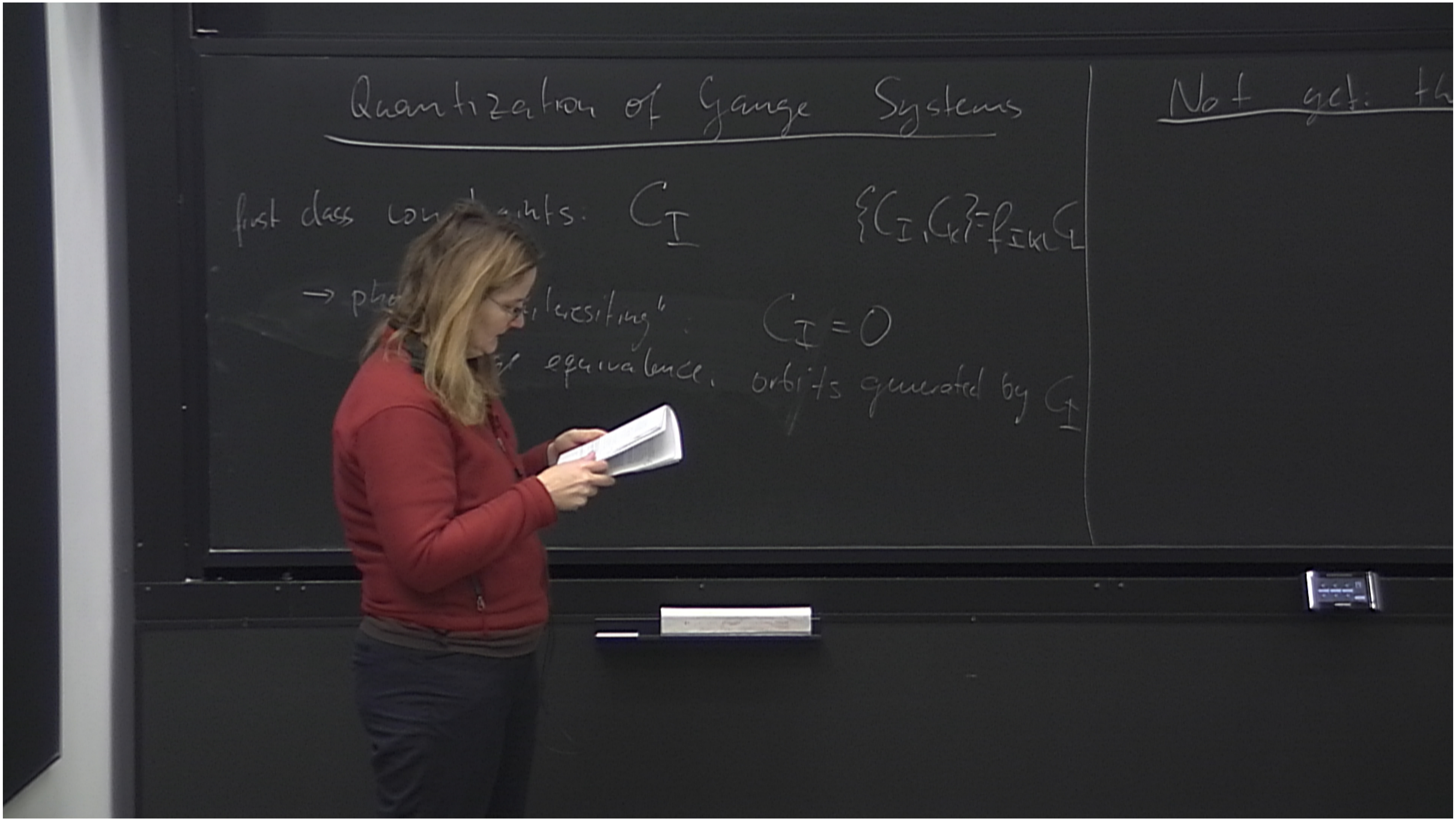
Date: Feb 25, 2014 10:15 AM

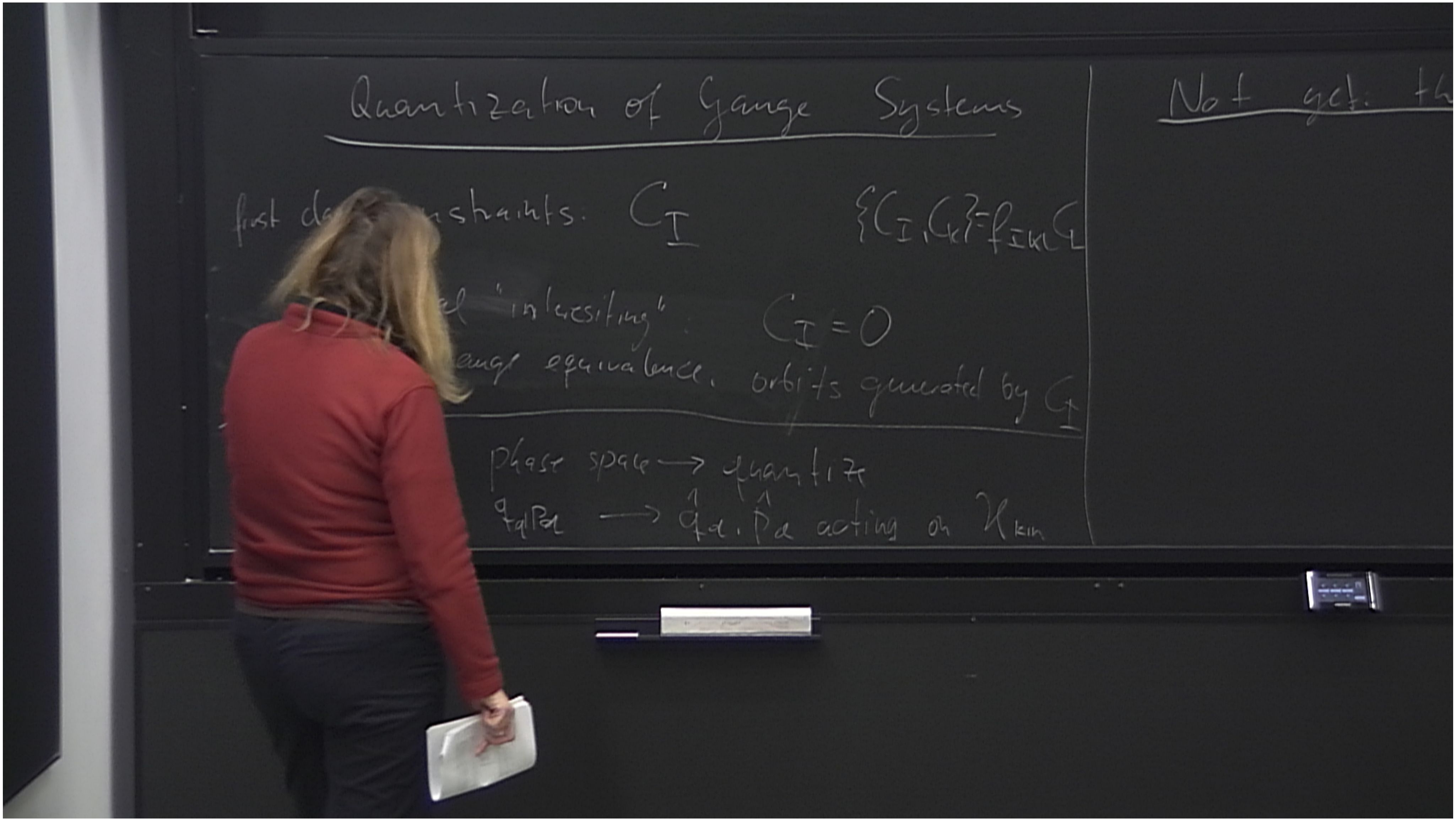
URL: http://pirsa.org/14020068

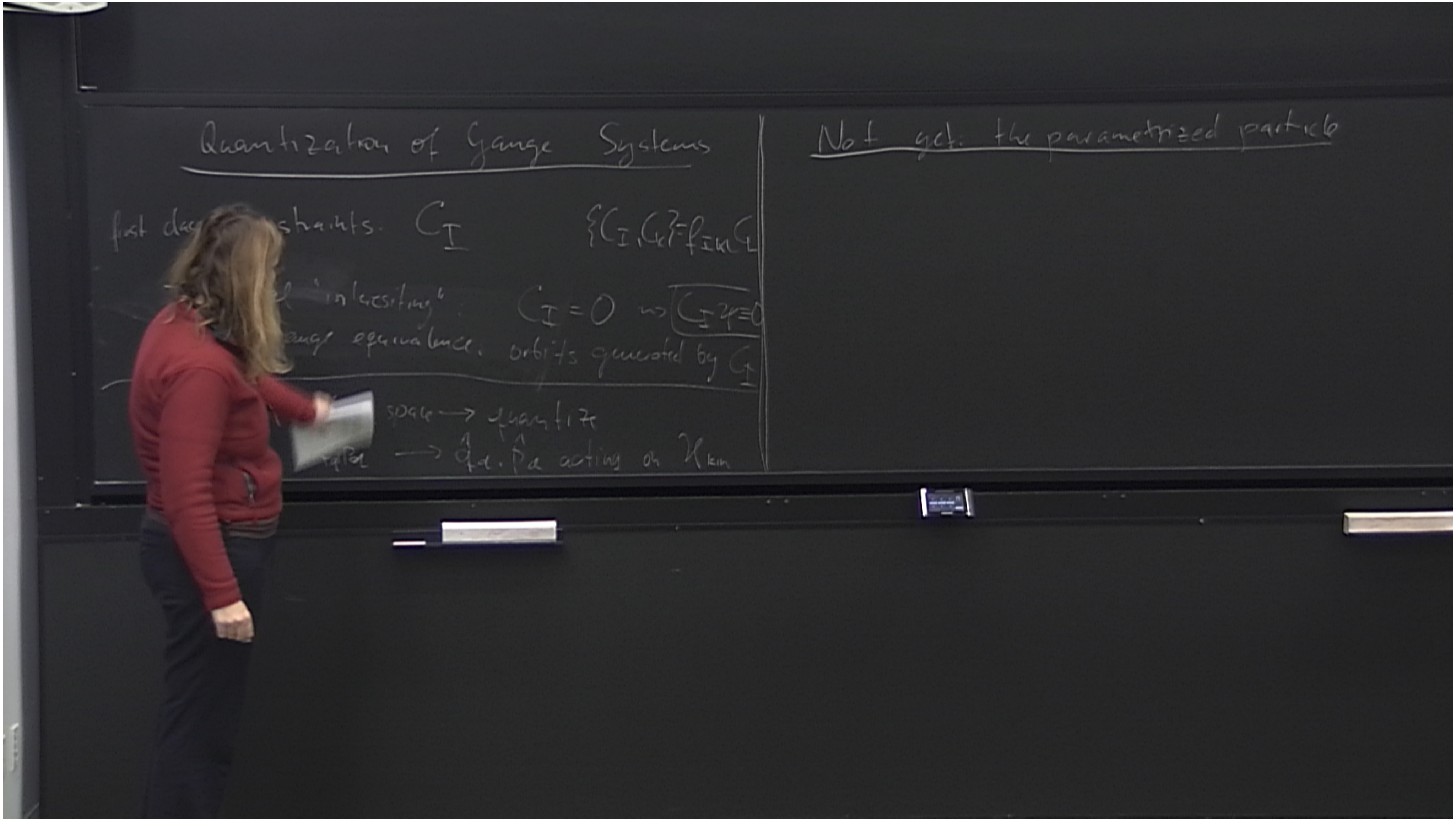
Abstract:

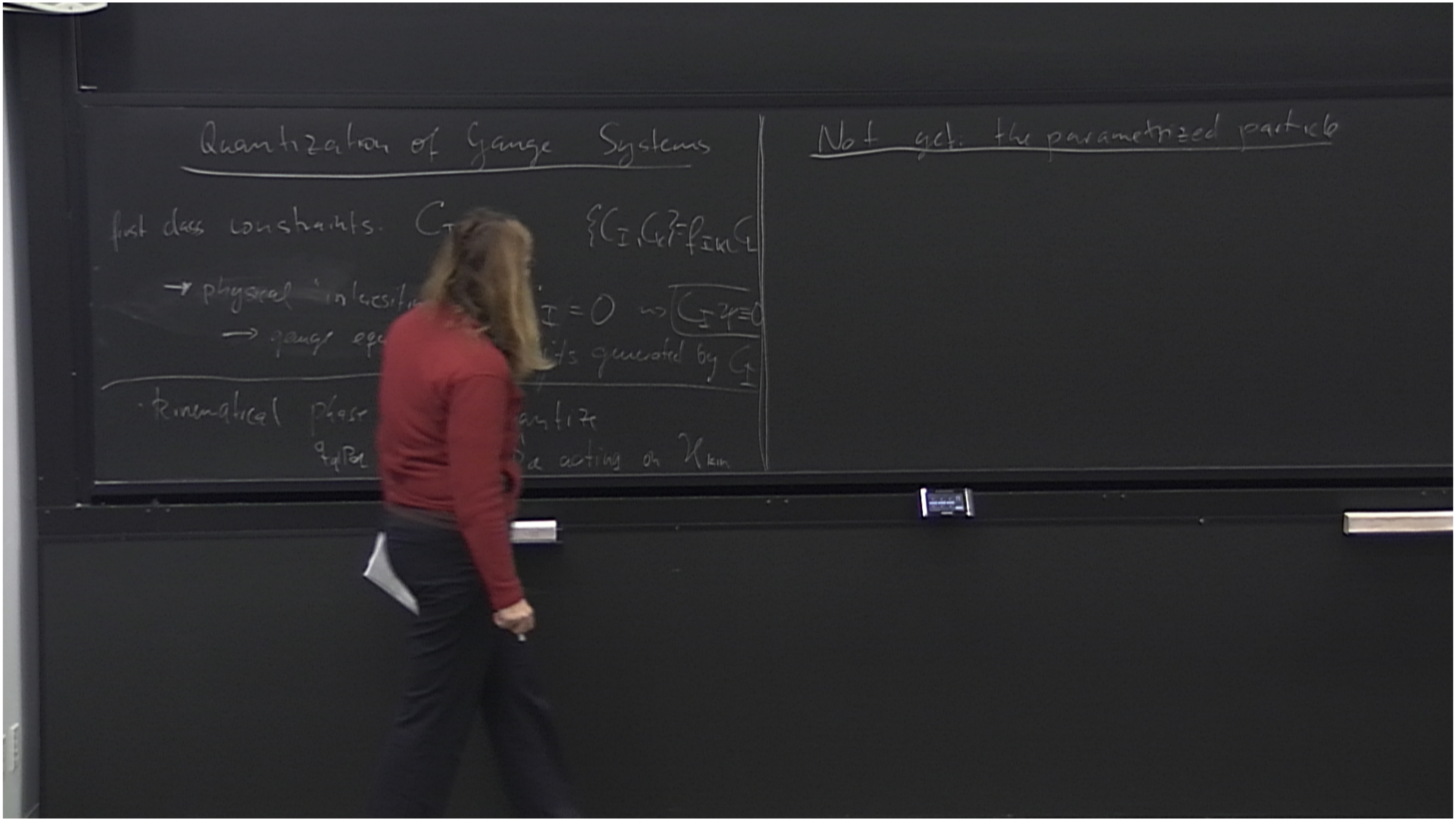

Quantization of Gauge Systems
Not

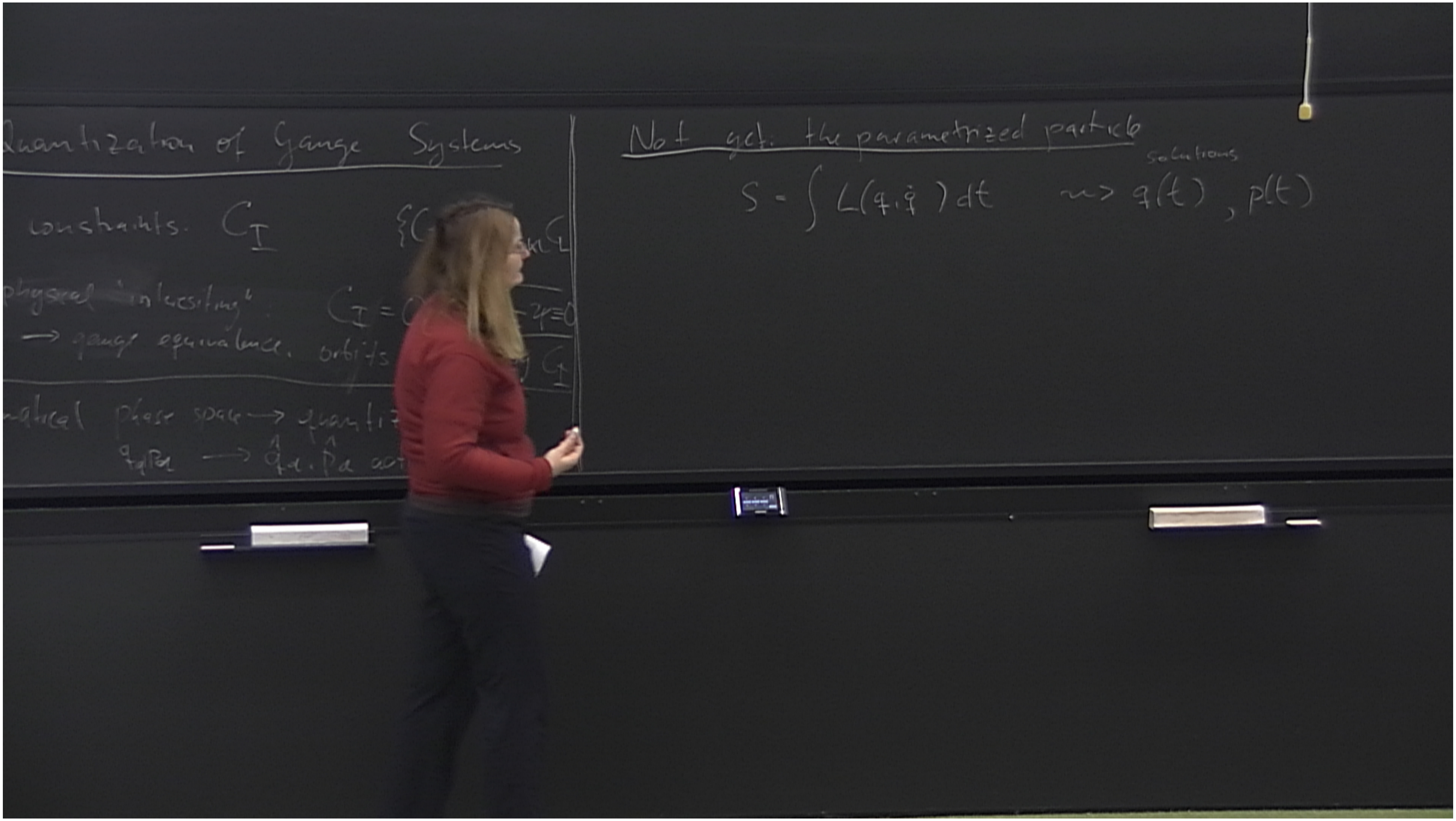

Quantization of Gauge Systems
C_I
$\{C_I, C_K\} = f_{IKL} C_L$
Not yet:

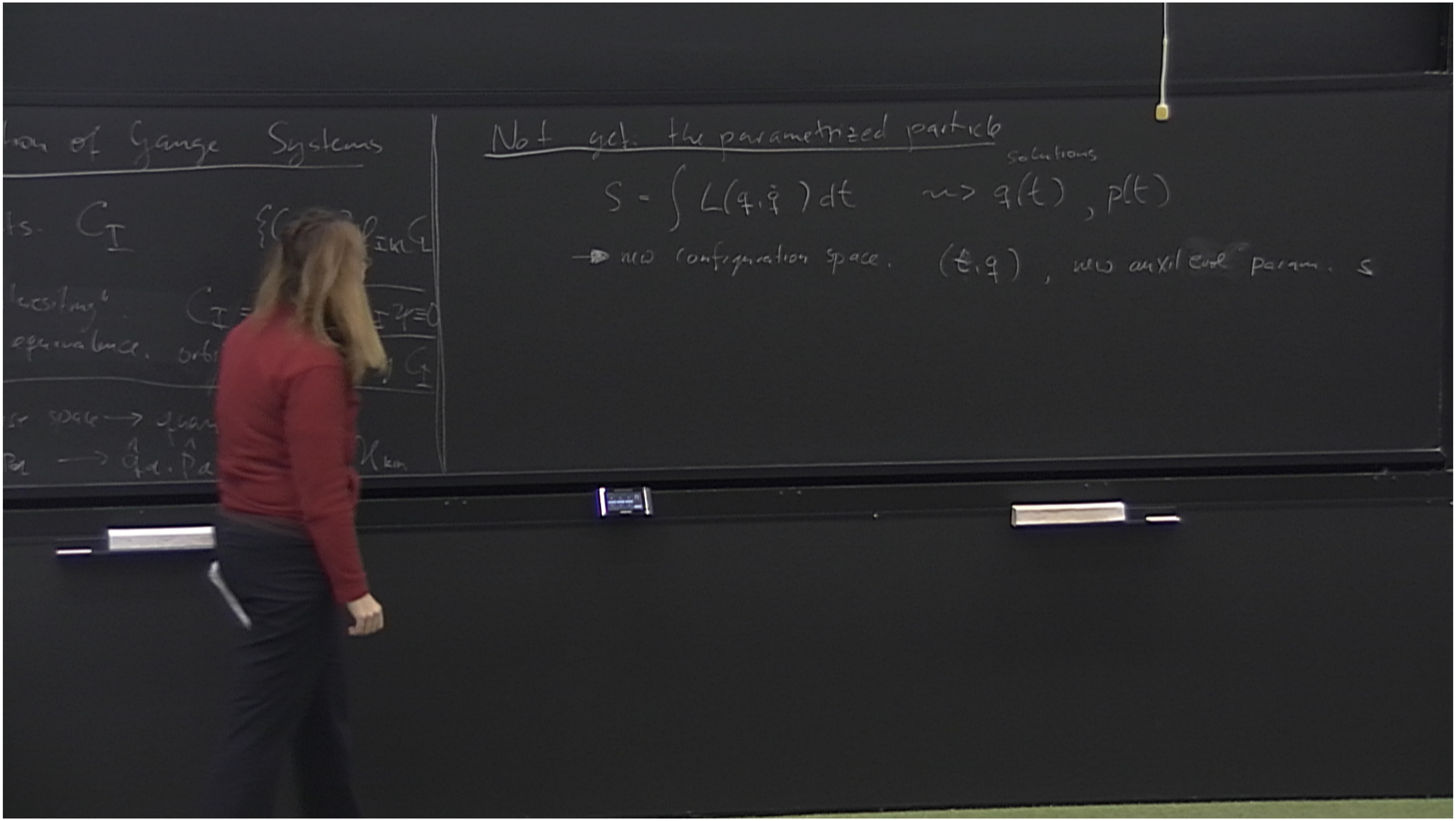

Quantization of Gauge Systems
first class
C_I
$\{C_I, C_K\} = f_{IKL} C_L$
$C_I = 0$
equivalence. orbits generated by C_I
Not yet:

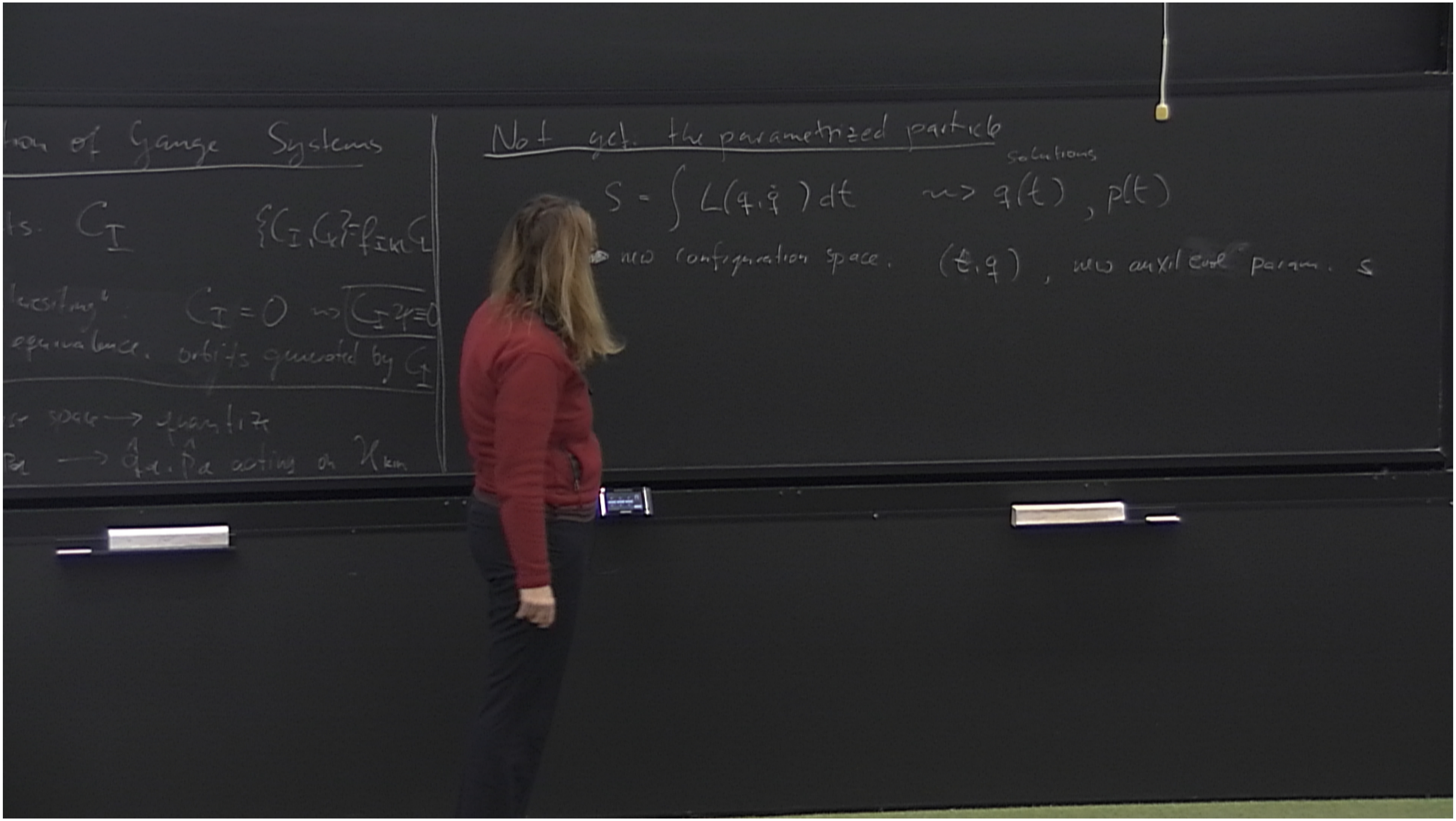

Quantization of Gauge Systems
Not yet:
'interesting': C_I = 0
orbits generated by C_I
phase space → quantize
acting on H_kin


Quantization of Gauge Systems
Not yet: the parametrized particle


Quantization of Gauge Systems
first class constraints.
$\{C_I, C_K\} = f_{IKL} C_L$
→ physical
→ gauge
$C_I \psi = 0$
generated by C_I
kinematical phase
acting on $\mathcal{H}_{kin}$
Not yet: the parametrized particle


Quantization of Gauge Systems
constraints. C_I
physical "interesting": $C_I = 0$
→ gauge equivalence. orbits
Not yet: the parametrized particle
solutions
$S = \int L(q,\dot q)\, dt \rightsquigarrow q(t), p(t)$


of Gauge Systems
Not yet: the parametrized particle
solutions
S = ∫ L(q, q̇) dt ~> q(t), p(t)
→ new configuration space. (t, q), new auxiliary param. s


Not yet: the parametrized particle
solutions
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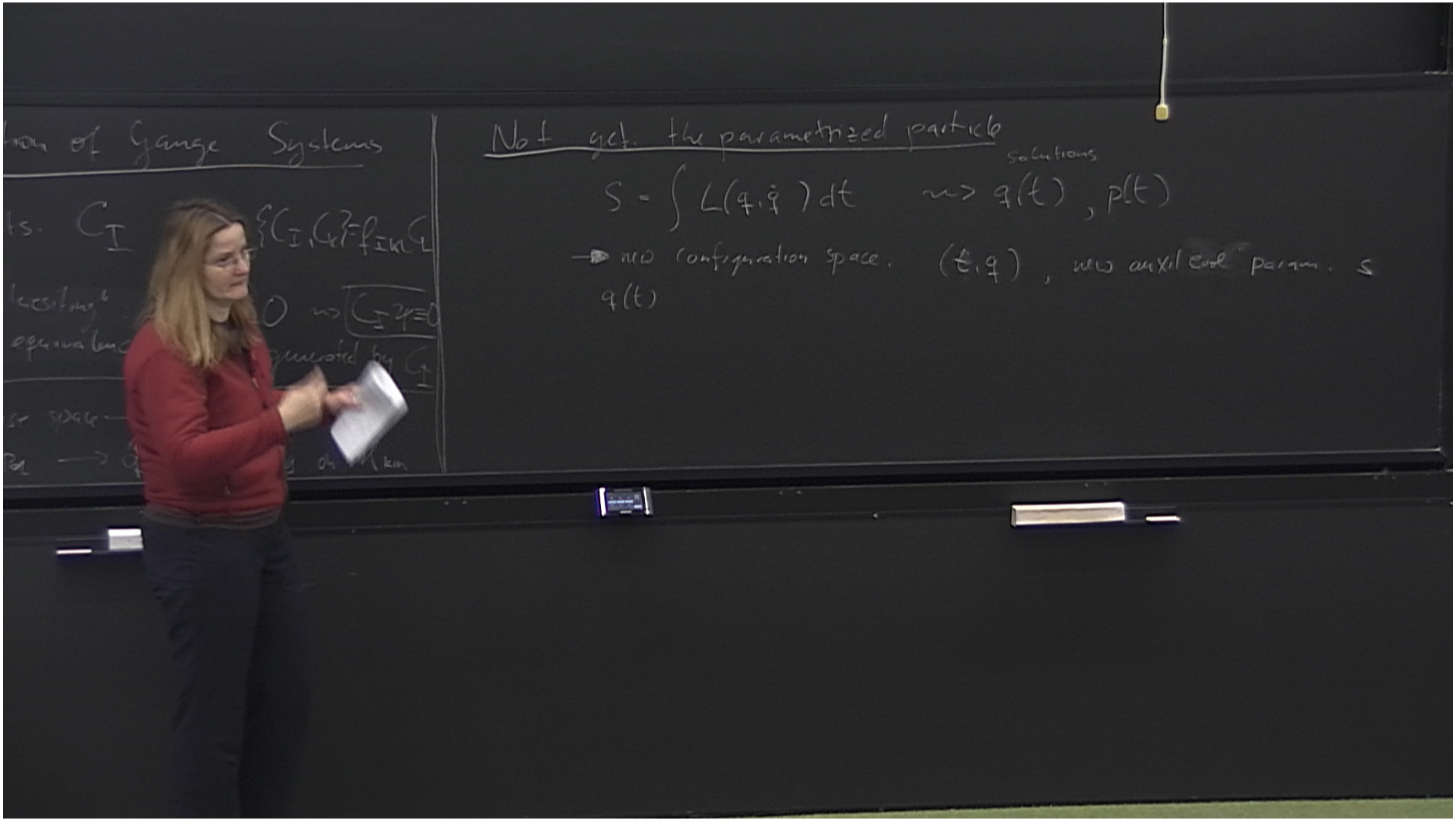
new configuration space. (t, q), new auxiliary param. s
tion of Gauge Systems
equivalence. orbits generated by C_I


