

Title: 13/14 PSI - Quantum Gravity Review - Lecture 4

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URL: <http://pirsa.org/14020064>

Abstract:

Canonical Formalism

Basics:

$$\int L(q, \dot{q}) dt = \int$$

Canonical Formalism

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$$S = \int L(q, \dot{q}) dt = \int p \dot{q}$$

$$p = \frac{\partial L}{\partial \dot{q}} = \frac{\delta S}{\delta \dot{q}}$$

Canonical Formalism

Basics: $S = \int L(q, \dot{q}) dt = \int (p\dot{q} - H(q, p)) dt$

$$\frac{\partial L}{\partial \dot{q}} = \frac{\delta S}{\delta \dot{q}}$$

Canonical Formalism

Basics:

$$S = \int L(q, \dot{q}) dt = \int (p \dot{q} - H(q, p)) dt$$

$$p = \frac{\partial L}{\partial \dot{q}} = \frac{\delta S}{\delta \dot{q}} \Rightarrow \text{find } \dot{q}(p, q)$$
$$\frac{\partial L}{\partial \dot{q} \partial \dot{q}} \neq 0$$

Canonical Formalism

Basics:

$$= \int L(q, \dot{q}) dt = \int [P \dot{q} - H(q, P)] dt = \boxed{\int_{t_0}^t [q, P]}$$

$$= \frac{\partial L}{\partial \dot{q}} = \frac{\delta S}{\delta \dot{q}} \Rightarrow \text{find } \dot{q}(P, q)$$

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{q}_i} \right] \neq 0$$

m. a. l. i. s. m.

$$\int_{t_0}^{t_1} (\dot{q} - H(q, p)) dt = \int_{t_0}^{t_1} [q, p]$$

find $\dot{q}(p, q)$

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{q}_i} \right] = 0$$

$$-\frac{\partial L}{\partial p} = \frac{\partial L}{\partial q}$$

$$\pi_q = \frac{\delta S}{\delta \dot{q}} = p$$

$$\pi_p = \frac{\delta S}{\delta \dot{p}} = -q$$

m. Galism

$$dt = \int [P\dot{q} - H(q,p)]dt = \int_{t_0}^{t_1} [q,p]$$

$$\frac{\delta S}{\delta \dot{q}} \Rightarrow \text{find } \dot{q}(p,q)$$

$$dt \left[\frac{\partial L}{\partial \dot{q}_i \partial \dot{q}_j} \right] \neq 0$$

$$q = \frac{\partial L}{\partial \dot{q}} \frac{\partial q}{\partial p} - \frac{\partial L}{\partial p} \frac{\partial q}{\partial \dot{q}}$$

$$\pi_q = \frac{\delta S}{\delta \dot{q}} = p$$

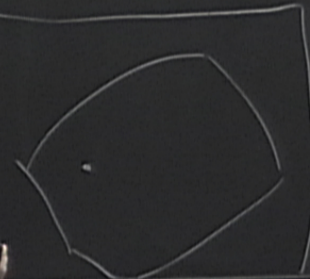
$$\pi_p = \frac{\delta S}{\delta \dot{p}} = -q$$

$$1 = \{q, \pi_q\}$$

$$1 = \{p, \pi_p\}$$

EDM

$$\left. \begin{aligned} \dot{q} &= \{q, t\} \\ p &= \{p, t\} \end{aligned} \right\} \dot{f} = \{f, t\}$$



m. a. l. s. m.

$$dt = \int [P\dot{q} - H(q, p)] dt = \int_{t_0}^{t_1} [q, p]$$

$$\frac{\delta S}{\delta \dot{q}} \Rightarrow \text{find } \dot{q}(p, q)$$

$$dt \left[\frac{\partial L}{\partial \dot{q}_i \partial \dot{q}_j} \right] \neq 0$$

$$q = \frac{\partial L}{\partial \dot{q}} \frac{\partial q}{\partial p} - \frac{\partial L}{\partial p} \frac{\partial q}{\partial \dot{q}}$$

$$\pi_q = \frac{\delta S}{\delta \dot{q}} = p$$

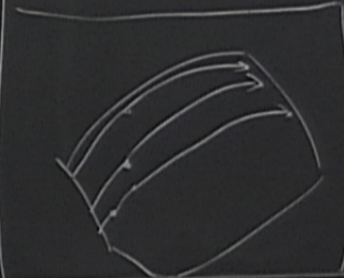
$$\pi_p = \frac{\delta S}{\delta \dot{p}} = -q$$

$$\{q, \pi_q\} = \{q, p\}$$

$$\{p, \pi_p\} = \{p, -q\}$$

EDM.

