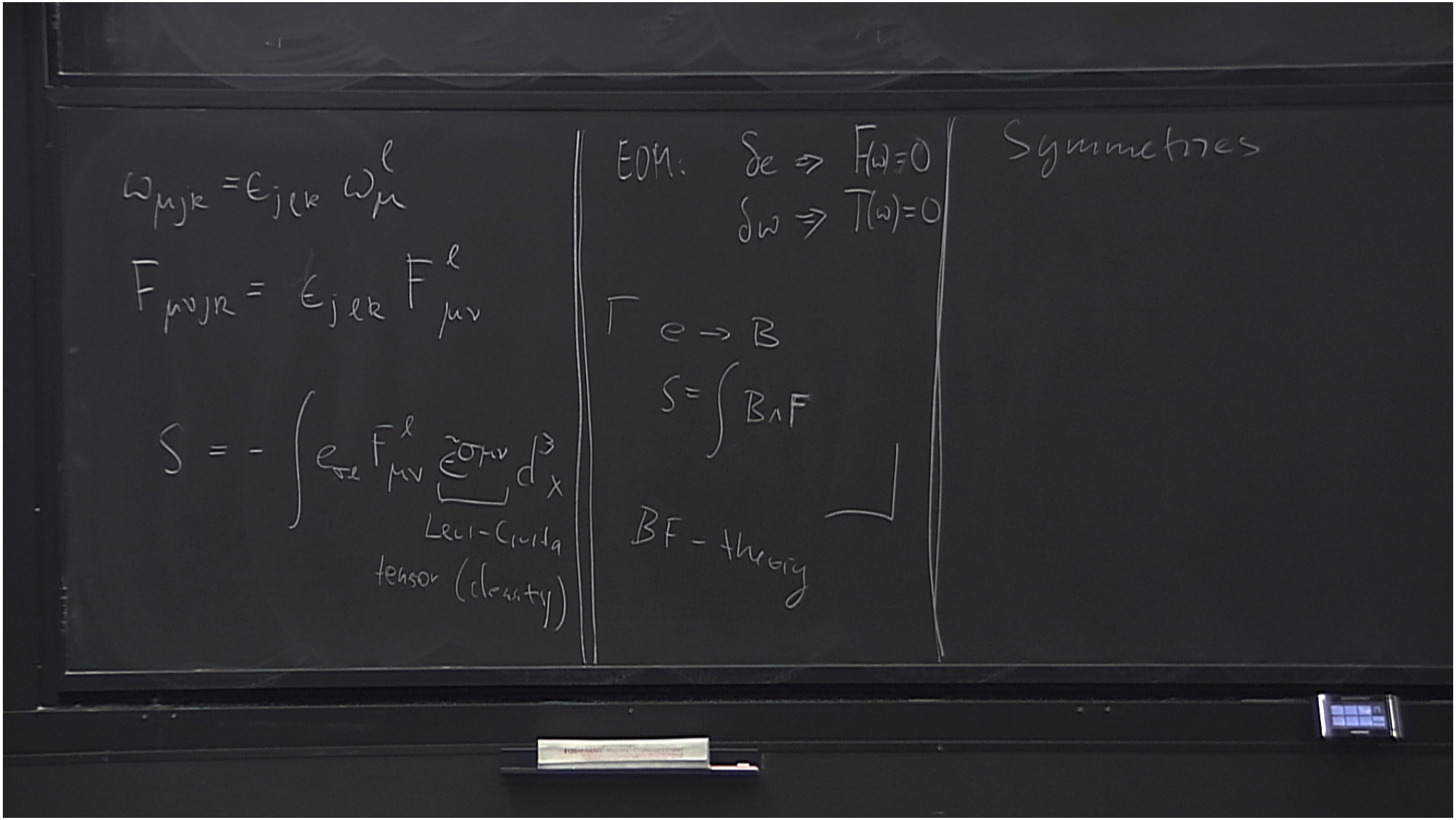
Title: 13/14 PSI - Quantum Gravity Review - Lecture 3

Date: Feb 20, 2014 10:15 AM

URL: http://pirsa.org/14020063

Abstract:


$\omega_{\mu jk} = \epsilon_{jlk}\,\omega^{l}_{\mu}$
$F_{\mu\nu jk} = \epsilon_{jlk}\,F^{l}_{\mu\nu}$
$S = -\int e_{ik}\,F^{l}_{\mu\nu}\,\tilde{\epsilon}^{0\mu\nu}\,d^{3}x$
Levi-Civita tensor (density)
EOM: $\delta e \Rightarrow F(\omega)=0$
$\delta\omega \Rightarrow T(\omega)=0$
Γ
$e \to B$
$S = \int B \wedge F$
BF-theory
Symmetries

$\Gamma(\omega)=0$

$\Gamma(\omega)=0$

Symmetries of the action

- global symmetries

$$S=\int \varphi \,\Box\, \varphi \; d^D x$$

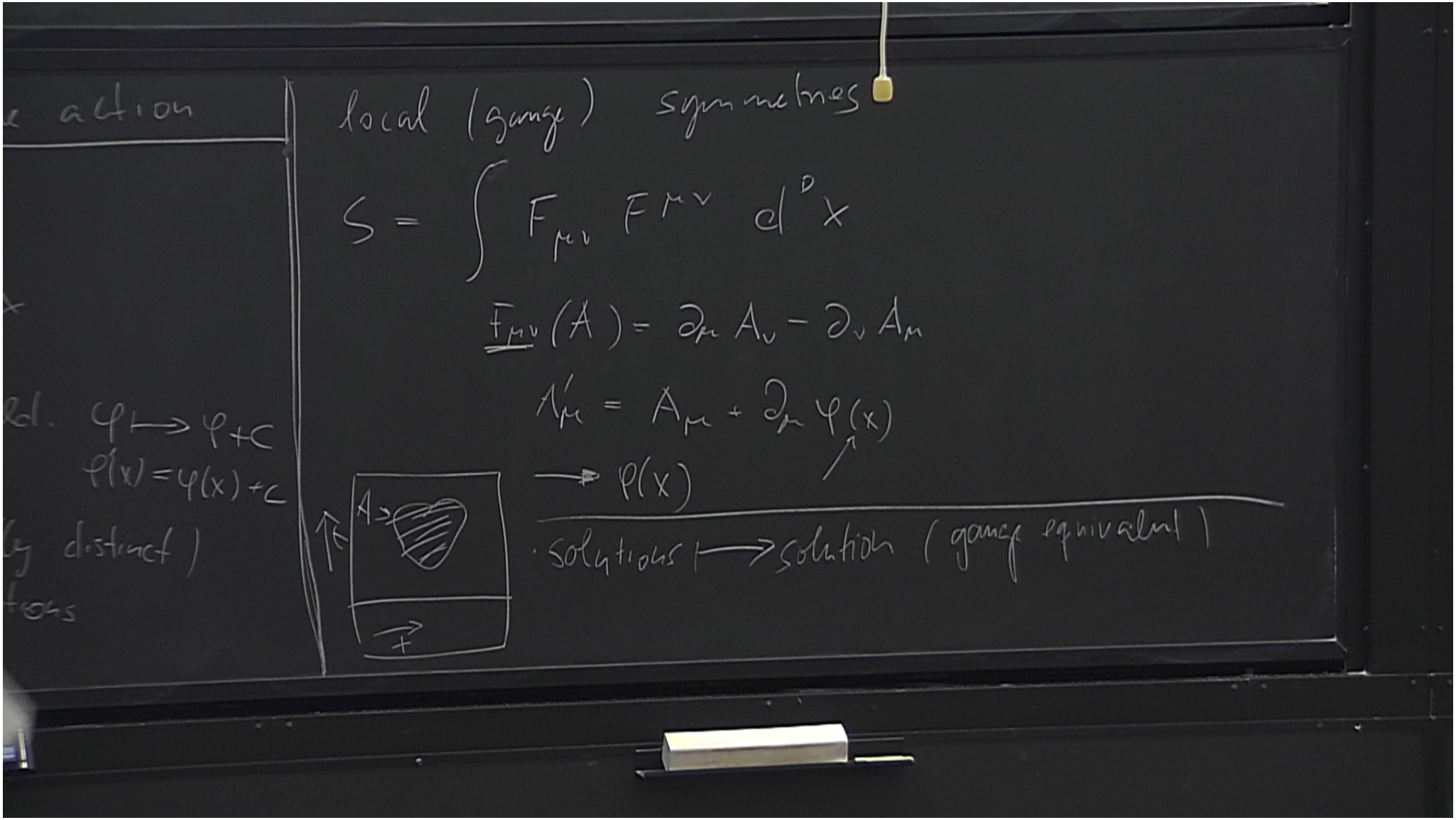
translation of the field. $\varphi \longmapsto \varphi + c$

