Title: 13/14 PSI - Quantum Gravity Review - Lecture 5

Date: Feb 24, 2014 10:15 AM

URL: http://pirsa.org/14020067

Abstract:

Time evolution & Gauge trafo's

$$\mathcal{H} = -\int (e_0^j F_j + A_0^j G_j)\, d^2x$$

$$G[\Lambda] := \int \Lambda_j G_j \, d^2x$$

$$F[N] := \int N^j F_j \, d^2x$$

$$\delta_{\mathcal{H}} f = \frac{d}{ds} f(s) = \{ f , \mathcal{H} \}(s)$$

Gauss - constraints

$$\delta_\Lambda E^a_j(x) = \{ E^a_j(x), \int \Lambda^k (\partial_b E^b_k + \epsilon_{klm} A^l_b E^{bm})(y)\, d^2y \} = -\Lambda^k \delta^k_j \delta^a_b \epsilon_{klm} E^{bm}$$

$$= -\Lambda^l \epsilon_{jlm} E^{am}(x)$$

$$\delta_\Lambda A^j_a(x) = \{ A^j_a(x), \int -(\partial_b \Lambda^k) E^b_k + \Lambda^k \epsilon_{klm} A^l_b E^{bm}(y)\, d^2y \}$$

$$= (-\partial_a \Lambda^j - \epsilon^j{}_{lm} A^l_a \Lambda^m)(x)$$

Gauss - constraints

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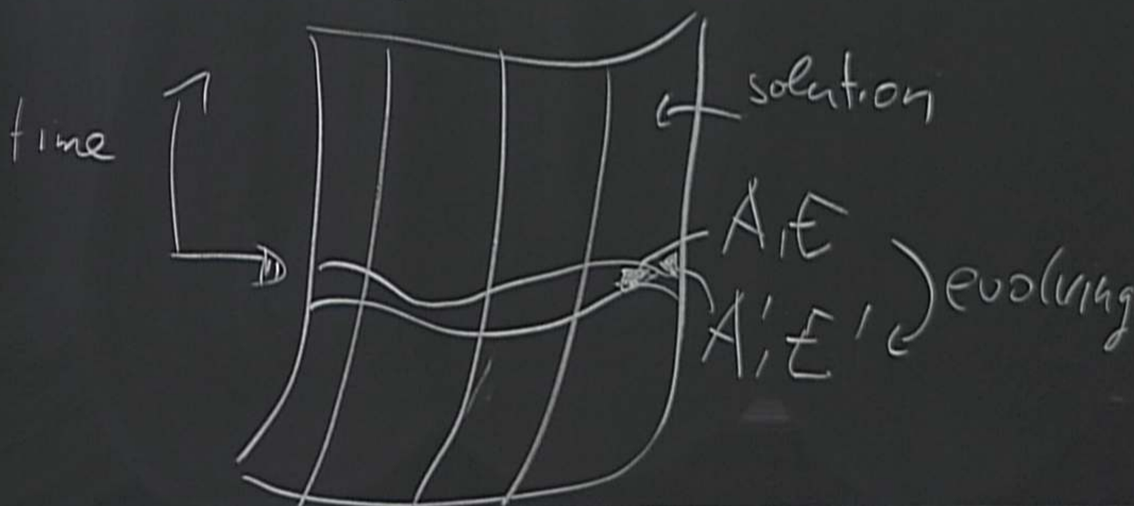
$$= (-\partial_a \Lambda^j - \epsilon^j{}_{lm} A^l_a \Lambda^m)(x).$$

Gauß constraints induce rotation on internal index.

⇒ • time evolutions are generated by constraints.

• time evolution is a gauge transformation.



$\dot{A}$

$\dot{E}$

eom

are generated by

is a gauge

ation

E

E, E) evolving

E

$$\dot{A} = \{A, \mathcal{H}\}$$
$$\dot{E} = \{A, \mathcal{H}\}$$

eom. $T_{a0} = 0$

$F_{a0} = 0$

<u>Constrain</u>

$\dot{G} \simeq 0$

$\{G[\Lambda],$

on internal index.

$, \mathcal{H}\}$

$\Lambda, \mathcal{H}\}$

$T_{a0} = 0$

$F_{a0} = 0$

Constraint Algebra

$\dot{G} \approx 0 = \{G[\Lambda], \mathcal{H}\}$ = combin. of constraints

$\{F[N], \mathcal{H}\} = -$ " $-$

$$\{G[\Lambda], G[\Lambda']\} = \int \epsilon_{jlm} (\Lambda')^j \Lambda^l G^m \, d^2x$$

$\{A, \mathcal{H}\}$

$\{A, \mathcal{H}\}$

$T_{a0} = 0$

$F_{a0} = 0$

Constraint Algebra

$\dot{G} \approx 0 = \{G[\Lambda], \mathcal{H}\}$ = combin. of constraints

$\{F[N], \mathcal{H}\}$ = - " -

$$\{G[\Lambda], G[\Lambda']\} = \int \boxed{\epsilon_{jlm} (\Lambda')^j \Lambda^l} G^m \, d^2x$$

"Algebra of constraints"