$$\begin{aligned} \dot{q} &= \{q, H\} \\ \dot{p} &= \{p, H\} \end{aligned} \quad \vec{f} = \{f, H\}$$



m. Galism

$$dt = \int [P\dot{q} - H(q,p)]dt = \int_{t_0}^{t_1} [q,p]$$

$$\frac{\delta S}{\delta \dot{q}} \Rightarrow \text{find } \dot{q}(p,q)$$

$$dt \left[\frac{\partial L}{\partial \dot{q}_i \partial \dot{q}_j} \right] \neq 0$$

$$g = \frac{\partial L}{\partial \dot{q}} \frac{\partial \dot{q}}{\partial p} - \frac{\partial L}{\partial p} \frac{\partial \dot{q}}{\partial \dot{q}}$$

$$\pi_q = \frac{\delta S}{\delta \dot{q}} = p$$

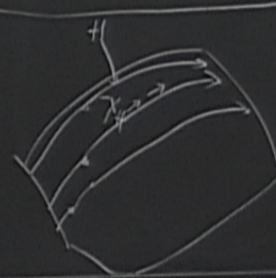
$$\pi_p = \frac{\delta S}{\delta \dot{p}} = -\dot{q}$$

$$q, \pi_p = \{q, p\}$$

$$p, \pi_q = \{p, -\dot{q}\}$$

EDM:

$$\begin{aligned} \dot{q} &= \{q, t\} \\ \dot{p} &= \{p, t\} \end{aligned} \quad \left. \begin{aligned} \dot{q} &= \{q, t\} \\ \dot{p} &= \{p, t\} \end{aligned} \right\} \quad \begin{aligned} \dot{q} &= \{q, t\} \\ \dot{p} &= \{p, t\} \end{aligned}$$



for any function $G \rightarrow \chi_G$



m. Galism

$$dt = \int [P\dot{q} - H(q, p)] dt = \int_{T_0}^T [q, p]$$

$$\Rightarrow \text{find } \dot{q}(p, q)$$

$$dt \left[\frac{\partial L}{\partial \dot{q}_i \partial \dot{q}_i} \right] \neq 0$$

$$\frac{\partial q}{\partial p} - \frac{\partial p}{\partial q}$$

$$\pi_q = \frac{\delta S}{\delta \dot{q}} = p$$

$$\pi_p = \frac{\delta S}{\delta \dot{p}} = -q$$

$$1 = \{q, \pi_p\} = \{q, p\}$$

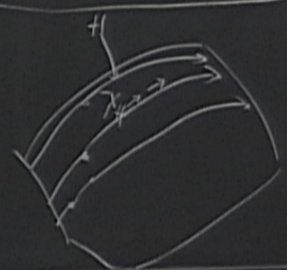
$$1 = \{p, \pi_q\} = \{p, -q\}$$

$$\begin{aligned} \vec{f} &= \{f, G\} \\ \vec{q} &= \{q, G\} \\ &= \{f, G\}_1 = \{f, G\}_2 \end{aligned}$$

$$f(A) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \{f, H\}_r$$

EDM

$$\begin{aligned} \dot{q} &= \{q, H\} \\ \dot{p} &= \{p, H\} \end{aligned} \quad \left. \begin{aligned} \vec{f} &= \{f, H\} \\ \chi_f(t) &= \{f, H\} \end{aligned} \right\}$$



for any function
 $G \rightarrow \chi_G$



$$\{q, p\} = 1, \quad \{f, g\} = \frac{\partial f}{\partial q} \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial g}{\partial q}$$

$$f = \{f, \omega\}, \quad f(A) = \sum_{i=0}^{\infty} \frac{f^i}{i!} \\ \tilde{f} = \{f, \omega\} = \{f, \omega\}_2 = \{f, \omega\}_2$$

Canonical Analysis of 3D FO Action

$$S = \frac{1}{2} \int e_{\sigma\tau} F_{\mu\nu}^2 \tilde{E}^{\sigma\mu\nu} d^3x \\ = \int d^3x$$

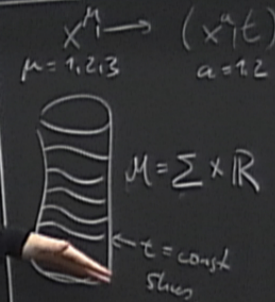
$$\{q, p\} = 1, \quad \{f, g\} = \frac{\partial f}{\partial q} \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial g}{\partial q}$$

$$f = \{f, g\}, \quad f(A) = \sum_{i=1}^n \frac{f_i}{A_i}$$

Canonical Analysis of 3D FO Action.