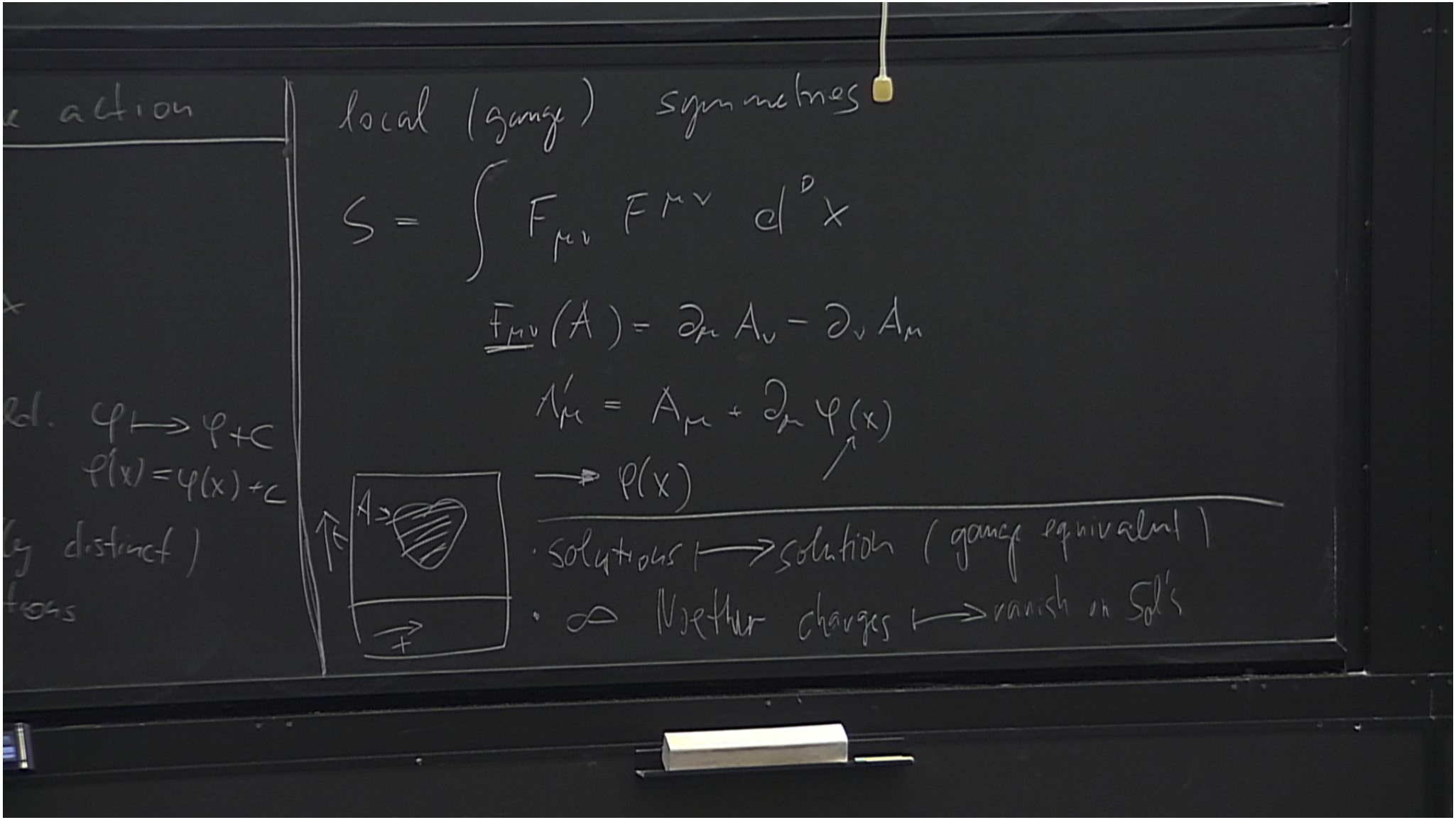
$\varphi'(x)=\varphi(x)+c$

- solutions $\longrightarrow$ (physically distinct) solutions
- Noether charges

local (gauge) symmetries

$$S=\int F_{\mu\nu} F^{\mu\nu} \; d^D x$$


action
local (gauge) symmetries
$S = \int F_{\mu\nu} F^{\mu\nu} \, d^D x$
$F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu$
$A'_\mu = A_\mu + \partial_\mu \varphi(x)$
$\varphi(x)$
$\varphi \longmapsto \varphi + c$
$\varphi(x) = \varphi(x) + c$
distinct)
A
t
solutions $\longmapsto$ solution (gauge equivalent)


action
local (gauge) symmetries
$S = \int F_{\mu\nu} F^{\mu\nu} \, d^D x$
$F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu$
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$\varphi(x)$
$\varphi \longmapsto \varphi + c$
$\varphi(x) = \varphi(x) + c$
distinct)
A
t
$\rightarrow \rho(x)$
· solutions $\longmapsto$ solution (gauge equivalent)
· ∞ Noether charges $\longmapsto$ vanish on Sol's

Gauge symmetries of Palatini action (3D)

1. Frame rotations $R^i{}_j(x)$

$e'^{j'}_{\mu}(x) = R^{j'}{}_j(x)\, e^j_\mu(x)$

$R_{ij} = \delta_{ij} + \varepsilon\, \epsilon_{ijk} \Lambda^k$

$$\delta^R_\Lambda e^j_\mu = \frac{d}{d\varepsilon} e''^j_\mu = \epsilon^j{}_{ki} e^k_\mu \Lambda^i$$

Gauge symmetries of Palatini action (3D)

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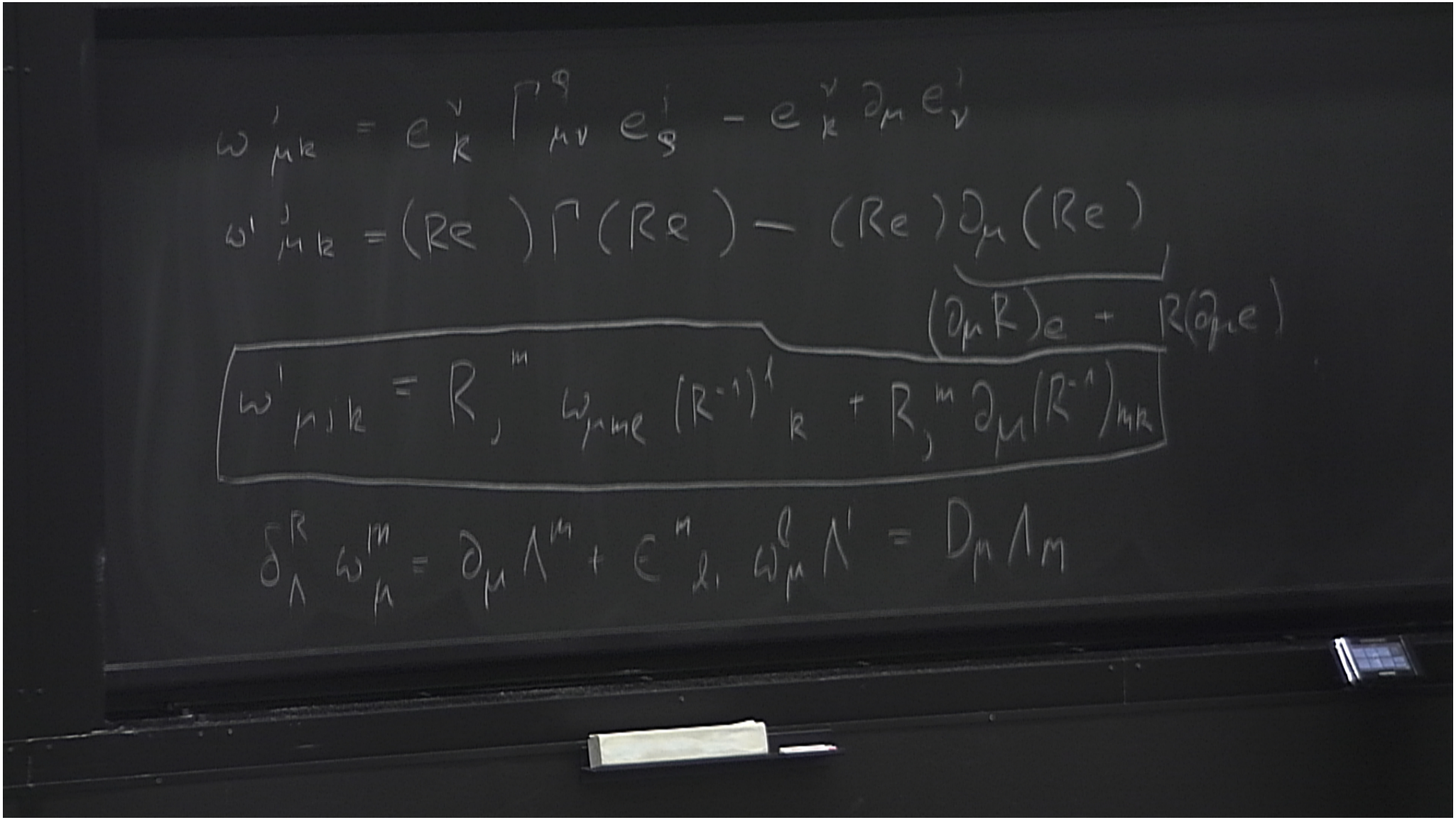
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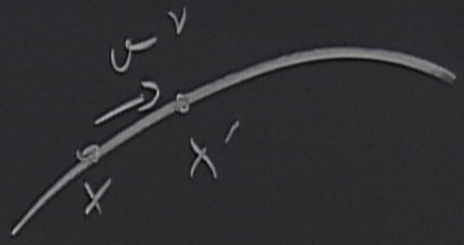
$$\omega'^{j}_{\mu k} = e^{\nu}_{k}\,\Gamma^{\rho}_{\mu\nu}\,e^{j}_{\rho} - e^{\nu}_{k}\,\partial_\mu e^{j}_{\nu}$$