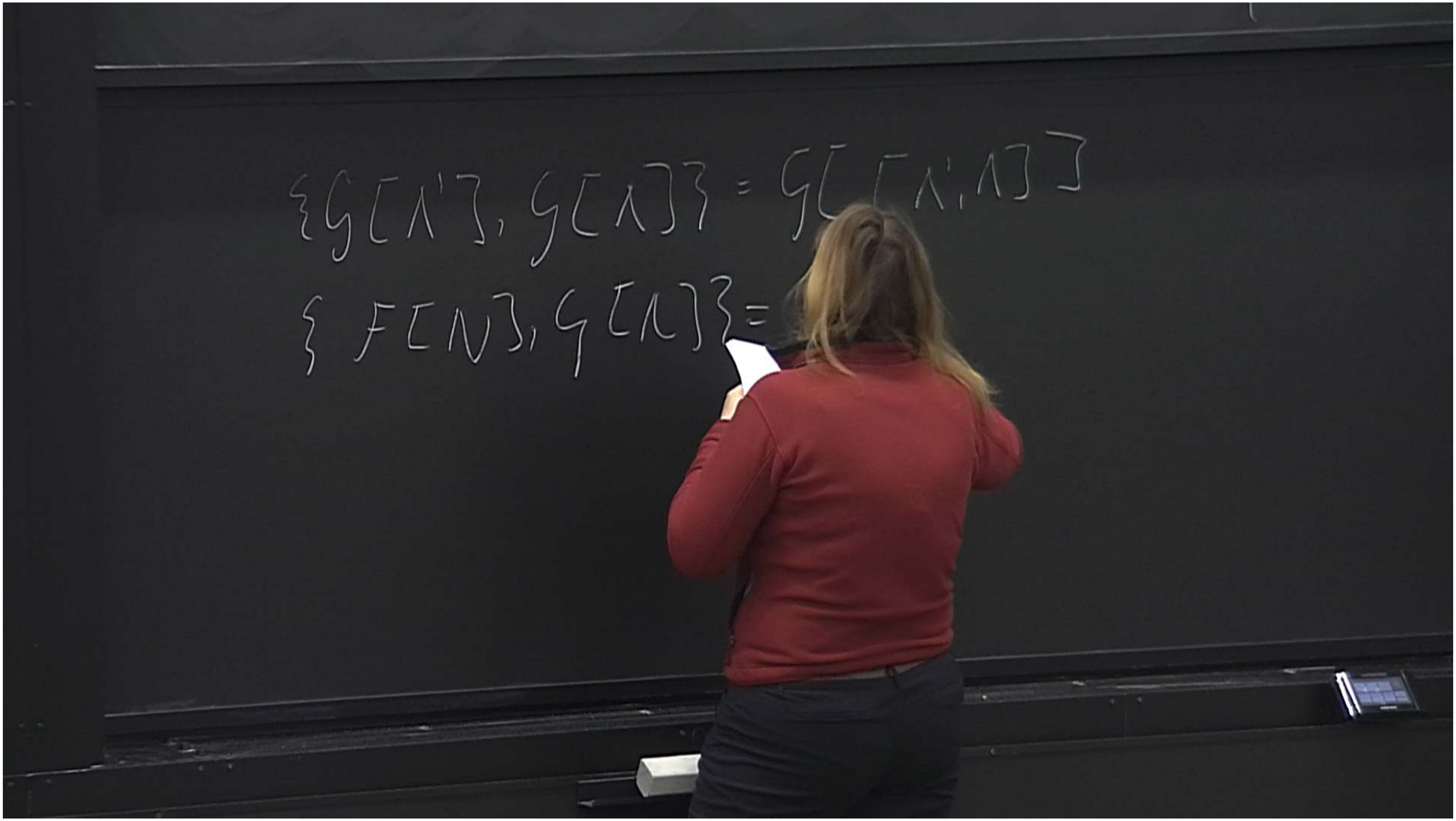
$$= G[[\Lambda', \Lambda]]$$

$$[\Lambda', \Lambda] = [(\Lambda')^j T_j, \Lambda^k T_k] = \epsilon_{jk}{}^{i} (\Lambda')^j (\Lambda)^k T_i$$