$$S = \frac{1}{2} \int e_{\sigma\tau} F_{\mu\nu}^2 \tilde{e}^{\sigma\mu\nu} d^3x$$

$$= \int$$



$$\{q, p\} = 1, \quad \{f, g\} = \frac{\partial f}{\partial q} \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial g}{\partial q}$$

$$f = \{f, g\}, \quad f(A) = \sum_{i=0}^{\infty} \frac{f^i}{i!} \\ \vec{q} = \{q, g\} \\ = \{f, g\}, \omega = \{f, g\}_2$$

Canonical Analysis of 3D FO Action.

$$S = \frac{1}{2} \int e_{\sigma\mu} F_{\mu\nu}^2 \tilde{e}^{\sigma\mu\nu} d^3x$$

$$= \int d^2x dt$$

$$x^\mu \rightarrow (x^a, t) \\ \begin{matrix} 3d & 2d \\ \downarrow & \downarrow \\ \mathcal{M} = \sum x \cdot \mathbb{R} \end{matrix}$$

const

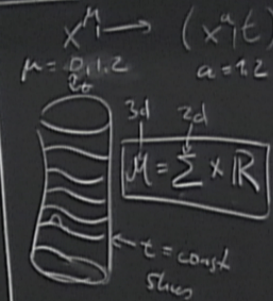
$$\{q, p\} = 1, \quad \{f, g\} = \frac{\partial f}{\partial q} \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial g}{\partial q} \quad \left[\frac{\partial q}{\partial t}, \frac{\partial q}{\partial t} \right] = 0$$

$$\begin{aligned} \vec{f} &= \{f, \vec{g}\}, & f(A) &= \sum_{i=0}^{\infty} \frac{f^{(i)}(A)}{i!} \\ \vec{f} &= \{f, \vec{g}\} \\ &= \{f, \vec{g}\}_1 = \{f, \vec{g}\}_2 \end{aligned}$$

Canonical Analysis of 3D FO Action.

$$S = \frac{1}{2} \int e_{\sigma\tau} F_{\mu\nu}^2 \tilde{e}^{\sigma\mu\nu} d^3x$$

$$d^2x dt$$



$$\tilde{e}^{ab} = \tilde{e}^{0ab}$$

$$F_{\mu\nu}^{\ell} = \partial_{\mu} \omega_{\nu}^{\ell} - \partial_{\nu} \omega_{\mu}^{\ell} + \epsilon^{\ell j k} \omega_{\mu j} \omega_{\nu k}$$

$$F_{\mu\nu}^l = \partial_\mu \overset{A}{\omega_\nu^l} - \partial_\nu \omega_\mu^l + \epsilon^{ljk} \omega_\mu \omega_\nu$$

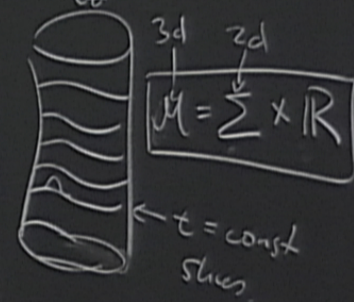
$$F_{0a} = \partial_0 A_a - \partial_a A_0 + \epsilon^{1j2} \omega_0 \omega_a$$

Canonical Analysis of 3D FO Action.

$$S = \frac{1}{2} \int e_{\sigma\alpha} F_{\mu\nu}^{(\alpha)} \tilde{\epsilon}^{\sigma\mu\nu} d^3x$$

$$= \int e_b^i \partial_a A_{ij} \tilde{\epsilon}^{ab} + e_{aj} \frac{1}{2} F_{ab}^j \tilde{\epsilon}^{ab} \\ + A_0^j \left(\partial_a e_{bj} + \epsilon_{jlm} A_a^l e_b^m \right) \tilde{\epsilon}^{ab} \\ d^3x dt$$

$$x^\mu \rightarrow (x^a t) \\ \mu = 0, 1, 2 \quad a = 1, 2$$



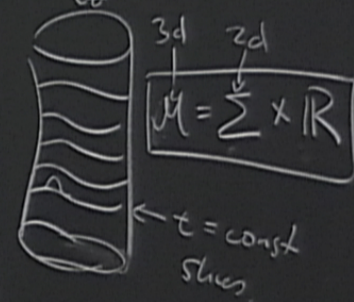
$$\tilde{\epsilon}^{ab} = \tilde{\epsilon}^{0ab}$$

Canonical Analysis of 3D FO Action.