$$\omega'^{j}_{\mu k} = (Re)\,\Gamma\,(Re) - (Re)\,\underbrace{\partial_\mu (Re)}_{(\partial_\mu R)e + R(\partial_\mu e)}$$

$$\boxed{\omega'^{j}_{\mu k} = R^{m}_{j}\,\omega_{\mu m e}\,(R^{-1})^{e}_{k} + R^{m}_{j}\,\partial_\mu (R^{-1})_{mk}}$$


$\omega'_{\mu k} = e^{\nu}_{k} \Gamma^{\rho}_{\mu\nu} e^{i}_{\rho} - e^{\nu}_{k} \partial_\mu e^{i}_{\nu}$
$\omega'^{j}_{\mu k} = (Re)\Gamma(Re) - (Re)\partial_\mu(Re)$
$(\partial_\mu R)e + R(\partial_\mu e)$
$\omega'_{\mu jk} = R^{m}_{j}\, \omega_{\mu me} (R^{-1})^{l}_{k} + R^{m}_{j}\, \partial_\mu (R^{-1})_{mk}$
$\delta^{R}_{\Lambda}\, \omega^{m}_{\mu} = \partial_\mu \Lambda^{m} + \epsilon^{m}_{\ell i}\, \omega^{\ell}_{\mu} \Lambda^{i} = D_\mu \Lambda_m$

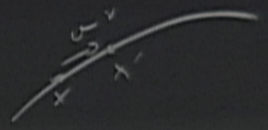
orphisms



$x' = \varphi(x)$; $v = v^\nu$ = vector field

$= \mathcal{L}_v e^j_\mu = v^\nu \partial_\nu e^j_\mu + e^j_\nu \partial_\mu v^\nu$

2 Diffeomorphisms



$x \longmapsto x' = \varphi(x)$; v^ν = vector field

$$\delta^D_v e^j_\mu = \mathcal{L}_v e^j_\mu = v^\nu \partial_\nu e^j_\mu + e^j_\nu \partial_\mu v^\nu$$

$$\delta^D_v \omega^j_\mu = \mathcal{L}_v \omega^j_\mu = v^\nu \partial_\nu \omega^j_\mu + \omega^j_\nu \partial_\mu v^\nu$$

tensor (density)

energy

• Noether charges

3. Translational symmetry

$$\delta_N^T e^j_\mu = D_\mu N^j = \partial_\mu N^j + \epsilon^j{}_{mk}\, \omega^m_\mu N^k$$

$$\delta_N^T \omega^j_\mu = 0$$

tensor (density)