of Gauge Systems
Not yet: the parametrized particle
solutions
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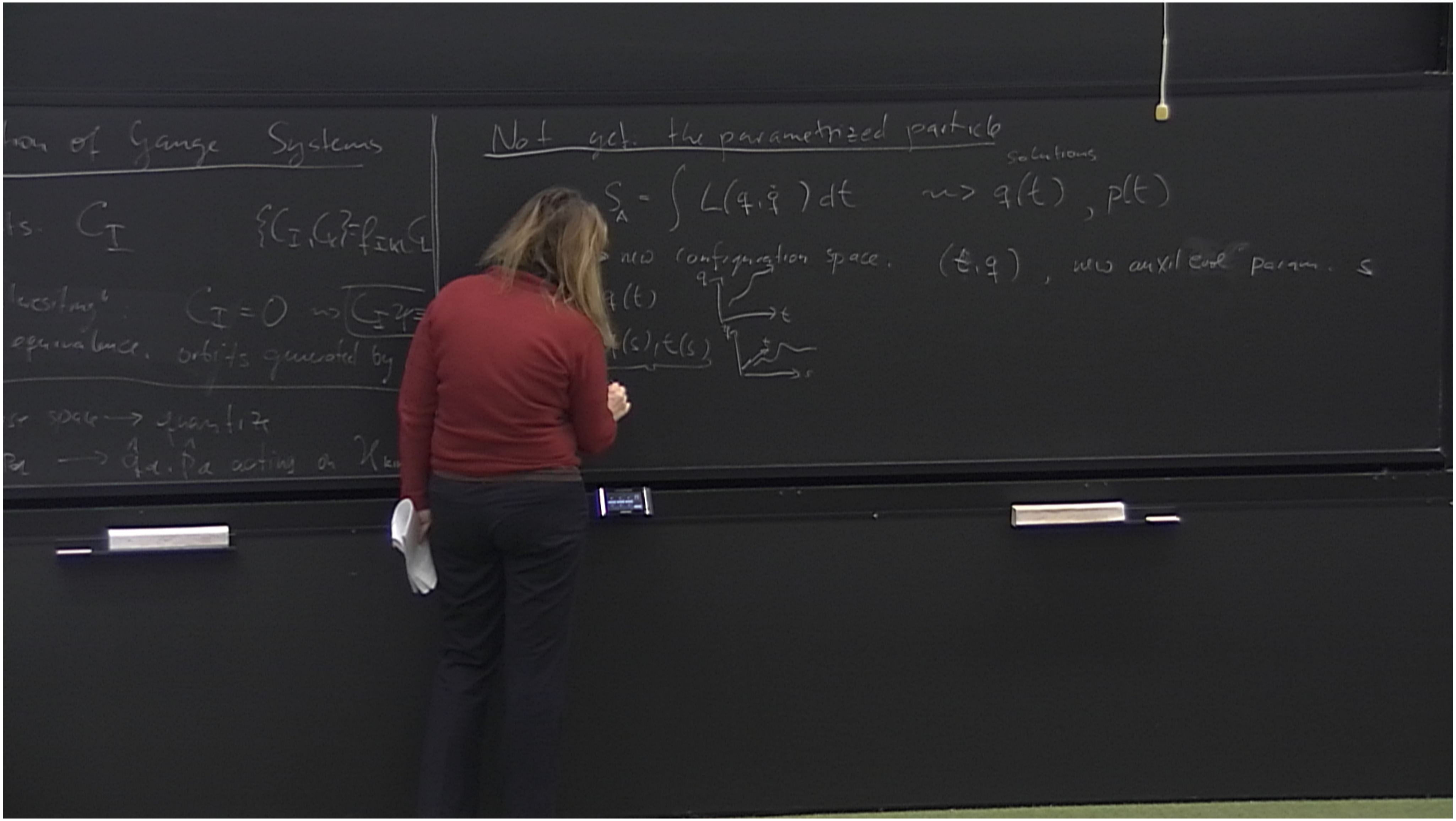
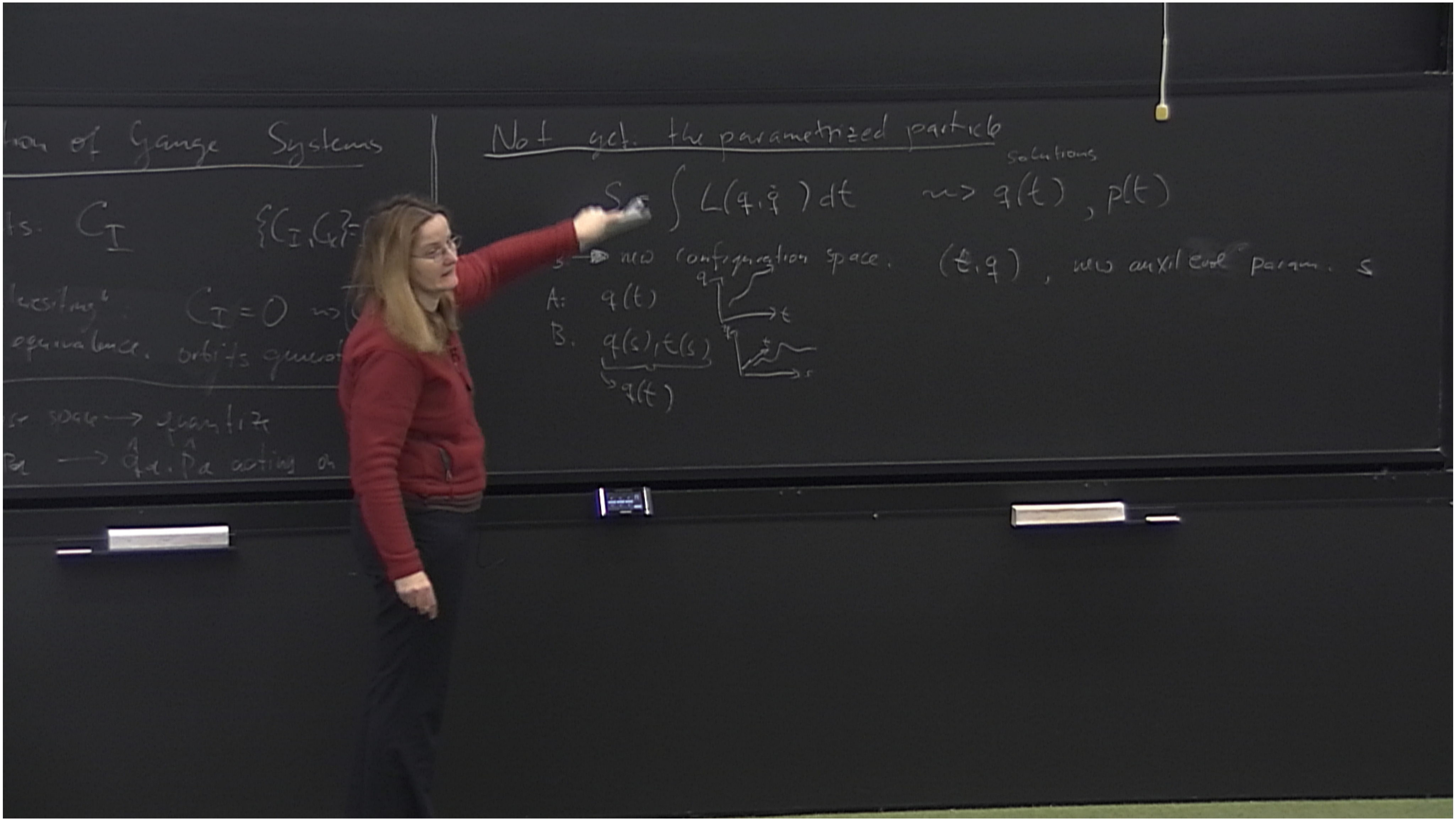
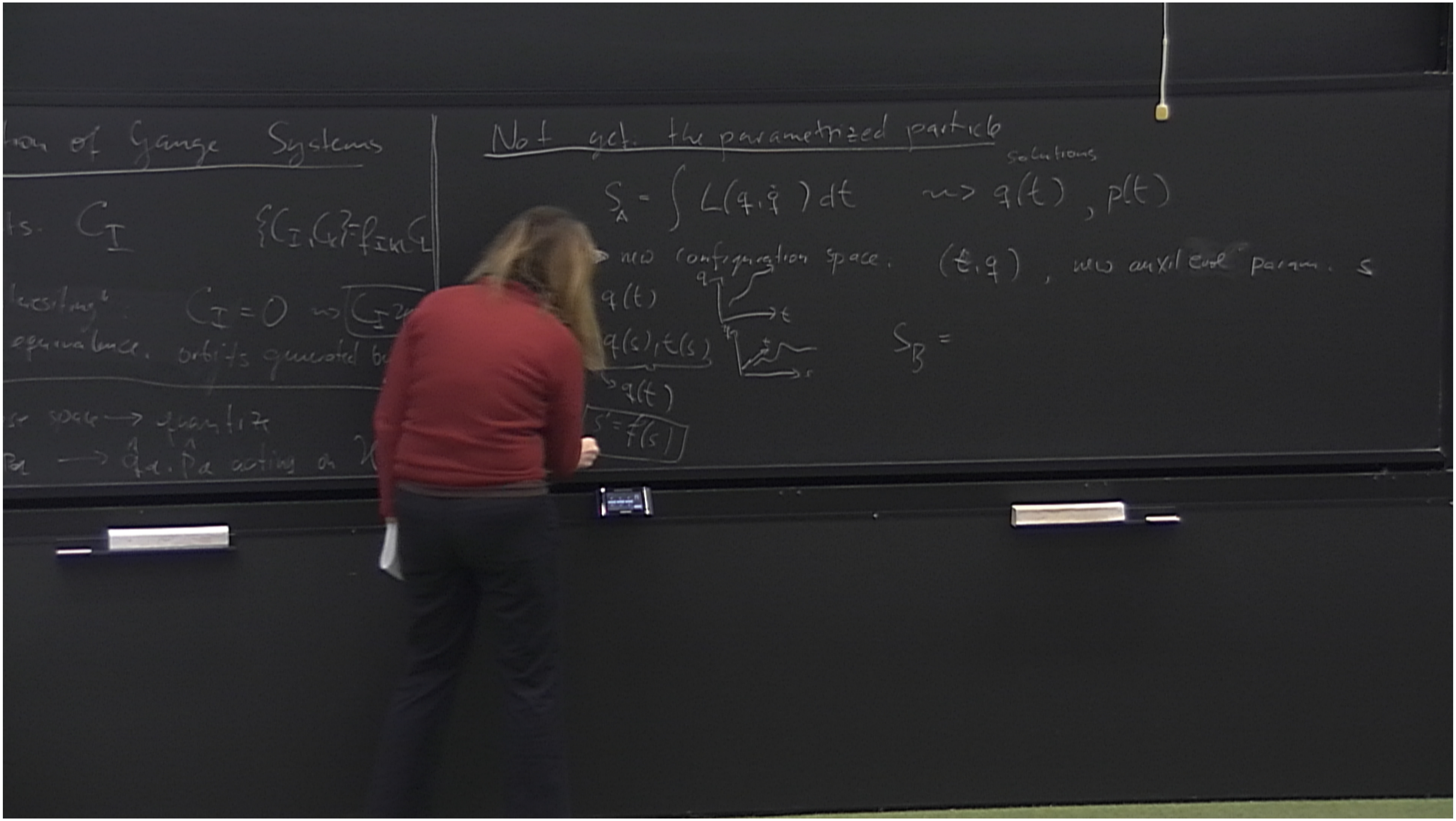
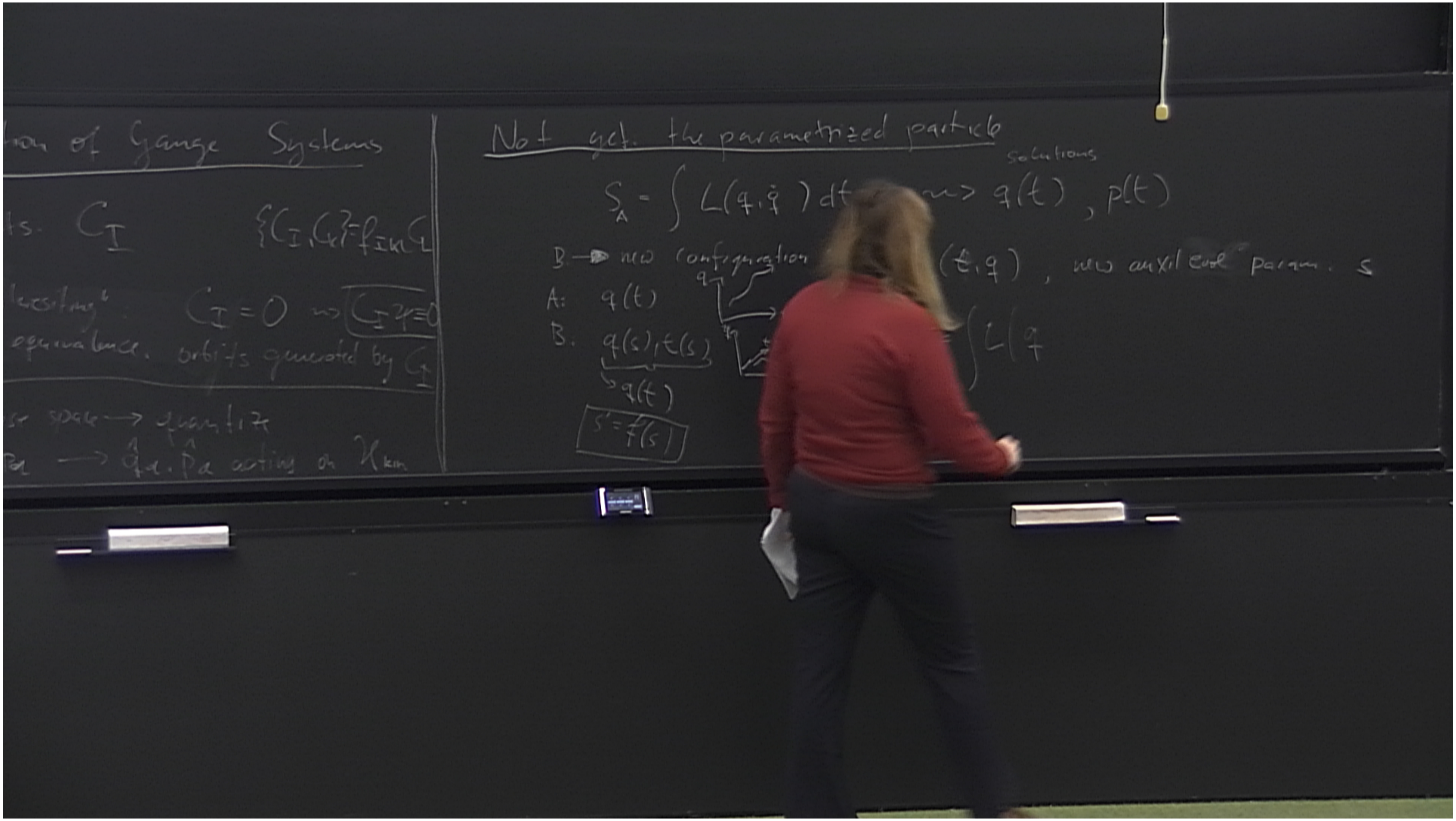
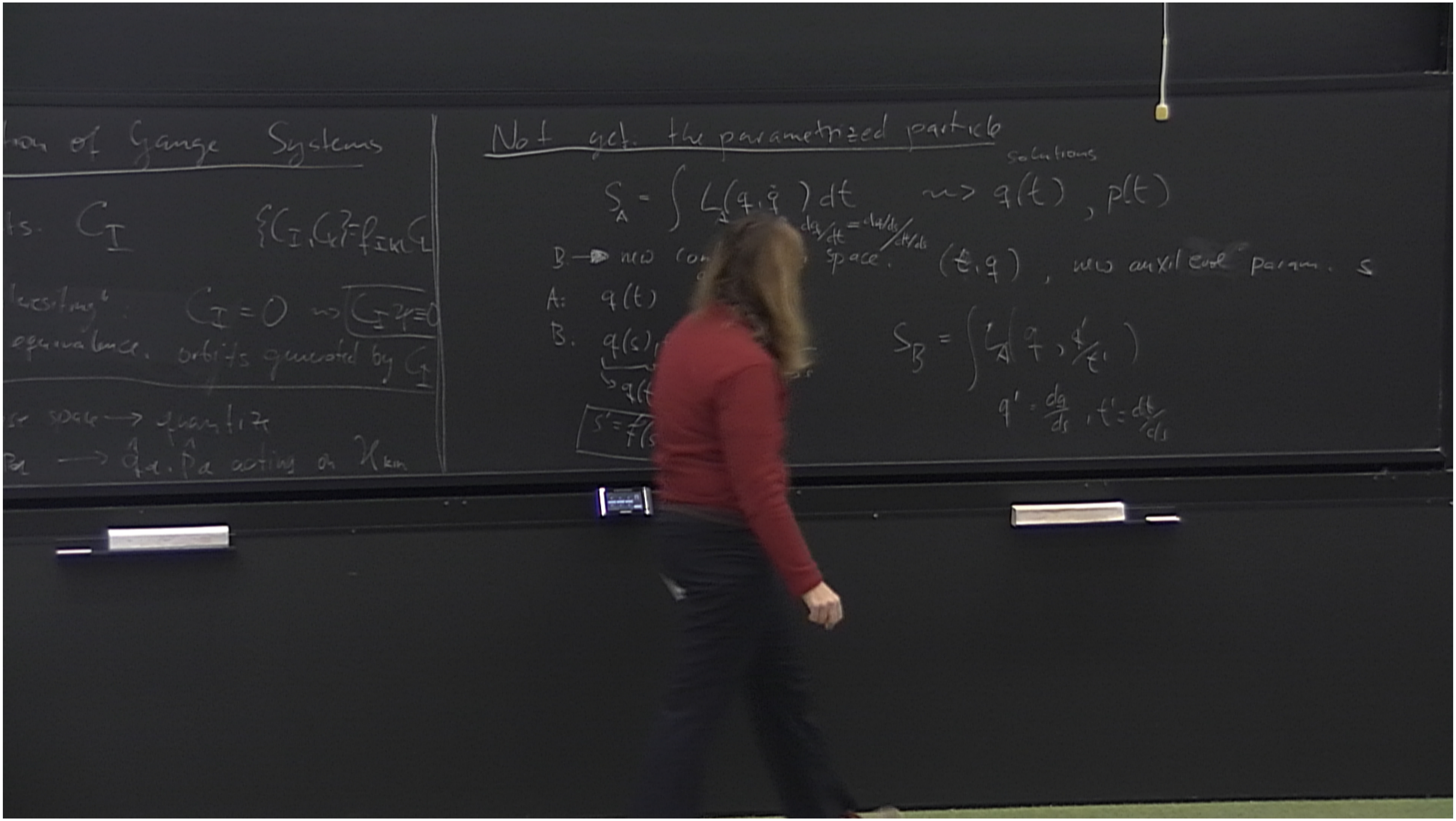
→ new configuration space. (t, q), new auxiliary param. s
q(t)
generated by C_I

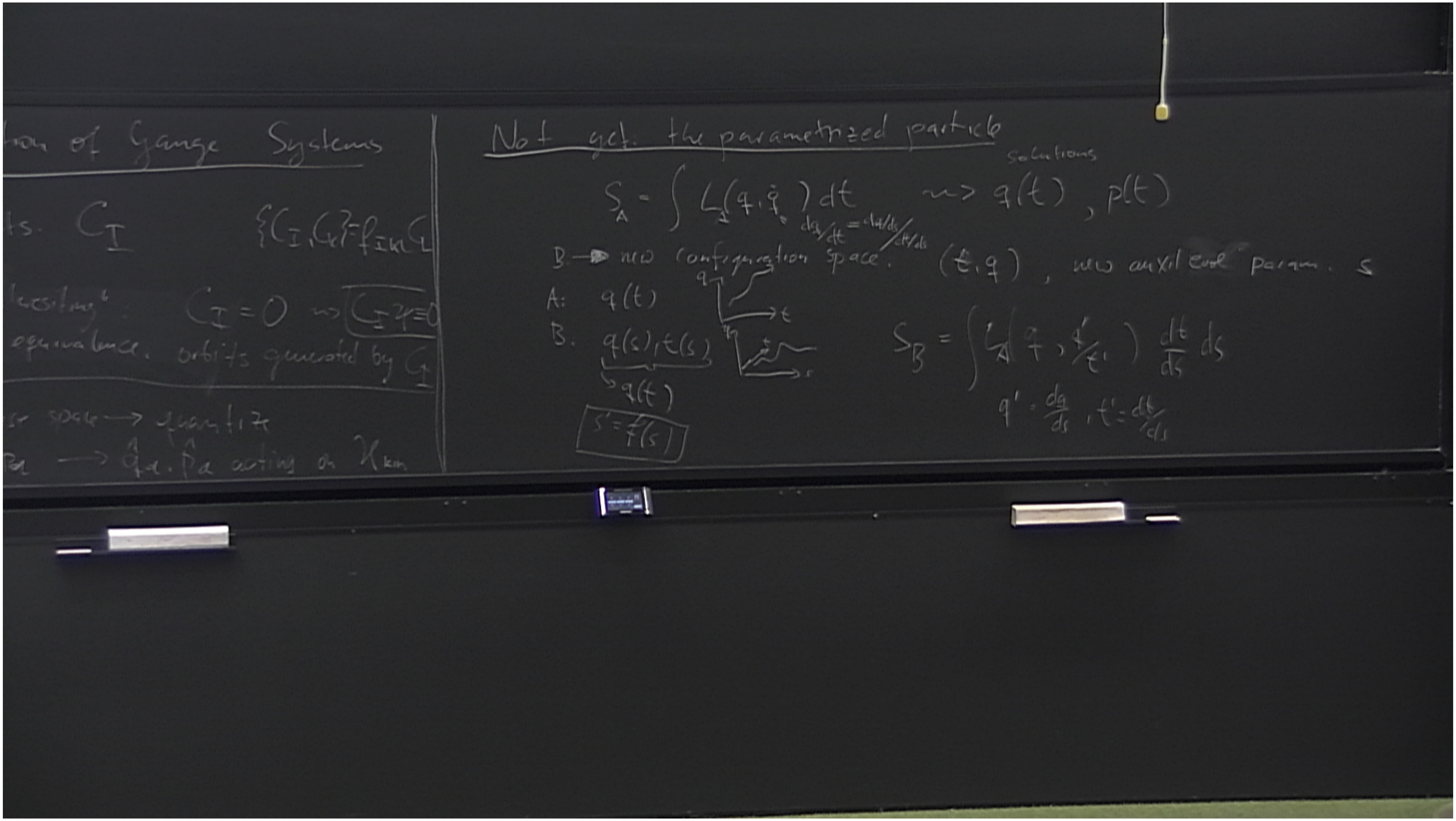

of Gauge Systems
Not yet: the parametrized particle
solutions
new configuration space. (t, q), new auxiliary param. s
orbits generated by


of Gauge Systems
Not yet: the parametrized particle
solutions
S = ∫ L(q, q̇) dt ~> q(t), p(t)
new configuration space. (t, q), new auxil evol param. s
A: q(t)
B. q(s), t(s)
q(t)
space → quantize

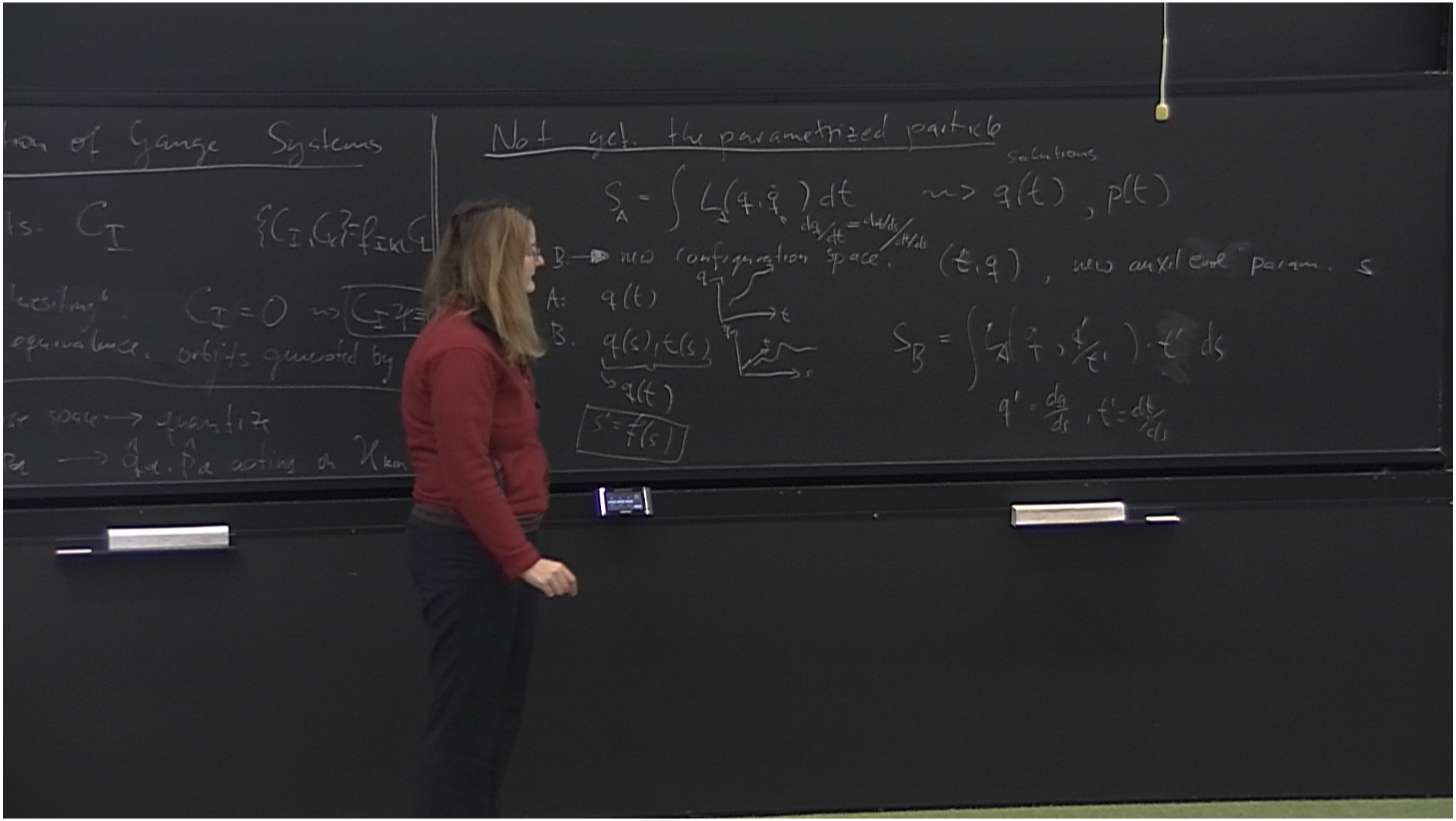

Gauge Systems
Not yet: the parametrized particle
solutions
new configuration space
q(t)
q(s), t(s)
quantize

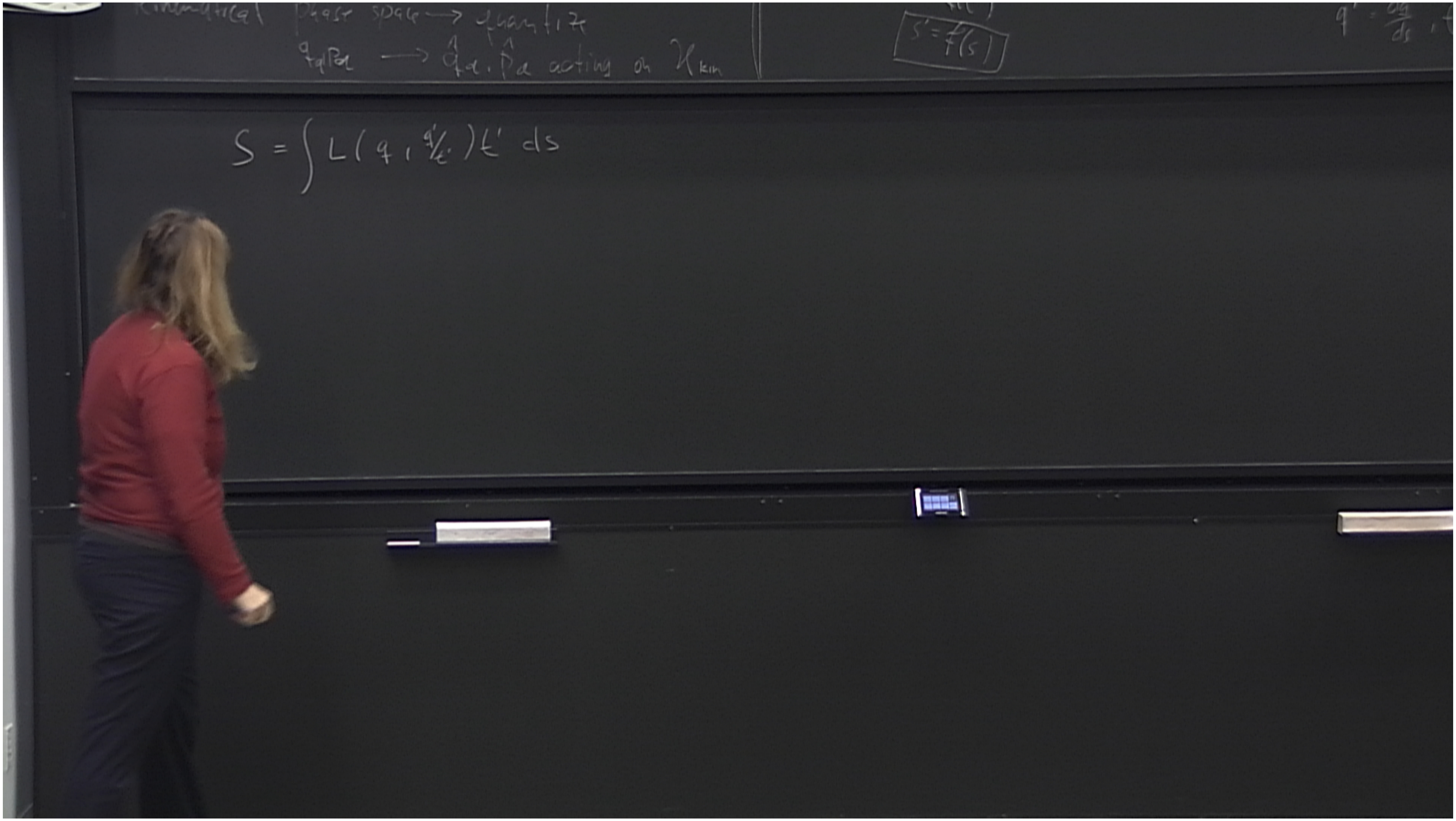

of Gauge Systems
Not yet: the parametrized particle
solutions
q(t), p(t)
new configuration
new auxiliary param. s
A: q(t)
B: q(s), t(s)
q(t)
s' = f(s)
orbits generated by

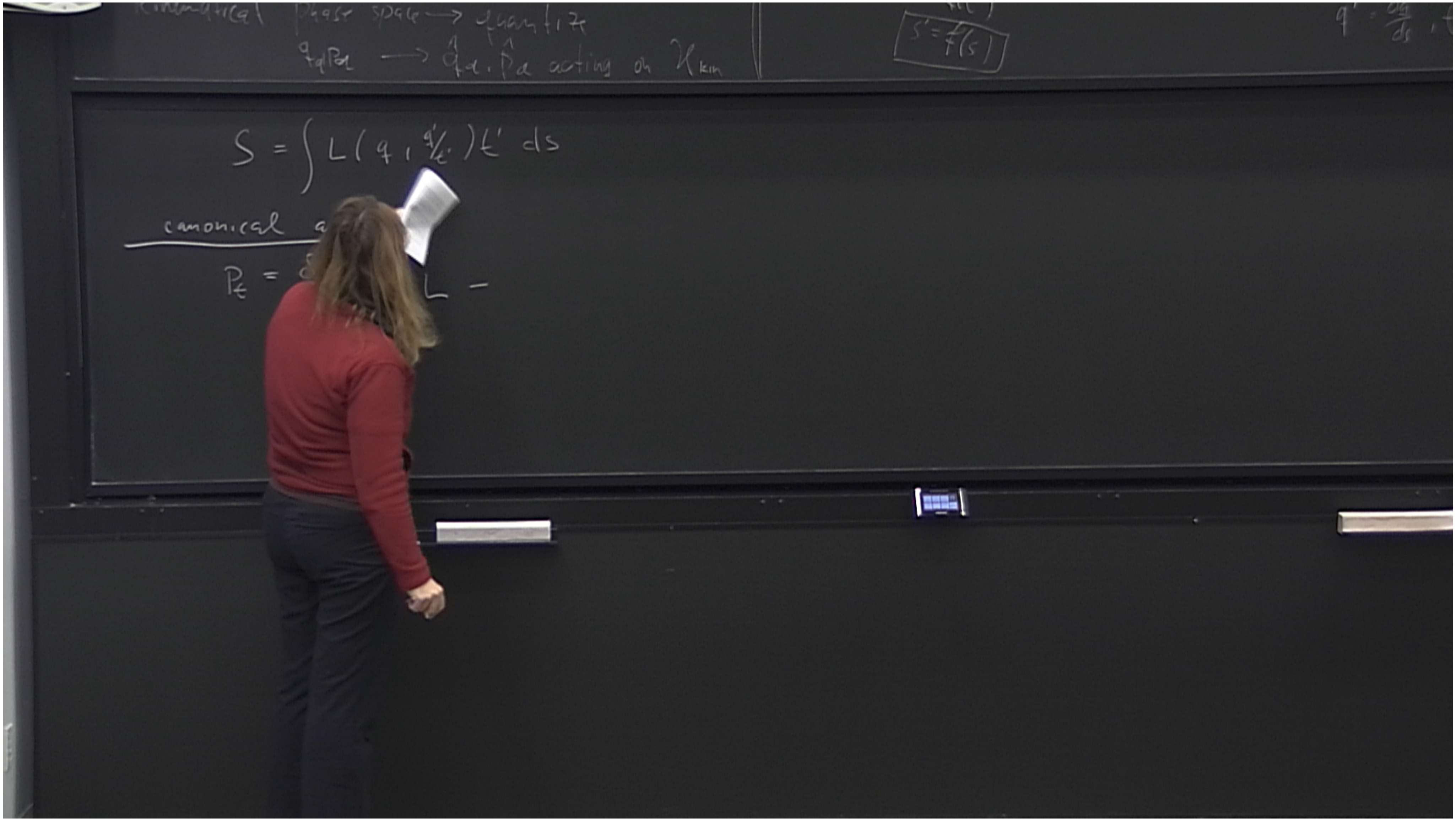

of Gauge Systems
Not yet: the parametrized particle
solutions
space
new auxiliary param. s
orbits generated by
quantize
acting on

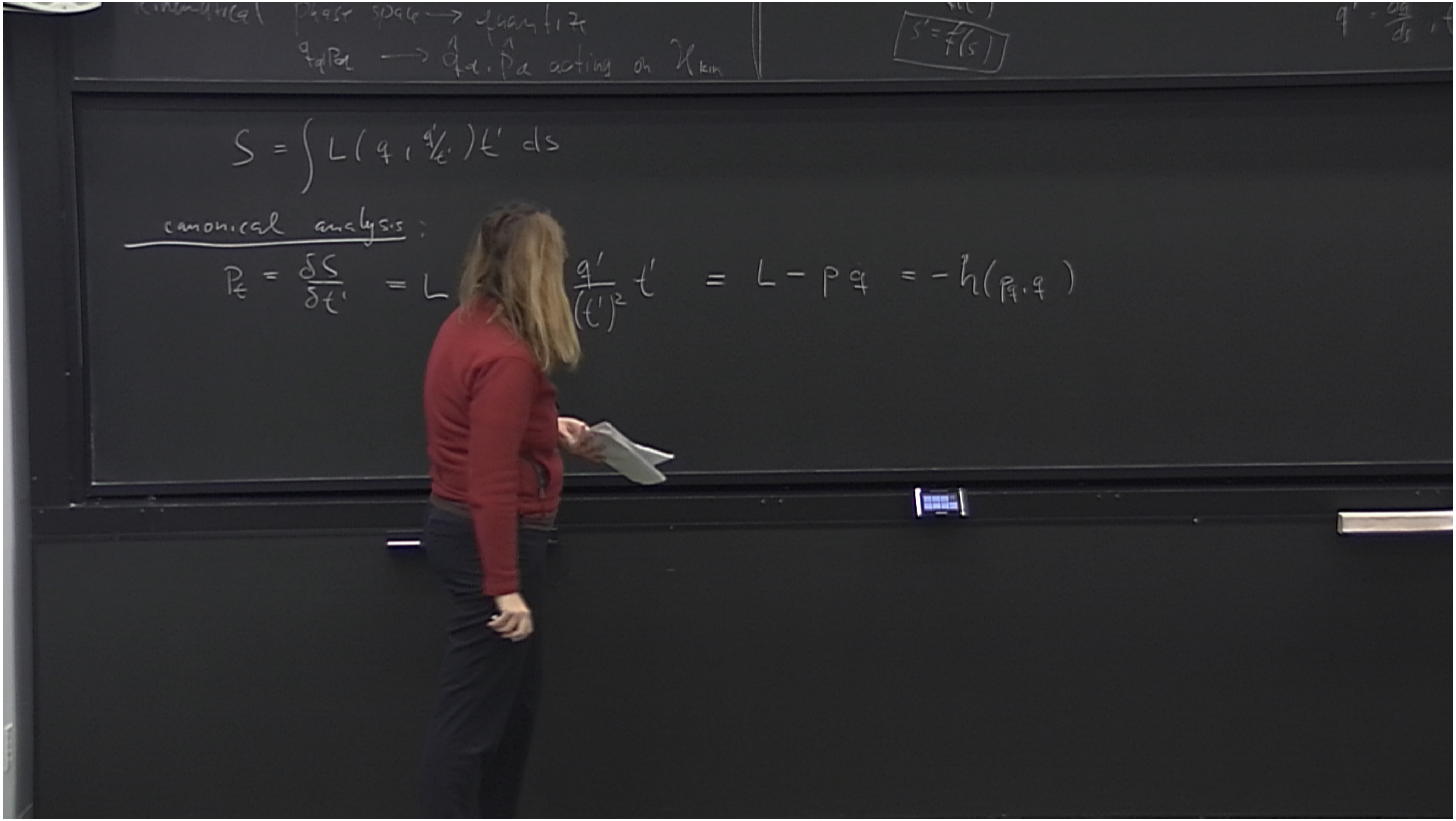


of Gauge Systems
Not yet: the parametrized particle
solutions
$S_A = \int L(q, \dot{q}) dt \rightsquigarrow q(t), p(t)$
B: new configuration space: (t, q), new auxiliary param. s
A: q(t)
B: q(s), t(s)
q(t)
s' = f(s)
$S_B = \int L_A(q, \frac{q'}{t'}) \frac{dt}{ds} ds$
$q' = \frac{dq}{ds}, t' = \frac{dt}{ds}$
equivalence: orbits generated by C_I
space → quantize