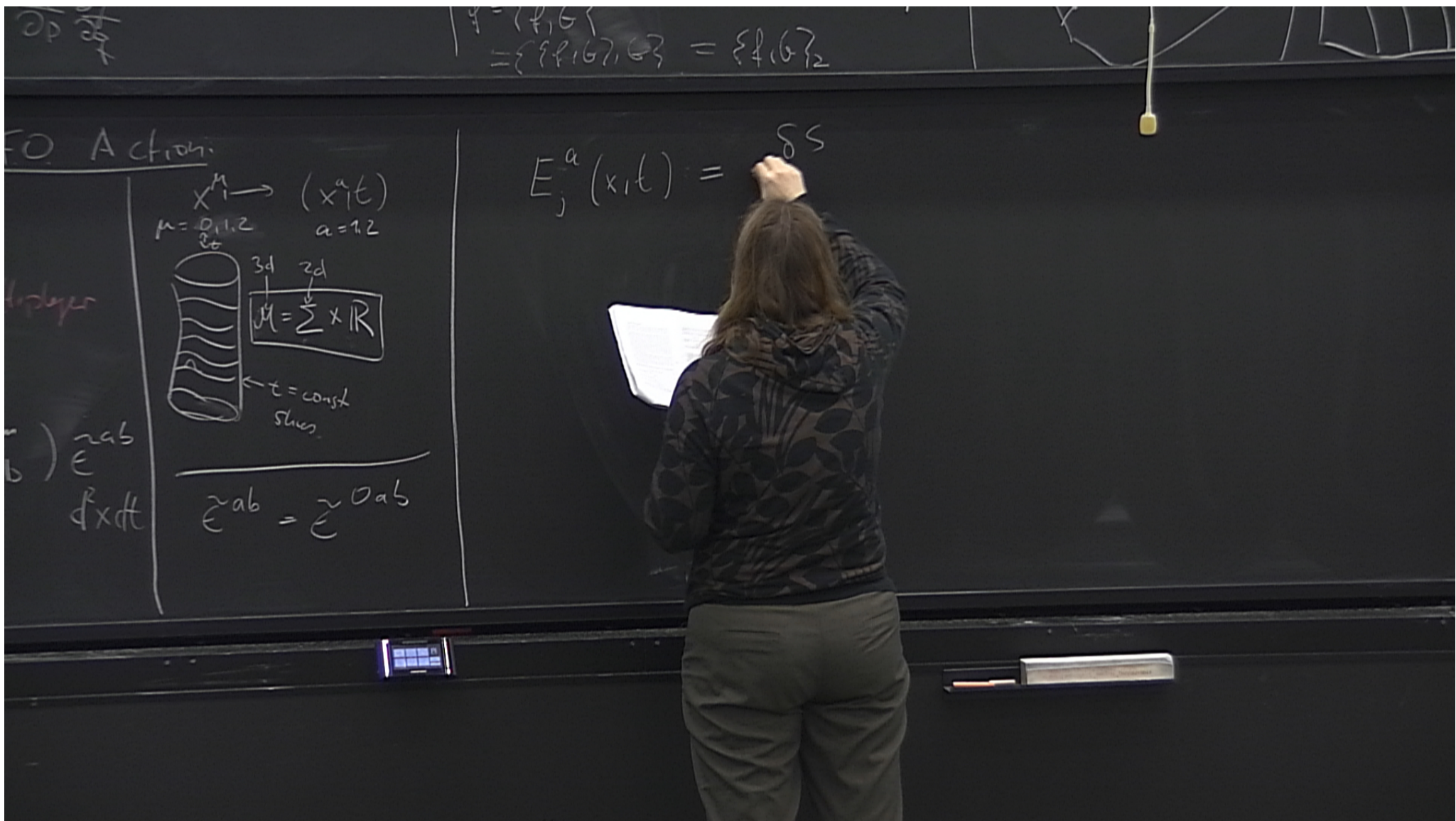
$$S = \frac{1}{2} \int e_{\sigma\alpha} F_{\mu\nu}^{(\alpha)} \tilde{\epsilon}^{\sigma\mu\nu} d^3x$$