energy

• Noether charges

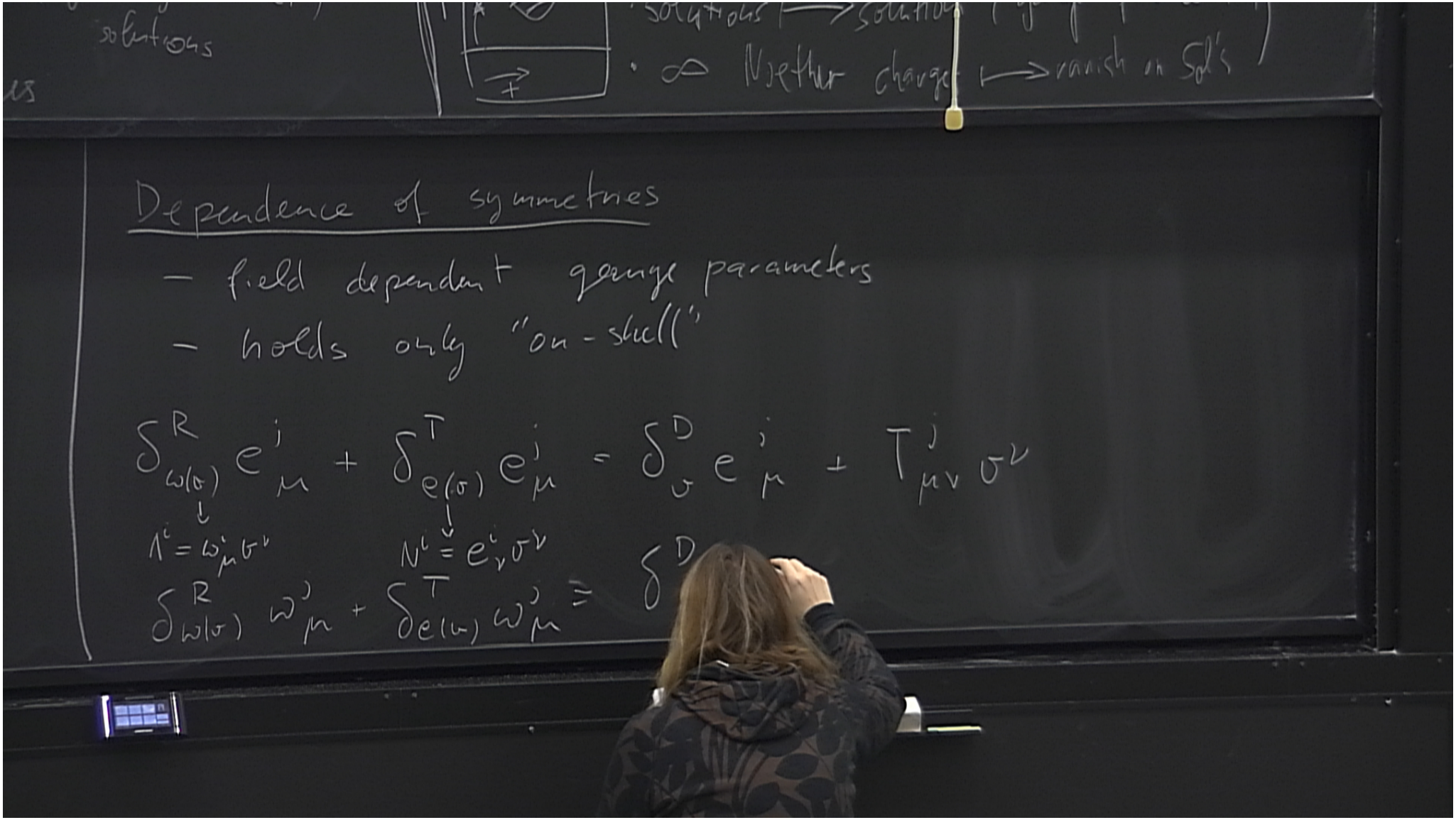
solutions

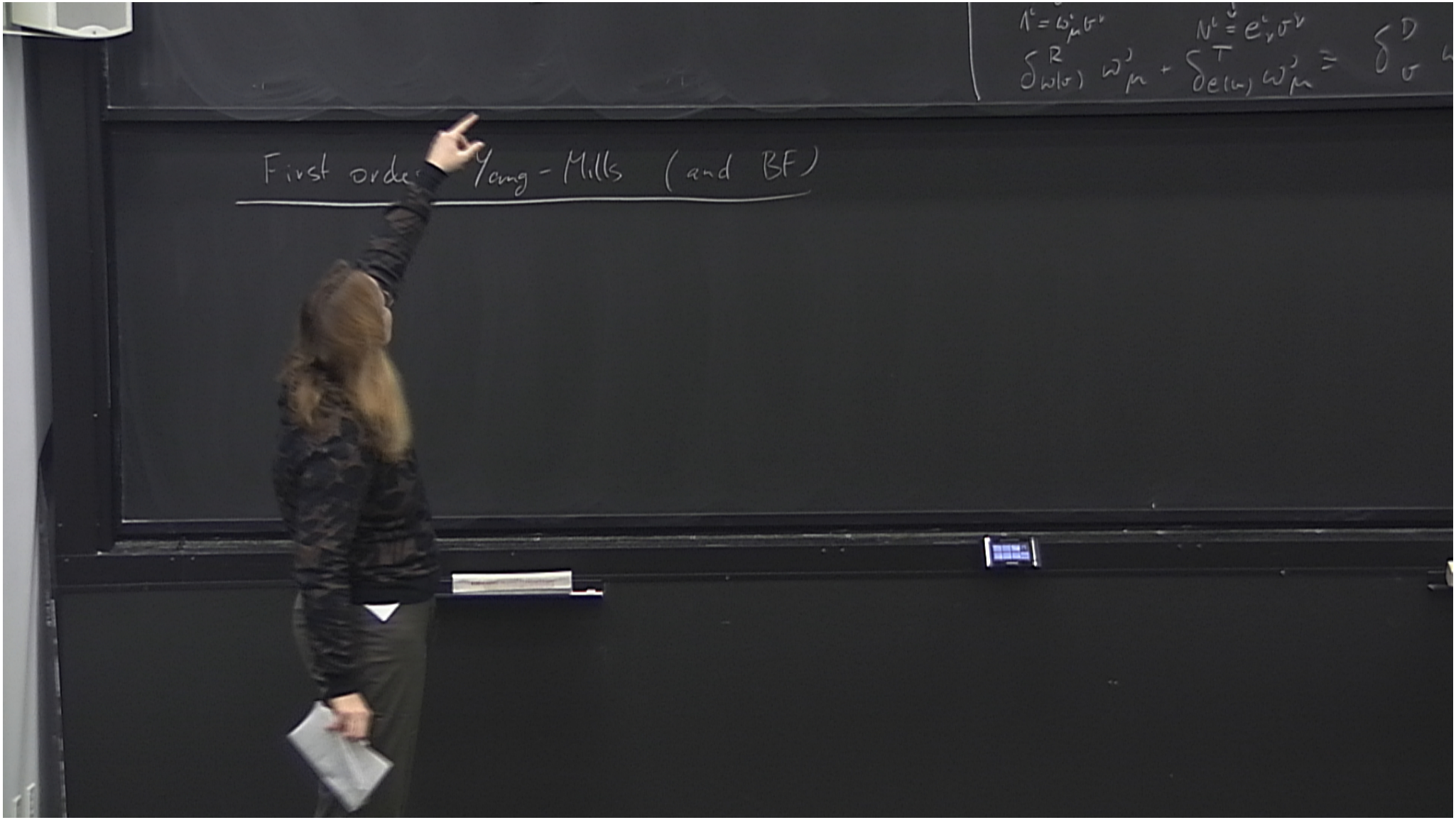
3. Translational symmetry

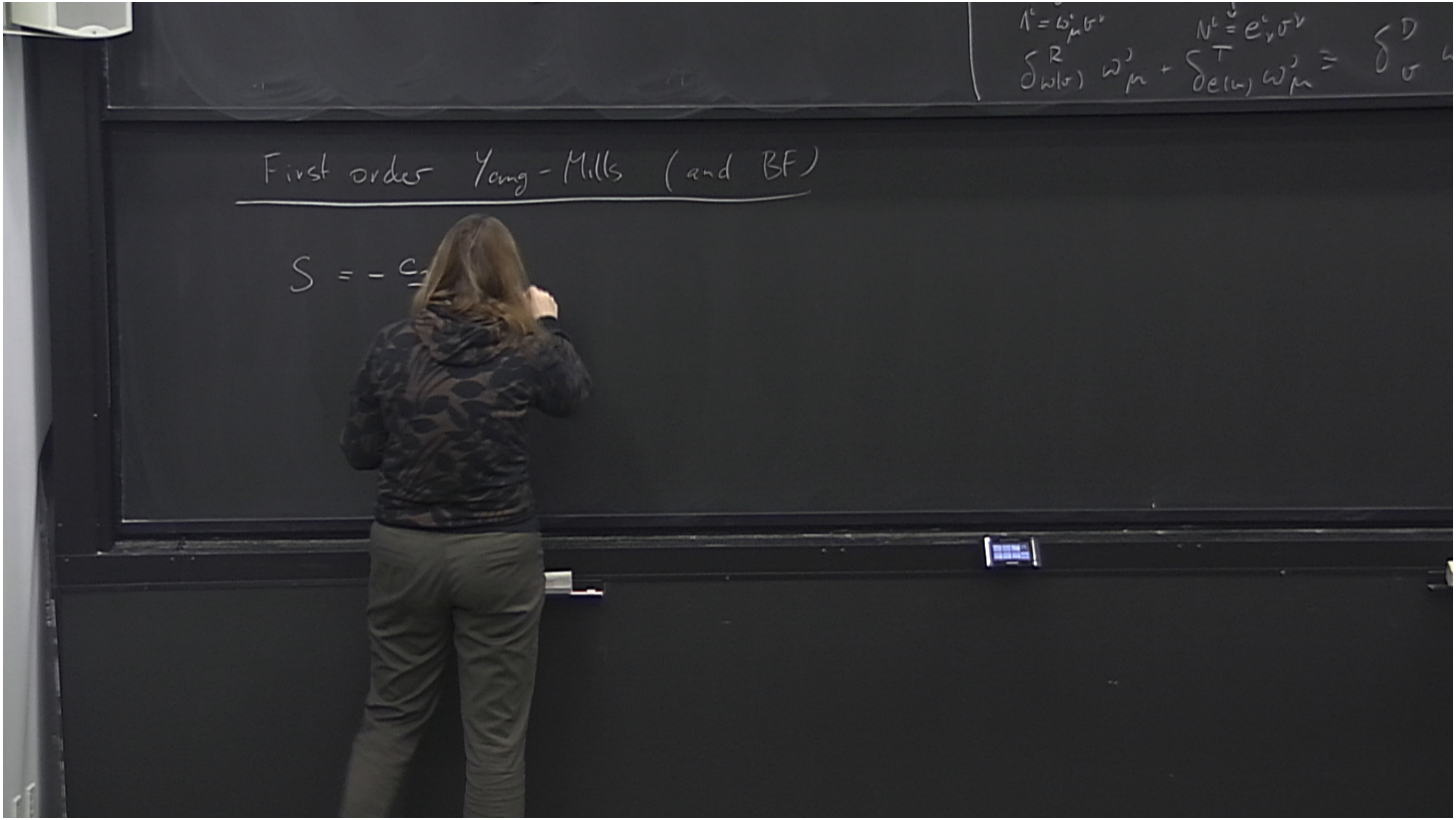
$$\delta^T_N e^j_\mu = D_\mu N^j = \partial_\mu N^j + \epsilon^j{}_{mk}\, \omega^m_\mu N^k$$

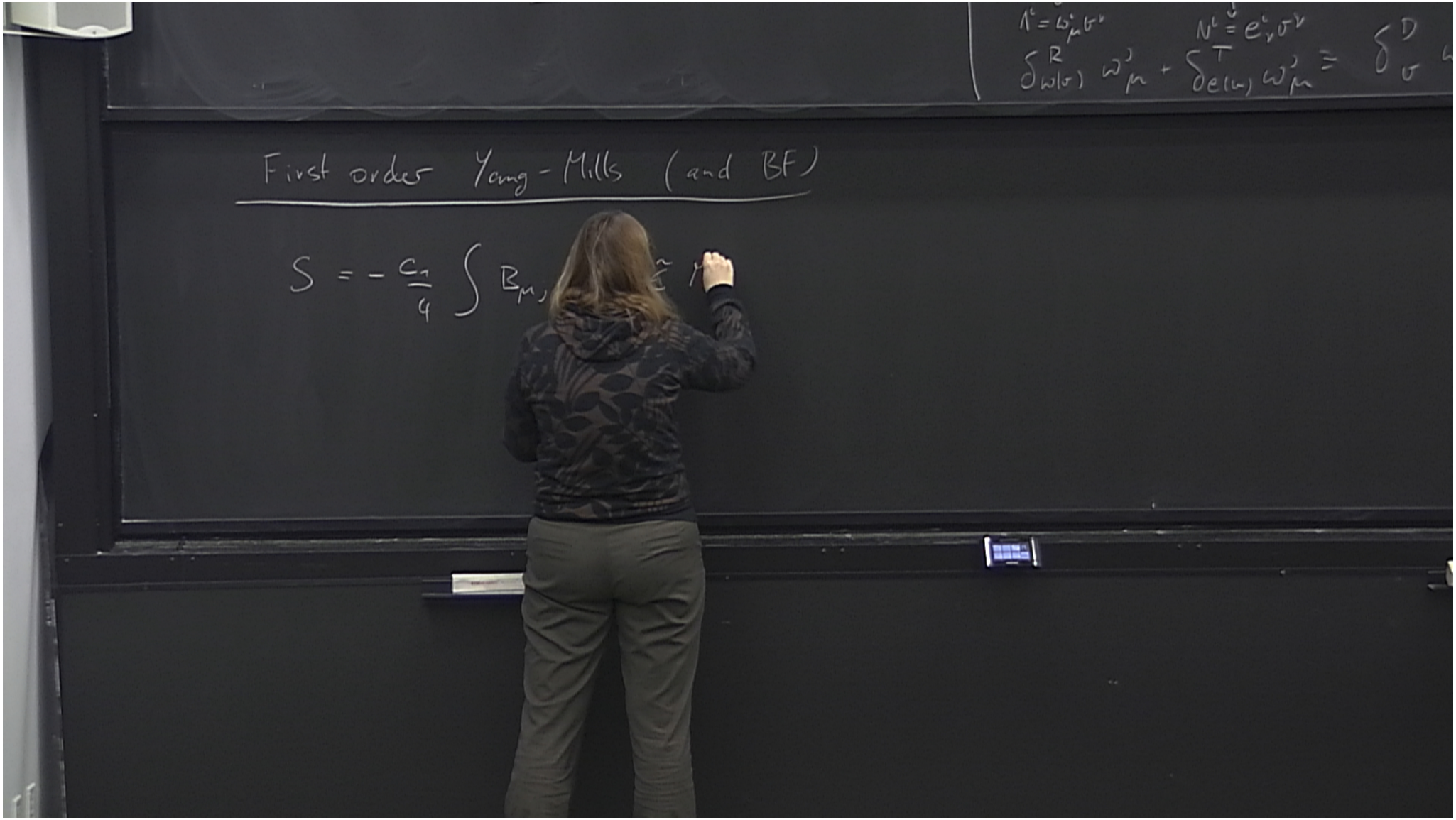
$$\delta^T_N \omega^j_\mu = 0 \qquad \text{(due to the Bianchi identity)}$$