$$\{ F[N], g[\Lambda] \} = - \int N^j \epsilon_{jmk} F^m \Lambda^k d^3x$$

$$= F[[N, \Lambda]]$$

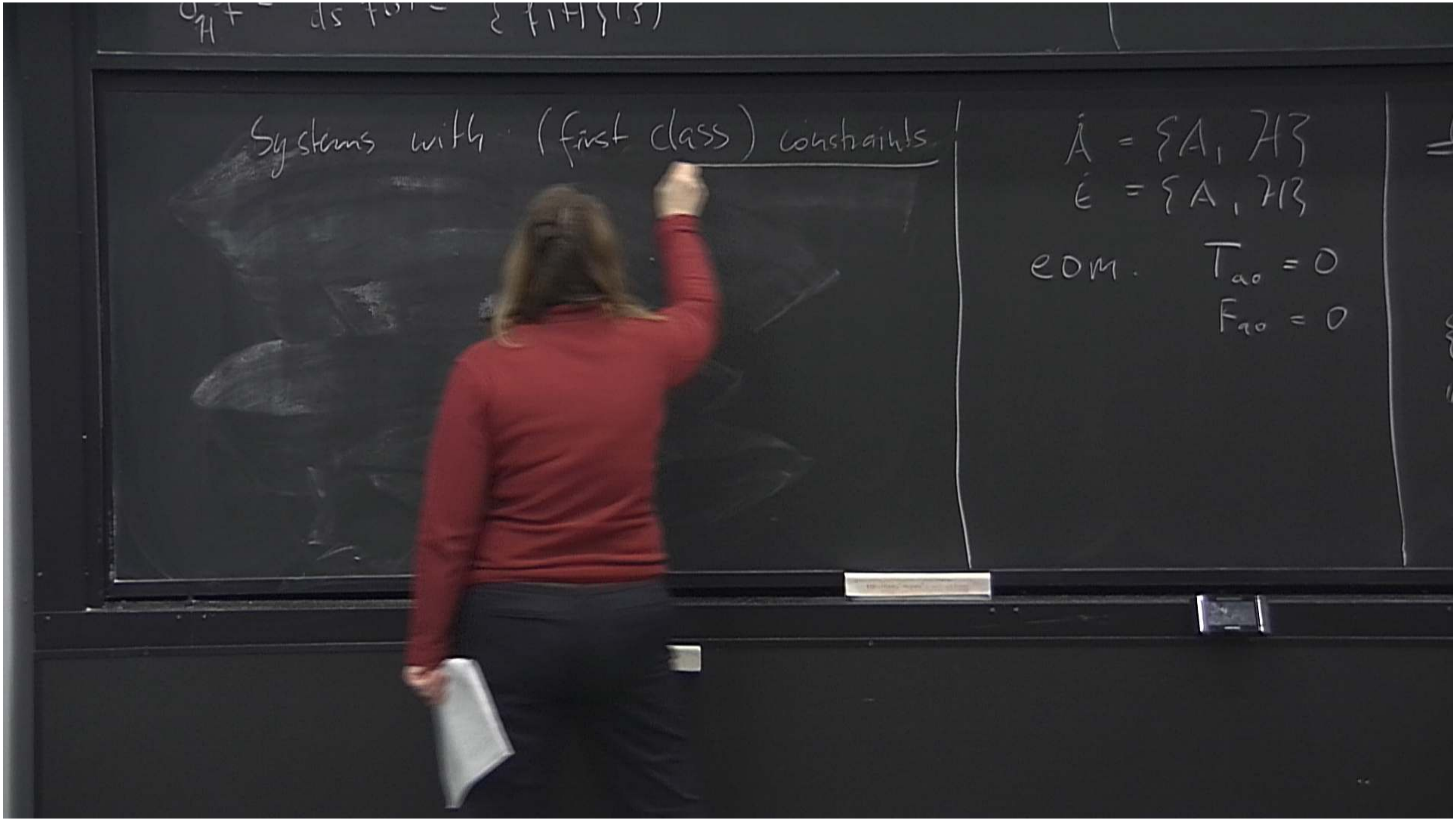
$$\{ F[N'], F[N] \} = 0$$

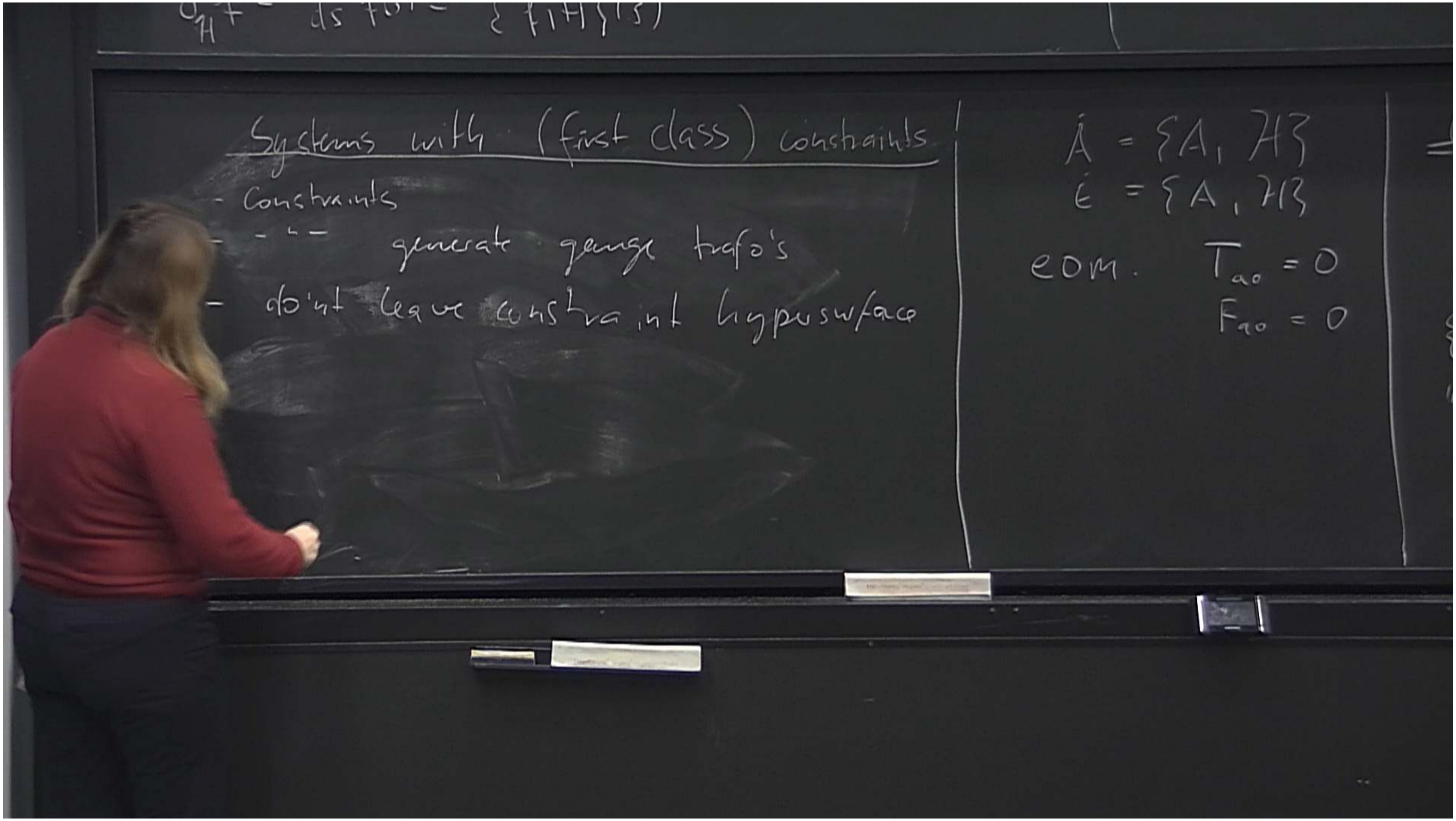

{G[Λ'], G[Λ]} = G[[Λ', Λ]]
{F[N], G[Λ]} =

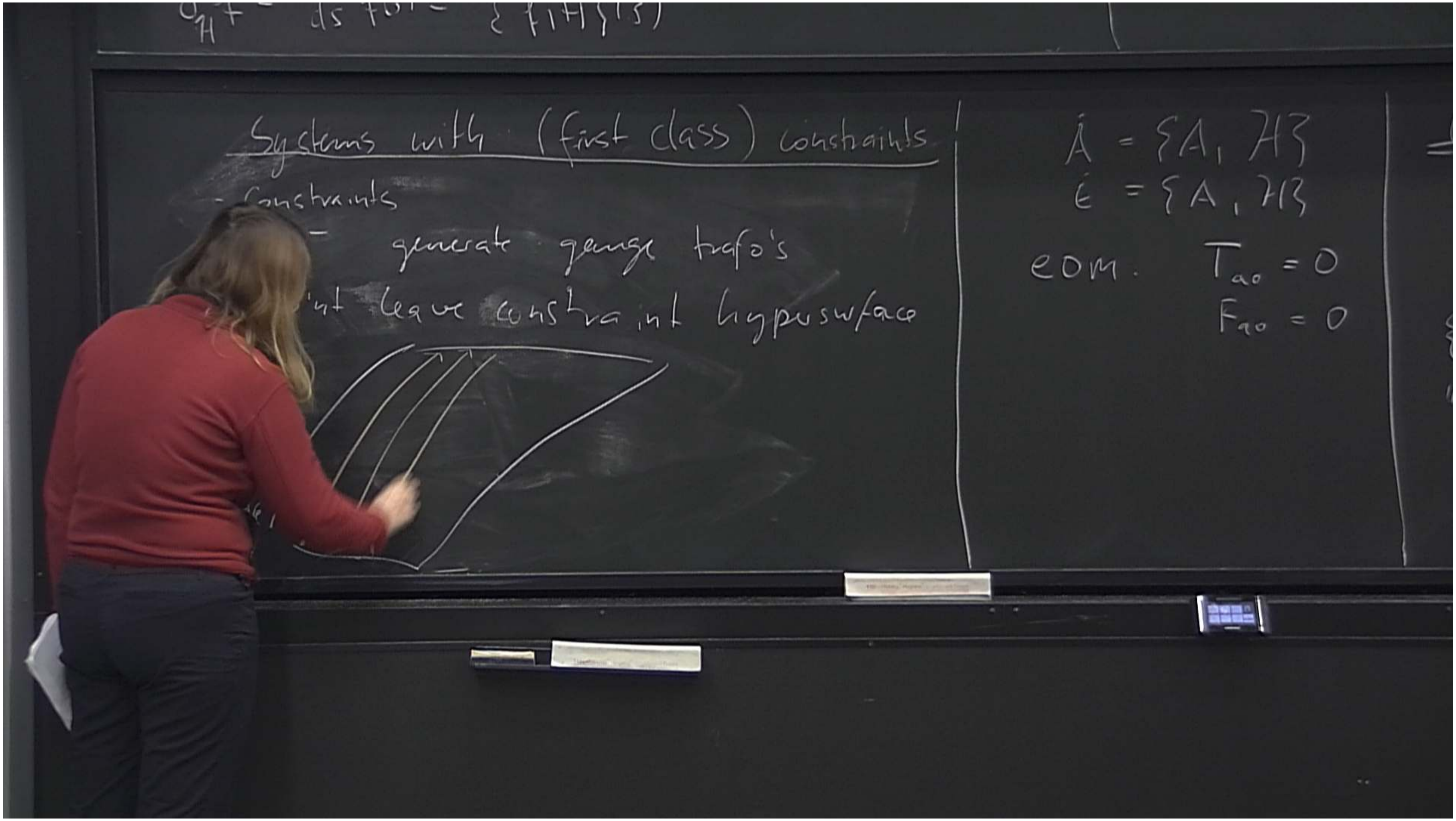
$$\{G[\Lambda'], G[\Lambda]\} = G[[\Lambda', \Lambda]]$$

$$\{F[N], G[\Lambda]\} = F[[N, \Lambda]]$$

$$\{F[N'], F[N]\} = 0$$


Systems with (first class) constraints
Ȧ = {A, H}
Ė = {A, H}
eom. T_a0 = 0
F_a0 = 0


Systems with (first class) constraints
- Constraints
- " generate gauge trafo's
- don't leave constraint hypersurface
Ȧ = {A, H}
Ė = {A, H}
eom. T_ao = 0
F_ao = 0