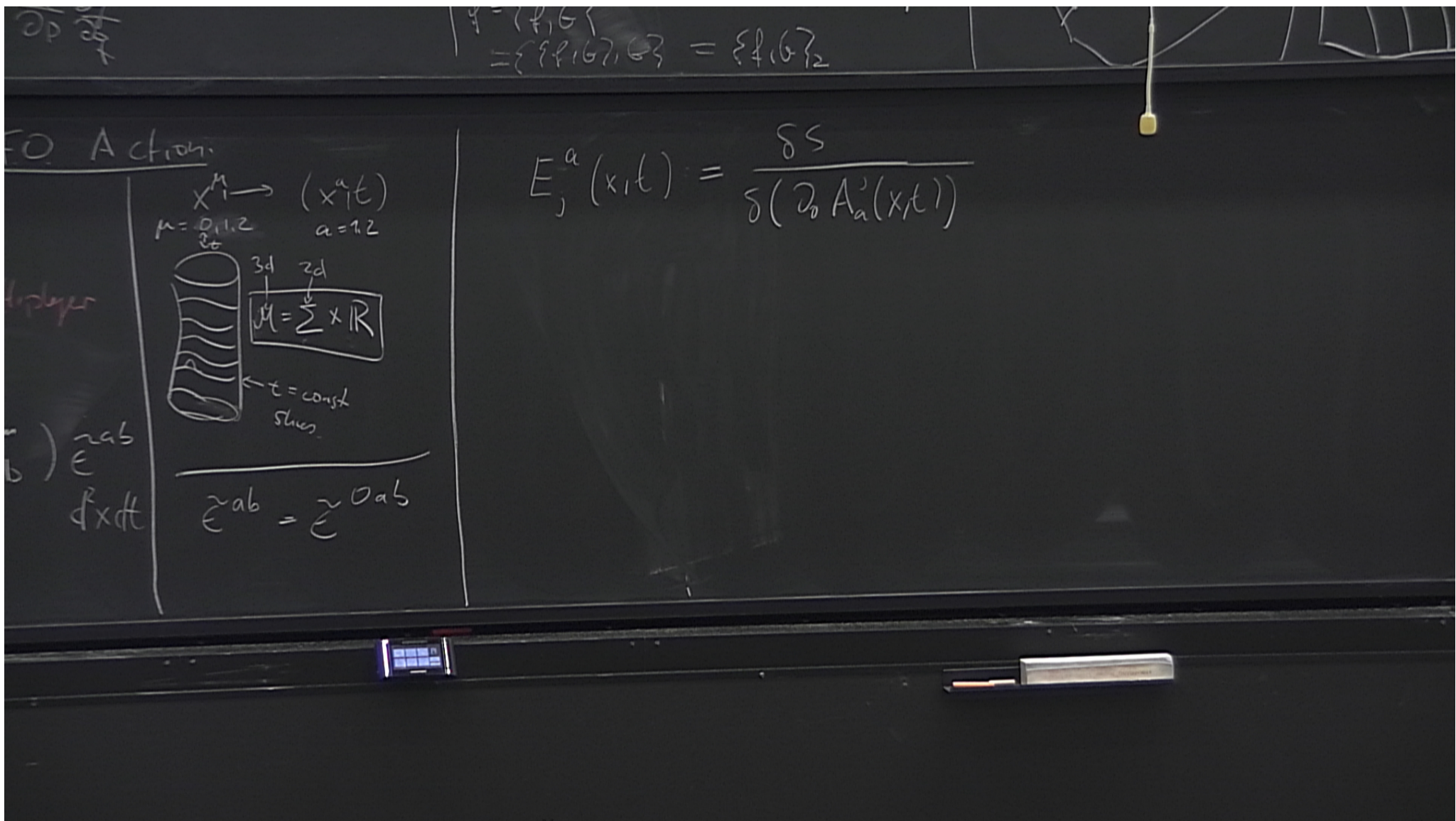
$$S = \int \left(e_b^j \partial_a A_j \right) \tilde{\epsilon}^{ab} + e_a^j \frac{1}{2} F_{ab}^j \tilde{\epsilon}^{ab} + A_0' \left(\partial_a e_b^j + \epsilon_{jlm} A_a^l e_b^m \right) \tilde{\epsilon}^{ab} d^3x dt$$