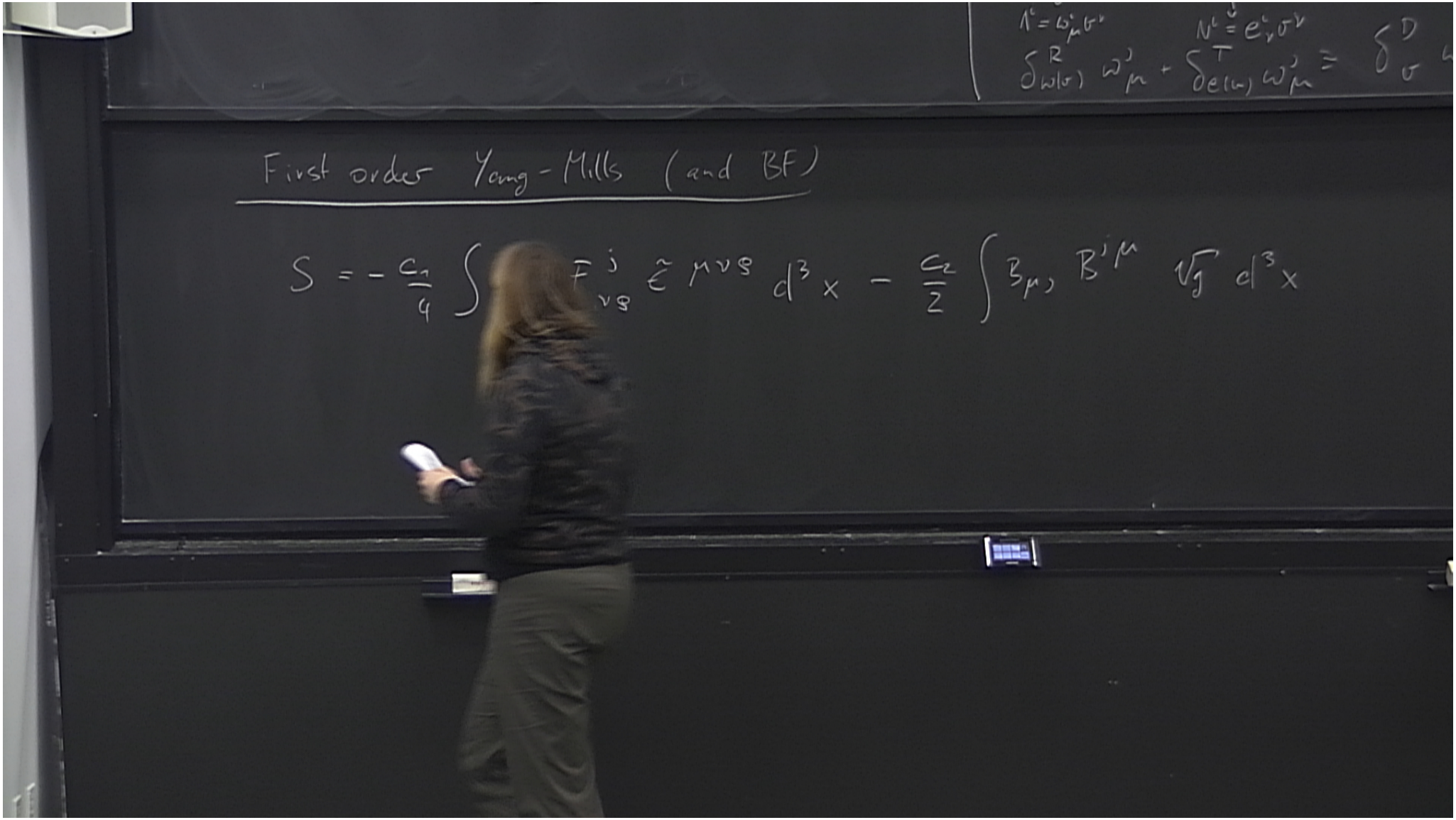
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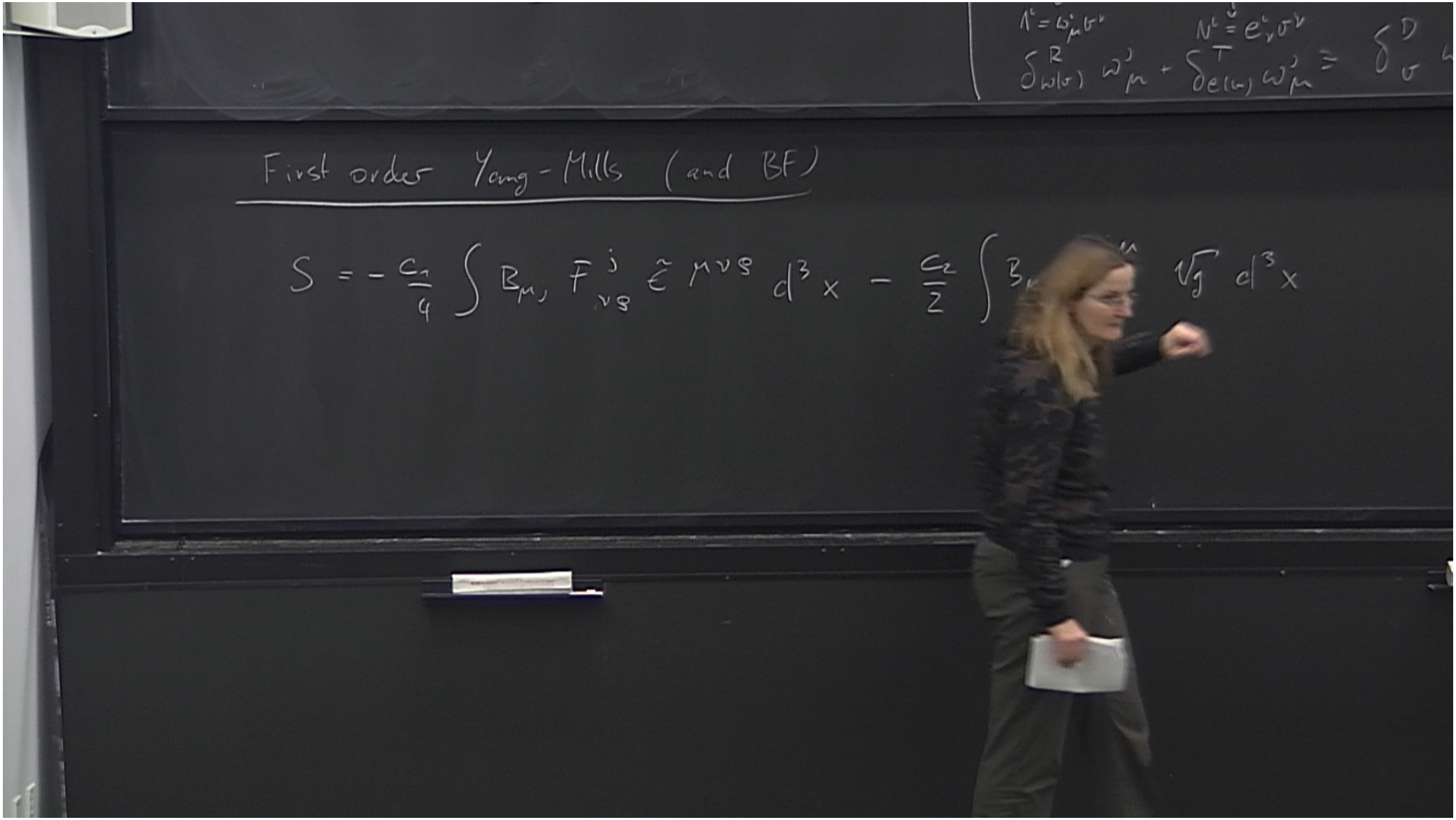

