

Title: 13/14 PSI - Quantum Gravity Review - Lecture 1

Date: Feb 18, 2014 10:15 AM

URL: <http://pirsa.org/14020061>

Abstract:

Quantum gravity and Quantum Geometry

Basic: Gambini, Pullin. "A first course in loop quantum gravity" Oxford 2011

More Advanced: A. Perez. "Intro. to loop quantum gravity and spin foams"
gr-qc/0409061

Even more Advanced: T. Thiemann, book, Cambridge

ometry

Quantum Gravity I

Oxford 2011

gravity

and

Quantum
Theory

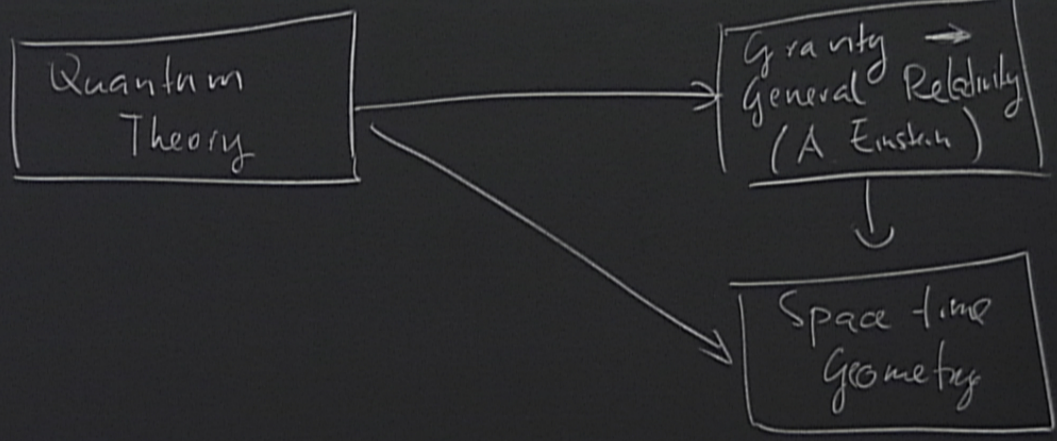
ometry

Oxford 2011

gravity

bridge

Quantum Gravity I

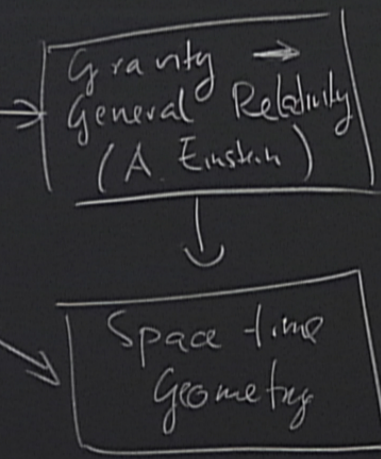


Challenge:

Standard:



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Challenge:

Standard: QFT on a fixed space-time

Overview:

I. Classical Part

- action, variation, symmetries
- first order action
(tetrads/triads, spin connection)
- canonical part $\rightarrow$
gauge symmetry $\Rightarrow$ (first class) constraints

II Quantum Part

- Quantization of gauge systems
- Quantum geometry

- Quantization:

- Edges
- Triangles
- Tetrahedra $\rightarrow$ Dynamics

} Kinematics

II Quantum Part

- Quantization of gauge systems
- Quantum geometry

- Quantization:

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} Kinematics

- wavefunction of the (3D) Universe

- Path integral

4th Part

Quantization of gauge systems
in geometry

- Quantization:
- Edges
 - Triangles
 - Tetrahedra $\rightarrow$ Dynamics
- } Kinematics

Function of the (3D) Universe

- Path integral quant.