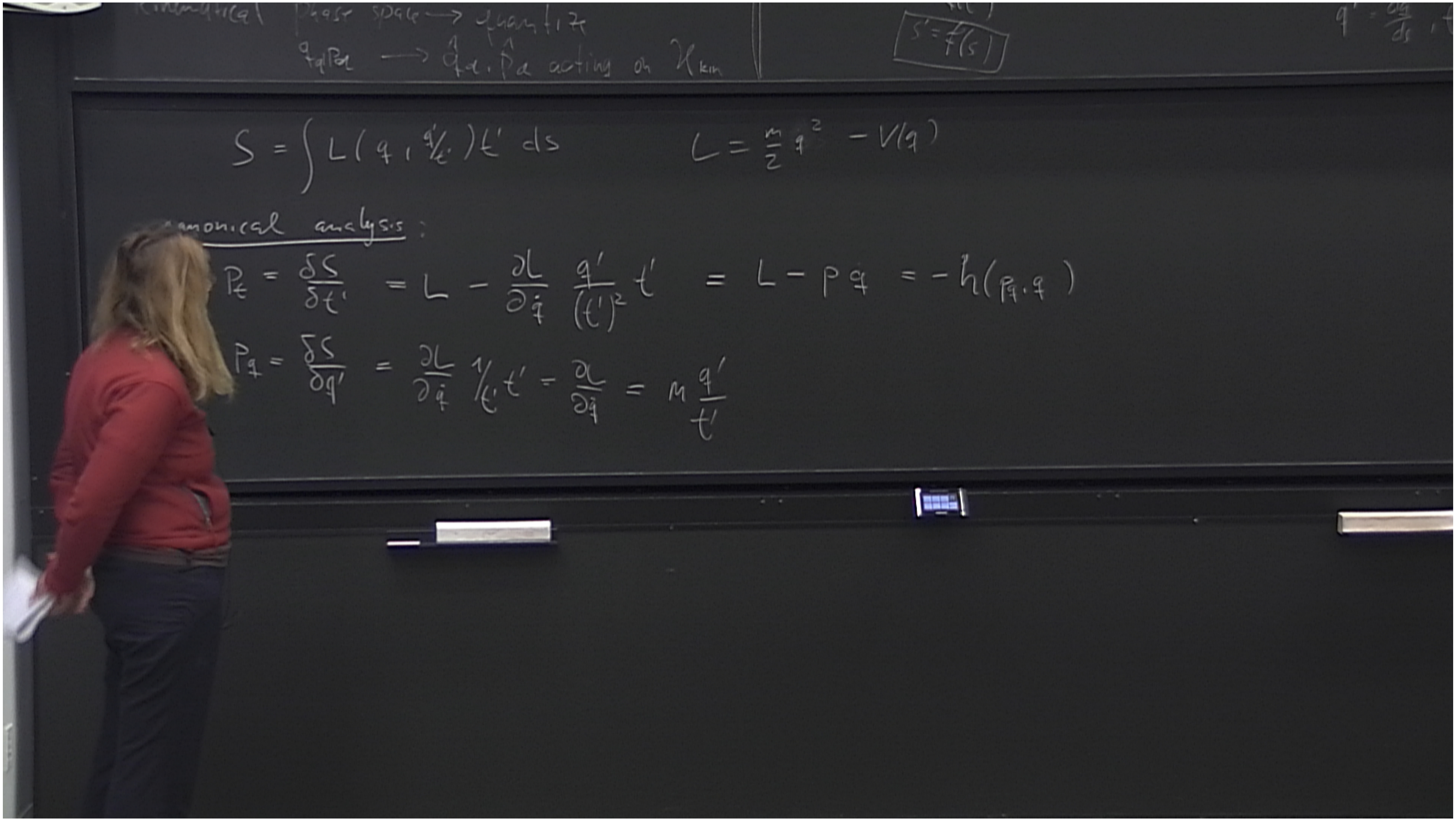

of Gauge Systems
Not yet: the parametrized particle
solutions
$S_A = \int L_A(q, \dot{q})\, dt \rightsquigarrow q(t), p(t)$
B. → new configuration space: (t, q), new auxil evol param. s
A: $q(t)$
B: $q(s), t(s)$
$\hookrightarrow q(t)$
$s' = f(s)$
$S_B = \int L_A(q, \frac{q'}{t'})\, t'\, ds$
$q' = \frac{dq}{ds}, t' = \frac{dt}{ds}$
$C_I = 0$
orbits generated by
space → quantize

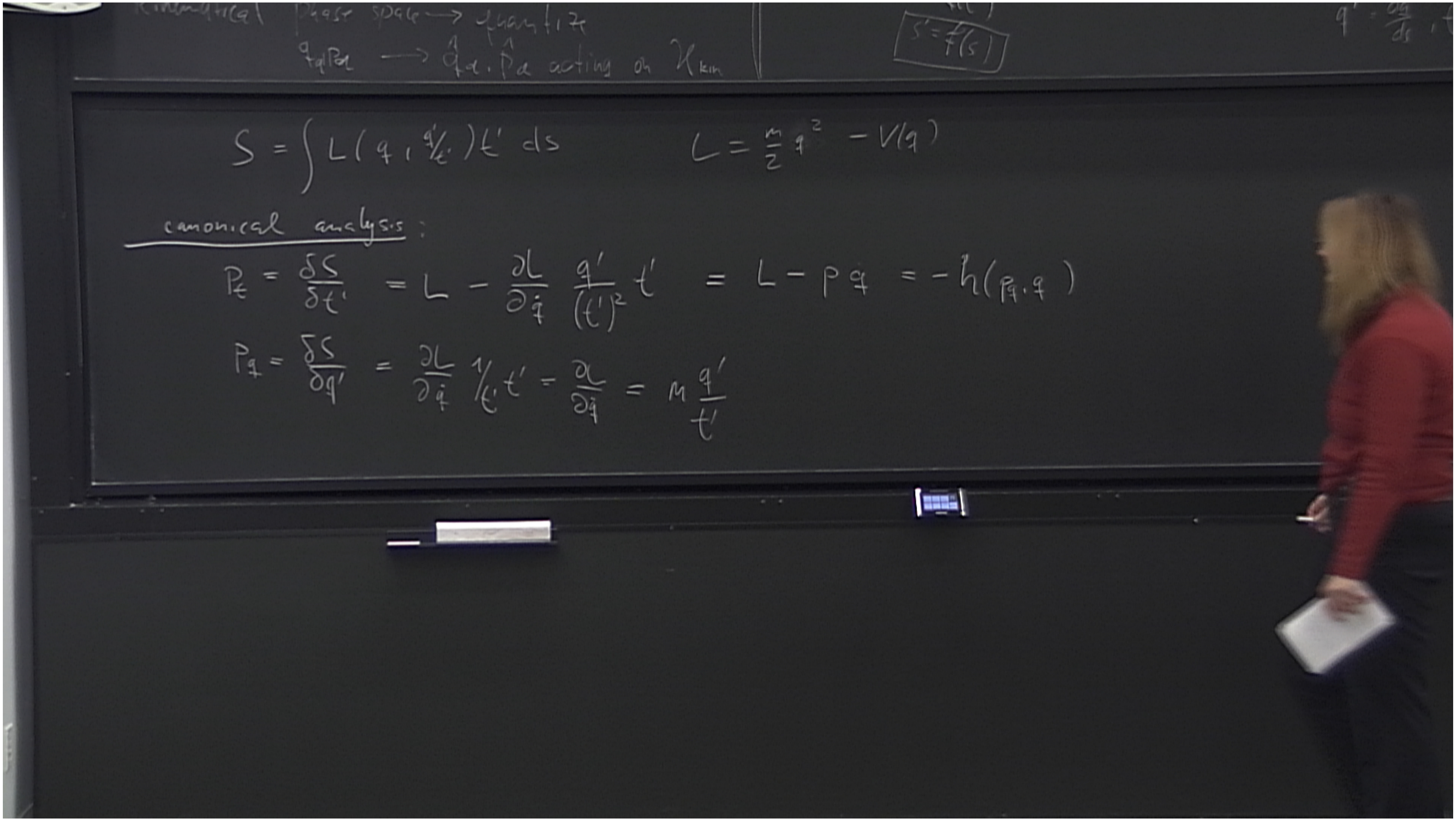

kinematical phase space → quantize
q_α p_α → q̂_α, p̂_α acting on H_kin
s' = f(s)
S = ∫ L(q, q'/t') t' ds

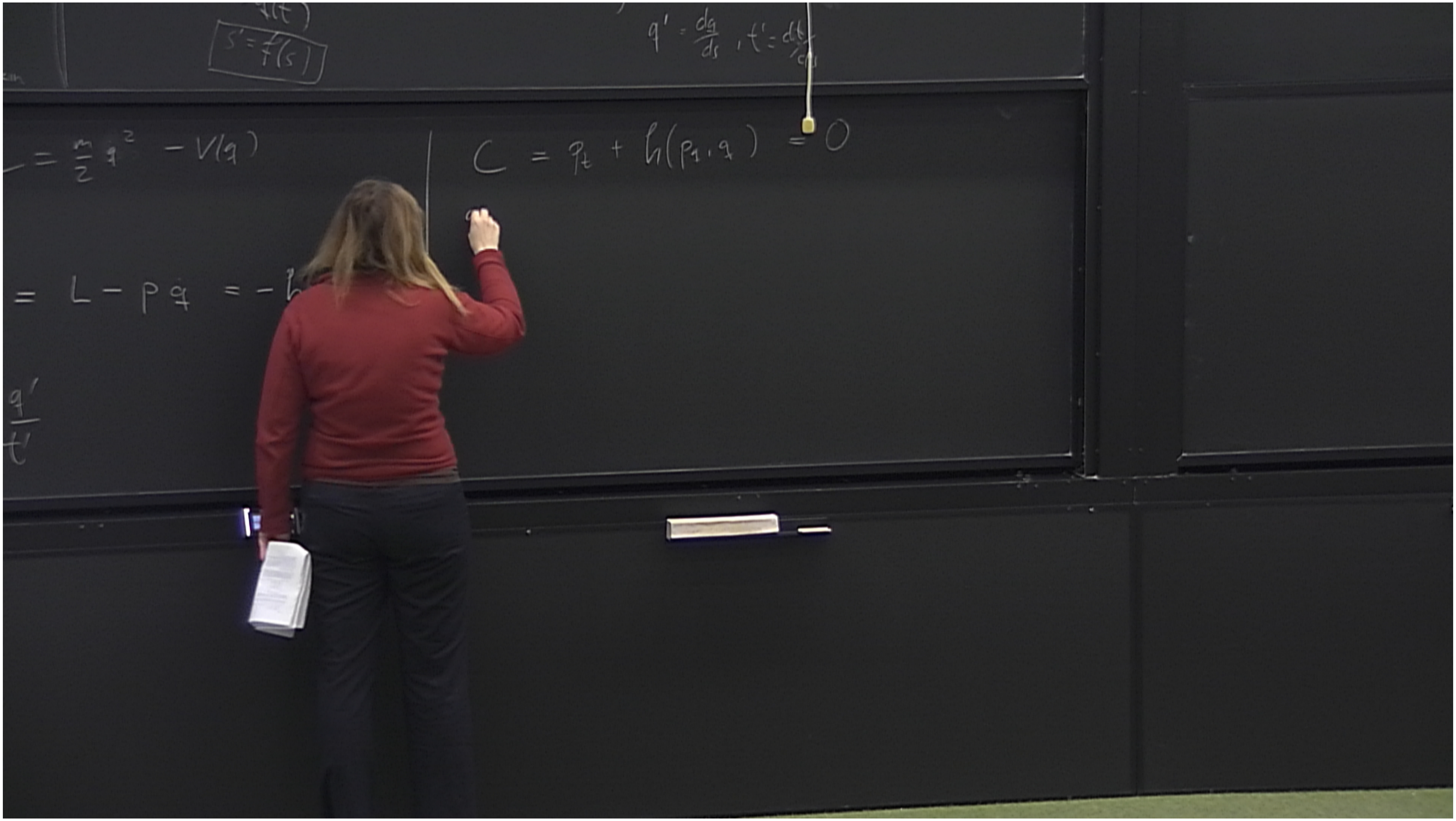

phase space → quantize
s' = f(s)
S = ∫ L (q , q'/t') t' ds
canonical



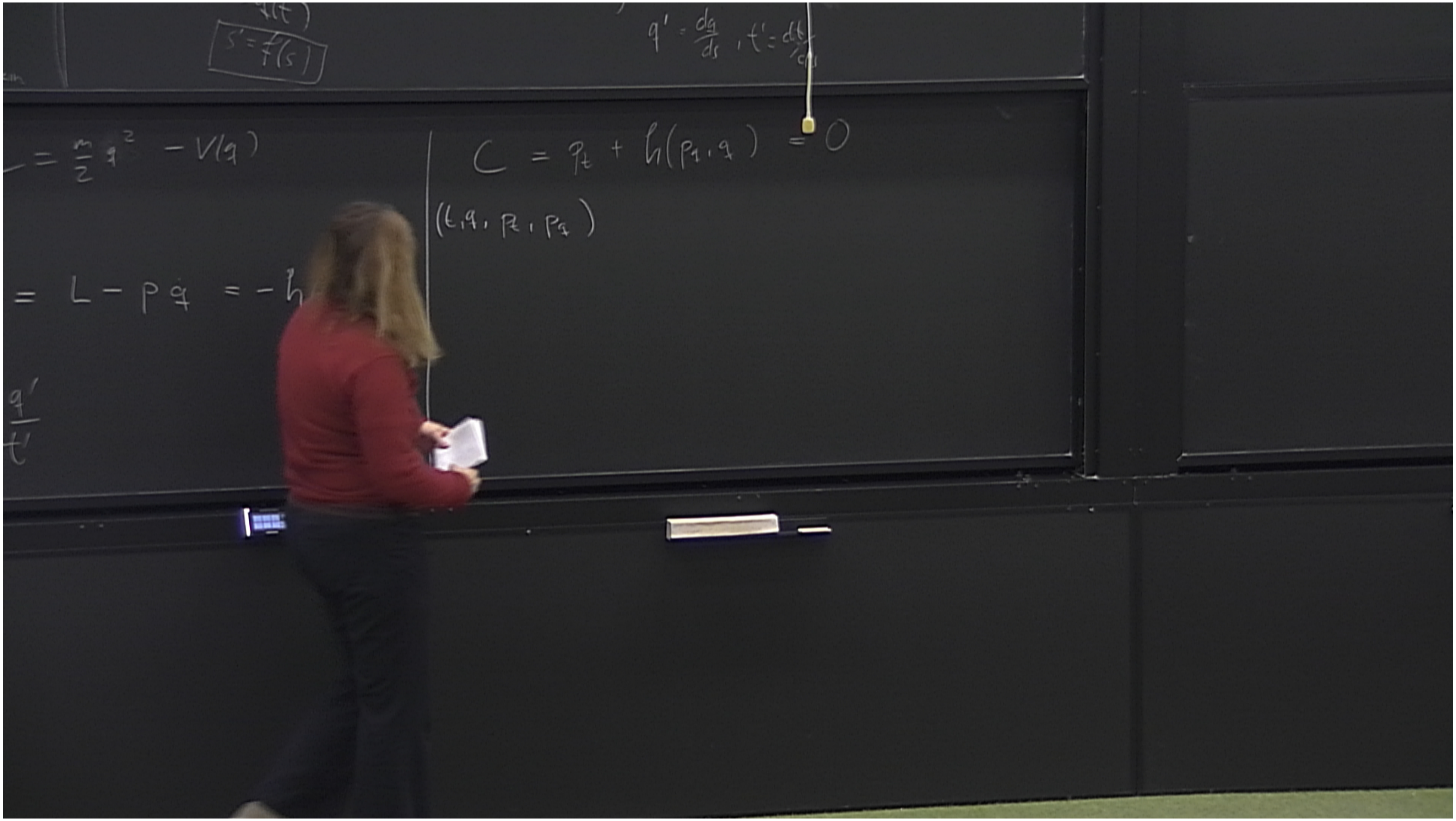
kinematical phase space → quantize
$S = \int L(q, q'/t') t' \, ds$
canonical analysis:
$p_t = \frac{\delta S}{\delta t'} = L$
$= L - p\dot{q} = -h(p_q, q)$

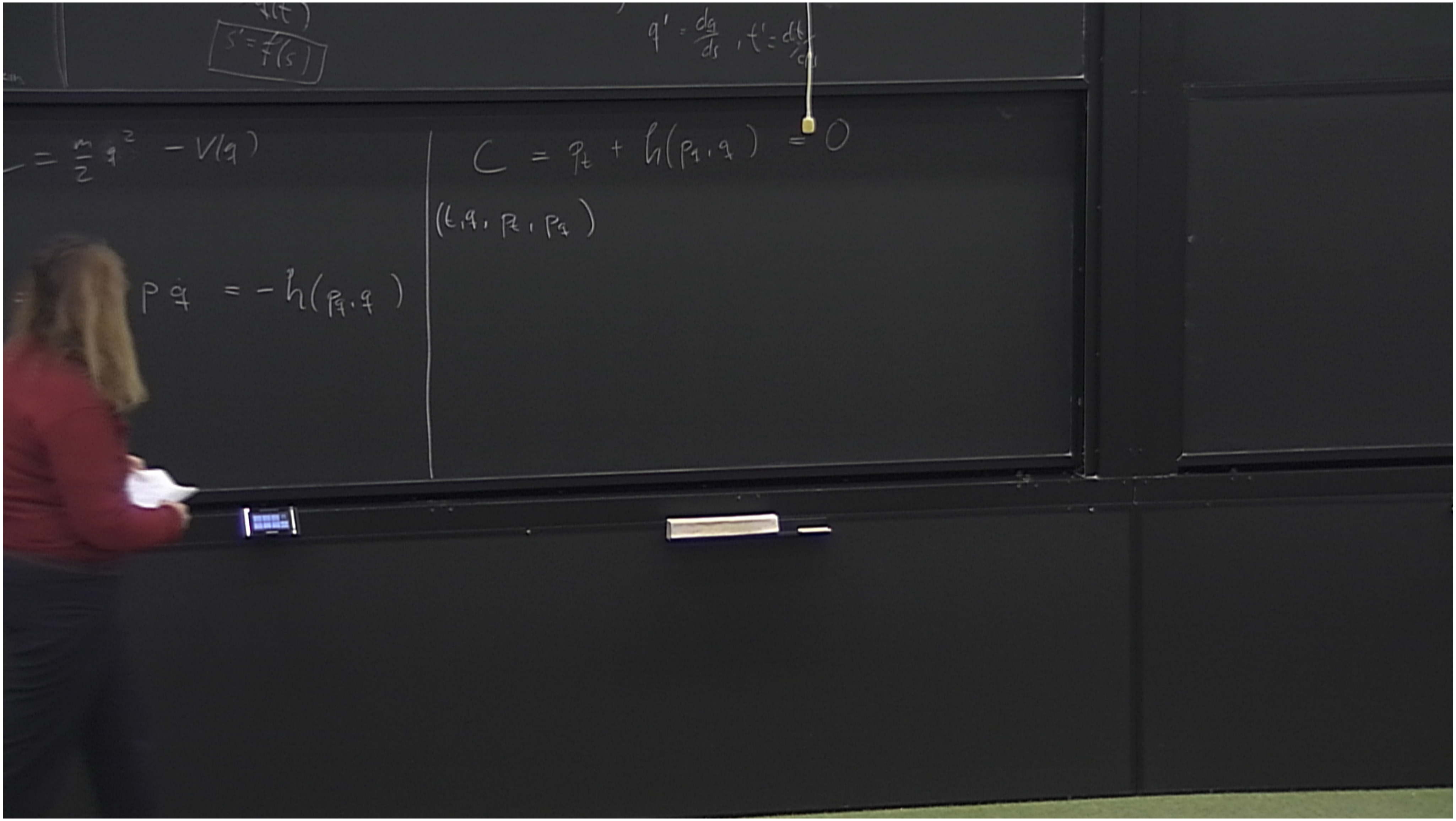

phase space → quantize
S = ∫ L(q, q'/t') t' ds
L = m/2 q² − V(q)
canonical analysis:
P_t = δS/δt' = L − ∂L/∂q̇ q'/(t')² t' = L − p q̇ = −h(p_q, q)
P_q = δS/δq' = ∂L/∂q̇ 1/t' t' = ∂L/∂q̇ = m q'/t'

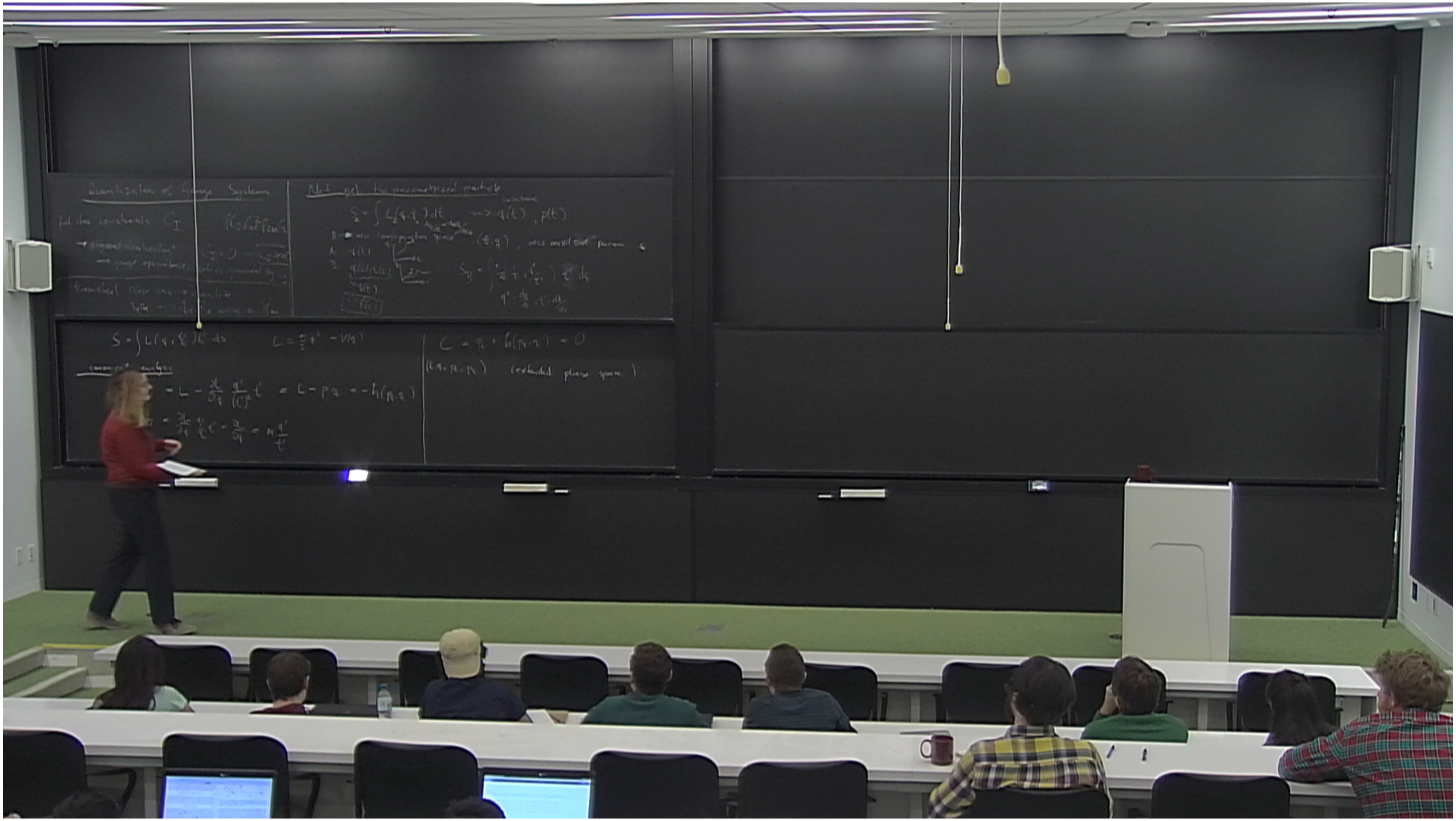

phase space → quantize
$q_\alpha, p_\alpha \to \hat{q}_\alpha, \hat{p}_\alpha$ acting on $\mathcal{H}_{kin}$
$s' = f(s)$
$S = \int L(q, q'/t')\, t'\, ds$
$L = \frac{m}{2}\dot{q}^2 - V(q)$
canonical analysis:
$p_t = \frac{\delta S}{\delta t'} = L - \frac{\partial L}{\partial \dot{q}} \frac{q'}{(t')^2} t' = L - p\dot{q} = -h(p_q, q)$
$p_q = \frac{\delta S}{\delta q'} = \frac{\partial L}{\partial \dot{q}} \frac{1}{t'} t' = \frac{\partial L}{\partial \dot{q}} = m \frac{q'}{t'}$

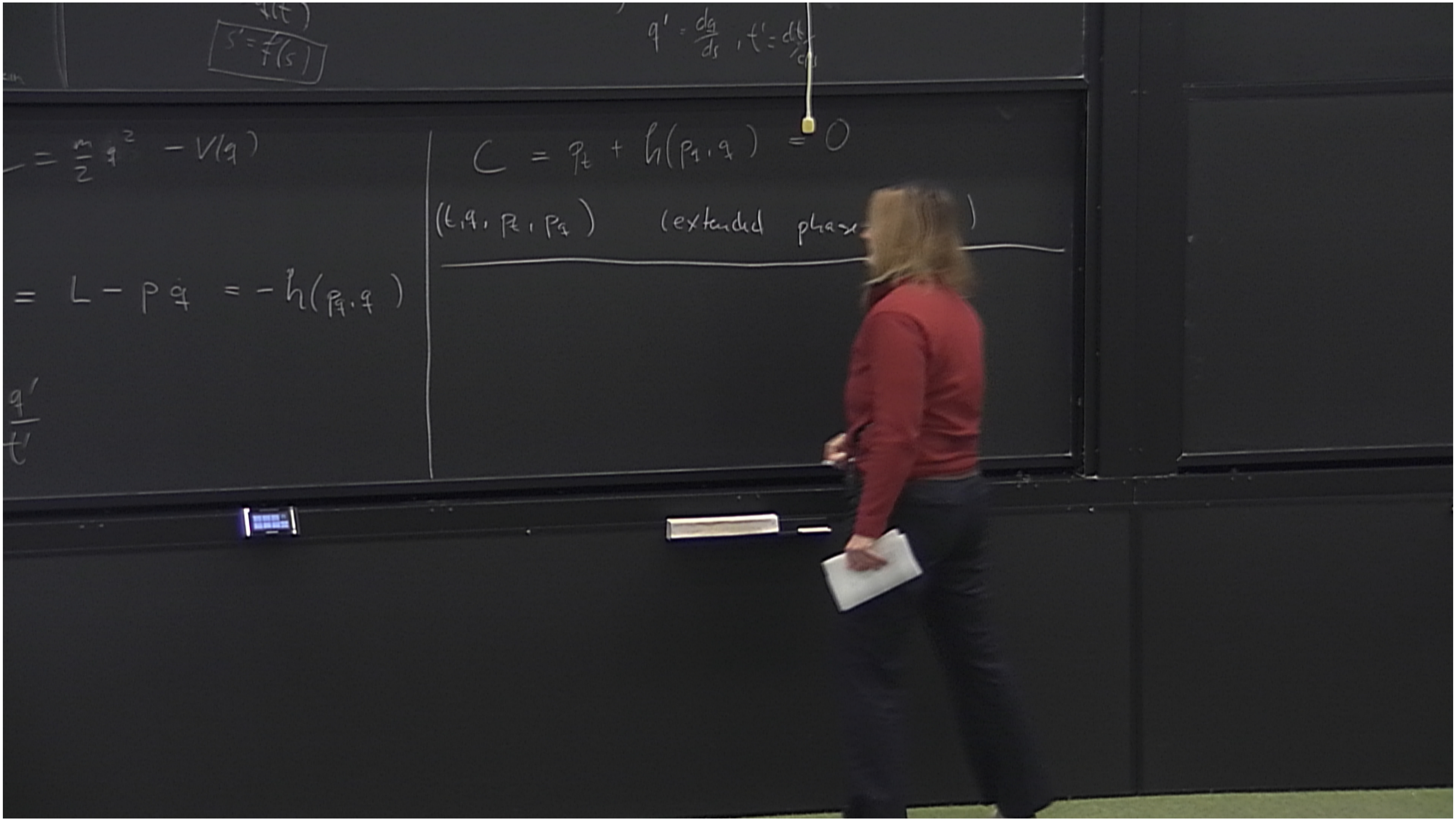


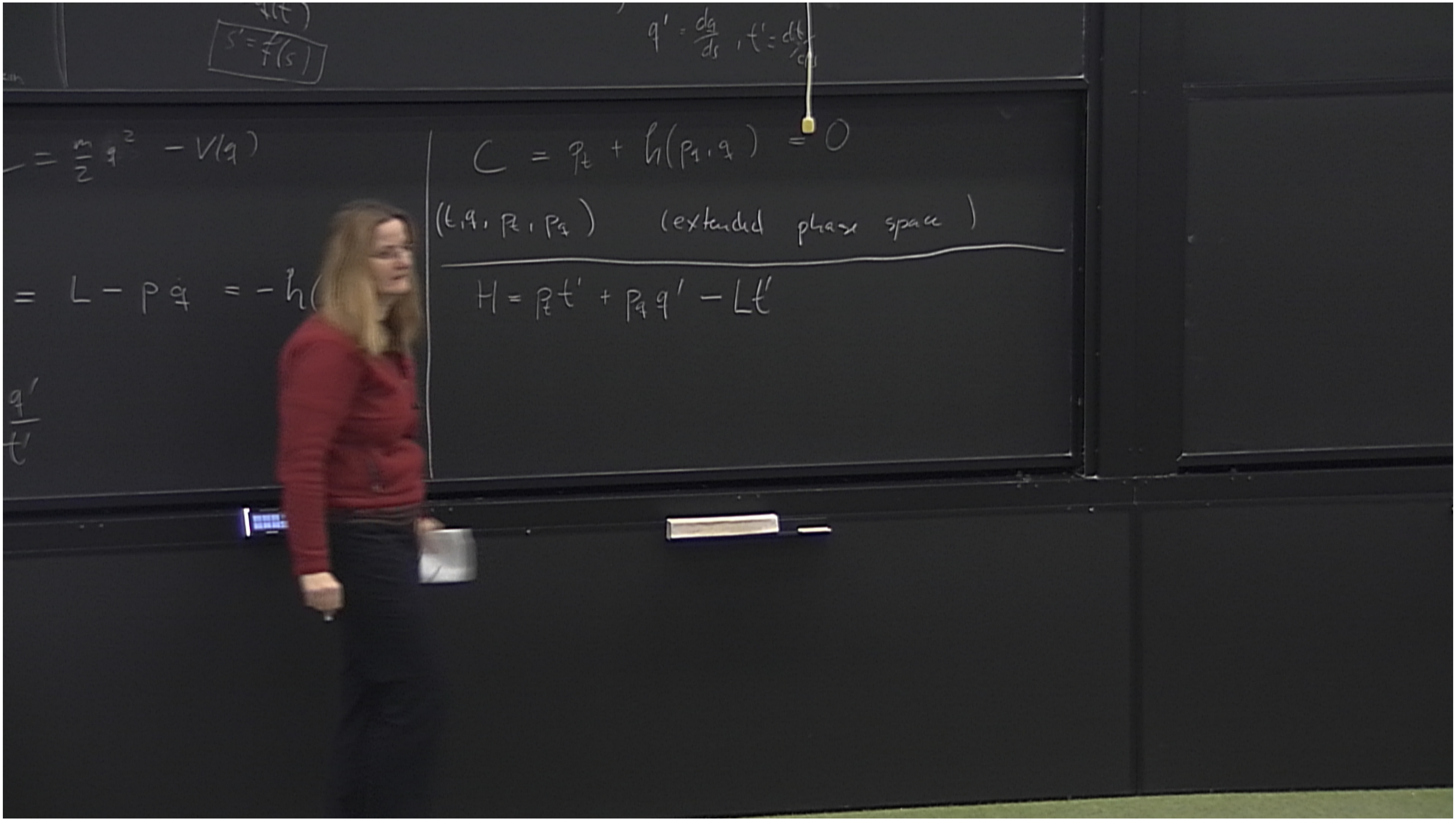
s' = f(s)
q' = dq/ds , t' = dt/ds
= m/2 q² − V(q)
C = p_t + h(p_q, q) = 0
= L − p q̇ = −h
q'/t'