solutions
solutions ⟶ solutions
∞ Noether charges ⟶ vanish on Sol's
Dependence of symmetries
– field dependent gauge parameters
– holds only "on-shell"
$\delta^R_{\omega(\sigma)} e^i_\mu + \delta^T_{e(\sigma)} e^i_\mu = \delta^D_\sigma e^i_\mu + T^\nu_{\mu\nu}\sigma^\nu$
$\Lambda^i = \omega^i_{\mu\nu}\sigma^\nu$
$N^i = e^i_\nu \sigma^\nu$
$\delta^R_{\omega(\sigma)} \omega^i_\mu + \delta^T_{e(\sigma)} \omega^i_\mu = \delta^D$

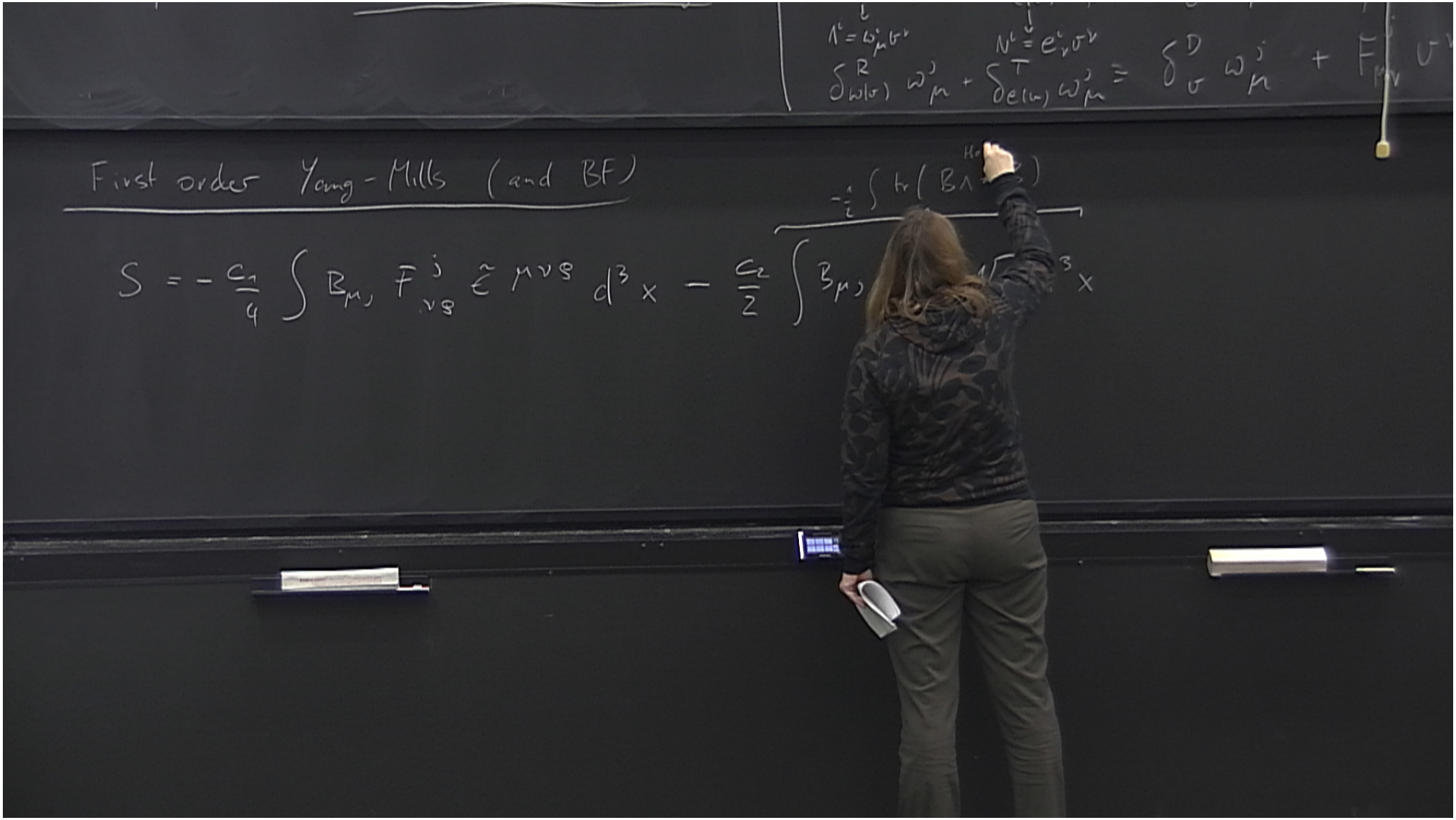

First order Yang-Mills (and BF)