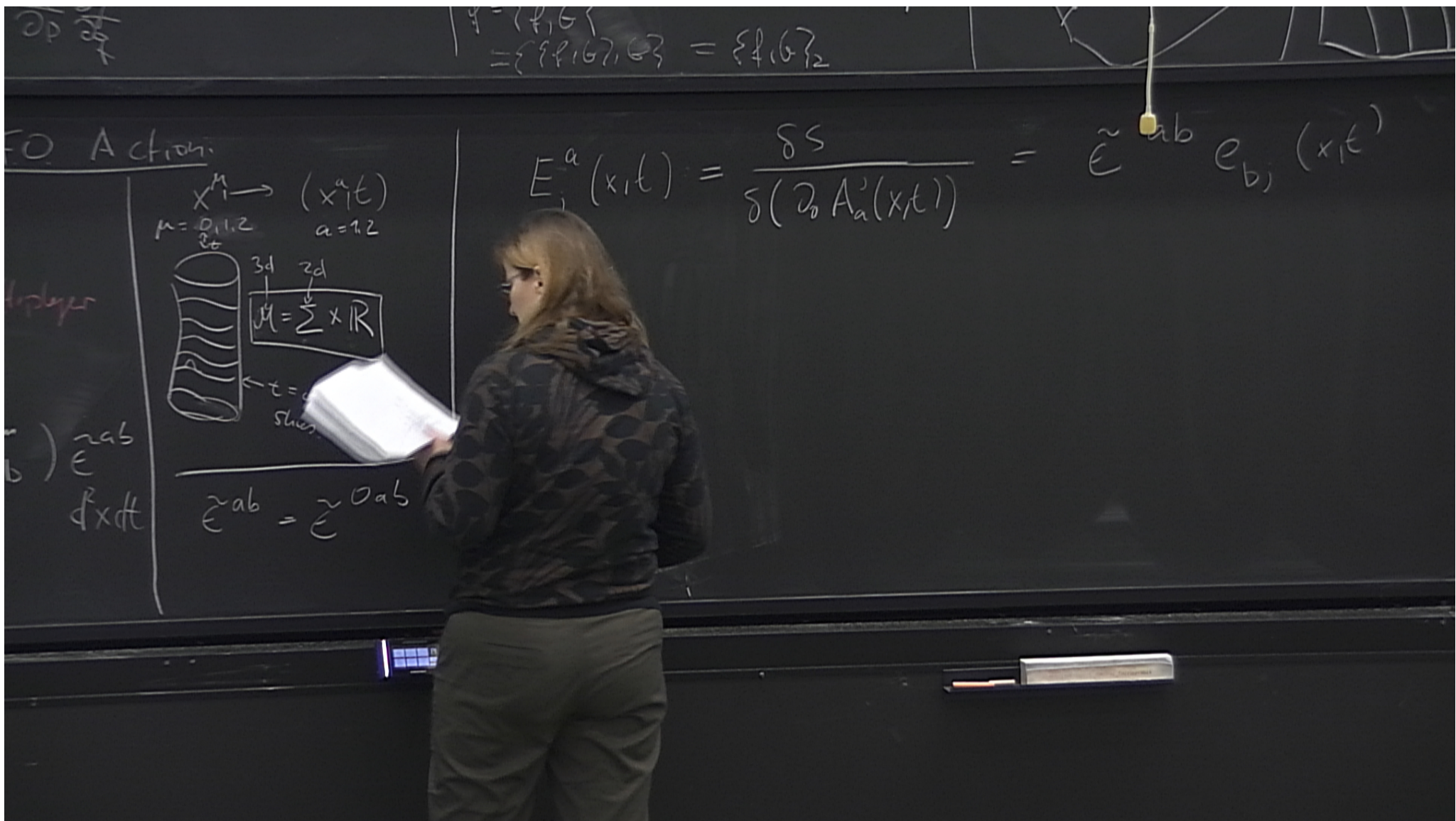
$$x^\mu \rightarrow (x^a, t) \\ \mu = 0, 1, 2 \quad a = 1, 2$$



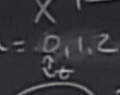
$$\tilde{\epsilon}^{ab} = \tilde{\epsilon}^{0ab}$$