- 4D

Actions

$$S_{EH} = \frac{1}{\kappa} \int R \sqrt{g} d^D x$$

$D=4$

$$\kappa = 1, \quad \kappa_{4D} = \frac{16\pi G}{c^3}$$

Actions

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Actions

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$$\kappa = 1, \quad \kappa_{4D} = \frac{16\pi G}{c^3}$$

$$g_{\mu\nu}(x), \quad g^{\mu\nu}(x)$$

$D=4$

$$\delta S_{EH} = \int$$

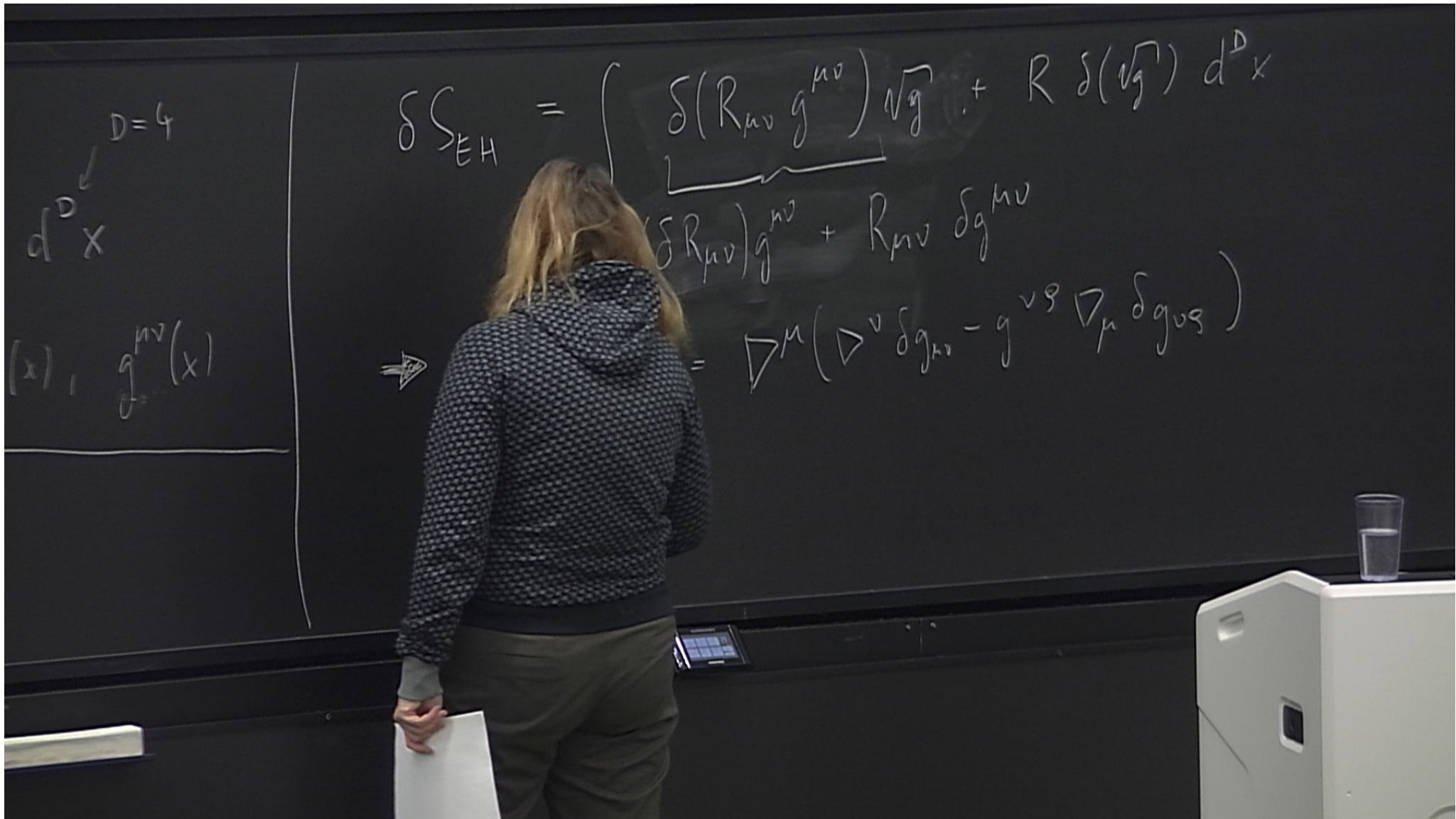
$D=4$

$$\delta S_{EH} = \int (\delta R) \cdot \sqrt{g} \, d^D x + \int R \, \delta(\sqrt{g}) \, d^D x$$