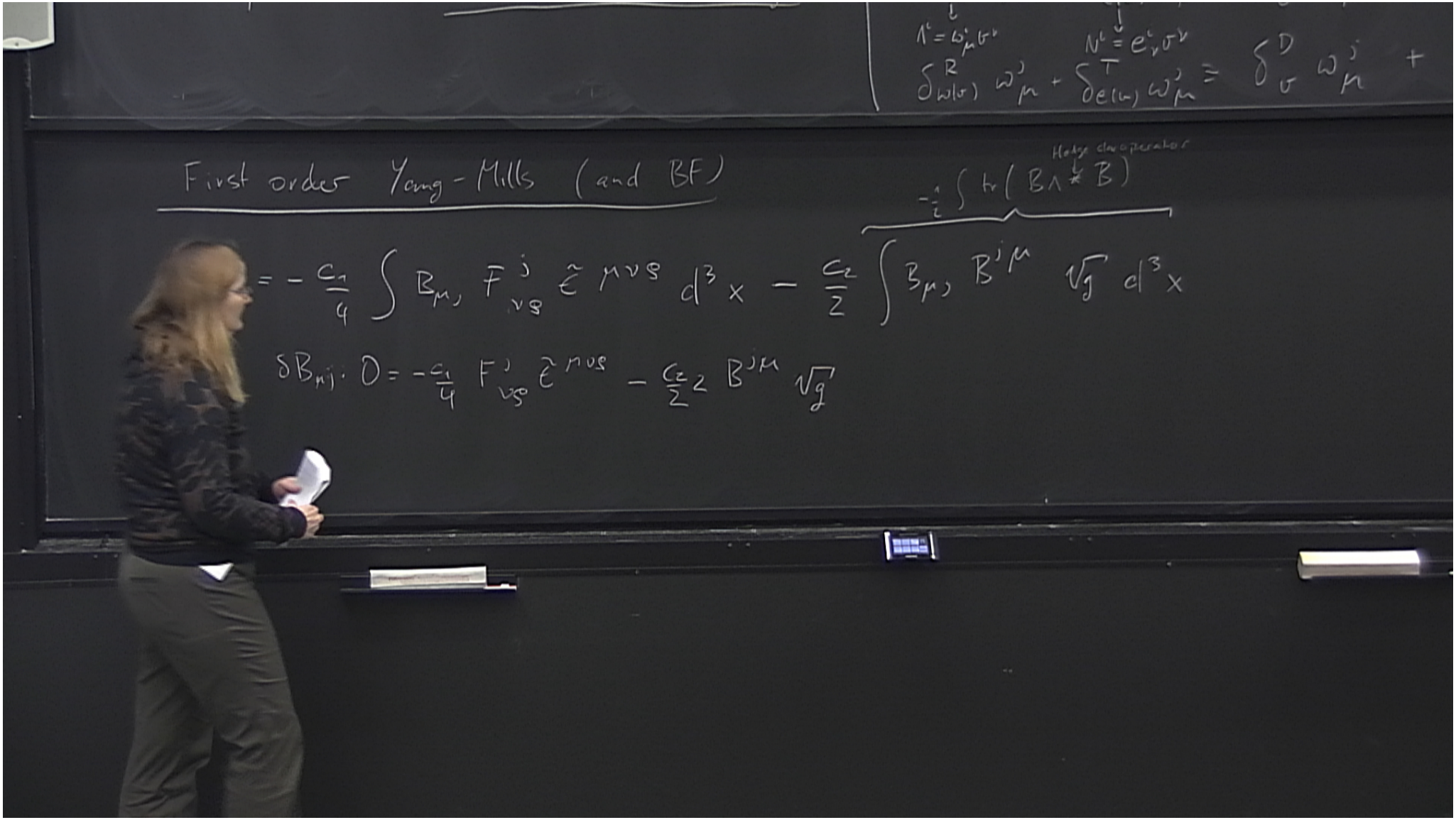

First order Yang-Mills (and BF)
S = -

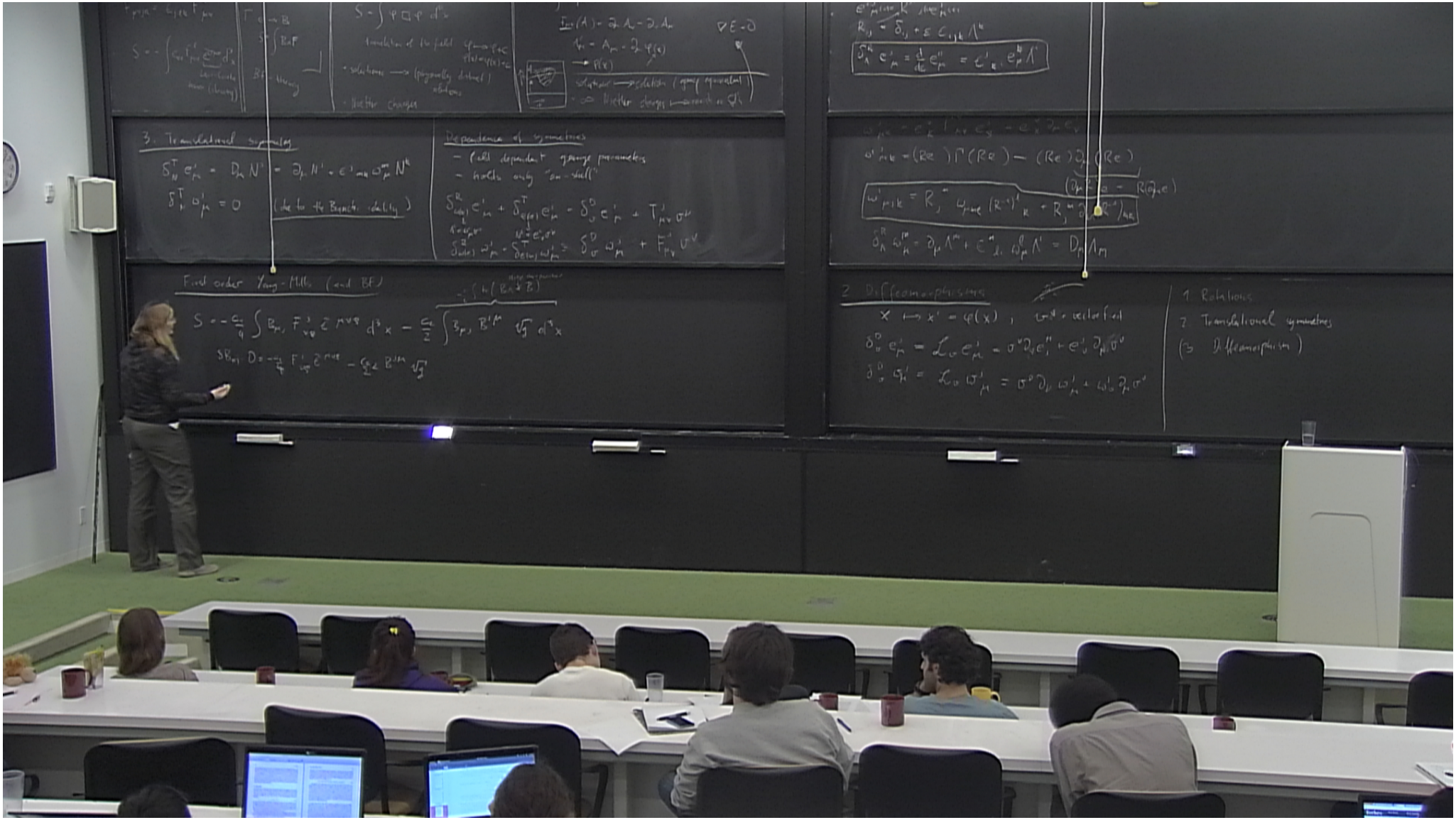

First order Yang-Mills (and BF)

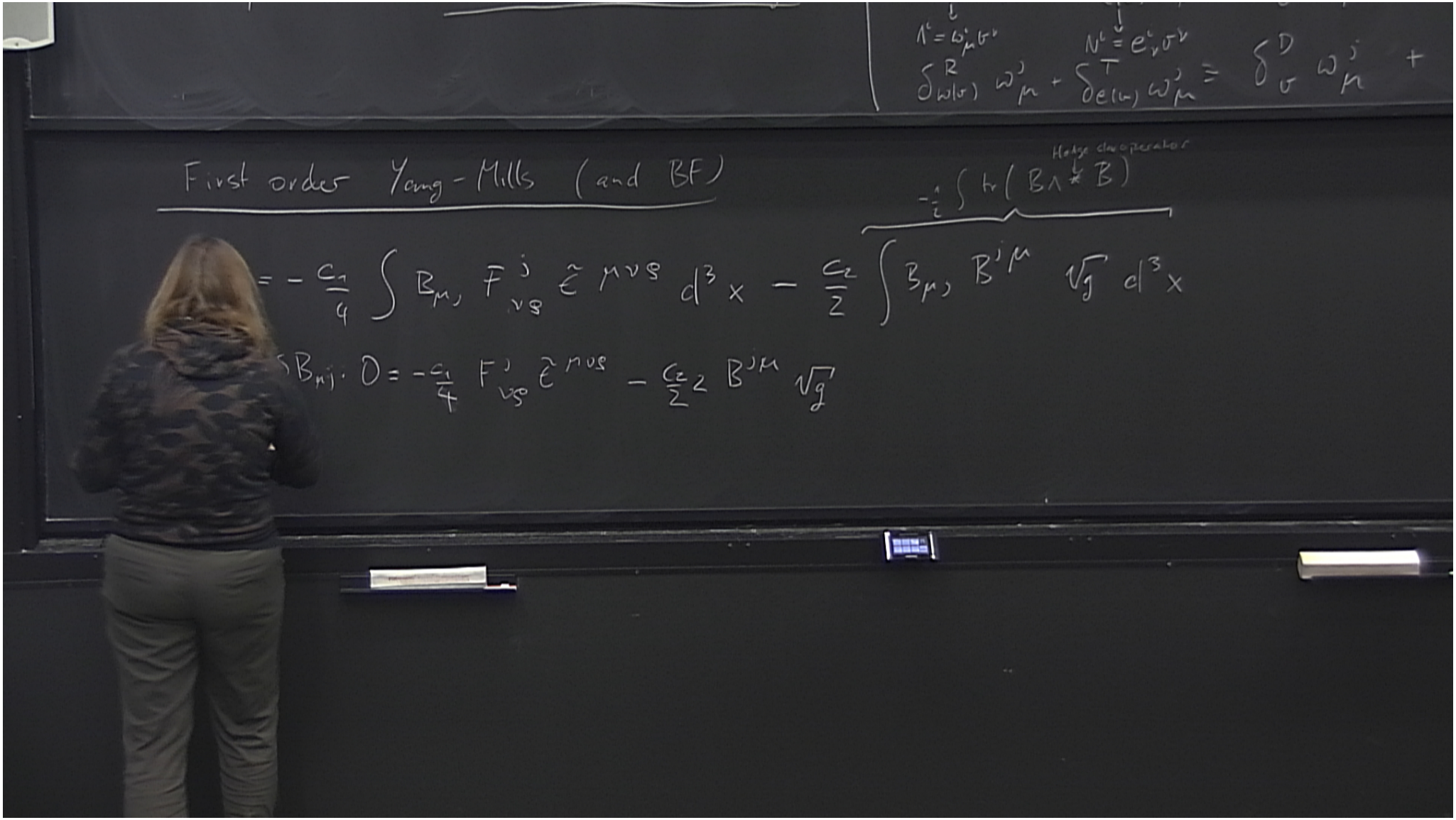

First order Yang-Mills (and BF)