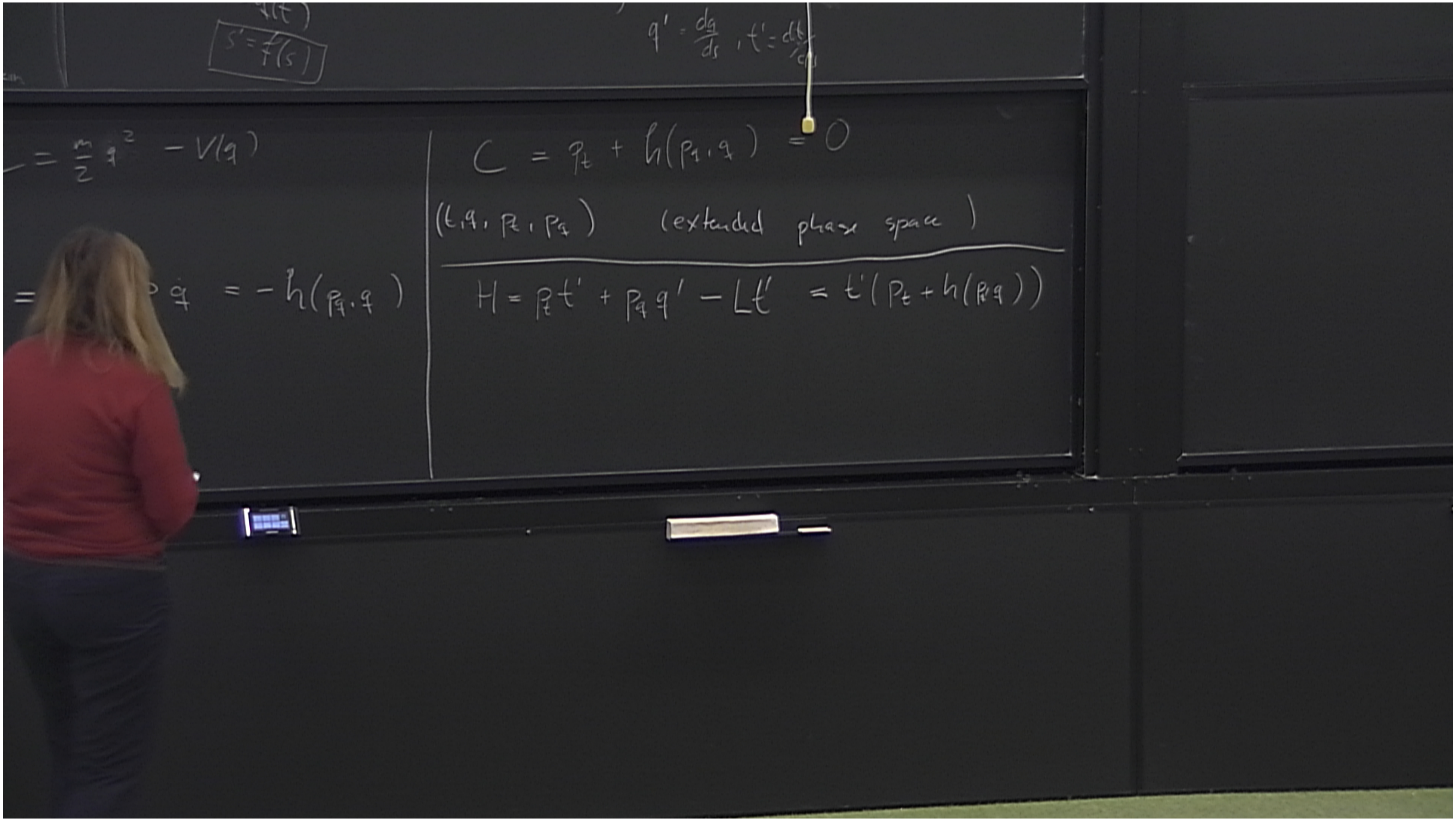

C = p_t + h(p_q, q) = 0
(t, q, p_t, p_q)
= L − p q̇ = − h

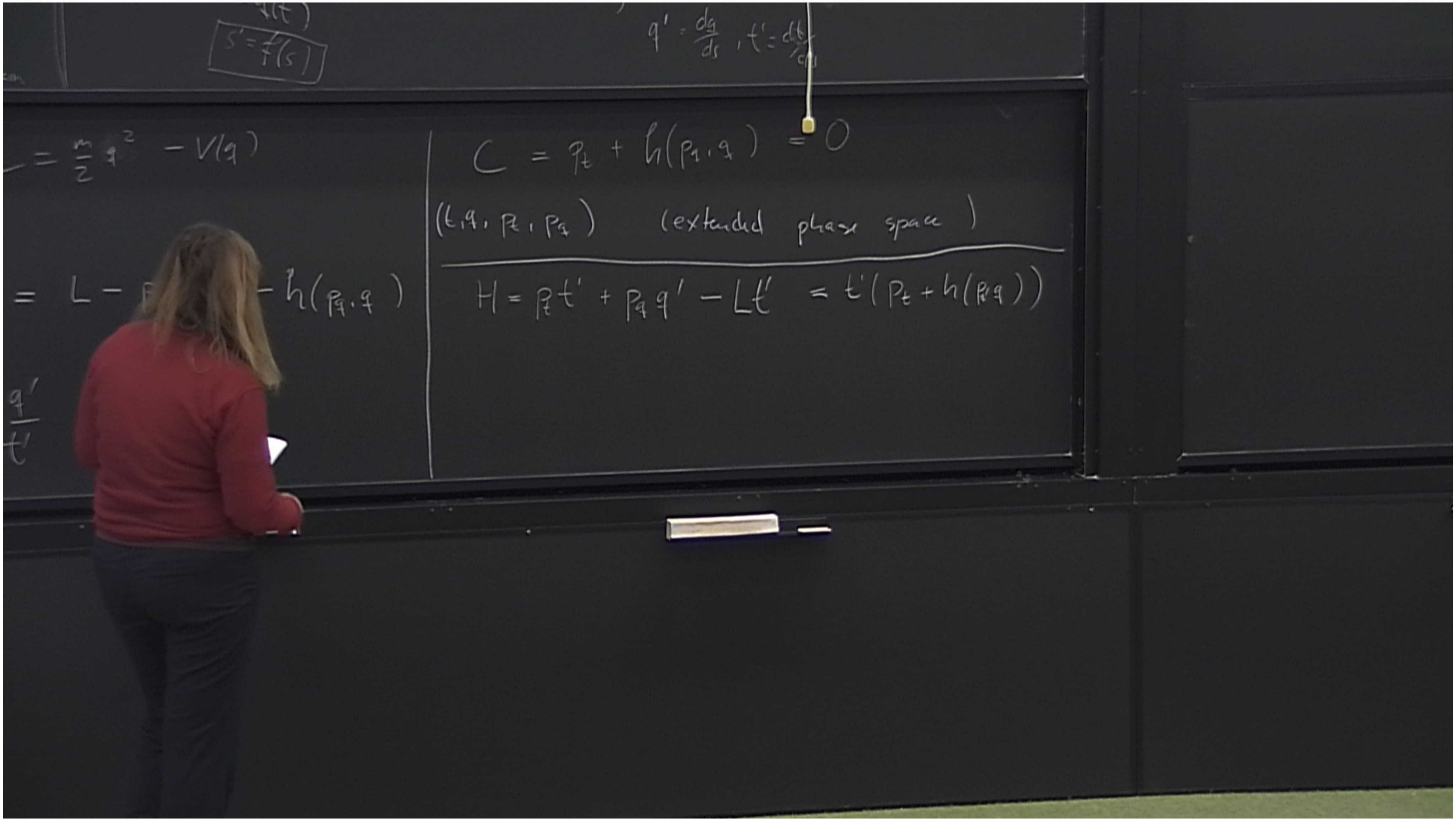

s' = f(s)
q' = dq/ds , t' = dt/ds
= m/2 q² − V(q)
C = p_t + h(p_q, q) = 0
(t, q, p_t, p_q)
p q̇ = −h(p_q, q)

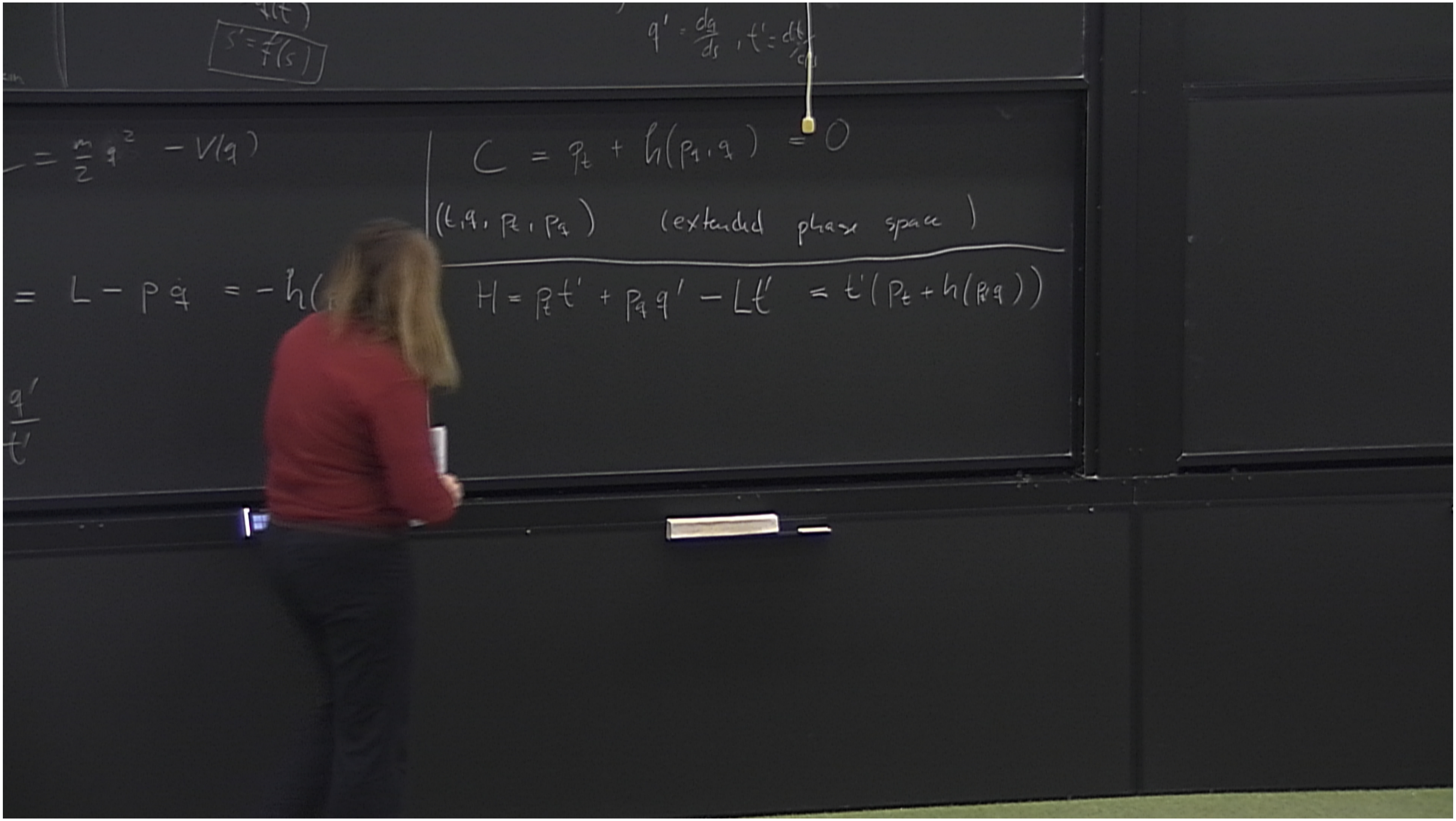

Quantization of Gauge Systems
(extended phase space)


s' = f(s)
q' = dq/ds , t' = dt/ds
= m/2 q̇² − V(q)
C = p_t + h(p_q, q) = 0
(t, q, p_t, p_q)
(extended
= L − p q̇ = −h(p_q, q)
q'/t'

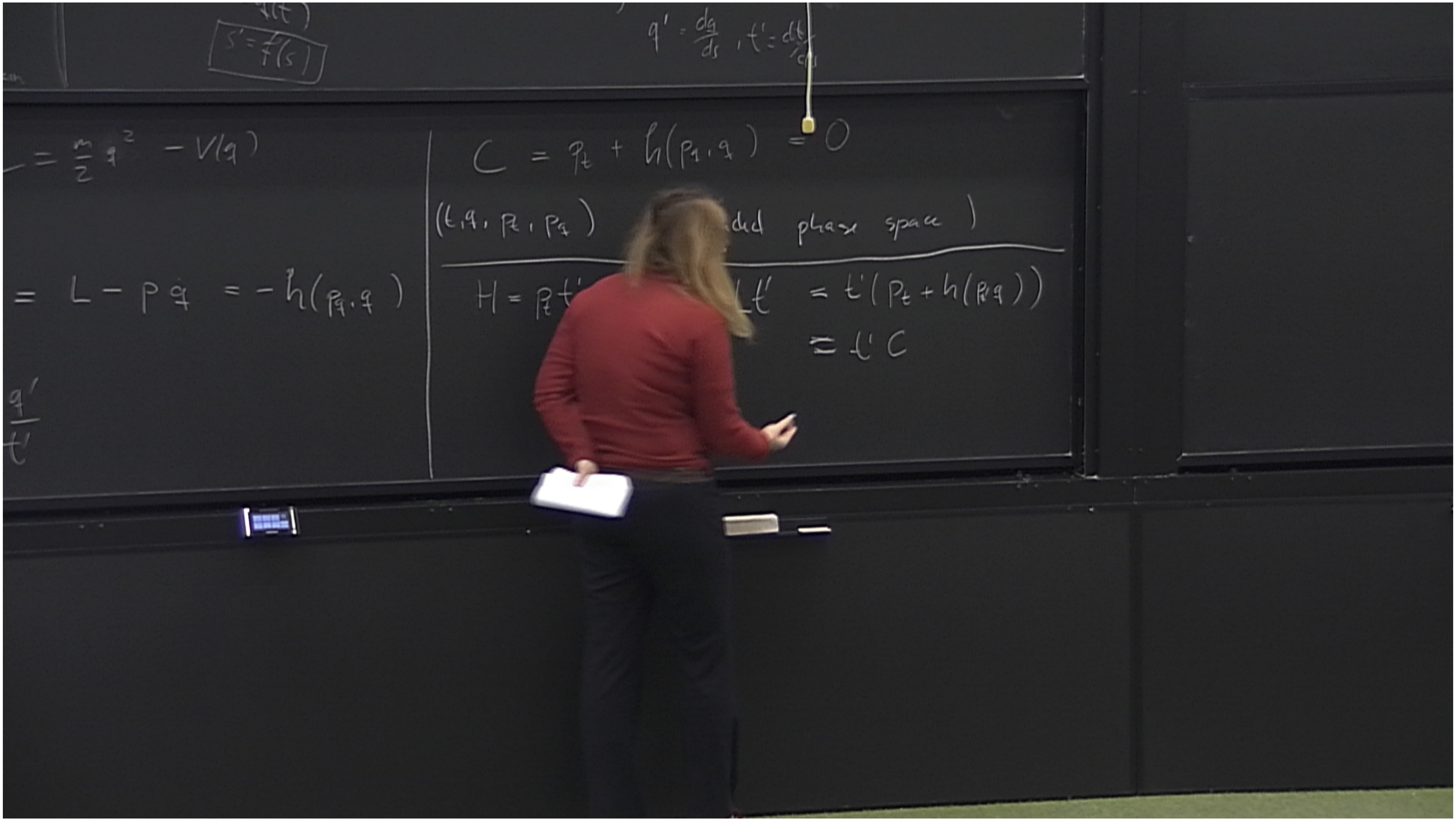

C = p_t + h(p_q, q) = 0
(t, q, p_t, p_q) (extended phase space)
H = p_t t' + p_q q' − L t'
= L − p q̇ = − h
q' = dq/ds , t' = dt/ds
s' = f(s)


s' = f(s)
q' = dq/ds , t' = dt/ds
= m/2 q̇² − V(q)
C = p_t + h(p_q, q) = 0
(t, q, p_t, p_q) (extended phase space)
= −h(p_q, q)
H = p_t t' + p_q q' − L t' = t'(p_t + h(p_q, q))

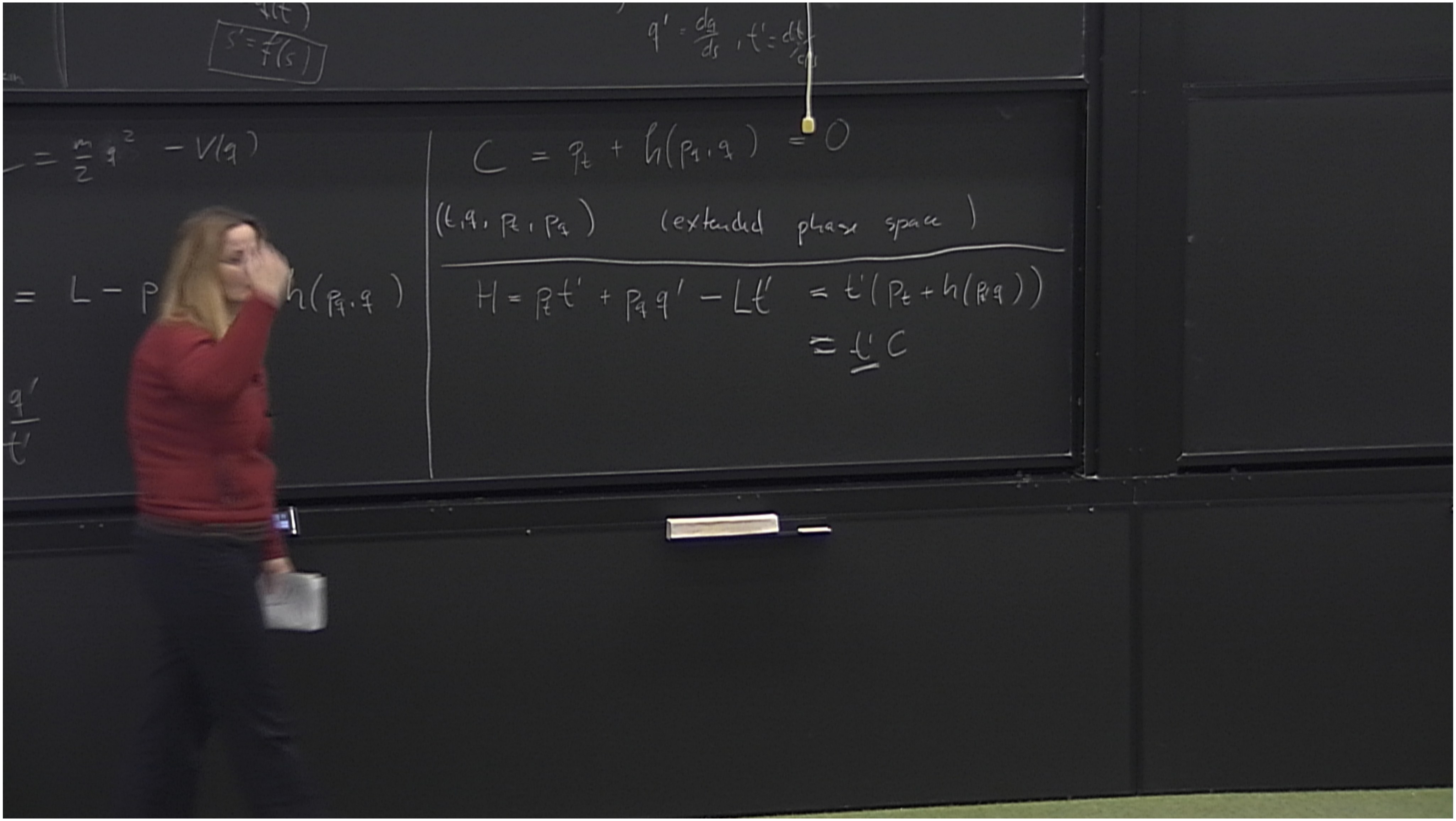

s' = f(s)
q' = dq/ds , t' = dt/ds
C = p_t + h(p_q, q) = 0
(t, q, p_t, p_q) (extended phase space)
H = p_t t' + p_q q' − L t' = t'(p_t + h(p_q, q))
= m/2 q² − V(q)
= L −
h(p_q, q)
q'/t'

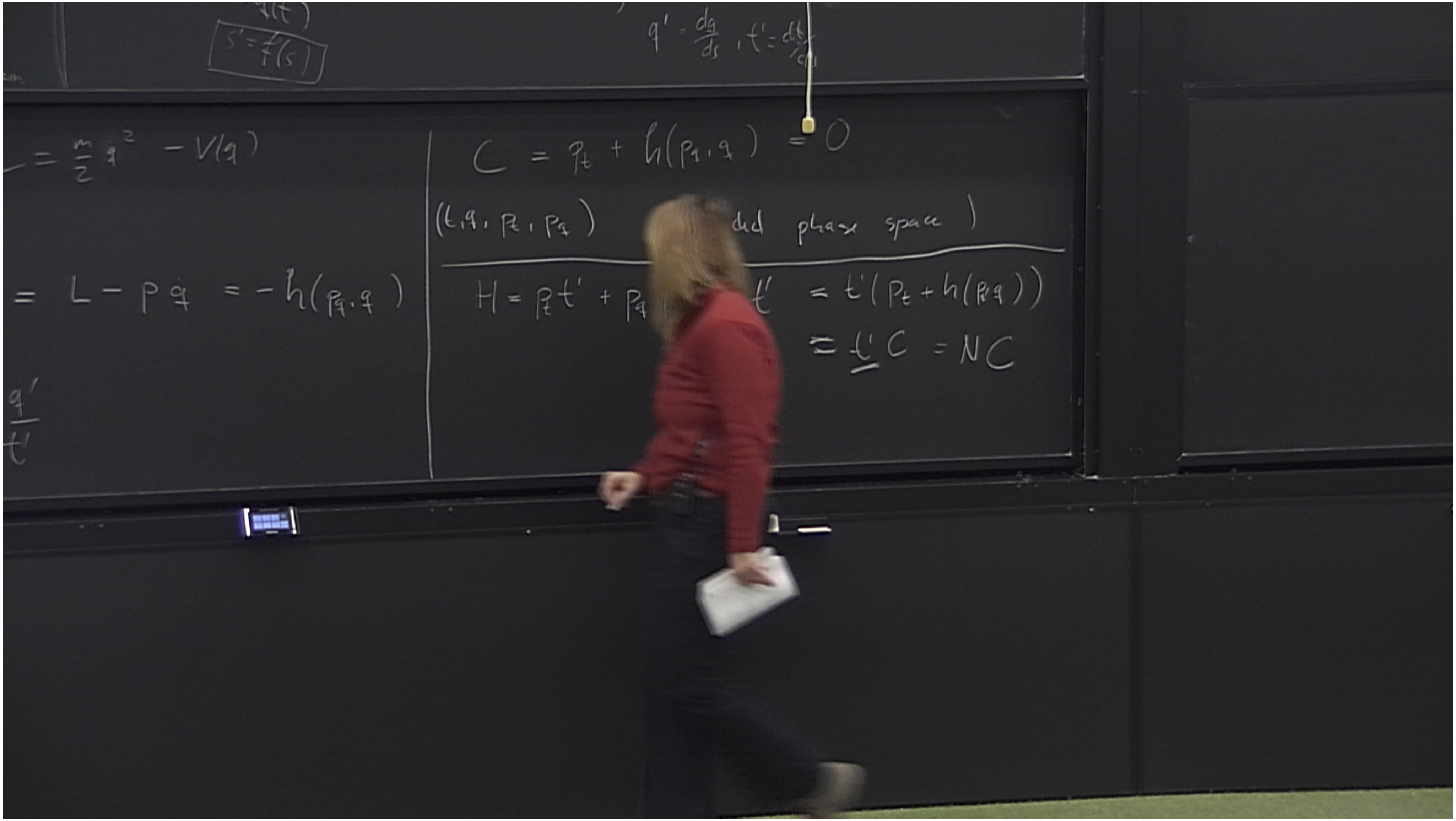


s' = f(s)
q' = dq/ds , t' = dt/ds
= m/2 q^2 − V(q)
C = p_t + h(p_q , q) = 0
(t, q, p_t , p_q) (extended phase space)
= L − p q̇ = −h(
H = p_t t' + p_q q' − L t' = t'(p_t + h(p_q q))
q'/t'

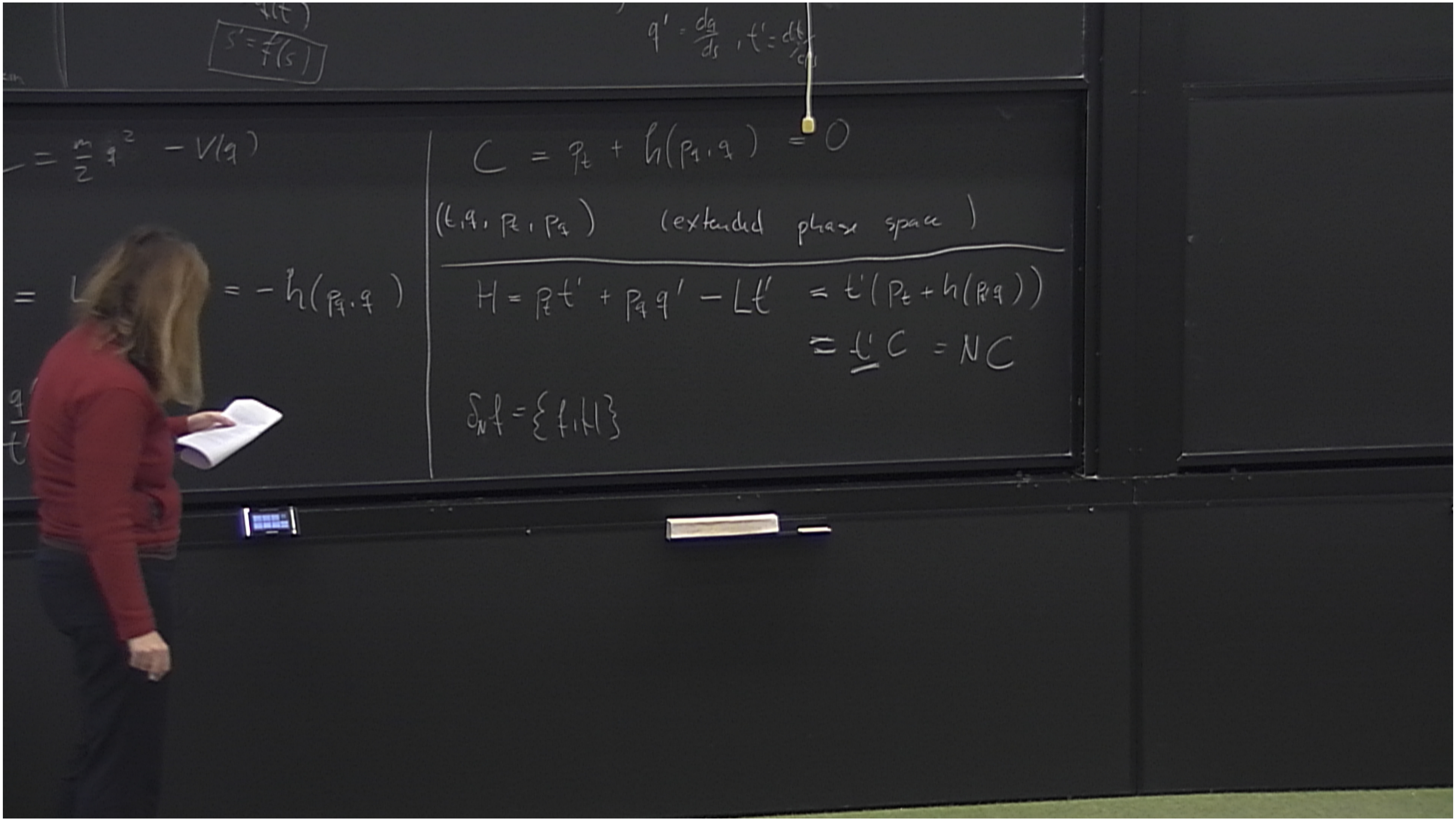


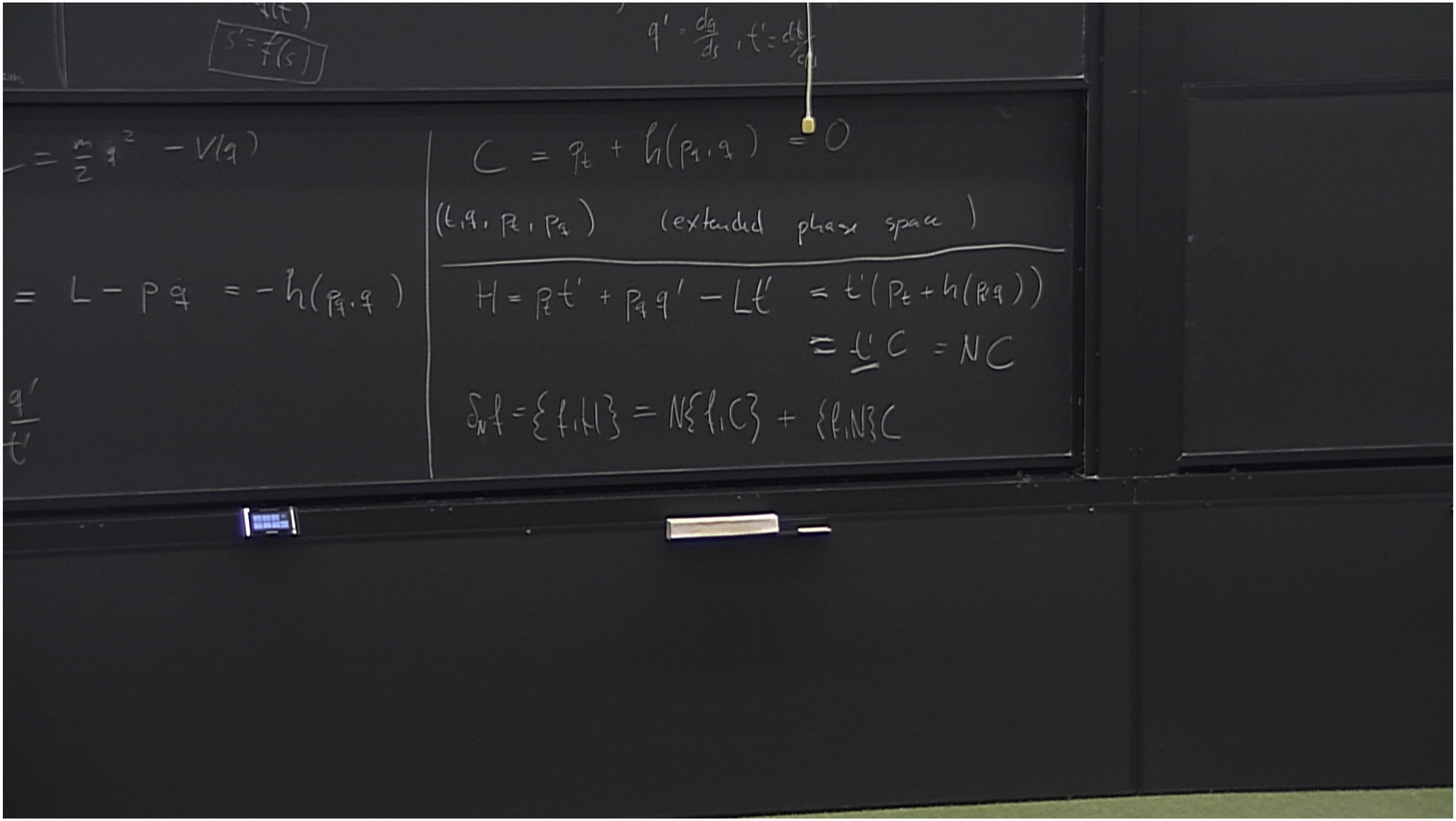
s' = f(s)
q' = dq/ds , t' = dt/ds
= m/2 q² − V(q)
C = p_t + h(p_q, q) = 0
(t, q, p_t, p_q)
phase space)
= L − p q̇ = −h(p_q, q)
= t'(p_t + h(p_q, q))
= t' C
q'/t'


s' = f(s)
q' = dq/ds , t' = dt/ds
= m/2 q^2 − V(q)
C = p_t + h(p_q, q) = 0
(t, q, p_t, p_q) (extended phase space)
= L − p
h(p_q, q)
H = p_t t' + p_q q' − L t' = t'(p_t + h(p_q, q))
= t' C
q'
t'

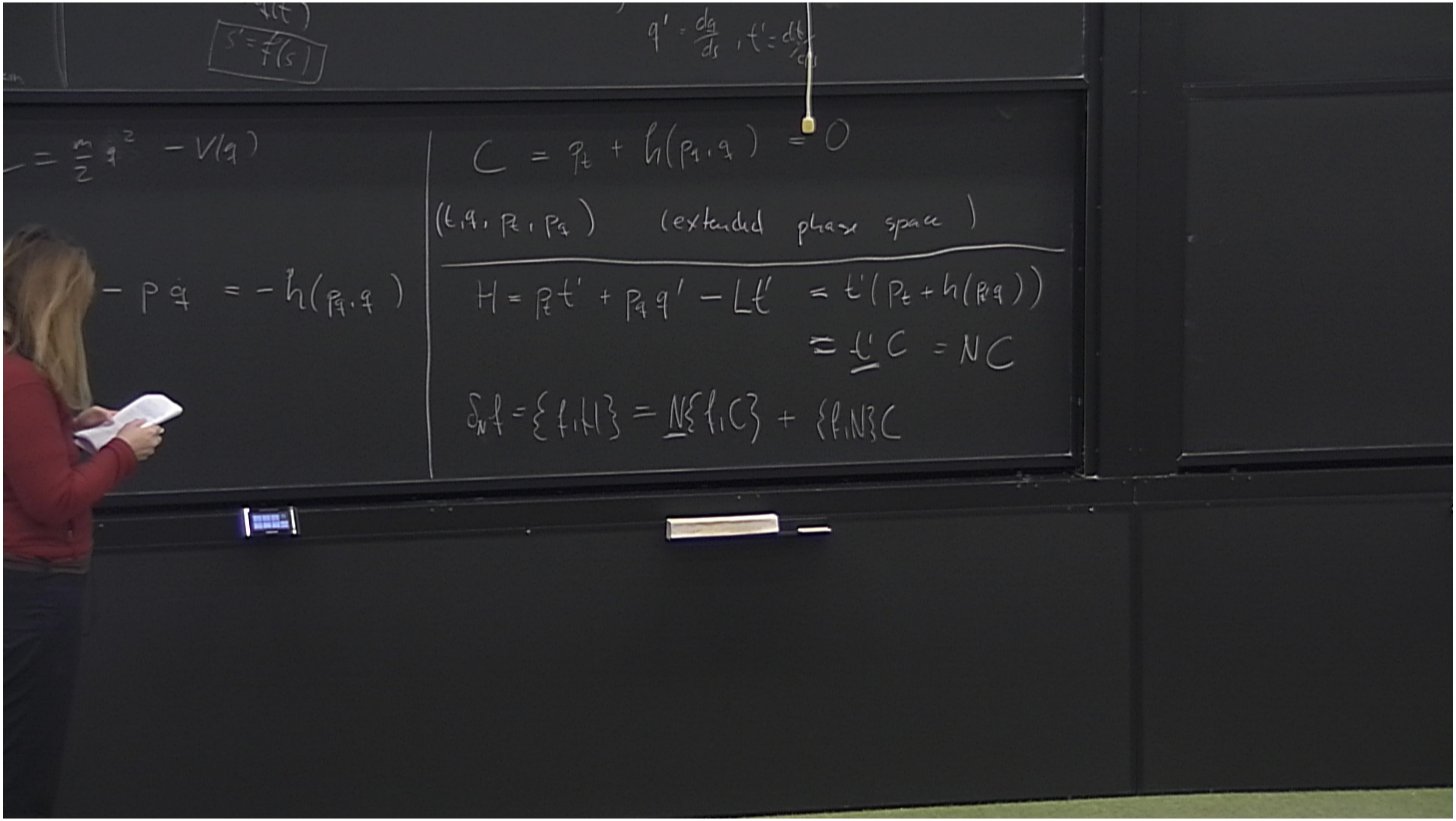


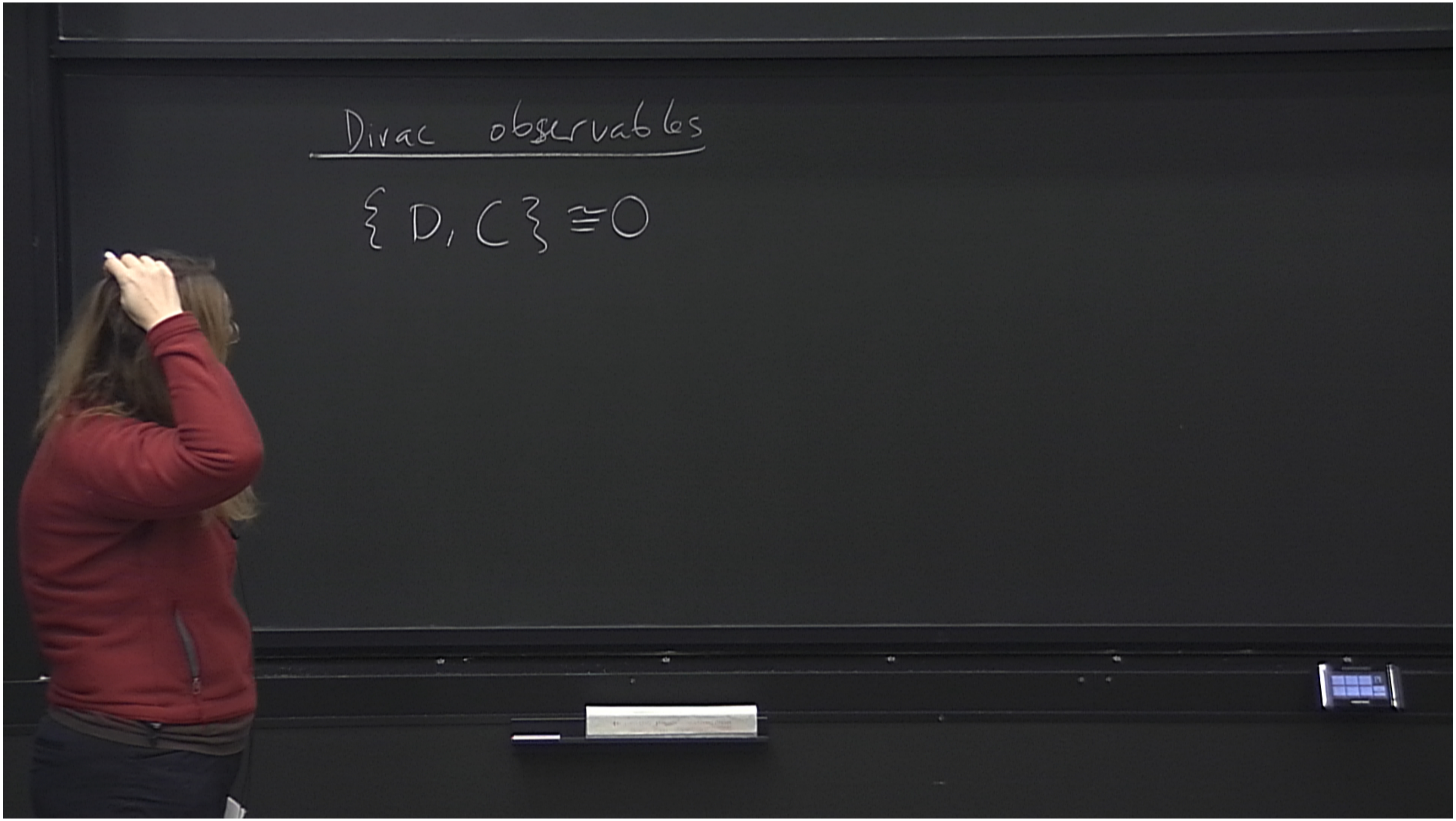
s' = f(s)
q' = dq/ds , t' = dt/ds
C = p_t + h(p_q, q) = 0
(t, q, p_t, p_q)
phase space)
= L − p q̇ = −h(p_q, q)
H = p_t t' + p_q
= t'(p_t + h(p_q, q))
= t' C = N C