Systems with (first class) constraints
Constraints
generate gauge trafo's
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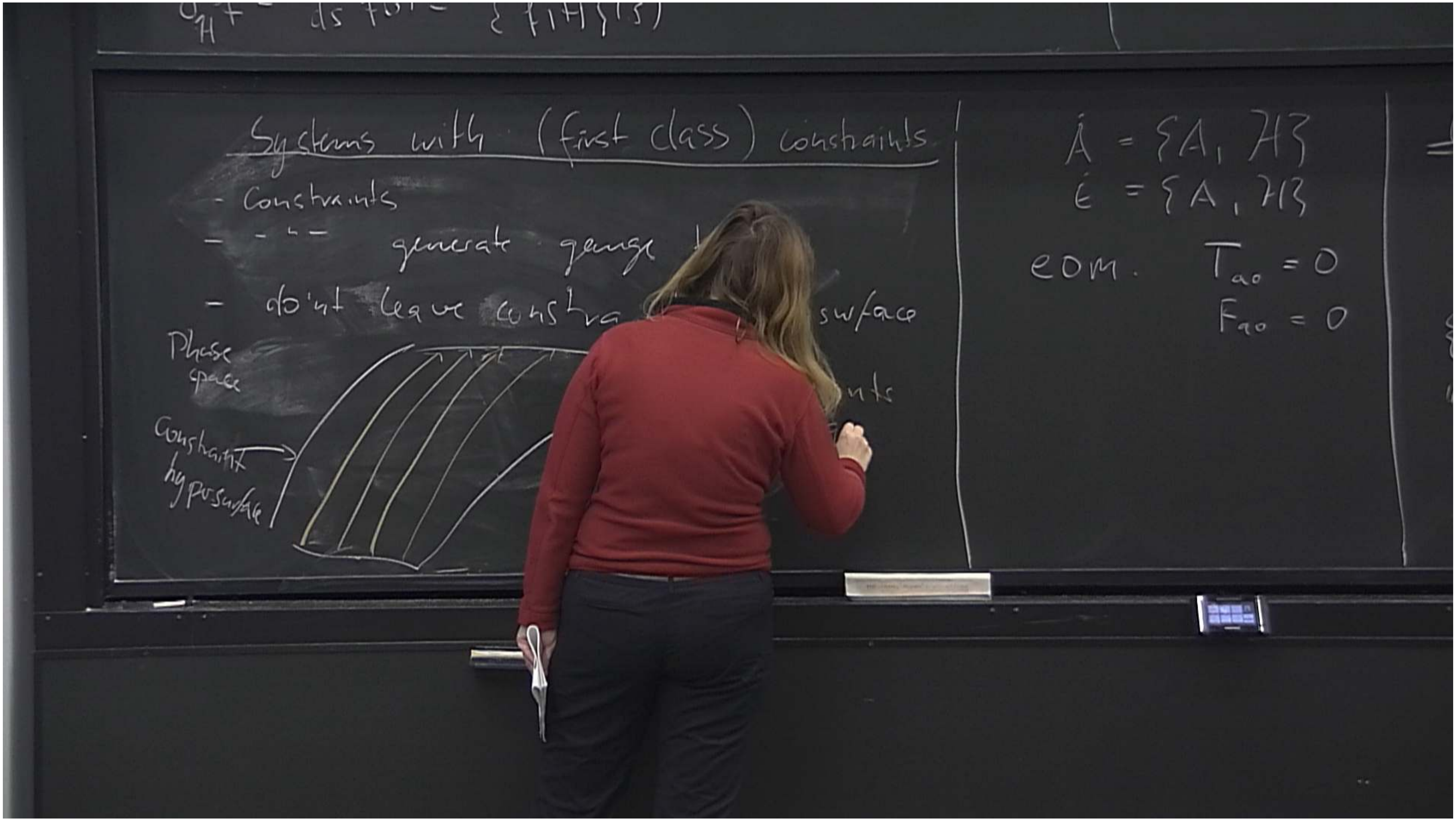