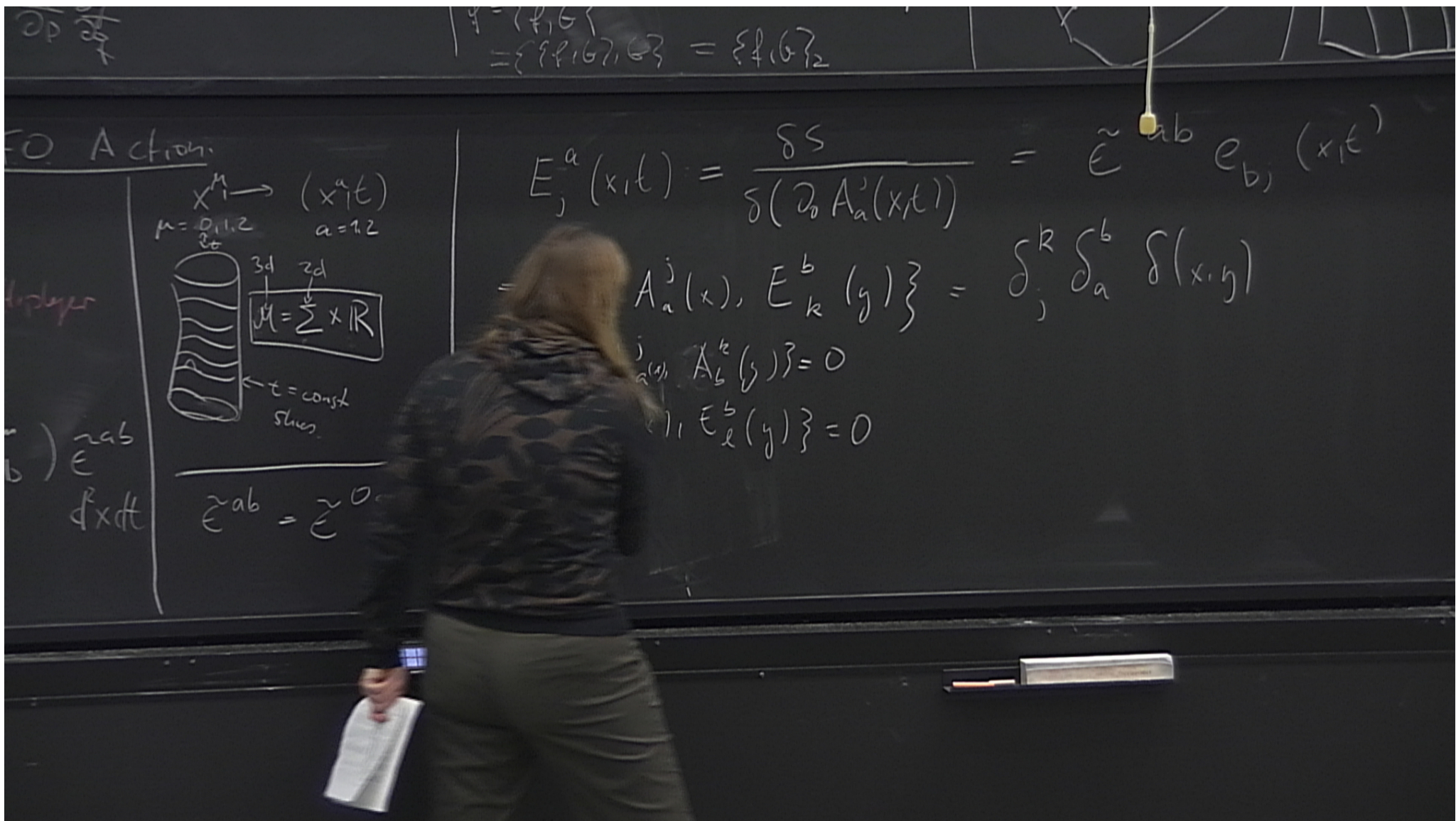
FO Action:

$x_1^m \rightarrow (x_1^a | t)$
 $\mu = 0.1, 2$ $a = 1, 2$


$$\tilde{\epsilon}^{ab} = \tilde{\epsilon}^{0ab}$$

$$E_j^a(x, t) = \frac{\delta S}{\delta(\partial_0 A_a^j(x, t))} = \tilde{E}^{ab} e_{bj}(x, t)$$

$$\Rightarrow \{A_a^j(x), E_k^b(y)\} = \delta^k_j$$



$$g_j = \partial_a E_j^a + \epsilon_{jlm} A_a^l E^{am}$$



$$g_{ij} = \partial_a E^a_j + \epsilon_{jlm} A^l_a E^{am} = \frac{1}{2} T_{ab,j} \tilde{E}^{ab} \stackrel{!}{=} 0$$

$$G_j = \partial_a E_j^a + \epsilon_{jlm} A_a^l E^{am} = \frac{1}{2} T_{abj} \tilde{E}^{ab} \stackrel{!}{=} 0$$