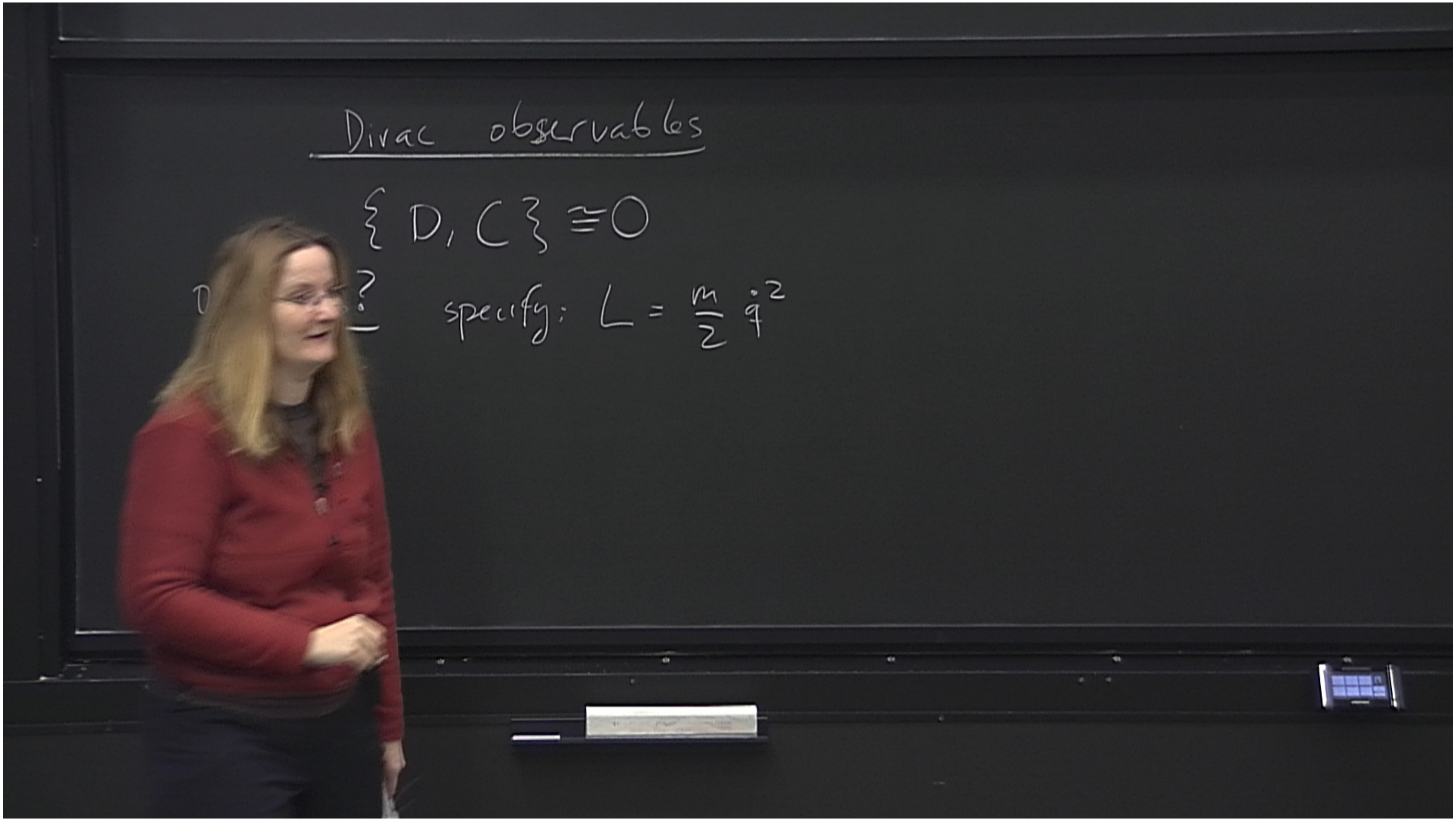

s' = f(s)
q' = dq/ds , t' = dt/ds
= m/2 q² − V(q)
= −h(pq, q)
C = pt + h(pq, q) = 0
(t, q, pt, pq) (extended phase space)
H = pt t' + pq q' − Lt' = t'(pt + h(pq, q))
= t' C = N C
δN f = {f, H}

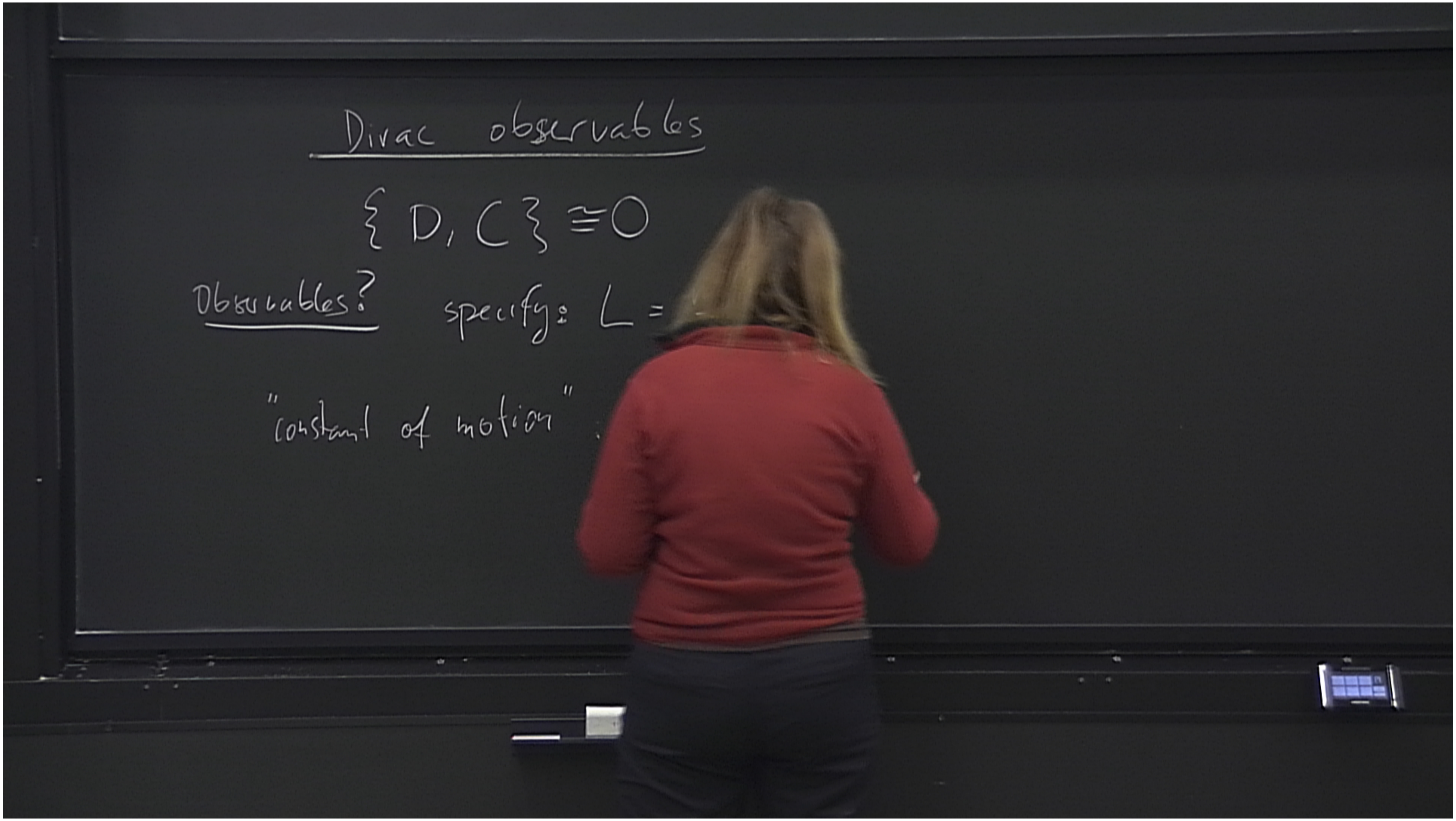


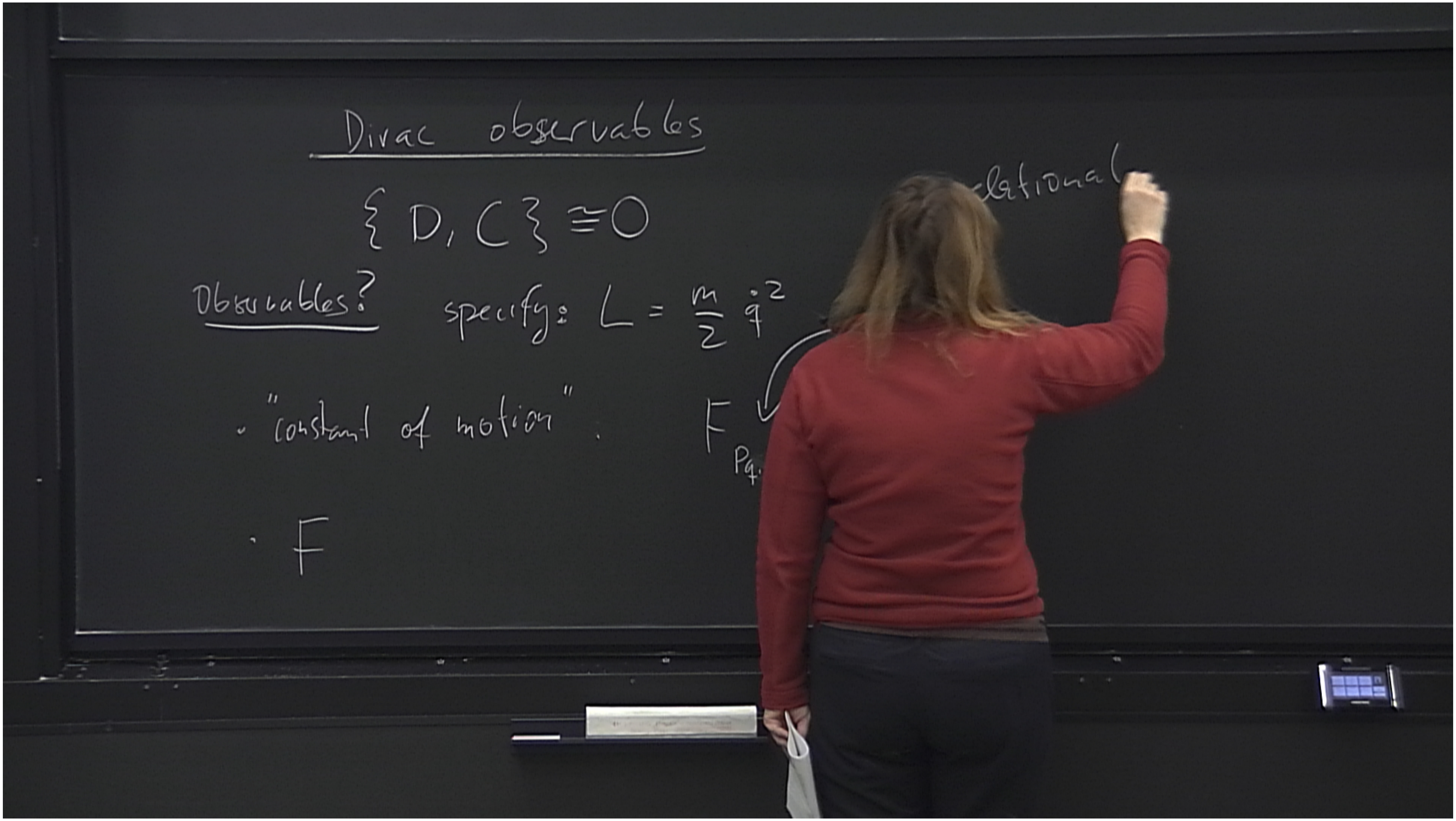
s' = f(s)
q' = dq/ds , t' = dt/ds
L = m/2 q̇² − V(q)
= L − p q̇ = −h(p_q, q)
C = p_t + h(p_q, q) = 0
(t, q, p_t, p_q) (extended phase space)
H = p_t t' + p_q q' − L t' = t'(p_t + h(p_q, q))
= t' C = N C
δ_N f = {f, H} = N{f, C} + {f, N} C


s' = f(s)
q' = dq/ds , t' = dt/ds
= m/2 q² − V(q)
− p q̇ = − h(p_q, q)
C = p_t + h(p_q, q) = 0
(t, q, p_t, p_q) (extended phase space)
H = p_t t' + p_q q' − L t' = t'(p_t + h(p_q, q))
= t' C = N C
δ_N f = {f, H} = N{f, C} + {f, N} C


Dirac observables
{D, C} ≅ 0


Dirac observables
{D, C} ≅ 0
specify: L = m/2 q̇²


Dirac observables
{ D, C } ≅ 0
Observables?
specify: L =
"constant of motion"


Dirac observables
{D, C} ≅ 0
Observables?
specify: L = m/2 q̇²
"constant of motion"
F
F

Dirac observables

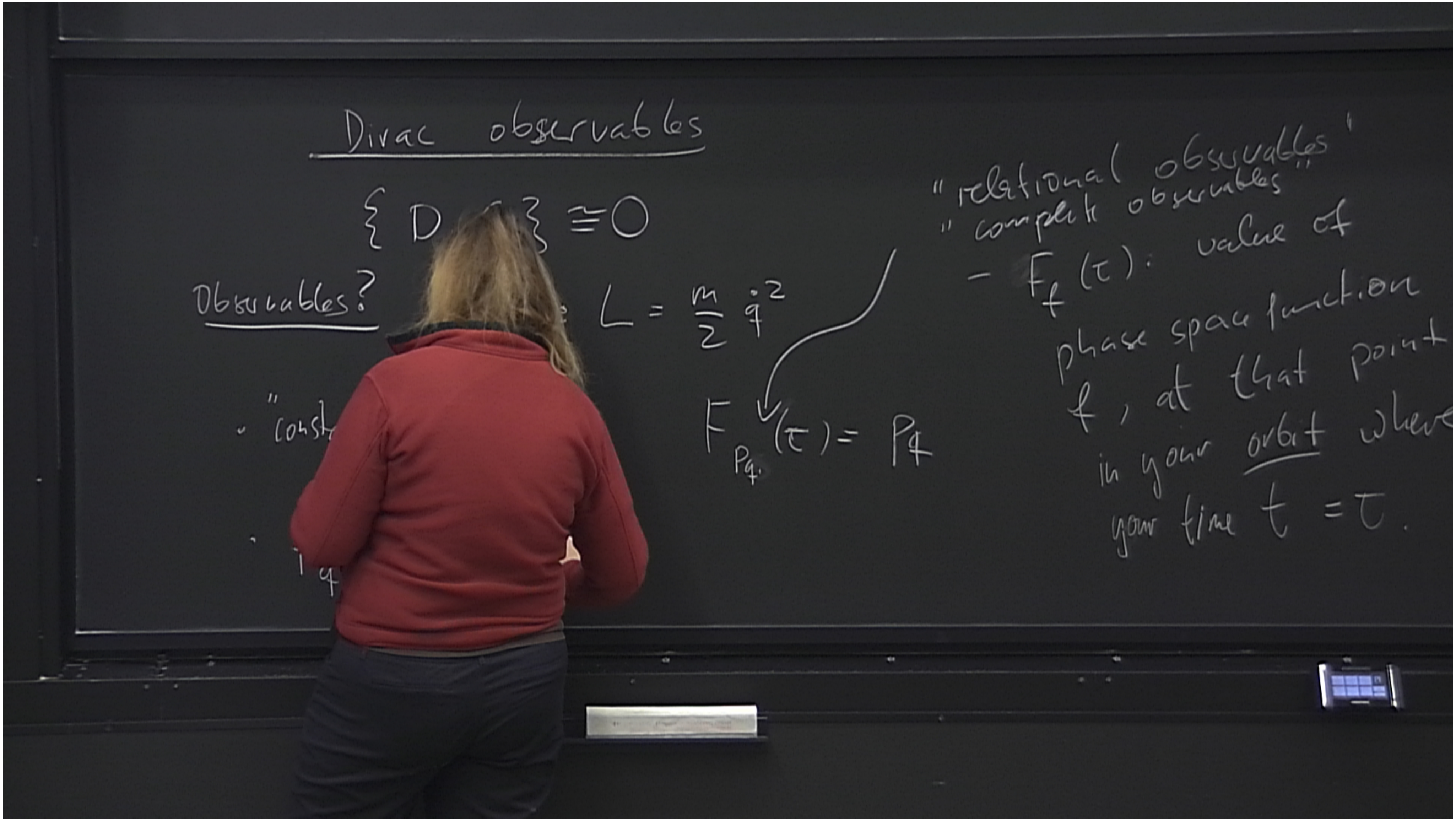
$$\{ D, C \} \cong 0$$

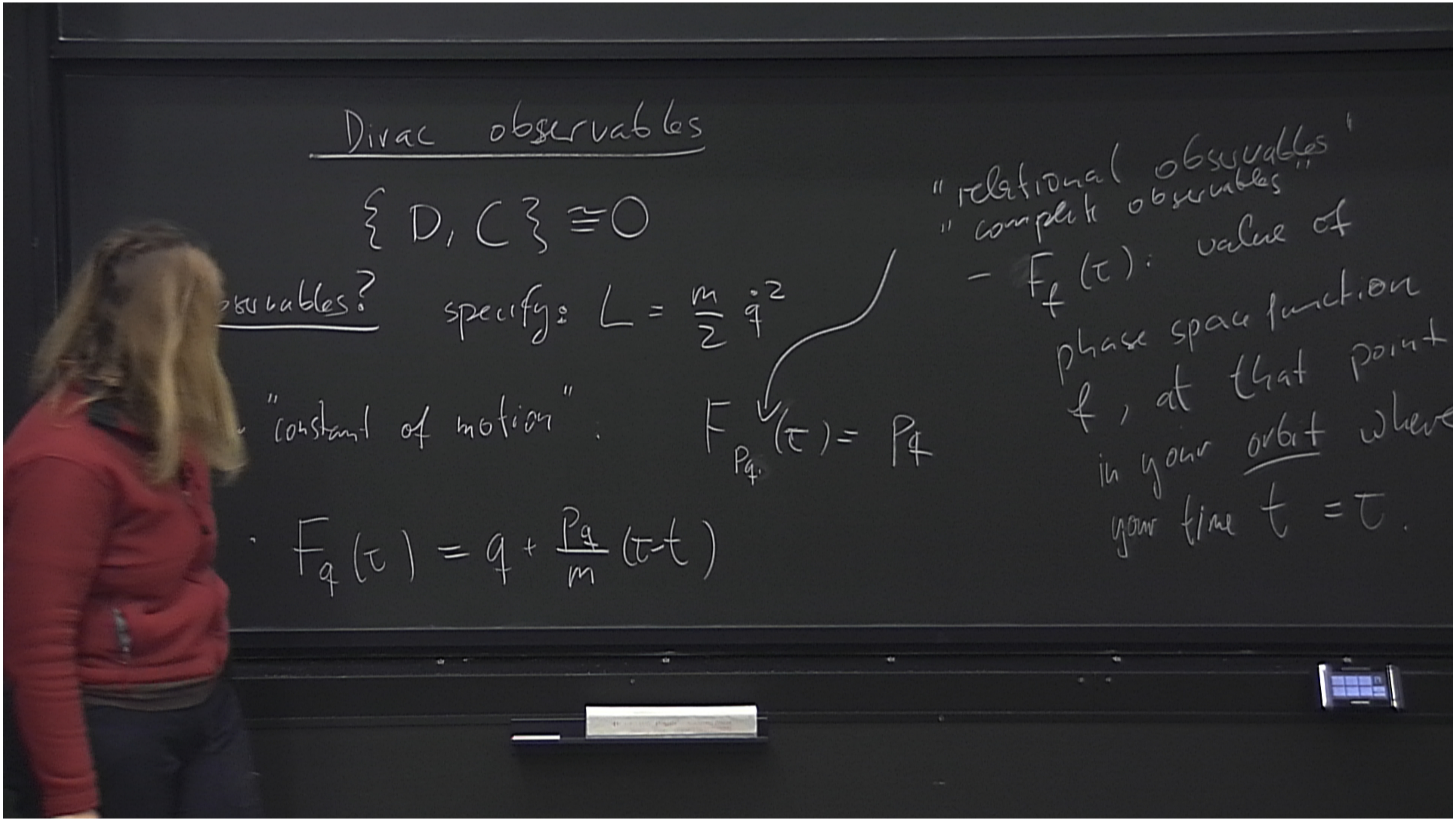
Observables? specify: $L = \frac{m}{2} \dot{q}^2$

- "constant of motion": $F_{p_q}(\tau) = p_q$
- F

"relational observables"
"complete observables"

- $F_f(\tau)$: value of phase space function f, at that point in your orbit


Dirac observables
Observables?
L = m/2 q̇²
F_pq (τ) = pq
"relational observables"
"complete observables"
- F_f (τ): value of
phase space function
f, at that point
in your orbit where
your time t = τ.


Dirac observables
{ D, C } ≅ 0
"relational observables"
"complete observables"
– F_f (τ): value of phase space function f, at that point in your orbit where your time t = τ.
specify: L = m/2 q̇²
"constant of motion":
F_{p_q}(τ) = p_q
F_q(τ) = q + p_q/m (τ – t)

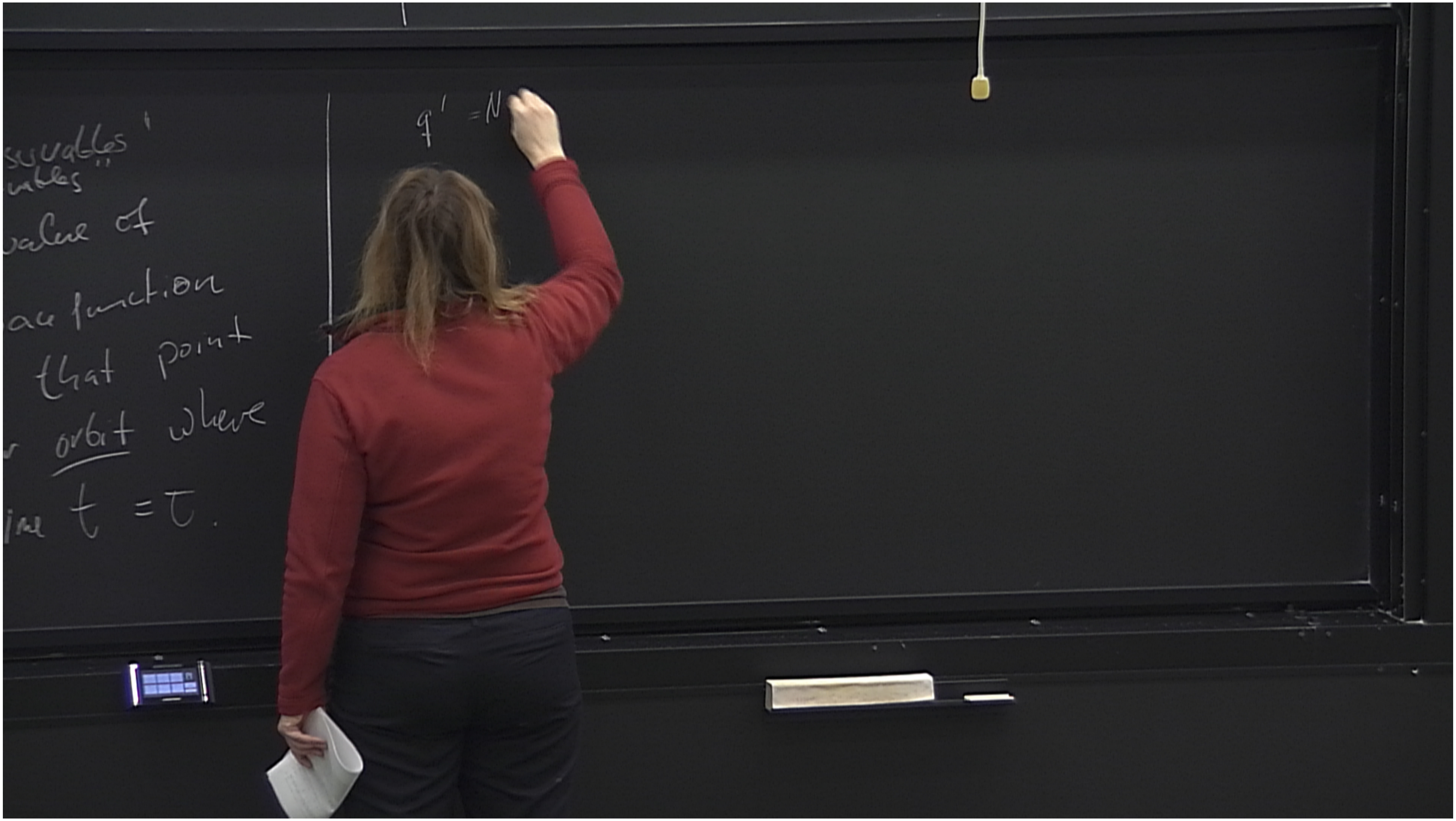
Dirac observables

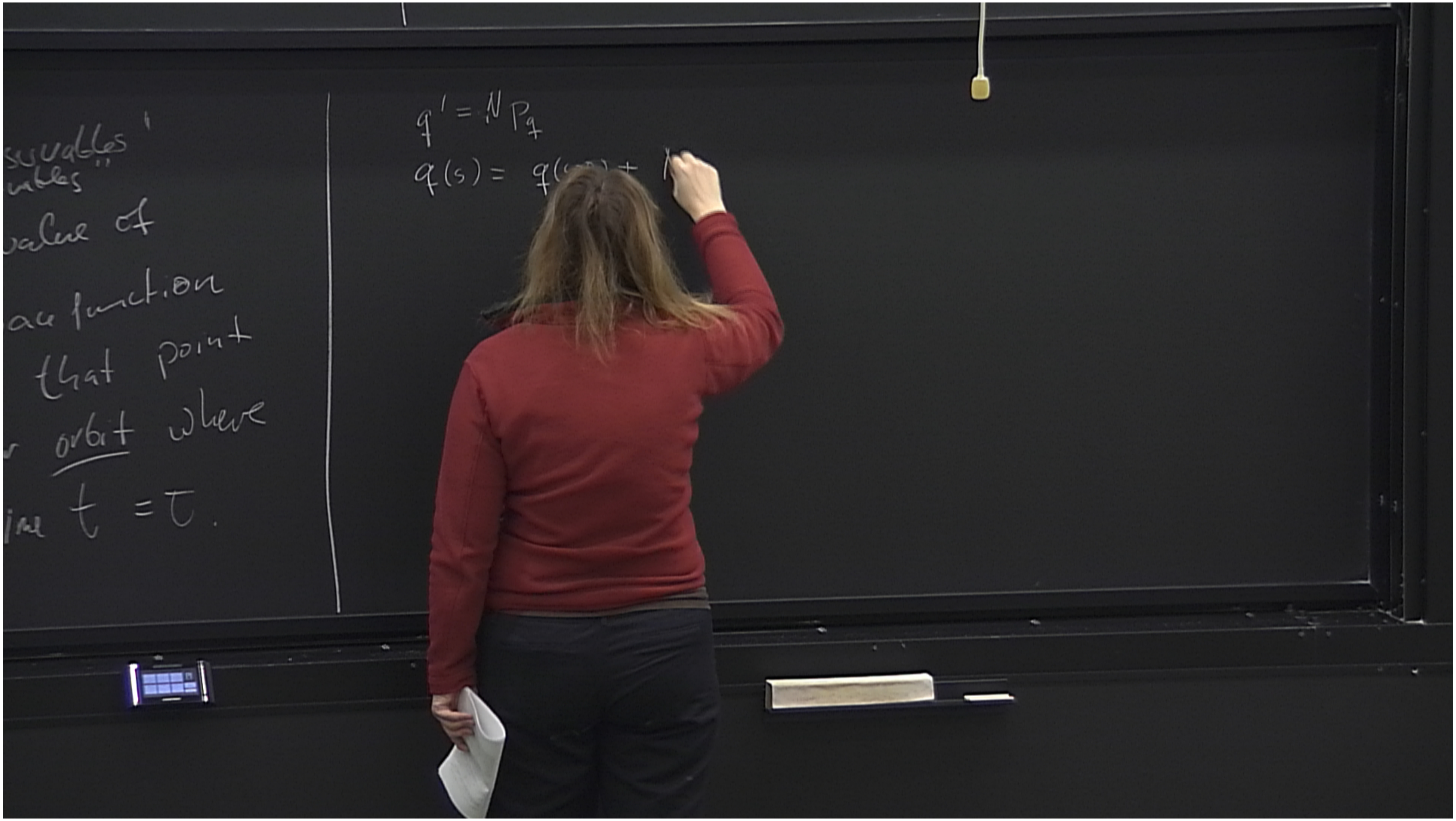
$$\{ D, C \} \approx 0$$

Observables? specify: $L = \frac{m}{2} \dot{q}^2$

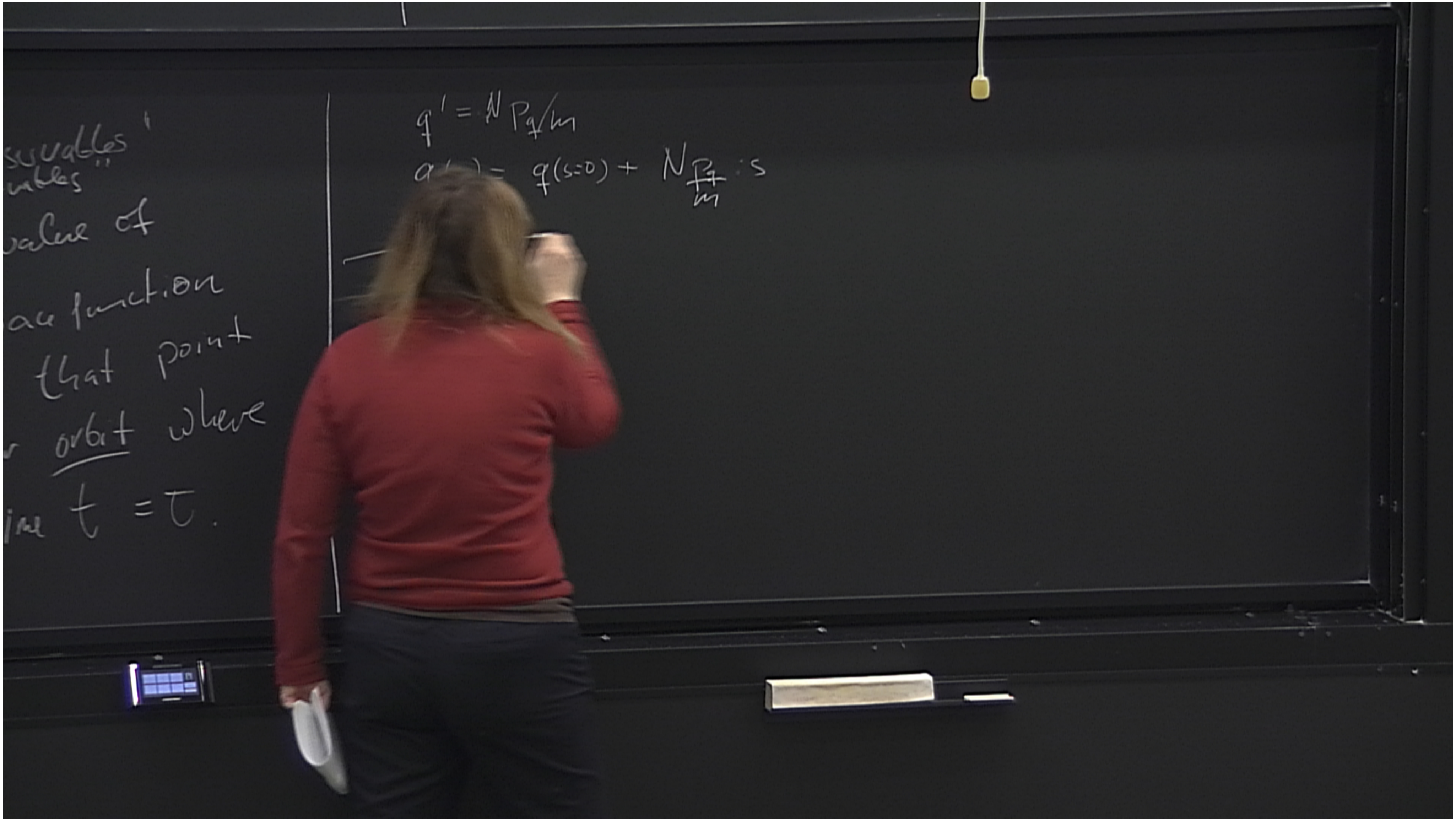
- "constant of motion": $F_{p_q}(\tau) = p_q$
- $F_q(\tau) = q + \frac{p_q}{m}(\tau - t)$

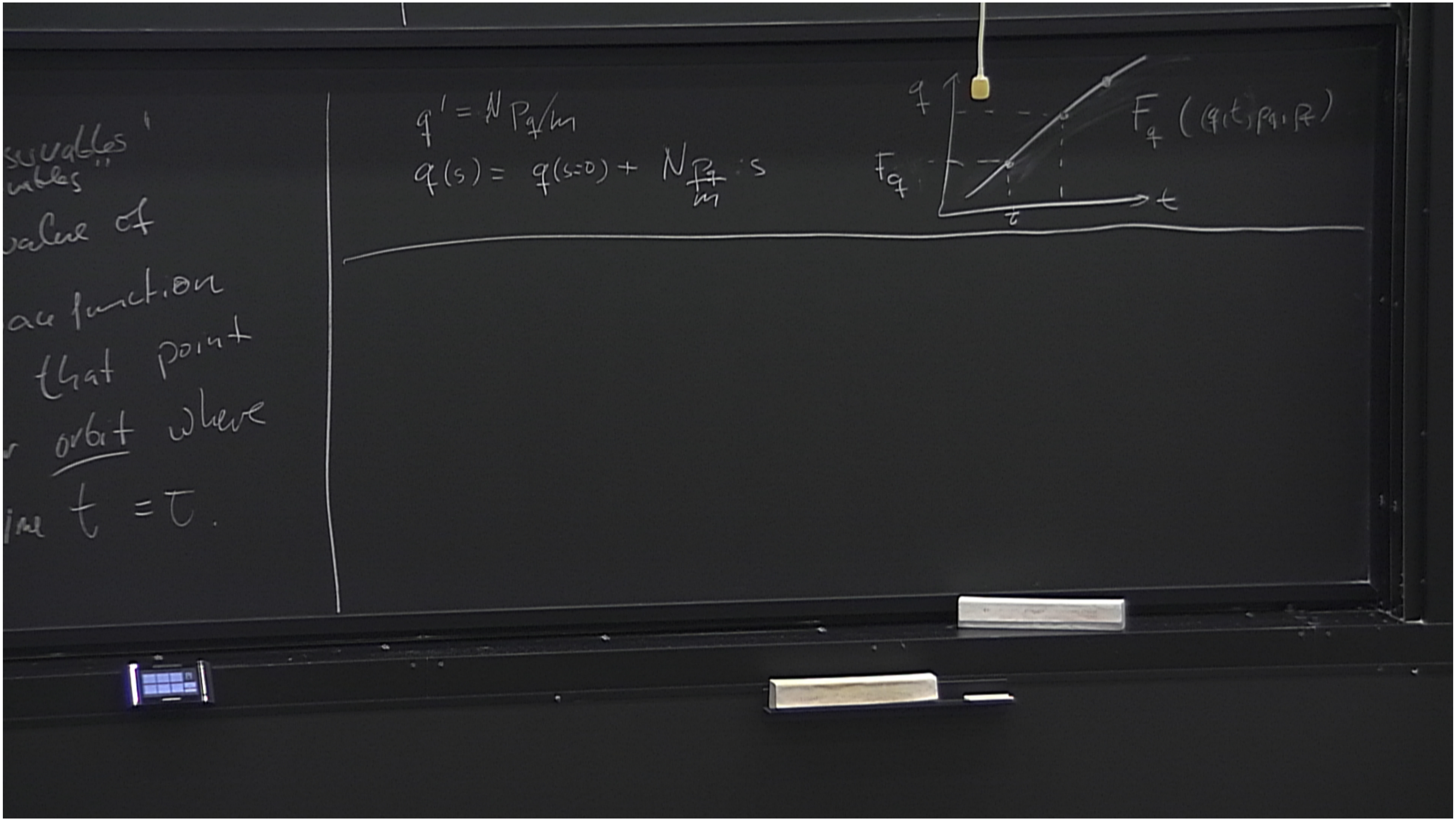
"relational observables"
"complete observables"
– $F_f(\tau)$: value of phase space function f, at that point in your orbit where your time $t = \tau$.