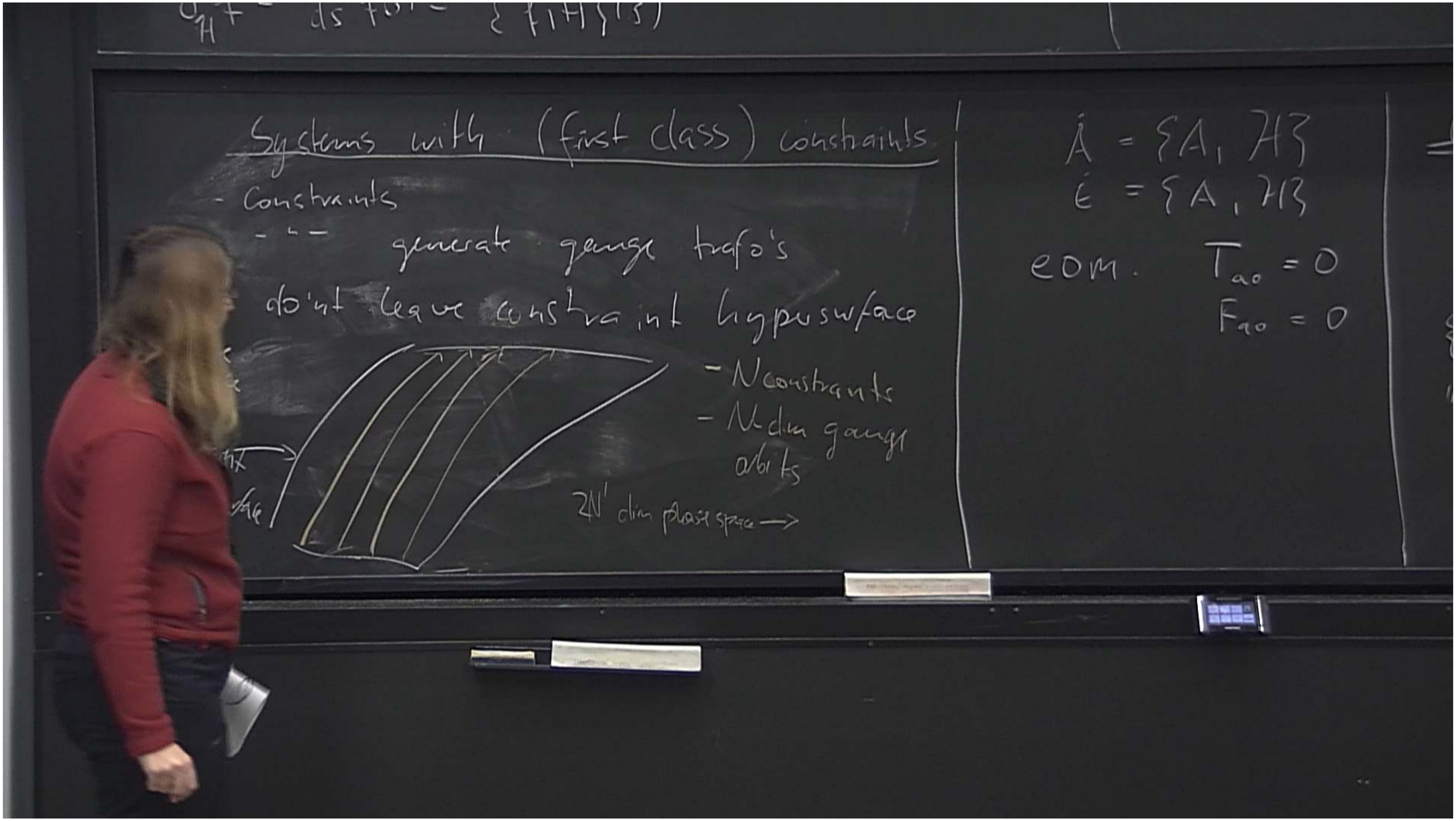
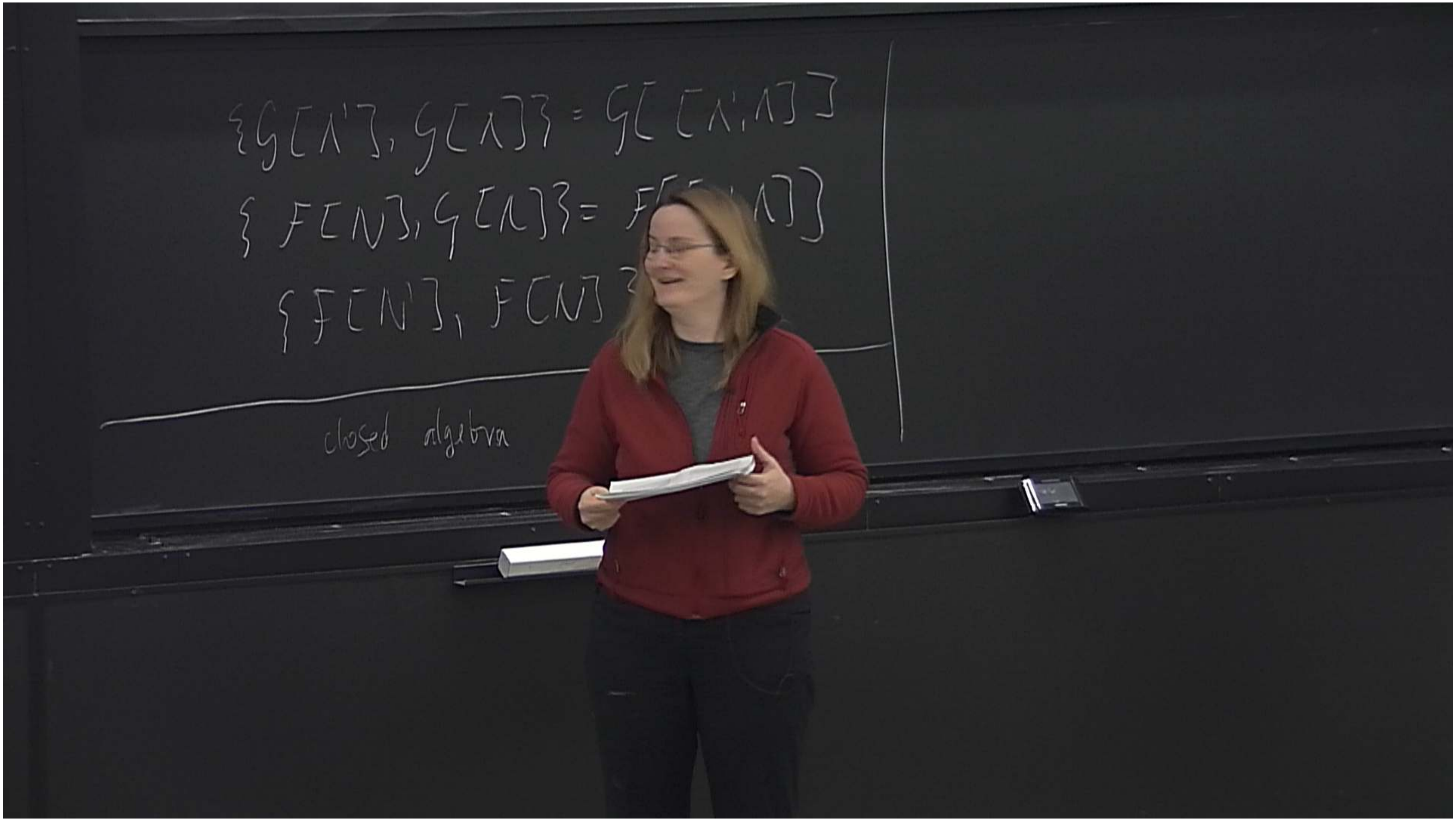
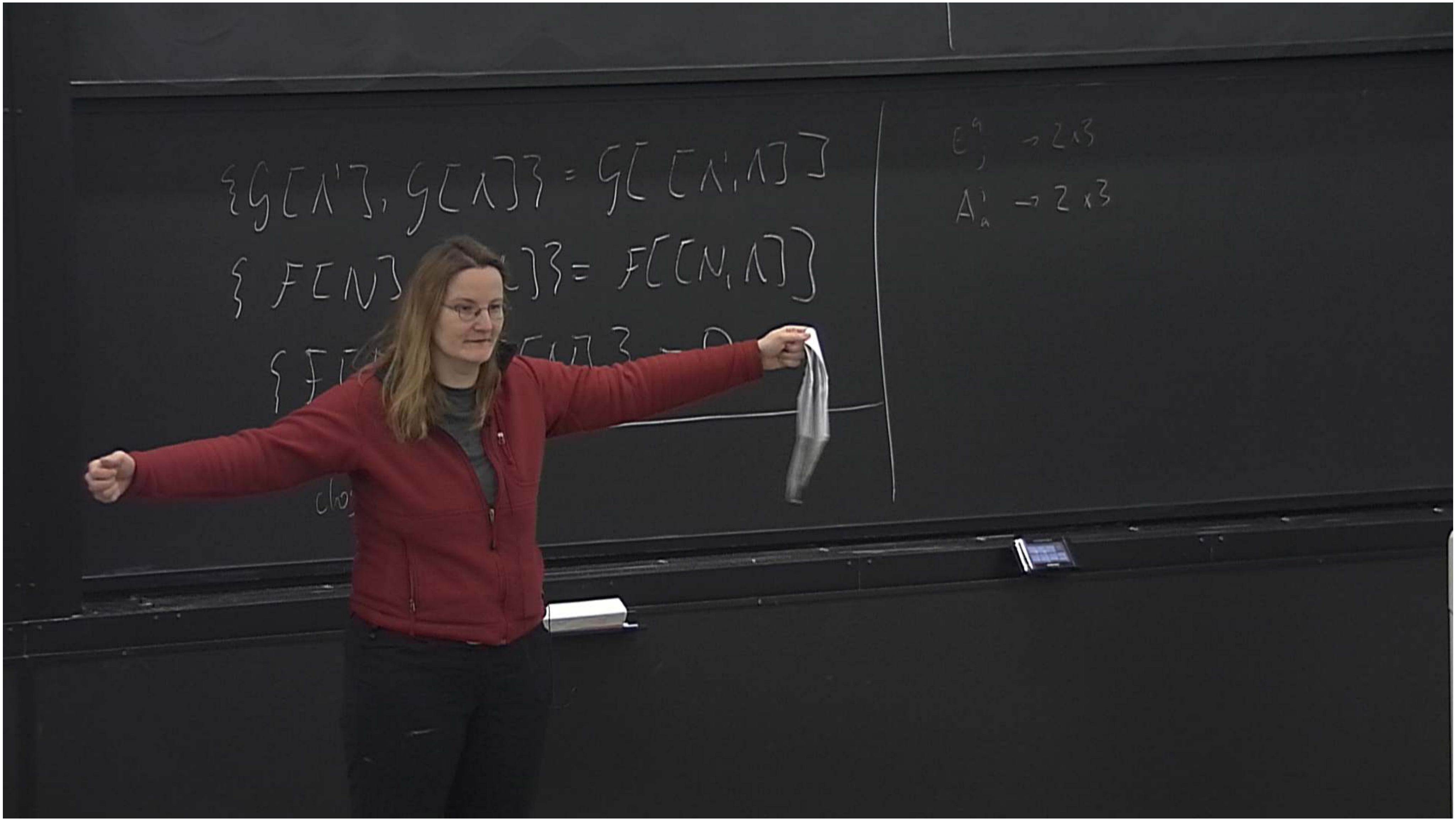
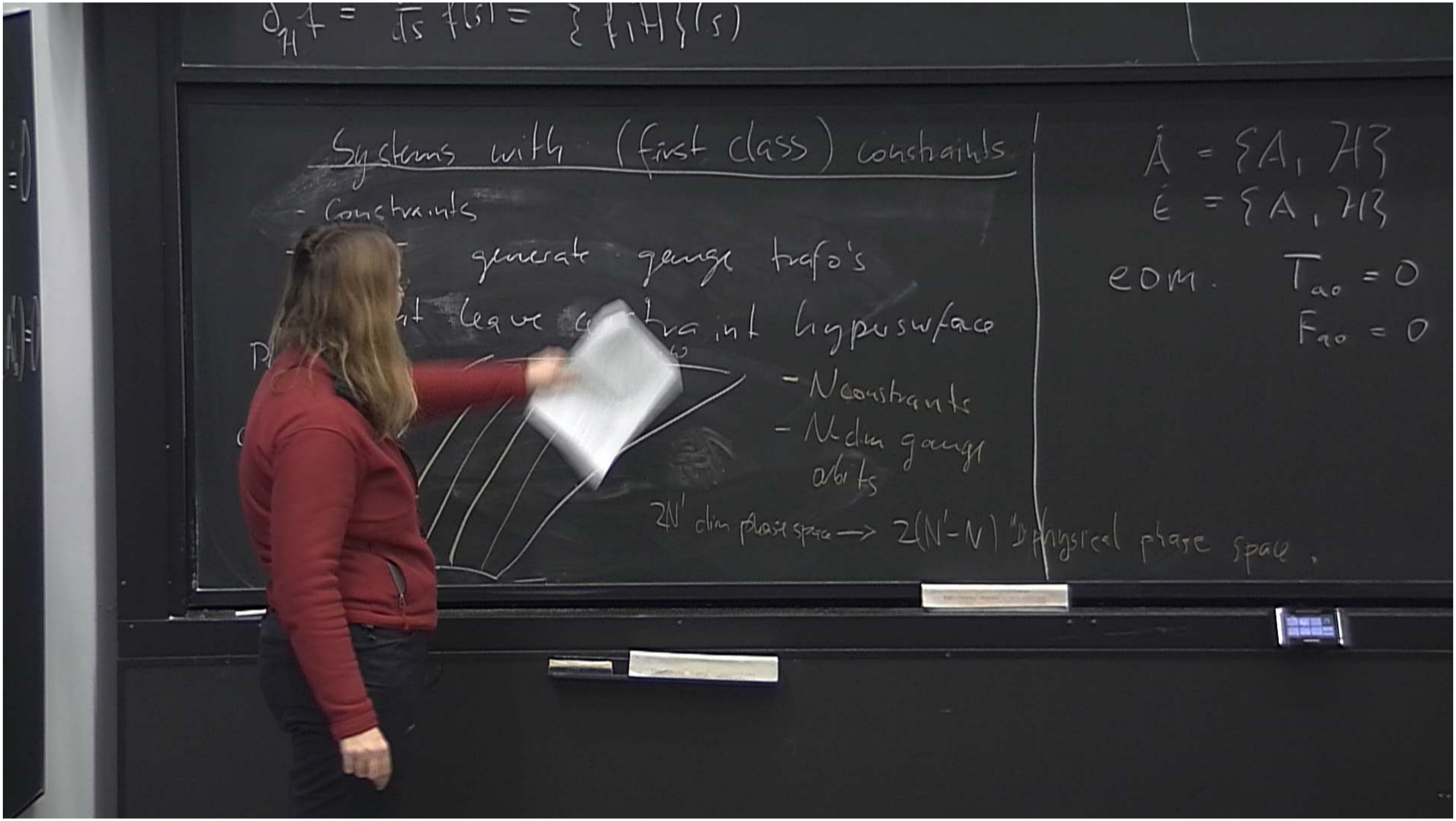