$$F_j = \tilde{E} \left(\partial_a A_j^a + \frac{1}{2} \epsilon_{jkl} A_a^k A_b^l \right) = \frac{1}{2} F_{ab} \tilde{E}^{ab}$$

$$G_j = \partial_a E_j^a + \epsilon_{jlm} A_a^l E^{am} = \frac{1}{2} T_{abj} \tilde{E}^{ab} \stackrel{!}{=} 0$$

$$G_j = \tilde{E} \left(\partial_a A_b^j + \frac{1}{2} \epsilon_{k\ell} A_a^k A_b^\ell \right) = \frac{1}{2} F_{ab}^j \tilde{E}^{ab} \stackrel{!}{=} 0$$

constraints

$$G_j = \partial_a E_j^a + \epsilon_{jlm} A_a^l E^{am} = \frac{1}{2} T_{abj} \tilde{E}^{ab} \stackrel{!}{=} 0$$

$$F^j = \tilde{E} \left(\partial_a A_b^j + \frac{1}{2} \epsilon_{k\ell}^j A^k A^\ell \right) = \frac{1}{2} F_{ab}^j \tilde{E}^{ab} \stackrel{!}{=} 0$$

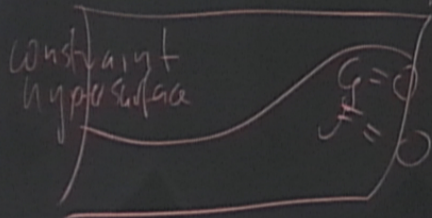
constraints: conditions on y and A

$$G_j = \partial_a E_j^a + \epsilon_{jlm} A_a^l E^{am} = \frac{1}{2} T_{ab} \tilde{E}^{ab} \stackrel{!}{=} 0$$

$$F^j = \tilde{E} \left(\partial_a A_b^j + \frac{1}{2} \epsilon^j_{kl} A_a^k A_b^l \right) = \frac{1}{2} F_{ab}^j \tilde{E}^{ab} \stackrel{!}{=} 0$$

constraints: conditions on your initial data

phase space



$$\tilde{E}^{ab} = 0$$

$$F_{ab} \tilde{E}^{ab} = 0$$

$$A_a^{b}(x), E_R^{b}(y)$$

$$\tilde{e}^{ab} = 0$$

$$F_{ab} \tilde{e}^{ab} = 0$$

$$A_a^i(x), E_b^j(y)$$

$$\mathcal{H} = - \int \left(\underbrace{e_o^j}_{N^j} \mathcal{F}_j + \underbrace{A_o^j}_{\Lambda^j} \mathcal{G}_j \right) d^2x$$

→ Hamiltonian = combination of constraints

$$\dot{\bar{e}}^{ab} = 0$$

$$F_{ab} \bar{e}^{ab} = 0$$

$$A_a^i(x), E_b^j(y)$$

$$H[e_o^i, A_o^i] = - \int \left(\underbrace{e_o^i}_{\substack{\downarrow \\ N^i}} \bar{F}_i + \underbrace{A_o^j}_{\downarrow \lambda^j} \bar{G}_j \right) d^2x$$

(6 x # spatial points)

$$H[e_o^i, A_o^i] \equiv 0$$

inside constraint
hyper surface?

→ Hamiltonian = combination of constraints
totally constrained system

Constraint Algebra

$$\{H[N, \Lambda], H[N', \Lambda']\} = \text{comb. of } \mathcal{H}'\text{'s}$$
$$\mathcal{H}'[N''[N, N', \Lambda, \Lambda'], \Lambda''[N, N', \Lambda, \Lambda']]$$

Constraint Algebra

$$\{H[N, \Lambda], H[N', \Lambda']\} = \text{comb. of } H\text{'s}$$
$$H[N''[N, N', \Lambda, \Lambda'], \Lambda''[N, N', \Lambda, \Lambda']]$$

$$\tilde{E}^{ab} \equiv 0$$

Gauß-constraints

$$F_{ab} \tilde{E}^{ab} \equiv 0$$

Flatness constraints

$$A_a^i(x), E_R^b(y)$$

$$H[e_o^i, A_o^i] = - \int \left(\underbrace{e_o^i}_{\downarrow N^i} \mathcal{F}_i + \underbrace{A_o^j}_{\downarrow A^j} \mathcal{G}_j \right) d^2x$$

(6 x #spatial points)

$$H[e_o^i, A_o^i] \equiv 0$$

inside constraint
hyper surface?

→ Hamiltonian = combination of constraints
totally constrained system