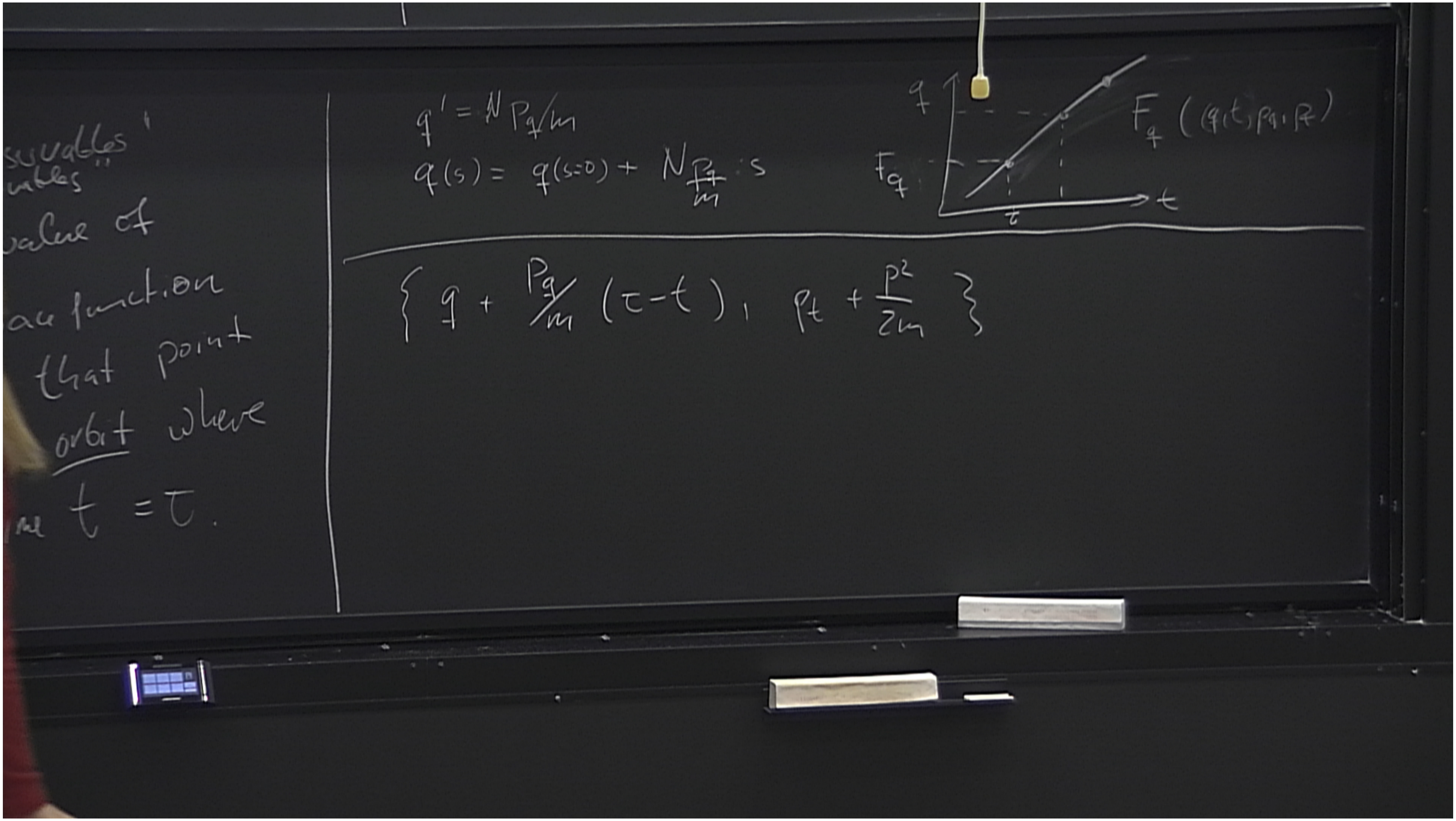

q' = N
value of
function
that point
orbit where
t = τ.

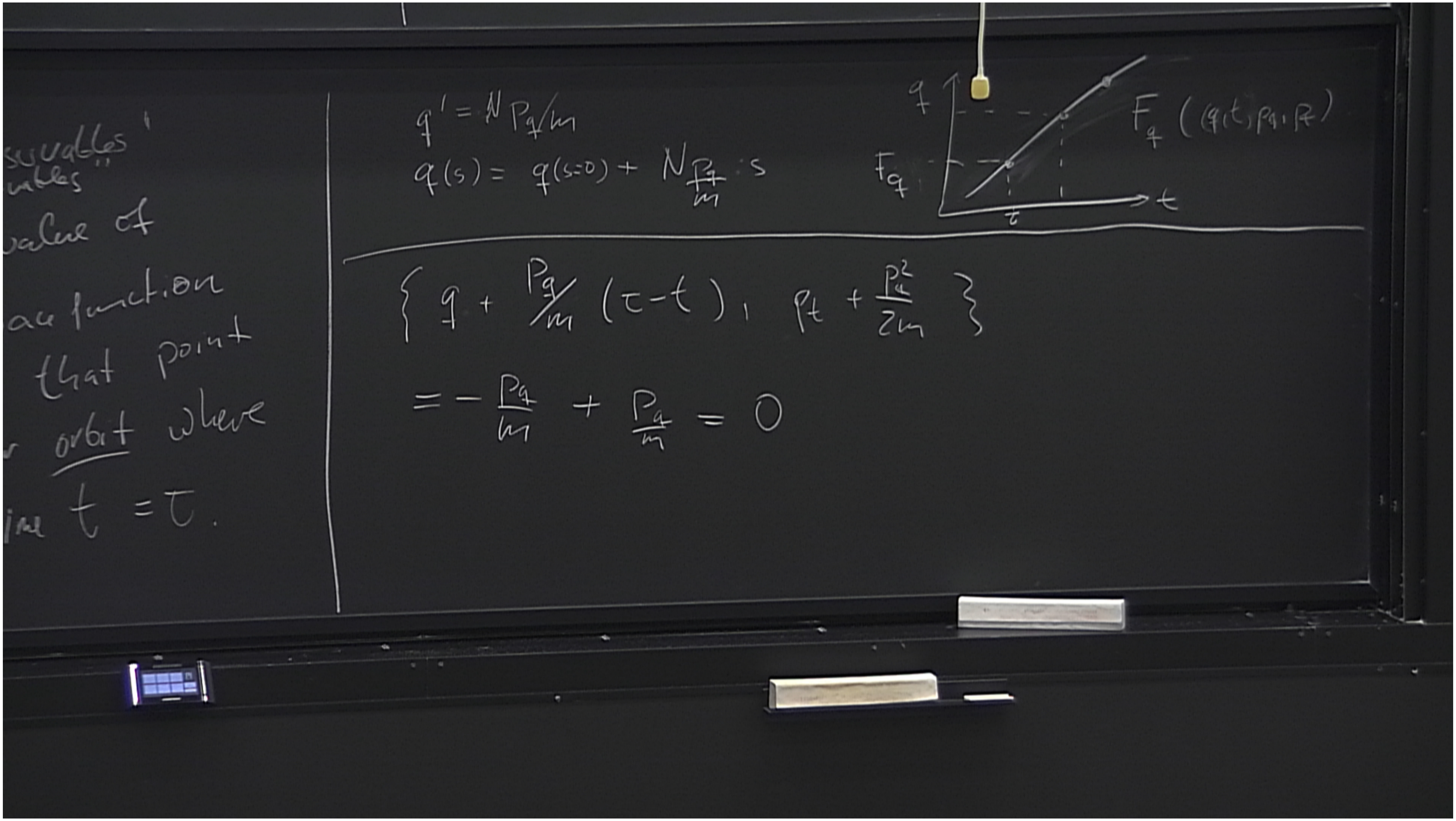


q' = N P_q
value of
function
that point
orbit where
t = τ

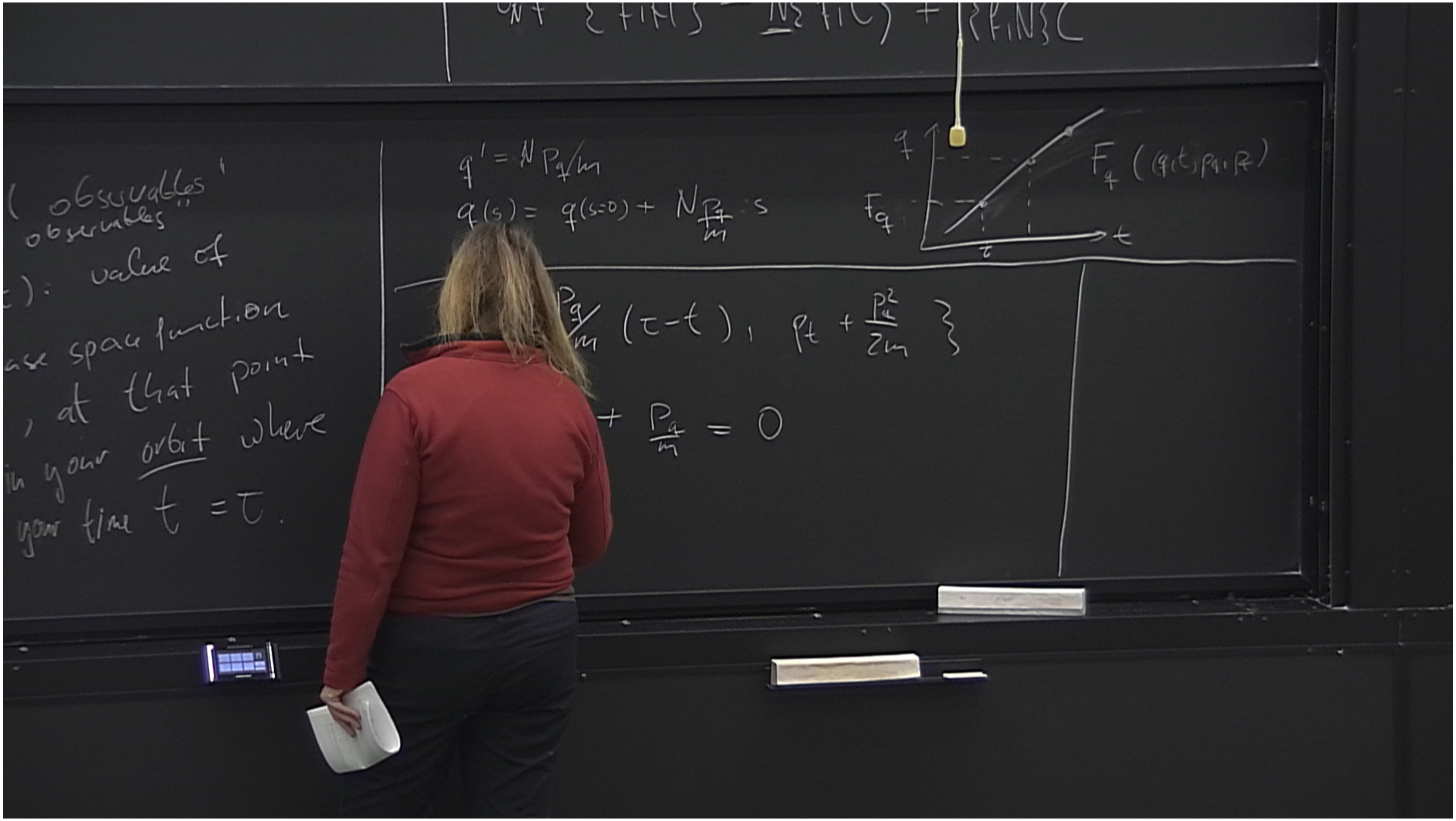

q' = N p_q/m
q(s=0) + N p_q/m · s
value of
function
that point
orbit where
t = τ.

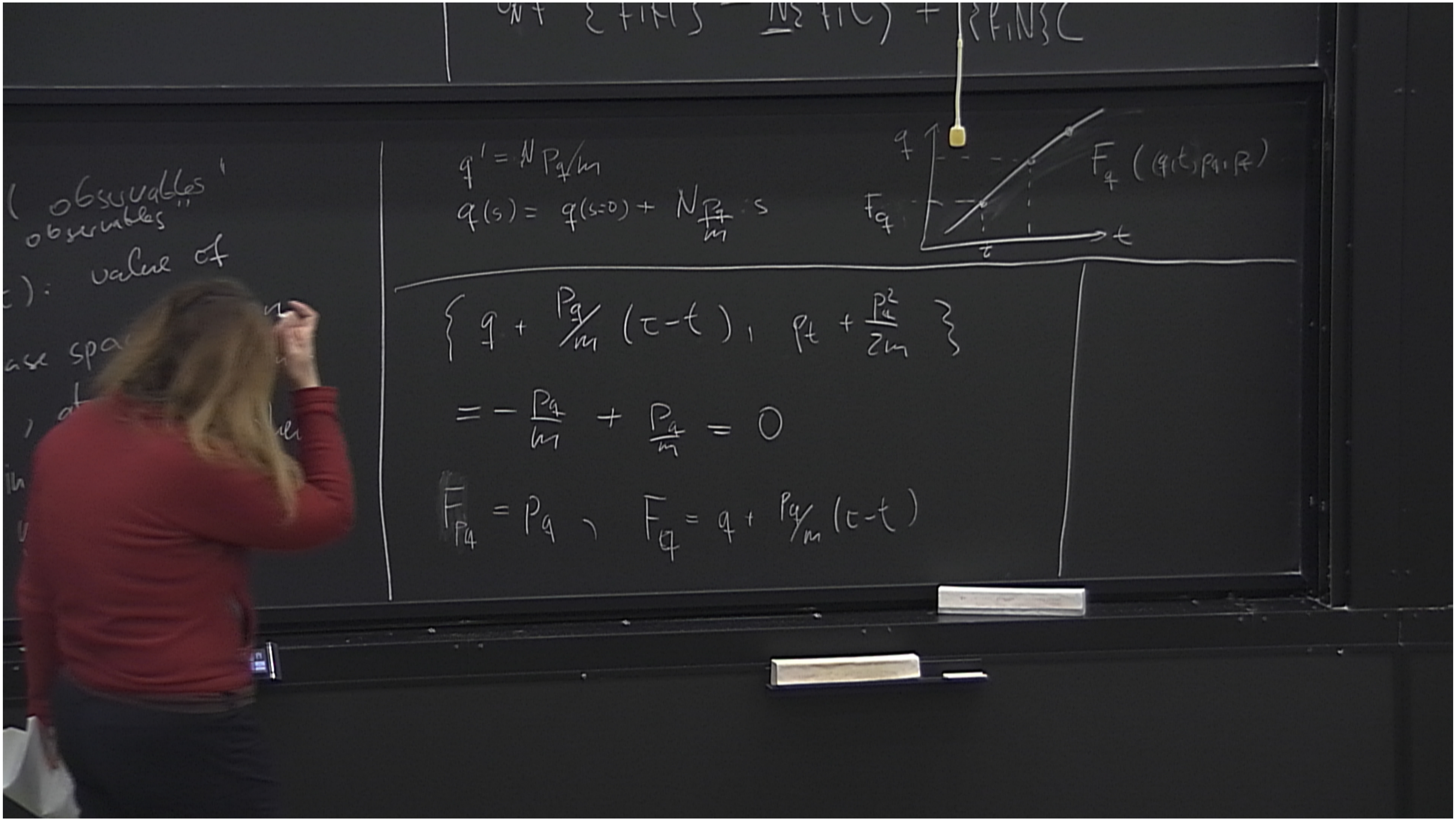

value of
function
that point
orbit where
t = τ
q' = N p_q/m
q(s) = q(s=0) + N p_q/m · s
F_q ((q,t), p_q, p_t)
q
t
τ
F_q

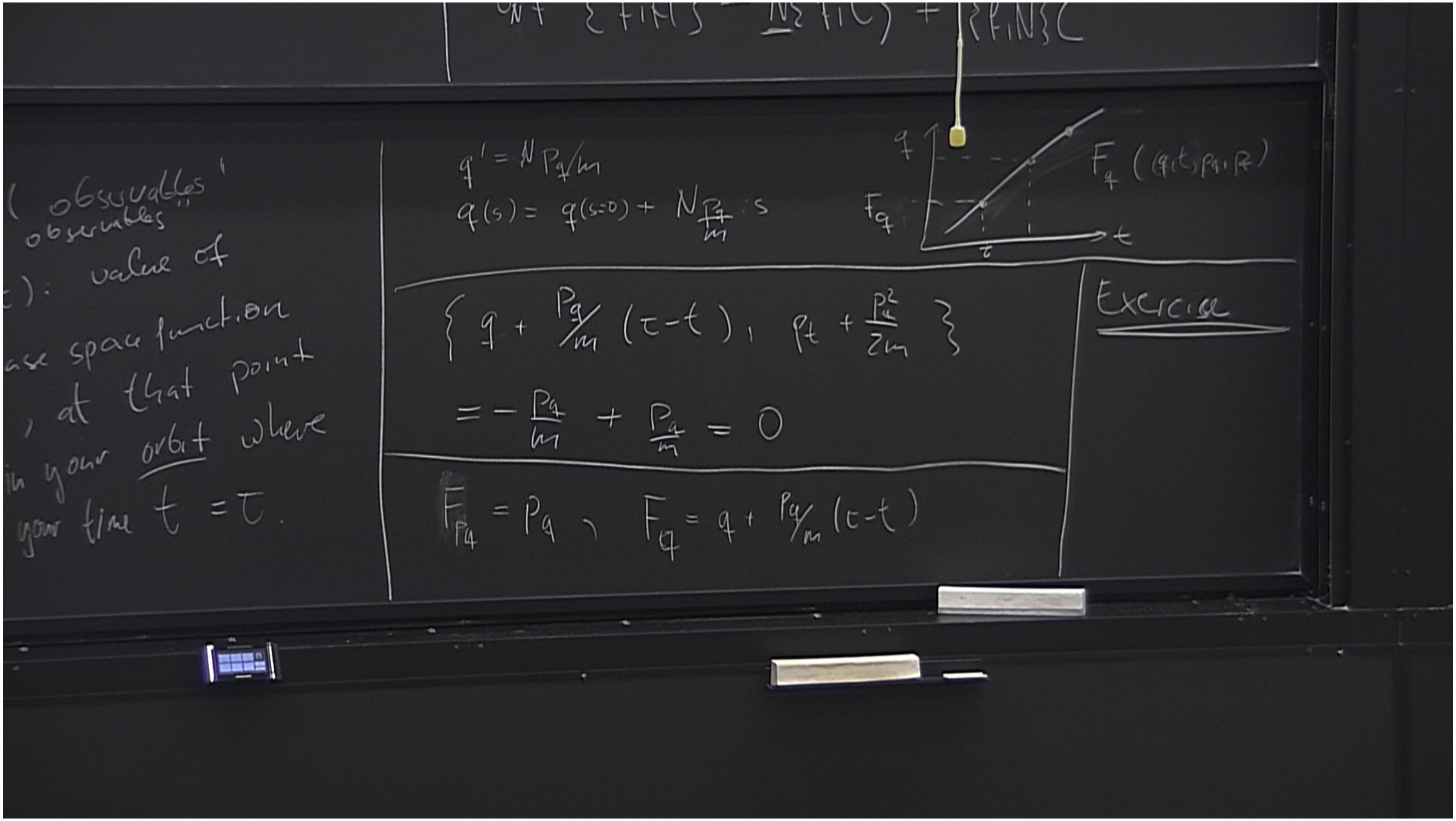

q' = N p_q/m
q(s) = q(s=0) + N p_q/m · s
F_q ((q,t), p_q, p_t)
{ q + p_q/m (τ − t) , p_t + p²/2m }
value of
a function
that point
orbit where
t = τ

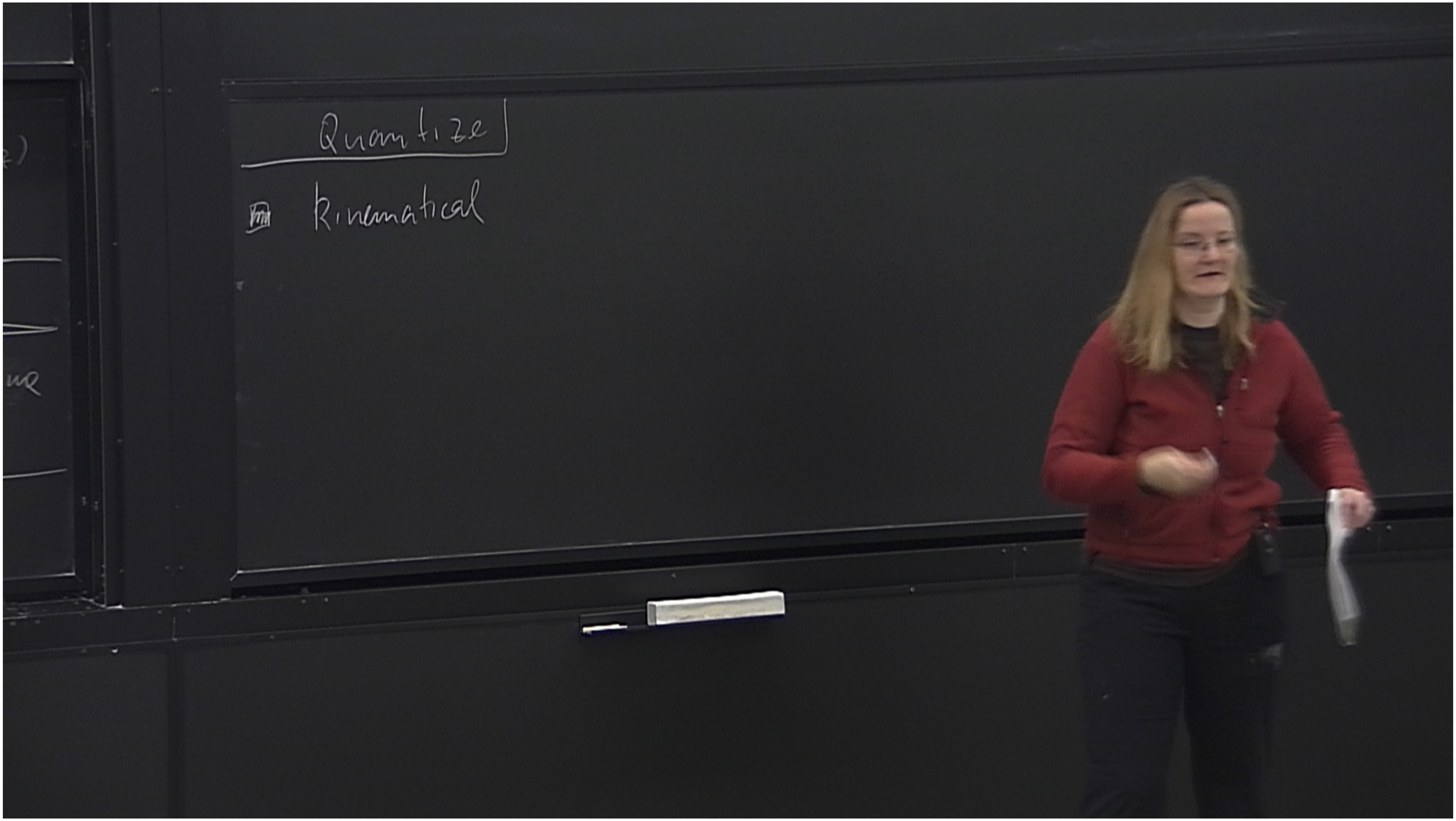


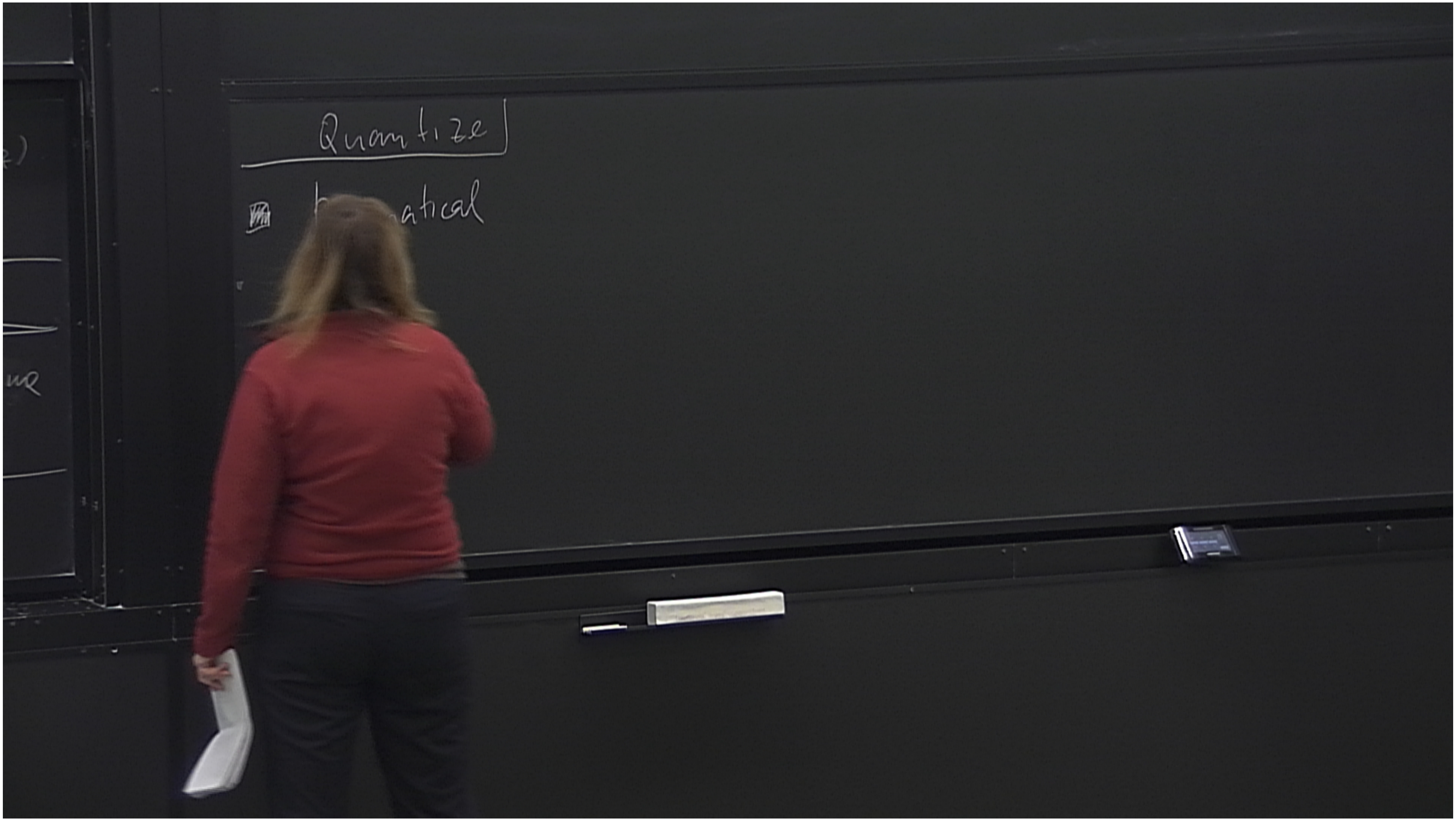
value of
function
that point
orbit where
$t = \tau$.
$q' = N p_q/m$
$q(s) = q(s=0) + N \frac{p_q}{m} s$
$F_q ((q,t), p_q, p_t)$
F_q
q
t
τ
$\{ q + \frac{p_q}{m}(\tau - t), p_t + \frac{p_q^2}{2m} \}$
$= -\frac{p_q}{m} + \frac{p_q}{m} = 0$

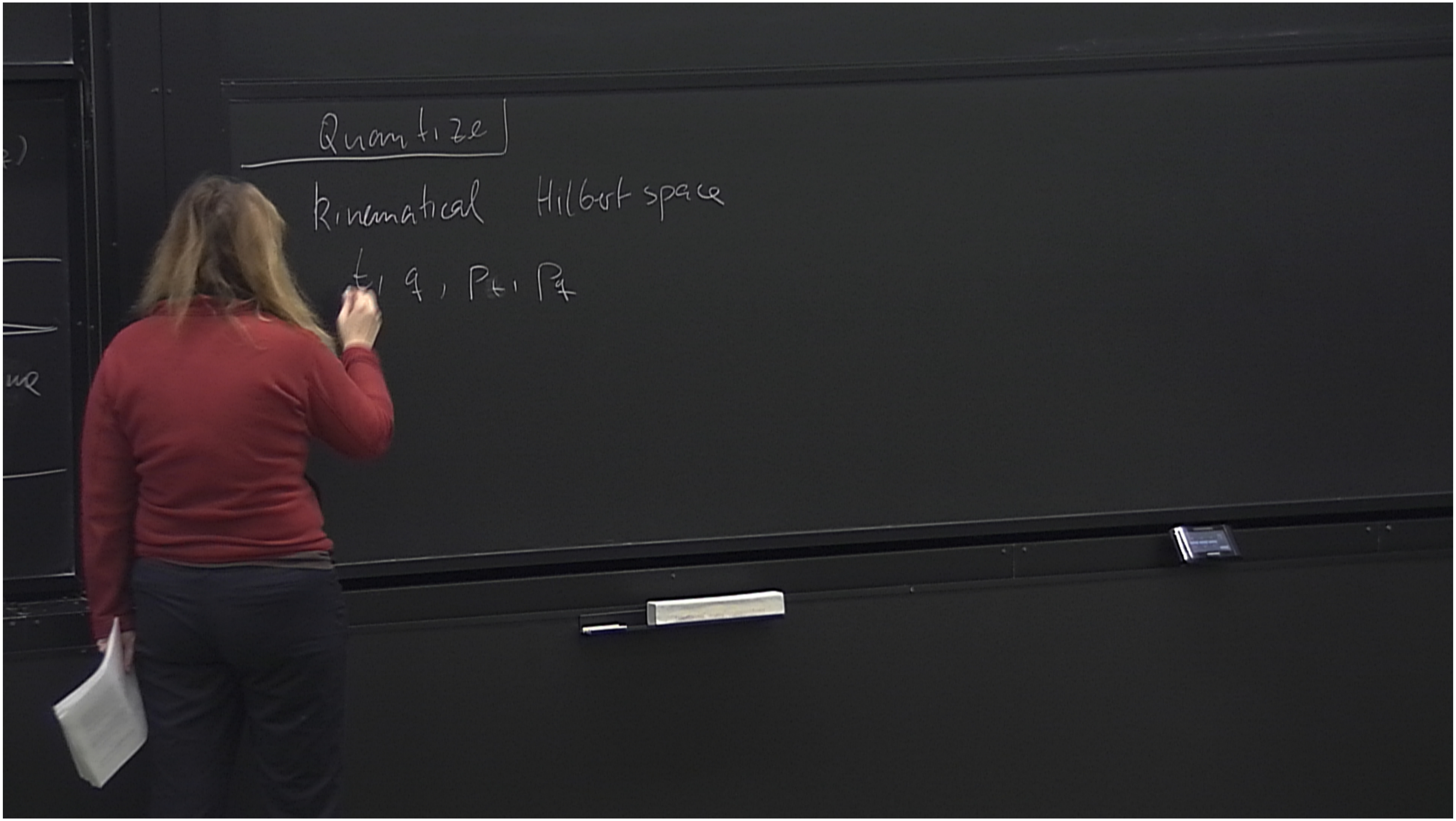

q' = N P_q/m
q(s) = q(s=0) + N P_q/m s
F_q ((q,t),p_q,p_t)
observables
value of
phase space function
, at that point
in your orbit where
your time t = τ.
(τ − t), p_t + p_q^2/2m }
+ P_q/m = 0