Systems with (first class) constraints
- Constraints
- generate gauge
- don't leave constra surface
Phase space
Constraint hypersurface
Ȧ = {A, H}
Ė = {A, H}
eom. T_ao = 0
F_ao = 0

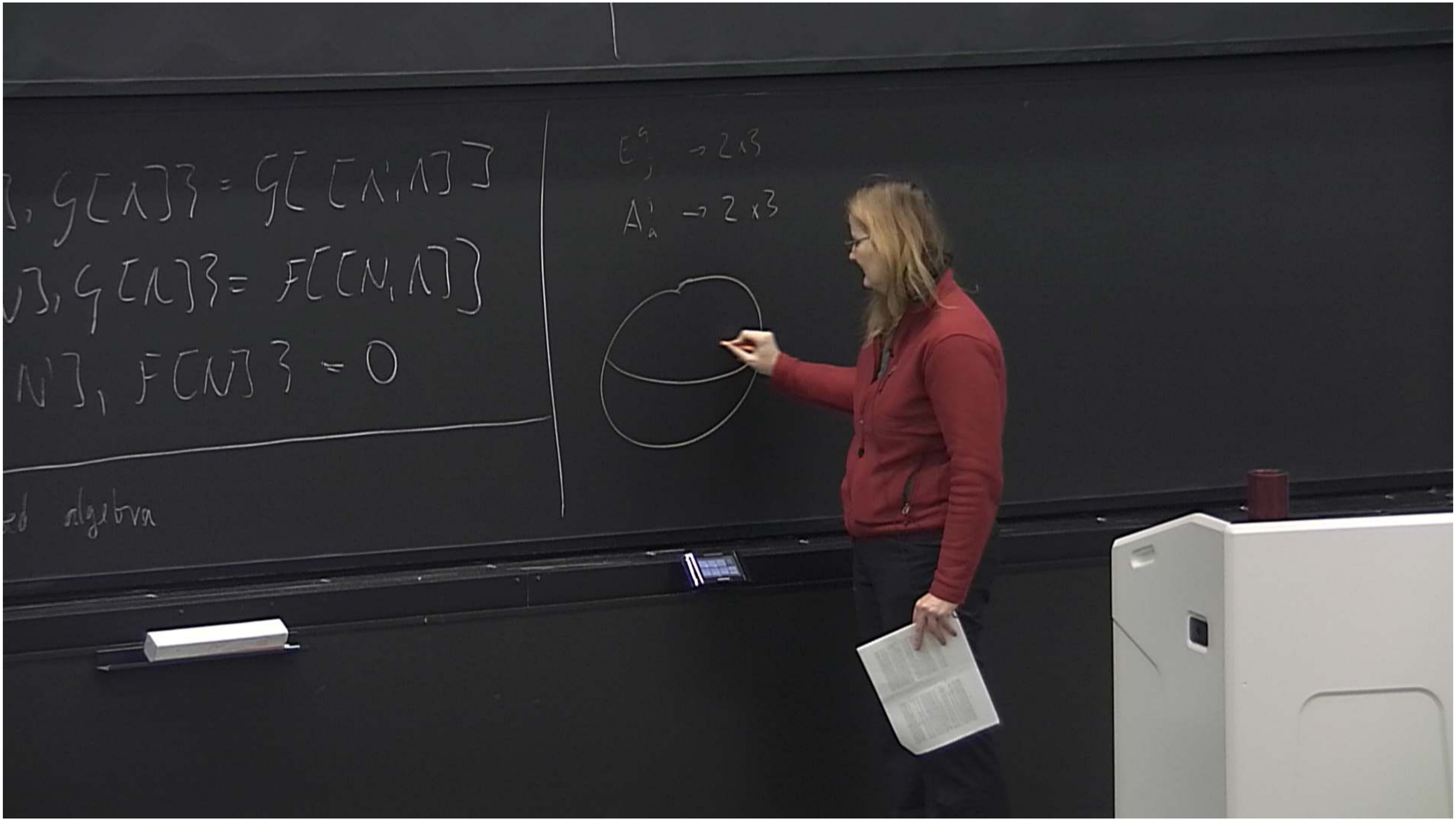

Systems with (first class) constraints
- Constraints
- " - generate gauge trafo's
don't leave constraint hypersurface
- N constraints
- N dim gauge orbits
2N' dim phase space →
Ȧ = {A, H}
Ė = {A, H}
eom. T_ao = 0
F_ao = 0

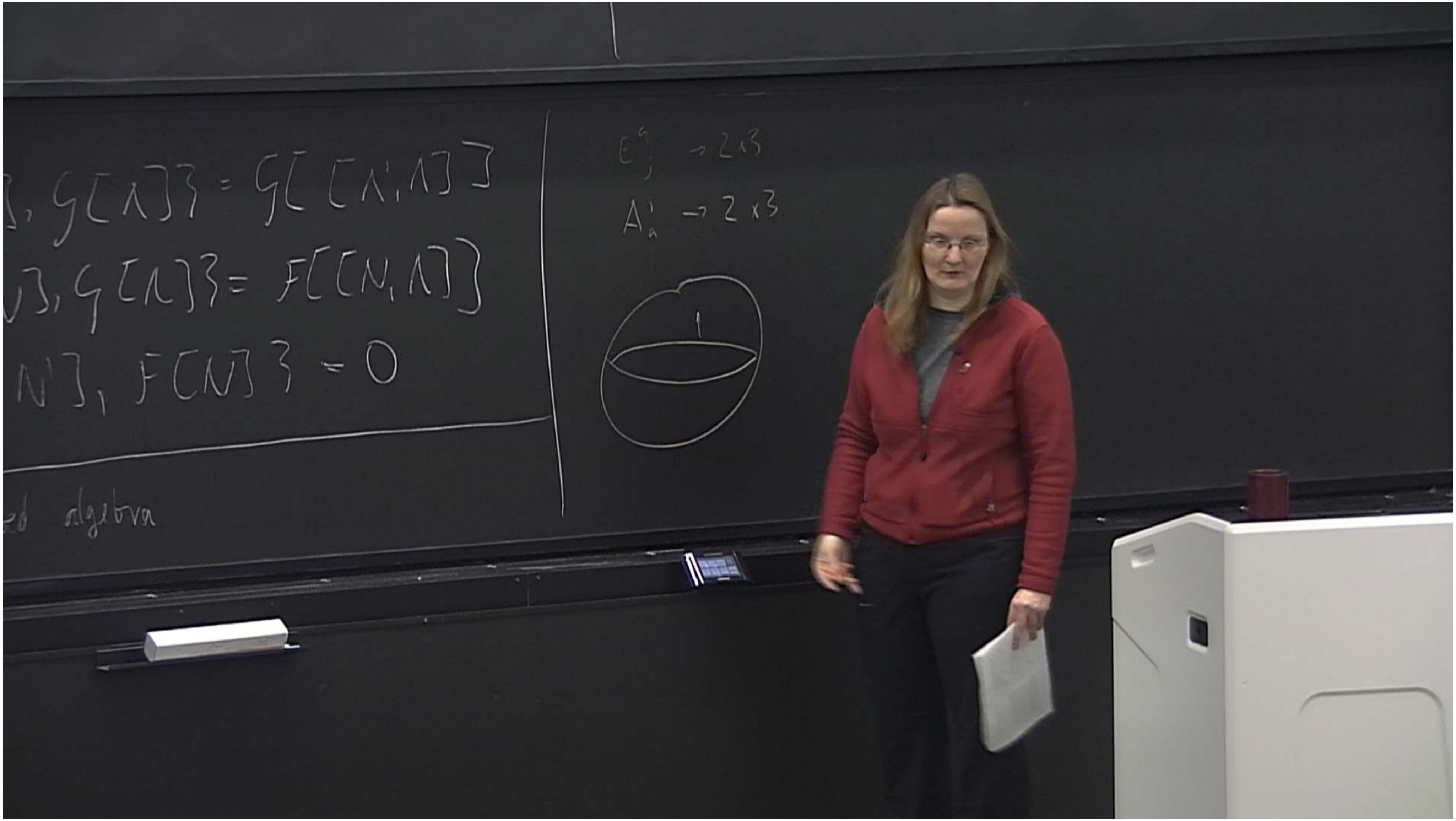

{G[Λ'], G[Λ]} = G[[Λ', Λ]]
{F[N], G[Λ]} =
{F[N'], F[N]}
closed algebra


{G[Λ'], G[Λ]} = G[[Λ', Λ]]
{F[N], [Λ]} = F[[N, Λ]]
$E^a_j \to 2\times3$
$A^j_a \to 2\times3$


$\{G[\Lambda'], G[\Lambda]\} = G[[\Lambda', \Lambda]]$
$\{F[N], G[\Lambda]\} = F[[N, \Lambda]]$
$\{F[N'], F[N]\} = 0$
closed algebra
$E^a_j \to 2 \times 3$
$A^i_a \to 2 \times 3$


$\partial_H f = \frac{d}{ds} f(s) = \{f, H\}(s)$
Systems with (first class) constraints
- Constraints
generate gauge trafo's
leave
hypersurface
- N constraints
- N dim gauge orbits
2N' dim phase space → 2(N'-N) dim physical phase space.
$\dot{A} = \{A, H\}$
$\dot{E} = \{A, H\}$
eom. $T_{a0} = 0$
$F_{a0} = 0$