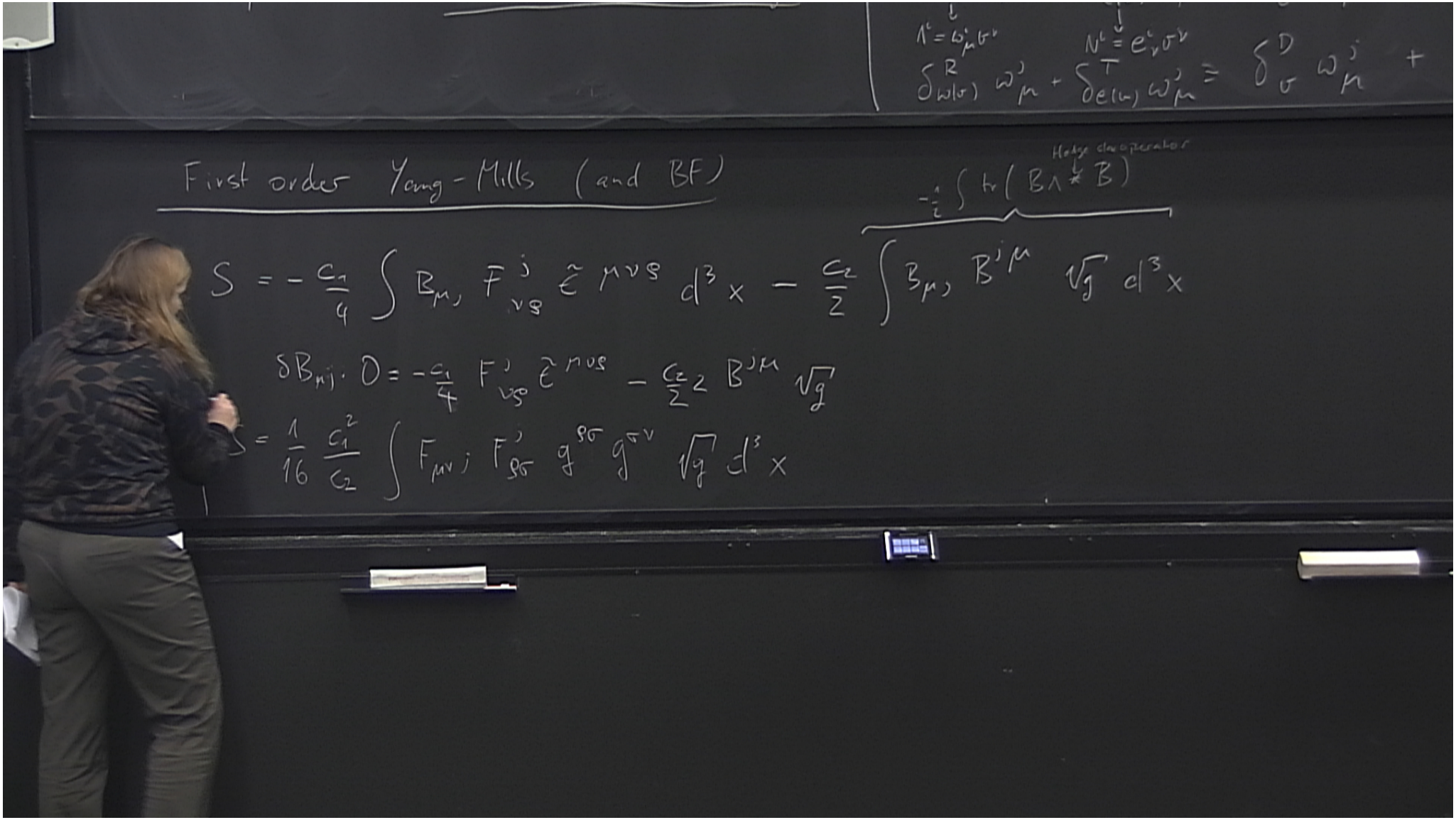

First order Yang-Mills (and BF)

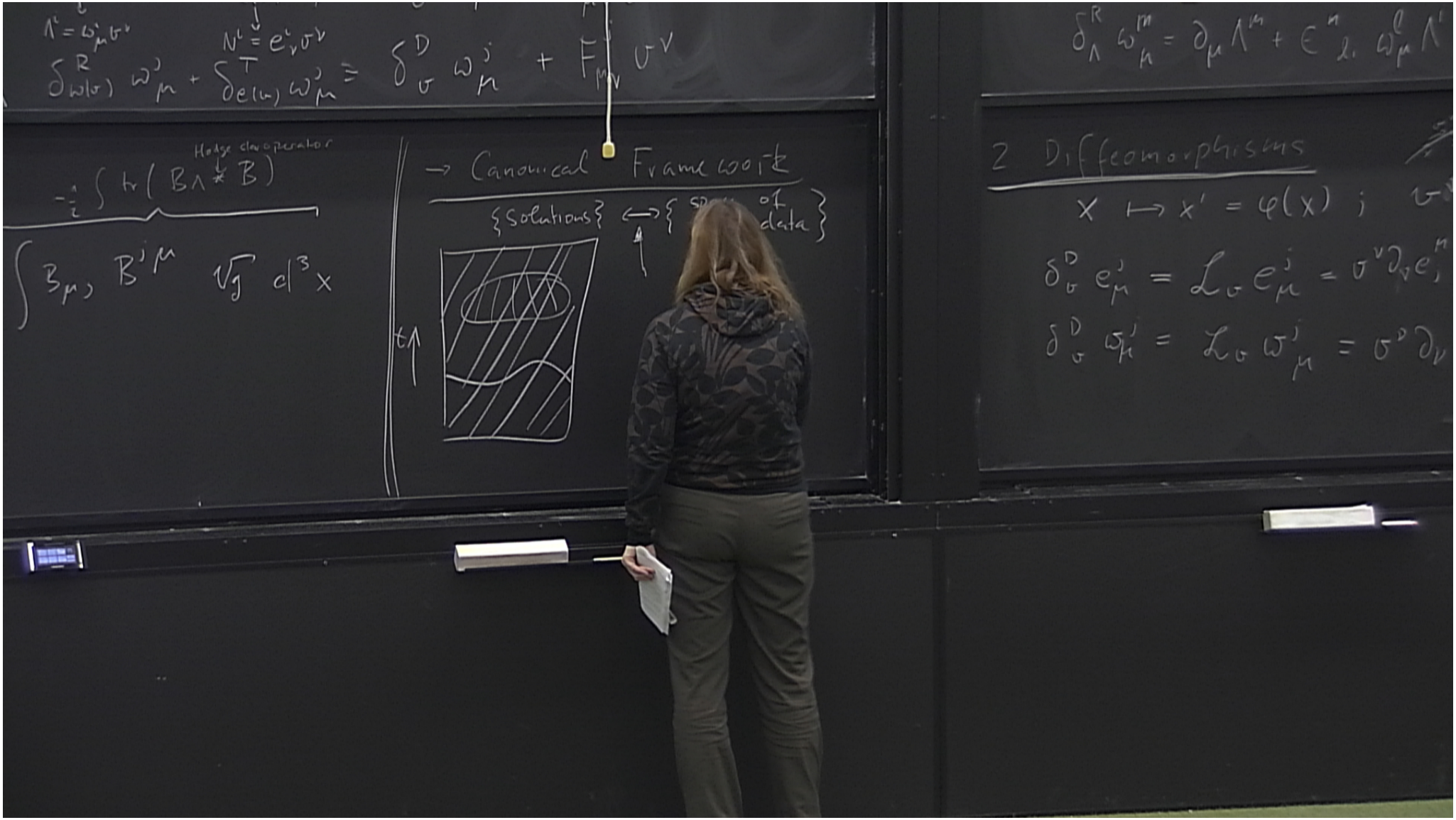

First order Yang-Mills (and BF)


First order Yang-Mills (and BF)
Hodge star operator


3. Translational symmetry
Independence of symmetries
First order Yang-Mills (and BF)
2. Diffeomorphisms
1. Rotations
2. Translational symmetries
(3. Diffeomorphisms)


First order Yang-Mills (and BF)
Hodge star operator


$\Lambda^i = \omega^i_\mu e^\mu$
$N^i = e^i_\nu e^\nu$
$\delta^R_{\omega(\omega)} \omega^j_\mu + \delta^T_{e(\omega)} \omega^j_\mu = \delta^D_\omega \omega^j_\mu +$
First order Yang-Mills (and BF)
Hodge star operator
$-\frac{1}{2}\int \mathrm{tr}(B \wedge {}^{\star} B)$
$S = -\frac{c_1}{4}\int B_{\mu j} F^j_{\nu\rho} \tilde{\epsilon}^{\mu\nu\rho} d^3x - \frac{c_2}{2}\int B_{\mu j} B^{j\mu} \sqrt{g}\, d^3x$
$\delta B_{\mu j} \cdot 0 = -\frac{c_1}{4} F^j_{\nu\rho} \tilde{\epsilon}^{\mu\nu\rho} - \frac{c_2}{2} B^{j\mu} \sqrt{g}$
$S = \frac{1}{16}\frac{c_1^2}{c_2}\int F_{\mu\nu j} F^j_{\rho\sigma} g^{\rho\sigma} g^{\nu\nu} \sqrt{g}\, d^3x$


Hodge star operator
→ Canonical Framework
{solutions}
t
2 Diffeomorphisms