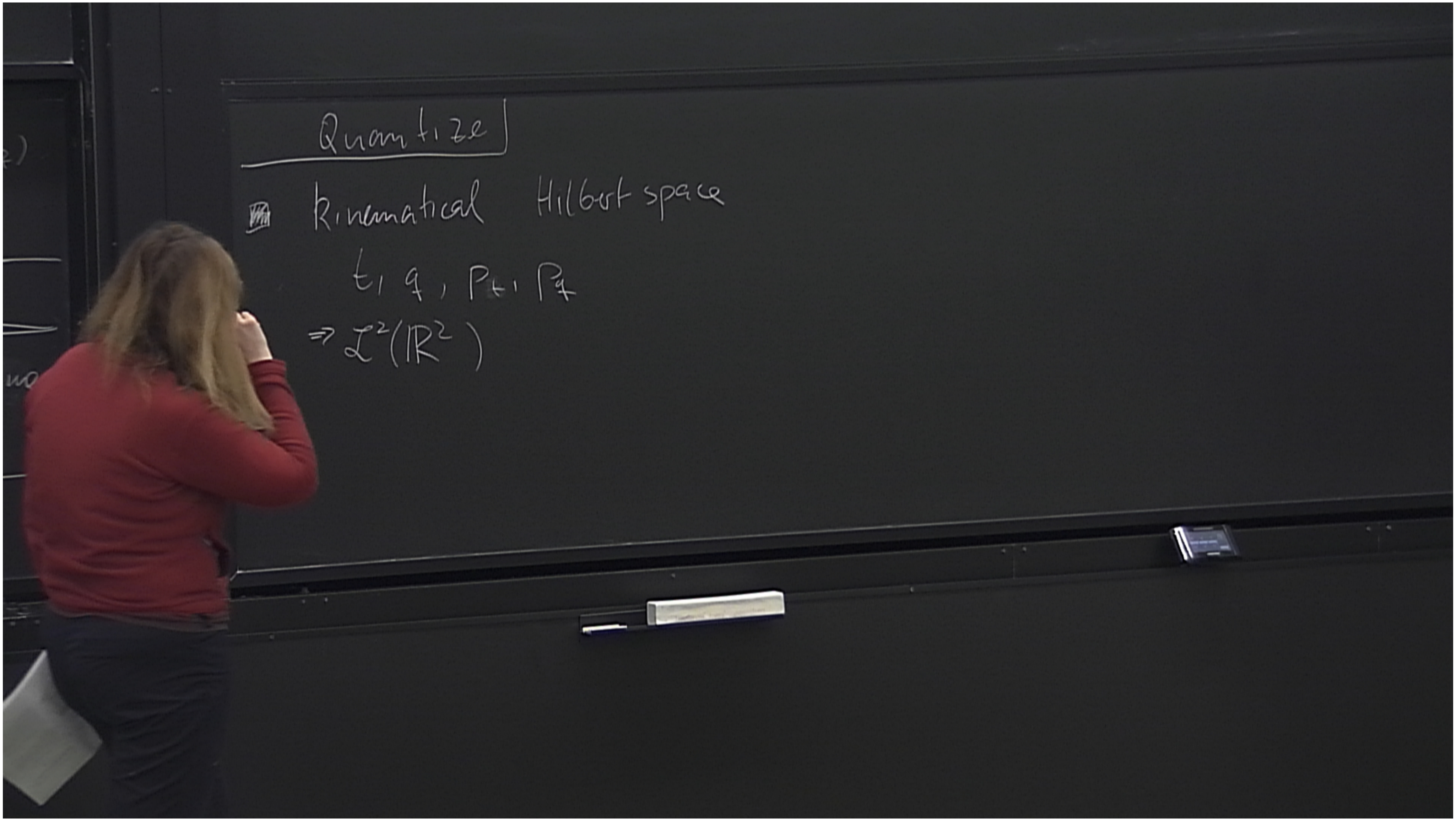

(observables'
observables"
): value of
ase spa
q' = N Pq/m
q(s) = q(s=0) + N Pq/m · s
Fq
t
Fq ((q,t),pq,pt)
{ q + Pq/m (τ − t) , pt + Pq²/2m }
= − Pq/m + Pq/m = 0
FPq = Pq , Fq = q + Pq/m (τ − t)

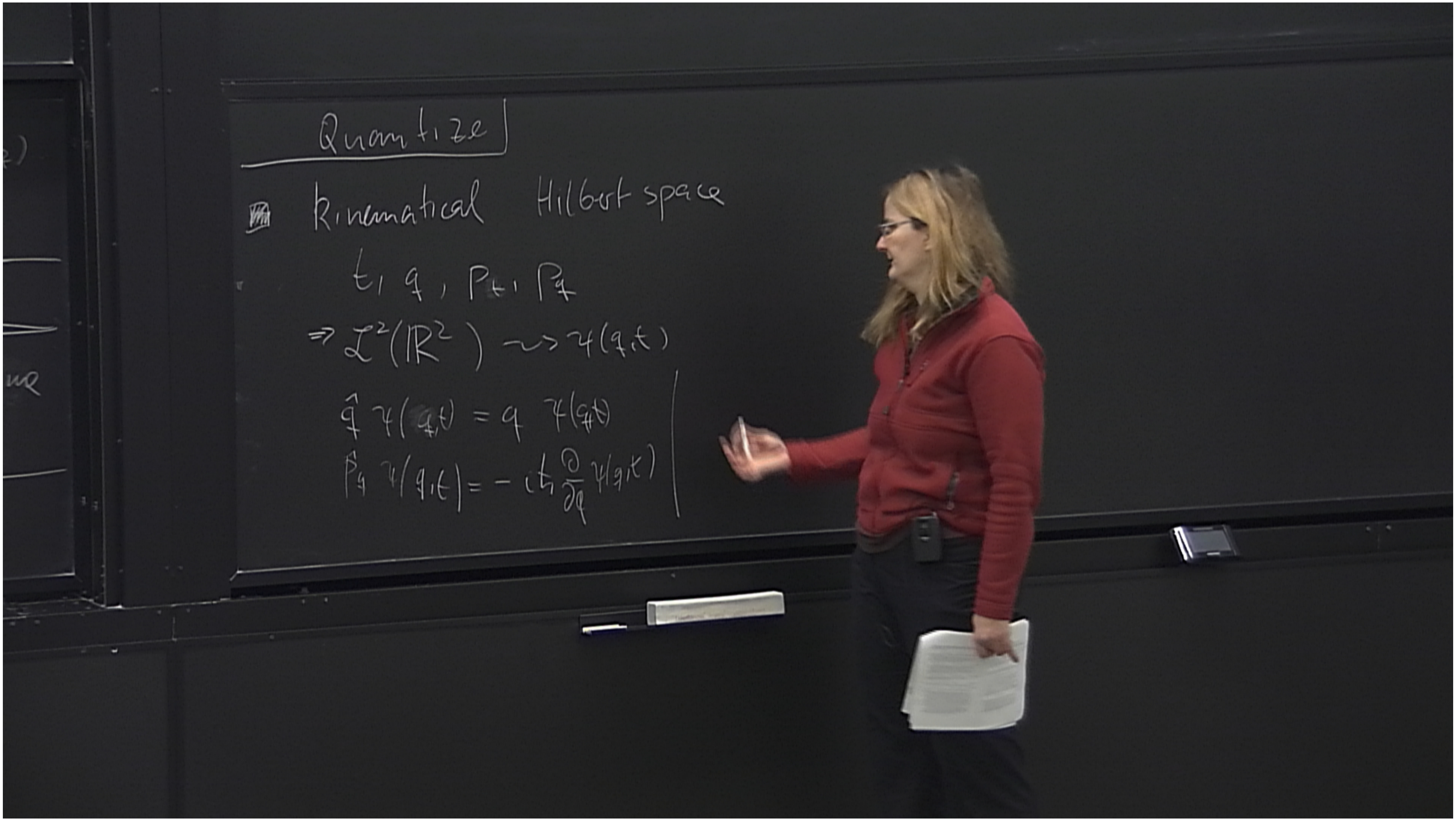

Exercise
q' = N p_q/m
q(s) = q(s=0) + N p_q/m · s
{ q + p_q/m (τ − t) , p_t + p_q^2/2m }
= − p_q/m + p_q/m = 0
F_{p_q} = p_q , F_q = q + p_q/m (τ − t)
F_q (q, t, p_q, p_t)
observables
value of
phase space function
at that point
in your orbit where
your time t = τ.

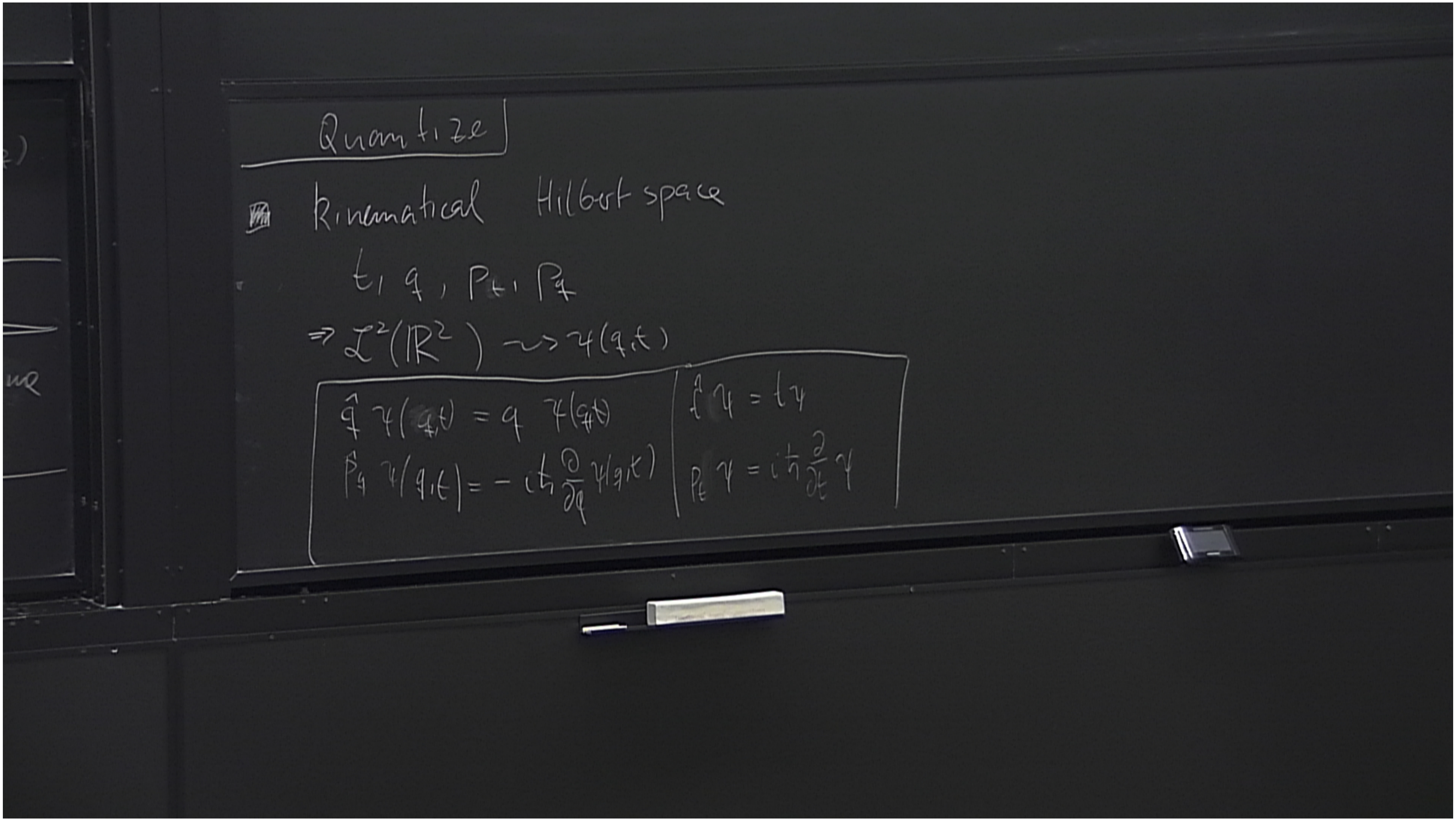

Quantize
Kinematical

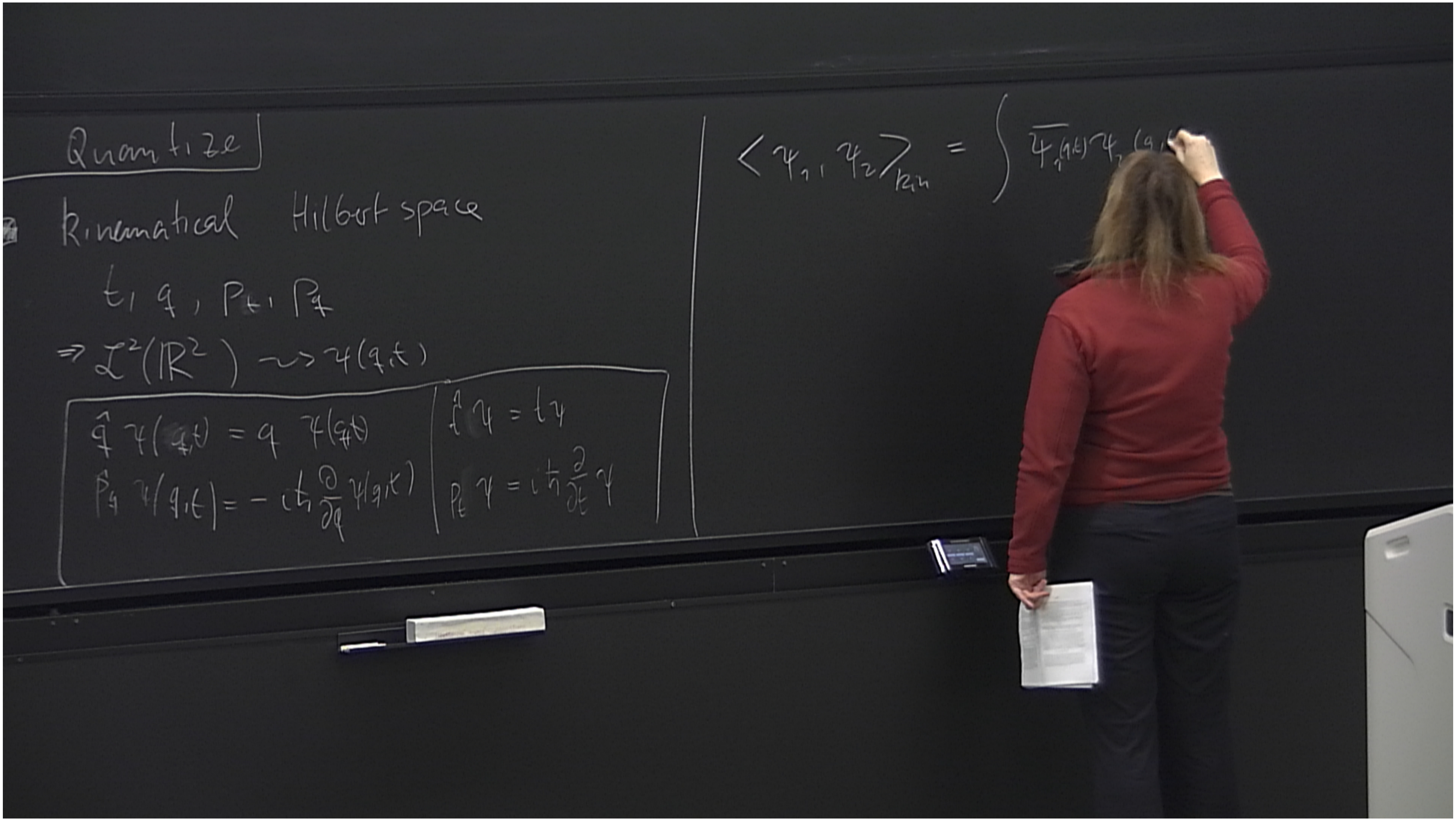

Quantize

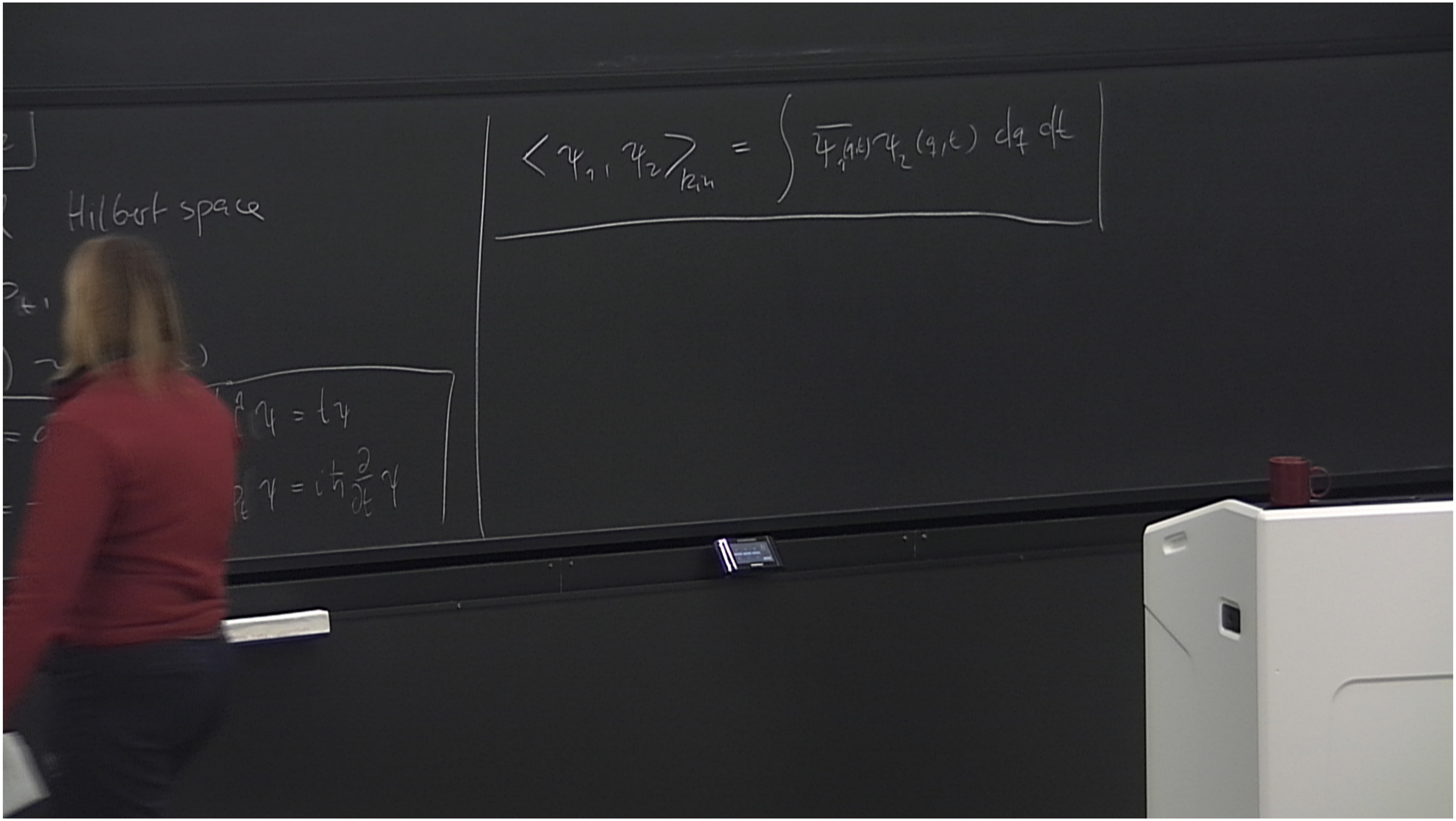

Quantize
Kinematical Hilbert space
t, q, pt, pq

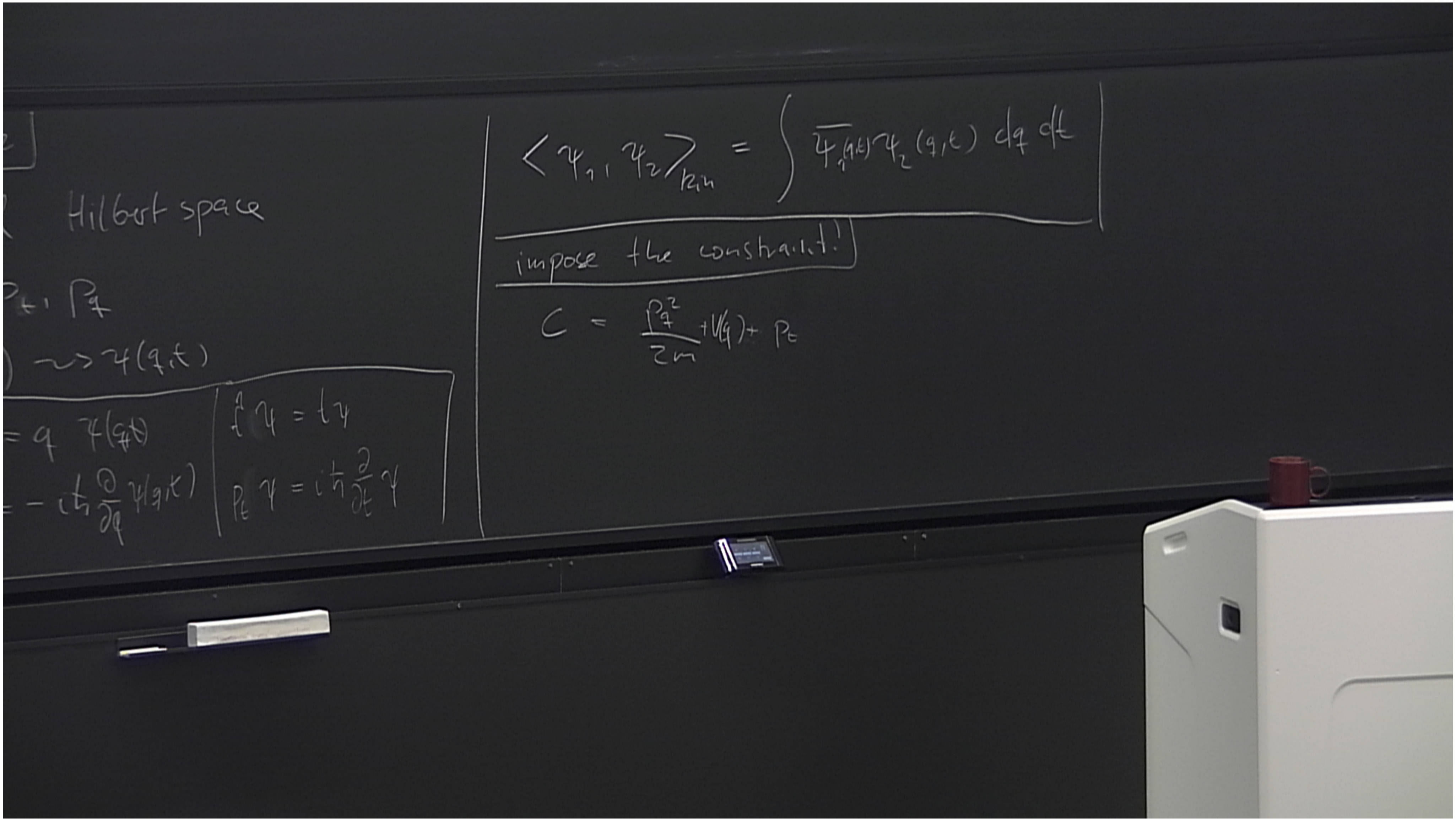

Quantize
Kinematical Hilbert space
t, q, p_t, p_q
$\Rightarrow \mathcal{L}^2(\mathbb{R}^2)$

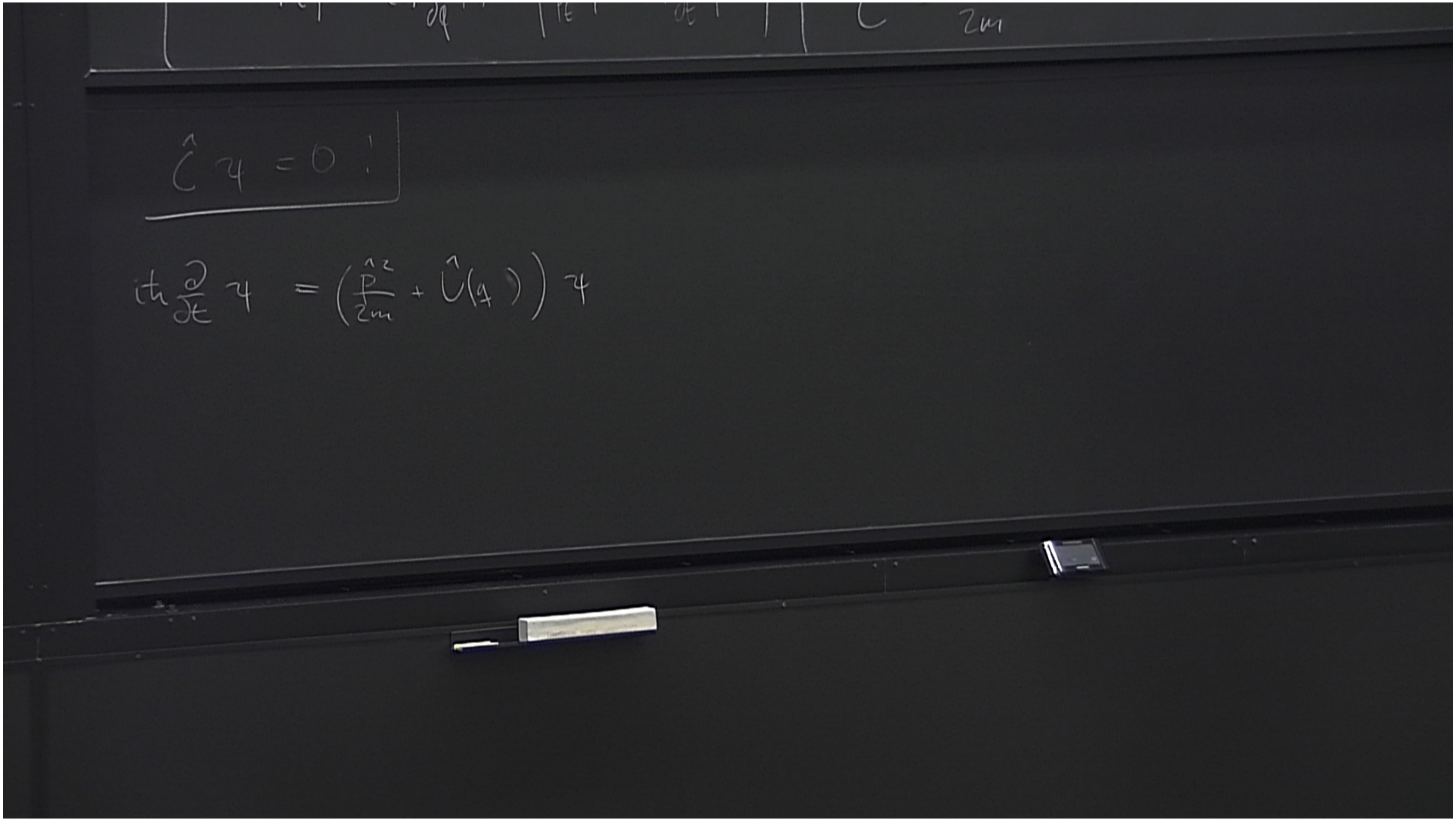

Quantize
Kinematical Hilbert space
t, q, p_t, p_q
$\Rightarrow \mathcal{L}^2(\mathbb{R}^2) \rightsquigarrow \psi(q,t)$
$\hat{q}\,\psi(q,t) = q\,\psi(q,t)$
$\hat{p}_q\,\psi(q,t) = -i\hbar \frac{\partial}{\partial q}\psi(q,t)$

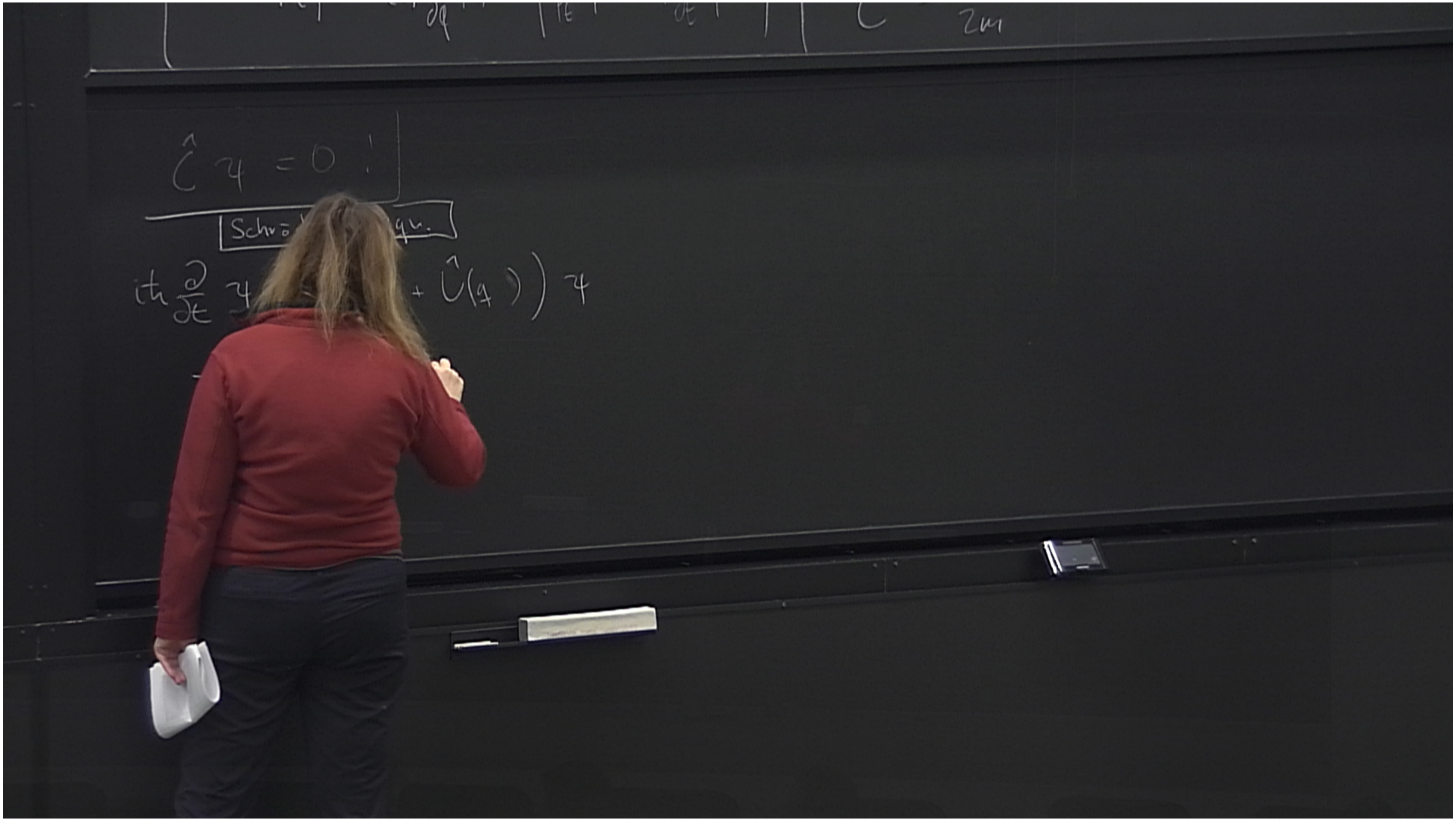

Quantize
Kinematical Hilbert space
t, q, p_t, p_q
⇒ L²(ℝ²) ⇝ ψ(q,t)
q̂ ψ(q,t) = q ψ(q,t)
p̂_q ψ(q,t) = − iħ ∂/∂q ψ(q,t)
t̂ ψ = t ψ
p_t ψ = iħ ∂/∂t ψ

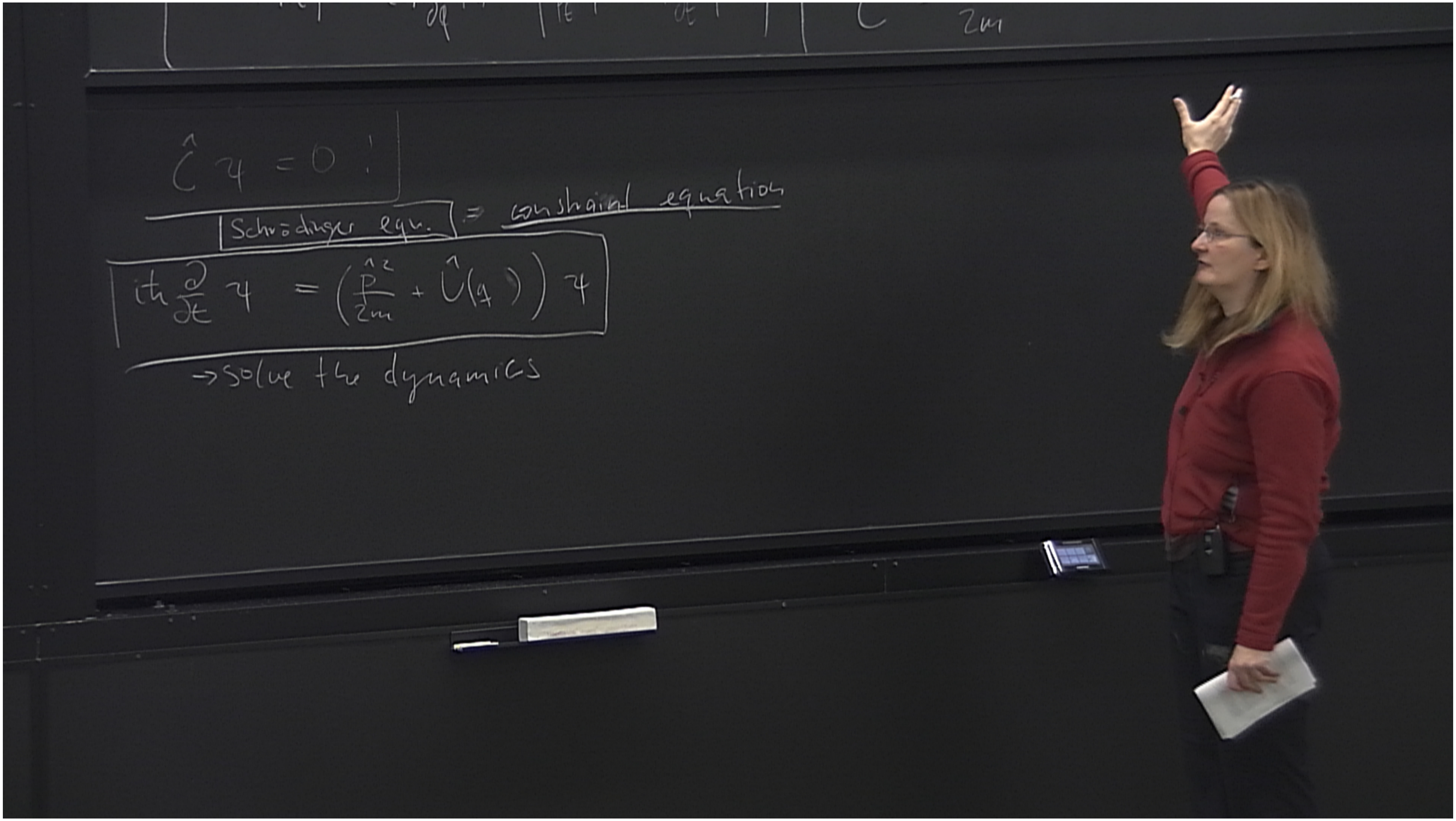

Quantize
Kinematical Hilbert space
t, q, P_t, P_q
⇒ L²(ℝ²) ⇝ ψ(q,t)
q̂ ψ(q,t) = q ψ(q,t)
p̂_q ψ(q,t) = −iħ ∂/∂q ψ(q,t)
t̂ ψ = t ψ
p_t ψ = iħ ∂/∂t ψ
⟨ψ₁, ψ₂⟩_kin = ∫ ψ̄₁(q,t) ψ₁(q,t