$E^a_i \to 2 \times 3$
$A^i_a \to 2 \times 3$
algebra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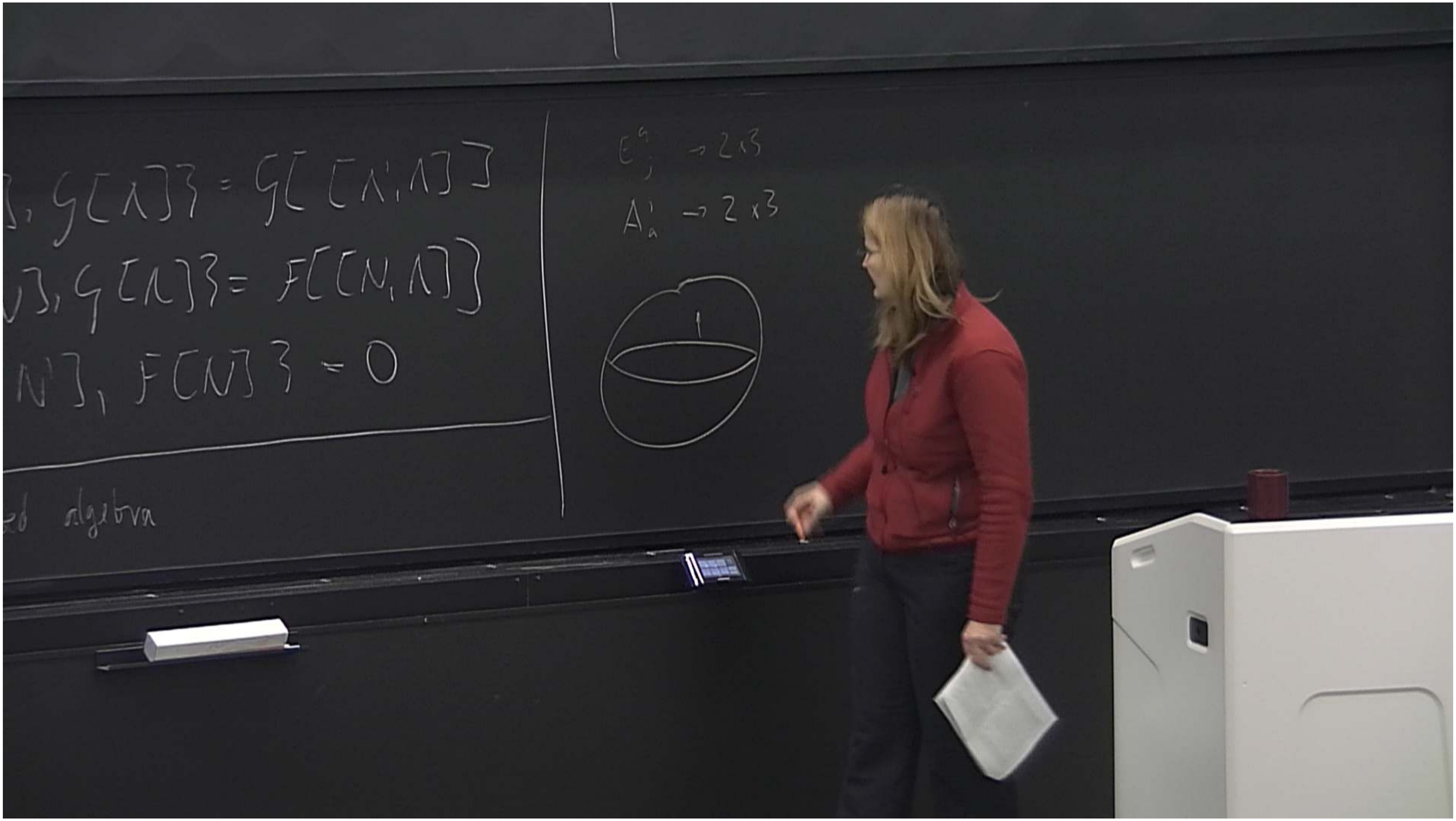
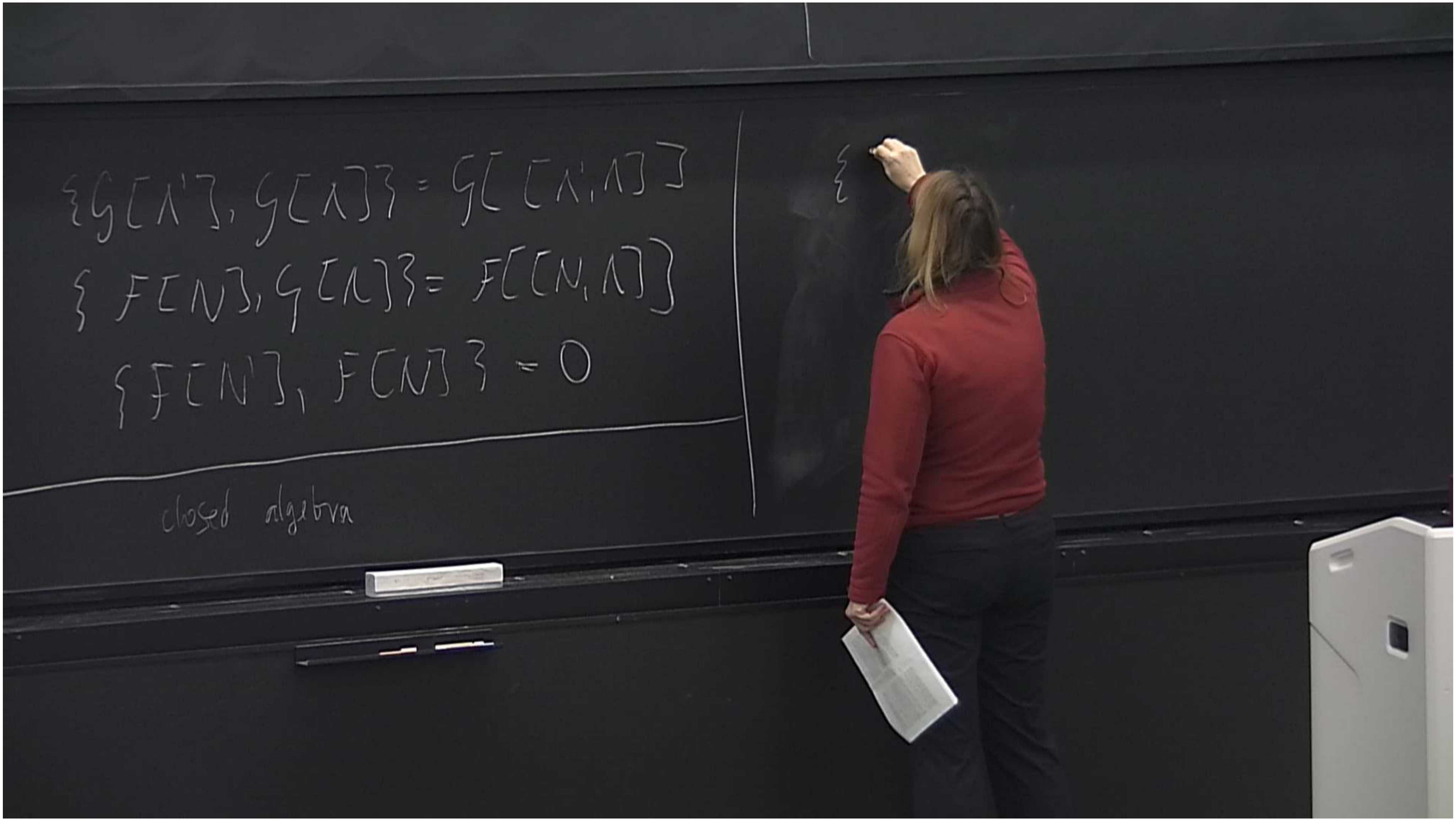

algebra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