$\gamma(x)$

$D=4$
 $d^D x$
 $(x), g^{\mu\nu}(x)$

$$\delta S_{EH} = \int \underbrace{\delta(R_{\mu\nu} g^{\mu\nu}) \sqrt{g}}_{(\delta R_{\mu\nu}) g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu}} + R \delta(\sqrt{g}) d^D x$$



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$$\Rightarrow (\delta R_{\mu\nu}) g^{\mu\nu} = \nabla^\mu (\nabla^\nu \delta g_{\mu\nu} - g^{\nu\rho} \nabla_\mu \delta g_{\rho\sigma})$$

Palatini equation

$D=4$
 $d^D x$

$g^{\mu\nu}(x)$

$$\delta S_{EH} = \int \underbrace{\delta(R_{\mu\nu} g^{\mu\nu})}_{(\delta R_{\mu\nu}) g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu}} \sqrt{g} + R \delta(\sqrt{g}) d^D x$$

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Palatini equation

$$\delta(\sqrt{g}) = \frac{1}{2} \frac{1}{\sqrt{g}} \delta g = -\frac{1}{2} \sqrt{g} g_{\mu\nu} \delta g^{\mu\nu}$$

$$g = \det(g_{\mu\nu})$$

$$\delta g =$$

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$$\boxed{\delta g = g g^{\mu\nu} \delta g_{\mu\nu}} \quad 0 = \delta(g_{\mu\nu} g^{\nu\alpha}) = (\delta g_{\mu\nu}) g^{\nu\alpha} + g_{\mu\nu} \delta g^{\nu\alpha}$$

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$$\delta S_E$$

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$\delta g^{\mu\nu}$

$$\delta S_{EH} = \int \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) \delta g^{\mu\nu} \sqrt{g} d^D x$$

$$g_{\mu\nu} = 0 \iff R_{\mu\nu} = 0$$

3D gravity

3D gravity

$$R_{\mu\nu} = 0 \quad \Rightarrow \quad R_{\mu\nu\alpha\beta} = 0$$

3D gravity w/out matter/cosm. constant
is locally flat!

3D gravity

$$R_{\mu\nu} = 0 \quad \boxed{D=3} \quad \Rightarrow \quad R_{\mu\nu\alpha\beta} = 0$$

3D gravity w/out matter/cosm. constant
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Riemann ten

Riemann tensor

1. antisymmetric in first two and last two ind.
2. symmetric

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2. symmetric. $R_{\mu\nu\alpha\beta} = R_{\alpha\beta\mu\nu}$

3. $R_{[\mu\nu\alpha\beta]} = 0$

$\mu\nu, \alpha\beta$

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$$R_{\underbrace{\mu\nu}} \underbrace{\alpha\beta}$$

$\mu \rightarrow \nu \quad \alpha \rightarrow \beta$

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$\epsilon^{\mu\nu\sigma} \rightarrow \sigma$ $\rightarrow \sigma$

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3D gravity

$R_{\mu\nu\alpha\beta}$ $\Rightarrow$ $R_{\mu\nu\alpha\beta} = 0$
gravity w/out matter / cosm. constant
locally flat! $\Rightarrow$ "topological field theory"

Riemann tensor

1. antisymmetric
2. symmetric
3. $R_{\mu\nu\alpha\beta}$

3D gravity

$$R_{\mu\nu} \quad \boxed{D=3} \Rightarrow R_{\mu\nu\alpha\beta} = 0$$

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2D

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$$R_{\underbrace{\mu\nu}} \underbrace{\alpha\beta} \\ \epsilon^{\mu\nu\sigma} \rightarrow \sigma \quad \rightarrow \sigma$$

3. $R[\mu\nu\alpha\beta] = 0$

2D: tors



Riemann tensor

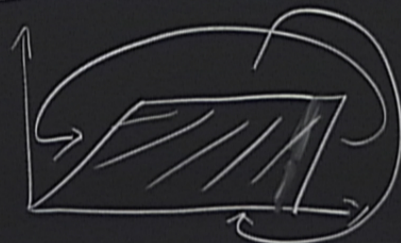
1. antisymmetric in first two and last two ind. $\rightarrow$