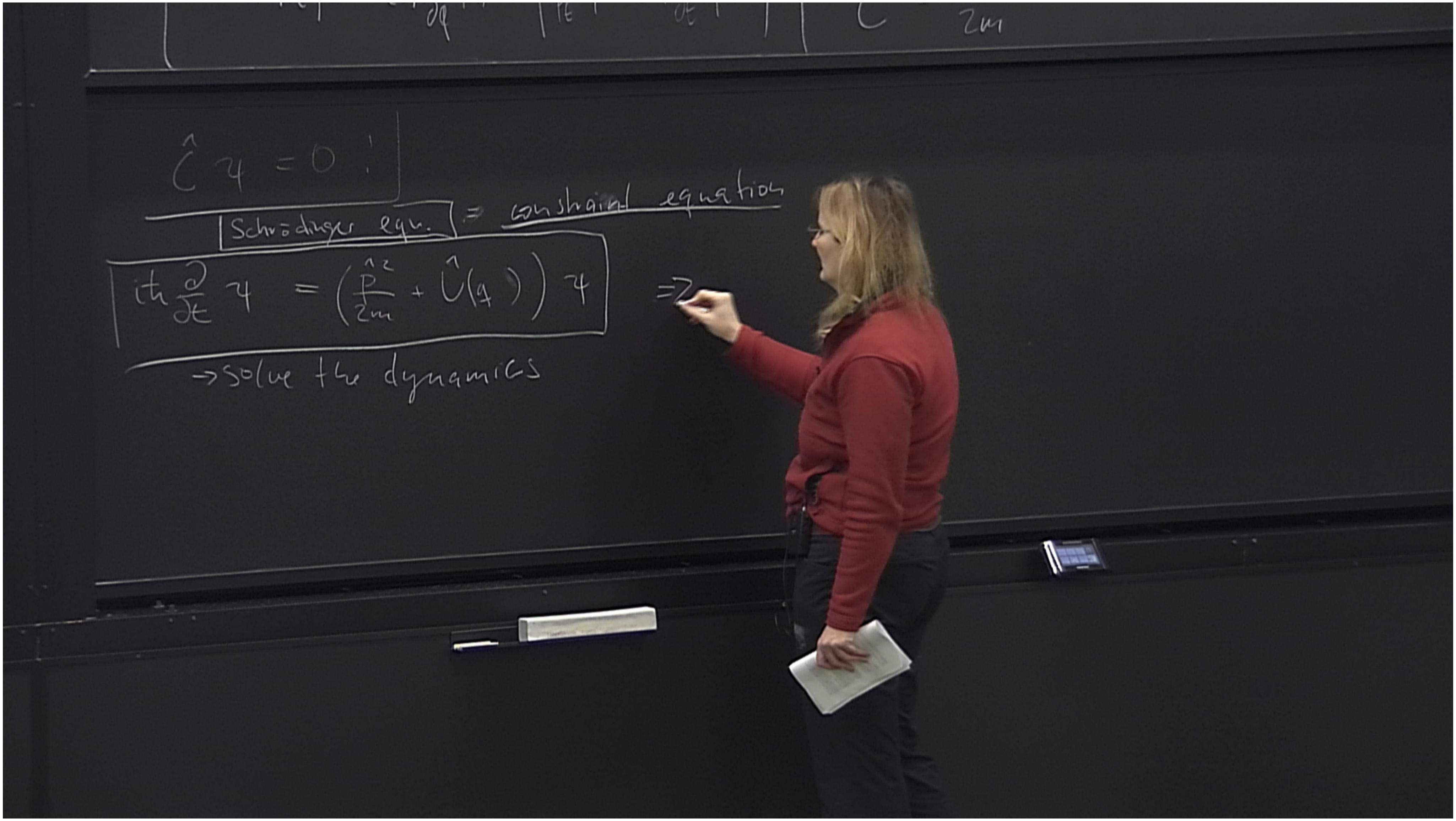

⟨ψ₁, ψ₂⟩kin = ∫ ψ̄₁(q,t) ψ₂(q,t) dq dt
Hilbert space

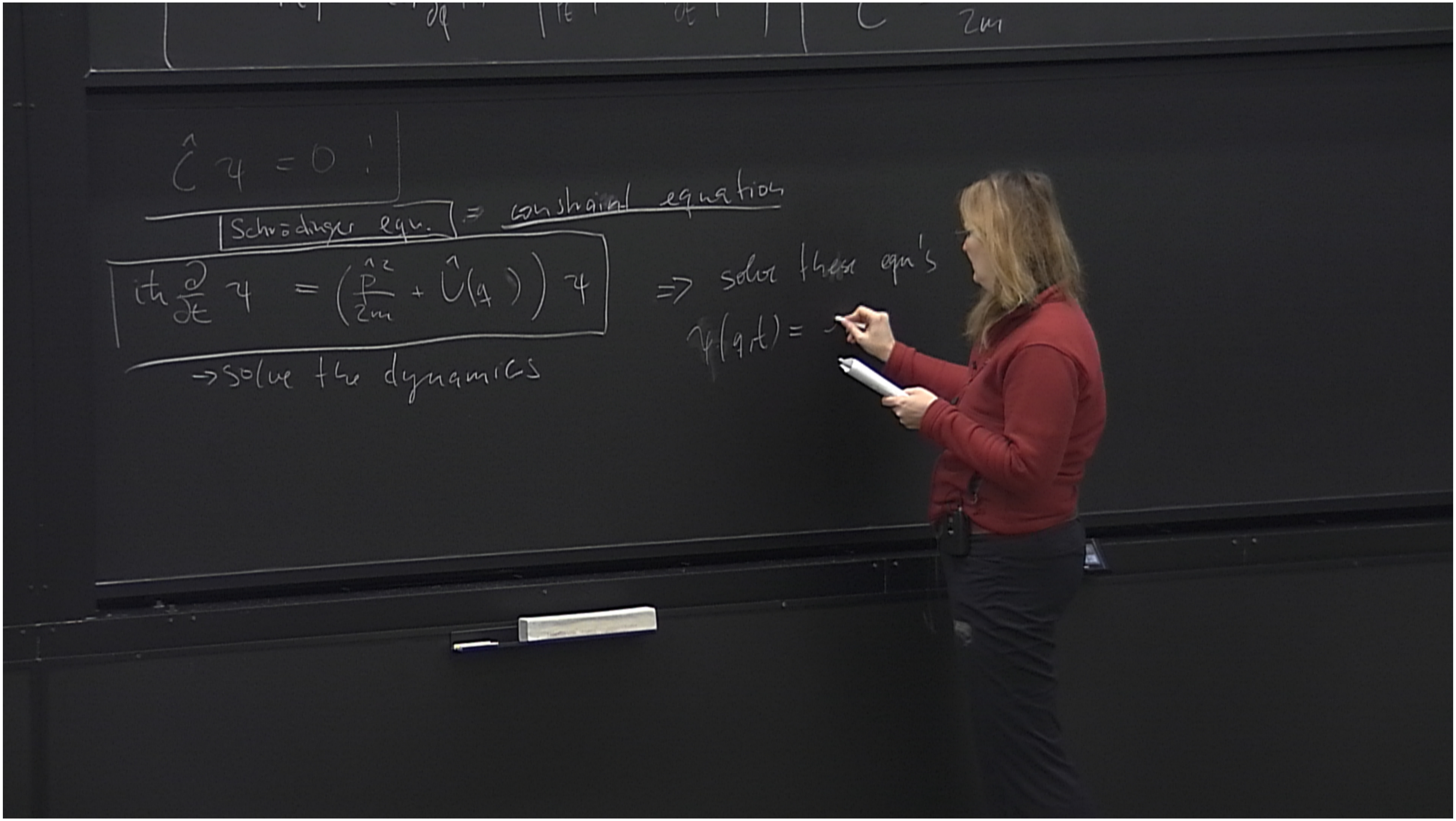

Hilbert space
p_t , p_q
$\rightsquigarrow \psi(q,t)$
$= q \, \psi(q,t)$
$= -i\hbar \frac{\partial}{\partial q} \psi(q,t)$
$\hat{t} \psi = t \psi$
$p_t \psi = i\hbar \frac{\partial}{\partial t} \psi$
$\langle \psi_1 , \psi_2 \rangle_{kin} = \int \overline{\psi_1(q,t)} \psi_2(q,t) \, dq \, dt$
impose the constraint!
$C = \frac{p_q^2}{2m} + V(q) + p_t$


Ĉ ψ = 0 !
iħ ∂/∂t ψ = (p̂²/2m + Û(q̂)) ψ

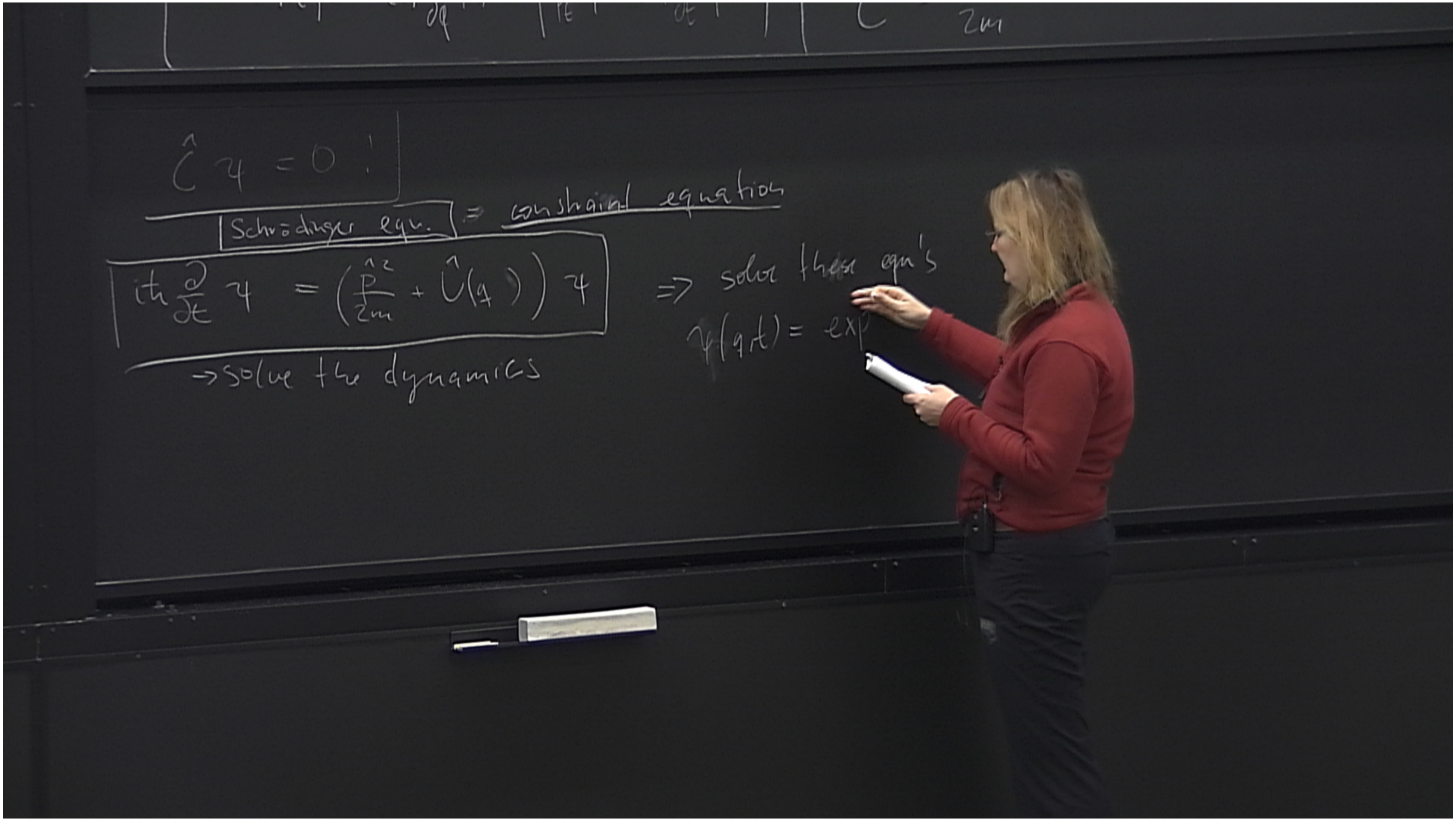

Ĉ ψ = 0 !
Schrö eqn.

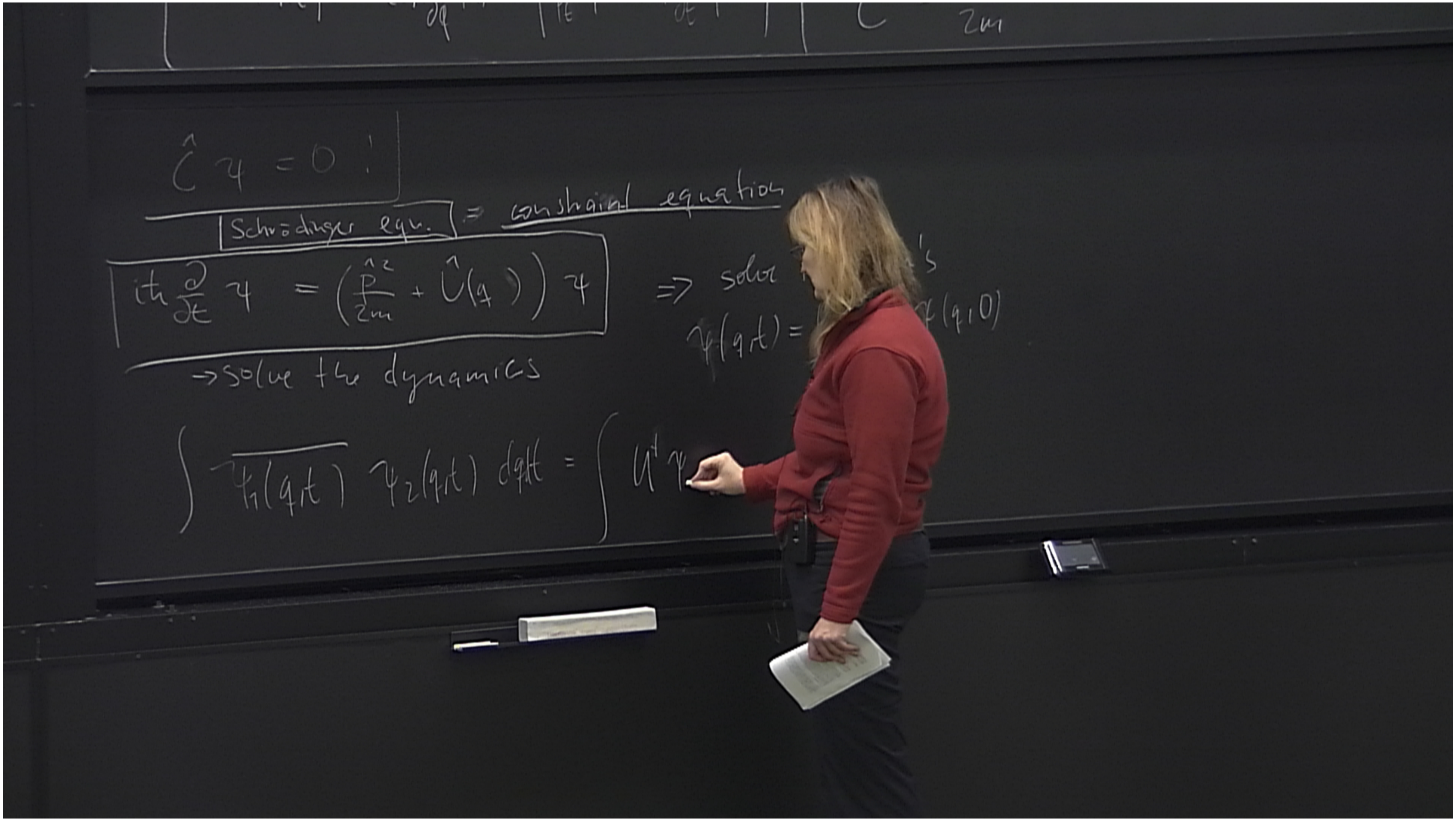

Ĉ ψ = 0 !
Schrödinger eqn. ⇒ constraint equation
iħ ∂/∂t ψ = (p̂²/2m + V̂(q)) ψ
→ solve the dynamics

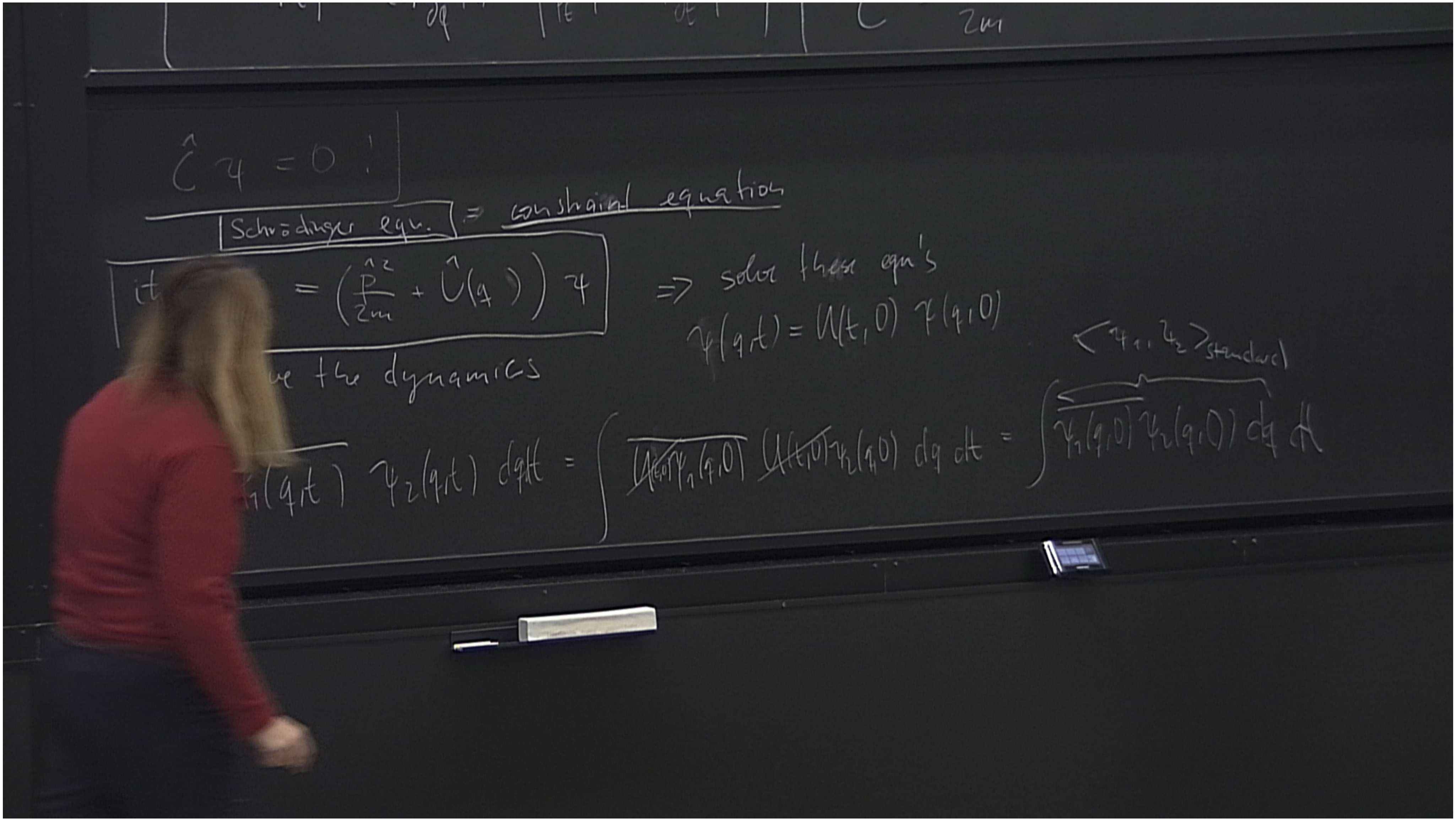

Ĉ ψ = 0 !
Schrödinger eqn.
⇒ constraint equation
iħ ∂/∂t ψ = (p̂²/2m + Û(q)) ψ
→ solve the dynamics

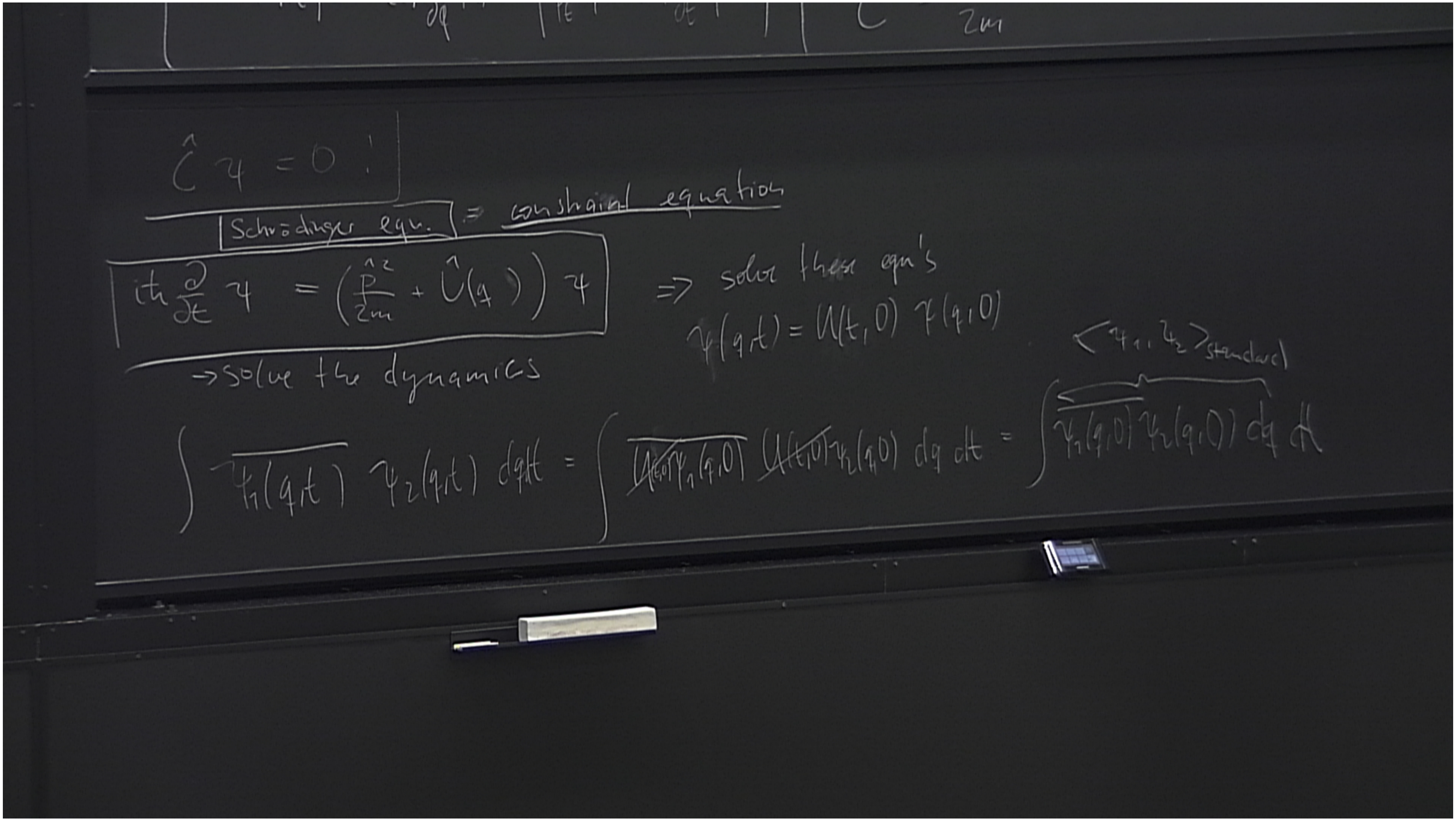


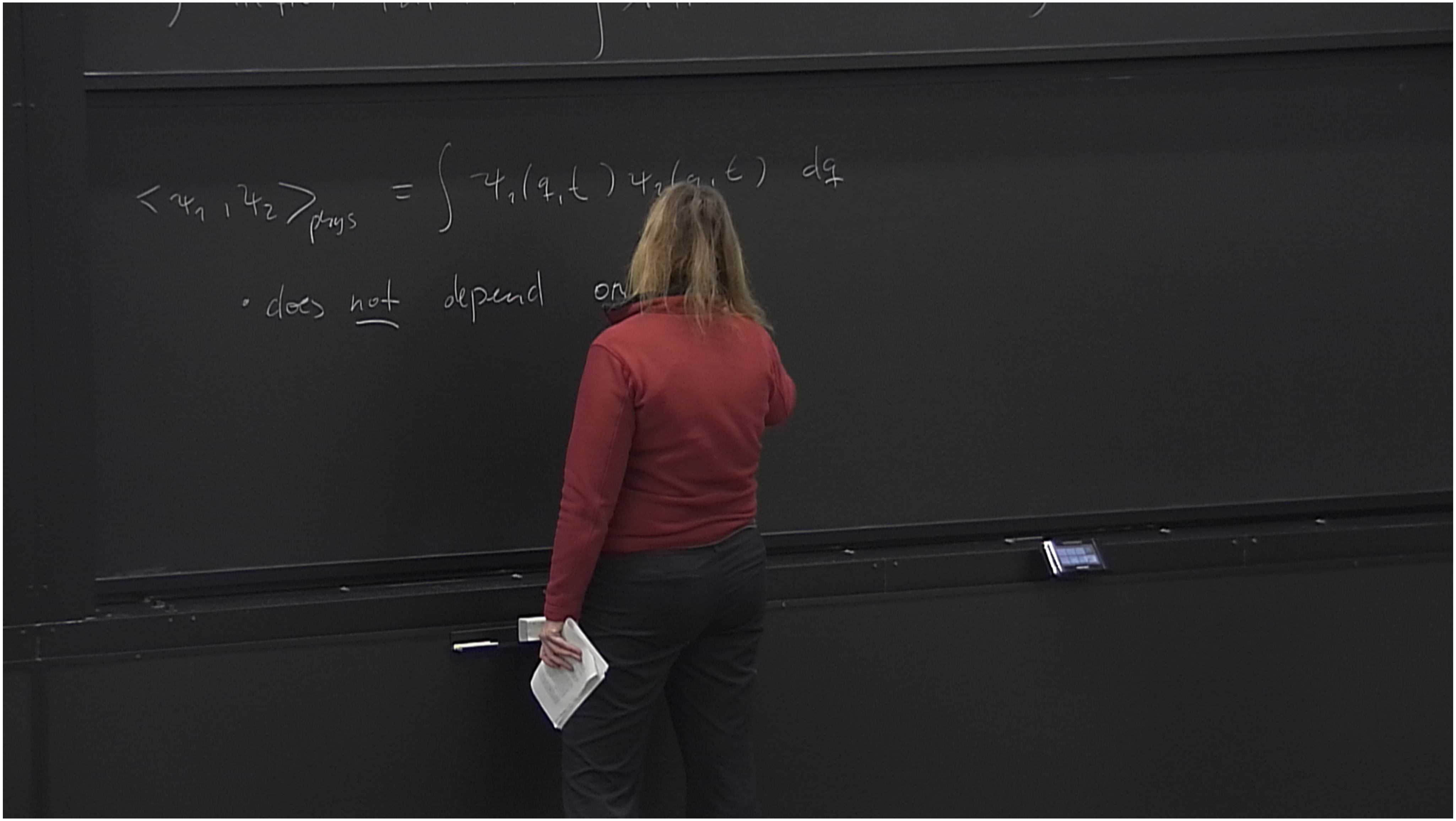
Ĉ ψ = 0 !
Schrödinger eqn.
⇒ constraint equation
iħ ∂/∂t ψ = (p̂²/2m + Û(q)) ψ
→ solve the dynamics
⇒ solve these eqn's
ψ(q,t) =

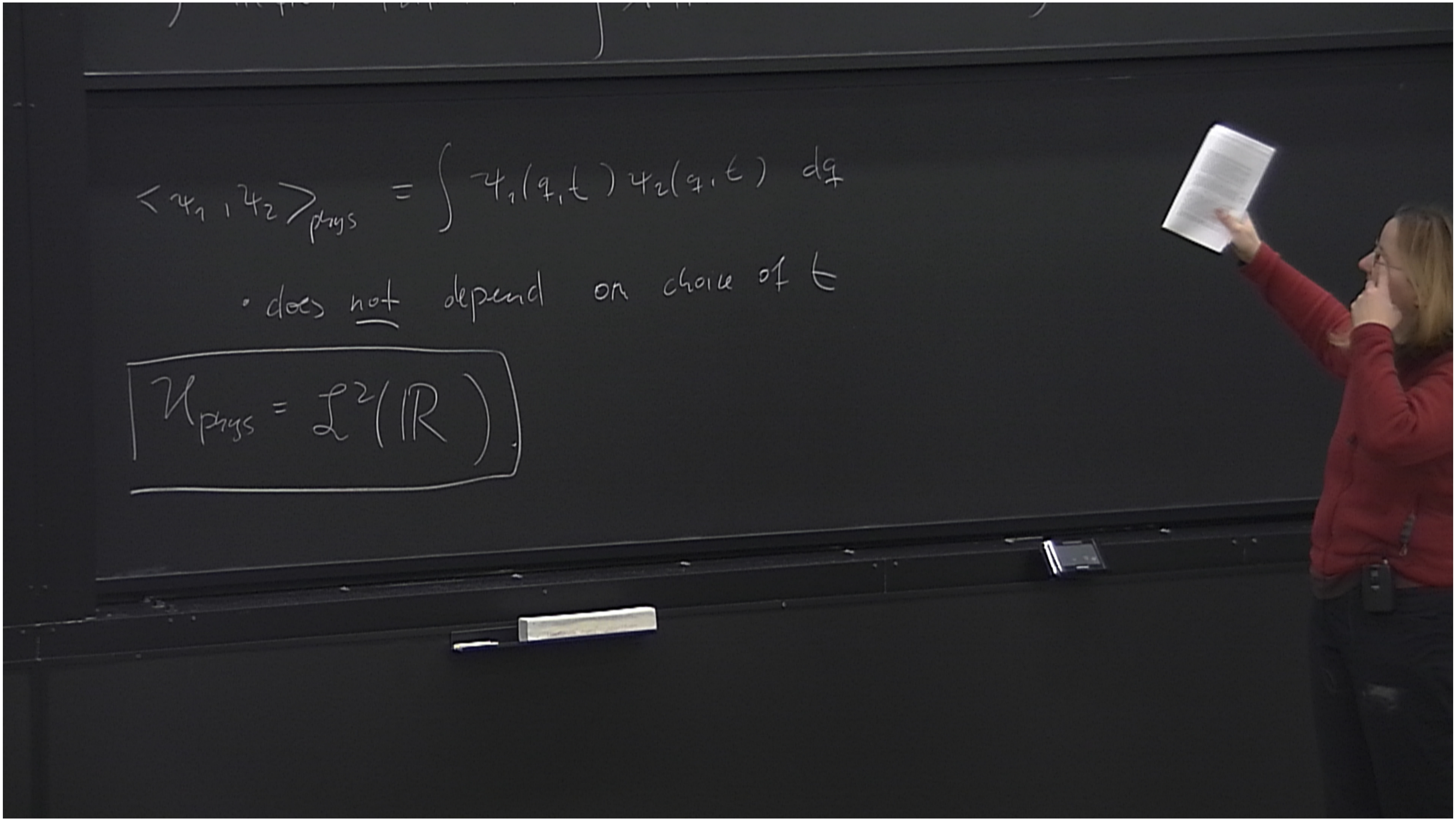

Ĉ ψ = 0 !
Schrödinger equ.
⇒ constraint equation
iħ ∂/∂t ψ = (p̂²/2m + Û(q)) ψ
→ solve the dynamics
⇒ solve these equ's
ψ(q,t) = exp


Ĉ ψ = 0 !
Schrödinger equ.
= constraint equation
iħ ∂/∂t ψ = (p̂²/2m + Û(q)) ψ
→ solve the dynamics
⇒ solve
ψ(q,t) =
ψ(q,0)
∫ ψ̄₁(q,t) ψ₂(q,t) dq dt = ∫ U† ψ


Ĉψ = 0 !
Schrödinger equ. ⇒ constraint equation
⇒ solve these equ's
ψ(q,t) = U(t,0) ψ(q,0)
the dynamics
⟨ψ1, ψ2⟩standard


Ĉψ = 0 !
Schrödinger equ. ⇒ constraint equation
iħ ∂/∂t ψ = (p̂²/2m + Û(q)) ψ
→ solve the dynamics
⇒ solve these equ's
ψ(q,t) = U(t,0) ψ(q,0)
⟨ψ1, ψ2⟩ standard
∫ ψ1(q,t)* ψ2(q,t) dq dt = ∫ (U(t,0)ψ1(q,0))* U(t,0)ψ2(q,0) dq dt = ∫ ψ1(q,0)* ψ2(q,0) dq dt


$\langle \psi_1, \psi_2 \rangle_{phys} = \int \psi_1(q,t)\, \psi_2(q,t)\; dq$
· does not depend on


⟨ψ₁, ψ₂⟩phys = ∫ ψ₁(q, t) ψ₂(q, t) dq
• does not depend on choice of t
Hphys = L²(ℝ)