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2D: tors



Riemann tensor

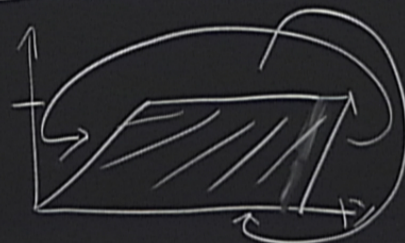
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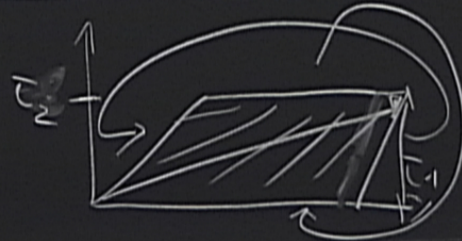
2D: tors



Riemann tensor

1. antisymmetric in first two and last two ind. $\rightarrow R_{\mu\nu} a_\beta$
 $\epsilon^{\mu\nu} \rightarrow \sigma \quad \rightarrow \sigma$
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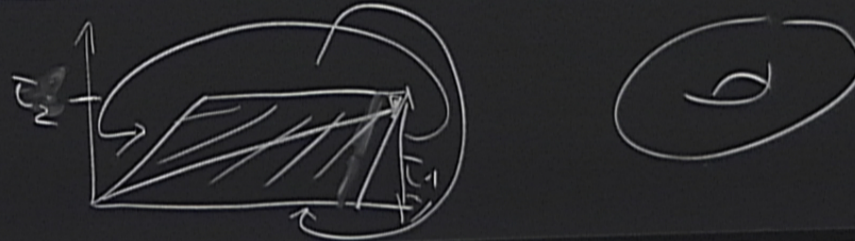
2D: tors



Riemann tensor

1. antisymmetric in first two and last two ind. $\rightarrow R_{\mu\nu} a_\beta$
 $\epsilon^{\mu\nu\sigma} \rightarrow \sigma \quad \epsilon^{\alpha\beta\gamma} \rightarrow \gamma$
2. symmetric. $R_{\mu\nu} a_\beta = R_{\alpha\beta\mu\nu}$
3. $R[\mu\nu\alpha\beta] = 0$

2D: tors



Palatini action (First order action)

Palatini equation

Tetrads / Triads / Vielbeins



Tetrads / Triads / Vielbeins

$$g_{\mu\nu} \longrightarrow (e_i^{\mu}, \omega)$$

$$x \rightarrow p \quad (x|p)$$

Tetrads / Triads / Vielbeins

$$x \rightarrow p \quad (x|p)$$

$$g_{\mu\nu} \longrightarrow (e_i^\mu, \omega_{\mu}^i)$$

Vielbein $\rightarrow$ ON frame :


$$e_i^\mu$$

$i = 1, 2, 3$

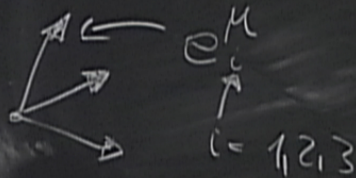
$$(g_{\mu\nu} e_i^\mu) e_j^\nu =$$

Tetrads / Triads / Vielbeins

$x \rightarrow p \quad (x|p)$

$$g_{\mu\nu} \longrightarrow (e^{\mu}_{\ i}, \omega^i_{\ \mu\nu})$$

Vielbein $\rightarrow$ ON frame



Triads $\cdot (g_{\mu\nu} e^{\mu}_{\ i}) e^{\nu}_{\ j} = e_{\ i} e_j = \delta_{ij}$

$$e^i_{\ \nu} = e^{\mu}_{\ j} g_{\mu\nu} \delta^{ij} \quad \text{co-triads}$$

$$g_{\mu\nu} = e^i_{\ \mu} e^j_{\ \nu} \delta_{ij}$$